MAT 281E – Homework 4

Due 03.12.2010

1. Let V be a k-dimensional subspace of $\mathbb{R}^n$. Show that $V^\perp$ is a subspace.

2. Does there exist a matrix whose row space contains $\begin{pmatrix} 1 & -2 & 1 \end{pmatrix}$ and whose null-space contains $\begin{pmatrix} -1 & 2 & 1 \end{pmatrix}$? If there exist such matrices, provide one. If not, explain why not.

3. In $\mathbb{R}^2$, describe two subspaces V_1, V_2 that are not orthogonal but such that any $x \in \mathbb{R}^2$ can be written as $x = x_1 + x_2$ where $x_1 \in V_1$ and $x_2 \in V_2$.

4. Let x, y be any two vectors. Show that
$$(x^T\, y)^2 \leq (x^T\, x)\,(y^T\, y). \tag{1}$$
Hint : Consider $\left\| x - \frac{y^T\,x}{y^T\,y}\, y \right\|^2$.
Note : This inequality is usually written as $\langle x, y\rangle \leq \|x\|\,\|y\|$, is very useful to know and is called __________ inequality.

5. Find the matrix that projects every point in $\mathbb{R}^3$ to the intersection of the planes $x + y + 2z = 0$ and $x + z = 0$.

6. Let P be the projection matrix that projects any vector onto a subspace V. What is the projection matrix for the subspace $V^\perp$? Please explain your answer.

7. (a) Let A be a $k \times k$ matrix whose rank is equal to k. If $A^2 = A$, show that actually $A = I$.
   (b) Let P be the projection matrix for a subspace V of $\mathbb{R}^n$. What is the condition on V such that P is invertible?