

Algorytmy metaheurystyczne 2

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1 Teoretyczna złożoność

Warunkiem wyjścia w naszym algorytmie było przekroczenie $15n$ iteracji, gdzie $n = |V|$ lub n ruchów bez zmiany na lepsze rozwiązanie. Rozważmy najgorszy możliwy przypadek, gdzie wykonujemy $15n$ iteracji. Niech k to długość listy tabu. Implementacja listy tabu za pomocą VecDeq pozwala na dostęp do i -tego elementu w czasie stałym, a usuwanie i dodawanie elementów w czasie liniowym.

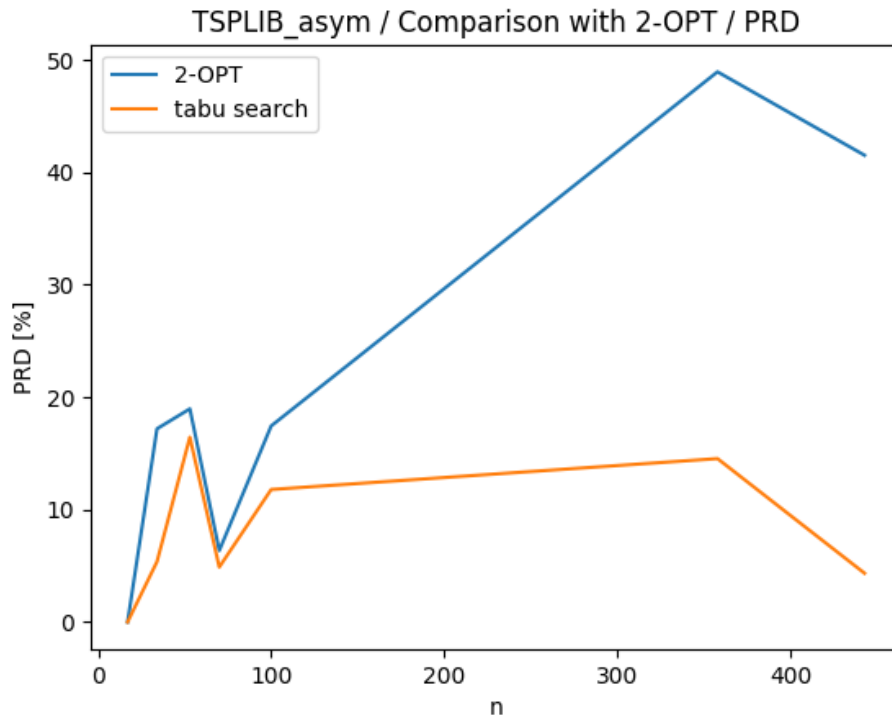
W każdym kroku algorytmu przeglądamy wszystkich $\frac{n(n-1)}{2}$ sąsiadów danego rozwiązania i dla każdego sprawdzamy z $O(k)$ czy jest na liście tabu. Sprawdzenie o ile sąsiad zmienia wartość permutacji jest stały (dla *invert* liczymy wcześniej z $O(n^2)$ pomocnicze tablice. Niech $O(l)$ to złożoność przybliżenia początkowego (dla *2-opt* $O(n^3)$). Wybieramy najlepszego z nich. Ostatecznie mamy (dla k stałego i 2-opta):

$$O\left(15n \cdot \frac{n(n-1)}{2} \cdot k + l\right) = O(n^3)$$

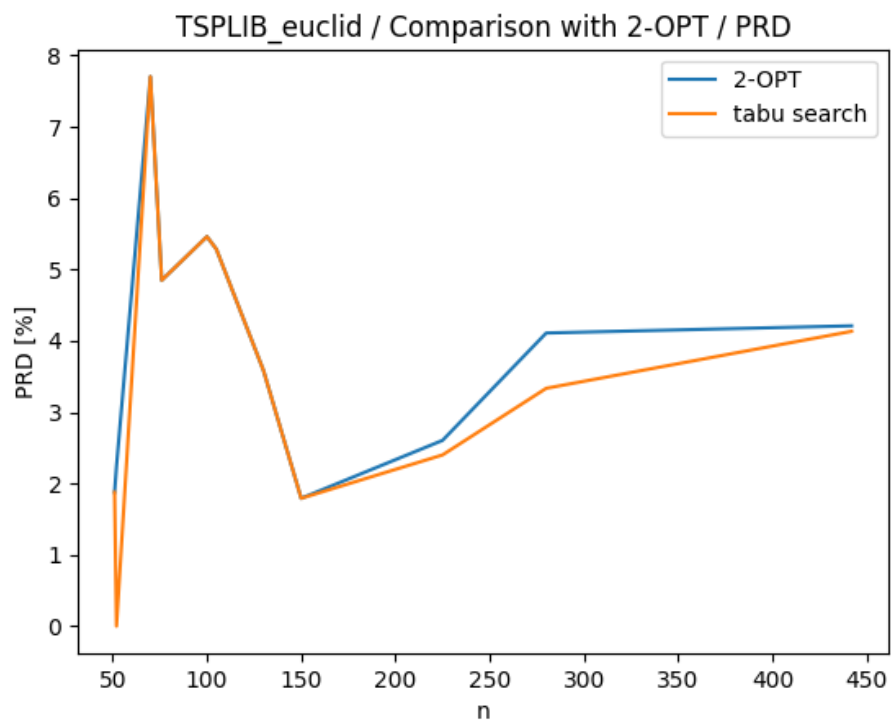
2 Porównanie Tabu Search z 2-OPT

2.1 Dane z TSPLIB

2.1.1 Grafy asymetryczne

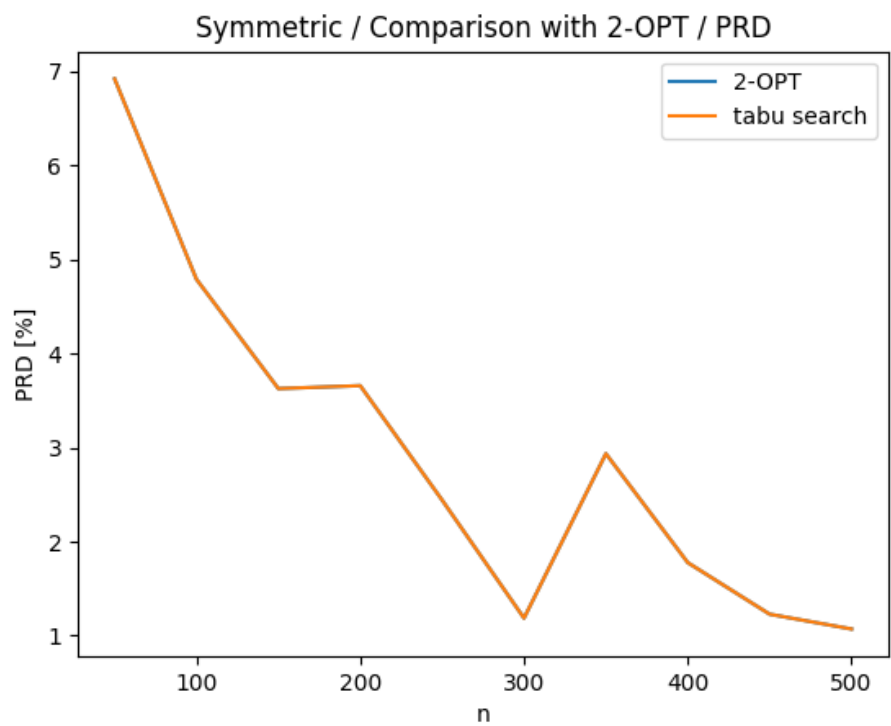


2.1.2 Grafy euklidesowe

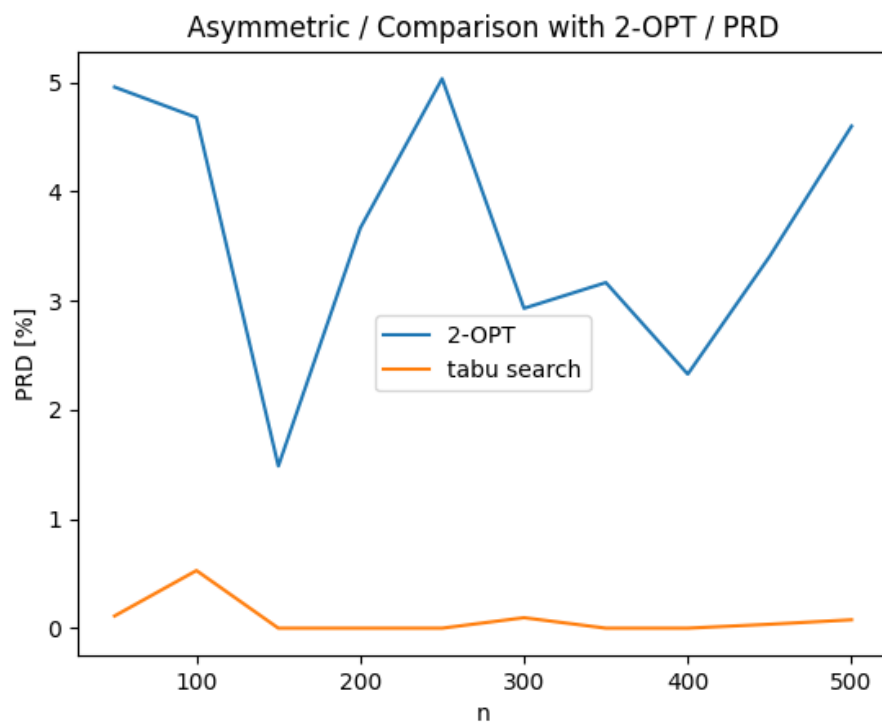


2.2 Dane generowane przez nas

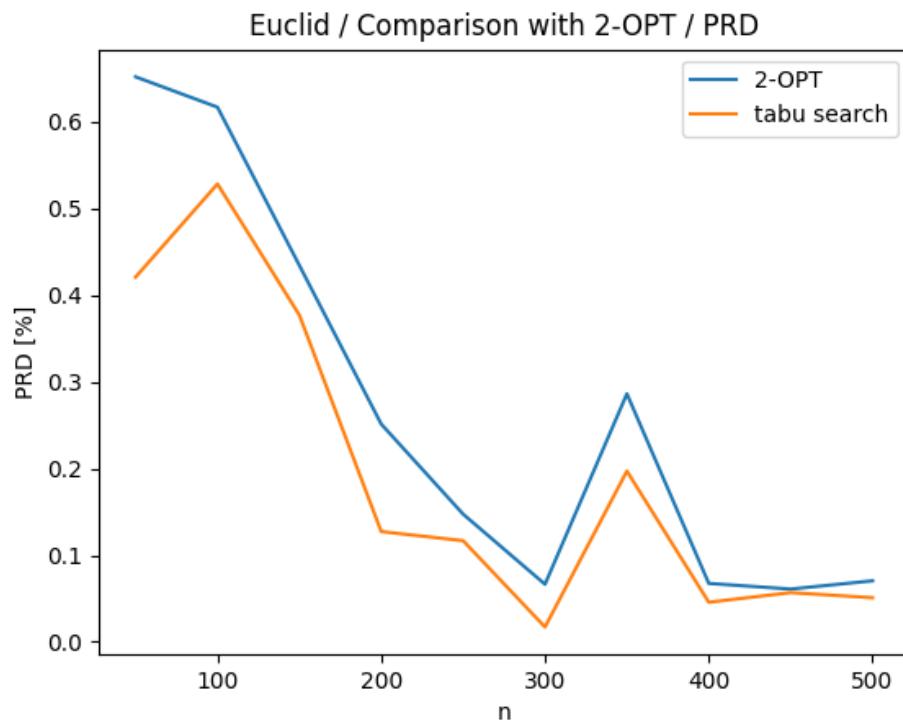
2.2.1 Grafy symetryczne



2.2.2 Grafy asymetryczne



2.2.3 Grafy euklidesowe



2.3 Obserwacje

2.4 Tabele

Asymmetric / Comparison with 2-OPT / PRD		
n	2-OPT	Tabu Search
50	0.11191047162270185	0.11191047162270185
100	0.52807735217553	0.52807735217553
150	0.0	0.0
200	0.0	0.0
250	0.0	0.0
300	0.09599191647019198	0.09599191647019198
350	0.0	0.0
400	0.0	0.0
450	0.03581064508216826	0.03581064508216826
500	0.0766979513573519	0.0766979513573519

Euclid / Comparison with 2-OPT / PRD		
n	2-OPT	Tabu Search
50	0.4207549708890032	0.4207549708890032
100	0.5282007435256711	0.5282007435256711
150	0.3769823007501678	0.3769823007501678
200	0.1271843910979231	0.1271843910979231
250	0.11667189906458164	0.11667189906458164
300	0.017076672487704073	0.017076672487704073
350	0.19697835915506468	0.19697835915506468
400	0.04564711832989831	0.04564711832989831
450	0.056766273647667766	0.056766273647667766
500	0.050901971434817714	0.050901971434817714

Symmetric / Comparison with 2-OPT / PRD		
n	2-OPT	Tabu Search
50	6.917692987893891	6.917692987893891
100	4.787518110638111	4.787518110638111
150	3.6259066537639213	3.6259066537639213
200	3.65693454118884	3.65693454118884
250	2.4413015900221464	2.4413015900221464
300	1.1869594271627262	1.1869594271627262
350	2.9343241182614768	2.9343241182614768
400	1.7767045918185538	1.7767045918185538
450	1.2285915175711815	1.2285915175711815
500	1.0711060843000912	1.0711060843000912

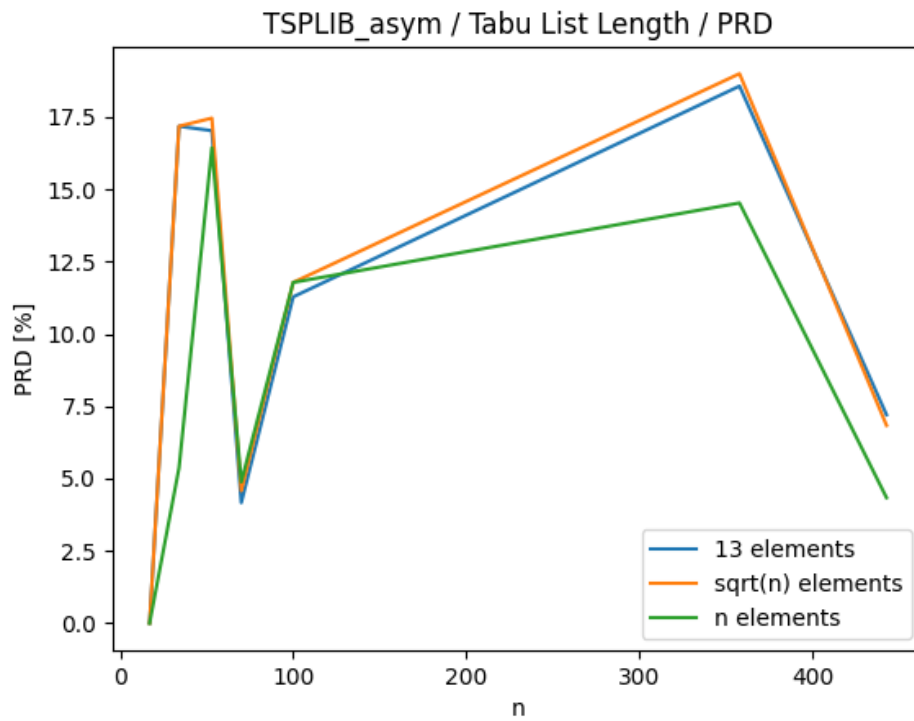
TSPLIB asym / Comparison with 2-OPT / PRD		
n	2-OPT	Tabu Search
17	0.0	0.0
34	5.365474339035769	5.365474339035769
53	16.437364228819696	16.437364228819696
70	4.881958989475862	4.881958989475862
100	11.783052718741374	11.783052718741374
358	14.531384350816854	14.531384350816854
443	4.338235294117647	4.338235294117647

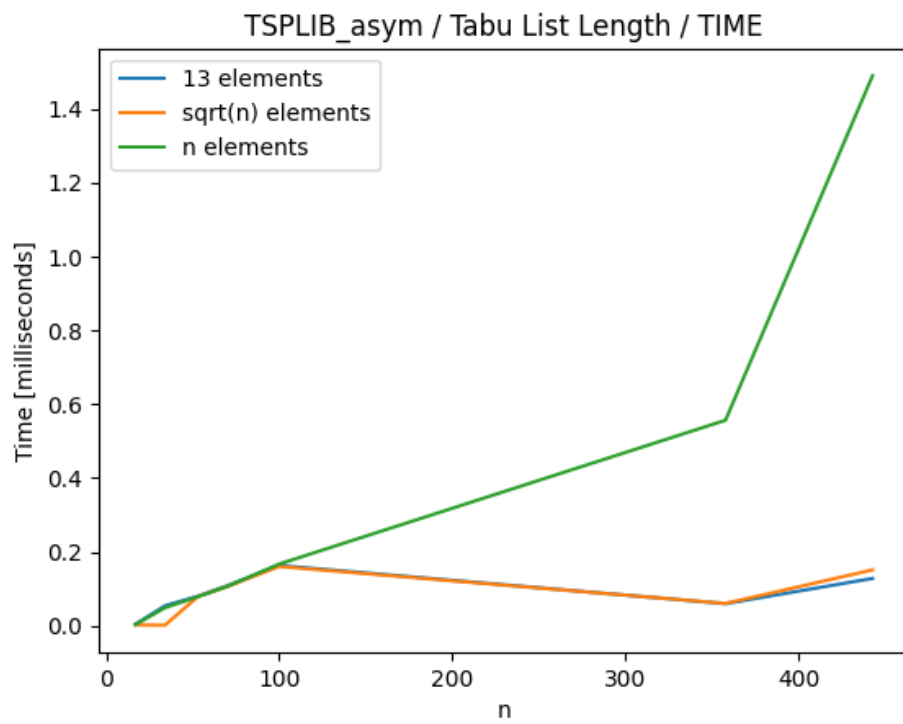
TSPLIB euclid / Comparison with 2-OPT / PRD		
n	2-OPT	Tabu Search
51	1.8779342723004695	1.8779342723004695
52	0.0	0.0
70	7.703703703703704	7.703703703703704
76	4.849342172172451	4.849342172172451
100	5.461441213653603	5.461441213653603
105	5.285485777870505	5.285485777870505
130	3.5842880523731586	3.5842880523731586
150	1.7922794117647058	1.7922794117647058
225	2.400408580183861	2.400408580183861
280	3.3346258239627766	3.3346258239627766
442	4.133679940131553	4.133679940131553

3 Porównanie różnych długości listy tabu

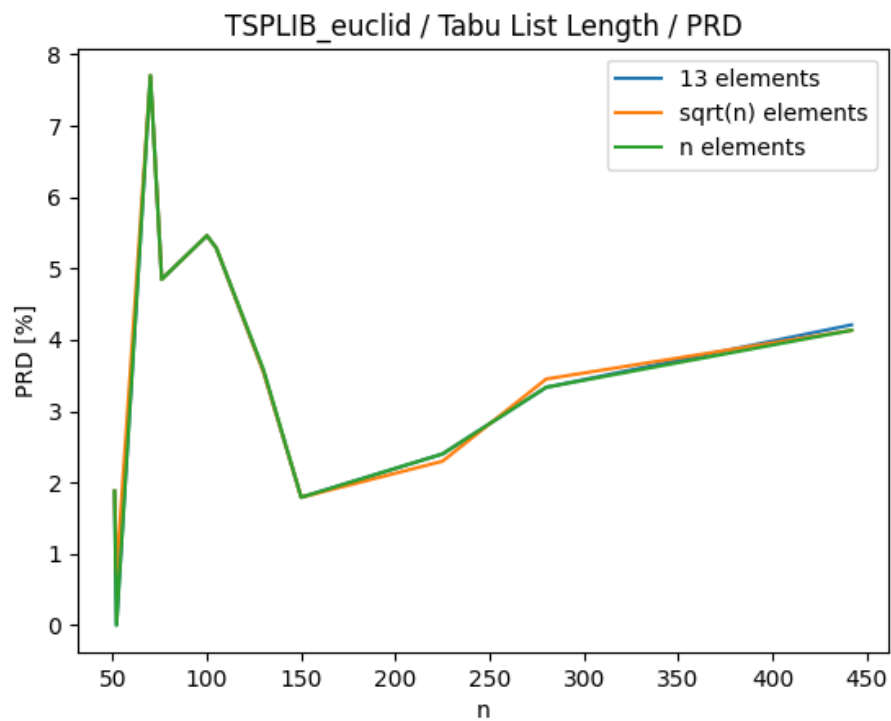
3.1 Dane z TSPLIB

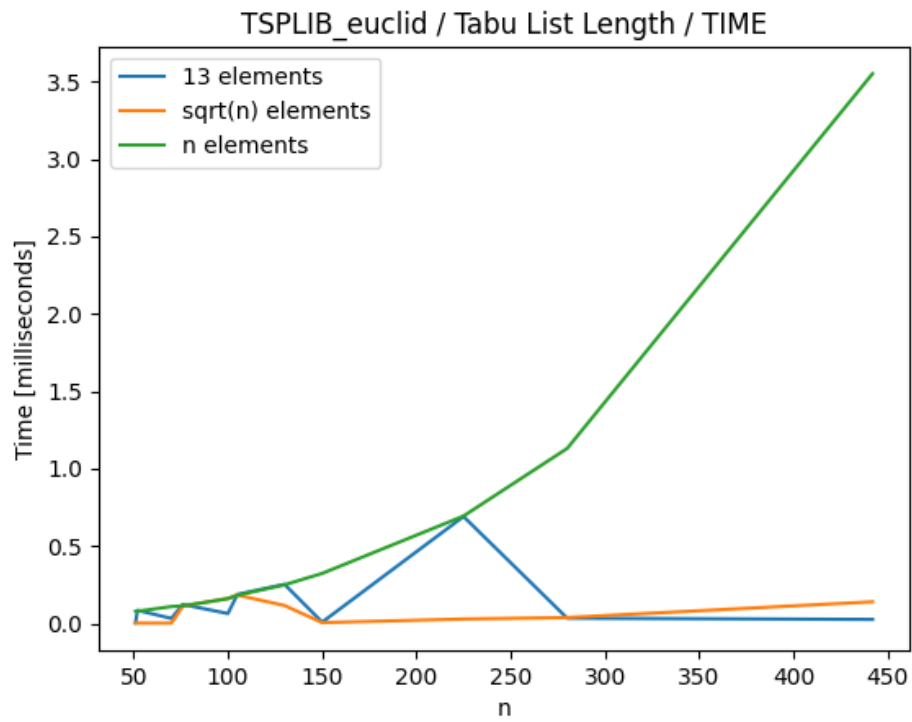
3.1.1 Grafy asymetryczne





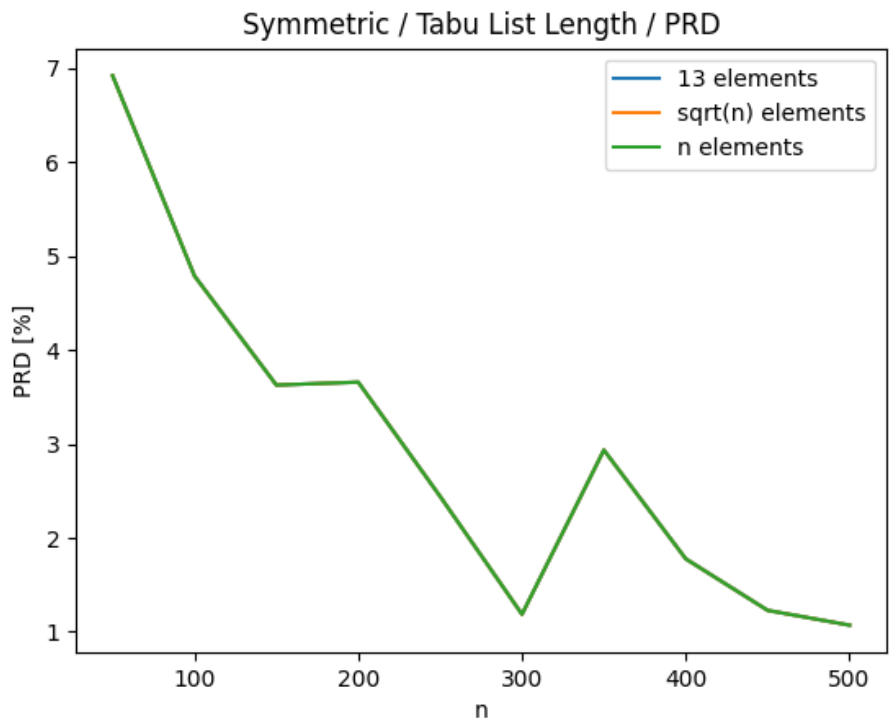
3.1.2 Grafy euklidesowe

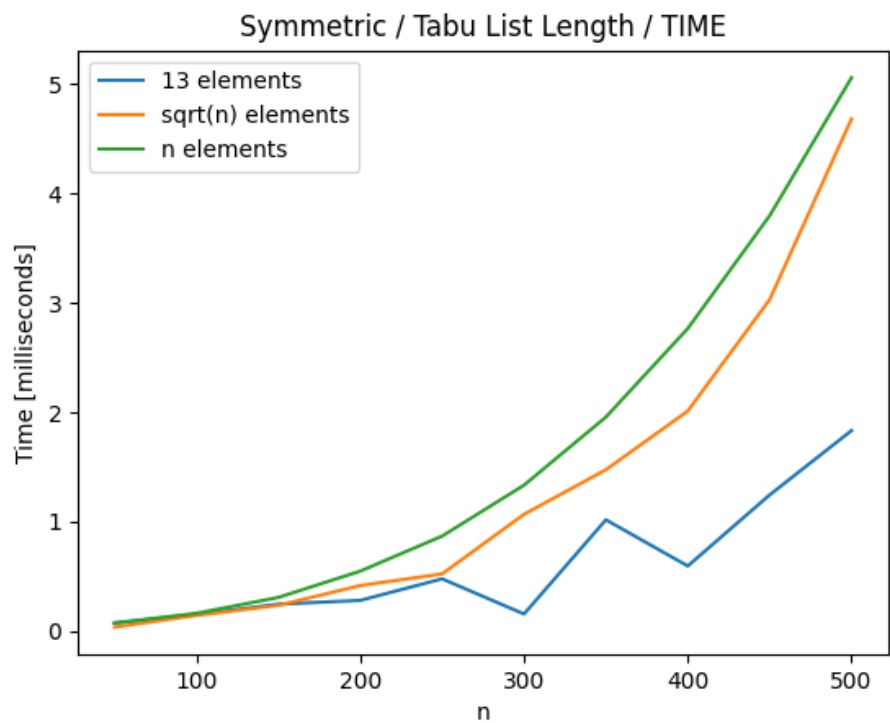




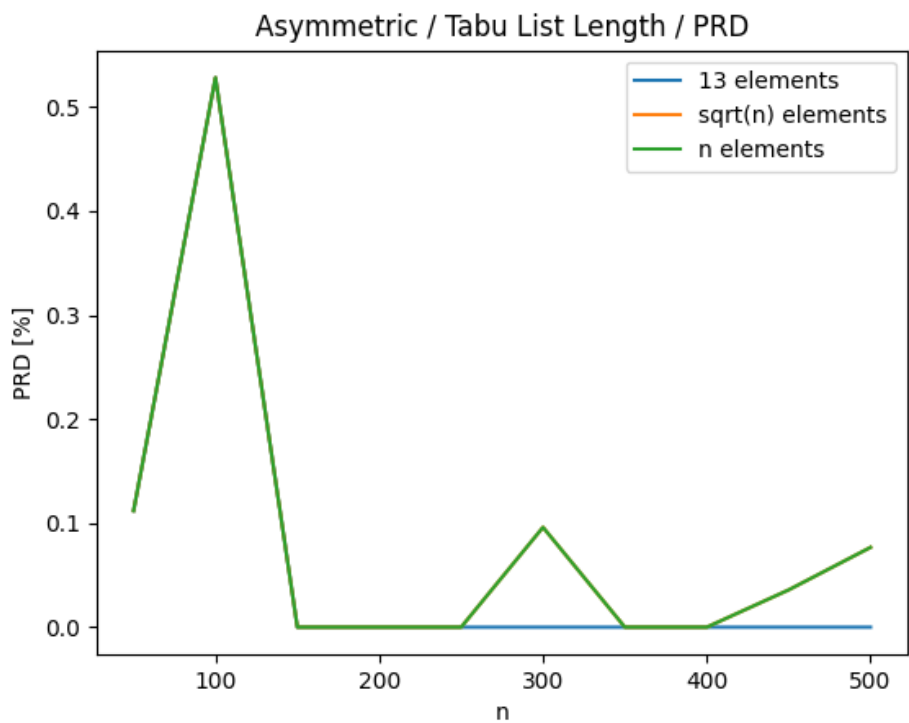
3.2 Dane generowane przez nas

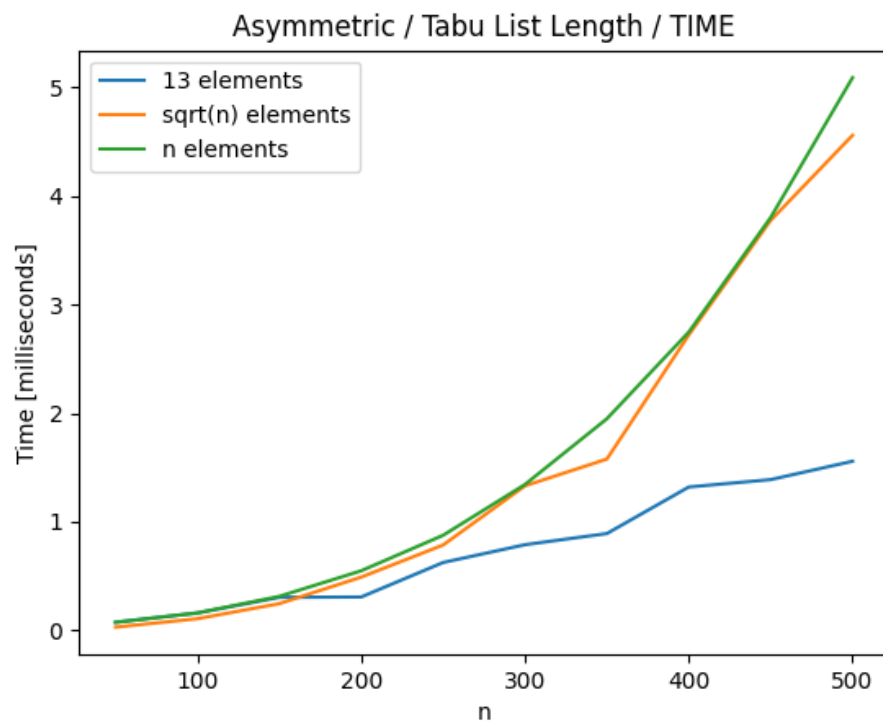
3.2.1 Grafy symetryczne



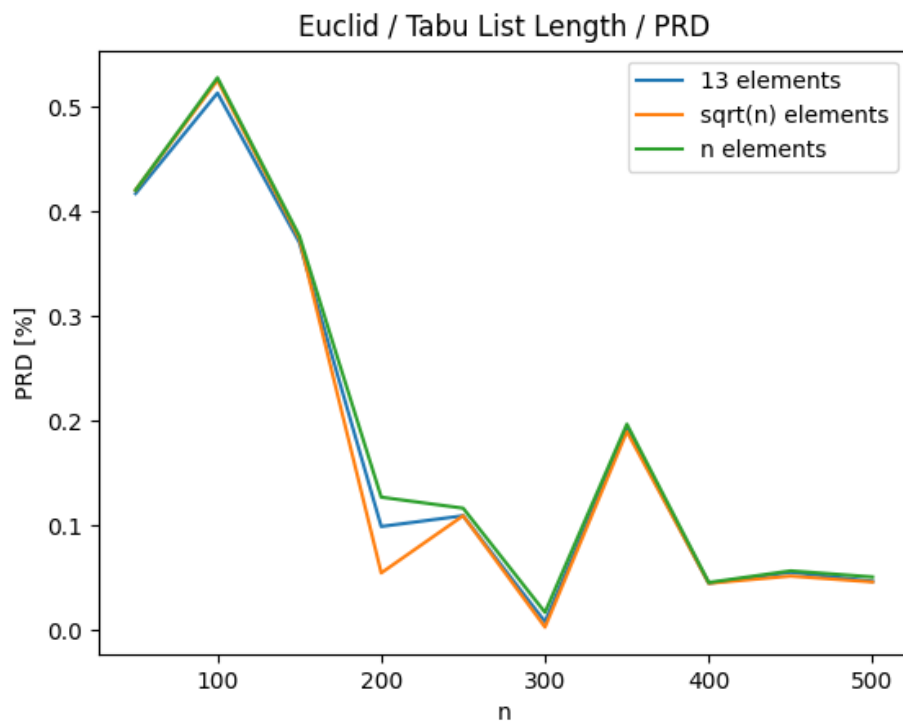


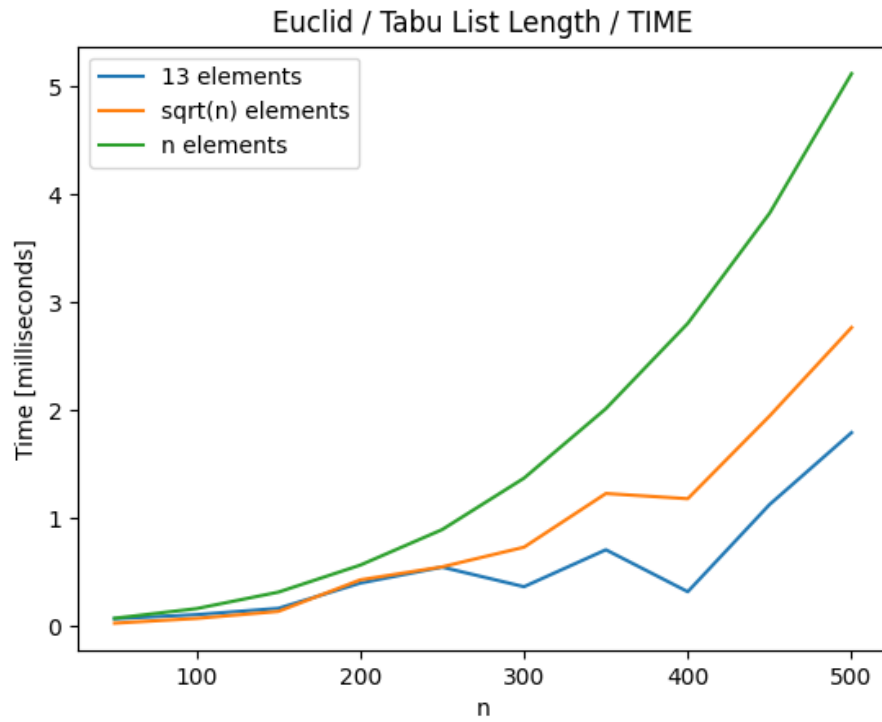
3.2.2 Grafy asymetryczne





3.2.3 Grafy euklidesowe





3.3 Obserwacje

3.4 Tabele

Asymmetric / Tabu list length / PRD			
n	13	\sqrt{n}	n
50	0.11191047162270185	0.11191047162270185	0.11191047162270185
100	0.52807735217553	0.52807735217553	0.52807735217553
150	0.0	0.0	0.0
200	0.0	0.0	0.0
250	0.0	0.0	0.0
300	0.0	0.09599191647019198	0.09599191647019198
350	0.0	0.0	0.0
400	0.0	0.0	0.0
450	0.0	0.03581064508216826	0.03581064508216826
500	0.0	0.0766979513573519	0.0766979513573519

Asymmetric / Tabu list length / Time			
n	13	\sqrt{n}	n
50	0.07385802999999999	0.028744079999999995	0.07376404
100	0.15944619999999998	0.10658849	0.16007074999999998
150	0.3033876	0.24521492999999994	0.31249241000000005
200	0.30667507999999993	0.48976515000000004	0.54841901
250	0.62445054	0.7853915999999999	0.8751694499999999
300	0.78834145	1.3305461199999997	1.34324706
350	0.8908741500000001	1.57709417	1.9484998399999998
400	1.31986308	2.72034879	2.74655025
450	1.38779908	3.7751664	3.8018252900000001
500	1.5568623	4.5591865	5.08990898

Euclid / Tabu list length / PRD			
n	13	\sqrt{n}	n
50	0.41747089863941367	0.4207549708890032	0.4207549708890032
100	0.513520849011387	0.5257261085343324	0.5282007435256711
150	0.3706918377833429	0.3742384401382869	0.3769823007501678
200	0.09916957017990424	0.05476201991278836	0.1271843910979231
250	0.10943361971724283	0.10943361971724283	0.11667189906458164
300	0.008143056907276255	0.002919636991800686	0.017076672487704073
350	0.19258345724128464	0.18991027309708547	0.19697835915506468
400	0.04483850563308839	0.04483850563308839	0.04564711832989831
450	0.055414537386798336	0.05175307680208402	0.056766273647667766
500	0.04684067228013384	0.04612221554882399	0.050901971434817714

Euclid / Tabu list length / Time			
n	13	\sqrt{n}	n
50	0.06892951	0.02786071	0.07344694
100	0.10688875	0.07182816	0.16274169
150	0.16533177	0.13686594	0.3153742
200	0.39902903	0.4290532799999999	0.5656857900000001
250	0.5470563900000001	0.5496629700000001	0.89462399
300	0.3658269	0.7317911899999998	1.37161821
350	0.70801814	1.2293225500000002	2.0154079099999995
400	0.31940638	1.1810165700000002	2.8054042600000004
450	1.1279154999999998	1.94813256	3.8271469
500	1.79325678	2.76704791	5.11919333

Symmetric / Tabu list length / PRD			
n	13	\sqrt{n}	n
50	6.917692987893891	6.917692987893891	6.917692987893891
100	4.787518110638111	4.787518110638111	4.787518110638111
150	3.6259066537639213	3.6259066537639213	3.6259066537639213
200	3.65693454118884	3.65693454118884	3.65693454118884
250	2.4413015900221464	2.4413015900221464	2.4413015900221464
300	1.1869594271627262	1.1869594271627262	1.1869594271627262
350	2.9343241182614768	2.9343241182614768	2.9343241182614768
400	1.7767045918185538	1.7767045918185538	1.7767045918185538
450	1.2285915175711815	1.2285915175711815	1.2285915175711815
500	1.0711060843000912	1.0711060843000912	1.0711060843000912

Symmetric / Tabu list length / Time			
n	13	\sqrt{n}	n
50	0.07311498	0.03475836	0.07261012
100	0.15076814	0.14221727999999997	0.16149818
150	0.24539080000000002	0.23409864999999996	0.30667559000000005
200	0.27950255	0.4152889	0.54723579
250	0.4766900100000001	0.5219849499999999	0.8675596299999999
300	0.15507567	1.06678467	1.33364641
350	1.0168580200000001	1.4747576800000002	1.95661837
400	0.59430354	2.01161023	2.7663511699999996
450	1.2444358499999997	3.0308625499999997	3.79851777
500	1.8332309300000003	4.68484663	5.06103543

TSPLIB asym / Tabu list length / PRD			
n	13	\sqrt{n}	n
17	0.0	0.0	0.0
34	17.1850699844479	17.1850699844479	5.365474339035769
53	17.031136857349747	17.465604634322954	16.437364228819696
70	4.165696997905515	4.57683655263362	4.881958989475862
100	11.283466740270494	11.783052718741374	11.783052718741374
358	18.572656921754085	19.002579535683576	14.531384350816854
443	7.205882352941176	6.838235294117648	4.338235294117647

TSPLIB asym / Tabu list length / Time			
n	13	\sqrt{n}	n
17	0.0043833	0.0021721	0.0034579
34	0.0543166	0.0016518	0.0488397
53	0.0799956	0.0790441	0.0787211
70	0.10852	0.10629	0.1066343
100	0.1633046	0.1606365	0.1671848
358	0.0598049	0.060473	0.556717
443	0.128276	0.1514229	1.4899777

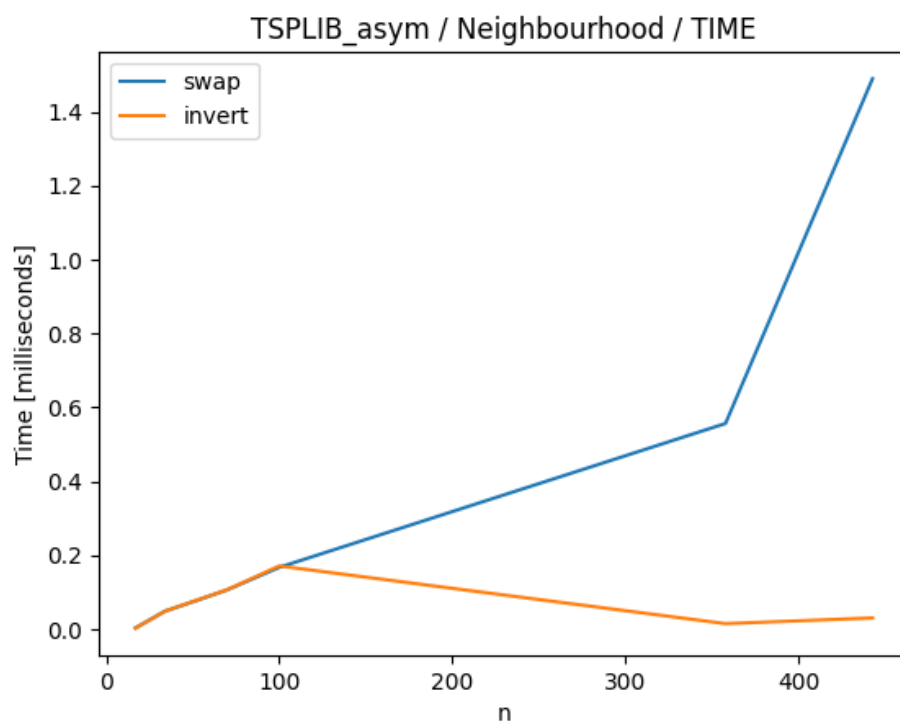
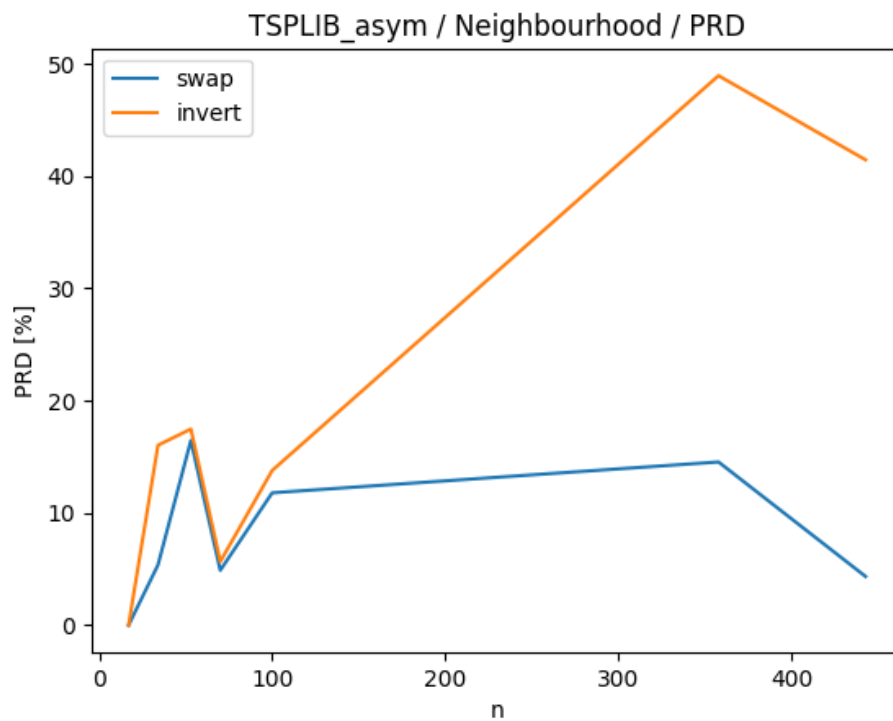
TSPLIB euclid / Tabu list length / PRD			
n	13	\sqrt{n}	n
51	1.8779342723004695	1.8779342723004695	1.8779342723004695
52	0.0	0.7159904534606205	0.0
70	7.703703703703704	7.703703703703704	7.703703703703704
76	4.849342172172451	4.849342172172451	4.849342172172451
100	5.461441213653603	5.461441213653603	5.461441213653603
105	5.285485777870505	5.285485777870505	5.285485777870505
130	3.551554828150573	3.551554828150573	3.5842880523731586
150	1.7922794117647058	1.7922794117647058	1.7922794117647058
225	2.400408580183861	2.29826353421859	2.400408580183861
280	3.3346258239627766	3.450949980612641	3.3346258239627766
442	4.210484855646146	4.133679940131553	4.133679940131553

TSPLIB euclid / Tabu list length / Time			
n	13	\sqrt{n}	n
51	0.0052827	0.0024253	0.0792794
52	0.0851107	0.002831	0.0793002
70	0.0332155	0.0026489	0.1098126
76	0.1232844	0.1115039	0.113911
100	0.0646367	0.1623564	0.1608226
105	0.1881329	0.1837667	0.1828318
130	0.2533528	0.11668	0.2498604
150	0.009127	0.0059214	0.3236019
225	0.6909736	0.0292159	0.6941704
280	0.0345763	0.0388206	1.1307793
442	0.0266773	0.1404226	3.5502116

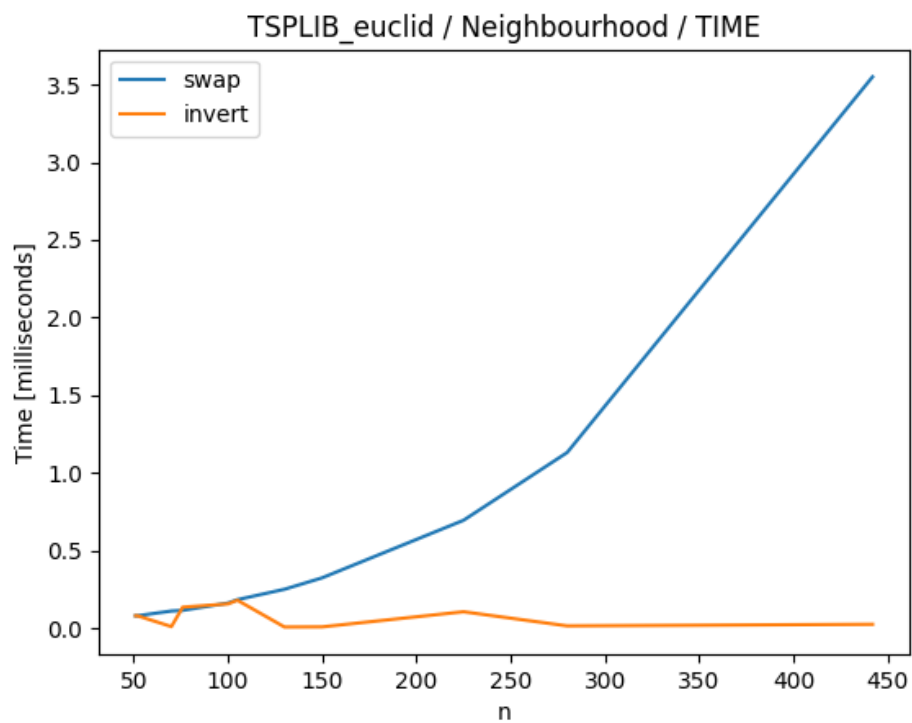
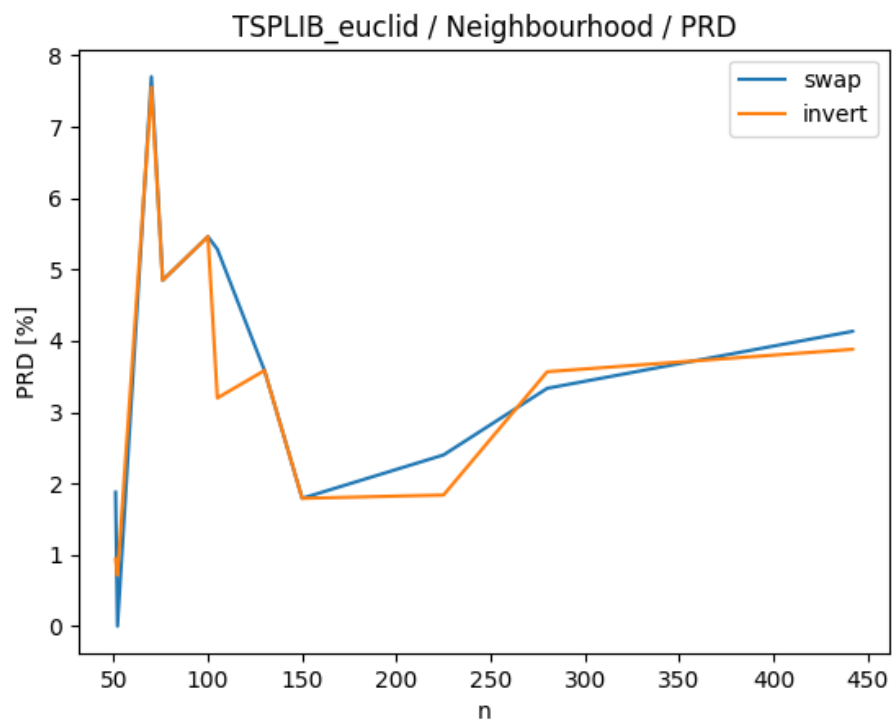
4 Porównanie sąsiedztwa insert i swap

4.1 Dane z TSPLIB

4.1.1 Grafy asymetryczne

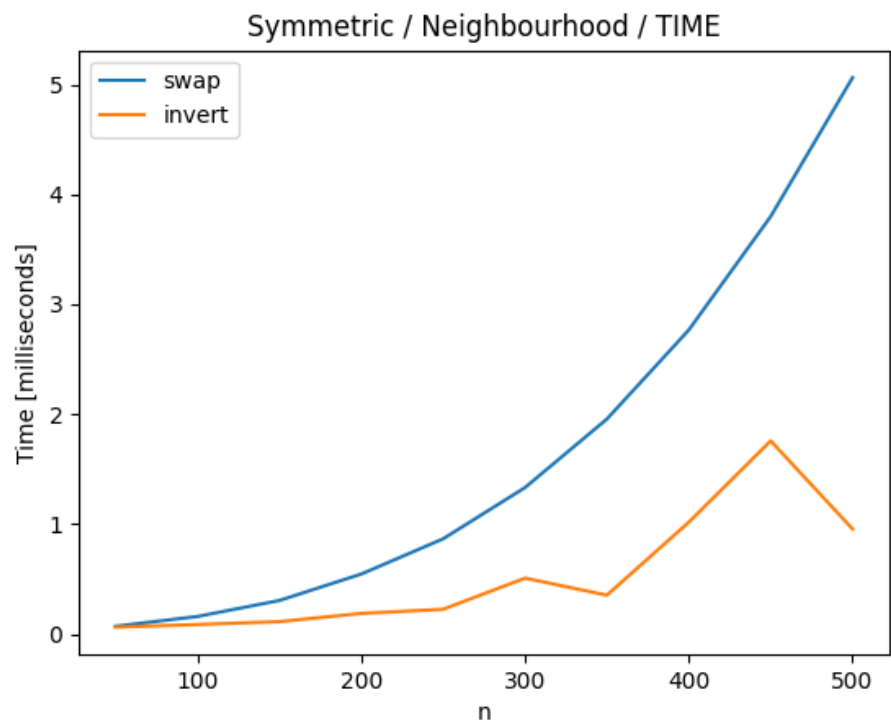
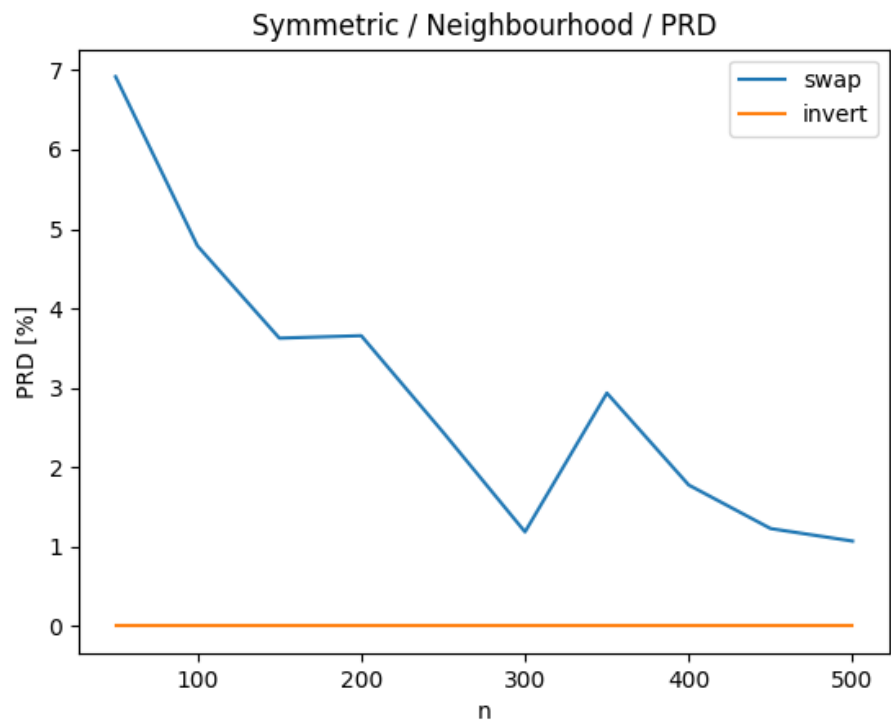


4.1.2 Grafy euklidesowe

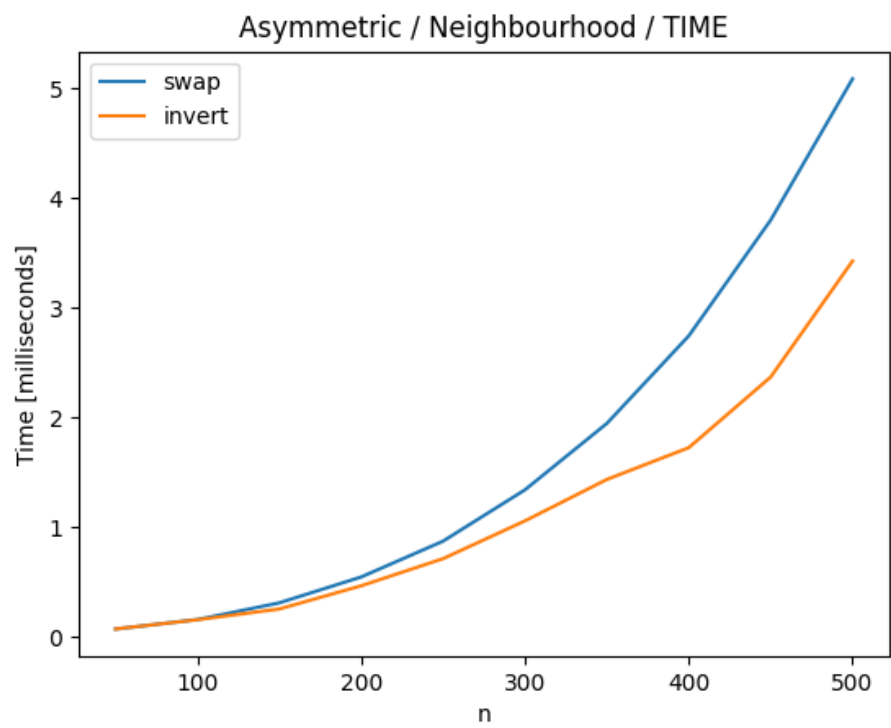
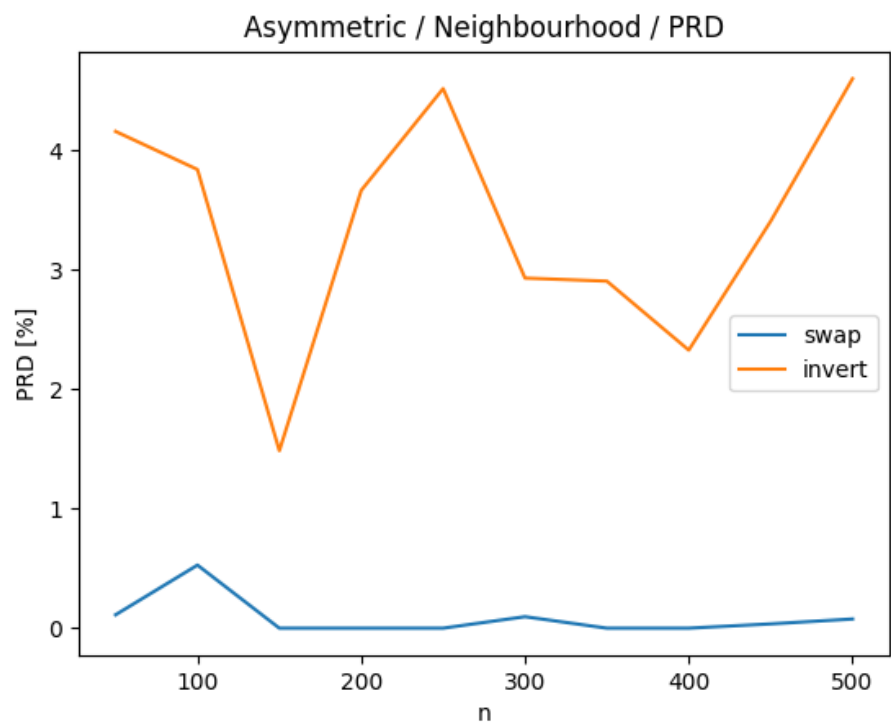


4.2 Dane generowane przez nas

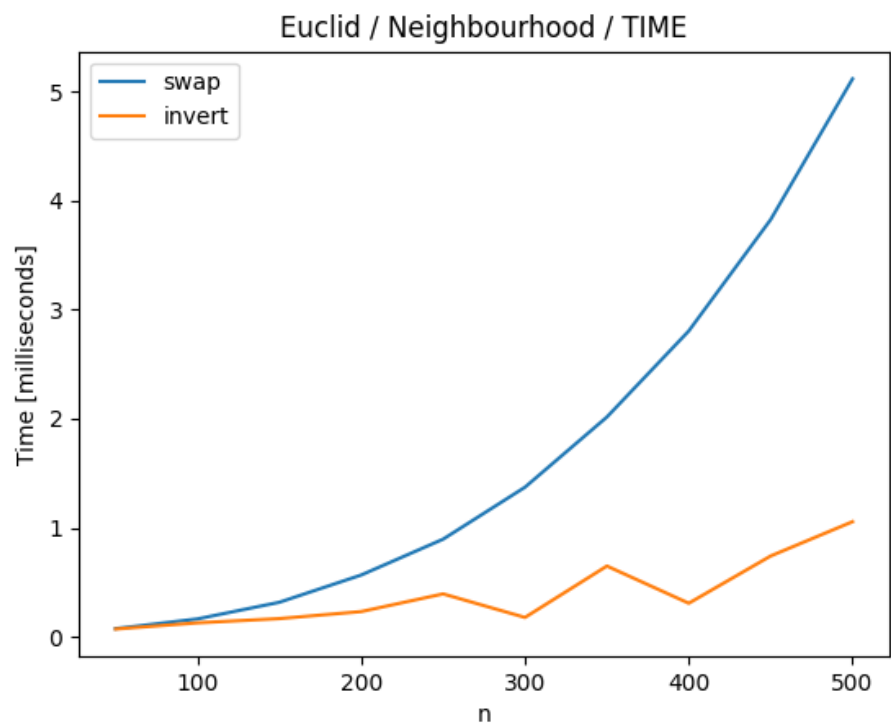
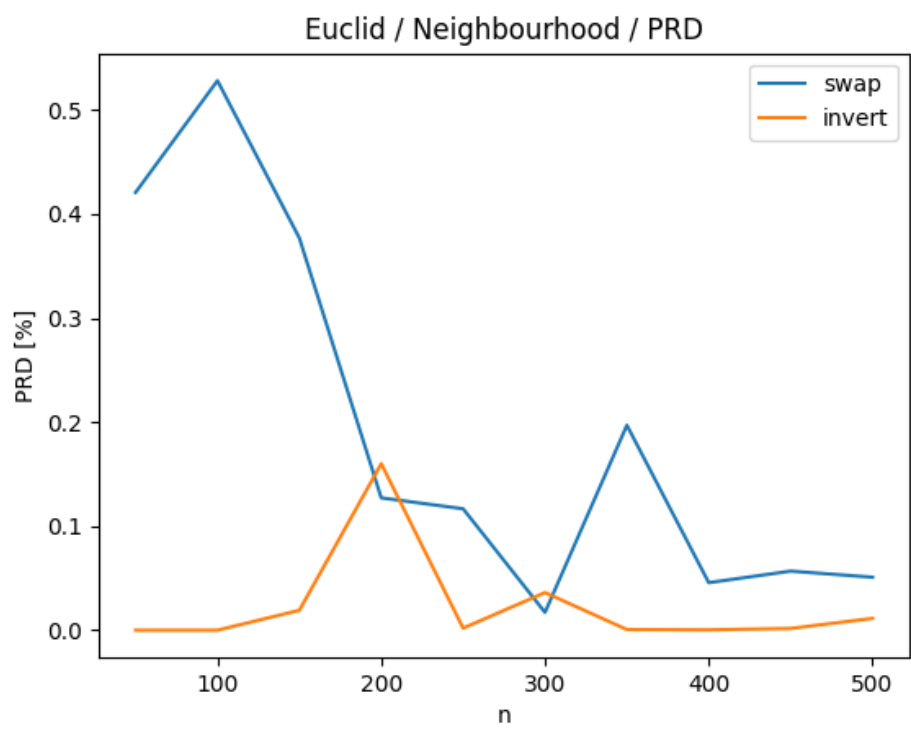
4.2.1 Grafy symetryczne



4.2.2 Grafy asymetryczne



4.2.3 Grafy euklidesowe



4.3 Obserwacje

4.4 Tabele

Asymmetric / Neighbourhood / PRD		
n	Swap	Invert
50	0.11191047162270185	4.158695674522066
100	0.52807735217553	3.8398777948595333
150	0.0	1.4858723898734574
200	0.0	3.6649730337912074
250	0.0	4.5168141129407395
300	0.09599191647019198	2.930695727694611
350	0.0	2.9052223793423044
400	0.0	2.32776759746732
450	0.03581064508216826	3.4065936038225146
500	0.0766979513573519	4.6002407510123104

Asymmetric / Neighbourhood / Time		
n	Swap	Invert
50	0.07376404	0.07431734000000001
100	0.16007074999999998	0.1585202
150	0.31249241000000005	0.25652474
200	0.54841901	0.46782404
250	0.8751694499999999	0.71564498000000001
300	1.34324706	1.05975427
350	1.9484998399999998	1.4376728399999998
400	2.74655025	1.72702253
450	3.8018252900000001	2.37098031
500	5.08990898	3.4289974899999995

Euclid / Neighbourhood / PRD		
n	Swap	Invert
50	0.4207549708890032	0.0
100	0.5282007435256711	0.0
150	0.3769823007501678	0.019084422822491107
200	0.1271843910979231	0.1599885829684126
250	0.11667189906458164	0.001928268414963363
300	0.017076672487704073	0.03614358129615766
350	0.19697835915506468	0.0005993048064245475
400	0.04564711832989831	0.0003221130616846514
450	0.056766273647667766	0.0016375098932889388
500	0.050901971434817714	0.011337910696104788

Euclid / Neighbourhood / Time		
n	Swap	Invert
50	0.4207549708890032	0.0
50	0.07344694	0.07000031999999999
100	0.16274169	0.12673666
150	0.3153742	0.16559282
200	0.5656857900000001	0.23007239999999998
250	0.89462399	0.39235521999999995
300	1.37161821	0.17630289
350	2.0154079099999995	0.64804025
400	2.8054042600000004	0.30653431
450	3.8271469	0.74014913
500	5.11919333	1.0551518899999999

Symmetric / Neighbourhood / PRD		
n	Swap	Invert
50	6.917692987893891	0.0
100	4.787518110638111	0.0
150	3.6259066537639213	0.0
200	3.65693454118884	0.0
250	2.4413015900221464	0.0
300	1.1869594271627262	0.0
350	2.9343241182614768	0.0
400	1.7767045918185538	0.0
450	1.2285915175711815	0.0
500	1.0711060843000912	0.0

Symmetric / Neighbourhood / Time		
n	Swap	Invert
50	0.07261012	0.06425642
100	0.16149818	0.08848346
150	0.30667559000000005	0.11450532
200	0.54723579	0.18917302
250	0.8675596299999999	0.22676869000000002
300	1.33364641	0.51011874
350	1.95661837	0.35503858
400	2.7663511699999996	1.01861845
450	3.79851777	1.75846868
500	5.06103543	0.95719657

TSPLIB asym / Neighbourhood / PRD		
n	Swap	Invert
17	0.0	0.0
34	5.365474339035769	16.018662519440124
53	16.437364228819696	17.465604634322954
70	4.881958989475862	5.662865565122955
100	11.783052718741374	13.773116202042507
358	14.531384350816854	48.92519346517627
443	4.338235294117647	41.43382352941177

TSPLIB asym / Neighbourhood / Time		
n	Swap	Invert
17	0.0034579	0.0027282
34	0.0488397	0.0485128
53	0.0787211	0.0796033
70	0.1066343	0.1066752
100	0.1671848	0.1711747
358	0.556717	0.0149537
443	1.4899777	0.0304211

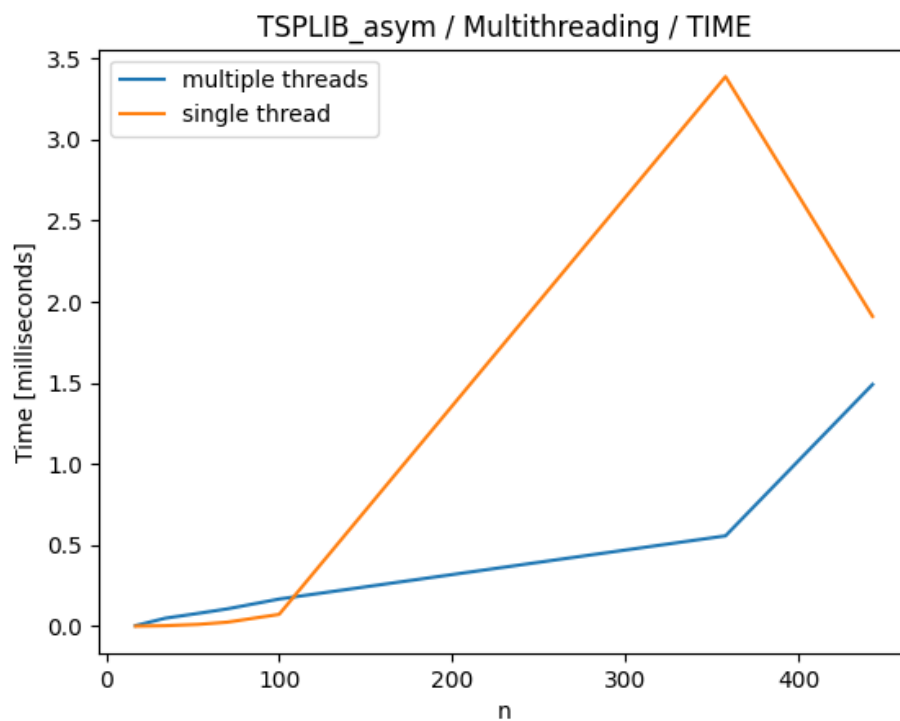
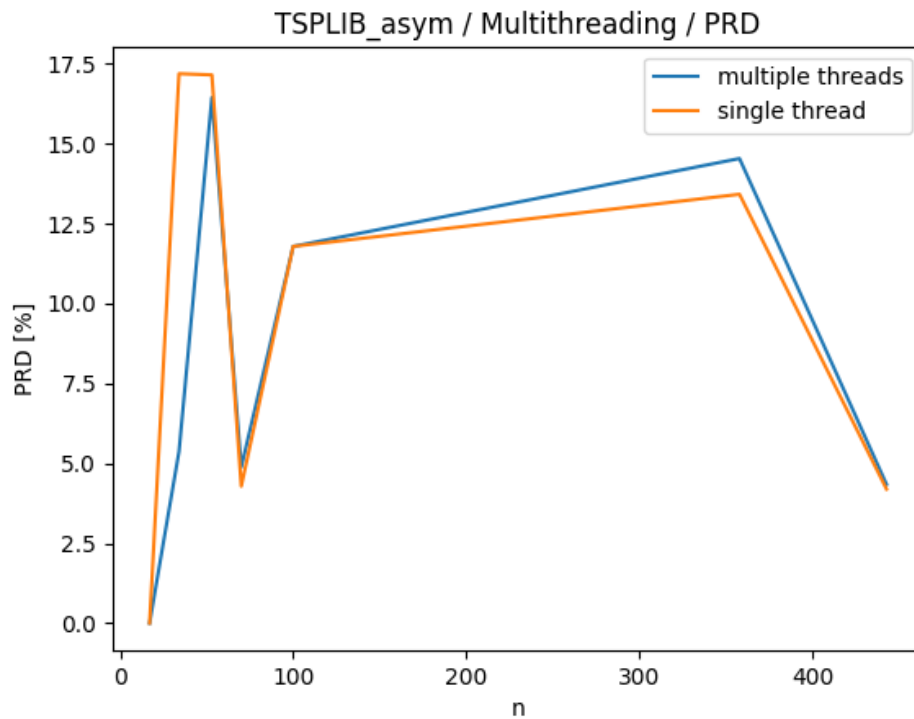
TSPLIB euclid / Neighbourhood / PRD		
n	Swap	Invert
51	1.8779342723004695	0.9389671361502347
52	0.0	0.7159904534606205
70	7.703703703703704	7.555555555555555
76	4.849342172172451	4.849342172172451
100	5.461441213653603	5.461441213653603
105	5.285485777870505	3.1991098129216216
130	3.5842880523731586	3.5842880523731586
150	1.7922794117647058	1.7922794117647058
225	2.400408580183861	1.8386108273748722
280	3.3346258239627766	3.5672741372625048
442	4.133679940131553	3.881602268699043

TSPLIB euclid / Neighbourhood / Time		
n	Swap	Invert
51	0.0792794	0.0792618
52	0.0793002	0.0792362
70	0.1098126	0.0094275
76	0.113911	0.1344349
100	0.1608226	0.155615
105	0.1828318	0.1793799
130	0.2498604	0.0066768
150	0.3236019	0.0076471
225	0.6941704	0.1050844
280	1.1307793	0.0139994
442	3.5502116	0.0242614

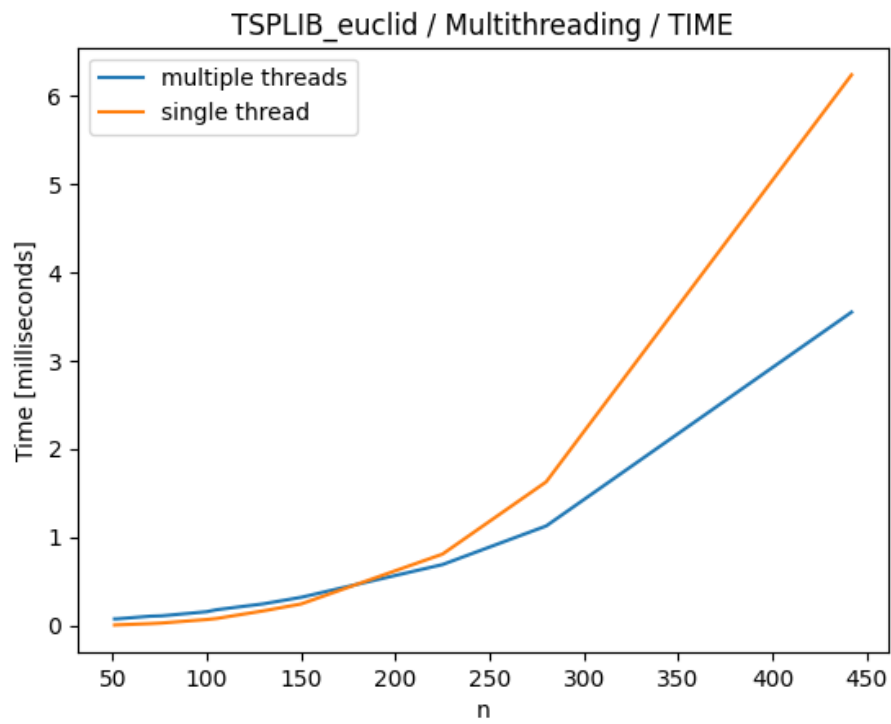
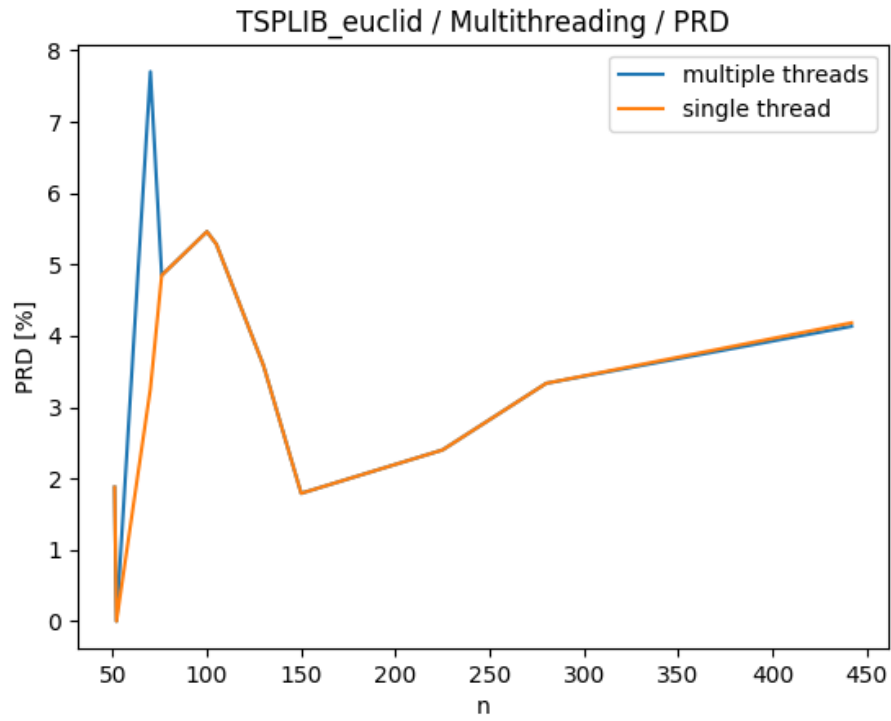
5 Porównanie wersji wielowątkowej i wersji jednowątkowej

5.1 Dane z TSPLIB

5.1.1 Grafy asymetryczne

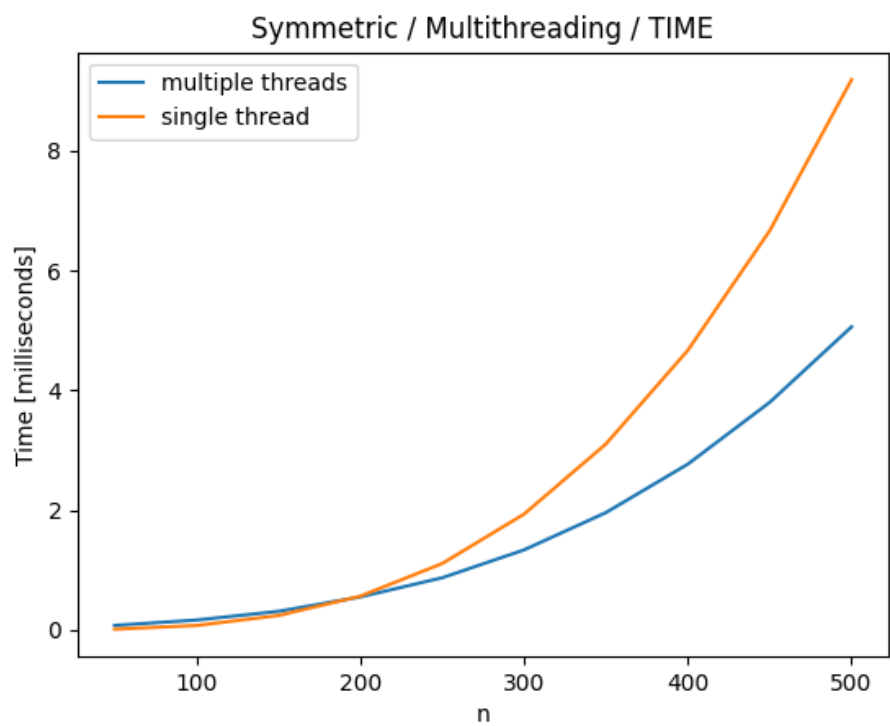
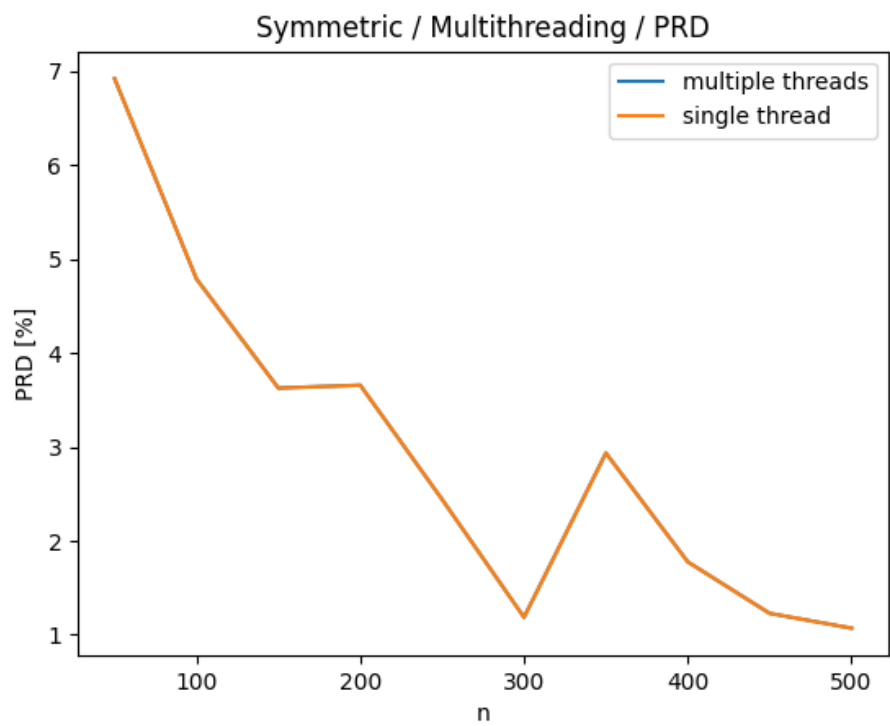


5.1.2 Grafy euklidesowe

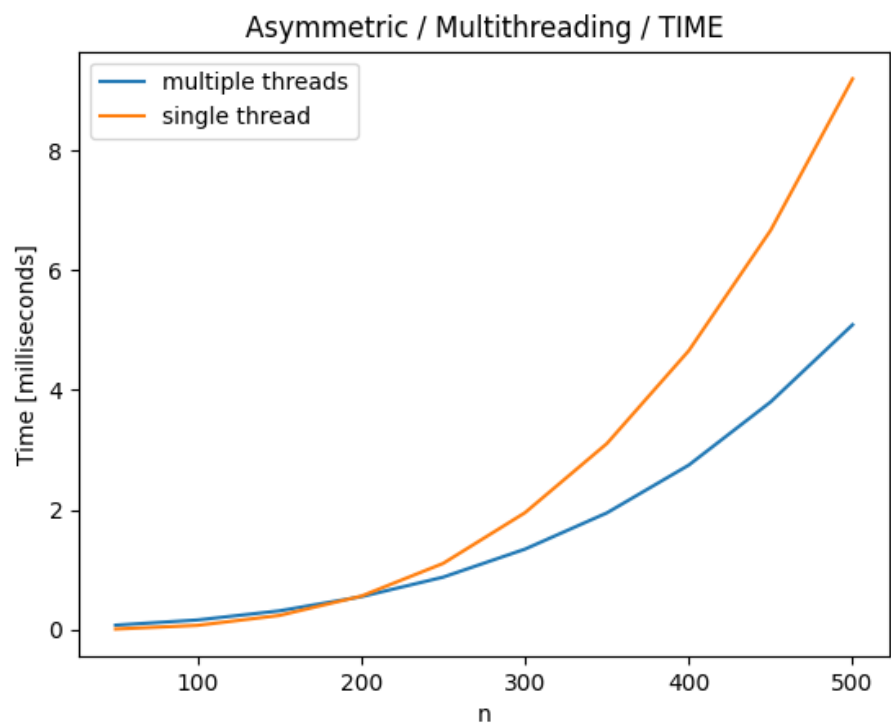
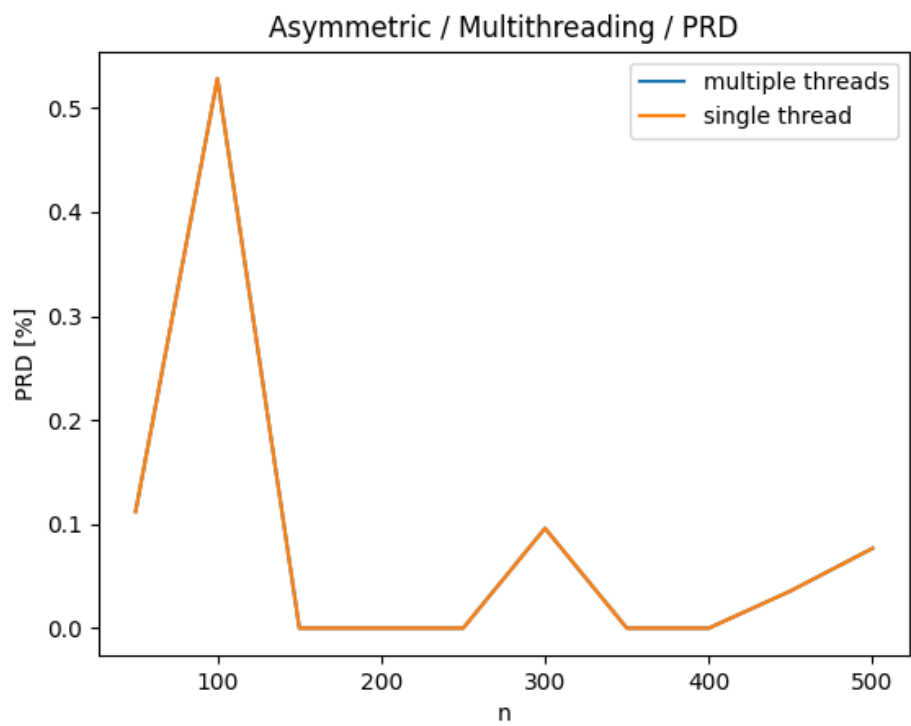


5.2 Dane generowane przez nas

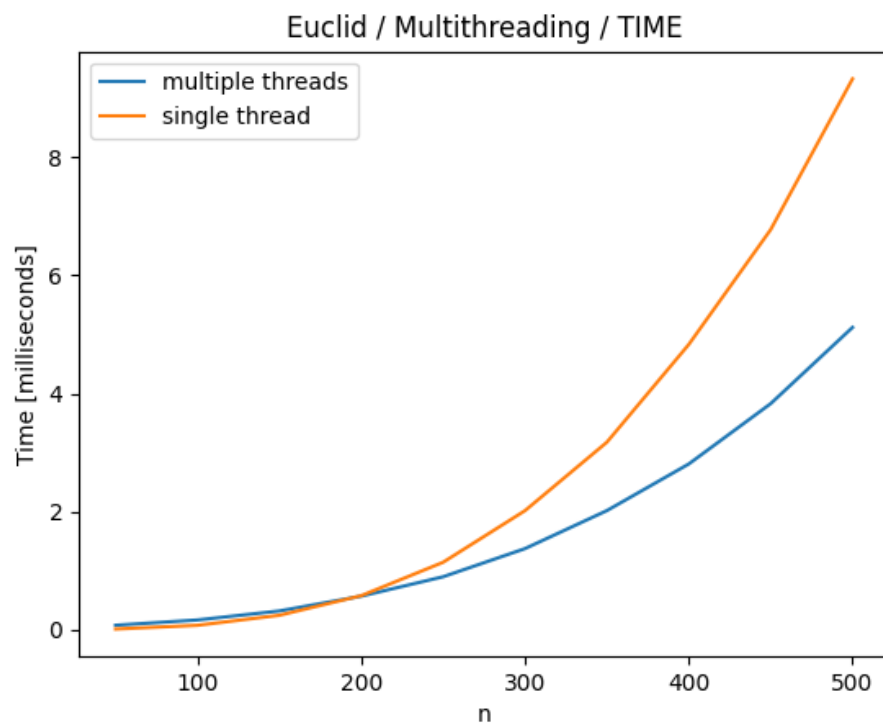
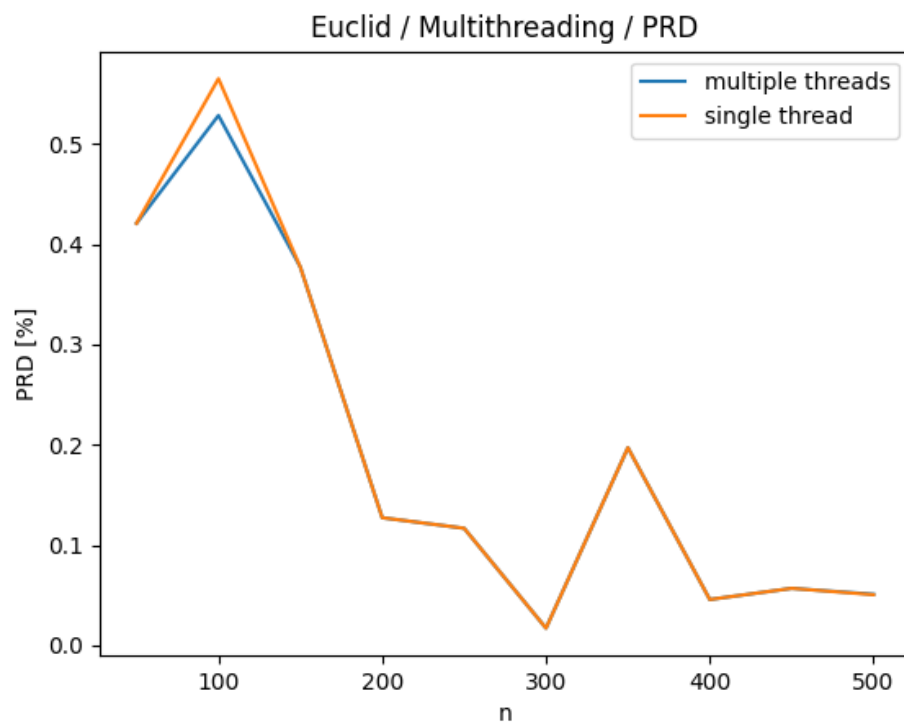
5.2.1 Grafy symetryczne



5.2.2 Grafy asymetryczne



5.2.3 Grafy euklidesowe



5.3 Obserwacje

5.4 Tabele

Asymmetric / Multithreading / PRD		
n	Multiple threads	Single thread
50	0.11191047162270185	0.11191047162270185
100	0.52807735217553	0.52807735217553
150	0.0	0.0
200	0.0	0.0
250	0.0	0.0
300	0.09599191647019198	0.09599191647019198
350	0.0	0.0
400	0.0	0.0
450	0.03581064508216826	0.03581064508216826
500	0.0766979513573519	0.0766979513573519

Asymmetric / Multithreading / Time		
n	Multiple threads	Single thread
50	0.07376404	0.00903833
100	0.16007074999999998	0.07002836999999999
150	0.31249241000000005	0.23549518999999997
200	0.54841901	0.5584251
250	0.8751694499999999	1.10466398
300	1.34324706	1.951844
350	1.9484998399999998	3.1064202999999995
400	2.74655025	4.65695869
450	3.8018252900000001	6.6676786
500	5.08990898	9.196990920000001

Euclid / Multithreading / PRD		
n	Multiple threads	Single thread
50	0.4207549708890032	0.4207549708890032
100	0.5282007435256711	0.5648531564761904
150	0.3769823007501678	0.3769823007501678
200	0.1271843910979231	0.1271843910979231
250	0.11667189906458164	0.11667189906458164
300	0.017076672487704073	0.017076672487704073
350	0.19697835915506468	0.19697835915506468
400	0.04564711832989831	0.04564711832989831
450	0.056766273647667766	0.056766273647667766
500	0.050901971434817714	0.05042549734309646

Euclid / Multithreading / Time		
n	Multiple threads	Single thread
50	0.07344694	0.009324530000000001
100	0.16274169	0.07212905
150	0.3153742	0.24179654
200	0.56568579000000001	0.57428542
250	0.89462399	1.14006875
300	1.37161821	2.01475883000000003
350	2.0154079099999995	3.17420528
400	2.80540426000000004	4.82983258
450	3.8271469	6.774824010000001
500	5.11919333	9.329994919999999

Symmetric / Multithreading / PRD		
n	Multiple threads	Single thread
50	6.917692987893891	6.917692987893891
100	4.787518110638111	4.787518110638111
150	3.6259066537639213	3.6259066537639213
200	3.65693454118884	3.65693454118884
250	2.4413015900221464	2.4413015900221464
300	1.1869594271627262	1.1869594271627262
350	2.9343241182614768	2.9343241182614768
400	1.7767045918185538	1.7767045918185538
450	1.2285915175711815	1.2285915175711815
500	1.0711060843000912	1.0711060843000912

Symmetric / Multithreading / Time		
n	Multiple threads	Single thread
50	0.07261012	0.00909796
100	0.16149818	0.070703260000000002
150	0.306675590000000005	0.23711314
200	0.54723579	0.55973583
250	0.8675596299999999	1.10501948
300	1.33364641	1.93088376
350	1.95661837	3.10280147
400	2.7663511699999996	4.66704174
450	3.79851777	6.665332530000001
500	5.06103543	9.190356060000001

TSPLIB asym / Multithreading / PRD		
n	Multiple threads	Single thread
17	0.0	0.0
34	5.365474339035769	17.1850699844479
53	16.437364228819696	17.14699493120927
70	4.881958989475862	4.282057249243659
100	11.783052718741374	11.783052718741374
358	14.531384350816854	13.413585554600171
443	4.338235294117647	4.191176470588235

TSPLIB asym / Multithreading / Time		
n	Multiple threads	Single thread
17	0.0034579	4.27e-05
34	0.0488397	0.0030225
53	0.0787211	0.011016
70	0.1066343	0.0245621
100	0.1671848	0.0727892
358	0.556717	3.387291
443	1.4899777	1.9098416

TSPLIB euclid / Multithreading / PRD		
n	Multiple threads	Single thread
51	1.8779342723004695	1.8779342723004695
52	0.0	0.0
70	7.703703703703704	3.259259259259259
76	4.849342172172451	4.849342172172451
100	5.461441213653603	5.461441213653603
105	5.285485777870505	5.285485777870505
130	3.5842880523731586	3.5842880523731586
150	1.7922794117647058	1.7922794117647058
225	2.400408580183861	2.400408580183861
280	3.3346258239627766	3.3346258239627766
442	4.133679940131553	4.182913860333215

TSPLIB euclid / Multithreading / Time		
n	Multiple threads	Single thread
51	0.0792794	0.0099841
52	0.0793002	0.0112551
70	0.1098126	0.0257936
76	0.113911	0.0326535
100	0.1608226	0.072437
105	0.1828318	0.0835306
130	0.2498604	0.1702319
150	0.3236019	0.2482995
225	0.6941704	0.8124654
280	1.1307793	1.6300746
442	3.5502116	6.2351401

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