TEL 311E – Homework 6

Due 27.12.2010

1. Compute the DFTs of the following length-N signals (where N is even).

   (a) $x(n) = \delta(n)$ for $0 \le n \le N-1$

   (b) $x(n) = \delta(n-m)$ for $0 \le n \le N-1$, where $0 \le m \le N-1$

   (c) $x(n) = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

   (d) $x(n) = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$

   (e) $x(n) = \begin{cases} 0 & \text{if } 0 \le n \le N/2-1 \\ 0 & \text{if } N/2 \le n \le N-1 \end{cases}$

   (f) $x(n) = a^n$ for $0 \le n \le N-1$

   (g) $x(n) = \exp\left(j\frac{2\pi}{N}\,m\,n\right)$ for $0 \le n \le N-1$, where $0 \le m \le N-1$

   (h) $x(n) = \sin\left(\frac{2\pi}{N}\,m\,n\right)$ for $0 \le n \le N-1$, where $0 \le m \le N-1$

2. Let $x_1(n)$, $x_2(n)$ be length-N signals, given as,

$$x_1(n) = \begin{cases} n+1 & \text{for } 0 \le n \le 4 \\ 0 & \text{for } 5 \le n \le N-1, \end{cases}$$
$$x_2(n) = \delta(n) + 2\,\delta(n-3).$$

   Let $X_1(k)$, $X_2(k)$ denote their length-N DFTs. Suppose we define $Y(k) = X_1(k)\,X_2(k)$ and let $y(n)$ be the inverse DFT of $Y(k)$. Determine and sketch $y(n)$ for

   (a) $N = 6$,

   (b) $N = 8$.

   What is the minimum value of N such that $y(n)$ is equal to the linear convolution of $x_1(n)$ and $x_2(n)$?

3. Let $x(n)$ be a length-N signal for N even, and let $X(k)$ denote its length-N DFT. Suppose we define

$$y(n) = \begin{cases} x(2n) & \text{for } 0 \le n \le N/2-1 \\ 0 & \text{for } N/2 \le n \le N-1, \end{cases}$$

   Let $Y(k)$ denote the length-N DFT of $y(n)$. Express $Y(k)$ in terms of $X(k)$.

4. Let $x(n)$ be a length-N signal for N even, and let $X(k)$ denote its length-N DFT. Suppose we set $y(n) = X(n)$ for $0 \le n \le N-1$. Express $y(n)$ in terms of $x(n)$.