

# Measurement of Faraday Rotation and Calculation of the Verdet Constant for a SF-59 Glass Rod

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(Dated: October 23, 2014)

## Abstract

We observed the Faraday effect, a shift in the polarization of light as it passes through medium subject to a magnetic field. Polarized light from a 650 nm laser was sent through a SF-59 glass rod surrounded by a solenoid. The light then passed through an rotatable polarizer and was detected by a photodiode. We calculated the Verdet constant for the rod by varying the magnetic fields and measuring the relative polarization shift and the voltage shift at a particular polarizer angle. From two separate calculations, we found a Verdet constant of  $19.9 \pm 1.0 \text{ rad/T} \cdot \text{m}$  and  $17.54 \pm 1.12 \text{ rad/T} \cdot \text{m}$ . Other colleagues performing the experiment found the Verdet constant to be between 19 and 23  $\text{rad/T} \cdot \text{m}$  from using the first method and between 19 and 21  $\text{rad/T} \cdot \text{m}$  from using the second method. In our paper, we address a possible systematic error that lead to our Verdet constant being too low from the second method.

### Comments on Abstract:

1. You should decide on whether you want to call it the Faraday Effect or the Faraday Rotation, or if you want to use both terms, and define faraday rotation in terms of the faraday effect.
2. The relative roles of the rotatable polarizer, solenoid, and rod of birefringent glass aren't clear here. Add a sentence or two describing how you used a second polarizer to determine the angle of polarization (or relative angle of polarization, or shift in polarization) of the light from the change in light intensity after it passed through the material.
3. Need a sentence or two explaining (1) how you detected the relative intensity of the polarized light by measuring the current output from a photodiode (or more specifically, the voltage drop across a 1 Kohm resistor in series with the photodiode) and (2) that the voltage output was proportional to the light intensity (as verified by experiment), and that (3) unwanted voltage offsets and fluctuations in measured light intensity with changes in room lighting were eliminated using by (square-wave) modulation of the laser output and phase-sensistive detection of the transmitted light at the modulation frequency (lock-in amplifier) or some similar language
4. You've explained what you varied to ultimately calculate a verdet coefficient, but you haven't explained what it is (!) nor have you given a clear reason why your would measure it. I assume it is because the Verdet constant characterizes the degree of rotation (or strength

of the Faraday effect) for a particular material **for a particular wavelength of light**. Thinking about what the Verdet constant tells you should help you with your description. By the way, this so-called constant actually depends on the frequency of light passing through the material.

5. Your description of the two experiments as two different calculations is misleading. It suggests that you found  $2 + 3$  by two different calculational methods and came up with two different results! More accurately, you used two different *experimental* methods. To improve the clarity of your description, I suggest distinguishing between what you varied, what you measured, and what you calculated. For example, in the first of your two experiments, you didn't measure the phase shift in an intensity versus angle plot, you measured the sinusoidal variation of polarized light intensity as a function of polarizer angle for a series of magnetic fields ranging from -x tesla to +y tesla, and from that you determined the magnetic-field-dependent phase shift in polarization (or, if you prefer, the magnetic-field dependent change in polarization angle) upon passage through the glass.

## I. AIMS

- (a) To demonstrate the Faraday effect of the rotation of the plane of polarization of light as it travels through magnetic fields of different magnitudes.
- (b) To calculate the Verdet constant ( $V_c$ ) relating the change in polarization angle to the change in magnetic field.

$$\Delta\phi = V_c \Delta B L_{\text{sample}} \quad (1)$$

comments on aims

Aim (a) is hard to follow. I think it actually describes two aims: (1) qualitative: demonstrating that the angle of polarization of light passing through a birefringent material can be altered by applying a magnetic field – that is, the angle of polarization rotates (continuously) as it passes through the material, but only if a field is present, and (2) quantitative: demonstrating that the magnitude of the change in phase angle/polarization angle scales linearly with magnetic field. Doubling the field strength doubles the change in polarization angle. Try listing these separately to improve the clarity and specificity of what you have written.

Aim (b) is clearer. It is the 3rd of your aims. Namely, to measure the constant of proportionality for the linear relationship between  $\Delta\phi$  and  $B$ .

Finally, I realize that in lab for the purposes of calculating  $v_c$  from the slope  $\Delta\phi/\Delta B$  of a line passing through  $\phi$  versus  $B$  data, we used equation (1) as written, it isn't clear theoretically from eq.(1) as written that  $\Delta\phi = 0$  if  $B = 0$ . For that reason, when defining the Verdet coefficient  $v_c$  theoretically, as in eq (1), it would be better to write

$$\Delta\phi = v_c B L_{\text{sample}}$$

notice also that  $L$  should be written in math mode rather than in text mode and that I have written  $v_c$  instead of  $V_C$  so as to distinguish the coefficient from the symbol for voltage.

## II. INTRODUCTION

The Faraday effect is a magneto-optical phenomenon in which light of a single wavelength, traveling through certain materials subject to a magnetic field, experiences a shift in the

plane of polarization. not clear from this that the magnetic field is the cause of the shift, or that it experiences a shift in polarization proportional to the strength of the magnetic field. These materials, called birefringent, have different refractive indices for the right and left circular polarizations of light. As light travels through the material, the relative phase angle between the two polarizations changes, rotating the overall polarization plane<sup>2</sup>. consider instead “causing the overall polarization plane to rotate as it passes through the material”.

The Faraday effect was discovered by Michael Faraday coincidence? in 1845 and was the first experimental demonstration of the interrelation of electricity and magnetism. It has a strong historical significance and is a benchmark in the development of electromagnetic theory<sup>2</sup>. It also is of primary importance in many contemporary research fields in physics. For example, the Faraday Effect is currently being used to probe the properties of systems of interacting quantum spins, or spin-liquids.<sup>5</sup>it is also used to create “optical diodes” and “light switches” for fiber-optics communications. I wonder if this paragraph might go better in the conclusion section. Alternatively, it could be the first paragraph in this section. As it is, it interrupts the logical and narrative flow from the 1st paragraph to the 3rd.

As shown in Eq.(I) ,  $V_c$  is a constant of proportionality that relates the change in polarization angle to the change in the magnetic field.<sup>4</sup> The value of  $V_c$  depends on the medium and the wavelength of the light. The intensity of the light is given by  $I = I_0 \cos^2(\theta)$ , where  $\theta$  is the polarization angle with respect to the angle of maximum intensity. not exactly. The light leaving the laser is polarized, and has a certain angle of orientation. The rotatable polarizer also has an angle of orientation.  $\theta$  is the difference in angle between the two; the intensity of the light transmitted through the rotatable polarizer is a maximum when angle of orientation of the polarizer matches that of the incident light. When light experiences the Faraday effect What do you mean by “when light experiences the Faraday effect?” do you mean “when a magnetic field is applied” parallel to the direction of light transmission?, there will be a shift in the polarization so the intensity will be given by  $I = I_0 * \cos^2(\theta + \phi)$  instead.

### III. PROCEDURE

Polarized light came from a 650 nm laser, driven by a 400 kHz square wave from a function generator. A 15.2 cm long solenoid with 1400 turns and a DC resistance of 1.6

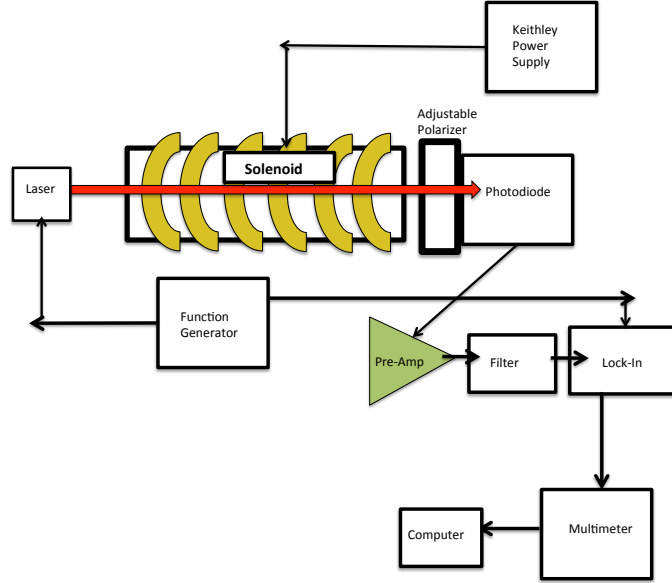


FIG. 1. A flow diagram of our experimental setup

$\Omega$  was the source of our magnetic field. The current for the solenoid was provided by a Keithley 2230-30-1 DC Power Supply. Channel 1 and Channel 2 were connected in parallel to provide sufficient current. The calibration of the solenoid was found by measuring the magnetic field at the center of the solenoid for a current of 0 A to 2.25 A in steps of 0.25 A. The calibration constant was calculated to be  $10.6 \text{ mT/A} \pm 0.05 \text{ mT}$ . A  $10.2 \pm 0.5 \text{ cm}$  SF-59 glass rod, was centered inside the solenoid. SF-59 glass is a heavy flint glass of refractive index  $1.95^1$ . The light is passed through a polarizer and detected by a photodiode, with the setting at  $1 \text{ k}\Omega$ . The photodiode outputs a voltage signal proportional to the detected light intensity<sup>3</sup>. A diagram of the experimental setup is shown in Fig. 1.

A lock-in amplifier was used so undesirable noise such as light from the environment and drift from the photodiode, can be removed from the signal. The output of the photodiode was connected to a band pass filter with a center frequency of 400 kHz and a Q of 20. This extracted the first harmonic of the square wave, a 400 kHz sine wave. The output of the filter was connected to a lock-in amplifier of gain 5. The reference signal for the lock-in amplifier was a 400 kHz sine wave with the same phase as the square wave driving the laser. It was generated by the same function generator for the square wave. The voltage output signal from the lock-in was measured by a Keithley 2100 Digital Multimeter.

Voltage readings from the multimeter were recorded onto the computer using LabView

program (Keithley DC Incremental Write.vi). Each recorded voltage was an average of 16 measurements. We took two types of measurements, so the Verdet constant can be calculated in two ways. The first method recorded the voltages as the polarizer was rotated in increments of  $5^\circ$ . We used this method with magnetic field values of -10.6 mT, 0 mT, 15.9 mT, and 21.2 mT. The negative field was generated by reversing the connection of the two leads of the power supply to the solenoid. The second method set the polarizer at an angle of  $\theta = 45^\circ$ , as it is the most sensitive angle. [can you explain what you mean by “most sensitive angle?”](#) Perhaps “largest change in voltage for a 1 degree change in polarization” or “slope of  $V(\theta)$  is greatest” or “angle corresponding to greatest sensitivity  $|dV/d\theta|$  to small changes in polarization angle” etc ? The voltages at field values of  $-31.8$ – $31.8$  mT [what?](#), in increments of 5.3 mT were measured.[try “... was taken for fields ranging from -31.8 mT to +31.8 mT...”](#)

## IV. RESULTS

Figs. 2–5 show the voltages plotted against the polarizer angle for different magnetic fields. Fig. 6 shows the voltages measured at an angle of  $\theta = 45^\circ$  for different magnetic fields.

[Since your results section is just 2 sentences, perhaps you should combine the results and analysis sections into a single “Results and Analysis” section.](#)

## V. ANALYSIS

### A. Method 1: Measurement of Photodiode Voltage for Various Polarizer Angles

A cosine-squared fit was performed on each our voltage measurements corresponding to a different magnetic field. The curve fits are shown in Figs. 2–5. One point in the curve fit for the  $B=0$  dataset was excluded from the curve fit because we measured the voltage twice for one angle. In all data sets, the experimental measurements of voltage for polarizer angles around  $305^\circ$ – $340^\circ$  were systematically higher than the values predicted by the sinusoidal curve fit. After a polarizer angle of  $340^\circ$ , we noticed that our experimental data values began to agree once again with the sinusoidal fit. In the Discussion section, we discuss possible causes for this systematic error. We masked the values corresponding to angles of

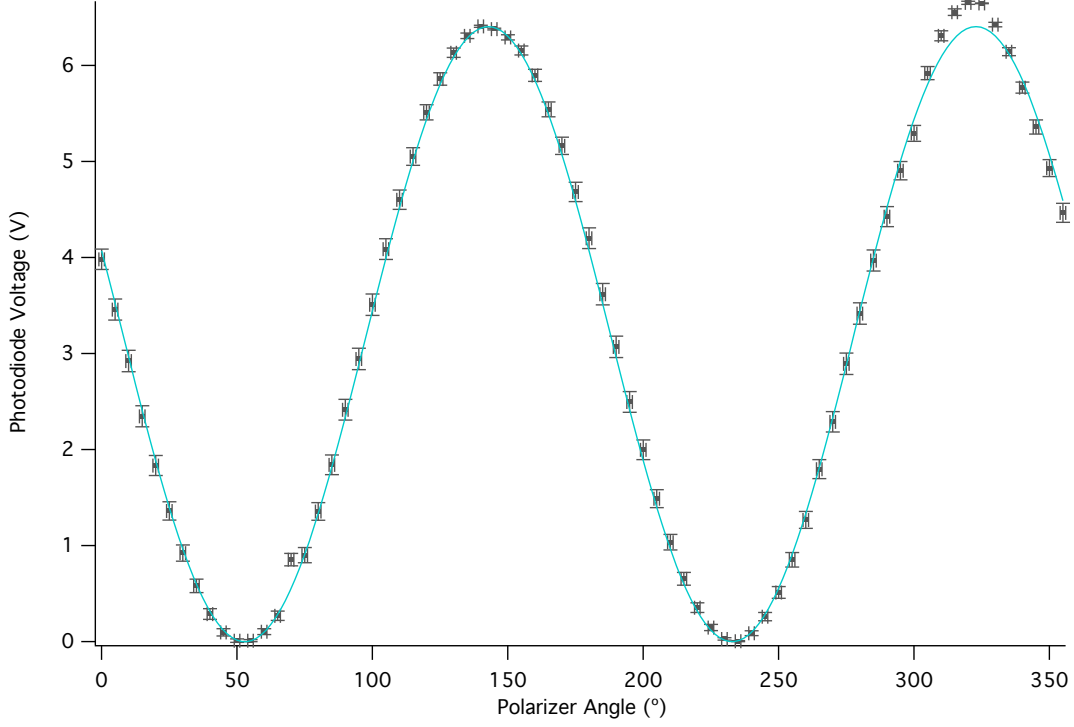


FIG. 2. Photodiode voltage measured for no magnetic field.

305°–340° from the curve fits as well from datasets with a nonzero magnetic field so they would not skew our calculation of the Verdet constant.

Our dominant source of error in these measurements is from setting the polarizer at the correct angle. We were able to set the polarizer within 1° of the desired angle. This error in the polarizer angle was propagated to an error in the voltage. We neglected voltage reading error from the multimeter as it was many magnitudes smaller than the error due to the angle setting.

The results of the curve fits are listed in Table I. We plotted  $\Delta\phi$  against  $\Delta B$  in Fig. 6. A linear fit was performed to obtain  $\Delta\phi/\Delta B = 1.79 \pm 0.07$  rad/T. The Verdet constant was calculated by rearranging Eq. (I) to

$$V_c = \frac{1}{L} \frac{\partial \theta}{\partial B}. \quad (2)$$

We calculated  $V_c$  to be  $17.54 \pm 1.12$  rad/T · m.



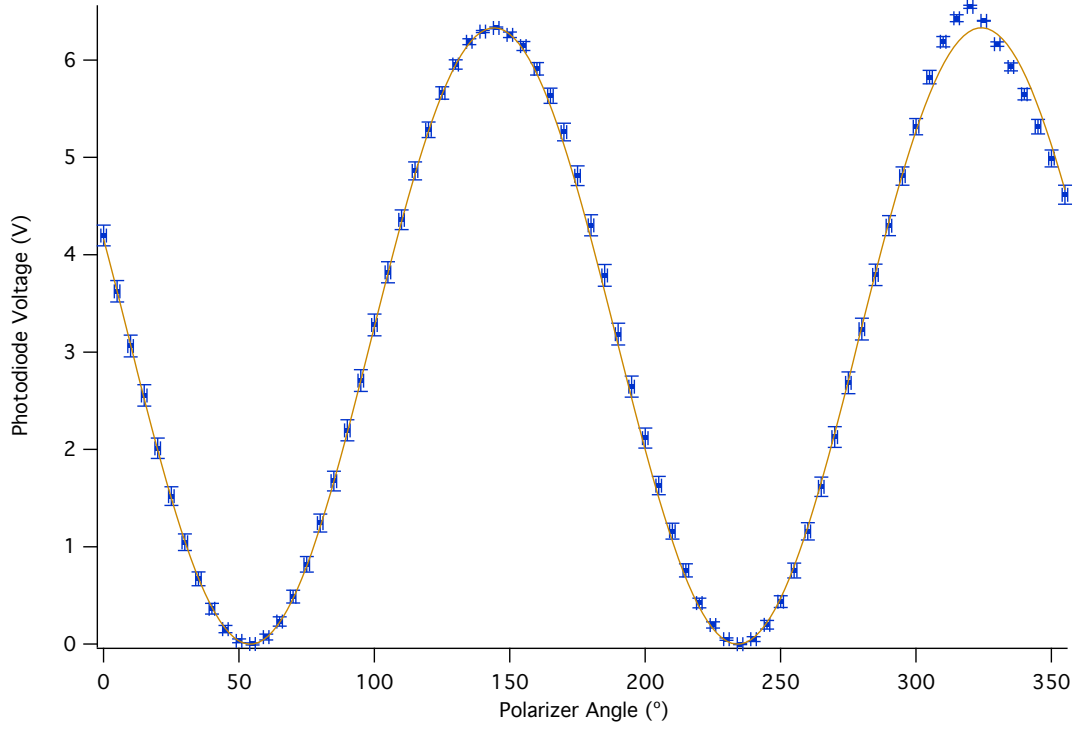


FIG. 3. Photodiode voltage measured for a magnetic field of -10.6 mT.

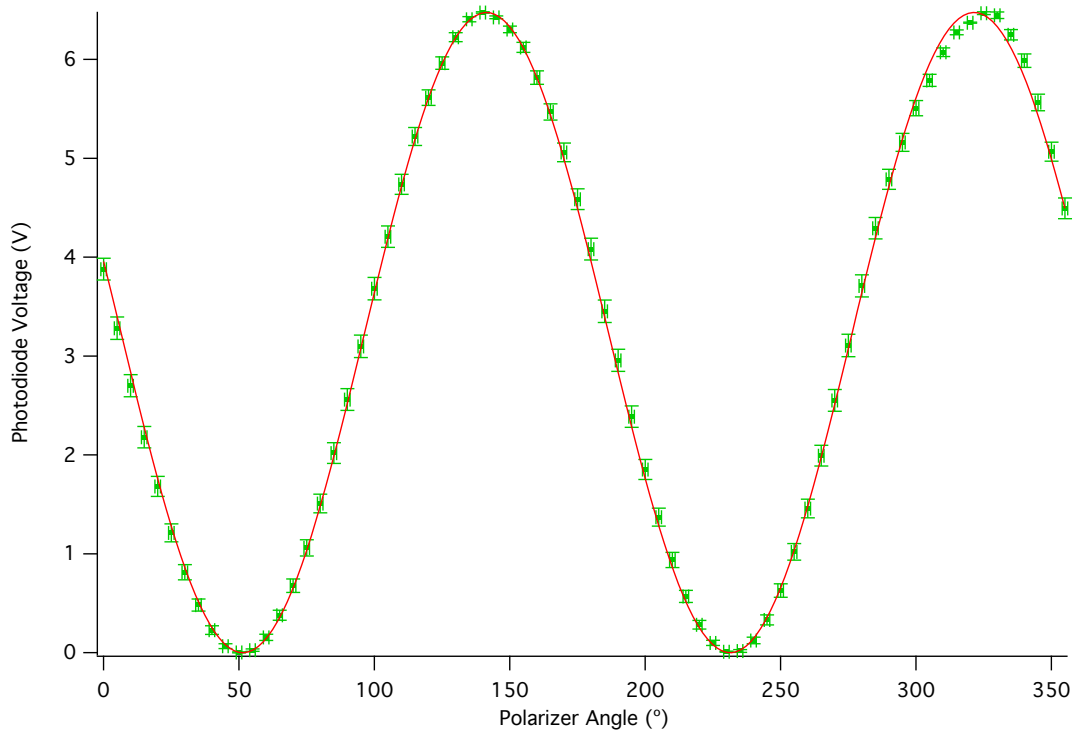


FIG. 4. Photodiode voltage measured for a magnetic field of 15.9 mT.

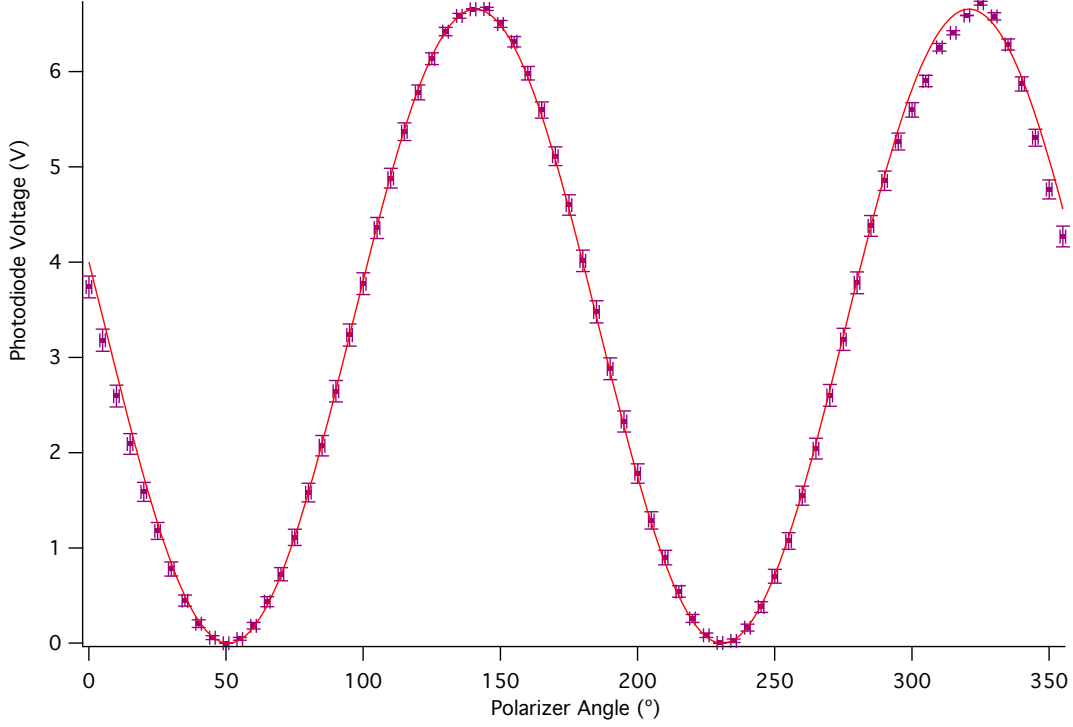


FIG. 5. Photodiode voltage measured for a magnetic field of 21.2 mT.

$\Delta B$ (mT)	Error $\Delta B$ (mT)	$\Delta\phi$ (rad)	Error $\Delta\phi$ ( $10^{-5}$ rad)	Reduced $\chi^2$
-10.6	0.05	-0.022691	187	0.76
0	0.05	$-4.77 \cdot 10^{-5}$	218	0.74
15.9	0.05	0.024996	219	0.50
21.2	0.05	0.035233	184	1.17

TABLE I. The shift in polarization due to a change in the magnetic field was found from a sinusoidal fit. The reduced  $\chi^2$  is a measure of the goodness of fit.

### B. Method 2: Measurement of Voltage at $\theta = 45^\circ$ for Various B

We calculated  $V_c$  using

$$\frac{\partial V}{\partial B} = \frac{\partial V}{\partial \theta} \frac{\partial \theta}{\partial B} \bigg|_{\theta=45^\circ}. \quad (3)$$

Recall that  $I = I_0 \cos^2 \theta$ . Since the voltage from the photodiode is proportional to  $I$ ,  $V = V_0 \cos^2 \theta$ . At  $\theta = 45^\circ$ ,  $\frac{\partial V}{\partial \theta} = V_0$ . We performed a linear fit of our data in Fig. 6 and found  $\frac{\partial V}{\partial B} = 0.0028 \pm 0.0006$  V/T. We used  $V_0 = 6.405$  obtained from the curve fit on the

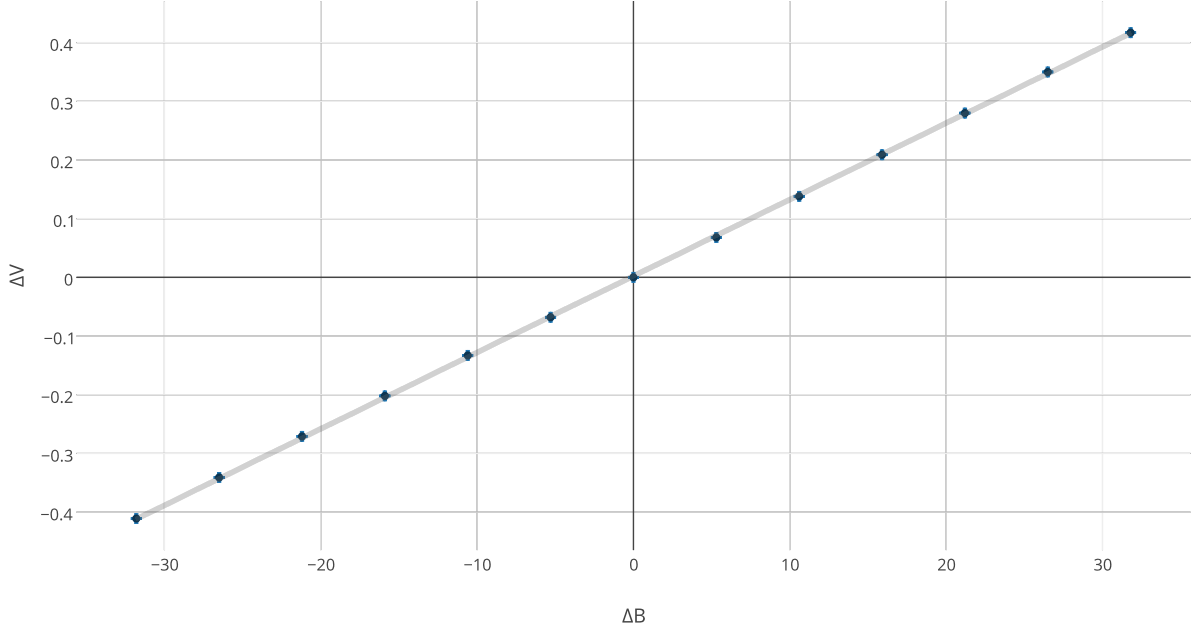


FIG. 6. The measured change in photodiode voltage for a change in the magnetic field at a polarizer angle  $45^\circ$  from the maximum voltage output. A linear fit was used to determine  $\Delta V/\Delta B$ . **this is a lovely graph, but it needs error bars for the data points, or some indication that the error in the data is smaller than the size of the data points**

data set for no magnetic field. The source of error for this method are error in setting the polarizer at  $\theta = 45^\circ$ , which leads to an error in  $\frac{\partial V}{\partial \theta}$  of 0.04 V/rad **assuming an error in angle setting of ??? degrees?**. We ignored the error in  $V_0$  from the curve fit because it was much smaller than the error due to the angle. From Eq. (I) and Eq. (3), we get

$$V_c = \frac{1}{L} \frac{\partial V / \partial B}{\partial V / \partial \theta} \quad (4)$$

Using Eq. (4), we obtained a value of  $19.9 \pm 1.0$  rad/T · m for the Verdet constant.

## VI. DISCUSSION

We compared our values for the Verdet Constant with those found by colleagues performing the same experiment. With Method 1, our colleagues found the value of the  $V_c$  to be between 19 and 23 rad/T · m. Our value of  $17.54 \pm 1.12$  rad/T · m is within this range for two standard deviations. Still, we consider systematic errors present in our data which would have generated an inaccurate value using Method 1. The most likely parameter which

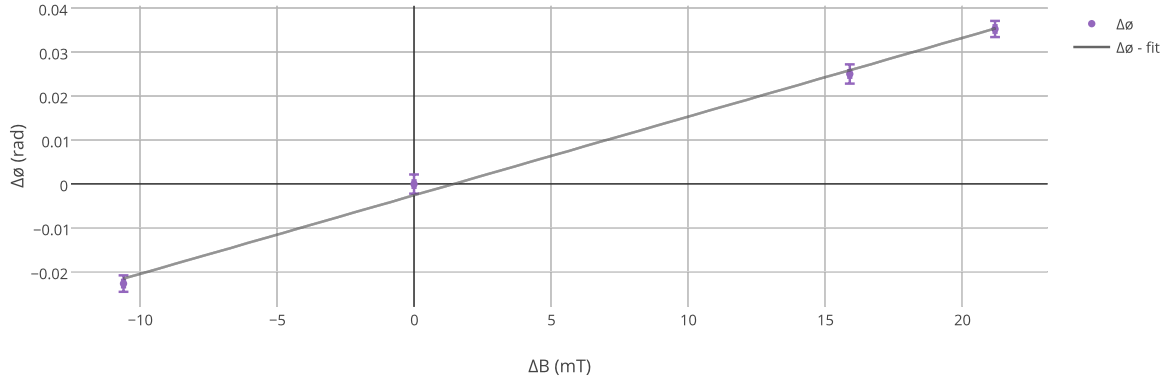


FIG. 7. The relative polarization shift for various magnetic fields. A linear fit was used to determine  $\Delta\phi/\Delta B$ .

went into Equation 3 that could exhibit a systematic error is our calculation of  $\partial\theta/\partial B$ . It is possible a systematic error was present in our measurement of the phase shift offset. We remarked earlier that the  $305^\circ$ – $340^\circ$  regions of all four of our datasets were offset from the sinusoidal fit, due to what appears to be a systematic error in data collection. In order to rectify this error, we had excluded those points from our curve fit.

We now explore possible causes of this systematic error. Angle values in the  $305^\circ$ – $340^\circ$  range were more difficult to set on the Faraday Rotator as they were partially obstructed by the solenoid. Using the human eye to set these values and attempting to look beyond the solenoid may have resulted in a systematic error in the angle setting due to parallax. What looked like  $305^\circ$  might have actually been  $303^\circ$ – $307^\circ$  due to this parallax effect. We have also considered the possibility that our Faraday Rotator was defective around the problematic values. Perhaps the angle markings on the device were improperly applied in the factory and what should have been a certain angle was not actually that angle. In order to confirm this, we would need to check the angle settings on the rotator with a very precise protractor. We masked data values in this range in order to eliminate data values rendered invalid due to systematic error but perhaps we might have included several points which appeared fine but were also subject to systematic error. Perhaps we should have thrown out all points beyond the  $305^\circ$  mark as they might have all been intrinsically shifted from the sinusoidal curve they should have followed. Including incorrect data points in the fit which generated

our phase offset which was critical for the calculation in Method 1 would have resulted in an incorrect offset and thus incorrect value for the Verdet Constant.

Using Method 2, our colleagues found the  $V_c$  to be between 19 and 21 rad/T · m. Our  $V_c$  using Method 2,  $19.9 \pm 1.0$  rad/T · m is thus in good agreement with the values of our fellow researchers as it falls within that range, within error. Method 2 involved parameters which did not depend on the accuracy of the phase offset and sinusoidal fit and thus our systematic error did not affect the calculations from Method 2. Our Verdet constant from Method 2 is thus the one to be reported as accurate. It was in agreement with the values reported by our colleagues, as mentioned earlier.

## VII. CONCLUSION

- (a) We were able to demonstrate Faraday Rotation for 3 different values of magnetic field.

As can be seen from the graphs, each plot of rotation angle vs. voltage has a different offset from the graph of rotation angle vs. voltage for a zero-field case. We can conclude that the overall plane of polarization for incident light has been rotated due to its travel through a magnetic field.

- (b) We calculated the Verdet Constant using two different methods of calculation. The first method gave us a constant of  $19.9 \pm 1.0$  rad/T · m, which is in good agreement with the values our colleagues calculated. The second method was likely affected by a systematic error in our data recording and ended up being  $17.54 \pm 1.12$  rad/T · m, not within agreement of the values calculated by our colleagues. **you should also note whether your two methods are in agreement with each other, given your uncertainty.  $19.9 - 17.54 = 2.36$  while  $1.0 + 1.12 = 2.12$ , so the difference is (just barely) significant. If you were able to improve your signal size or reduce your uncertainty in some way for measurement method 2, that would give you more confidence. Can you think of a way to do improve your measurement? larger fields? better angle control? Modulating the field about a particular value  $B = B_0 + B_1 \sin(\omega t)$  and using a 2nd lock-in for detection instead of plotting  $V(B)$  and finding the slope for different constant values of  $B_0$ , or ???**

**This would also be a good place to discuss how you might modify the experiment to measure the wavelength dependence, as well as to mention important technological ap-**

plications of this effect (for example in chemistry and/or fiber optic telecommunications).

## VIII. REFERENCES

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