MAT 271E – Homework 3

Due 09.03.2011

1. A gambler tosses a coin three times. Each time a 'Head' comes, he gains 1 TL and each time a 'Tail' comes he loses 2 TL. Suppose that the coin is biased and the probability of 'Heads' is p. Assume also that the tosses are independent. Let X denote the amount of money (in TL) that the gambler gains (we say that he gains a negative amount if he loses money).

   (a) Find the PMF for X.

   (b) Compute $\mathbb{E}(X)$.

2. A gambler tosses a fair coin until the first 'Head' appears. If the number of tosses is n, she receives 2^n TL.

   (a) What is the expected gain of the gambler?

   (b) Suppose the gambler was asked to pay 3 TL to play this game once. She will decide to play if her probability of winning money is greater than than her probability of losing money. Will she play?

3. Suppose there are 20 boxes and in one of them is a ring. The ring is equally likely to be in any of the boxes. You open the boxes in any order you like until you find the ring. What is the expected number of boxes you open?

4. A gambler tosses a coin until the first 'Head' appears. Suppose the coin is biased and the probability of a 'Head' is p. Assume that the tosses are independent. Let X be the number of tosses. Find the mean and variance of X.

5. A troubled rabbit takes a test of 10 questions. Suppose it is known from past experience that the expected number of correct answers is 0.3.

   (a) Let A be the event defined as $A = \{$all answers are wrong$\}$. Find a lower bound for $P(A)$, i.e., find some $p > 0$ such that $P(A) \geq p$.

   (b) Let B be the event defined as $B = \{2$ or more answers are correct$\}$. Find an upper bound for $P(B)$ that is less than $1 - p$, i.e., find some $q < (1 - p)$ such that $P(B) \leq q$.