MAT 271E – Homework 7

Due 13.04.2011

1. We are given a biased coin and we are told that the probability of Heads is P. Assume that P is a random variable whose pdf is

$$f_P(t) = \begin{cases} 2t & \text{if } t \in [0,1], \\ 0 & \text{if } t \notin [0,1]. \end{cases}$$

   Suppose we start tossing the coin. Assume that the tosses are independent.

   (a) What is the probability that the first toss is a Head?

   (b) Given that the first toss is a Head, compute the conditional pdf of P.

   (c) Given that the first toss is a Head, compute the probability that the second toss is also a Head.

2. Let X and Y be independent random variables, uniformly distributed on $[0,1]$. Find the pdf of $Z = X/Y$.

3. Let X and Y be independent random variables, uniformly distributed on $[0,1]$. Find the pdf of $Z = X + Y$.

4. Let X and Y be independent random variables, uniformly distributed on $[0,1]$. Find the pdf of $Z = |X - Y|$.

5. Let X be a standard Gaussian random variable. Compute $\mathbb{E}(X^3)$ and $\mathbb{E}(X^4)$.