

Data Science & ML Course

Lesson #24 Deep Learning Fundamentals I

Ivanovitch Silva
December, 2018



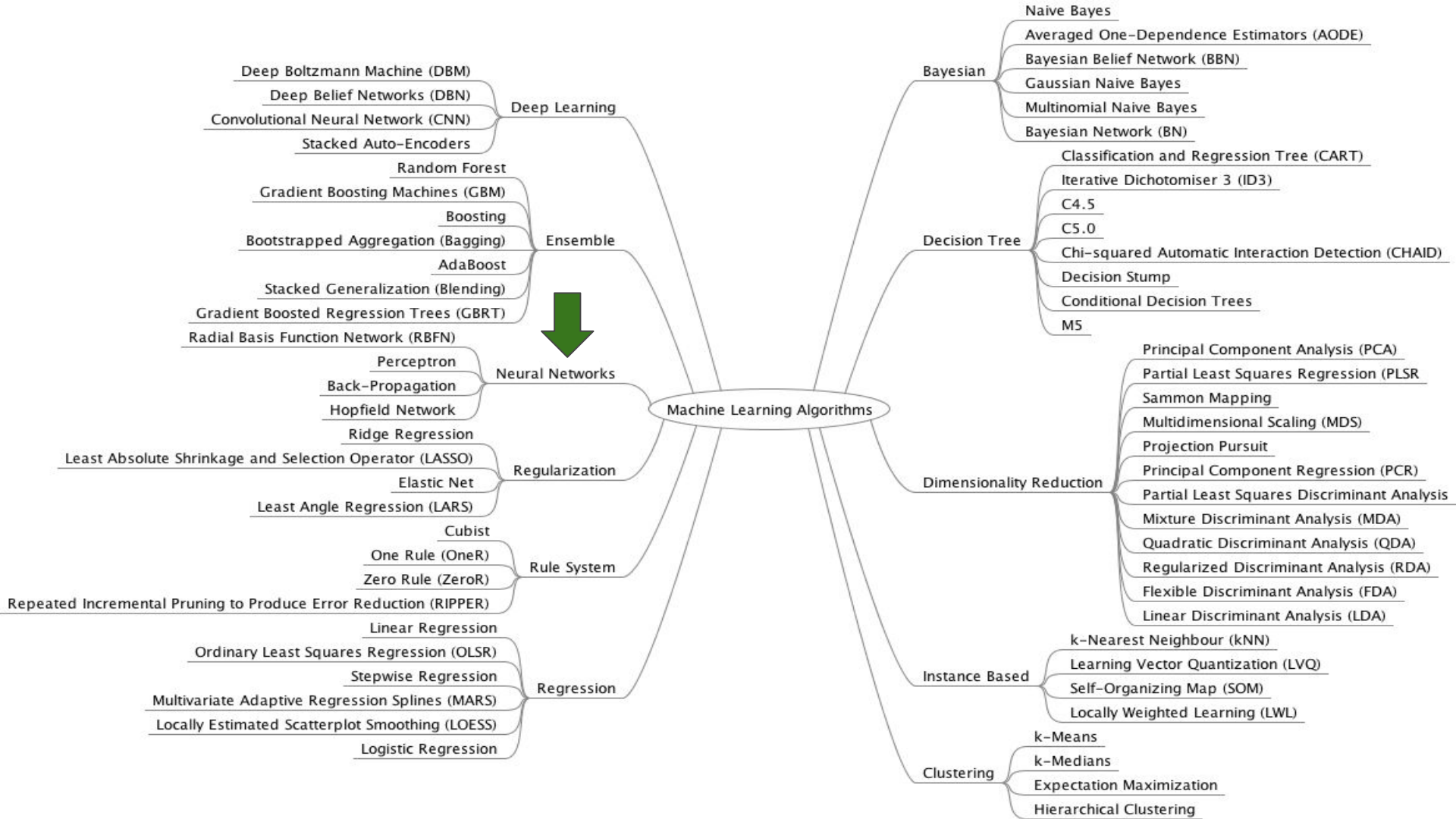
Update from repository

```
git clone https://github.com/ivanovitchm/datascience2machinelearning.git
```

Or

```
git pull
```





Agenda

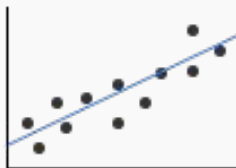
1. Representing neural network
2. Nonlinear activation functions
3. Hidden Layers
4. Case study: build a handwritten digit classifier

k-nearest neighbors



None
(nonexistent
training process)

linear regression



$$\hat{y} = 3x_1 + 10$$

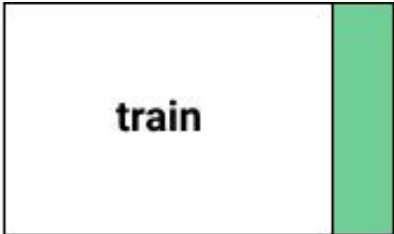
logistic regression

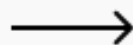


$$\hat{p} = \frac{e^{3x}}{1 + e^{3x}}$$

decision trees &
random forests



`model.fit(`  `)`

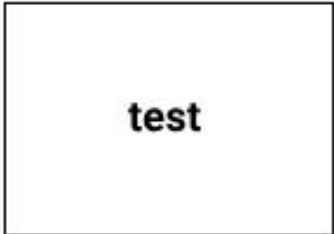


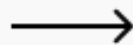
regression

$$y = 3x_1 + 4x_2$$

decision tree





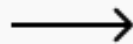
`model.predict(`  `)`



predictions



`error(`   `)`



MSE

600.25

RMSE

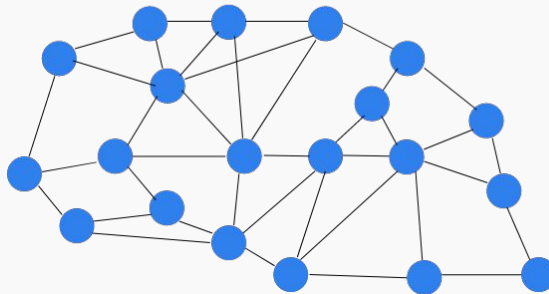
24.5

AUC

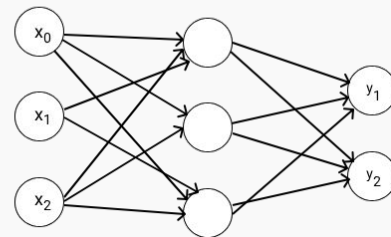
0.7

Representing a Neural Network

Biological Neural Network



Artificial Neural Network

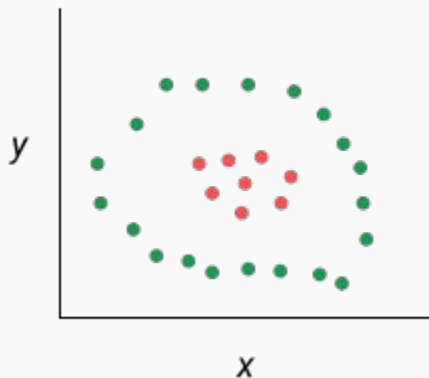
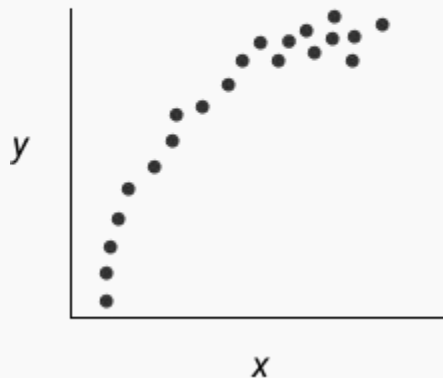


Neuron

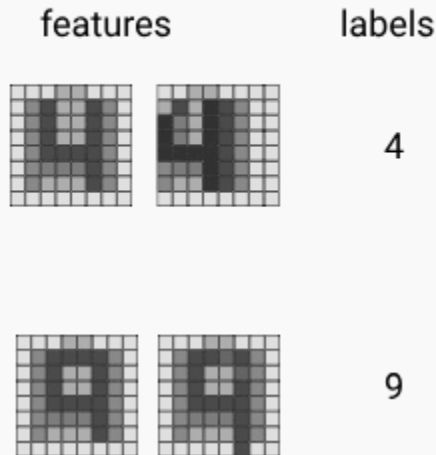
Neural network models were inspired by the structure of neurons in our brain and message passing between neurons

Deep Neural Network

nonlinear relationship between x and y



no obvious relationship between pixel values and labels



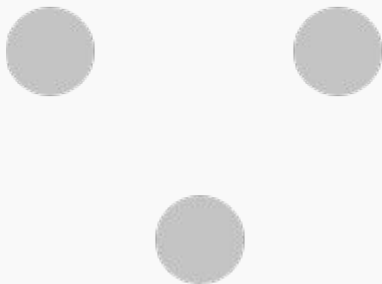
A **deep neural network** is a specific type of **neural network** that excels at capturing nonlinear relationships in data

How **neural networks** are represented and how to represent **linear regression** and **logistic regression** models in that representation

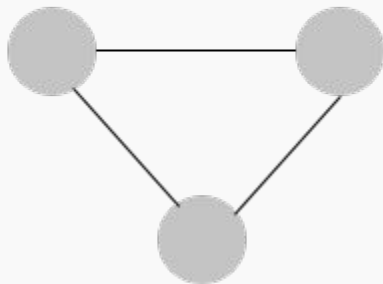
Introduction to Graphs

Neural networks are usually represented as graphs.

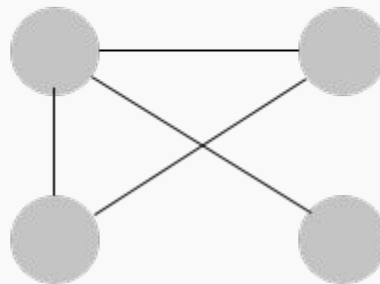
A graph with
3 nodes ● and
0 edges —



A graph with
3 nodes ● and
3 edges —



A graph with
4 nodes ● and
4 edges —



Computational Graphs

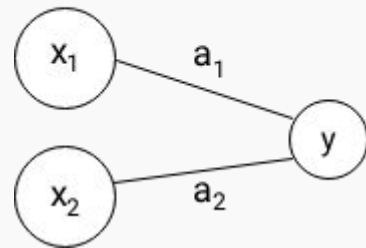
In the context of neural networks, graphs let us compactly express a pipeline of functions that we want to be executed in succession

$$\text{stage 1} \quad \sigma \left(\underset{X}{\begin{bmatrix} 100 \times 3 \end{bmatrix}} \underset{a_1^T}{\begin{bmatrix} 3 \times 6 \end{bmatrix}} \right) = \underset{L_1}{\begin{bmatrix} 100 \times 6 \end{bmatrix}}$$



$$\text{stage 2} \quad \sigma \left(\underset{L_1}{\begin{bmatrix} 100 \times 6 \end{bmatrix}} \underset{a_2^T}{\begin{bmatrix} 6 \times 1 \end{bmatrix}} \right) = \underset{L_2}{\begin{bmatrix} 100 \times 1 \end{bmatrix}}$$

$$y = a_1 x_1 + a_2 x_2$$



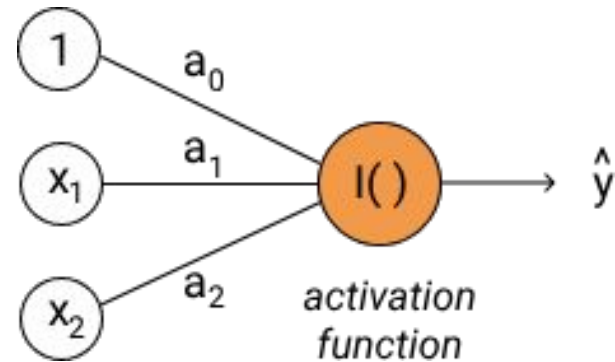
A neural network that performs a linear regression

$$\hat{y} = a_0 + a_1x_1 + a_2x_2 + \dots + a_nx_n$$

$$Xa^T = \hat{y}$$

$$\begin{bmatrix} 1 & 2.3 & 0.2 \\ 1 & 3.1 & 0.9 \\ 1 & 1.1 & 0.5 \end{bmatrix} \begin{bmatrix} 0.9 \\ 0.7 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.61 \\ 3.52 \\ 1.92 \end{bmatrix}$$

$X \qquad a^T \qquad \hat{y}$



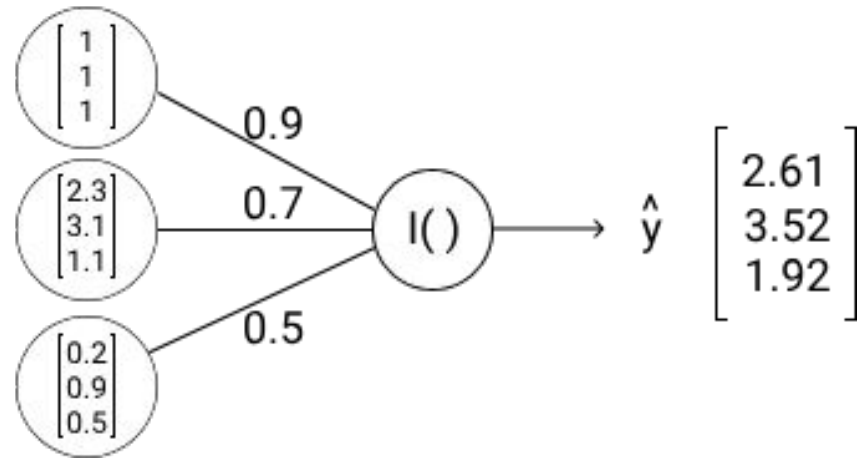
A neural network that performs a linear regression

Linear Algebra

$$I \left(\begin{matrix} 1 & 2.3 & 0.2 \\ 1 & 3.1 & 0.9 \\ 1 & 1.1 & 0.5 \end{matrix} \right) \begin{matrix} \begin{bmatrix} 0.9 \\ 0.7 \\ 0.5 \end{bmatrix} \end{matrix} = \begin{matrix} \begin{bmatrix} 2.61 \\ 3.52 \\ 1.92 \end{bmatrix} \end{matrix}$$

X a^T \hat{y}

Neural Network



Generate yourself dataset

Scikit-learn contains the following convenience functions for generating data:

- [sklearn.datasets.make_regression\(\)](#)
- [sklearn.datasets.make_classification\(\)](#)
- [sklearn.datasets.make_moons\(\)](#)

Generating regression data

```
from sklearn.datasets import make_regression
import pandas as pd

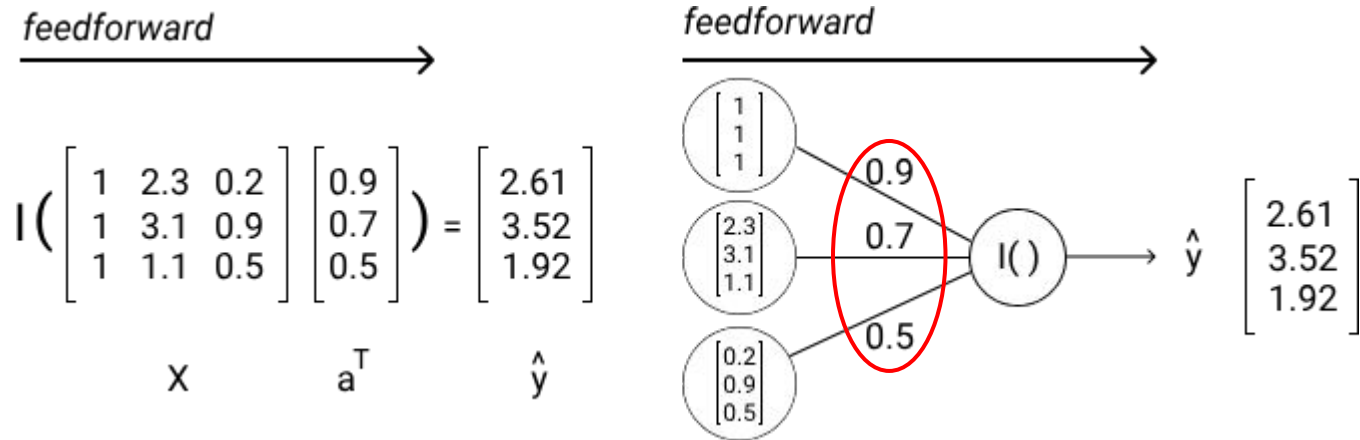
# make_regression return a tuple
# data[0].shape (1000,3) -> features
# data[1].shape (1000,) -> target
data = make_regression(n_samples=100,
                      n_features=3, random_state=1)

features = pd.DataFrame(data[0])
labels = pd.Series(data[1])
```

```
0    -10.378660
1     25.512450
2     19.677056
3    149.502054
4   -121.652109
dtype: float64
```

	0	1	2
0	1.293226	-0.617362	-0.110447
1	-2.793085	0.366332	1.937529
2	0.801861	-0.186570	0.046567
3	0.129102	0.502741	1.616950
4	-0.691661	-0.687173	-0.396754

Fitting a linear regression neural network



```
from sklearn.linear_model import SGDRegressor
lr = linear_model.SGDRegressor()
lr.fit(X,y)
```


Fitting a linear regression neural network

```
# generate the dataset
data = make_regression(n_samples=100,
                      n_features=3,
                      random_state=1)

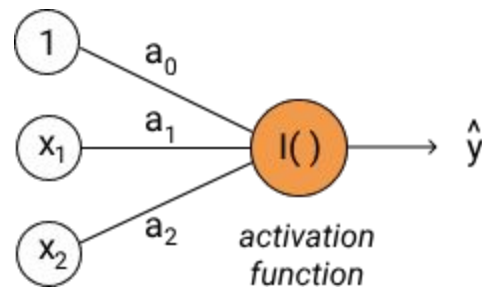
features = pd.DataFrame(data[0])
labels = pd.Series(data[1])
```

```
# configure the bias
features["bias"] = 1
```

```
train_weights = train(features, labels)
linear_predictions = feedforward(features,
                                  train_weights)
```

```
def train(features, labels):
    lr = SGDRegressor()
    lr.fit(features, labels)
    # Returns a nested NumPy array of weights.
    weights = lr.coef_
    return weights

def feedforward(features, weights):
    predictions = np.dot(features, weights.T)
    return predictions
```



Generating a classification data

```
from sklearn.datasets import make_classification
class_data = make_classification(n_samples=1000,
                                n_features=4,
                                random_state=1)
```

Features

```
class_features = class_data[0]
class_features[:5]
```

```
array([[ 1.91518414,  1.14995454, -1.52847073,  0.79430654],
       [ 1.4685668 ,  0.80644722, -1.04912964,  0.74652026],
       [ 1.47102089,  0.90060386, -1.20228498,  0.57845433],
       [ 1.07642824, -0.1813636 ,  0.49116807,  1.95642108],
       [-5.34139911, -2.29763222,  2.77907005, -3.87463248]])
```

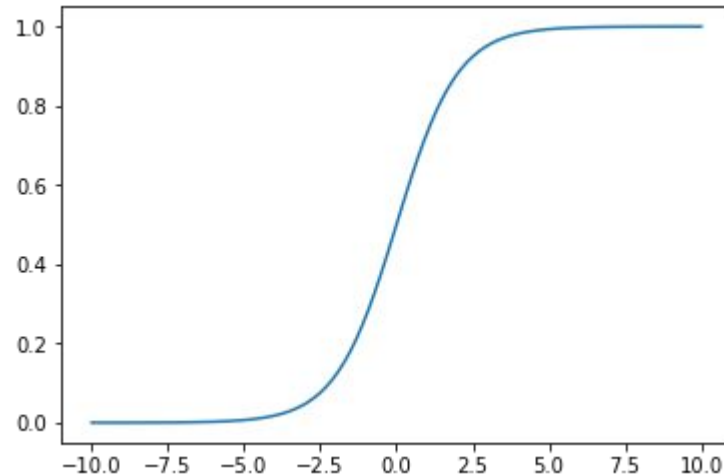
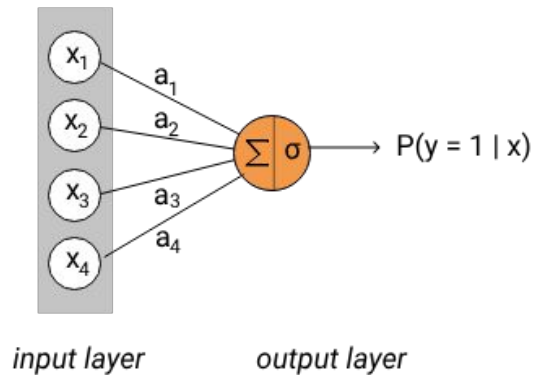
Labels

```
class_labels = class_data[1]
class_labels[:5]
```

```
array([1, 1, 1, 1, 0])
```

Implementing a neural network that performs classification

Binary Classification



$$\begin{bmatrix} 100 \times 4 \\ X \end{bmatrix} \begin{bmatrix} 4 \times 1 \\ a^T \end{bmatrix} = \begin{bmatrix} 100 \times 1 \\ P(y = 1 | x) \end{bmatrix}$$

$$\hat{y} = \sigma(Xa^T)$$

$$P(y = 1 | x) > 0.5$$


$$P(y = 0 | x) < 0.5$$

Implementing a Logistic Regression Model

```
# generate classification dataset
```

```
class_data = make_classification(n_samples=100,  
                                n_features=4,  
                                random_state=1)
```

```
class_features = class_data[0]  
class_labels = class_data[1]
```


$$\hat{y} = \sigma(Xa^T)$$


```
log_train_weights = log_train(class_features,  
                               class_labels)  
log_predictions = log_feedforward(class_features,  
                                   log_train_weights)
```

```
def log_train(class_features, class_labels):  
    sg = SGDClassifier()  
    sg.fit(class_features, class_labels)  
    return sg.coef_
```

```
def sigmoid(linear_combination):  
    return 1/(1+np.exp(-linear_combination))
```

```
def log_feedforward(class_features, log_train_weights):  
    linear_combination = np.dot(class_features,  
                                log_train_weights.T)  
    log_predictions = sigmoid(linear_combination)  
    log_predictions[log_predictions >= 0.5] = 1  
    log_predictions[log_predictions < 0.5] = 0  
    return log_predictions
```

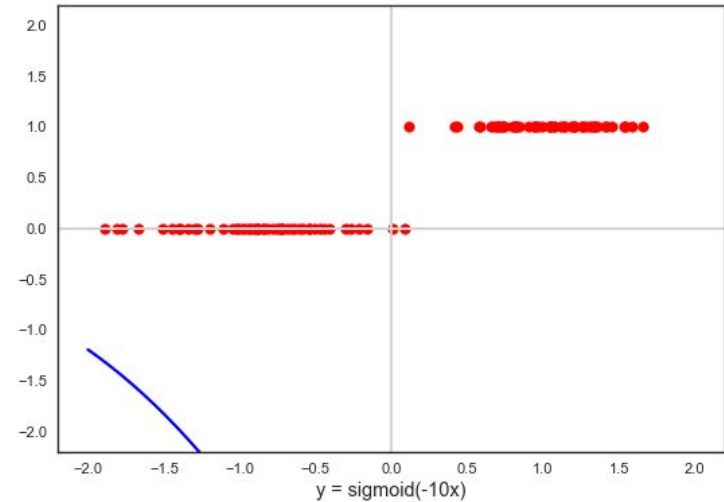
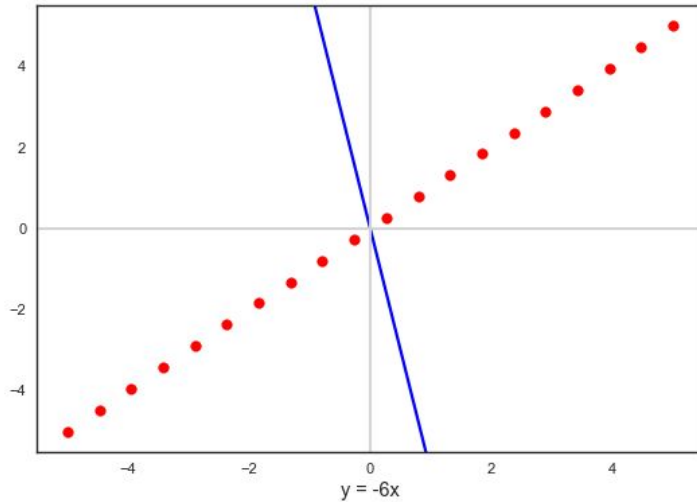


$$P(y = 1|x) > 0.5$$

$$P(y = 0|x) < 0.5$$

Activation functions

Nonlinear Activation Functions



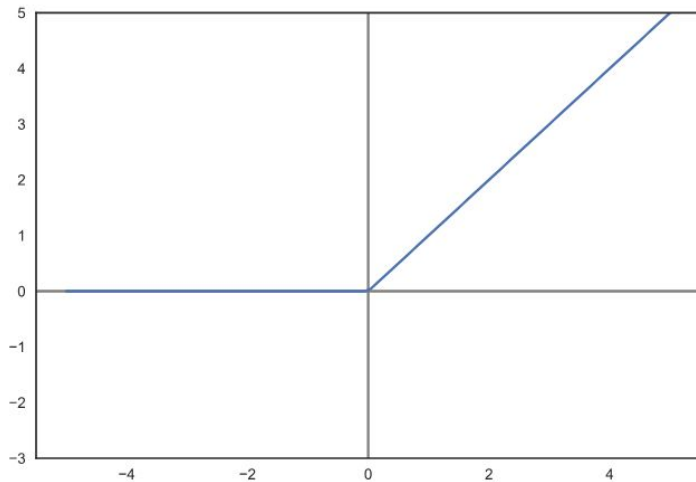
Neural Networks - Activation Functions

The three most commonly used activation functions in neural networks are:

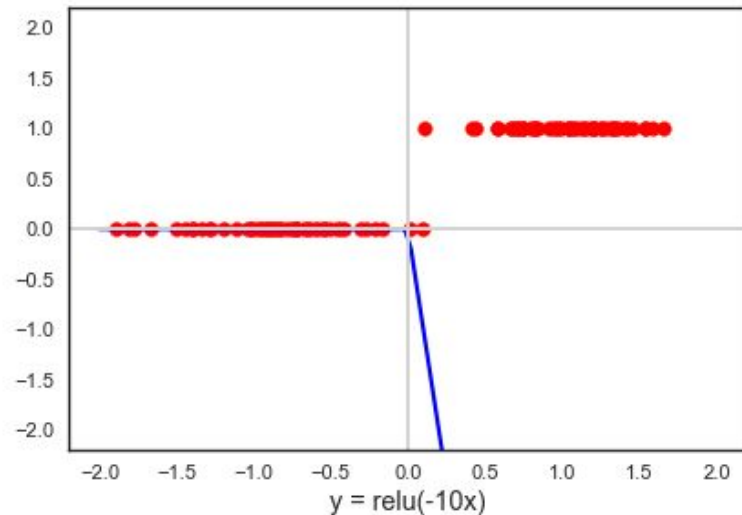
- the sigmoid function
- the ReLU function
- the tanh function

ReLU Function - Rectifier Linear Unit

```
relu = lambda x: np.maximum(0,x)  
x=np.linspace(-10,10,100)  
plt.plot(x,relu(x))
```

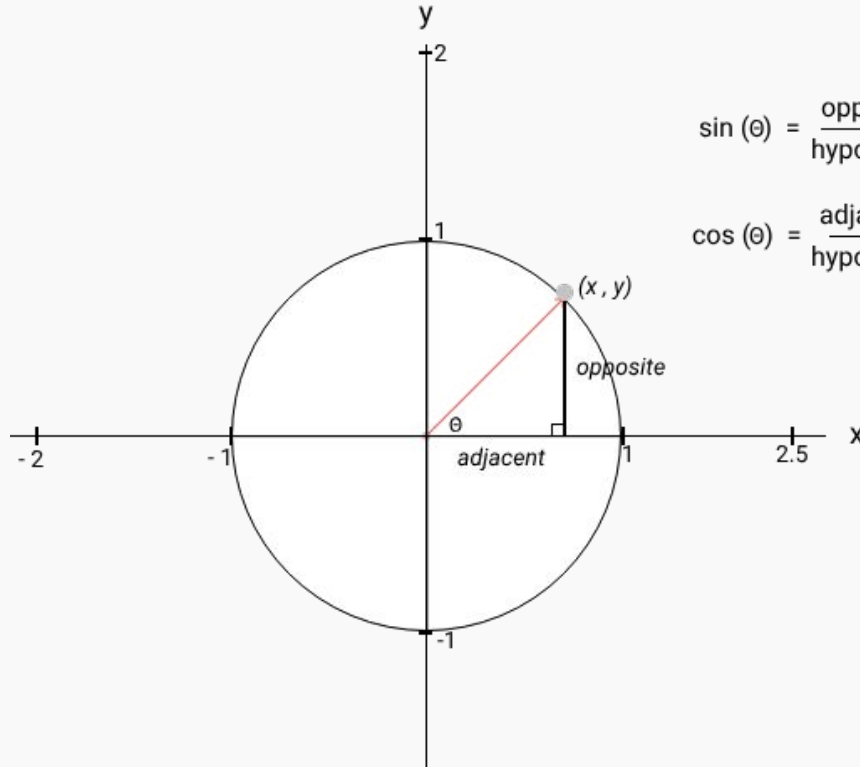


$$\text{ReLU} = \max(0, x)$$



ReLU is a commonly used activation function in neural networks for solving regression problems.

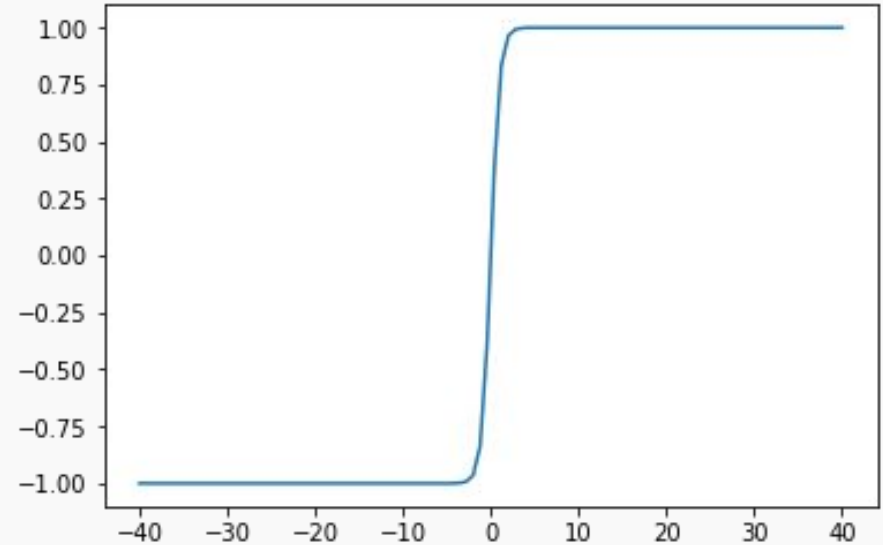
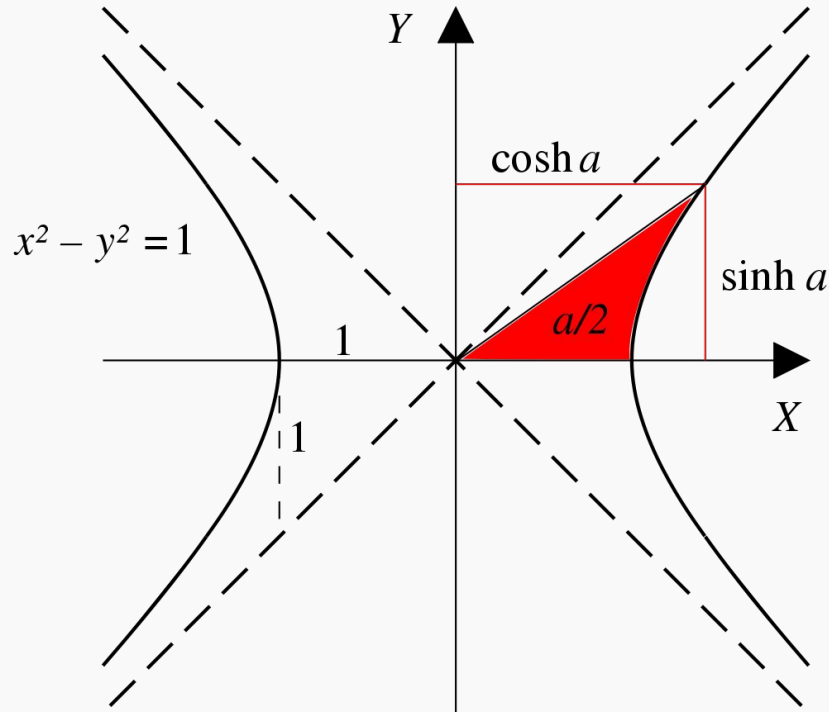
Trigonometric Functions



$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \text{opposite} = y$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \text{adjacent} = x$$

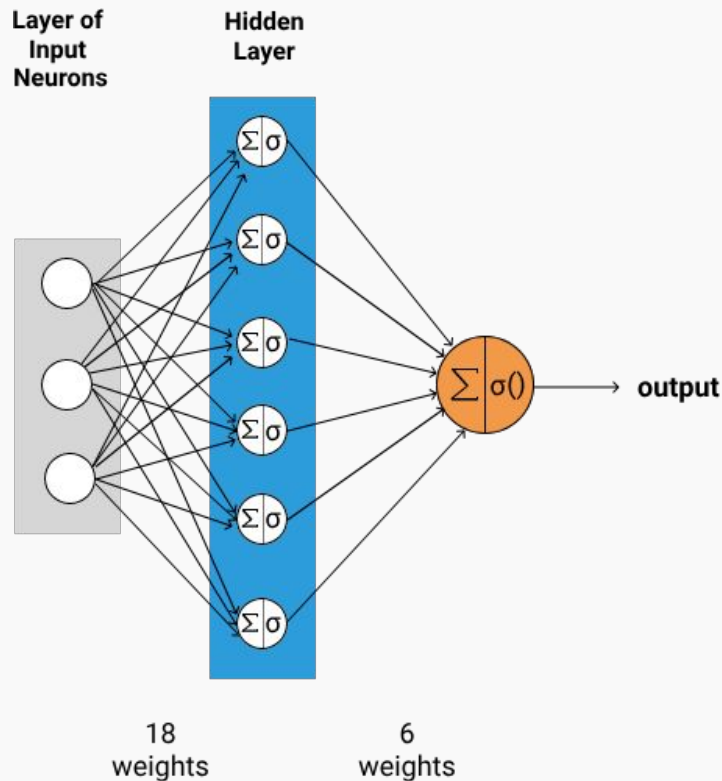
Hyperbolic Tangent Function (tanh)



It is commonly used in neural networks for classification tasks.

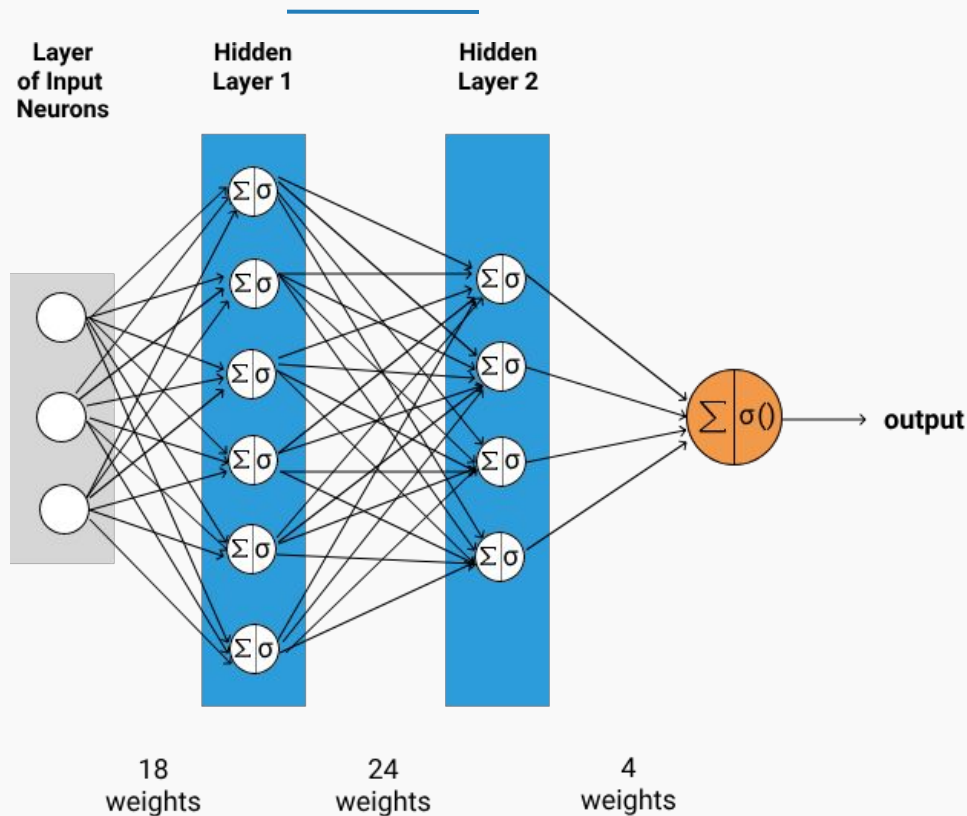
Hidden Layers

Multi-layers networks (deep neural networks)



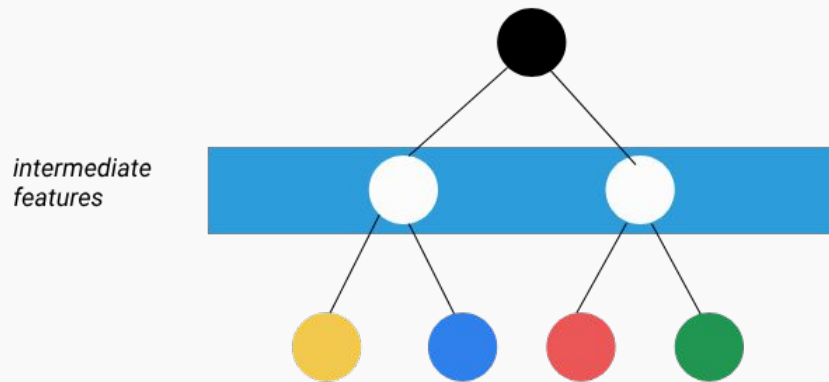
- This kind of models are able to better capture nonlinearity in the data
- Choosing the number of neurons in this layer is a bit of an art
- We can think of each hidden layer as intermediate features that are learned during the training process.

Multi-layers networks (deep neural networks)

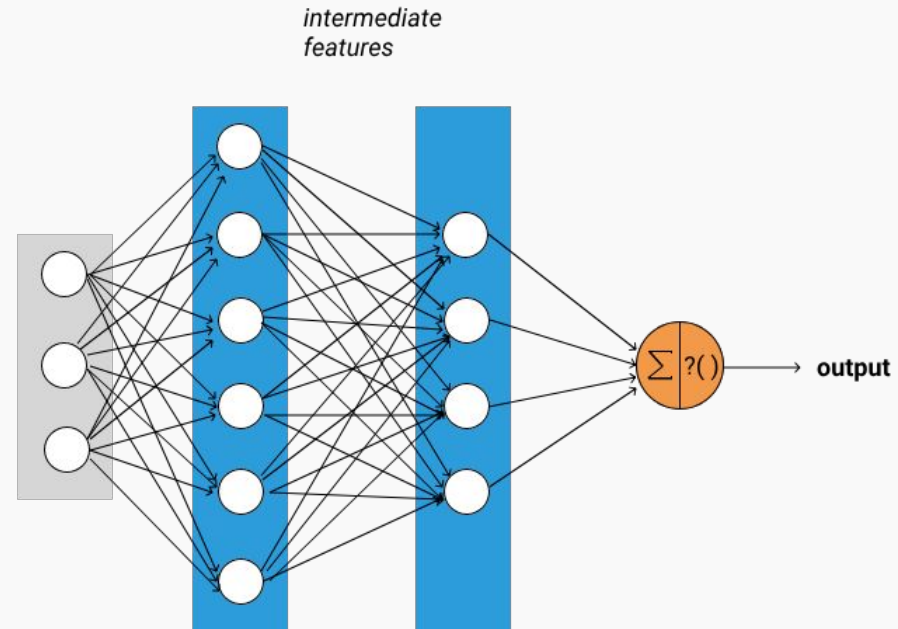


Comparison with Decision Tree Models

Decision Tree



Deep Neural Network

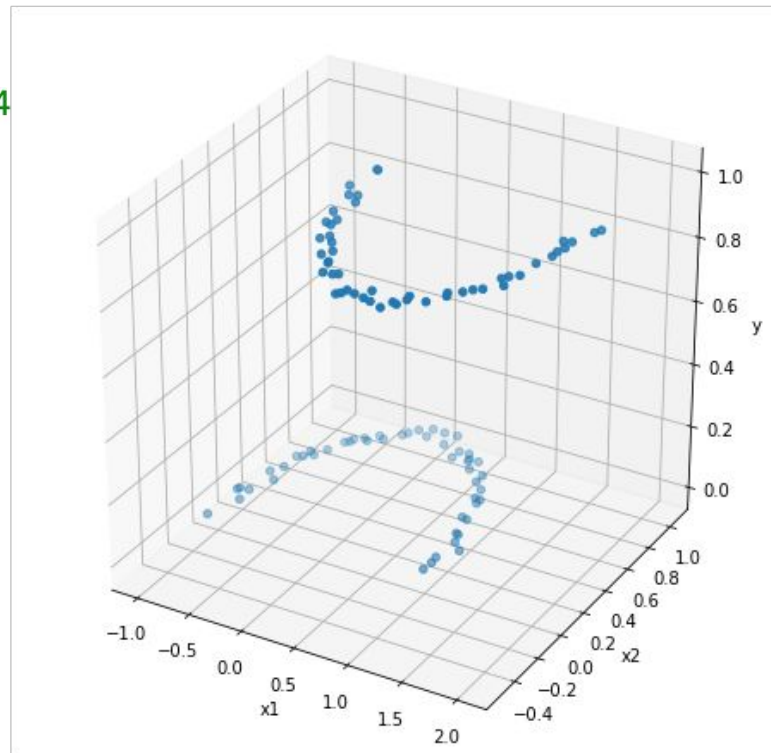


Generating data that contains nonlinearity

```
data = make_moons(100, random_state=3, noise=0.04)
features = pd.DataFrame(data[0])
labels = pd.Series(data[1])
```

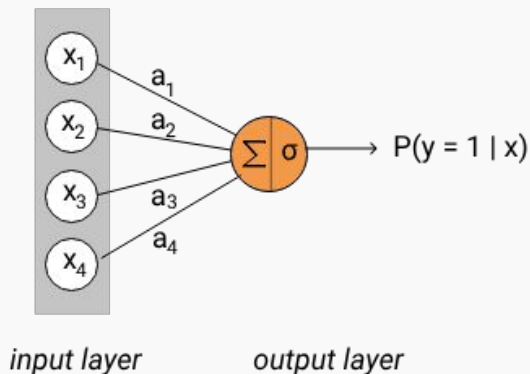
```
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')
```

```
ax.scatter(features[0], features[1], labels)
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y')
```

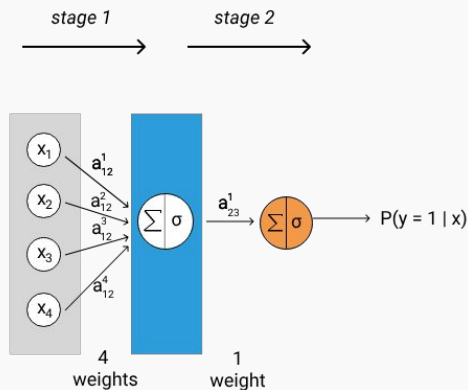


Hidden Layer with a single neuron

Binary Classification



$$\begin{bmatrix} 100 \times 4 \\ X \end{bmatrix} \begin{bmatrix} 4 \times 1 \\ a^T \end{bmatrix} = \begin{bmatrix} 100 \times 1 \\ P(y = 1 | x) \end{bmatrix}$$



$$\text{stage 1} \quad \sigma \left(\begin{bmatrix} 100 \times 4 \\ X \end{bmatrix} \begin{bmatrix} 4 \times 1 \\ a_1^T \end{bmatrix} \right) = \begin{bmatrix} 100 \times 1 \\ L_1 \end{bmatrix}$$

$$\text{stage 2} \quad \sigma \left(\begin{bmatrix} 100 \times 1 \\ L_1 \end{bmatrix} \begin{bmatrix} 1 \times 1 \\ a_2^T \end{bmatrix} \right) = \begin{bmatrix} 100 \times 1 \\ L_2 \end{bmatrix}$$

Training a neural network using scikit-learn

Scikit-learn contains two classes for working with neural networks:

- [MLPClassifier](#)
- [MLPRegressor](#)

```
from sklearn.neural_network import MLPClassifier
mlp = MLPClassifier()
mlp.fit(X_train, y_train)
predictions = mlp.predict(X_test)
```

```
mlp = MLPClassifier(hidden_layer_sizes=(6,), activation='logistic')
```

```
data = make_moons(1000, random_state=3, noise=0.04)
features = pd.DataFrame(data[0])
labels = pd.Series(data[1])
features["bias"] = 1

train_x, test_x, train_y, test_y = train_test_split(features,
                                                    labels,
                                                    test_size=0.30,
                                                    random_state=42)
```

```
mlp = MLPClassifier(hidden_layer_sizes=(4,),
                    activation='logistic', max_iter=10000)
mlp.fit(train_x, train_y)
nn_predictions = mlp.predict(test_x)
```

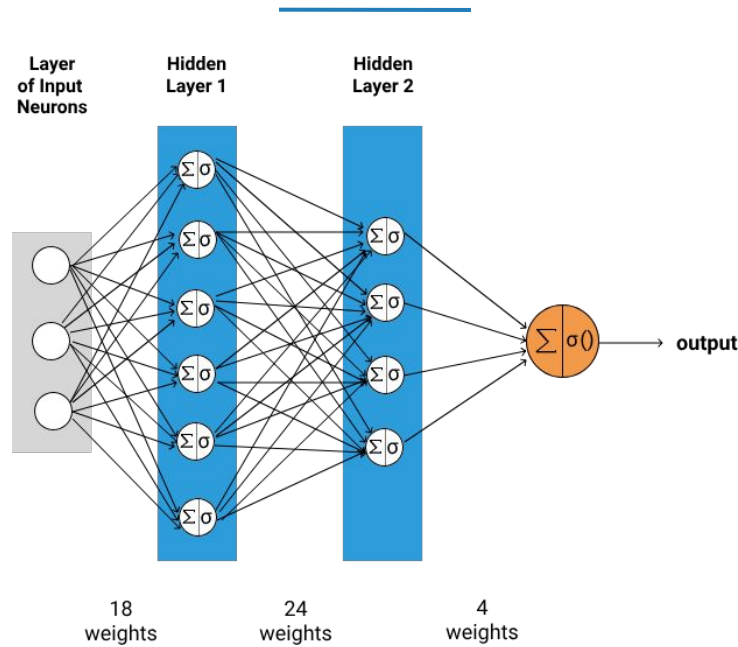
```
lr = LogisticRegression(solver='lbfgs')
lr.fit(train_x, train_y)
log_predictions = lr.predict(test_x)
```

0.88

0.88

```
nn_accuracy = accuracy_score(test_y, nn_predictions)
log_accuracy = accuracy_score(test_y, log_predictions)
```

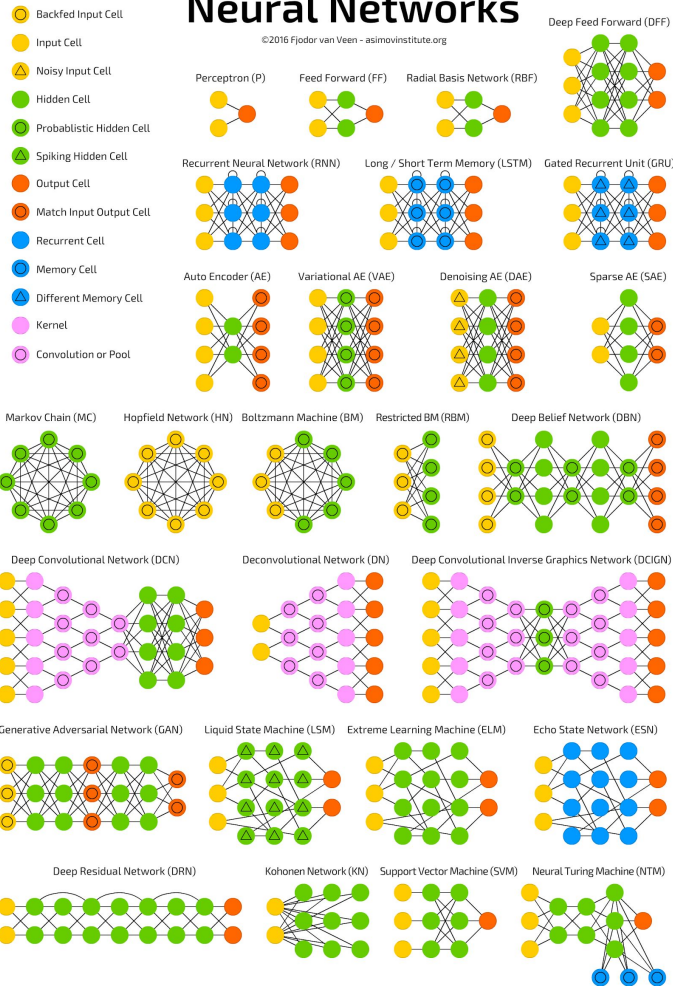
Multiple Hidden Layers



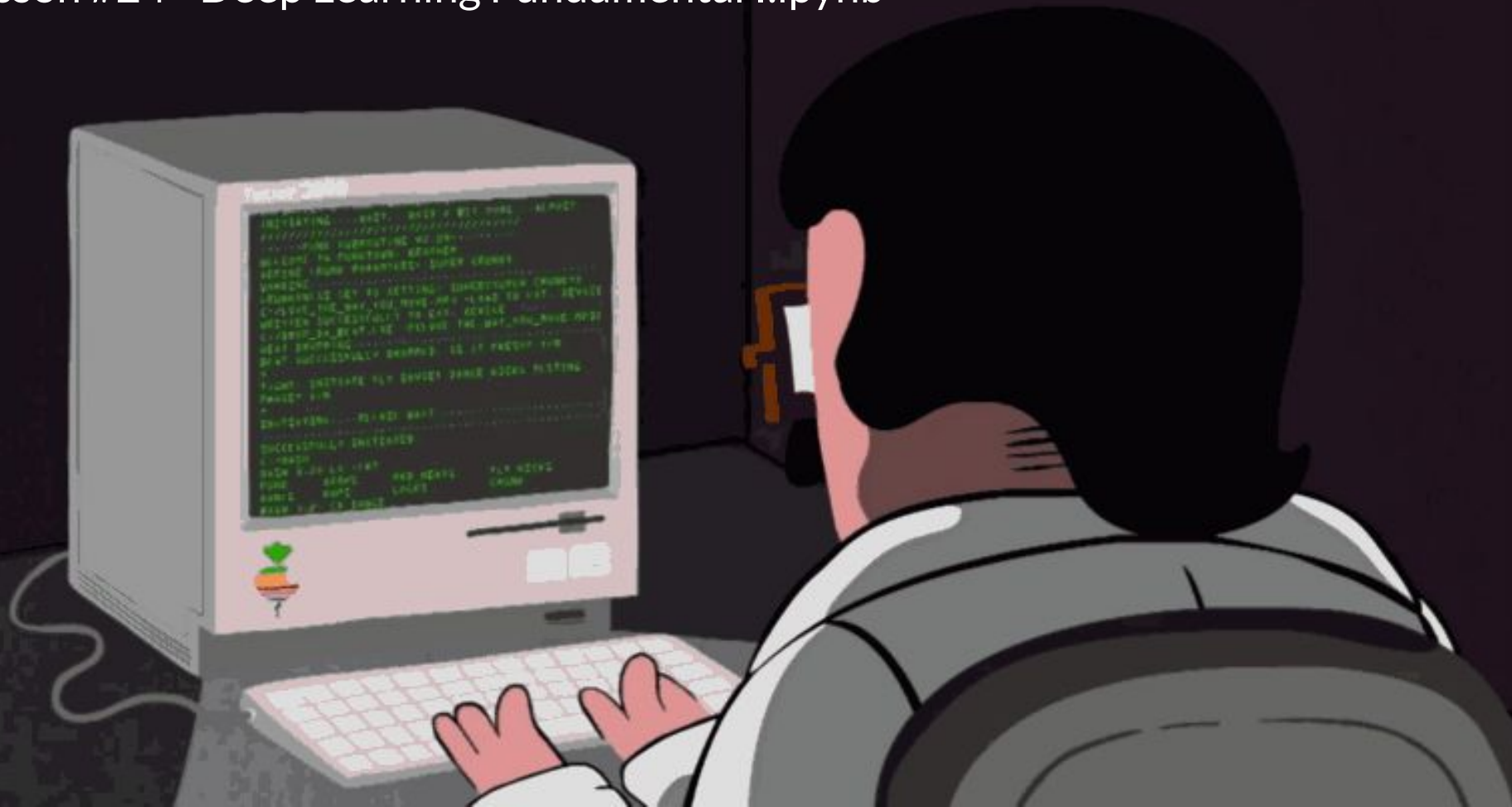
```
mlp = MLPClassifier(hidden_layer_sizes=(n,k))
```

Neural Networks

©2016 Fjodor van Veen - asimovinstitute.org



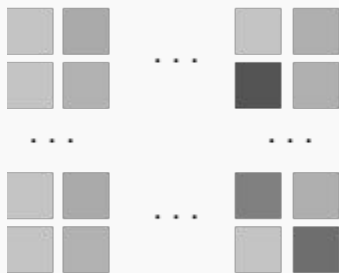
Lesson #24 - Deep Learning Fundamental I.ipynb



Case Study: building a handwritten digits classifier

Why is image classification a hard task?

Single Image in the Dataset



rendered image

50	90	...	70	90
50	82	...	180	70
...
50	120	50
50	50	120

pixel values

thousands or millions of columns

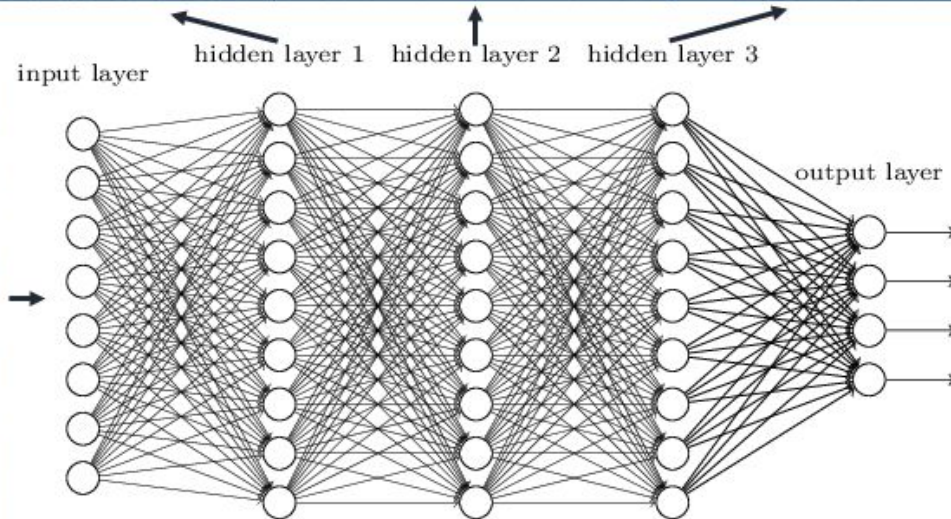
50	90	...	70	90	50	82	...	50	120
----	----	-----	----	----	----	----	-----	----	-----

single observation in the data

A 128 x 128 image has 16384 features

Why is deep learning effective in image classification?

Deep neural networks learn hierarchical feature representations

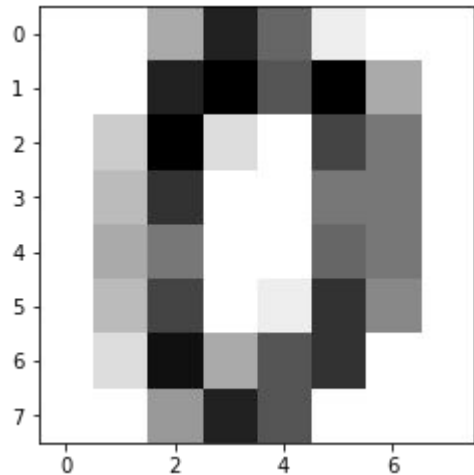


Working with image data

```
from sklearn.datasets import load_digits
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

```
digits = load_digits()
df_digits = pd.DataFrame(digits["data"])
labels = pd.Series(digits["target"])
```

```
first_image = df_digits.iloc[0].values.reshape(8,8)
plt.imshow(first_image, cmap='gray_r')
```

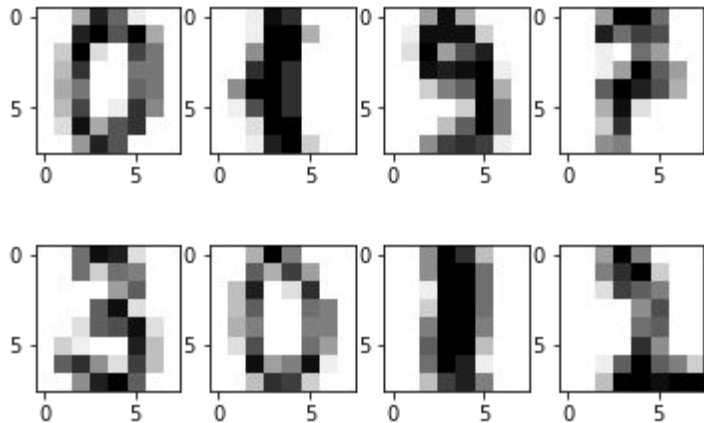


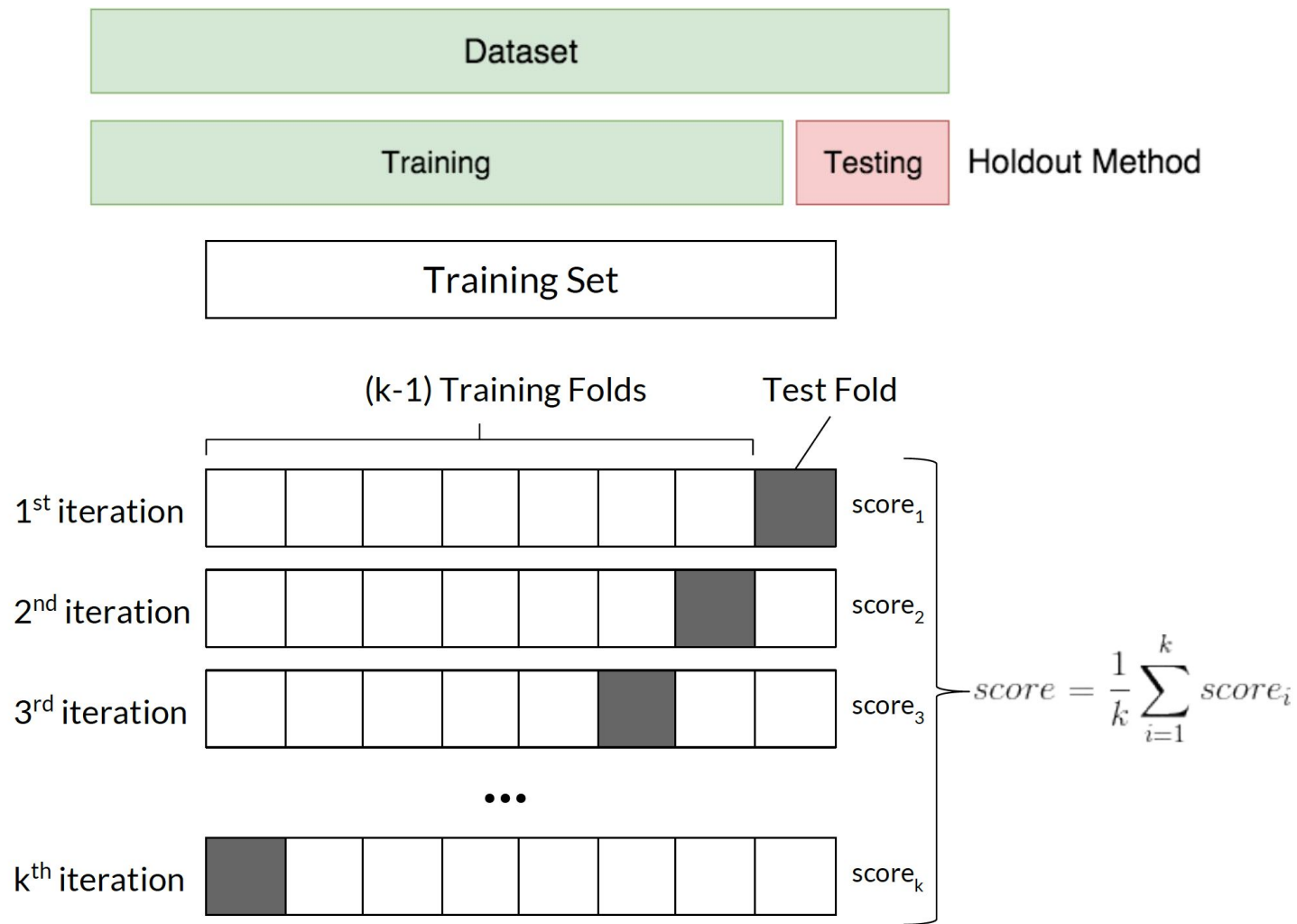
Working with image data

```
fig, ax = plt.subplots(2, 4)
```

```
selected_rows = [0, 99, 199, 299, 999, 1099, 1199, 1299]
```

```
for i, index in enumerate(selected_rows):  
    ax[i//4, i%4].imshow(df_digits.iloc[index].values.reshape(8,8), cmap='gray_r')
```





Cross-Validation - Step #1



Split dataset into train and test

```
train_x, test_x, train_y, test_y = train_test_split(df_digits,
                                                    labels,
                                                    test_size=0.30,
                                                    random_state=42)
```

df_digits + labels = Dataset

Cross-Validation - Step #2

```
# k-fold validation
# df_digits is our original data
kf = KFold(n_splits = 4, random_state=2)

for train_index, test_index in kf.split(train_x):

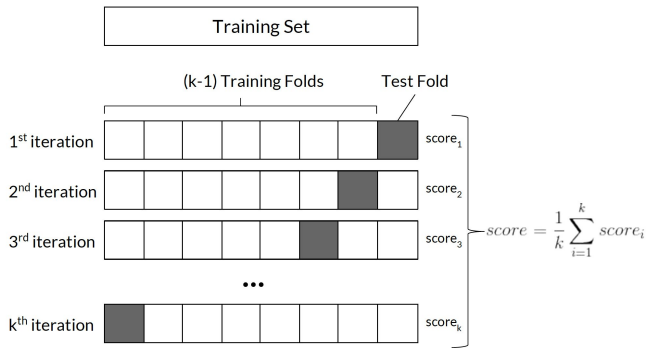
    # split each fold into train and test

    # create a model

    # make predictions

    # evaluate accuracy

np.mean(fold accuracies)
```



K-Nearest Neighbor Model

```
# number of neighbors  
neighbors = 3
```

```
for train_index, test_index in kf.split(train_x):
```

```
    # split each fold into train and test
```

```
    train_features, test_features = train_x.iloc[train_index], train_x.iloc[test_index]
```

```
    train_labels, test_labels = train_y.iloc[train_index], train_y.iloc[test_index]
```

```
    # create a knn classifier model
```

```
    knn = KNeighborsClassifier(n_neighbors = neighbors)
```

```
    knn.fit(train_features, train_labels)
```

```
    # make predictions
```

```
    predictions = knn.predict(test_features)
```

```
    # evaluate accuracy
```

```
    overall_accuracy = accuracy_score(test_labels, predictions)
```

```
    fold_accuracies.append(overall_accuracy)
```

```
np.mean(fold_accuracies)
```

```
[0.9809523809523809,  
 0.9840764331210191,  
 0.9968152866242038,  
 0.9745222929936306]
```

```
0.9840915984228086
```



K-Nearest Neighbor - Final Model

number of neighbors

neighbors = 3

create a knn classifier model

knn = KNeighborsClassifier(n_neighbors = neighbors)

knn.fit(train_x, train_y)

make predictions

predictions = knn.predict(test_x)

evaluate accuracy

overall_accuracy = accuracy_score(test_y, predictions)

overall_accuracy

0.9888888888888889

Neural Network with one hidden layer

architecture of network
one hidden layer with 64 neurons
 nn_one_neurons = (64,)

```
for train_index, test_index in kf.split(train_x):
```

```
    # split each fold into train and test
```

```
    train_features, test_features = train_x.iloc[train_index], train_x.iloc[test_index]
```

```
    train_labels, test_labels = train_y.iloc[train_index], train_y.iloc[test_index]
```

```
    # create a MLP classifier model
```

```
    mlp = MLPClassifier(hidden_layer_sizes=nn_one_neurons,  
                        max_iter=500, activation="logistic",  
                        solver='adam')
```

```
    mlp.fit(train_features, train_labels)
```

```
    # make predictions
```

```
    predictions = mlp.predict(test_features)
```

```
    # evaluate accuracy
```

```
    overall_accuracy = accuracy_score(test_labels, predictions)
```

```
    fold_accuracies.append(overall_accuracy)
```

```
[0.9555555555555556,  
 0.9585987261146497,  
 0.9904458598726115,  
 0.9426751592356688]
```

```
0.9618188251946214
```


NN with one hidden layer - Final Model

```
from sklearn.neural_network import MLPClassifier
from sklearn.model_selection import KFold
```

```
# architecture of network
# one hidden layer with 64 neurons
nn_one_neurons = (64,)
```

0.9796296296296296

```
# create a MLP classifier model
mlp = MLPClassifier(hidden_layer_sizes=nn_one_neurons,
                    max_iter=500, activation="logistic",
                    solver='adam')
mlp.fit(train_x, train_y)
```

```
# make predictions
predictions = mlp.predict(test_x)
```

```
# evaluate accuracy
overall_accuracy = accuracy_score(test_y, predictions)
```

NN with two or three layers hidden layers

```
# architecture of network
```

```
# two hidden layers with 128 neurons
```

```
nn_two_neurons = (128,128)
```

```
# three hidden layers with 128 neurons
```

```
nn_three_neurons = (128,128,128)
```

?