MAT 281E – Homework 5

Due 10.12.2010

1. (a) Find a vector x that minimizes $\|A\,x - b\|$ where

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 5 \\ 0 & 2 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   (b) Is the vector x you found in part (a) unique or can you find $\tilde{x} \neq x$ such that $\|A\,\tilde{x} - b\| = \|A\,x - b\|$? If x is not unique, provide such a $\tilde{x}$. If it is unique, explain why.

2. Consider two lines l_1, l_2, described by $l_1 = \big(x,\, 2x,\, x\big)$, $l_2 = \big(y,\, 3y,\, -1\big)$.

   (a) Find two points p, q where $p \in l_1$, $q \in l_2$ such that $\|p - q\|$ is minimized.

   (b) Are the points you found in part (a) unique – that is, can you find $\tilde{p} \in l_1$, $\tilde{q} \in l_2$ such that $\tilde{p} \neq p$ or $\tilde{q} \neq q$ but $\|\tilde{p} - \tilde{q}\| = \|p - q\|$? Please explain your answer.

3. If Q_1 and Q_2 are orthogonal matrices, show that $Q_1\,Q_2$ is also orthogonal.

4. We showed in class that if Q has orthonormal columns, then it preserves the lengths of vectors, i.e. $\|Q\,x\| = \|x\|$ for every x. Show that the converse is also true. That is, show that if $\|Q\,x\| = \|x\|$ for every x, then Q has orthonormal columns.

   Hint : Suppose that $Q = \begin{bmatrix} q_1 & q_2 & \dots & q_k \end{bmatrix}$ does not have orthonormal columns and construct an x such that $\|Q\,x\| \neq \|x\|$. (Why is this equivalent to what you are trying to show?)

5. Find the $Q\,R$ decomposition of

$$A = \begin{bmatrix} 1 & -1 & 0 & -3 \\ 1 & 1 & 2 & 1 \\ 1 & -1 & -2 & 1 \\ 1 & 1 & 0 & -3 \end{bmatrix}.$$