BYM 510E – Homework 1

Due 24.02.2011

1. Consider the system given by,

$$y(n) = \sum_{k=-\infty}^{\infty} h(n+k)\, x(k),$$

where $x(n)$ is the input and $y(n)$ is the output. Assume that $h(n) = 0$ for $n < 0$ and $n > 50$. Specify whether the system is
(a) Memoriless, (b) Linear, (c) Time-invariant, (d) Causal, (e) Stable.
Please explain your answers. If information is insufficient, write 'insufficient information' (and explain why you think so).

2. Repeat the first question for the system given by

$$y(n) = h(n)\, x(n),$$

where $x(n)$ is the input and $y(n)$ is the output. Assume now that $h(n)$ is not a constant but $|h(n)| < M$ for some real number M.

3. Let $H(e^{j\omega})$ be the ideal filter with cutoff at $\pi/2$ given by,

$$H(e^{j\omega}) = \begin{cases} 0 & \text{for} \quad -\pi \le \omega < -\pi/2, \\ 1 & \text{for} \quad -\pi/2 \le \omega < \pi/2, \\ 0 & \text{for} \quad \pi/2 \le \omega < \pi. \end{cases}$$

(a) Compute the periodic convolution of $H(e^{j\omega})$ with itself, i.e.,

$$G(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H\big(e^{j(\omega-\theta)}\big)\, H\big(e^{j\theta}\big)\, d\theta.$$

Sketch $H(e^{j\omega})$ and $G(e^{j\omega})$.

(b) Derive the discrete-time sequence $h(n)$ associated with $H(e^{j\omega})$ through the inverse DTFT relation. Specify the inverse-DTFT of $G(e^{j\omega})$. (Hint: Make use of the DTFT theorems.)

4. Let $x(n)$ and $g(n)$ be two sequences whose DTFTs are denoted by $X(e^{j\omega})$ and $G(e^{j\omega})$ respectively. Consider the periodic convolution of $X(e^{j\omega})$ and $G(e^{j\omega})$:

$$Y(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\big(e^{j(\omega-\theta)}\big)\, G\big(e^{j\theta}\big)\, d\theta.$$

Show that the inverse DTFT of $Y(e^{j\omega})$ is $x(n)\, g(n)$.