TEL 311E – Homework 2

Due 18.10.2010

1. Let $H(e^{j\omega})$ be the ideal filter with cutoff at $\pi/2$ given by,

$$H(e^{j\omega}) = \begin{cases} 0 & \text{for} \quad -\pi \leq \omega < -\pi/2, \\ 1 & \text{for} \quad -\pi/2 \leq \omega < \pi/2, \\ 0 & \text{for} \quad \pi/2 \leq \omega < \pi. \end{cases}$$

   (a) Compute the convolution of $H(e^{j\omega})$ with itself, i.e.,

$$G(e^{j\omega}) = \int_{-\pi}^{\pi} H(e^{j(\omega-\theta)})\, H(e^{j\theta})\, d\theta.$$

   Sketch $H(e^{j\omega})$ and $G(e^{j\omega})$.

   (b) We derived in class the discrete-time sequence $h(n)$ associated with $H(e^{j\omega})$ through the inverse DTFT relation. Specify the inverse-DTFT of $G(e^{j\omega})$. (Hint: Make use of the DTFT theorems.)

2. In this question, you will derive a more general form of Parseval's relation, that states

$$\sum_{n=-\infty}^{\infty} x(n)\, y^*(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, Y^*(e^{j\omega})\, d\omega. \tag{1}$$

   You can show this result in two steps.

   (a) Using the convolution theorem and the symmetry properties of DTFT, determine, in terms of $x(n)$ and $y(n)$, the sequence $z(n)$, whose DTFT is $X(e^{j\omega})\, Y^*(e^{j\omega})$.

   (b) Using the result of part (a) and the inverse DTFT relation, deduce eqn.(1). (Hint : Consider a particular sample of $z(n)$.)

3. Let $x(n) = (3)^n\, u(n+2) - (1/2)^n\, u(-n)$. Find the z-transform of $x(n)$. Sketch the pole-zero diagram and specify the ROC on the diagram.

4. Suppose that $x(n)$ is a causal finite-duration sequence with $x(n) = 0$ for $n > 3$. Suppose we also know that $X(e^{j\pi/4}) = X(e^{j\pi}) = X(e^{-j\pi/4}) = 0$ and $X(e^{j\,0}) = 1$. What is $X(z)$? Sketch the pole-zero plot.
   (Hint : How do we express a polynomial in terms of its roots? – See also the 'Fundamental Theorem of Algebra'.)