Hybrid dynamical modeling and control of public transport systems

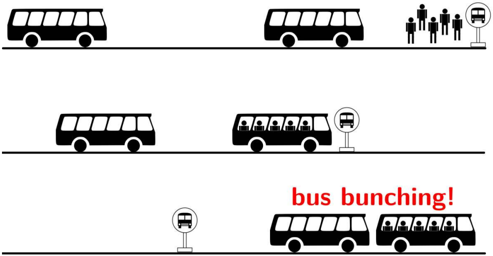
Işık İlber Sırmatel, Nikolas Geroliminis

Urban Transport Systems Laboratory, EPFL

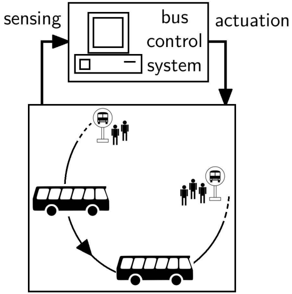
hEART2016, 15.09.2016

Motivation

Problem: Inefficiency



Solution: Control



Literature review - Control of bus systems[1]

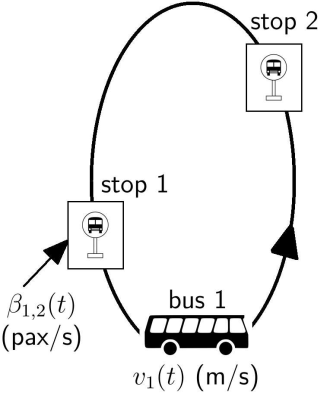
- **Station control** (only at some stops)
  - Holding
    - Eberlein, Wilson, and Bernstein 2001
    - Daganzo 2009
  - Stop-skipping
    - Fu, Liu, and Calamai 2003
    - Cortés et al. 2010
- **Inter-station control** (while buses are moving)
  - Traffic signal priority
    - Liu, Skabardonis, and Zhang 2003
    - Van Oort, Boterman, and Van Nes 2012
  - **Bus speed control (focus of the talk)**
    - Daganzo and Pilachowski 2011
    - Ampountolas and Kring 2015

[1] OJ Ibarra-Rojas et al. (2015). *Transportation Research Part B: Methodological* 77, pp. 38–75.

Hybrid modeling - States

Continuous states

- Distance of bus 1 from stop 1 at time t: $x_1(t) \in \mathbb{R}$
- No. of pax on bus 1 at time t: $n_1(t) \in \mathbb{R}$
- No. of pax at stop 1 at time t: $m_1(t) \in \mathbb{R}$



Binary states

- Is bus 1 holding at stop 2 at time t?

$$\delta_{1,2}(t) = \begin{cases} 0 & \rightarrow \quad \text{no} \\ 1 & \rightarrow \quad \text{yes} \end{cases}$$

- Is bus 1 cruising to stop 2 at time t?

$$\gamma_{1,2}(t) = \begin{cases} 0 & \rightarrow \quad \text{no} \\ 1 & \rightarrow \quad \text{yes} \end{cases}$$

Hybrid modeling - Continuous dynamics

- **Bus position**

$$x_1(t+1) = \overbrace{(\gamma_{1,1}(t)+\gamma_{1,2}(t))(x_1(t)+T_s v_1(t))}^{\text{cruising}} + \dots$$
$$\overbrace{\delta_{1,2}(t)x_1(t)}^{\text{holding}} + \overbrace{\delta_{1,1}(t)0}^{\text{reset}}$$

- **Bus accumulation**

$$n_1(t+1) = n_1(t) + \overbrace{\delta_{1,1}(t)q^{\text{in}}_{1,2}(t)}^{\text{boarding}} - \overbrace{\delta_{1,2}(t)q^{\text{out}}_{1,2}(t)}^{\text{alighting}}$$

- **Stop accumulation**

$$m_1(t+1) = m_1(t) + \overbrace{T_s\beta_{1,2}(t)}^{\text{accumulating}} - \overbrace{\delta_{1,1}(t)q^{\text{in}}_{1,2}(t)}^{\text{alighting}}$$

Hybrid modeling - Events

- **“Bus nonempty” event**

$$e_1^n(t) = \begin{cases} 0 & \text{if } n_1(t) = 0 \\ 1 & \text{otherwise} \end{cases}$$

- **“Stop reached” event**

$$e_{1,2}^x(t) = \begin{cases} 0 & \text{if } x_1(t) < D_2 \\ 1 & \text{otherwise} \end{cases}$$

Hybrid modeling - Binary dynamics

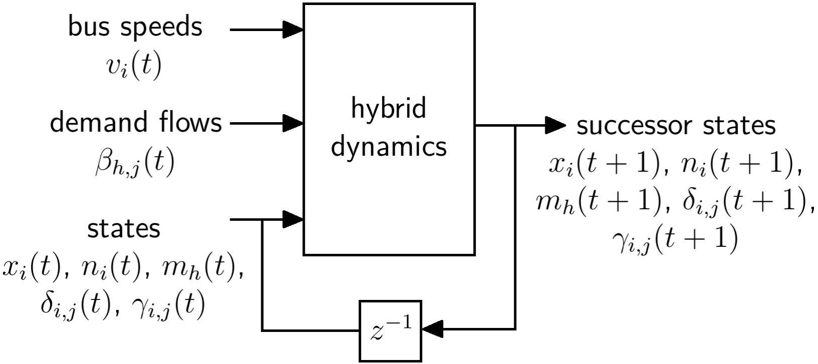
- **Holding state**

$$\delta_{1,2}(t+1) = \overbrace{\Big(\gamma_{1,2}(t) \wedge e^{x}_{1,2}(t)\Big)}^{\text{start holding}} \vee \overbrace{\Big(\delta_{1,2}(t) \wedge e^{n}_{1}(t)\Big)}^{\text{keep holding}}$$

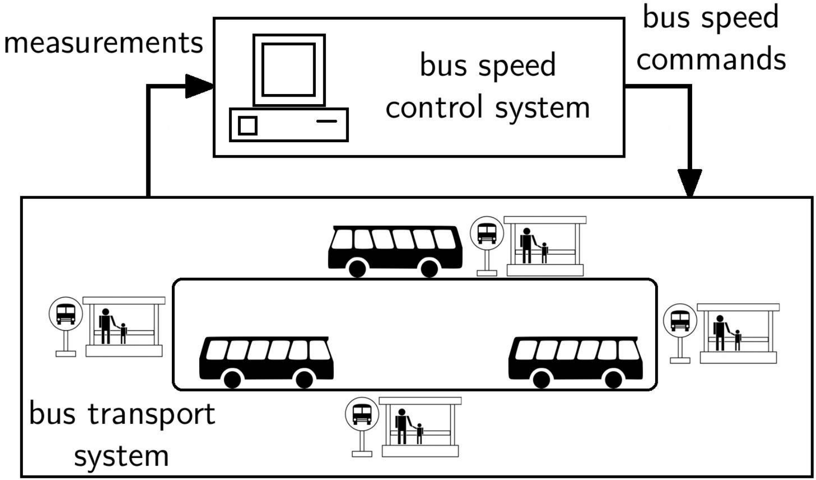
- **Cruising state**

$$\gamma_{1,1}(t+1) = \overbrace{\Big(\delta_{1,2}(t) \wedge \neg e^{n}_{1}(t)\Big)}^{\text{start cruising}} \vee \overbrace{\Big(\gamma_{1,1}(t) \wedge \neg e^{x}_{1,1}(t)\Big)}^{\text{keep cruising}}$$

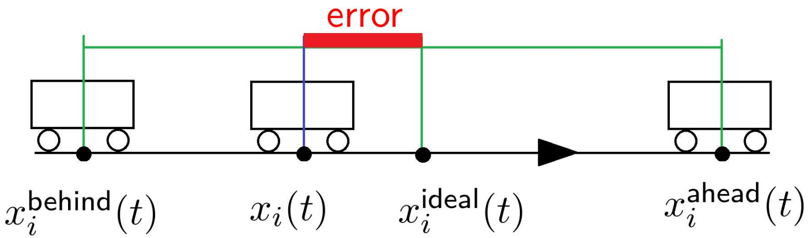
Hybrid modeling - Simulation



Control - Bus speed control



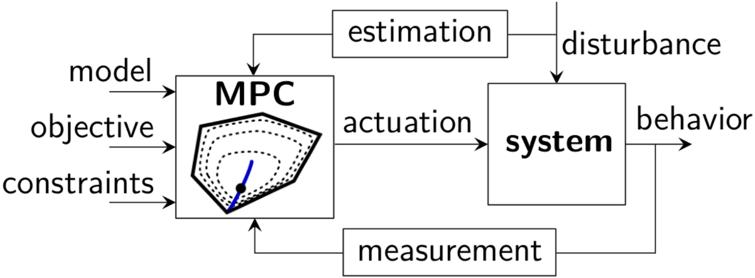
Control - Double control

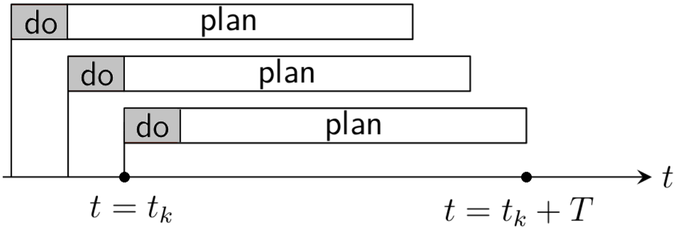


$$x_i^{\text{ideal}}(t) = \frac{x_i^{\text{ahead}}(t) + x_i^{\text{behind}}(t)}{2}$$

$$v_i(t+1) = v_i(t) + K_{\text{DC}} \cdot \underbrace{\boxed{\left(x_i^{\text{ideal}}(t) - x_i(t)\right)}}_{\text{error}}$$

Control - Intro to model predictive control





Control - Linear MPC for bus speed control

regularize headways

drive as fast as possible

$$\underset{\{v_i(k)\}_{k=0}^{N-1}}{\text{minimize}} \quad \sum_{i=1}^{K_b}\left(\boxed{\sum_{k=1}^{N}(x_i(k)-\hat{x}_i(k))^2} + \sigma \boxed{\sum_{k=0}^{N-1}(v_i(k)-v_{i,\mathsf{max}}(k))^2}\right)$$

subject to for $i = 1, \ \ldots, \ K_b$:

$$\boxed{x_i(0) = \tilde{x}_i(t_c)}$$

measurement as initial state

for $k = 1, \ \ldots, \ N$:

ideal position of bus i

$$\boxed{\hat{x}_i(k) = 0.5(x_i^{\mathsf{a}}(k) + x_i^{\mathsf{b}}(k))}$$

for $k = 0, \ \ldots, \ N-1$:

dynamical model of bus motion

$$\boxed{x_i(k+1) = x_i(k) + T_p v_i(k)}$$

bounds on bus speed

$$\boxed{v_{i,\mathsf{min}}(k) \le v_i(k) \le v_{i,\mathsf{max}}(k)}$$

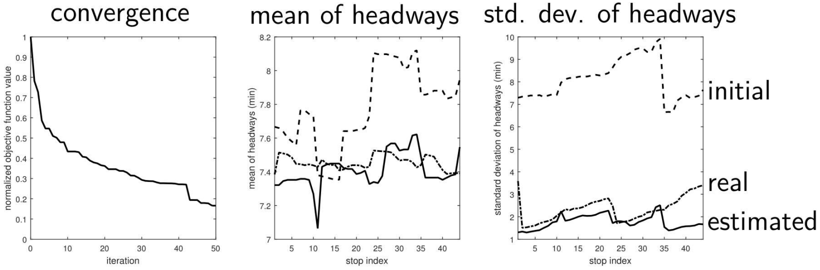
Case study - Setup

- **Bus system description**
  - Bus line 2 of Fribourg (Switzerland) bus network
  - 9 buses, 44 stops, 15 km loop
  - Demands estimated from bus data of 2 months
  - Bus speed bounds extracted from same data
- **Compared schemes**
  - No control, holding inactive (NC-HI)
  - No control, holding active (NC-HA)
  - Double control (DC)
  - Linear model predictive control (LMPC)

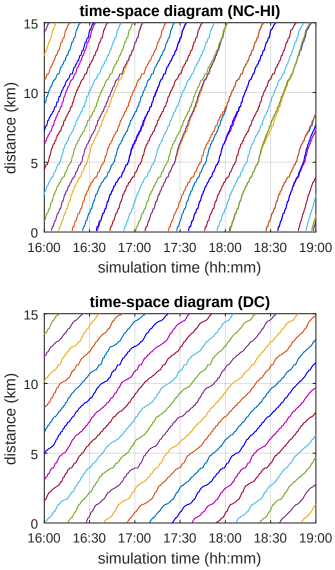
Case study - Demand estimation

$$\underset{\boldsymbol{\beta}}{\text{minimize}} \quad \sum_{j=1}^{K_s} \left(\overbrace{\left(\mu_j^{H,\mathsf{d}} - \mu_j^{H,\mathsf{s}}\left(\{\tilde{\boldsymbol{x}}\}_{t=0}^{t_f}\right)\right)^2}^{\text{match headway mean of stop } j} + \overbrace{\left(\sigma_j^{H,\mathsf{d}} - \sigma_j^{H,\mathsf{s}}\left(\{\tilde{\boldsymbol{x}}\}_{t=0}^{t_f}\right)\right)^2}^{\text{match headway std. dev. of stop } j} \right)$$

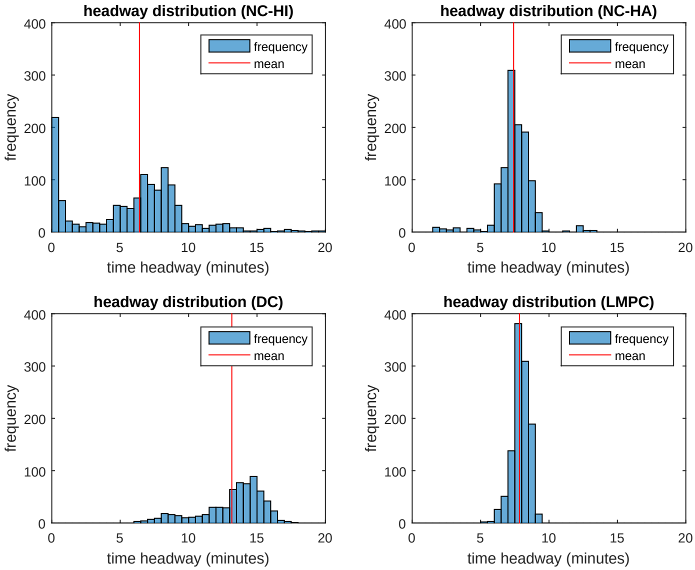
$$\text{subject to} \quad \overbrace{\{\tilde{\boldsymbol{x}}(t)\}_{t=1}^{t_f+1} = \boldsymbol{f}_{\tilde{\boldsymbol{x}}}(\tilde{\boldsymbol{x}}(0), \{\boldsymbol{v}(t)\}_{t=0}^{t_f}, \boldsymbol{\beta})}^{\text{one simulation experiment with } \boldsymbol{\beta}}$$



Case study - Time-space diagrams



Case study - Headway distributions



Case study - Performance evaluation

control scheme	mean commercial speed (km/h)	mean travel time per pax (min)	mean of headways (min)	std. dev. of headways (min)
NC-HI	17.4	12.7	6.4	5
NC-HA	14.8	11.9	7.4	2.3
DC	8.2	22.1	13.2	2.4
LMPC	14.2	13.1	7.8	0.61

Conclusion

Contributions

- Hybrid dynamical bus system model
- LMPC scheme for bus speed control

Result

- Possible to regularize headways via LMPC

Ongoing/future work

- Develop hybrid MPC scheme
- Extend to larger (multi-loop) bus systems