

TEL 519E – Take Home Final Examination

Due 18.01.2011

- (50 pts) 1. Let $f(x, y)$ be a circularly-symmetric function (i.e. $f(x, y)$ is a function of $x^2 + y^2$), and let

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy.$$

For $g(x) = e^{-(x-1)(x+1)}$, determine $f(x, y)$.

- (50 pts) 2. Let

$$h(x, y) = e^{-(x^2+y^2)},$$

$$f_1(x, y) = (x + 2y) e^{-(x^2+y^2)},$$

$$f_2(x, y) = (2x + y) e^{-(x^2+y^2)}.$$

Find the function $g(x, y)$ that minimizes

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |h(x, y) - g(x, y)|^2 dx dy.$$

subject to

$$\langle g(x, y), f_1(x, y) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f_1(x, y) dx dy = 1,$$

$$\langle g(x, y), f_2(x, y) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f_2(x, y) dx dy = 3.$$