Algorytmy i struktury danych - Lista $4\,$

P. Cegieła, W. Sęk

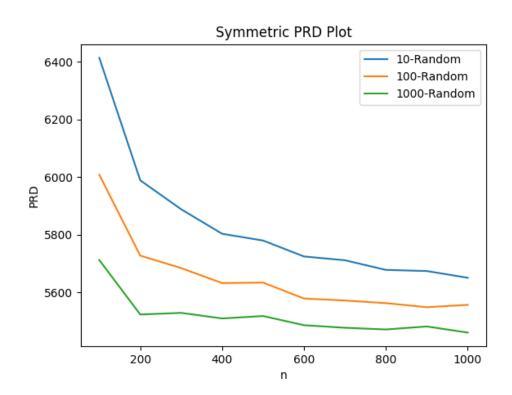
 $31~\mathrm{marca}~2022$

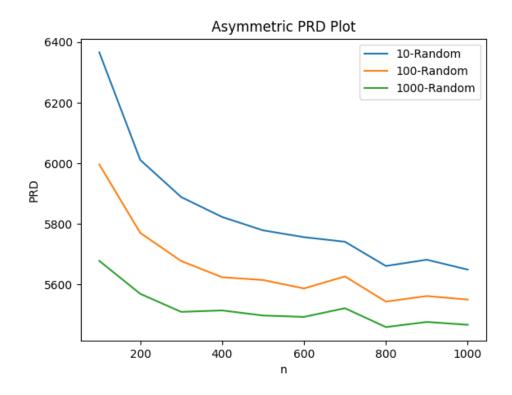
1 Algorytm k-random

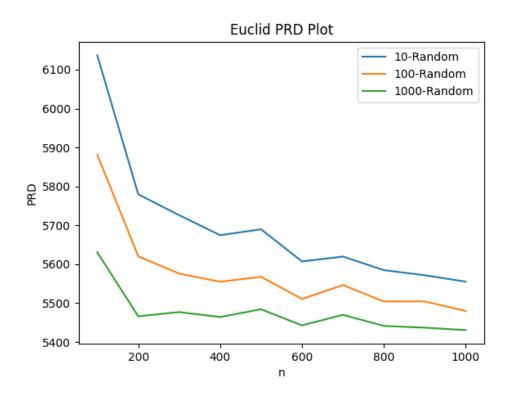
1.1 Tabela

| | Average PRD | | | | | |
|------|--------------------|-------------------|--------------------|--|--|--|
| n | 10-Random | 100-Random | 1000-Random | | | |
| 100 | 6413.9156401721075 | 6007.918586928952 | 5711.806977951733 | | | |
| 200 | 5988.615372379403 | 5727.247304868426 | 5522.69171607988 | | | |
| 300 | 5888.036831784668 | 5683.792028830031 | 5528.337270981829 | | | |
| 400 | 5802.963142088209 | 5631.843379656301 | 5508.994282851 | | | |
| 500 | 5779.552537965469 | 5633.505579085746 | 5517.331574583659 | | | |
| 600 | 5724.137341197624 | 5578.206097637371 | 5485.583014224162 | | | |
| 700 | 5711.069467439124 | 5571.201094319976 | 5476.566562989776 | | | |
| 800 | 5677.560472641102 | 5562.185079759854 | 5470.719995332991 | | | |
| 900 | 5673.623396583947 | 5548.007479574144 | 5481.1123195262335 | | | |
| 1000 | 5650.144622314967 | 5556.232385763668 | 5459.875331845722 | | | |

1.2 Wykresy







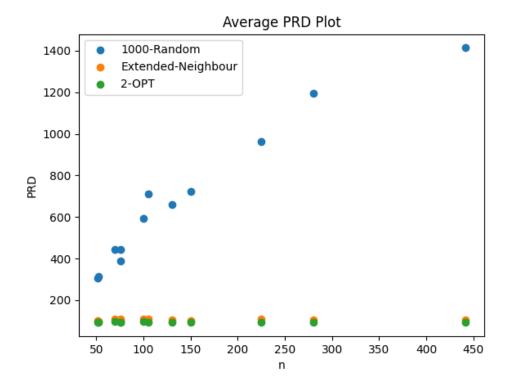
2 Porównanie algorytmów na grafach euklidesowych z TSPLIB

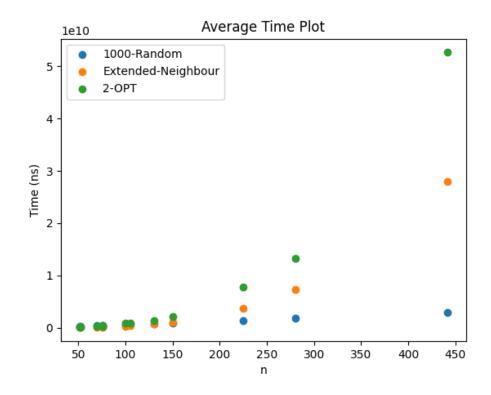
2.1 Tabele

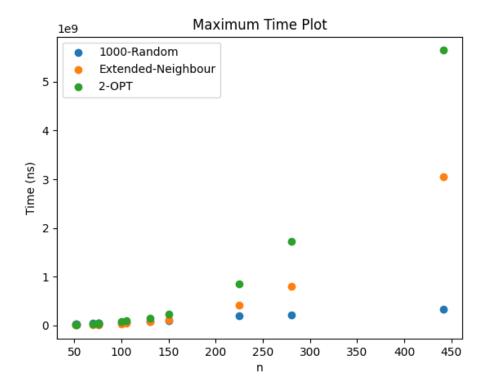
| | Average PRD | | | | | |
|-----|-------------|---------------------|--------|--|--|--|
| n | 1000-Random | Extended-Neighbours | 2-OPT | | | |
| 280 | 1190.481 | 105.355 | 94.110 | | | |
| 52 | 313.842 | 98.473 | 92.241 | | | |
| 130 | 655.514 | 105.463 | 93.669 | | | |
| 150 | 722.093 | 100.510 | 91.808 | | | |
| 51 | 304.930 | 103.146 | 91.878 | | | |
| 76 | 390.223 | 103.011 | 92.974 | | | |
| 105 | 719.383 | 107.776 | 95.285 | | | |
| 442 | 1410.979 | 106.094 | 94.210 | | | |
| 76 | 437.657 | 111.045 | 94.849 | | | |
| 100 | 599.637 | 109.201 | 95.515 | | | |
| 70 | 443.081 | 107.926 | 97.704 | | | |
| 225 | 958.394 | 110.576 | 94.255 | | | |

| | Average Time | | | | | |
|-----|--------------|---------------------|-------------|--|--|--|
| n | 1000-Random | Extended-Neighbours | 2-OPT | | | |
| 280 | 1824391000 | 7216115200 | 12956555100 | | | |
| 52 | 337466300 | 53224900 | 100680300 | | | |
| 130 | 847446300 | 776220800 | 1495083700 | | | |
| 150 | 1216246900 | 1413185300 | 2831431300 | | | |
| 51 | 397964700 | 70906500 | 135488700 | | | |
| 76 | 529495400 | 169370500 | 342460800 | | | |
| 105 | 826099200 | 562516800 | 1193324400 | | | |
| 442 | 2925433800 | 28520444700 | 53773197200 | | | |
| 76 | 451942500 | 153868400 | 348626900 | | | |
| 100 | 599398700 | 332370900 | 876795000 | | | |
| 70 | 435804700 | 123948500 | 285729600 | | | |
| 225 | 1336604800 | 3619145800 | 7337685000 | | | |

| | Maximal Time | | | | | |
|-----|--------------|---------------------|------------|--|--|--|
| n | 1000-Random | Extended-Neighbours | 2-OPT | | | |
| 280 | 226062700 | 800549100 | 1374390600 | | | |
| 52 | 36579000 | 5832400 | 11066200 | | | |
| 130 | 103836900 | 84908600 | 173113200 | | | |
| 150 | 196152300 | 201122200 | 433930400 | | | |
| 51 | 52224600 | 11076500 | 18398300 | | | |
| 76 | 66321000 | 20809800 | 38979800 | | | |
| 105 | 103035700 | 73946000 | 164836700 | | | |
| 442 | 330026000 | 3123459200 | 5680294300 | | | |
| 76 | 45770600 | 15989400 | 35452500 | | | |
| 100 | 61684700 | 34254200 | 90924800 | | | |
| 70 | 50222300 | 15401600 | 34864000 | | | |
| 225 | 137559000 | 367219200 | 757268500 | | | |







3 Porównanie algorytmów na grafach generowanych losowo

3.1 Tabele

3.1.1 Grafy asymetryczne

| | Average Time | | | | | |
|------|--------------|------------|---------------------|---------------|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | |
| 100 | 5215583333 | 28571666 | 2786506666 | 3001888333 | | |
| 200 | 10385900000 | 102848333 | 20872141666 | 21460786666 | | |
| 300 | 15575736666 | 230665000 | 69467785000 | 71687228333 | | |
| 400 | 20725138333 | 400460000 | 161340206666 | 164356600000 | | |
| 500 | 25681436666 | 630796666 | 312382090000 | 319592263333 | | |
| 600 | 31847411666 | 893138333 | 539254811666 | 546289266666 | | |
| 700 | 37347566666 | 1216818333 | 852635371666 | 864857386666 | | |
| 800 | 43085846666 | 1589101666 | 1276754056666 | 1285017576666 | | |
| 900 | 48517803333 | 2017996666 | 1807960408333 | 1825393600000 | | |
| 1000 | 53376455000 | 2458155000 | 2488077001666 | 2527648053333 | | |

| | Maximum Time | | | | | |
|------|--------------|------------|---------------------|-------------|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | |
| 100 | 53740500 | 296200 | 28250900 | 31605000 | | |
| 200 | 110770900 | 1068300 | 222382100 | 217436400 | | |
| 300 | 156366900 | 2430200 | 699352300 | 730365300 | | |
| 400 | 218396100 | 4173300 | 1630130200 | 1658699700 | | |
| 500 | 272075700 | 6445900 | 3154009300 | 3248476300 | | |
| 600 | 326146900 | 9109900 | 5445648700 | 5518205200 | | |
| 700 | 392337400 | 12303600 | 8644185800 | 8705532900 | | |
| 800 | 442730100 | 16022200 | 12919136400 | 12989845300 | | |
| 900 | 497044800 | 20912700 | 18259475600 | 18320587400 | | |
| 1000 | 545819300 | 25097700 | 25367610900 | 26443099700 | | |

| | Average PRD | | | | | |
|------|--------------------|--------------------|---------------------|-------|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | |
| 100 | 144389.48040576265 | 5323.792664503988 | 19.841269841269842 | 0 | | |
| 200 | 264424.58297728904 | 5207.564744303786 | 0 | 0 | | |
| 300 | 344372.7648431962 | 5095.76765521116 | 99.36766034327009 | 0 | | |
| 400 | 454295.2612852427 | 5024.999051480909 | 0 | 0 | | |
| 500 | 542028.8049036451 | 3589.213599432774 | 103.10202617894926 | 0 | | |
| 600 | 699247.4253613068 | 4932.616806843915 | 0 | 0 | | |
| 700 | 755741.8205443585 | 4089.52732594919 | 0 | 0 | | |
| 800 | 808528.5198217536 | 4024.938451458384 | 0 | 0 | | |
| 900 | 926564.7954870298 | 4759.48286220438 | 0 | 0 | | |
| 1000 | 1000638.9406083454 | 3671.7679495233406 | 0 | 0 | | |

3.1.2 Grafy symetryczne

| | Average Time | | | | | |
|------|--------------|------------|---------------------|---------------|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | |
| 100 | 5118872000 | 27634000 | 2761260000 | 4705872000 | | |
| 200 | 10426454000 | 110419000 | 20948980000 | 33434436000 | | |
| 300 | 15504584000 | 235028000 | 69605889000 | 109212242000 | | |
| 400 | 20687462000 | 405624000 | 162160400000 | 246114934000 | | |
| 500 | 25238599000 | 626720000 | 315572915000 | 464210183000 | | |
| 600 | 33565792000 | 908487000 | 552476749000 | 793604803000 | | |
| 700 | 40401877000 | 1304289000 | 891778583000 | 1298559476000 | | |
| 800 | 44722603000 | 1664659000 | 1291840090000 | 1848839045000 | | |
| 900 | 48923905000 | 2028359000 | 1834605650000 | 2642838875000 | | |
| 1000 | 53613716000 | 2543461000 | 2531951057000 | 3497862690000 | | |

| | Maximum Time | | | | |
|------|--------------|----------|-------------|-------------|--|
| 100 | 53478300 | 287700 | 28774700 | 59023500 | |
| 200 | 118932100 | 1496800 | 225860300 | 374150100 | |
| 300 | 158825400 | 2666500 | 713698900 | 1210248700 | |
| 400 | 214809200 | 4154100 | 1651365900 | 2783661100 | |
| 500 | 259651300 | 6437400 | 3196615100 | 4989667500 | |
| 600 | 369051000 | 9376100 | 5731537000 | 8662696500 | |
| 700 | 453044000 | 15011700 | 9255213200 | 14112162500 | |
| 800 | 519034200 | 18486000 | 13176742300 | 20361935900 | |
| 900 | 542197200 | 21026000 | 18626545300 | 27632920000 | |
| 1000 | 587381000 | 28130500 | 26359530800 | 38144374100 | |

| | Average PRD | | | | | |
|------|--------------------|--------------------|---------------------|-------|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | |
| 100 | 169641.5152490044 | 8651.420983070748 | 2910.828178233857 | 0 | | |
| 200 | 336491.148021938 | 8695.594682621479 | 3759.5914230406697 | 0 | | |
| 300 | 467506.6707100455 | 9672.03524228703 | 3958.851600018442 | 0 | | |
| 400 | 626451.037540901 | 9242.735074436148 | 4352.368040212525 | 0 | | |
| 500 | 762030.9959416362 | 9491.651021903159 | 3688.834144036091 | 0 | | |
| 600 | 907750.4576801556 | 10564.279122298864 | 3896.5508009869172 | 0 | | |
| 700 | 999294.050859801 | 10511.632970306257 | 4405.698984443974 | 0 | | |
| 800 | 1177366.205236822 | 10932.69827564215 | 4323.395735797874 | 0 | | |
| 900 | 1312890.7423081866 | 9844.799687300712 | 4385.352415182208 | 0 | | |
| 1000 | 1445342.3876189813 | 9645.3501317382 | 4554.947937370816 | 0 | | |

3.1.3 Grafy euklidesowe

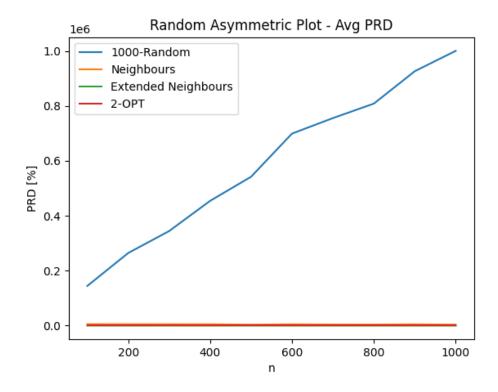
| | Average Time | | | | | |
|------|--------------|------------|---------------------|---------------|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | |
| 100 | 5430231666 | 29933333 | 2909718333 | 5610398333 | | |
| 200 | 11033958333 | 108678333 | 22288358333 | 47841571666 | | |
| 300 | 15708336666 | 238925000 | 71815146666 | 149125501666 | | |
| 400 | 21969775000 | 420680000 | 168567516666 | 340063338333 | | |
| 500 | 26306493333 | 644906666 | 327083776666 | 689031216666 | | |
| 600 | 33492466666 | 941325000 | 568170671666 | 1224798638333 | | |
| 700 | 40003946666 | 1280896666 | 907097273333 | 1902227616666 | | |
| 800 | 46368281666 | 1705881666 | 1352010363333 | 2914841420000 | | |
| 900 | 49810670000 | 2115170000 | 1857726728333 | 3970601576666 | | |
| 1000 | 53497760000 | 2547503333 | 2548862593333 | 5568105215000 | | |

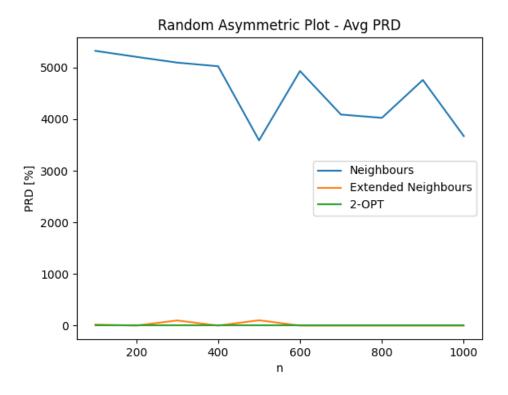
| | Maximum Time | | | | |
|------|--------------|----------|-------------|-------------|--|
| 100 | 56904000 | 321200 | 30141400 | 66528500 | |
| 200 | 113894700 | 1158400 | 234500000 | 519662900 | |
| 300 | 164992700 | 2442400 | 726804400 | 1549853100 | |
| 400 | 232275800 | 4327000 | 1701320800 | 3590054900 | |
| 500 | 286456400 | 6665100 | 3415103000 | 7359945700 | |
| 600 | 349887600 | 9601400 | 5758347300 | 13165101000 | |
| 700 | 426304400 | 13348700 | 9258026500 | 20301730000 | |
| 800 | 485403100 | 18255900 | 13873937600 | 31742944800 | |
| 900 | 510576800 | 21615300 | 18716893000 | 41209877500 | |
| 1000 | 555995400 | 26168200 | 25734825600 | 57516775300 | |

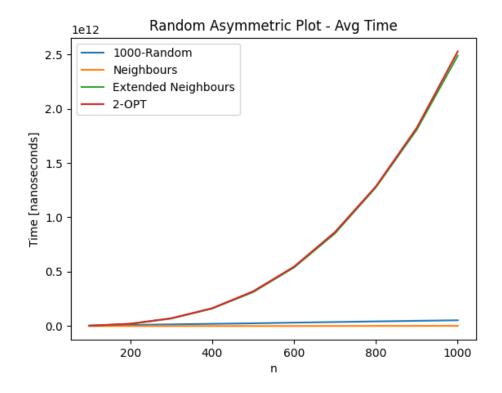
| Average PRD | | | | | | | |
|-------------|--------------------|--------------------|---------------------|-------|--|--|--|
| n | 1000-Random | Neighbours | Extended-Neighbours | 2-OPT | | | |
| 100 | 45057.17215979038 | 2590.834778478913 | 1152.047375520498 | 0 | | | |
| 200 | 75024.50161461842 | 2064.382965833475 | 1126.787169849889 | 0 | | | |
| 300 | 95586.47736955529 | 2089.941890785232 | 1319.4679540953857 | 0 | | | |
| 400 | 115379.31540464885 | 1936.2643602499013 | 1170.3148715360107 | 0 | | | |
| 500 | 129969.40868945165 | 1916.510105455013 | 1432.3355018206873 | 0 | | | |
| 600 | 147132.99203529008 | 1877.0634799785885 | 1325.9108020407148 | 0 | | | |
| 700 | 158377.07578069592 | 1780.3974674441433 | 1334.865053976823 | 0 | | | |
| 800 | 173366.5990915691 | 2018.4858325151736 | 1443.038050374855 | 0 | | | |
| 900 | 184435.2199324318 | 1859.5850439135654 | 1345.825159240164 | 0 | | | |
| 1000 | 197381.2850131005 | 2066.702637095026 | 1479.7361632179882 | 0 | | | |

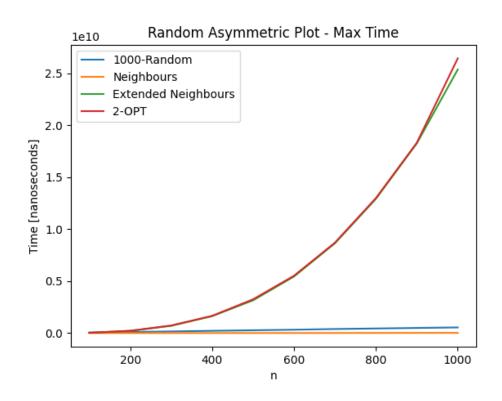
3.2 Wykresy

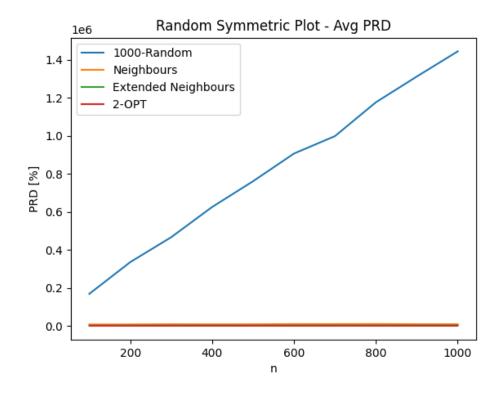
3.2.1 Grafy asymetryczne

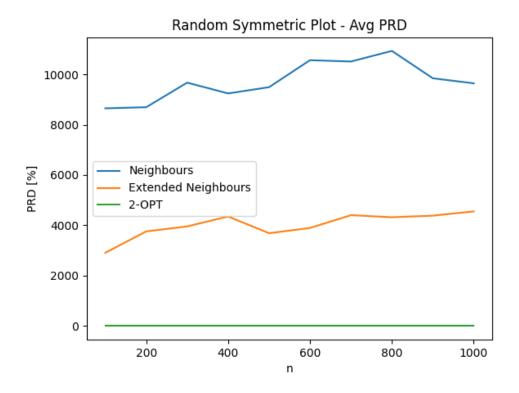


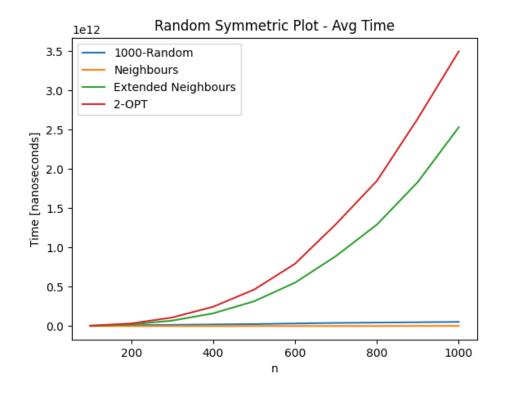


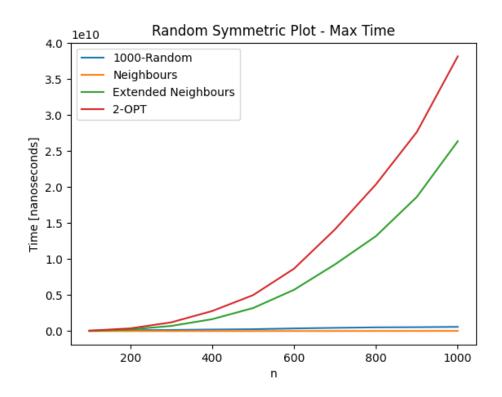


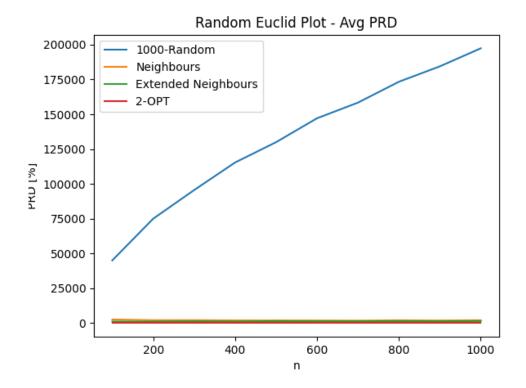


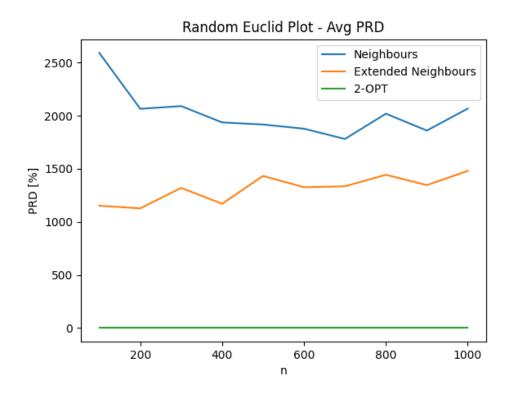


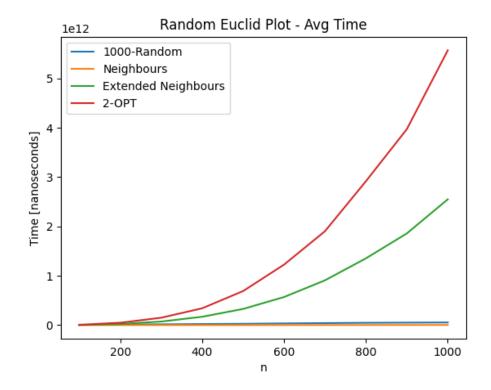


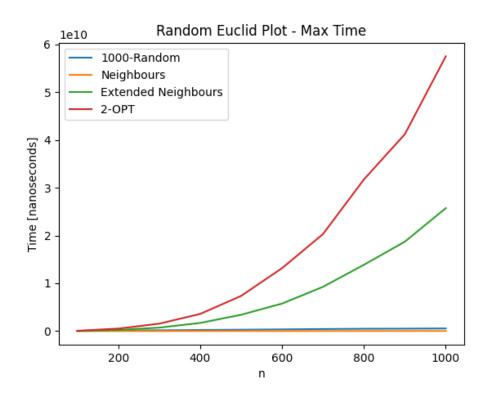












3.3 Wnioski

4 Porównanie 2-OPT w zależności od przybliżenia początkowego

4.1 Tabele

4.1.1 Grafy asymetryczne

| Asymmetric Graphs with 2-OPT | | | | | | |
|------------------------------|-------------|----------------|--------------|----------------|--------------------|----------------|
| | AVG Time | | MAX Time | | AVG PRD | |
| n | 1000-Random | Ext-Neighbours | 1000-Random | Ext-Neighbours | 1000-Random | Ext-Neighbours |
| 50 | 48453200 | 6136500 | 3738398333 | 489811666 | 35597.04376268311 | 0 |
| 100 | 140853600 | 35301100 | 11970911666 | 3169770000 | 73004.9758129217 | 0 |
| 150 | 337295400 | 98910400 | 29113163333 | 9698358333 | 96256.98438783734 | 0 |
| 200 | 643440000 | 222620100 | 58731895000 | 21984178333 | 119059.65961691776 | 0 |
| 250 | 1392918200 | 438522000 | 123776605000 | 43217630000 | 144661.62881606517 | 0 |
| 300 | 1981728700 | 744836600 | 181790746666 | 72750526666 | 168466.84457767615 | 0 |
| 350 | 3127470800 | 1156490900 | 294949878333 | 114528933333 | 201839.59192018144 | 0 |
| 400 | 4895469300 | 1719545200 | 434607098333 | 170149210000 | 219001.69640333427 | 0 |
| 450 | 6997186800 | 2429447700 | 638500593333 | 241024308333 | 231538.9713570598 | 0 |
| 500 | 10230351100 | 3378707200 | 905001025000 | 331310758333 | 249834.60481496935 | 0 |

4.1.2 Grafy symetryczne

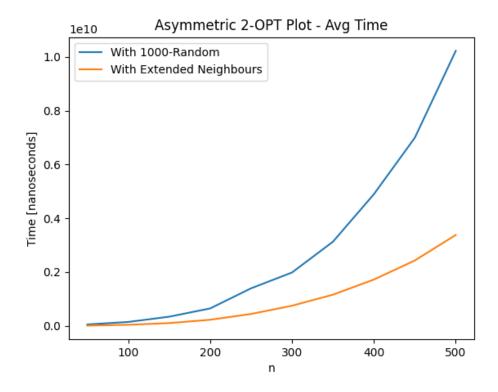
| Symmetric Graphs with 2-OPT | | | | | | | |
|-----------------------------|-------------|----------------|---------------|----------------|--------------------|-------------------|--|
| | AVG Time | | MAX Time | | AVG PRD | | |
| n | 1000-Random | Ext-Neighbours | 1000-Random | Ext-Neighbours | 1000-Random | Ext-Neighbours | |
| 50 | 54974700 | 8059300 | 4912670000 | 742475000 | 435.01603975448376 | 0 | |
| 100 | 263593200 | 79644600 | 24933430000 | 6215445000 | 296.5871346219297 | 76.26814793409217 | |
| 150 | 854941600 | 252056300 | 79812985000 | 21103678333 | 562.0643403667292 | 0 | |
| 200 | 1974940700 | 463551300 | 190022033333 | 43272870000 | 386.77781639316737 | 0 | |
| 250 | 3901226900 | 977426600 | 381697053333 | 91518833333 | 435.57918990772185 | 0 | |
| 300 | 6813259300 | 1703896100 | 653903618333 | 152749565000 | 527.1451034832014 | 0 | |
| 350 | 10818486000 | 2609217900 | 1042695165000 | 247398490000 | 371.06322593105233 | 0 | |
| 400 | 15996929900 | 4046430100 | 1577727018333 | 354304303333 | 351.7245426245308 | 0 | |
| 450 | 22753961100 | 5739451900 | 2246563521666 | 524977991666 | 510.1009193872297 | 0 | |
| 500 | 31306288200 | 7293158000 | 3088863130000 | 682899385000 | 423.6764076969968 | 0 | |

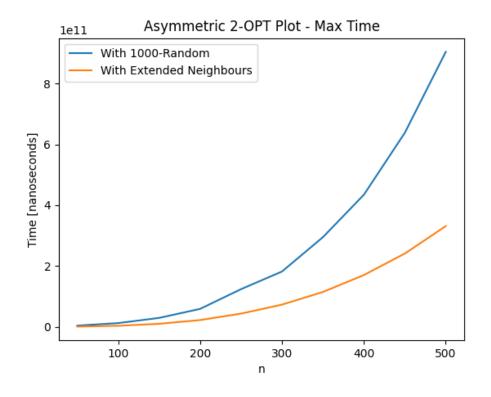
4.1.3 Grafy euklidesowe

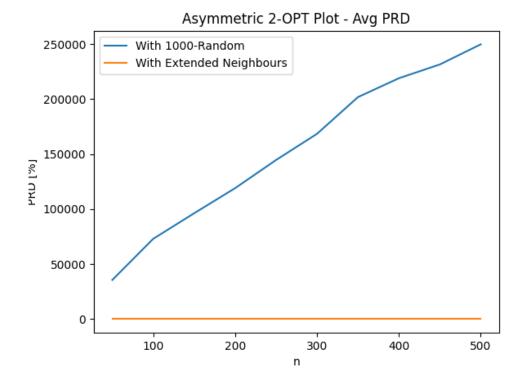
| Euclid Graphs with 2-OPT | | | | | | | |
|--------------------------|-------------|----------------|---------------|----------------|--------------------|----------------|--|
| | AVG Time | | MAX Time | | AVG PRD | | |
| n | 1000-Random | Ext-Neighbours | 1000-Random | Ext-Neighbours | 1000-Random | Ext-Neighbours | |
| 50 | 50845700 | 7325400 | 4806420000 | 695278000 | 2596.308721025397 | 0 | |
| 100 | 236384300 | 50275900 | 22503444000 | 4360152000 | 4683.4335972388535 | 0 | |
| 150 | 748642900 | 159204100 | 71871182000 | 15081932000 | 4791.206250105386 | 0 | |
| 200 | 1752181800 | 360983300 | 173144656000 | 33194196000 | 5703.476754080477 | 0 | |
| 250 | 3597519600 | 719419300 | 336032994000 | 66473034000 | 6640.879179142622 | 0 | |
| 300 | 6177012400 | 1247142900 | 584690090000 | 115264876000 | 8480.93216294145 | 0 | |
| 350 | 9811912400 | 2034889600 | 950499916000 | 179479596000 | 8406.706667218925 | 0 | |
| 400 | 14885090600 | 2743647200 | 1429223796000 | 263122394000 | 9456.626662228227 | 0 | |
| 450 | 20679005000 | 3990645200 | 2027474658000 | 374963440000 | 10298.344279214956 | 0 | |
| 500 | 30105890700 | 5752855600 | 2857272732000 | 527189884000 | 10733.905445781715 | 0 | |

4.2 Wykresy

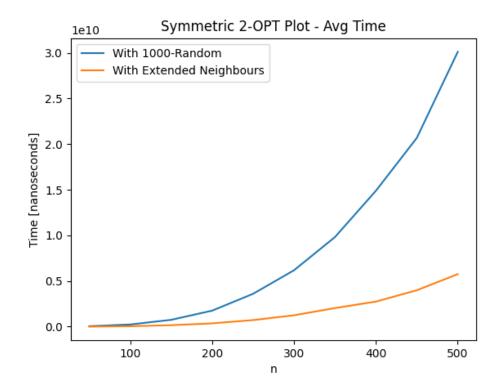
4.2.1 Grafy asymetryczne

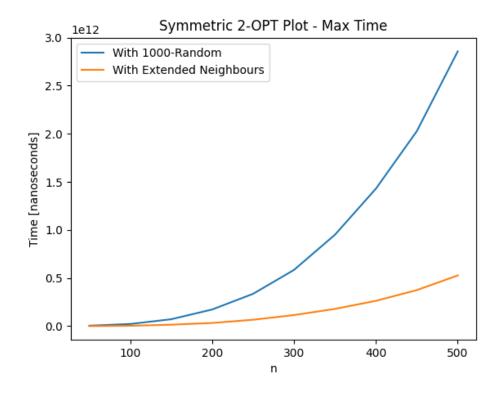


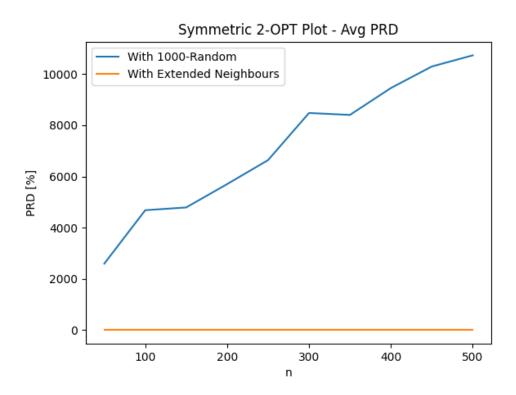


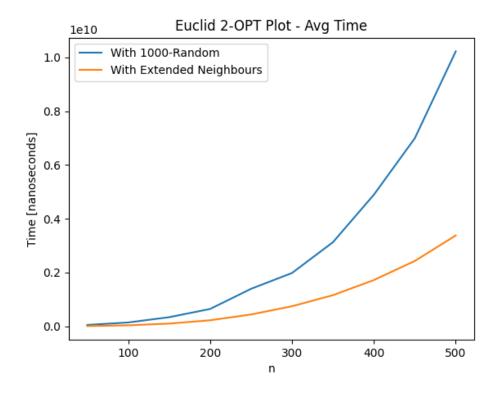


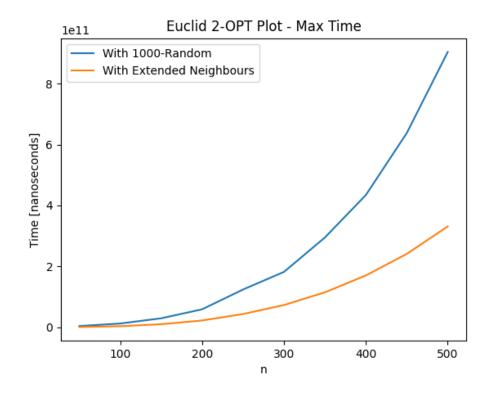
4.2.2 Grafy symetryczne

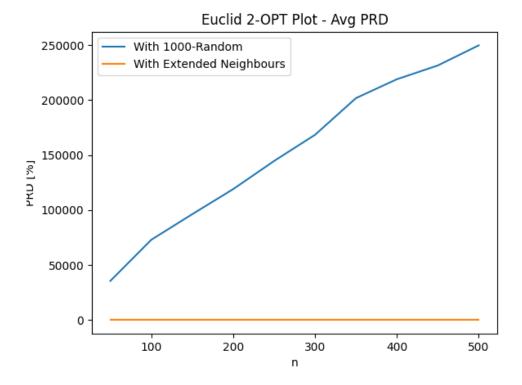












4.3 Wnioski