MAT 271E – Homework 4

Due 14.03.2011

1. Assume that the probability of a certain player winning a game of checkers is 0.6. Let X be the number of wins in n games. What is the PMF of X? What is the expected number of wins if the player plays n games?

2. We roll a fair die with four faces, twice. Assume that the rolls are independent. Let O_1 and O_2 denote the outcomes of the first and the second roll respectively. Also, let $X = \min(O_1, O_2)$ and $Y = \max(O_1, O_2)$.

   (a) Find the marginal PMFs of X and Y.

   (b) Find the joint PMF of X and Y.

3. Suppose that the chance of a new gambler winning a game is p. Assume that the games are independent until the gambler's first win. However, once the gambler wins a game, the playing partners start to pay more attention to him and the probability of him winning decreaes to $p/2$. Assume that the games following the gambler's first win are also independent.

   (a) What is the expected number of games that the gambler needs to play to win once?

   (b) What is the expected number of games that the gambler needs to play to win twice?

4. A collector saves the stamps from the envelopes of the letters he receives. Assume that there are 100 different stamps and each one of them is equally likely to appear on an envelope that the collector receives. Assuming also that the mails are independent, what is the expected number of letters that the collector should receive in order to collect all the 100 different stamps?

5. Let X and Y be discrete independent random variables. Also let $g(\cdot)$, $h(\cdot)$ be real-valued functions. Show that $Z_1 = g(X)$ and $Z_2 = h(Y)$ are also independent.

6. We roll a fair die twice. Assume that the rolls are independent. Let X and Y denote the outcomes of the first and the second roll respectively. Also, let $Z = X\,Y$. Compute the mean and variance of Z.