Moving horizon estimation for large-scale urban networks

Işık İlber Sırmatel, Nikolas Geroliminis

Urban Transport Systems Laboratory, EPFL

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Introduction - State estimation[1]

$$\text{dynamics:} \quad \dot{x}(t) = f(x(t), u(t)) + w(t)$$
$$\text{measurement:} \quad y(t) = g(x(t), u(t)) + v(t)$$

state estimation problem (at time t):

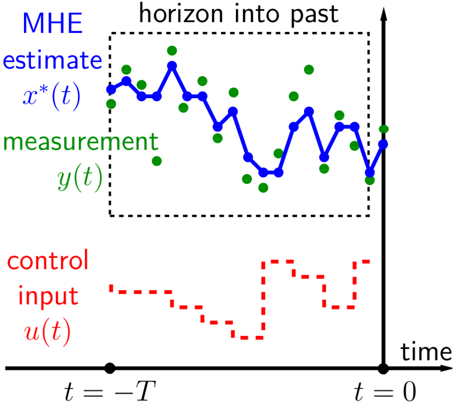
$$\textbf{given:} \ \{y(\tau), u(\tau)\}_{0 \leq \tau \leq t}$$
$$\textbf{estimate:} \ x(t + \delta)$$

three types of estimation:

- $\delta = 0$: filtering
- $\delta > 0$: prediction
- $\delta < 0$: smoothing

[1] Slide adapted from lecture slides of Prof. Manfred Morari.

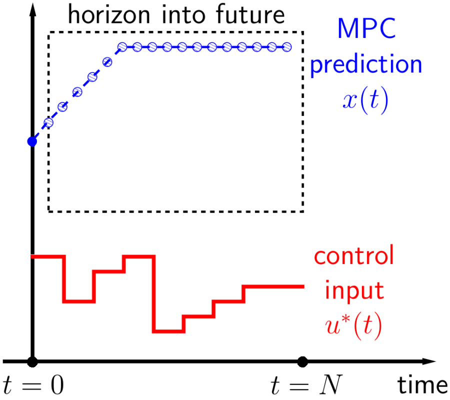
Moving horizon estimation (MHE)[2]



$$
\begin{aligned}
\min_{w} \quad & \int_{-T}^{0} \|w\|_Q^2 + \|v\|_R^2 \, dt \\
\text{s.t.} \quad & \dot{x} = f(x, u) + w \\
& y = g(x, u) + v \\
& x \in \mathcal{X} \\
& w \in \mathcal{W}
\end{aligned}
$$

[2]Slide adapted from lecture slides of Prof. James B. Rawlings.

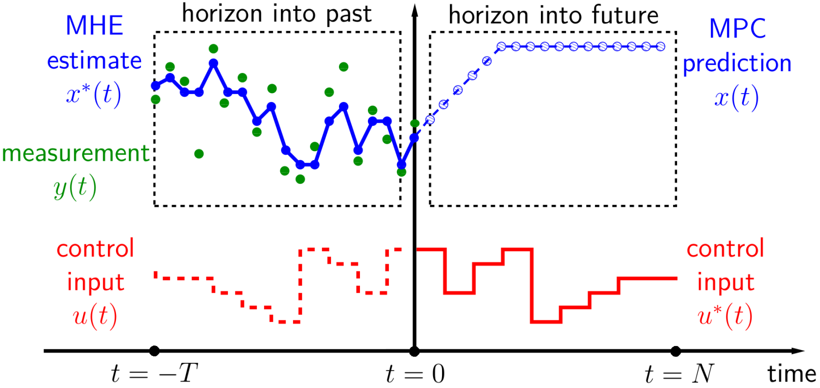
Model predictive control (MPC)[3]



$$
\begin{aligned}
\min_{u}. \quad & \int_0^N l(x, u)\ dt \\
\text{s.t.} \quad & x(0) = x_0 \\
& \dot{x} = f(x, u) \\
& x \in \mathcal{X} \\
& u \in \mathcal{U}
\end{aligned}
$$

[3]Slide adapted from lecture slides of Prof. James B. Rawlings.

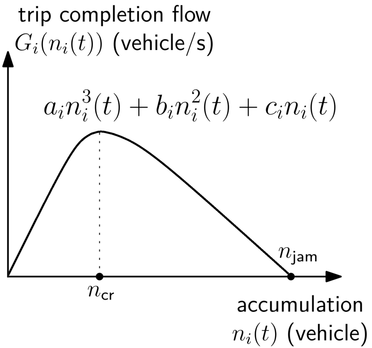
Combined MHE and MPC[4]



[4] Slide adapted from lecture slides of Prof. James B. Rawlings.

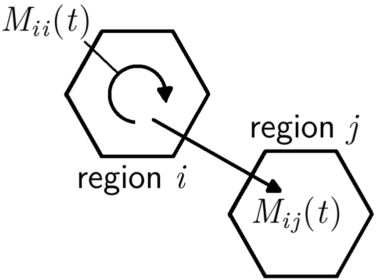
Modeling of an urban region

Macroscopic fundamental diagram:



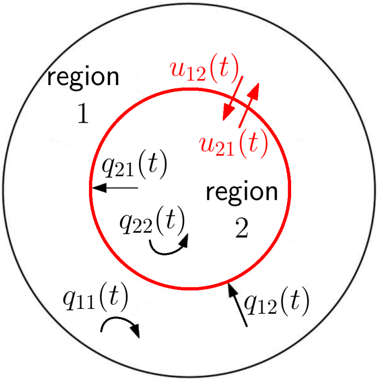
$$n_i(t) = \sum_{j \in \mathcal{R}} n_{ij}(t)$$

Relation with transfer flows:



$$M_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)} G_i(n_i(t))$$

Modeling of a two-region urban network



$$\dot{n}_{11} = q_{11} + u_{21} M_{21} - M_{11}$$
$$\dot{n}_{12} = q_{12} - u_{12} M_{12}$$
$$\dot{n}_{21} = q_{21} - u_{21} M_{21}$$
$$\dot{n}_{22} = q_{22} + u_{12} M_{12} - M_{22}$$

Signal uncertainty in urban networks

Uncertainty in inflow demands:

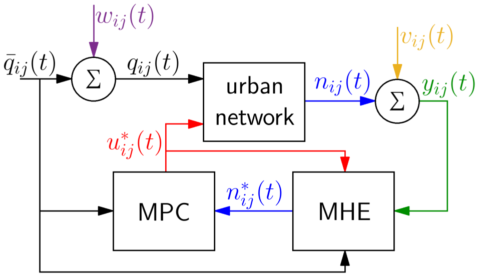
$$\overbrace{q_{ij}(t)}^{\text{actual demand}} = \overbrace{\bar{q}_{ij}(t)}^{\text{known demand}} + \overbrace{w_{ij}(t)}^{\text{demand uncertainty}}$$

Noise in accumulation measurements:

$$\overbrace{y_{ij}(t)}^{\text{measurement}} = \overbrace{n_{ij}(t)}^{\text{accumulation}} + \overbrace{v_{ij}(t)}^{\text{measurement noise}}$$

$$w_{ij}(t) \in \mathcal{N}(0, \sigma^2_{w_{ij}}) \qquad v_{ij}(t) \in \mathcal{N}(0, \sigma^2_{v_{ij}})$$

Combined MHE-MPC for urban networks



MHE

$$\min_{w} \quad \int_{-T}^{0} \|w\|_Q^2 + \|v\|_R^2 \, dt$$

$$\text{s.t.} \quad \dot{n} = f(\bar{q}, n, u) + w$$

$$y = n + v$$

MPC

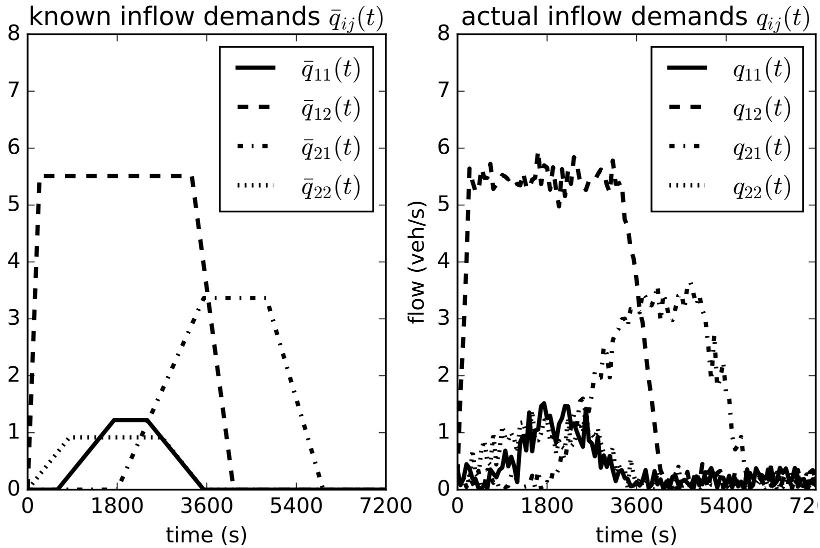
$$\min_{u} \quad \int_{0}^{N} \|n\|_1 \, dt$$

$$\text{s.t.} \quad n(0) = n_0^{*,\text{MHE}}$$

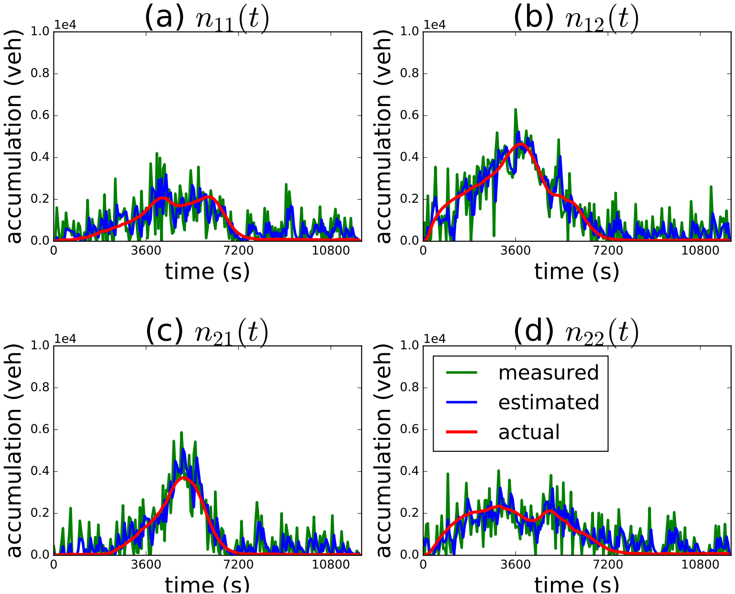
$$\dot{n} = f(\bar{q}, n, u)$$

$$n \in \mathcal{N}, \;\; u \in \mathcal{U}$$

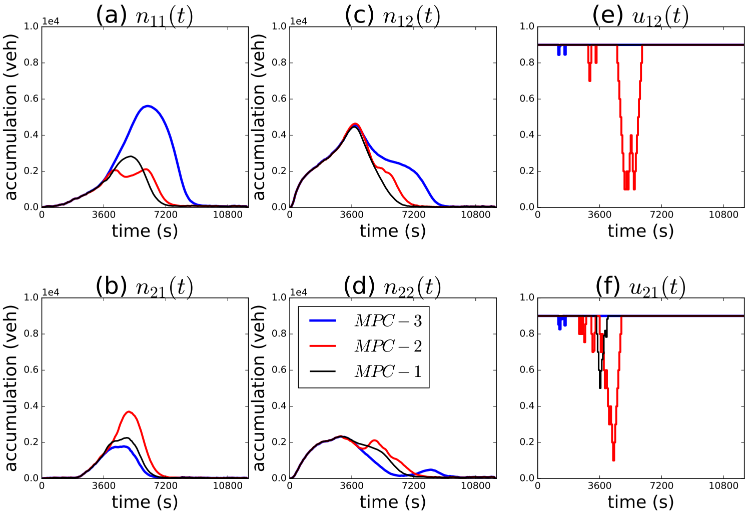
Results - Demand scenario



Results - Estimation



Results - Control



Conclusion

Contribution:

- A combined MHE-MPC scheme

Result:

- Potential in handling signal uncertainty

Ongoing work:

- Compare MHE with traditional methods
- Evaluate via more detailed simulations