# The Faraday Effect and Calculation of the Verdet Constant for a SF-59 Glass Rod

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## Abstract

The Faraday effect is a shift in the polarization of light as it passes through a medium subject to a magnetic field. We observed the Faraday effect in the following experiment. Polarized light from a 650 nm laser was sent through a SF-59 glass rod in a magnetic field produced by a solenoid. We then sent the light through a rotatable polarizer and measured intensity of light passing through with a photodiode. This allowed us to determine the shift in polarization as the intensity depends on the relative angle of polarization of the polarizer and the light. The voltage across a 1 k$\Omega$ resistor in series with the photodiode was measured. We experimentally confirmed it to be proportional to light intensity. The laser output was modulated so phase-sensitive detection could be used to remove voltage offsets and fluctuations due to variations in room lighting. The Verdet constant, a wavelength-dependent coefficient which relates the polarization angle rotation to the strength of the magnetic field, characterizes the strength of the Faraday effect for a specific material. We determined the Verdet constant for the rod using two experimental methods. For the first method, we measured the sinusoidal variation in the voltage as a function of polarizer angle for magnetic fields ranging from -10.6 mT to 21.2 mT. From this, we found the magnetic-field dependent phase shift in polarization from which we calculated a Verdet constant of $19.9 \pm 1.0$ rad/T$\cdot$m. The second method measured fixed the polarizer at an angle of 45° relative to light polarization when there was no field. This angle was used because it maximizes the change in light intensity, thus voltage, for a given shift in polarization. We measured the voltage for fields ranging from $-31.8$ mT to 31.8 mT and used this to calculate the Verdet constant of $17.54 \pm 1.12$ rad/T $\cdot$ m. Our colleagues performing these experiments found the Verdet constant to be between 19 and 23 rad/T $\cdot$ m and between 19 and 21 rad/T $\cdot$ m from using the first and second methods, respectively. We address possible causes of the discrepancy between our value from the second method from that of the first method and from the values obtained by our colleagues.

## I. AIMS

(a) To demonstrate the Faraday effect: a rotation of the polarization angle of light as it passes through a birefringent material subject to a magnetic field.

(b) To demonstrate that the magnitude of rotation of the polarization angle is proportional to the magnitude of the applied magnetic field.

(c) To determine the Verdet constant ($v_c$), the constant of proportionality relating the change in polarization angle $\Delta\phi$ to the applied magnetic field. The relationship between these quantities can be seen in

$$\Delta\phi = v_c B L_{\text{sample}}, \tag{1}$$

where $L_{\text{sample}}$ is the length of the material the light passes through.

## II. INTRODUCTION

The Faraday effect was discovered by Michael Faraday in 1845 and was the first experimental demonstration of the interrelation of electricity and magnetism. It has a strong historical significance and is a benchmark in the development of electromagnetic theory[2]. It also is of primary importance in many contemporary research fields in physics. For example, the Faraday Effect is currently being used to probe the properties of systems of interacting quantum spins, or spin-liquids.[5]

The Faraday effect is a magneto-optical phenomenon in which light of a single wavelength traveling through certain materials subject to a magnetic field, experiences a shift in polarization proportional to the magnetic field stregth. These materials, called birefringent, have different refractive indices for the right and left circular polarizations of light when a magnetic field is applied. As light travels through the material, the relative phase angle between the two polarizations changes, causing the overall polarization plane to rotate as it masses through the material[2].

As shown in Eq.(1) , $V_c$ is a constant of proportionality that relates the change in polarization angle to the change in the magnetic field.[4] The value of $V_c$ depends on the medium and the wavelength of the light. The intensity of the light is given by $I = I_0 \cos^2(\theta)$, where $\theta$ is the difference in angle between the initial polarization angle of the laser light and the

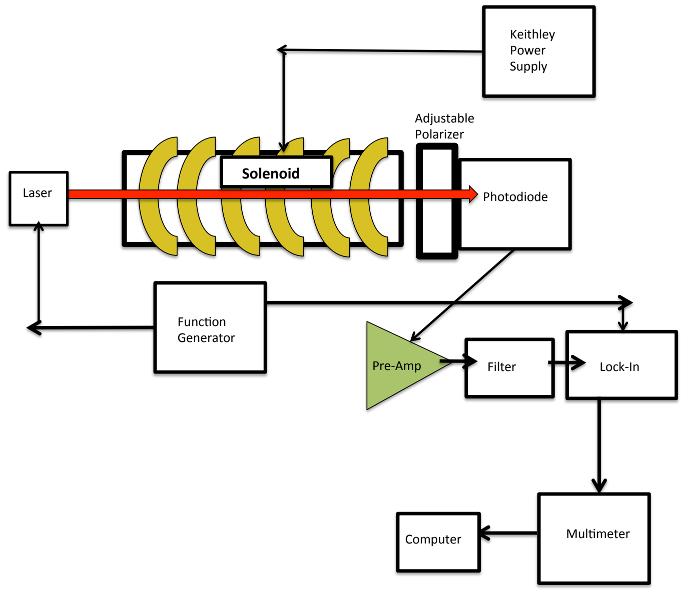


FIG. 1. A flow diagram of our experimental setup

angle of orientation of the polarizer. Maximum intensity of light occurs when these angles are the same. When light travels through a magnetic field applied parallel to the direction of light transmission, there will be a shift in the polarization so the intensity will be given by $I = I_0 * \cos^2(\theta + \phi)$ instead.

## III. PROCEDURE

Polarized light came from a 650 nm laser, driven by a 400 kHZ square wave from a function generator. A 15.2 cm long solenoid with 1400 turns and a DC resistance of 1.6 $\Omega$ was the source of our magnetic field. The current for the solenoid was provided by a Keithley 2230-30-1 DC Power Supply. Channel 1 and Channel 2 were connected in parallel to provide sufficient current. The calibration of the solenoid was found by measuring the magnetic field at the center of the solenoid for a current of 0 A to 2.25 A in steps of 0.25 A. The calibration constant was calculated to be 10.6 mT/A$\pm$0.05 mT. A 10.2 $\pm$ 0.5 cm SF-59 glass rod, was centered inside the solenoid. SF-59 glass is a heavy flint glass of refractive index 1.95[1]. The light is passed through a polarizer and detected by a photodiode, with the setting at 1 k$\Omega$. The photodiode outputs a voltage signal proportional to the detected light intensity[3]. A diagram of the experimental setup is shown in Fig. 1.

A lock-in amplifier was used so undesirable noise such as light from the environment and drift from the photodiode, can be removed from the signal. The output of the photodiode was connected to a band pass filter with a center frequency of 400 kHz and a Q of 20. This extracted the first harmonic of the square wave, a 400 kHz sine wave. The output of of the filter was connected to a lock-in amplifier of gain 5. The reference signal for the lock-in amplifier was a 400 kHz sine wave with the same phase as the square wave driving the laser. It was generated by the same function generator for the square wave. The voltage output signal from the lock-in was measured by a Keithley 2100 Digital Multimeter.

Voltage readings from the multimeter were recorded onto the computer using LabView program (Keithley DC Incremental Write.vi). Each recorded voltage was an average of 16 measurements. We took two types of measurements, so the Verdet constant can be calculated in two ways. The first method recorded the voltages as the polarizer was rotated in increments of 5°. We used this method with magnetic field values of -10.6 mT, 0 mT, 15.9 mT, and 21.2 mT. The negative field was generated by reversing the connection of the two leads of the power supply to the solenoid. The second method set the polarizer at an angle of $\theta = 45^\circ$, as it is the most sensitive angle, the angle where the slope of $V(\theta)$ is greatest and thus corresponds to the angle of greatest sensitivity in voltage change in response to changes in polarization angle. The voltages were taken for fields ranging from $-31.8$–$31.8$ mT, in increments of 5.3 mT.

## IV. RESULTS AND ANALYSIS

Figs. 2–5 show the voltages plotted against the polarizer angle for different magnetic fields. Fig. 6 shows the voltages measured at an angle of $\theta = 45^\circ$ for different magnetic fields.

### A. Method 1: Measurement of Photodiode Voltage for Various Polarizer Angles

A cosine-squared fit was performed on each our voltage measurements corresponding to a different magnetic field. The curve fits are shown in Figs. 2–5. One point in the curve fit for the B=0 dataset was excluded from the curve fit because we measured the voltage twice for one angle. In all data sets, the experimental measurements of voltage for polarizer angles

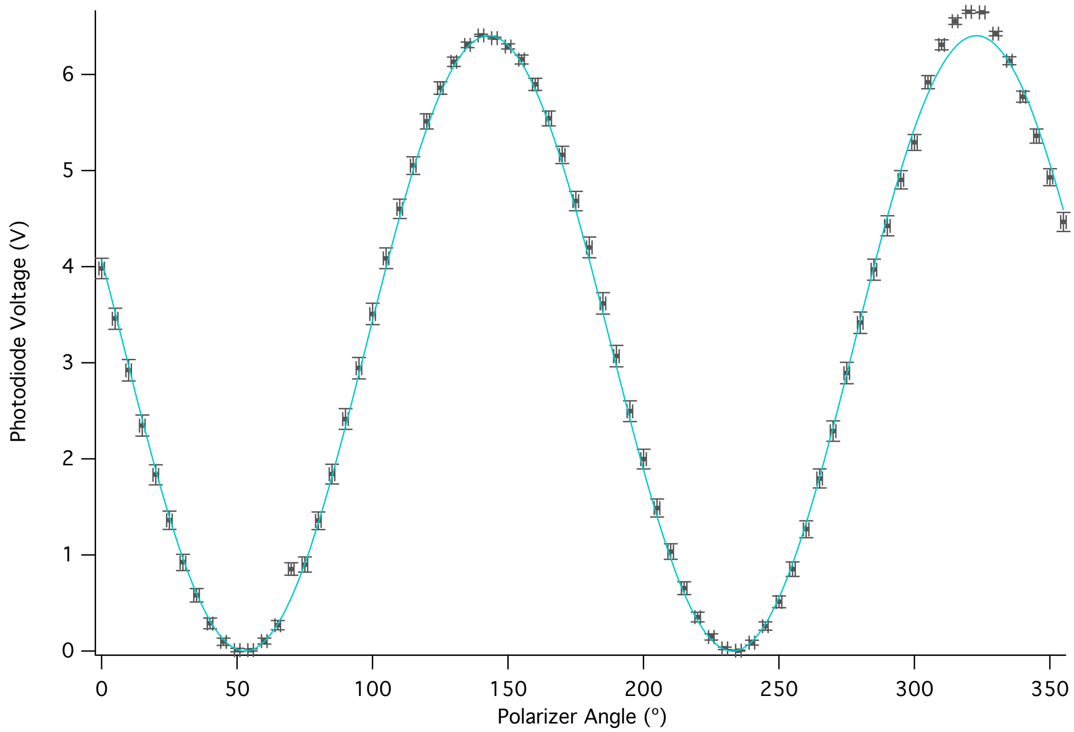


FIG. 2. Photodiode voltage measured for no magnetic field.

around 305°–340°were systematically higher than the values predicted by the sinusoidal curve fit. After a polarizer angle of 340°, we noticed that our experimental data values began to agree once again with the sinusoidal fit. In the Discussion section, we discuss possible causes for this systematic error. We masked the values corresponding to angles of 305°–340° from the curve fits as well from datasets with a nonzero magnetic field so they would not skew our calculation of the Verdet constant.

Our dominant source of error in these measurements is from setting the polarizer at the correct angle. We were able to set the polarizer within 1° of the desired angle. This error in the polarizer angle was propagated to an error in the voltage. We neglected voltage reading error from the multimeter as it was many magnitudes smaller than the error due to the angle setting.

The results of the curve fits are listed in Table I. We plotted $\Delta\phi$ agains $\Delta B$ in Fig. 6. A linear fit was performed to to obtain $\Delta\phi/\Delta B = 1.79 \pm 0.07$ rad/T. The Verdet constant was calculated by rearranging Eq. (1) to

$$V_c = \frac{1}{L}\frac{\partial\theta}{\partial B}. \tag{2}$$

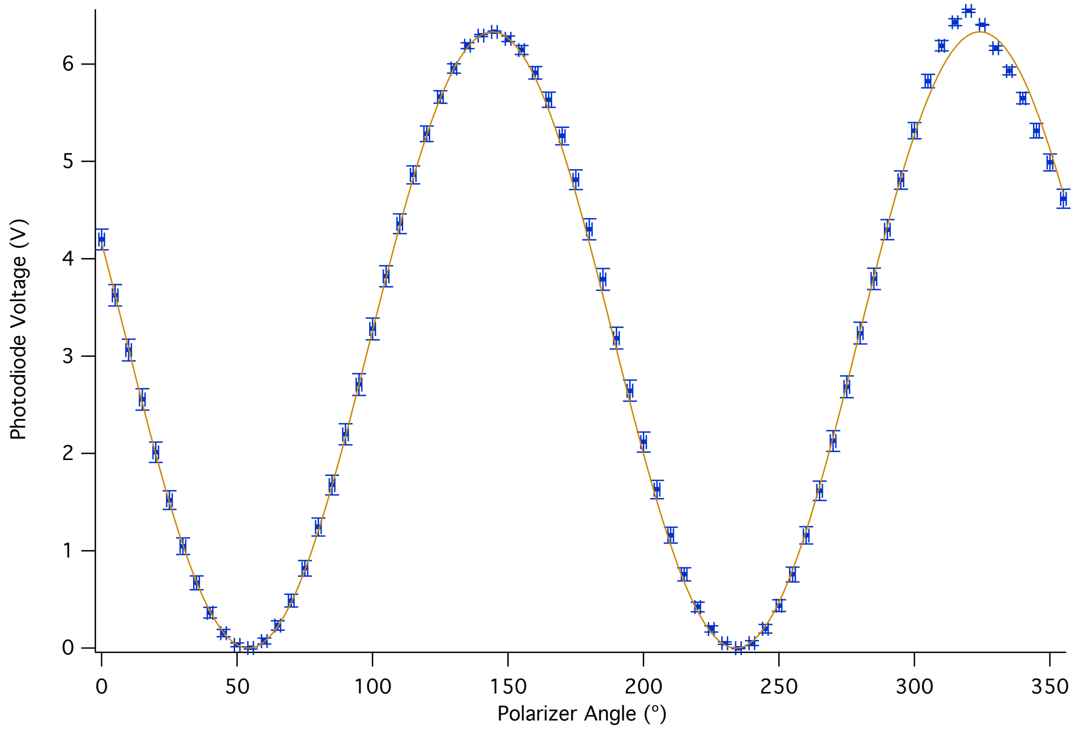


FIG. 3. Photodiode voltage measured for a magnetic field of -10.6 mT.

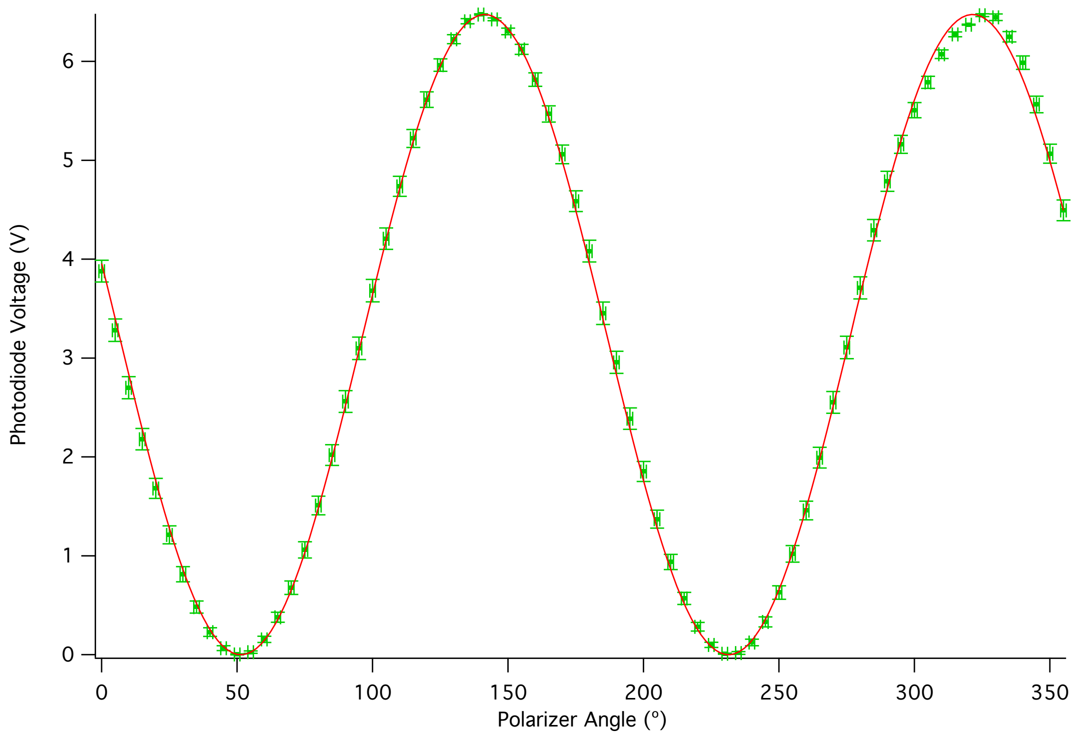


FIG. 4. Photodiode voltage measured for a magnetic field of 15.9 mT.

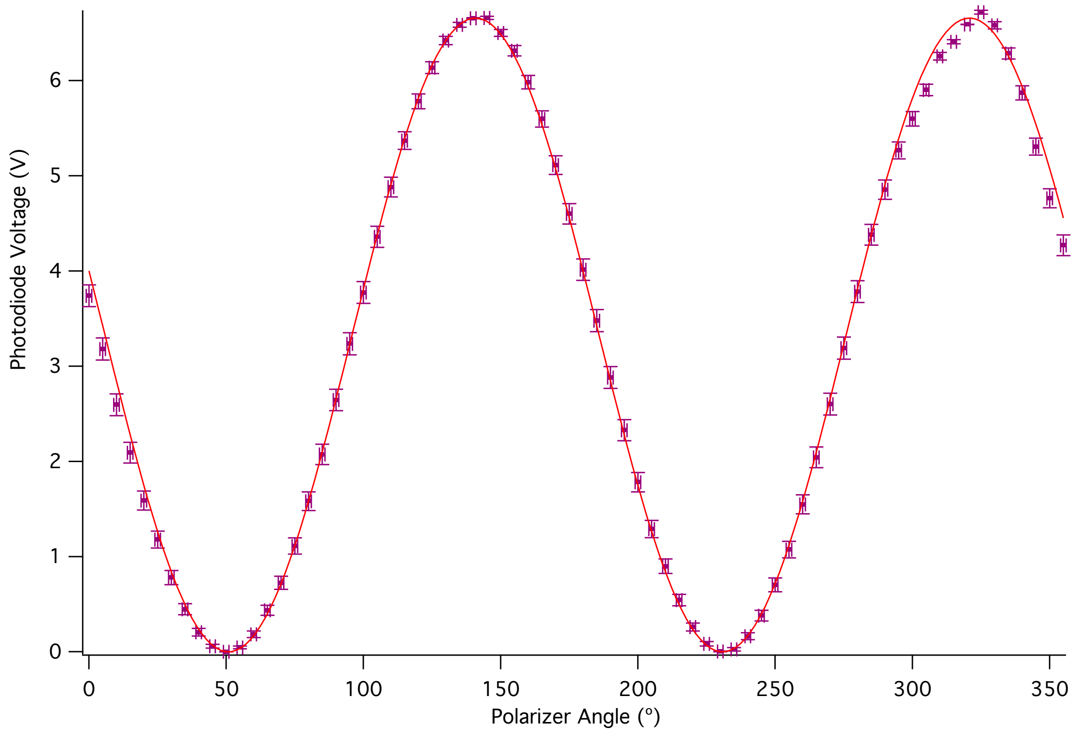


FIG. 5. Photodiode voltage measured for a magnetic field of 21.2 mT.

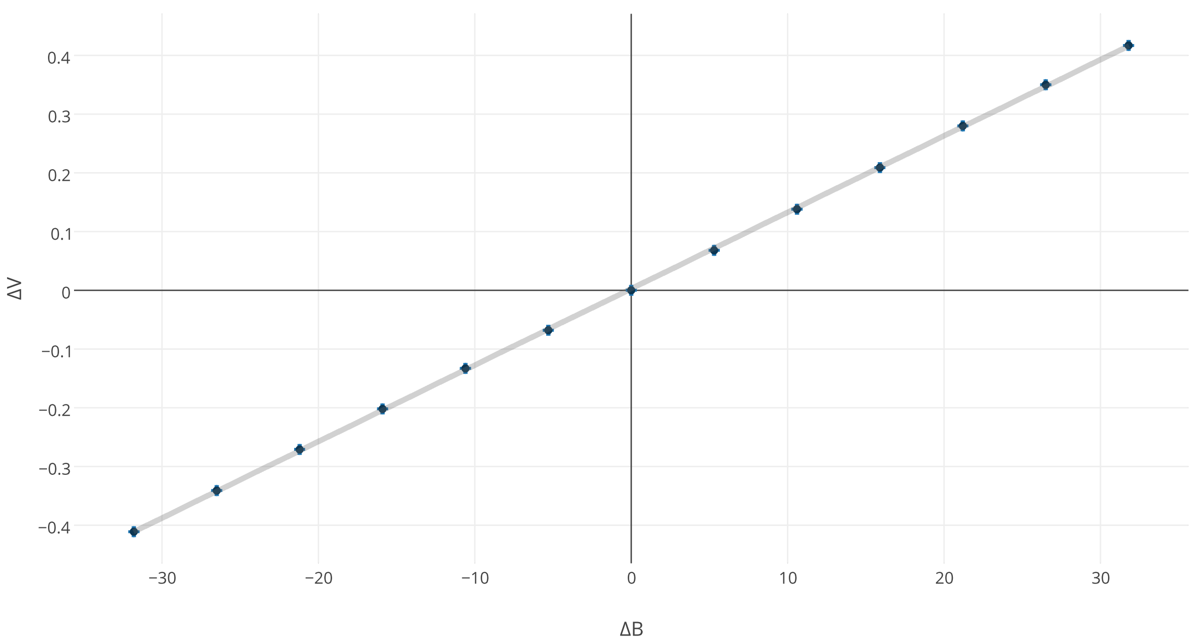


FIG. 6. The measured change in photodiode voltage for a change in the magnetic field at a polarizer angle 45° from the maximum voltage output. A linear fit was used to determine $\Delta V/\Delta B$.

| $\Delta$B (mT) | Error $\Delta$B (mT) | $\Delta\phi$ (rad) | Error $\Delta\phi$ ($10^{-5}$ rad) | Reduced $\chi^2$ |
|---|---|---|---|---|
| -10.6 | 0.05 | -0.022691 | 187 | 0.76 |
| 0 | 0.05 | -4.77$\cdot 10^{-5}$ | 218 | 0.74 |
| 15.9 | 0.05 | 0.024996 | 219 | 0.50 |
| 21.2 | 0.05 | 0.035233 | 184 | 1.17 |

TABLE I. The shift in polarization due to a change in the magnetic field was found from a sinusoidal fit. The reduced $\chi^2$ is a measure of the goodness of fit.

We calculated $V_c$ to be $17.54 \pm 1.12$ rad/T $\cdot$ m.

### B. Method 2: Measurement of Voltage at $\theta = 45^\circ$ for Various B

We calculated $V_c$ using

$$\frac{\partial V}{\partial B} = \frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial B}\bigg|_{\theta=45^\circ}. \tag{3}$$

Recall that $I = I_0 \cos^2\theta$. Since the voltage from the photodiode is proportional to $I$, $V = V_0 \cos^2\theta$. At $\theta = 45^\circ$, $\frac{\partial V}{\partial \theta} = V_0$. We performed a linear fit of our data in Fig. 6 and found $\frac{\partial V}{\partial B} = 0.0028 \pm 0.0006$ V/T. We used $V_0 = 6.405$ obtained from the curve fit on the data set for no magnetic field. The source of error for this method are error in setting the polarizer at $\theta = 45^\circ$, which leads to an error in $\frac{\partial V}{\partial \theta}$ of 0.04 V/rad. We ignored the error in $V_0$ from the curve fit because it was much smaller than the error due to the angle. From Eq. (1) and Eq. (3), we get

$$V_c = \frac{1}{L}\frac{\partial V/\partial B}{\partial V/\partial \theta} \tag{4}$$

Using Eq. (4), we obtained a value of $19.9 \pm 1.0$ rad/T $\cdot$ m for the Verdet constant.

## V. DISCUSSION

We compared our values for the Verdet Constant with those found by colleagues performing the same experiment. With Method 1, our colleagues found the value of the $V_c$ to be between 19 and 23 rad/T $\cdot$ m. Our value of $17.54 \pm 1.12$ rad/T $\cdot$ m is within this range for two standard deviations. Still, we consider systematic errors present in our data which

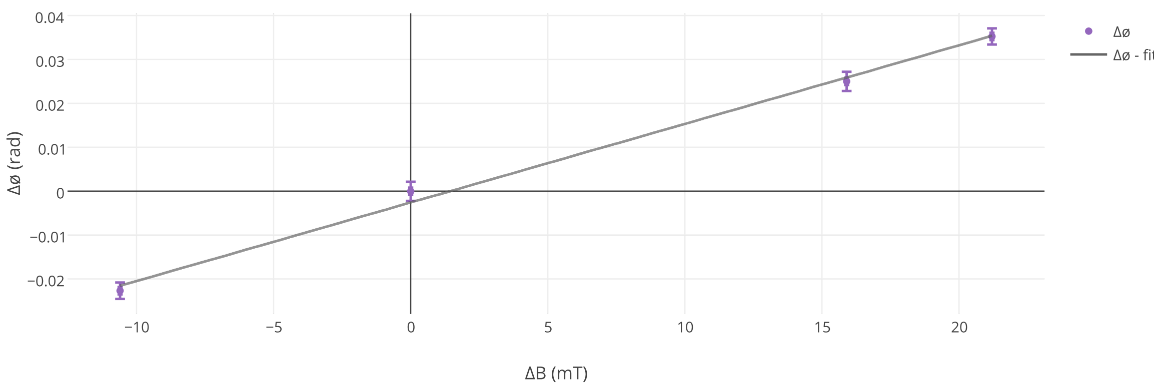


FIG. 7. The relative polarization shift for various magnetic fields. A linear fit was used to determine $\Delta\phi/\Delta B$.

would have generated an inaccurate value using Method 1. The most likely parameter which went into Equation 3 that could exhibit a systematic error is our calculation of $\partial\theta/\partial B$. It is possible a systematic error was present in our measurement of the phase shift offset. We remarked earlier that the 305°–340° regions of all four of our datasets were offset from the sinusoidal fit, due to what appears to be a systematic error in data collection. In order to rectify this error, we had excluded those points from our curve fit.

We now explore possible causes of this systematic error. Angle values in the 305°–340°range were more difficult to set on the Faraday Rotator as they were partially obstructed by the solenoid. Using the human eye to set these values and attempting to look beyond the solenoid may have resulted in a systematic error in the angle setting due to parallax. What looked like 305°might have actually been 303°–307°due to this parallax effect. We have also considered the possibility that our Faraday Rotator was defective around the problematic values. Perhaps the angle markings on the device were improperly applied in the factory and what should have been a certain angle was not actually that angle. In order to confirm this, we would need to check the angle settings on the rotator with a very precise protractor. We masked data values in this range in order to eliminate data values rendered invalid due to systematic error but perhaps we might have included several points which appeared fine but were also subject to systematic error. Perhaps we should have thrown out all points beyond the 305° mark as they might have all been intrinsically shifted from the sinusoidal curve they should have followed. Including incorrect data points in the fit which generated our phase offset which was critical for the calculation in Method 1 would have resulted in an incorrect offset and thus incorrect value for the Verdet Constant.

Using Method 2, our colleagues found the $V_c$ to be between 19 and 21 rad/T $\cdot$ m. Our $V_c$ using Method 2, $19.9 \pm 1.0$ rad/T $\cdot$ m is thus in good agreement with the values of our fellow researchers as it falls within that range, within error. Method 2 involved parameters which did not depend on the accuracy of the phase offset and sinusoidal fit and thus our systematic error did not affect the calculations from Method 2. Our Verdet constant from Method 2 is thus the one to be reported as accurate. It was in agreement with the values reported by our colleagues, as mentioned earlier.

## VI. CONCLUSION

(a) We were able to demonstrate Faraday Rotation for 3 different values of magnetic field. As can be seen from the graphs, each plot of rotation angle vs. voltage has a different offset from the graph of rotation angle vs. voltage for a zero-field case. We can conclude that the overall plane of polarization for incident light has been rotated due to its travel through a magnetic field.

(b) We calculated the Verdet Constant using two different methods of calculation. The first method gave us a constant of $19.9 \pm 1.0$ rad/T $\cdot$ m, which is in good agreement with the values our colleagues calculated. The second method was likely affected by a systematic error in our data recording and ended up being $17.54 \pm 1.12$ rad/T $\cdot$ m, not within agreement of the values calculated by our colleagues.

## VII. REFERENCES

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