Investigations of novel model predictive control structures for hybrid and adaptive control

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1. Model predictive control overview

2. Hybrid model predictive control of mixed integer-input linear systems

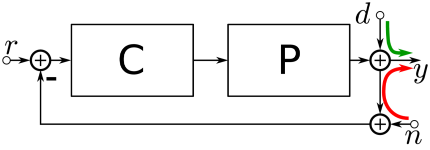
3. Adaptive model predictive control of multiple-input multiple-output systems

4. Conclusion

Section 1

Model predictive control overview

Classical control: Design C.

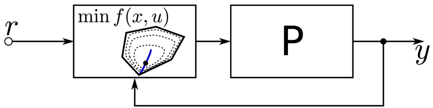


Dominant issues addressed:

- Disturbance rejection ($d \rightarrow y$)
- Noise insensitivity ($n \rightarrow y$)
- Model uncertainty

(usually in *frequency domain*)

MPC: Find $u(t)$ via real-time, repeated optimization.



Dominant issues addressed:

- Control constraints (limits)
- Process constraints (safety)

(usually in *time domain*)

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Constraints in control

All physical systems have **constraints**:

- Physical constraints, e.g. actuator limits
- Performance constraints, e.g. overshoot
- Safety constraints, e.g. temperature/pressure limits

Optimal operating points are often near constraints.

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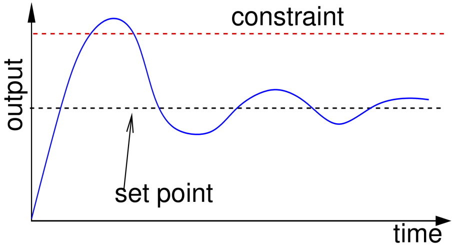
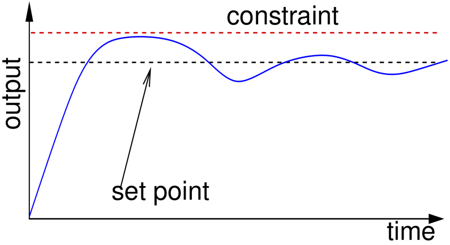
Classical control vs. MPC - Handling constraints

Classical control:



- Ad hoc constraint management
- Set point sufficiently far from constraints
- Suboptimal plant operation

MPC:



- **Constraints included in the design**
- Set point optimal
- Optimal plant operation

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General MPC problem

$$
\begin{array}{rlll}
u_N^*(x(t)) \triangleq \underset{u_N}{\operatorname{argmin}} & \displaystyle\sum_{k=0}^{N-1} J_{t+k}(x_{t+k},\ u_{t+k}) & & \text{objective function} \\
\text{subject to} & x_t = x(t) & & \text{measurement} \\
& x_{t+k+1} = f_{t+k}(x_{t+k},\ u_{t+k}) & & \text{system model} \\
& x_{t+k} \in \mathcal{X} & & \text{state constraints} \\
& u_{t+k} \in \mathcal{U} & & \text{input constraints} \\
& u_N \triangleq \{u_{t+k}\}_{k=0}^{N-1} & & \text{optimization variables}
\end{array}
$$

Problem is defined by

- **Objective** that is minimized,
  e.g., distance from origin, sum of squared/absolute errors, economic, ...
- Internal **system model** to predict behaviour,
  e.g., linear, nonlinear, single-/multi-variable, ...
- **Constraints** that have to be satisfied,
  e.g., on inputs, outputs, states, linear, quadratic, ...

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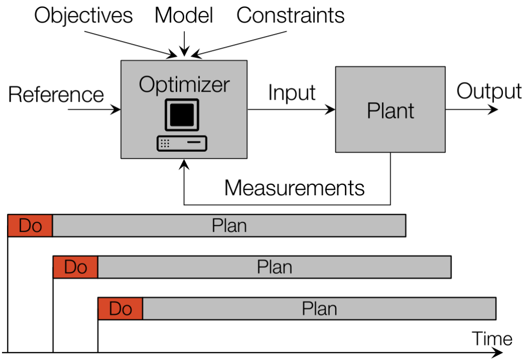
Standard linear MPC problem

$$
\begin{aligned}
u_N^*(x(t)) \triangleq \underset{u_N}{\text{argmin}} \quad & \sum_{k=0}^{N-1} \left(x_{t+k}^T Q x_{t+k} + u_{t+k}^T R u_{t+k} \right) && \text{objective function} \\
\text{subject to} \quad & x_t = x(t) && \text{measurement} \\
& x_{t+k+1} = A x_{t+k} + B u_{t+k} && \text{system model} \\
& C x_{t+k} \leq e && \text{state constraints} \\
& D u_{t+k} \leq g && \text{input constraints} \\
& u_N \triangleq \{u_{t+k}\}_{k=0}^{N-1} && \text{optimization variables}
\end{aligned}
$$

- Convex quadratic objective ($Q \succeq 0,\ R \succ 0$)
- Linear dynamics and affine constraints

standard linear MPC $\rightarrow$ convex Quadratic Program (QP)
can be solved reliably and efficiently!

MPC - Receding horizon control



Receding horizon strategy introduces **feedback**.

At each sample time (i.e., in receding horizon):

1. Measure/estimate current state $x(t)$
2. Find the *optimal input* sequence for the entire planning window (i.e., prediction horizon) N: $u_N^* \triangleq \{u_{t+k}^*\}_{k=0}^{N-1}$
3. Implement only the *first* control action u_t^*

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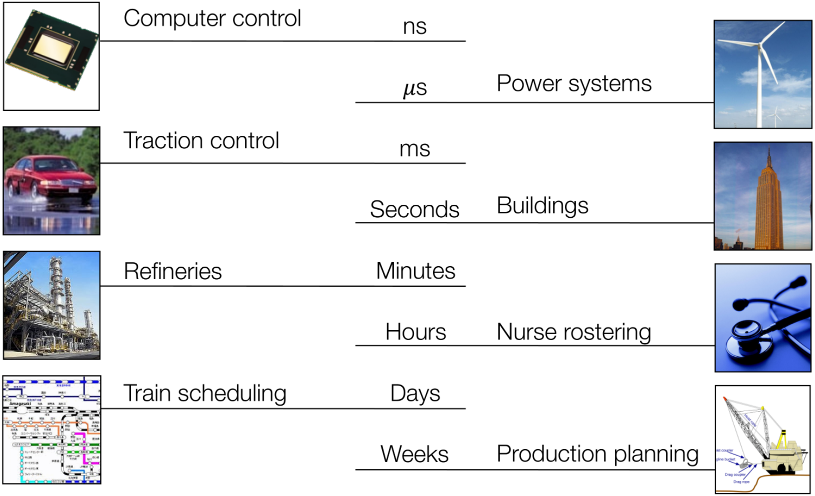
MPC - Applications



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MPC - Important aspects

Main advantages:

- Systematic approach for handling *constraints*
- High *performance* controller

Main challenges:

- *Implementation*: MPC problem has to be solved in real-time, i.e. within the sampling interval of the system, and with available hardware (storage, processor, ...)
- *Stability*: Closed-loop stability, i.e. convergence, is not automatically guaranteed
- *Robustness*: The closed-loop system is not necessarily robust against uncertainties or disturbances
- *Feasibility*: Optimization problem may become infeasible at some future time step, i.e. there may not exist a plan satisfying all constraints

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Section 2

Hybrid model predictive control of mixed integer-input linear systems

Hybrid MPC - Introduction

Motivation

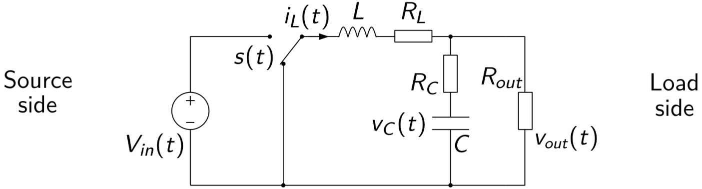
- Optimal control formulations for hybrid systems in discrete time usually require solving Mixed Integer Quadratic Programs (MIQPs).
- Real-time solution of MIQPs is computationally demanding and thus limited to control problems with slow sampling rates.

Aims

- We investigate the possibility of obtaining good feasible solutions without too much computational effort, through solving a single convex relaxation of an MIQP.
- For a special class of hybrid systems, namely mixed integer-input linear systems (MILSs), linear (QP) relaxations were previously shown to be promising. Here we investigate stronger convex (Semidefinite Program - SDP) relaxations.

A simple hybrid system - Buck converter

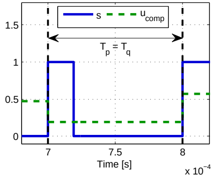
Takes an unregulated DC voltage as input, outputs a lower DC voltage and regulates it at a desired level against disturbances.



States: Inductor current $i_L(t)$, capacitor voltage $v_C(t)$ (continuous).
Control input: Switch position $s(t)$ (binary).

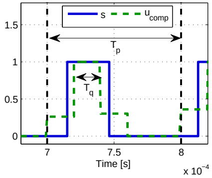
Averaged vs. hybrid models

Averaged model



- Switch position $s(t) \in \{0, 1\}$ $\rightarrow$ Duty cycle $\mathrm{u}(t) \in [0, 1]$.
- Linear dynamics; standard linear MPC can be used.
- Problem is convex QP; computationally cheap.

Hybrid model



- Switching dynamics included in the model as $s(t) \in \{0, 1\}$.
- Hybrid dynamics; standard linear MPC cannot be used.
- Problem is MIQP (nonconvex); computationally expensive.
- With special formulation, yields an MILS.

Hybrid MPC problem - QP

$$
\begin{array}{rlr}
u_N^*(x(t)) \triangleq \underset{u_N}{\operatorname{argmin}} & \displaystyle\sum_{k=0}^{N-1} \left(x_{t+k}^T Q x_{t+k} + u_{t+k}^T R u_{t+k}\right) & \text{objective function} \\
\text{subject to} & x_t = x(t) & \text{measurement} \\
& x_{t+k+1} = A x_{t+k} + B u_{t+k} & \text{system model} \\
& C x_{t+k} \leq e & \text{state constraints} \\
& D u_{t+k} \leq g & \text{input constraints} \\
& u_N \triangleq \{u_{t+k}\}_{k=0}^{N-1} & \text{optimization variables}
\end{array}
$$

- Convex quadratic objective ($Q \succeq 0,\ R \succ 0$)
- Linear dynamics and affine constraints

hybrid MPC with averaged model $\rightarrow$ QP (convex problem)
can be solved reliably and efficiently!

Hybrid MPC problem - MIQP

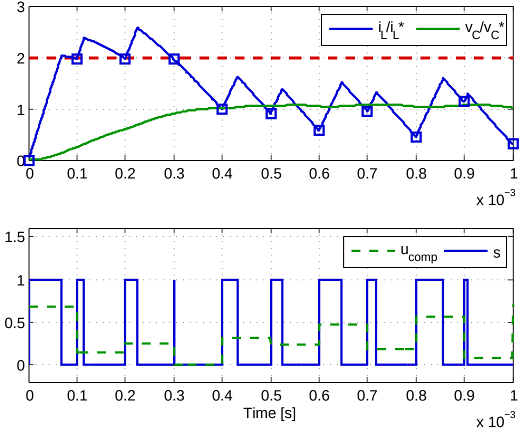
$$
\begin{array}{rll}
u_N^*(x(t)) \triangleq \underset{u_N}{\operatorname{argmin}} & \displaystyle\sum_{k=0}^{N-1} \left(x_{t+k}^T Q x_{t+k} + u_{t+k}^T R u_{t+k} \right) & \text{objective function} \\
\text{subject to} & x_t = x(t) & \text{measurement} \\
& x_{t+k+1} = A x_{t+k} + B u_{t+k} & \text{system model} \\
& C x_{t+k} \leq e & \text{state constraints} \\
& D u_{t+k} \leq g & \text{input constraints} \\
& u_N \triangleq \{u_{t+k}\}_{k=0}^{N-1} & \text{optimization variables} \\
& u_{t+k,i} \in \{0,1\},\ i \in \mathfrak{B} & \text{integrality constraints}
\end{array}
$$

- Convex quadratic objective ($Q \succeq 0,\ R \succ 0$)
- Hybrid dynamics and affine constraints
- Some optimization variables are **integers**

hybrid MPC with hybrid model $\rightarrow$ MIQP (nonconvex problem)
cannot be solved reliably and efficiently!

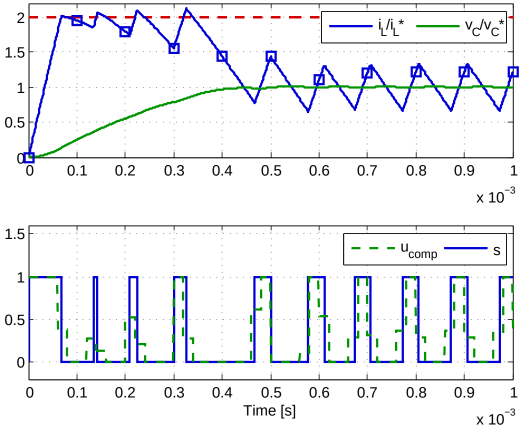
Operation of QP based hybrid MPC

- Step response of buck converter under hybrid MPC based on averaged model.
- Top: States (normalized).
- Bottom: Output of MPC (green dashed) and the binary input.
- Severe violation of constraint on $i_L(t)$.
- Some steady-state error in $v_C(t)$.



Operation of MIQP based hybrid MPC

- Step response of buck converter under hybrid MPC based on hybrid model.
- Top: States (normalized).
- Bottom: Output of MPC (green dashed) and the binary input.
- Mild violation of i_L constraint.
- Small steady-state error in v_C.



Improvement of quasi steady state with hybrid model

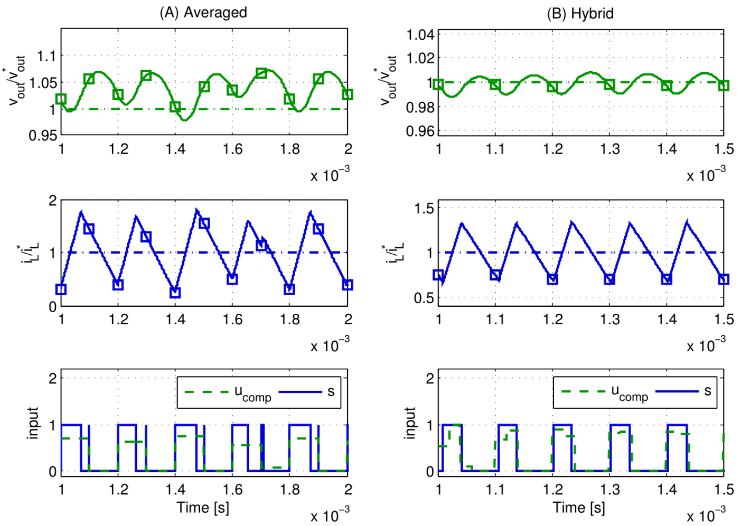


Figure 1: Quasi steady state behaviour of buck converter under hybrid MPC using (A) averaged model, and (B) hybrid model.

Relaxation-and-projection (RaP) method for hybrid MPC

- A single relaxed problem is much easier to solve than an MIQP.
- For MILSs it is easy to recover a feasible control sequence from the relaxed solution. We can
  1. **easily design a suitable projection** for recovering feasible solutions,
  2. **compensate** for the projection-induced state uncertainty by **robustification through contraction of the state constraints**.

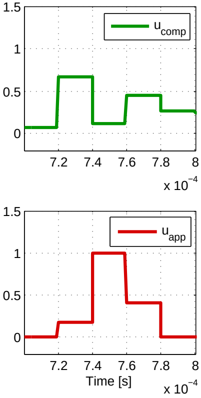
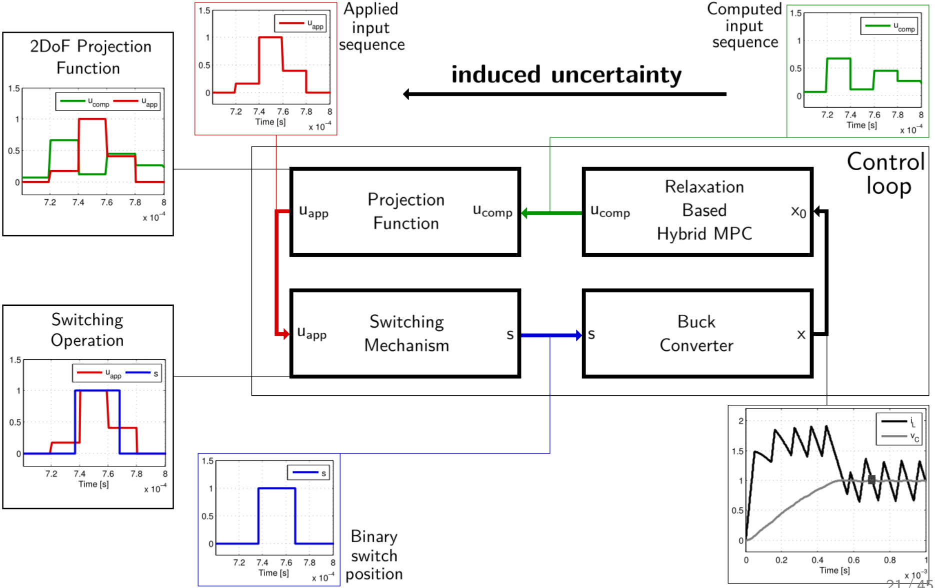


Figure 2: Typical relaxed solution and its projected version.

RaP method based hybrid MPC - Architecture



QP-RaP based hybrid MPC problem - QP

$$
\begin{aligned}
u_N^*(x(t)) \triangleq \underset{u_N}{\operatorname{argmin}} \quad & \sum_{k=0}^{N-1} \left(x_{t+k}^T Q x_{t+k} + u_{t+k}^T R u_{t+k} \right) && \text{objective function} \\
\text{subject to} \quad & x_t = x(t) && \text{measurement} \\
& x_{t+k+1} = A x_{t+k} + B u_{t+k} && \text{system model} \\
& C x_{t+k} \le e && \text{state constraints} \\
& D u_{t+k} \le g && \text{input constraints} \\
& u_N \triangleq \{u_{t+k}\}_{k=0}^{N-1} && \text{optimization variables} \\
& u_{t+k,i} \in [0,1], \ \ i \in \mathfrak{B} && \text{QP relaxation}
\end{aligned}
$$

- Convex quadratic objective ($Q \succeq 0, \ R \succ 0$)
- Hybrid dynamics (relaxed) and affine constraints

QP-RaP based hybrid MPC $\rightarrow$ QP (convex problem)
can be solved reliably and efficiently!

SDP-RaP based hybrid MPC problem - SDP

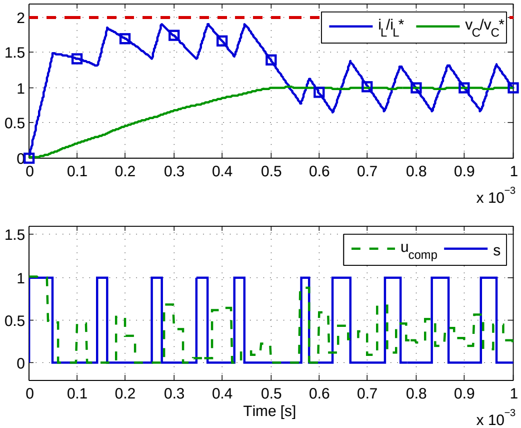
$$
\begin{array}{rlr}
u_N^*(x(t)) \triangleq \underset{u_N,\ U_N}{\operatorname{argmin}} & \displaystyle\sum_{k=0}^{N-1} \left(x_{t+k}^T Q x_{t+k} + u_{t+k}^T R u_{t+k} \right) & \text{objective function} \\
\text{subject to} & x_t = x(t) & \text{measurement} \\
& x_{t+k+1} = A x_{t+k} + B u_{t+k} & \text{system model} \\
& C x_{t+k} \leq e & \text{state constraints} \\
& D u_{t+k} \leq g & \text{input constraints} \\
& u_N \triangleq \{u_{t+k}\}_{k=0}^{N-1} & \text{optimization variables} \\
& \begin{bmatrix} U_N & u_N \\ u_N^T & 1 \end{bmatrix} \succeq 0,\ U_N \in \mathbb{S}_+^{Nm} & \text{SDP relaxation} \\
& U_{N,ii} = u_{t+k,i},\ i \in \mathfrak{B} &
\end{array}
$$

- Convex quadratic objective ($Q \succeq 0,\ R \succ 0$)
- Hybrid dynamics (relaxed) and affine constraints

SDP-RaP based hybrid MPC $\rightarrow$ SDP (convex problem)
can be solved reliably and efficiently!

Operation of QP- or SDP-RaP based hybrid MPC

- Step response of buck converter under QP- or SDP-RaP based hybrid MPC.
- Top: States (normalized).
- Bottom: Output of MPC (green dashed) and the binary input.
- Constraint on $i_L(t)$ satisfied.
- Small steady-state error in $v_C(t)$.



Performance comparison of hybrid MPC controllers

MPC	QP	MIQP	QP-RaP	SDP-RaP
RMS deviation (Volt)	1.55	0.20	0.24	0.22
Solver time, $N = 10(\cdot T_s)$	2 ms	12 ms	8 ms	2.438 s

Table 1: Summary of hybrid MPC performances, showing: RMS deviation of v_C from the reference v_C^* in quasi steady state and solver times for one instance with $N = 10$.

Section 3

Adaptive model predictive control of multiple-input multiple-output systems

Adaptive MPC - Introduction

Motivation

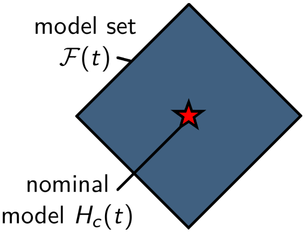
- A recently developed adaptive MPC algorithm enables constrained control of linear multiple-input multiple-output (MIMO) systems with unknown dynamics, via integrating real-time Set-Membership Identification (SMI) and MPC.
- The polytopic SMI engine of the algorithm is very simple and cannot handle time-varying systems.

Aims

To enhance the adaptive MPC algorithm, we investigate:

- Methods to improve performance of the polytopic SMI engine.
- Extensions to handle time-varying systems.
- Zonotopic SMI methods to reduce computational effort.

Adaptive MPC - Problem formulation



FIR model of length m:

$$y(t) = \sum_{k=1}^{m} u(t-k)h(k) + d(t)$$
$$= \varphi(t)^T H + d(t)$$

Regressor vector:

$$\varphi(t) \triangleq [u(t-1), \ldots, u(t-m)]^T$$

Measured output: $\tilde{y}(t) = y(t) + v(t)$

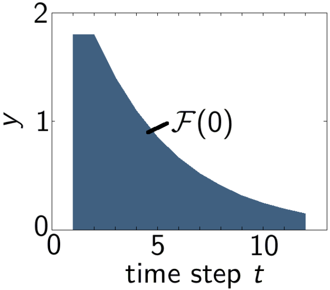
Adaptive MPC - Problem formulation

Prior assumption on disturbance and noise:

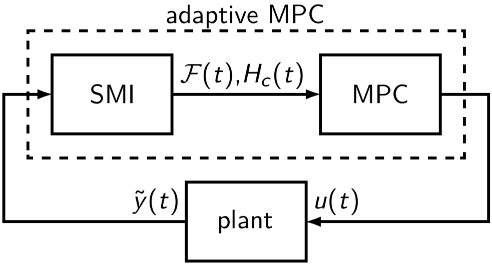
$$|d(t)| \leq \epsilon_d, \quad |v(t)| \leq \epsilon_v, \ \forall t \in \mathbb{Z}$$

Prior assumption on system:

True plant is inside $\mathcal{F}(0)$.



Adaptive MPC - Architecture



Real-time SMI engine:

- Recursively identify model set $\mathcal{F}(t)$.
- Remove redundant faces (LP).
- Compute nominal model (LP).

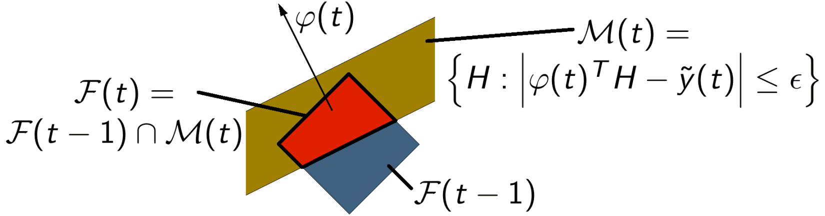
Properties of the algorithm:

- Offset free reference tracking for constant output disturbances.
- Robust output constraint satisfaction and recursive feasibility if the model set is non-expanding, i.e., $\mathcal{F}(t+1) \subseteq \mathcal{F}(t)$.

Robust MPC (QP):

- Minimize tracking error for nominal model $H_c(t)$.
- Enforce output constraints for all models inside the model set $\mathcal{F}(t)$.

Existing polytopic SMI - Basic polytopic update (PU)

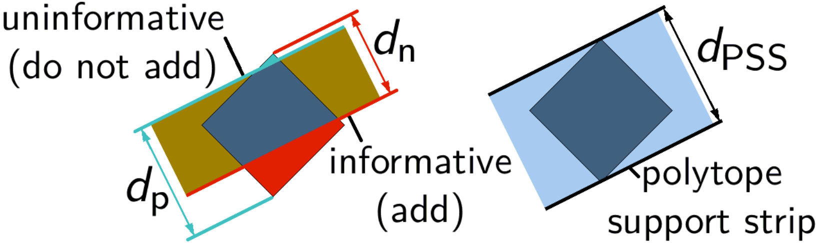


Weaknesses:

- Updates without considering informativeness of new faces.
- Cannot handle time-varying systems.
- Needs to bound number of faces: Stops updating when face number limit reached.

Improvement on polytopic SMI - Face filtering PU (FFPU)

Idea: Evaluate informativeness of new faces, add only if informative.

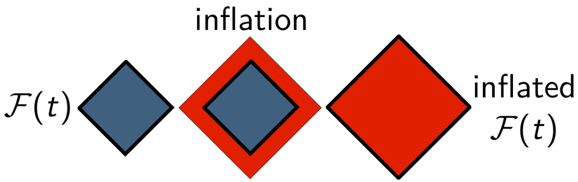


$$\text{Cut ratios:}\quad \kappa_p \triangleq \frac{d_p}{d_{PSS}}, \qquad \kappa_n \triangleq \frac{d_n}{d_{PSS}}$$

New face informative if $\kappa < \Gamma_A$.

Γ_A is a design parameter, $\Gamma_A \in [0, 1]$.

Idea: Forget past measurements by inflating polytope; track slowly varying dynamics.

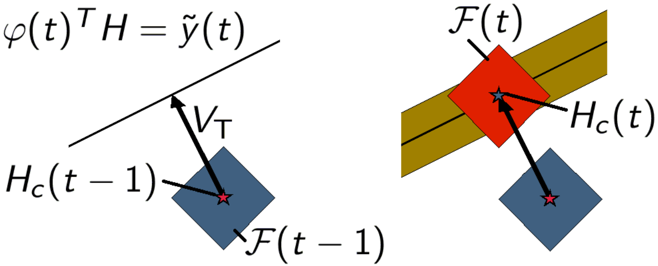


$$\left|\varphi(k)^T H - \tilde{y}(k)\right| \leq \Omega^{t-k}\epsilon, k = 0, \ldots, t$$

Inflation factor: $\Omega > 1$

Downside: $\mathcal{F}(t+1) \subseteq \mathcal{F}(t)$ does not hold; recursive feasibility lost. Need soft output contraints.

Idea: Shift polytope; track rapidly varying dynamics.

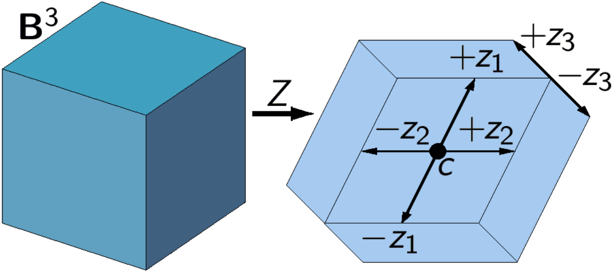


$$\text{Shifting vector: } V_{\mathrm{T}} = \varphi \frac{\tilde{y} - \varphi^T H_c(t-1)}{\varphi^T \varphi}$$

Downside: $\mathcal{F}(t+1) \subseteq \mathcal{F}(t)$ does not hold; recursive feasibility lost. Need soft output contraints.

Definition: An n-zonotope of order r is the linear image of an r-dimensional hypercube in $\mathbb{R}^n$.

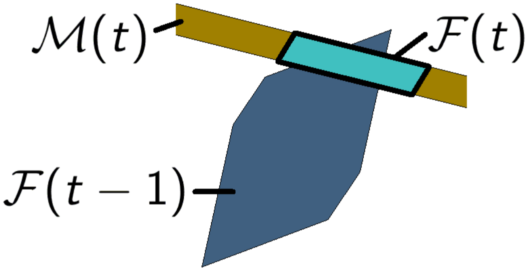
$$c \oplus Z\mathbf{B}^r = \{c + Zv : v \in \mathbf{B}^r\},$$
$$c \in \mathbb{R}^n,\ Z \in \mathbb{R}^{n \times r}$$



Advantages:

- Set fully defined by c and Z; no need to remove redundant faces or compute nominal model.
- Bounded complexity for constant r.

Basic zonotopic update (Bravo2006)



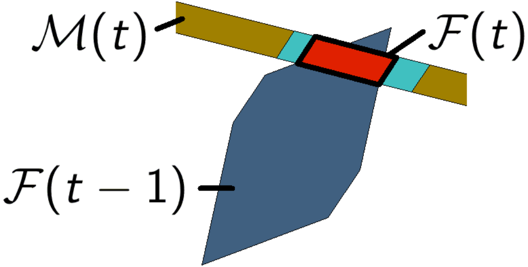
Advantage:

- r constant.

Disadvantages:

- Does not yield exact intersection; conservative.
- Collapsing into parallelotopes.
- $\mathcal{F}(t+1) \subseteq \mathcal{F}(t)$ does not hold; recursive feasibility lost. Need soft output contraints.

Improved zonotopic update (Chai2011)



Advantages:

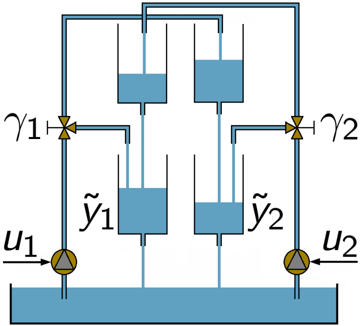
- r constant.
- Less conservative than Bravo2006.

Disadvantages:

- Computationally more expensive than Bravo2006.
- Collapsing into parallelotopes.
- $\mathcal{F}(t+1) \subseteq \mathcal{F}(t)$ does not hold; recursive feasibility lost. Need soft output contraints.

Simulation results - Model and measures

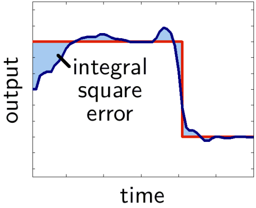
Quadruple-tank process:

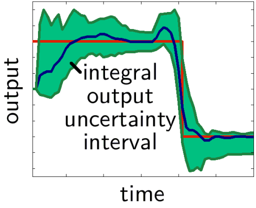


Nonlinear MIMO simulation model with a fixed LHP zero and a tunable zero that switches half-planes with varying valve constants γ_1 and γ_2.

Performance measures:

- Control: ISE
- Identification: IOUI





Simulation results - Scenario for sensitivity analyses

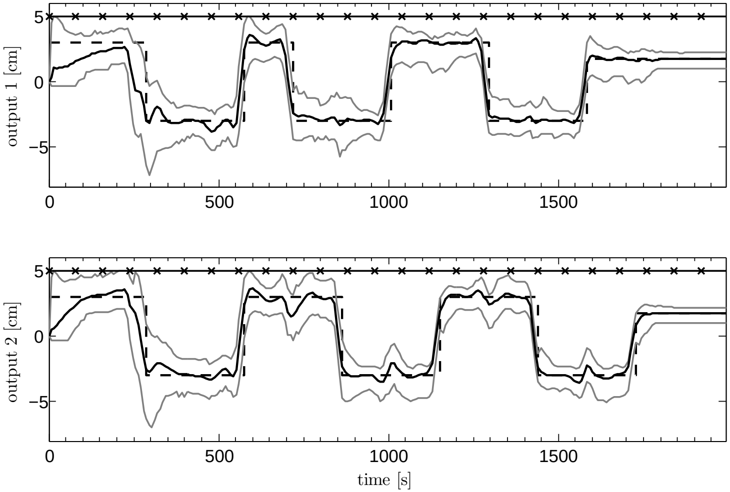
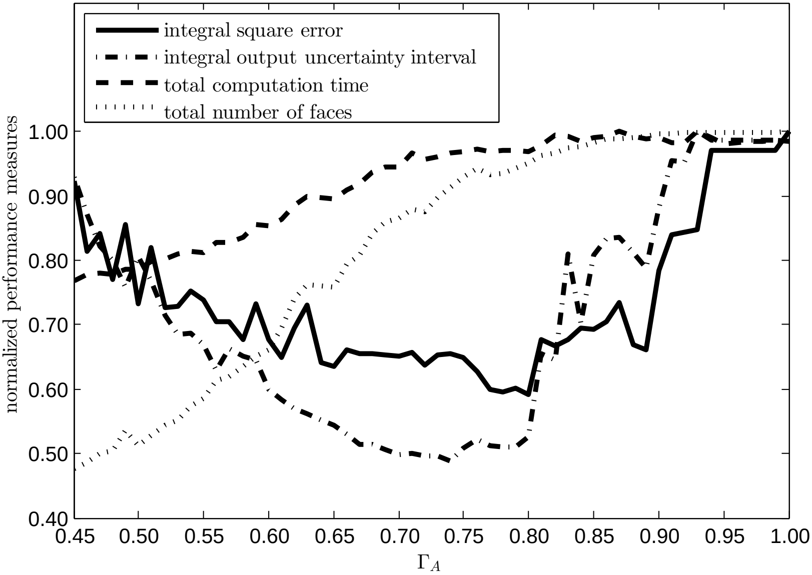


Figure 3: Simulation scenario used in the sensitivity analyses.

Sensitivity analysis of FFPU



Comparison of Basic ZU and Improved ZU

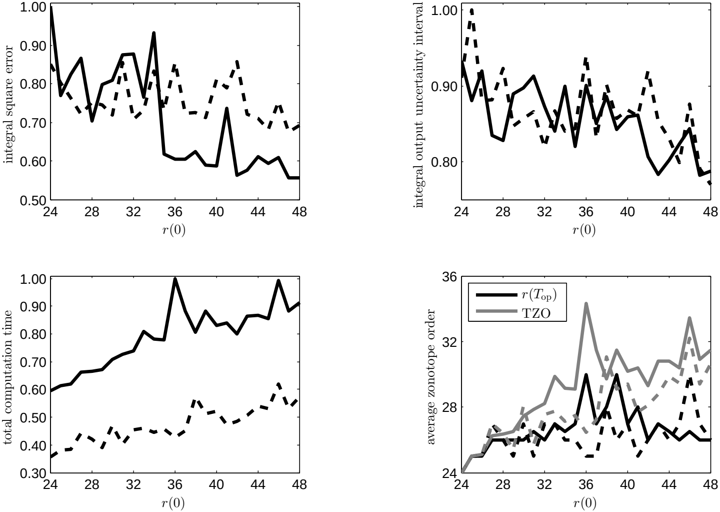


Figure 5: Comparison of basic ZU (dashed lines) and improved ZU (solid lines).

Simulation results - Scenario for time-varying system

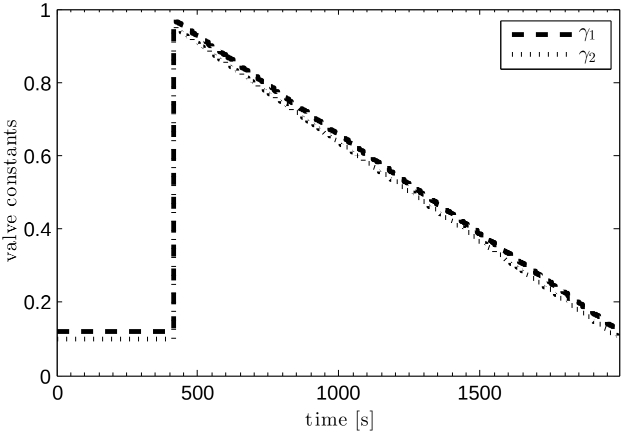


Figure 6: Variation of valve constants γ_1 and γ_2 with time.

Polytopic operation with time-varying system

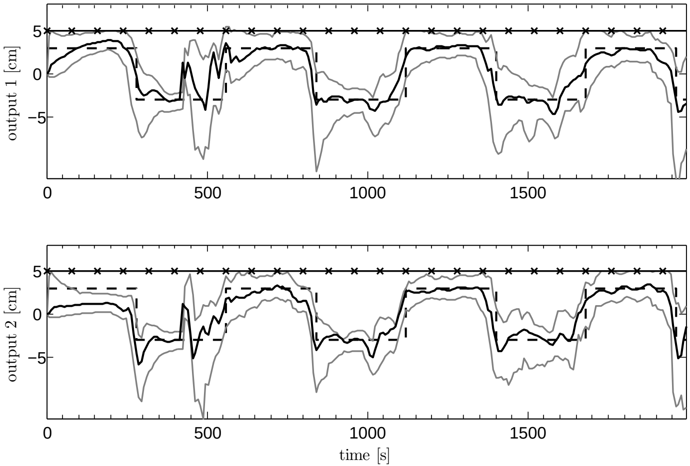


Figure 7: Polytopic operation with time-varying valve constants.

Average computation time per step: 3 seconds

Zonotopic operation with time-varying system

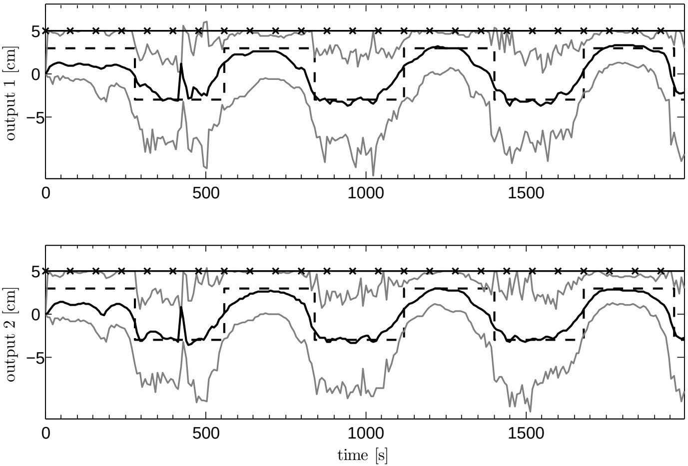


Figure 8: Zonotopic operation with time-varying valve constants.

Average computation time per step: 0.5 seconds

- Investigated SDP relaxation based relaxation-and-projection method for hybrid MPC of MILSs. Observed no clear advantage over QP relaxation for the buck converter case.
- Developed improvements for the SMI engine of the adaptive MPC of linear MIMO systems and extensions to handle time-varying systems. Observed improvements in performance and/or reductions in computational effort. Verified capability in handling time-varying systems.