

MAT 281E – Homework 1

Due 08.10.2010

1. Consider the linear system of equations,

$$\underbrace{\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \pi \\ \pi \\ \pi \\ \pi \end{bmatrix}}_{\mathbf{b}}$$

- (a) For A , what is the sum of the elements in row 1? row 2? row 3? row 4?
 (b) Find an \mathbf{x} that satisfies the system above.

2. Consider the linear system of equations,

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\mathbf{x}} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

Find an \mathbf{x} that satisfies the system above.

3. Let us say that an $n \times n$ matrix with integer entries has property- M if all its rows, columns and diagonals add to the same number and all of its entries are distinct. For example, for $n = 3$, a matrix that has property- M is,

$$\begin{bmatrix} 8 & 3 & 4 \\ 1 & 5 & 9 \\ 6 & 7 & 2 \end{bmatrix}. \quad (1)$$

Notice that all of its rows, columns and diagonals add to 15.

Suppose now that A is a 4×4 matrix with entries $\{2, 3, \dots, 17\}$ and it has property- M . What is the sum of one of its rows?

4. Let A be a 5×5 matrix. Write down the matrix B (multiplying A on the left) that subtracts $3 \times \text{row}_2$ from row_4 and leaves the rest of the rows unchanged. What is B^{-1} ?
 5. Consider the equation $AB = C$ where

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \quad C = \begin{bmatrix} 3a & 4b & 5c \\ 3d & 4e & 5f \\ 3g & 4h & 5i \end{bmatrix}.$$

- (a) Find B .
 (b) Compute BA .
 (c) Write, *in words*, the action of B when it multiplies A on the right (i.e. how AB relates to A); on the left (i.e. how BA relates to A).