MAT 281E – Homework 2

Due 22.10.2010

1. Let
$$A = \begin{bmatrix} -1 & -2 & 0 & 1 \\ 0 & 1 & 1 & -3 \\ -2 & 3 & 1 & 2 \\ 0 & -1 & -1 & 6 \end{bmatrix}.$$
Find the LU decomposition of A.

2. Let
$$A = \begin{bmatrix} 0 & -2 & 0 \\ -2 & 1 & -1 \\ 2 & -3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -3 & -4 \\ -3 & 7 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$
   (a) Using Gauss-Jordan elimination, find the inverse of A.
   (b) Using Gauss-Jordan elimination, find the matrix D such that $B\,D = C$.
   (Hint : Do not use the inverse of B. Use an augmented matrix of the form $\begin{bmatrix} B & V \end{bmatrix}$ where V is a 3×3 matrix. What should V be?)

3. Let
$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$
   (a) Find two permutation matrices P_1, P_2 such that,
$$P_1\,A\,P_2 = \begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix}.$$
   (b) Find two permutation matrices $\tilde{P}_1$, $\tilde{P}_2$ such that,
$$\tilde{P}_1\,A\,\tilde{P}_2 = \begin{bmatrix} b & c & a \\ e & f & d \\ h & i & g \end{bmatrix}.$$

4. (a) Suppose we are given a matrix A and $B = \begin{bmatrix} A & b \end{bmatrix}$ where b is a column vector (B has one more column than A). Let $C(A)$ and $C(B)$ denote the column spaces of A and B.
   Which is true in general – '$C(A) \subset C(B)$' or '$C(B) \subset C(A)$'? (If both are true in general, write so.) Please explain your answer.
   (b) Let A, B be given matrices and $D = \begin{bmatrix} A & A\,B \end{bmatrix}$ (the matrix AB is augmented to A). Let $C(A)$ and $C(D)$ denote the column spaces of A and D.
   Which is true in general – '$C(A) \subset C(B)$' or '$C(B) \subset C(A)$'? (If both are true in general, write so.) Please explain your answer.

5. Let x, y, z be vectors such that $x + y + z = 0$. Show that x and y span the same space as y and z. (Hint : Let A denote the space spanned by x and y and B denote the space spanned by y and z. Pick an element from A, show that it is in B. This implies that $A \subset B$(Why?). Then pick an element from B, show that it is in A. This implies that $A \subset B$. If $A \subset B$ and $B \subset A$ then it must be that $A = B$.)