TEL 311E – Homework 7

Due 11.01.2011

1. Consider the system given by,
$$y(n) = x(2n) + 1$$
where $x(n)$ is the input and $y(n)$ is the output. Specify whether the system is

   (a) Memoriless, (b) Linear, (c) Time-invariant, (d) Causal, (e) Stable in the BIBO sense.

   Please explain your answers.

2. Let T_1 be an LTI system with impulse response $h_1(n) = a^n\, u(n)$.

   (a) For which values of 'a' is T_1 stable in the BIBO sense?

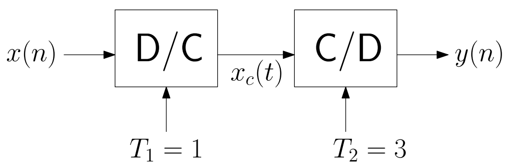
   (b) Assume that T_1 is BIBO stable. Suppose we input some $x(n)$ to T_1 and obtain $y(n)$, as shown below. Let T_2 be another LTI system with impulse response $h_2(n) = \delta(n) - a^{-1}\,\delta(n+1)$ and
$$x(n) \longrightarrow \boxed{T_1} \longrightarrow y(n) \longrightarrow \boxed{T_2} \longrightarrow z(n)$$
suppose we input $y(n)$ to this system to obtain $z(n)$. Express $z(n)$ in terms of $x(n)$.

3. Suppose that the z-transform of the step response (i.e. the response when a unit step function $u(n)$, is input to the system) of an LTI system is given by
$$X(z) = \frac{1}{1 + \frac{1}{4}\, z^{-1}} + \frac{1}{1 - z^{-1}}.$$
Let us denote the z-transform of the impulse response as $H(z)$.

   (a) If we know that the system is stable, what should be the region of convergence for $H(z)$?

   (b) Determine the impulse response $h(n)$ of this stable system.

4. Consider the system below which maps $x(n)$ to $y(n)$.



   (a) Express $y(n)$ in terms of $x(n)$.

   (b) Express $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$.

5. Let $x(n)$ be an N-point signal whose N-point DFT is denoted by $X(k)$. Suppose we circularly shift $X(k)$ by one sample to obtain $\tilde{X}(k)$, i.e.,
$$\begin{aligned} \tilde{X}(0) &= X(N-1), \\ \tilde{X}(k) &= X(k-1) \quad \text{for } 1 \le k \le N-1. \end{aligned}$$
Express $\tilde{x}(n)$, the IDFT of $\tilde{X}(k)$, in terms of $x(n)$.

6. Let $x(n)$ be a 10-point signal with 10-point DFT
$$X(k) = k^2 \quad \text{for} \quad 0 \le k \le 9.$$
Compute
$$s = \sum_{n=0}^{N} x(n) \left[\cos\left(\frac{\pi}{N}\, n\right) + 2 \sin\left(\frac{6\pi}{N}\, n\right) \right].$$