Dynamical Modeling and Predictive Control of Bus Transport Systems: A Hybrid Systems Approach

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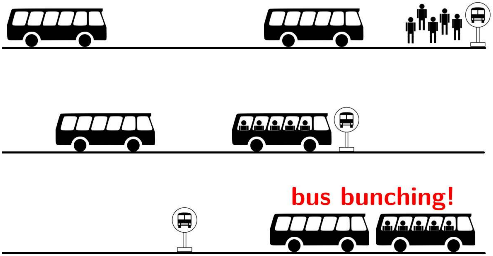
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

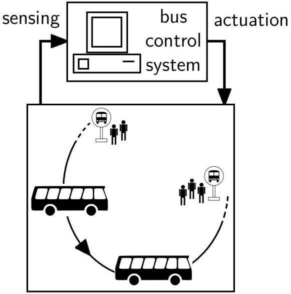
LUTS

Motivation

Problem: Irregularity/inefficiency



Solution: Control



Literature review - Control of bus systems[1]

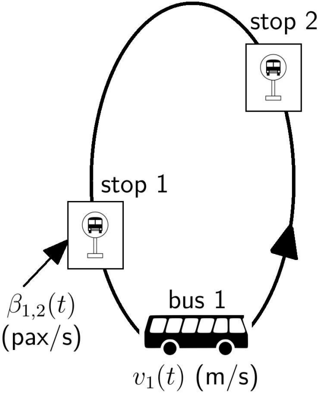
- **Station control** (only at some stops)
  - Holding
    - Eberlein, Wilson, and Bernstein 2001
    - Daganzo 2009
  - Stop-skipping
    - Fu, Liu, and Calamai 2003
    - Cortés et al. 2010
- **Inter-station control** (while buses are moving)
  - Traffic signal priority
    - Liu, Skabardonis, and Zhang 2003
    - Van Oort, Boterman, and Van Nes 2012
  - **Bus speed control (focus of the talk)**
    - Daganzo and Pilachowski 2011
    - Ampountolas and Kring 2015

[1] Ibarra-Rojas et al. 2015.

Mixed logical dynamical (MLD) modeling[2]

Continuous states

- Distance of bus 1 from stop 1 at time t: $x_1(t) \in \mathbb{R}$
- No. of pax on bus 1 at time t: $n_1(t) \in \mathbb{R}$
- No. of pax at stop 1 at time t: $m_1(t) \in \mathbb{R}$



Binary states

- Is bus 1 holding at stop 2 at time t?

$$\delta_{1,2}(t) = \begin{cases} 0 & \rightarrow \quad \text{no} \\ 1 & \rightarrow \quad \text{yes} \end{cases}$$

- Is bus 1 cruising to stop 2 at time t?

$$\gamma_{1,2}(t) = \begin{cases} 0 & \rightarrow \quad \text{no} \\ 1 & \rightarrow \quad \text{yes} \end{cases}$$

[2] Bemporad and Morari 1999.

MLD modeling - Continuous dynamics

- **Bus position**

$$x_1(t+1) = \overbrace{(\gamma_{1,1}(t) + \gamma_{1,2}(t))(x_1(t) + T_s v_1(t))}^{\text{cruising}} + \dots$$
$$\overbrace{\delta_{1,2}(t)x_1(t)}^{\text{holding}} + \overbrace{\delta_{1,1}(t)0}^{\text{reset}}$$

- **Bus accumulation**

$$n_1(t+1) = n_1(t) + \overbrace{\delta_{1,1}(t)q^{\text{in}}_{1,2}(t)}^{\text{boarding}} - \overbrace{\delta_{1,2}(t)q^{\text{out}}_{1,2}(t)}^{\text{alighting}}$$

- **Stop accumulation**

$$m_1(t+1) = m_1(t) + \overbrace{T_s\beta_{1,2}(t)}^{\text{accumulating}} - \overbrace{\delta_{1,1}(t)q^{\text{in}}_{1,2}(t)}^{\text{alighting}}$$

MLD modeling - Events

- **"Bus nonempty" event**

$$e_1^n(t) = \begin{cases} 0 & \text{if } n_1(t) = 0 \\ 1 & \text{otherwise} \end{cases}$$

- **"Stop reached" event**

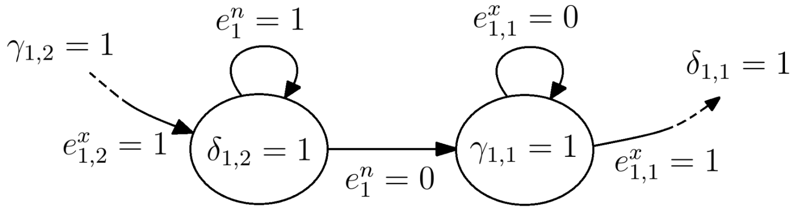
$$e_{1,2}^x(t) = \begin{cases} 0 & \text{if } x_1(t) < D_2 \\ 1 & \text{otherwise} \end{cases}$$

$$\left(\begin{array}{l} [f(x) \le 0] \leftrightarrow [\delta = 1] \text{ is true iff } \begin{cases} f(x) \le M(1-\delta) \\ f(x) \ge \varepsilon + (m - \epsilon)\delta \end{cases} \\ x \in \mathbb{R} \quad \delta \in \{0,\ 1\} \quad M \triangleq \max_{x \in \mathcal{X}} f(x) \quad m \triangleq \min_{x \in \mathcal{X}} f(x) \end{array} \right)$$

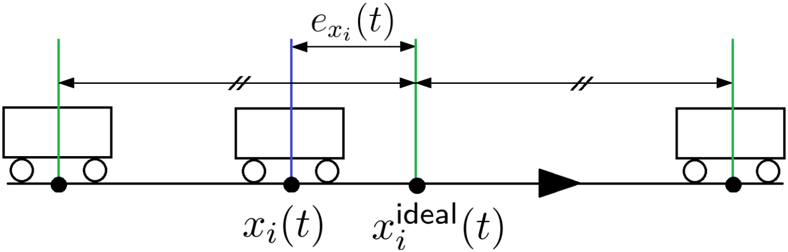
MLD modeling - Discrete dynamics

$$\overbrace{\delta_{1,2}(t+1)}^{\text{holding state}} = \overbrace{\left(\gamma_{1,2}(t) \wedge e^x_{1,2}(t)\right)}^{\text{start holding}} \vee \overbrace{\left(\delta_{1,2}(t) \wedge e^n_1(t)\right)}^{\text{keep holding}}$$

$$\overbrace{\gamma_{1,1}(t+1)}^{\text{cruising state}} = \overbrace{\left(\delta_{1,2}(t) \wedge \neg e^n_1(t)\right)}^{\text{start cruising}} \vee \overbrace{\left(\gamma_{1,1}(t) \wedge \neg e^x_{1,1}(t)\right)}^{\text{keep cruising}}$$



Control - PI-like bus speed controller



position error: $e_{x_i}(t) = x_i^{\text{ideal}}(t) - x_i(t)$

speed error: $e_{v_i}(t) = \bar{v}_i(t) - v_i(t-1)$

$$v_i(t) = v_i(t-1) + K_P\, e_{v_i}(t) + K_I\, e_{x_i}(t)$$

Control - Linear MPC (QP)

regularize headways

drive as fast as possible

$$\underset{v_{i,k}}{\text{minimize}} \quad \sum_{i=1}^{K_b} \sum_{k=0}^{N-1} \left(\boxed{\left(x^f_{i,k} - x^r_{i,k}\right)^2} + \sigma \boxed{\left(\bar{v}_{i,k} - v_{i,k}\right)^2} \right)$$

subject to for $i = 1, \; \ldots, \; K_b$:

$$\boxed{\begin{aligned} x^f_{i,1} &= x^f_i(t) \\ x^r_{i,1} &= x^r_i(t) \end{aligned}}$$

measurement of initial positions

for $k = 1, \; \ldots, \; N$:

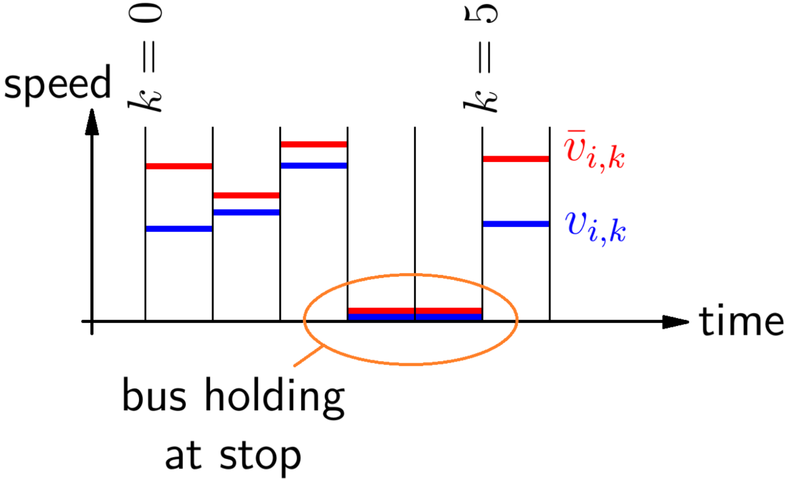
$$\boxed{\begin{aligned} x^f_{i,k+1} &= x^f_{i,k} + T\left(v_{i+1,k} - v_{i,k}\right) \\ x^r_{i,k+1} &= x^r_{i,k} + T\left(v_{i,k} - v_{i-1,k}\right) \end{aligned}}$$

position dynamics

speed bounds

$$\boxed{0 \leq v_{i,k} \leq \bar{v}_{i,k}}$$

Control - Linear MPC, speed bounds



Control - Hybrid MPC (MIQP)

$$\underset{v_{i,k}}{\text{minimize}} \quad \sum_{i=1}^{K_b} \sum_{k=0}^{N-1} \Bigg(\underbrace{\boxed{\left(x^f_{i,k} - x^r_{i,k}\right)^2}}_{\text{regularize headways}} + \sigma \underbrace{\boxed{\left(\bar{v}_{i,k} - v_{i,k}\right)^2}}_{\text{drive as fast as possible}} \Bigg)$$

subject to for $i = 1, \ \ldots, \ K_b$:

$$\boxed{\begin{aligned} x^f_{i,1} &= x^f_i(t) \\ x^r_{i,1} &= x^r_i(t) \\ x^u_{i,1} &= x^u_i(t) \end{aligned}} \quad \text{measurement of initial positions}$$

$$\boxed{c_{i,k} = 0} \quad \text{digital clock initialization}$$

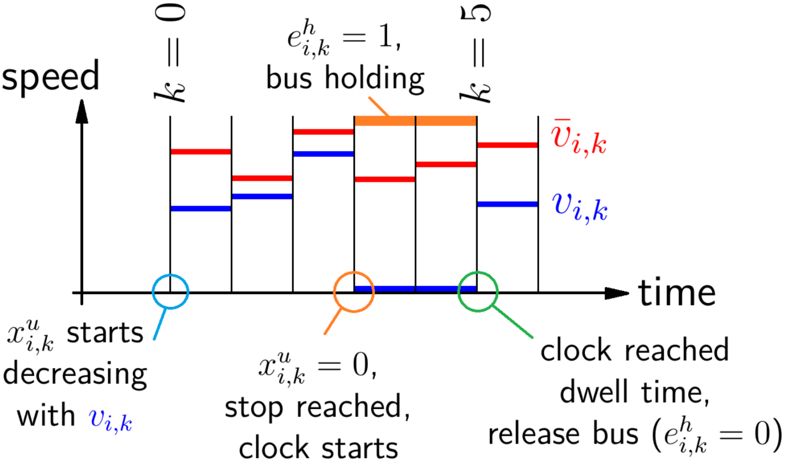
for $k = 1, \ \ldots, \ N$:

$$\boxed{\begin{aligned} x^f_{i,k+1} &= x^f_{i,k} + T(v_{i+1,k} - v_{i,k}) \\ x^r_{i,k+1} &= x^r_{i,k} + T(v_{i,k} - v_{i-1,k}) \\ x^u_{i,k+1} &= x^u_{i,k} - T v_{i,k} \end{aligned}} \quad \text{position dynamics}$$

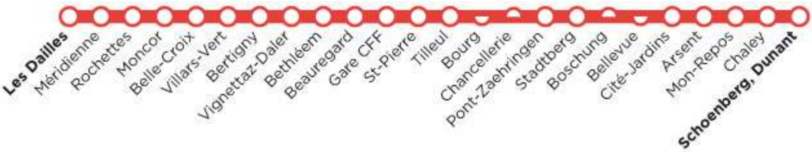
$$\text{digital clock dynamics} \quad \boxed{c_{i,k+1} = c_{i,k} + e^x_{i,k}}$$

$$\boxed{0 \le v_{i,k} \le \bar{v}_{i,k}(1 - e^h_{i,k})} \quad \text{speed bounds}$$

Control - Hybrid MPC, speed bounds

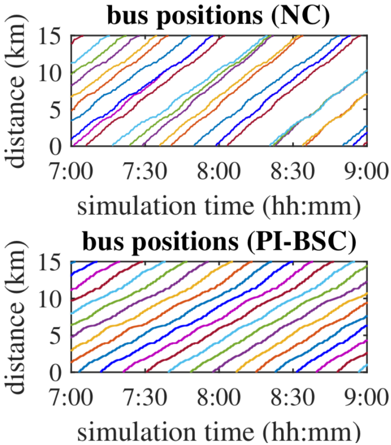


Results - Simulation setup

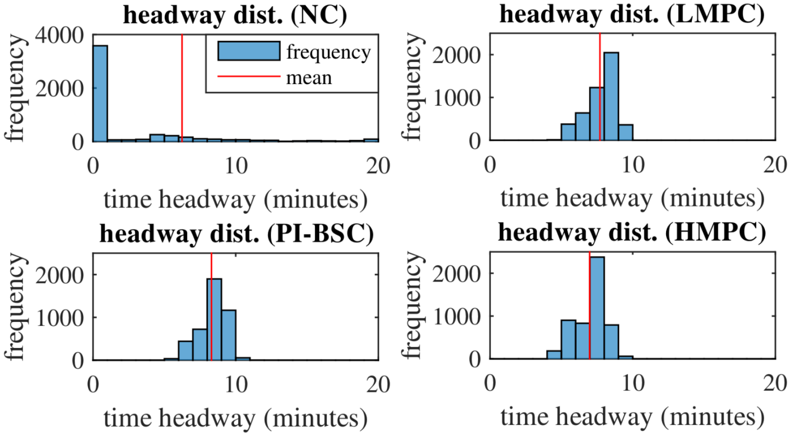


- **Bus system description**
  - 9 buses, 44 stops, 15 km loop
  - Demands and speed bounds from bus data of 2 months
- **Compared schemes**
  - No control (NC)
  - PI-like bus speed control (PI-BSC)
  - Linear model predictive control (LMPC)
  - Hybrid model predictive control (HMPC)

Results - Bus positions



Results - Headway distributions



Results - Performance evaluation

Control scheme	mean TSPP (min)	mean speed (km/h)	std. of hws. (min)	mean/max. CPU time (s)
NC	25.7	25.7	13.3	–
PI-BSC	31.2	17.6	0.95	–
LMPC	24.6	19.3	1.02	0.16/0.21
HMPC	18.6	18.7	1.03	0.47/0.68

Conclusion

Contributions

- MLD bus system model
- Linear and hybrid bus speed MPC

Results

- MLD model captures detailed dynamics
- MPC yields improvement

Ongoing/future work

- Consider pax flows/accumulations in MPC
- Extend to multi-loop bus systems

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