MAT 281E – Homework 6

Due 31.12.2010

1. True or False?

   (a) An $n \times n$ matrix always has n distinct eigenvalues.

   (b) An $n \times n$ matrix always has n, possibly repeating, eigenvalues.

   (c) An $n \times n$ matrix always has n eigenvectors.

   (d) Every matrix has at least 1 eigenvector.

   (e) If A and B have the same eigenvalues, they always have the same eigenvectors.

   (f) If A and B have the same eigenvectors, they always have the same eigenvalues.

   (g) If Q has $1/2$ as an eigenvalue, then it cannot be orthogonal.

   (h) If $A = S\,\Lambda\,S^{-1}$ where Λ is diagonal, then the rows of S have to be the eigenvectors of A.

   (i) If $A = S\,\Lambda\,S^{-1}$ where Λ is diagonal, then the columns of S have to be the eigenvectors of A.

   (j) An arbitrary matrix A can always be diagonalized as $A = S\,\Lambda\,S^{-1}$ where Λ is diagonal.

2. Let A be an $n \times n$ matrix with all entries equal to 1 (i.e. $a_{i,j} = 1$). For $n = 2, 3$, find the eigenvalues and eigenvectors of A.

3. Suppose that A is a 3×3 matrix with eigenvalues λ_1, λ_2, λ_3 where the corresponding eigenvectors are x_1, x_2, x_3. What are the eigenvalues and eigenvectors of $2A - I$?

4. Find the eigenvalues of the following matrices.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}.$$

5. Let $y(n) = 2\,y(n-1) + 3\,y(n-2)$. Suppose that $y(1) = 4$, $y(0) = 0$. Compute $y(101)$.