MAT 281E – Homework 7

Due 11.01.2011

1. Construct a 3×3 matrix whose column space contains $\begin{pmatrix}1 & 1 & 1\end{pmatrix}$ and $\begin{pmatrix}1 & 1 & 0\end{pmatrix}$ but not $\begin{pmatrix}1 & 0 & 1\end{pmatrix}$.

2. Consider the line l described as the intersection of the planes $x+y+z=0$ and $x+2y+z=0$. Construct, if you can, a 3×3 matrix A where $C(A) = l$.

3. Consider the line $l = \begin{pmatrix}\alpha & \alpha - 1 & 2\,\alpha\end{pmatrix}$. Construct, if you can, a 3×3 matrix A where $C(A) = l$.

4. Let $A = E_1\,R$ and $B = E_2\,R$ where E_1 and E_2 are invertible. We do not have further information about R. Below are four questions regarding the four fundamental subspaces. If you think that the information is not sufficient to answer the questions, write so.

   (a) Can you find a relation between $C(A)$ and $C(B)$?

   (b) Can you find a relation between $C(A^T)$ and $C(B^T)$?

   (c) Can you find a relation between $N(A)$ and $N(B)$?

   (d) Can you find a relation between $N(A^T)$ and $N(B^T)$?

5. (This was the last question in HW5) Find the $Q\,R$ decomposition of

$$A = \begin{bmatrix} 1 & -1 & 0 & -3 \\ 1 & 1 & 2 & 1 \\ 1 & -1 & -2 & 1 \\ 1 & 1 & 0 & -3 \end{bmatrix}.$$

6. Let λ_1, λ_2, λ_3, be the distinct non-zero eigenvalues of a 3×3 matrix B, where the associated eigenvectors are x_1, x_2, x_3. What are the eigenvalues and eigenvectors of B^{-1}?

7. Consider the plane P_1 in $\mathbb{R}^4$ described by $x_1+x_2-x_3 = 2$ and the line $l = \begin{pmatrix}\alpha, & \alpha+1, & -2\alpha, -\alpha\end{pmatrix}$. Find the points $p \in P_1$, $q \in l$ that minimize $\|p-q\|$. Are these points unique?

8. Let A be a 17×17 matrix where $A_{ij} = i - j$. Notice that $A^T = -A$. Let $x = \begin{bmatrix}1 & 2 & \dots & 17\end{bmatrix}^T$. What is $x^T\,A\,x$?

9. Let B be a 3×3 matrix and suppose that the eigenvectors x_1, x_2, x_3, with associated eigenvectors λ_1, λ_1, λ_2, span $\mathbb{R}^3$. Consider the matrix

$$A = \begin{bmatrix} B & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}.$$

   Find four vectors y_1, y_2, y_3 and y_4 that span $\mathbb{R}^4$ and are also eigenvectors of A.

10. Consider the matrix

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}.$$

Find a decomposition of A as $A = Q\Lambda Q^T$ where Q is orthogonal and Λ is diagonal.