MAT 281E – Homework 1

05.10.2012

1. Consider the linear system of equations

$$\begin{bmatrix} 2 & 3 \\ 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ b \end{bmatrix}.$$

   (a) Find a pair (a, b) so that the system has a unique solution.

   (b) Find a pair (a, b) so that the system has no solutions.

   (c) Find a pair (a, b) so that the system has infinitely many solutions.

2. Consider the linear system of equations

$$\begin{bmatrix} 3 & -3 & 2 & 0 \\ -3 & 3 & 1 & -3 \\ -15 & 10 & -10 & -3 \\ -12 & 17 & -2 & -4 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} = \begin{bmatrix} 6 \\ 0 \\ -19 \\ -21 \end{bmatrix}.$$

   Solve for $\mathbf{x}$ using Gaussian elimination and back-substitution.

3. Suppose A is an $n \times n$ matrix with inverse A^{-1}. Using A, we form the $2n \times 2n$ matrix C as,

$$C = \begin{bmatrix} A & A \\ A & A \end{bmatrix}$$

   Is C invertible? If so, find the inverse. If not, explain why not.

4. (a) Consider the matrix $M = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$. Find M^{-1} using the Gauss-Jordan procedure.

   (b) Let A be an $n \times n$ matrix. Also, let I denote the $n \times n$ identity matrix. Using A and I we form the $2n \times 2n$ matrix C as,

$$C = \begin{bmatrix} I & A \\ 0 & I \end{bmatrix}$$

   Is C invertible? If so, find the inverse. If not, explain why not.