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# Control Algorithm of Acceleration Curve for Stepper Motor

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**Abstract**-In some application situations, a stepper motor acceleration on a specific curve does not make full use of its acceleration performance. By analysing the motion characteristic of a stepper motor, this paper introduced an acceleration curve control algorithm suited to the torque-frequency characteristic of stepper motors. Compared to traditional S-curve acceleration, the new algorithm consisted of three movement stages: exponential increasing acceleration, constant acceleration, and decreasing acceleration. Simulations of the presented algorithm and the S-curve acceleration algorithm were conducted in MATLAB/Simulink, and the two algorithms were compared to each other in aspects of speed, acceleration, angular displacement, and number of steps. The simulation results showed that the presented acceleration curve control algorithm brought the driving capability of the stepper motor into full play, reduced starting time, and provided a favourable dynamic response. Under the control of the developed algorithm, the stepper motor accelerated faster in unit time when starting at 100 Hz and ending at 1200 Hz. The smooth curves of velocity and acceleration can avoid falling out of step, curb overshoot of the stepper motor, and reduce mechanical flexible impact.

**Keywords**- Acceleration Curve Control Algorithm; Acceleration Performance; S-curve Acceleration; Stepper Motor

## I. INTRODUCTION

With the increasing demand for processing speed and accuracy, acceleration and deceleration control are increasingly important [1]. Flexible acceleration and deceleration control makes machining processes stable and lowers impact, which can improve the dynamic performance of a lathe [2, 3].

Currently, there are four kinds of acceleration and deceleration curves in use including the trapezoid curve, linear curve, exponential curve, and S-curve [4-6]. A trapezoid curve tends to cause out-of-step and overshoot at acceleration jumps, and excites residual vibrations when the machine reaches the final position [7]. A segmental linear curve has acceleration jumps at transition points, which causes flexible impact. Due to the low processing speed of the control system, this curve is applicable to situations that do not require high acceleration [8]. An exponential curve has a strong tracking capability [9] but has weak stability at higher speeds, and the acceleration jumps at the start and end [9]. An S-curve responds quickly, and is applied in fast and high accuracy situations [10, 11]. S-curves can be applied in electric vehicles [12], stepper motors [13], and CNC machine tools [14]. The second order S-curve hardly reaches the desired position due to flexibility and vibration of the system [15, 16]. These four kinds of curves are used in different situations but cannot bring the driving capability of a stepper motor into full play. An improved and optimized acceleration/deceleration control method is needed to meet the demand for processing accuracy and efficiency.

While pointing out the downside of a trapezoid curve, Kim Doang [7] presented an S-curve motion trajectory using a special strategy of a triangle model that is applicable and effective but does not analyse the dynamic characteristic of acceleration. Xian Min [8] indicated the advantages and realization of an exponential curve but did not consider the advantages of other curves or present a solution for acceleration jumps. Zhan Li [9] presented the disadvantages of a traditional S-curve and provided the corresponding optimization to stabilize the motion but did not offer a faster way of acceleration at early starts. Shuang Hui [10], Huai Zhong [11], and Jin Hung [13] specified the advantages and disadvantages of the S-curve but did not do enough research on fast acceleration or analyse the acceleration process according to the motion characteristic of a stepper motor. Cheng Tzung [17] proposed an equal time discretization method for the speed-up curve of a stepper motor. However, it did not maximize the motion characteristics of a stepper motor, which can easily cause the loss of step during acceleration of the motor.

The motion characteristic of a stepping motor requires that the motor have a large acceleration, short acceleration time, and a steady acceleration process at the starting stage, which can reduce the loss of step. This paper presented an acceleration curve control algorithm suited to the torque-frequency characteristic of stepper motors consisting of three movement stages: exponential increasing acceleration stage, constant acceleration stage, and decreasing acceleration stage. This algorithm made full use of the driving ability of a stepper motor, accelerated to the specified speed in a relatively short period of time, and made the running process stable. When comparing the new algorithm to an S-shaped acceleration curve in MATLAB/Simulink under the same conditions, the results showed that the curve of the proposed method had a better acceleration performance.

This paper is organized as follows. Section II details the motion characteristics of a stepper motor. Section III presents the realization of the acceleration curve. Section IV presents the analysis of the simulation results. Finally, Section V concludes this paper.

## II. MOTION CHARACTERISTIC OF STEPPER MOTOR

### A. Torque-frequency and Acceleration Characteristics

In order to obtain the ideal operation curve, the motion characteristic of a stepper motor was analyzed. Fig. 1 shows the relationship between torque-frequency characteristic, acceleration, and frequency. In the figure,  $T$  is torque,  $J$  is rotational inertia,  $\varepsilon_m$  is the maximum angular acceleration,  $T_m$  is the maximum torque, and  $f_m$  is the maximum running frequency. The dotted curve of the torque-frequency characteristic in Fig. 1 indicates that the torque is higher in low frequency range than in high frequency range and decreases significantly at high frequency. When the stepper motor starts at a frequency that is higher than its start-up frequency, it may lose steps or fail to start. In an ideal state and with definite rotational inertia, it is harder for the motor to start as the load torque gets higher. The solid curve in Fig. 1 shows that when the inertia torque is lower than the electromagnetic torque, a higher angular acceleration  $\varepsilon$  during speed increment will bring a faster response. In order to run stably and to reduce impact,  $\varepsilon$  should not jump. The angular acceleration increases linearly from zero to  $f_1$ , stays invariant to  $f_2$ , and decreases linearly to zero when it reaches  $f_3$  [13].

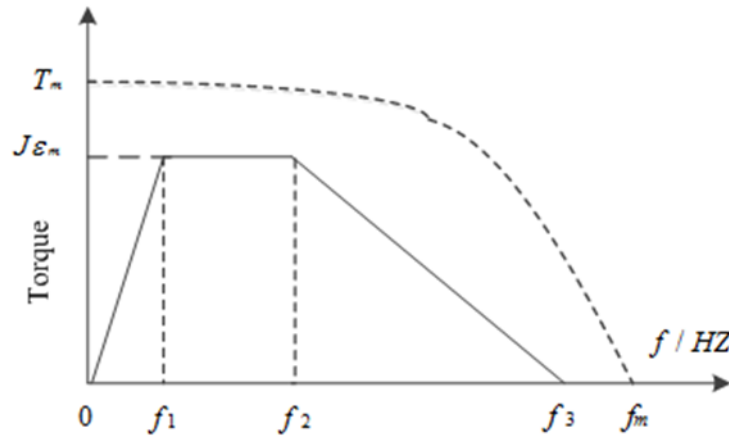


Fig. 1 Characteristic of torque-frequency and acceleration curve of stepper motor

## III. REALIZATION OF ACCELERATION CURVE

### A. Establishment of Acceleration Curve

In the control system of a stepper motor, one pulse drives the motor to turn a certain angle or advance one step, and the motor speed is proportional to the pulse frequency [14, 18, 19]. On the premise of not losing step, the angular acceleration is proportional to the differential of the pulse frequency versus time. The differential equation describing every segment of the acceleration curve is as follows:

$$\varepsilon = df/dt = A - B \quad (1)$$

where  $A$  and  $B$  are parameters related to the performance of the stepping motor,  $f$  is the frequency of the driving pulse, and  $\varepsilon$  is the angular acceleration of motor.

Assuming that the speed changes with the frequency of  $f_0$ , and using the method of Laplace transformation in Eq. (1), the following can be obtained:

$$F(s) = A/[s(s+B)] + f_0/(s+B) \quad (2)$$

By applying Laplace inverse transformation, the following can be obtained:

$$f(t) = A/B + (f_0 - A/B)\exp(-Bt) \quad (3)$$

In order to better describe the change law of the speed curve of a stepper motor, rewrite Formula (3) as the following:

$$f(t) = f_0 + C(1 - \exp(-Bt)) \quad (4)$$

where  $C = A/B - f_0$ .

In terms of Eq. (4), the change of pulse frequency  $f$  with time  $t$  is determined by constant  $B$ .  $B > 0$  is the first segment of the acceleration curve, and the velocity curve is convex.  $B = 0$  is the second segment of the acceleration curve, and the velocity curve is a straight line.  $B < 0$  is the third segment of the acceleration curve, and the velocity curve is concave.  $1/B$  is time constant, which determines accelerating speed, and is decided by the specific driving system.

According to the above analysis, the acceleration curve presented in this paper consisted of three segments. The first segment was the exponential curve, which produced fast responses and smoothness, and the torque-frequency characteristic was fully considered. The second segment was the straight line with simple calculation and high efficiency. The third segment was the parabola with a smooth transition point, which reduced vibration at high frequencies. The equations of these segments were as follow:

$$\begin{cases} f_1(t) = f_0 + C(1 - \exp(-Bt)) \\ f_2(t) = at + d \\ f_3(t) = -b(t - t_c)^2 + f_c \end{cases} \quad (5)$$

where  $f_0$  is the start-up frequency of the motor,  $f_c$  is the highest frequency,  $a$ ,  $b$ ,  $d$  are parameters to be determined and  $t_c$  is the acceleration time.

In order to avoid the jumps of acceleration and to keep the motor stable, the slope and the value of function should be equal at the transition point. Thus, time parameters of every segment can be obtained.

For the curve of  $f_1(t)$ ,  $f_2(t)$ , the following is used:

$$\begin{cases} f_0 + C(1 - \exp(-Bt_1)) = at_1 + d \\ a = BC \exp(-Bt_1) \end{cases} \quad (6)$$

In terms of Eq. (6), the following can be obtained:

$$t_1 = (f_0 + C - d - a/B)/a \quad (7)$$

As to curve of  $f_2(t)$ ,  $f_3(t)$  there is the following:

$$\begin{cases} at_2 + d = -b(t_2 - t_c)^2 + f_c \\ a = -2b(t_2 - t_c) \end{cases} \quad (8)$$

According to Eq. (8), the following can be obtained:

$$\begin{cases} t_2 = (f_c - d)/a - a/4b \\ t_c = t_2 + a/4b \end{cases} \quad (9)$$

By integrating  $f_1(t)$ ,  $f_2(t)$ , and  $f_3(t)$ , pulse numbers of every segment can be obtained; the total number is the sum of every segment.

### B. Discretization Method of Acceleration Curve

An efficient way of discretizing the control function curve is pulse counting. Contrary to the work of Cheng Tzung [17], the left endpoint was chosen as the sampling point and equal pulse dispersion was utilized instead of the equal time discretization. The discretizing process is shown in Fig. 2. Its essence is equal area discretization of  $f(t)$ . The initial acceleration takes more time to avoid slow response and desynchronizing. When running at high frequency, the pulse interval becomes small to meet the high speed requirement. The frequency values obtained with this method can be transformed to time constants of the counter and loaded into the register.

Assume the controller generates the  $N$ -th pulse at time  $t$ . Eq. (10) shows that  $N$  is the equation of  $t$ , the time related to the  $N$ -th pulse can be obtained using the Newton iteration method, which is shown in Eq. (11). After several iterations of Eq. (11), the time of the  $N$ -th pulse that the controller sends out can be precisely obtained, which is  $t(N)$ . Then, by putting  $t(N)$  into Eq. (5),  $f(N)$  can be obtained. Finally, pulse period  $T(N)$  can be obtained by the reciprocal relation.

When running at high speed, the time interval of two adjacent pulses is very short. However, the presented iteration takes a long time. In practical application, the pulse sequence  $T(N)$  needs to be calculated in advance and saved in ROM as a datasheet. By checking the datasheet while running, the calculating time can be reduced and the response speed can be higher. According to the characteristic of the stepper motor, the deceleration curve is the inversion of the acceleration curve, thus the datasheet should be followed backward during the deceleration process.

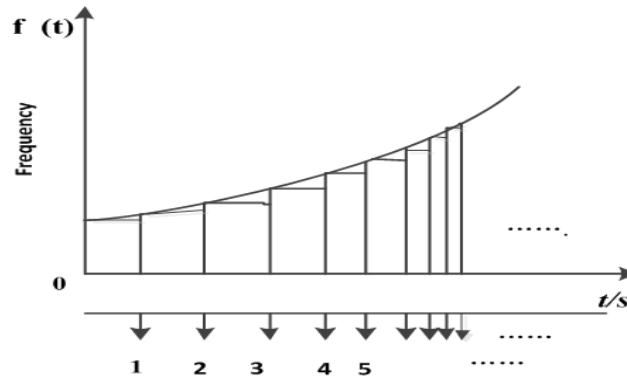


Fig. 2 Discretization of frequency curve

$$\begin{cases} N_1(t) = \int_0^t f_1(s) ds = (f_0 + C)t + C \exp(-Bt)/B - C/B \\ N_2(t) = \int_0^t f_2(s) ds = at^2/2 + dt \\ N_3(t) = \int_0^t f_3(s) ds = -b(t-t_c)^3/3 - bt_c^3/3 + f_c t \end{cases} \quad (10)$$

$$\begin{cases} t_k = t_{k-1} + \frac{(f_0 + C)t_{k-1} + C \exp(-Bt_{k-1})/B - C/B - N}{f_0 + C(1 - \exp(-Bt_{k-1}))} \\ t_k = t_{k-1} + \frac{at_{k-1}^2/2 + dt_{k-1} - N}{at_{k-1} + d} \\ t_k = t_{k-1} + \frac{-b(t_{k-1} - t_c)^3/3 - bt_c^3/3 + f_c t_{k-1} - N}{-b(t_{k-1} - t_c)^2 + f_c} \end{cases} \quad (11)$$

#### IV. ANALYSIS OF SIMULATION RESULTS

##### A. Simulation of Algorithm

The rotation speed of a stepper motor is proportional to the frequency, thus its speed was adjusted by controlling the frequency [20]. In order to prove the acceleration curve presented in this paper, the curves were simulated and compared in MATLAB/Simulink. The chosen stepper motor had a step angle of  $1.8^\circ$ , the total rotational inertia of the drive system was  $0.075 \text{ Kg/cm}^2$ , and the maximum static torque was  $1.2 \text{ N.m}$  constantly. The start-up frequency  $f_0$  and the maximum frequency were  $100 \text{ Hz}$  and  $1200 \text{ Hz}$ , respectively. Fig. 3 shows the simulation model of the proposed method built by Laplace transformation of Eq. (5). The desired curve was obtained according to the corresponding parameters and adjusted simulation time.

The simulation model of the S-curve acceleration was built under the same conditions as that shown in Fig. 4.  $J$  is the total rotational inertia of drive system,  $A$  and  $B$  are parameters that can be calculated according to the performance of the stepper motor,  $a$ ,  $b$ , and  $C$  are determined by Eq. (6), Eq. (8), and Eq. (4), respectively, and  $t_c$  is the acceleration time.

Fig. 5 illustrates that the presented acceleration curve matched the motion characteristic of the stepper motor perfectly, which can bring the driving capability of the stepper motor into full play. Compared to the S-curve, the presented curve responded faster at the first motion stage and effectively restrained desynchronizing.

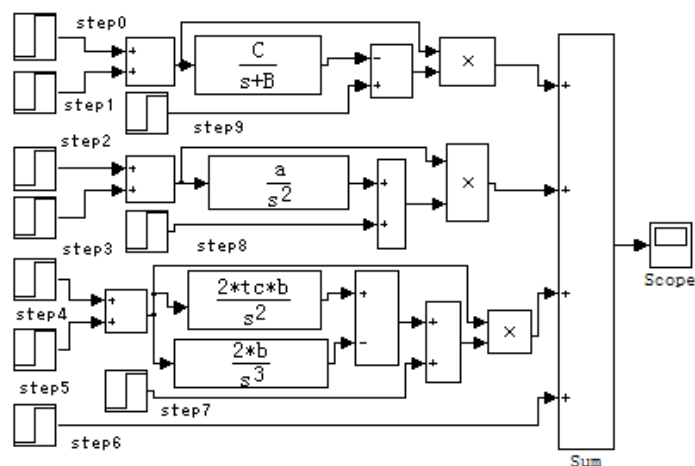


Fig. 3 Simulation model of proposed method

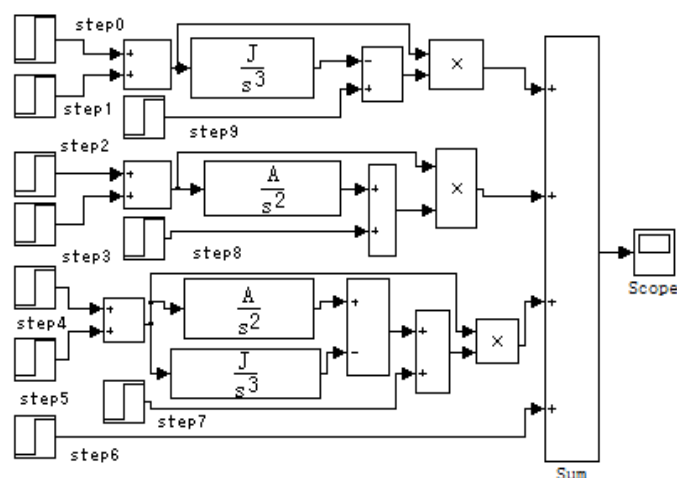


Fig. 4 Simulation model of S-curve acceleration

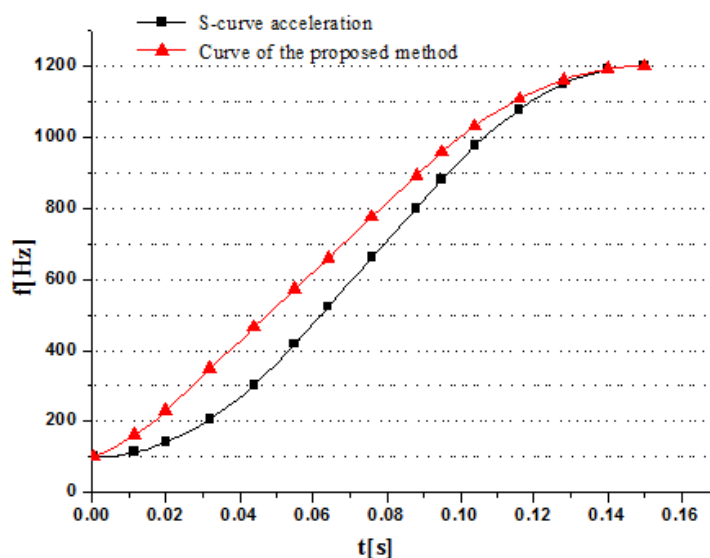


Fig. 5 Simulation of speed curve

In order to verify the continuity and the flexibility of the acceleration curve, the proposed method was compared to the S-curve, the results of which are shown in Fig. 6.

From Fig. 6, it can be seen that the accelerations of the proposed curve and the S-curve are continuous. Both of the curves allowed the acceleration process to remain stable. The proposed curve had a higher acceleration than the S-curve, so the stepper motor accelerated faster and reached the designated frequency for a shorter time, which met the requirement of the stepper motor operating in a high frequency, and easily avoided desynchronizing.

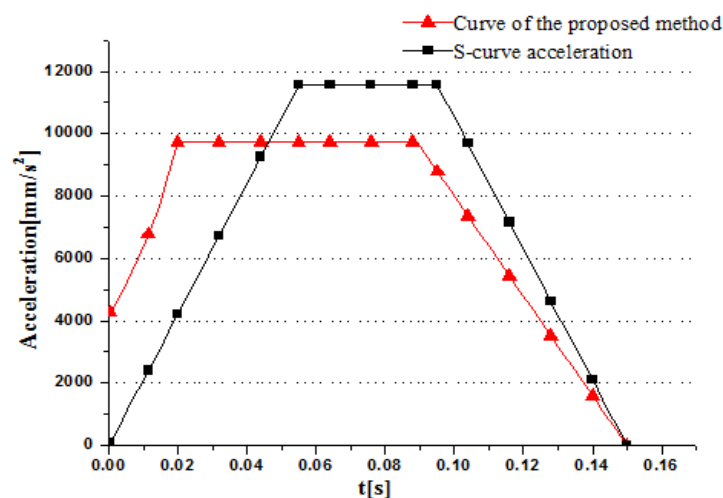


Fig. 6 Simulation of acceleration curve

In order to verify whether the machine could reach the designated position with the required precision under control of the proposed curve, simulation of the angular displacement was done. The results are shown in Fig. 7. Obviously, the proposed curve had good position accuracy compared to the S-curve acceleration.

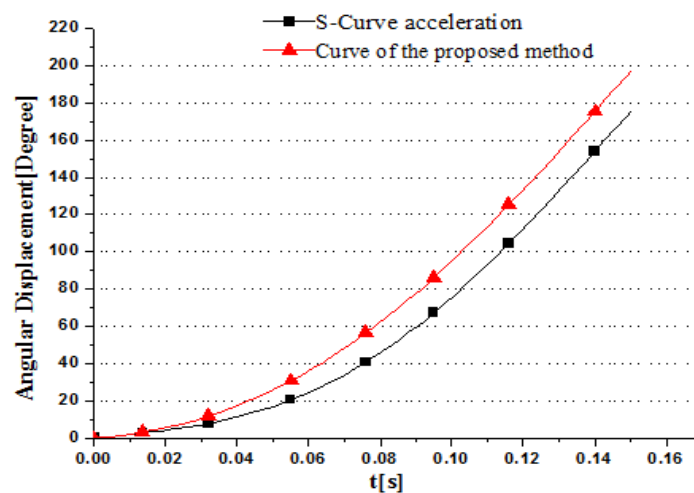


Fig. 7 Simulation of angular displacement curve

Discretization of the S-curve and the proposed curve are shown in Tables 1 and 2, where  $N$  represents the  $N$ -th pulse,  $\Delta T$  represents the pulse time interval, and the  $F$  indicates the corresponding frequencies.

TABLE 1 PULSE INTERVAL OF PROPOSED CURVE

$N$	$\Delta T$ [ $\mu s$ ]	$F/Hz$	$N$	$\Delta T$ [ $\mu s$ ]	$F/Hz$
1	7027.4	142.3	7	2911.2	343.5
2	5437.7	183.9	8	2696.9	370.8
3	4428.7	225.8	9	2524.6	396.1
4	4093.3	244.3	10	2378.7	420.4
5	3554.9	281.3	11	2258.4	442.8
6	3181.7	314.3	...	...	...

TABLE 2 PULSE INTERVAL OF S- CURVE ACCELERATION

$N$	$\Delta T$ [ $\mu s$ ]	$F/Hz$	$N$	$\Delta T$ [ $\mu s$ ]	$F/Hz$
1	9099.2	109.9	7	3425.8	291.9
2	7457.1	134.1	8	3098.9	322.7
3	6090.1	164.2	9	2843.3	351.7
4	5086.5	196.6	10	2621.9	381.4
5	4368.7	228.9	11	2443.8	409.2
6	3832.9	260.9	...	...	...

Data obtained using the Newton iteration method in Tables 1 and 2 are the time intervals of the pulse sequence at different frequencies. In consideration of accumulated errors and according to the tables, the number of steps during acceleration are shown in Fig. 8.

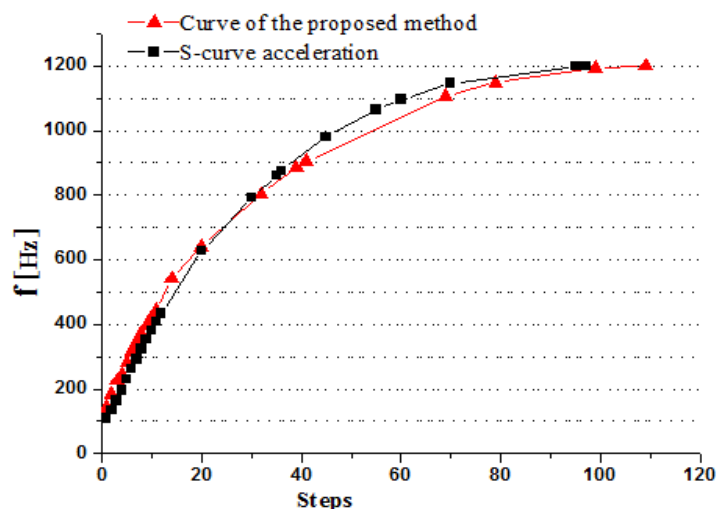


Fig. 8 Number of steps during acceleration

Fig. 8 illustrates that the curve of the proposed method resulted in more steps than the S-curve acceleration. The machine made more steps at high frequency than at low frequency, which perfectly met the requirement of high speed at high frequency. The control method avoided slow responses at low speed and met the requirement of dynamic performance.

## V. CONCLUSIONS

The presented acceleration curve control algorithm complemented the torque-frequency and acceleration characteristics of a stepper motor. This algorithm can be widely used on the premise of accurate controller speed. According to the comparison with the S-curve acceleration in the aspects of speed, acceleration, angular displacement, and number of steps, the proposed acceleration curve reduced accelerating time, brought the driving capability of the stepper motor into full play, responded quickly, and met the requirements of speed and precision. Simulation results showed that the stepper motor accelerated faster in unit time when starting at 100 Hz and ending at 1200 Hz. Moreover, the acceleration and speed curves that can be seen in the simulation diagram were smooth, meaning the vibration and mechanical flexible impact were reduced.

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