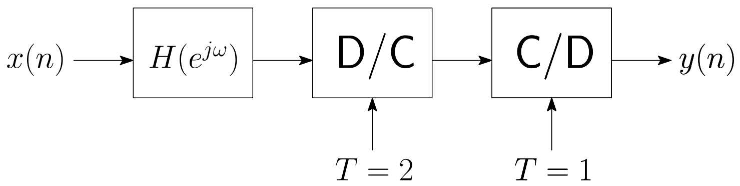
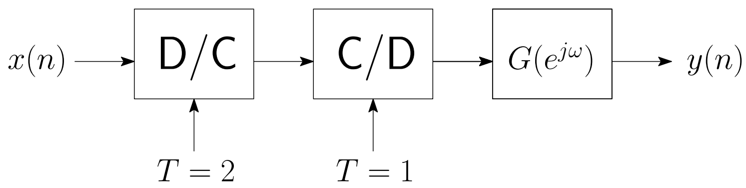
BYM 510E Take-Home Final Examination

Due 23.05.2011, 15:00

(40 pts) 1. Consider the following system

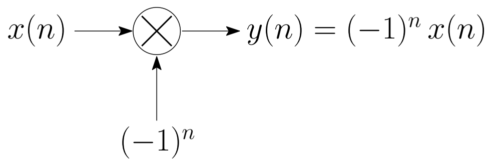


which filters a discrete-time signal, converts it to a continuous-time signal and resamples it at a different rate. Consider now the system below which places the digital filter after the C/D converter.



Find an expression for $G(e^{j\omega})$ in terms of $H(e^{j\omega})$ so that the two systems are equivalent.

(30 pts) 2. Consider the system below which modulates the input with $(-1)^n$.



Assume that the input signal $x(n)$ is a stationary stochastic process with autocorrelation function $R_x(k)$ and power spectral density $S_x(e^{j\omega})$.

(a) Is $y(n)$ wide-sense stationary? If so,

  (i) express its autocorrelation function $R_y(k)$ in terms of $R_x(k)$,

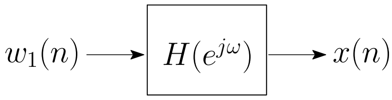
  (ii) express its power spectral density $S_y(e)$ in terms of $S_x(e^{j\omega})$.

(b) Consider the process $z(n) = x(n) + y(n)$. Is $z(n)$ wide-sense stationary? If so,

  (i) express its autocorrelation function $R_z(k)$ in terms of $R_x(k)$,

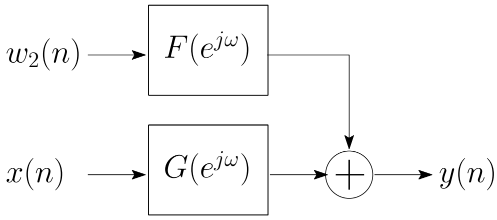
  (ii) express its power spectral density $S_z(e)$ in terms of $S_x(e^{j\omega})$.

(30 pts) 3. Let $x(n)$ be a stationary process obtained by LTI filtering a white noise process $w_1(n)$ as shown below.



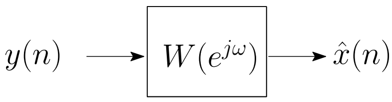
We know that $\sum_n h(n) = 0$ but we do not have further information regarding $h(n)$ or $x(n)$.

Consider now the system below which produces blurred and noisy observations of $x(n)$.



Assume that in the system above, we know that $w_2(n)$ is a white noise process, independent of $w_1(n)$ but we do not know its variance. Assume also that $\sum_n f(n) \neq 0$

We apply an LTI filter to estimate $x(n)$ as shown below.



Let $W(e^{j\omega})$ be the frequency response of the filter which minimizes the squared error $\mathbb{E}\left(\left[x(n) - \hat{x}(n)\right]^2\right)$.

(a) Express the variance of $w_2(n)$ in terms of $S_y(e^{j\omega})$ (the power spectral density of $y(n)$) and $F(e^{j\omega})$.

(b) Express $W(e^{j\omega})$ in terms of $S_y(e^{j\omega})$, $F(e^{j\omega})$ and $G(e^{j\omega})$.