

# MAT 271E – Homework 5

Due 30.03.2011

1. Let  $X$  be uniformly distributed on the interval  $[-3,2]$ . We define a discrete random variable  $Y$  as,

$$Y = \begin{cases} 1 & \text{if } X \geq 0, \\ 0 & \text{if } X < 0. \end{cases}$$

What is the PMF of  $Y$ ?

2. You just missed a bus and you're waiting for the next one. Suppose that the probability that the next bus arrives in  $T$  units of time (for a non-negative  $T$ !) is given by

$$\int_0^T c e^{-ct} dt$$

where ' $c$ ' is a constant. Let  $X$  be the amount of time you wait for the next bus to arrive.

- (a) What is the probability density function (pdf) of  $X$ ?
  - (b) What is the expected amount of time you need to wait?
  - (c) What is the variance of  $X$ ?
3. The taxis in a city are numbered from 1 to  $n$ , where  $n$  is the total number of taxis. You try to estimate  $n$  as follows. Suppose that after seeing  $i$  taxis, your estimate is  $E_i$  (and you start from  $E_0 = 0$ ). At your next observation of a taxi, whose number is, say  $X_{i+1}$ , you set  $E_{i+1} = \max(X_{i+1}, E_i)$ . Assume that, any time you observe a taxi, its number is equally likely to be any one in the set  $\{1, \dots, n\}$ , independent of previous observations.
    - (a) What is the CDF of  $E_2$ ?
    - (b) For  $i > 0$ , what is the CDF of  $E_i$ ?
    - (c) For  $i > 0$ , what is the PMF of  $E_i$ ?
    - (d) Compute  $\lim_{i \rightarrow \infty} \mathbb{E}(E_i)$  and  $\lim_{i \rightarrow \infty} \text{var}(E_i)$ .