MAT 281E – Homework 3

Due 01.11.2010

1. Which of the following subsets of $\mathbb{R}^3$ also form subspaces of $\mathbb{R}^3$? Please explain your answer.
   (a) All vectors $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$ with $x_2 = 0$.
   (b) All vectors $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$ with $x_1 = 1$.
   (c) The vector $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ alone.
   (d) All vectors $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$ with $x_2\, x_3 = 0$.
   (e) All vectors $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$ with $x_2 + x_3 = 1$.
   (f) All vectors $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}$ with $x_1 + 2\, x_3 = 0$.

2. Consider the system of equations

$$\underbrace{\begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 0 & 3 & 1 \\ 1 & 3 & 1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}}_{b}.$$

   (a) Describe $N(A)$, the nullspace of A (find the special solutions).
   (b) What is the rank of A?
   (c) What is the dimension of $N(A)$?
   (d) Describe the solution set of $A\,x = b$ (find a particular solution and use $N(A)$).

3. Find a 2×3 system $A\,x = b$ (i.e. find a 2×3 matrix A and a vector b) whose set of solutions is described by

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \alpha \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

   where α can be any real number.

4. Let A be an $m \times n$ matrix with full row rank. If the nullspace of A consists of

$$\alpha \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix},$$

   where α is an arbitrary scalar, what is m and n? Provide such a matrix A.

5. Suppose A is a $5 \times k$ matrix with $k \neq 5$ and it has full column rank. In this case, $C(A)$ is a subset of $\mathbb{R}^5$. Is it possible, for some choice of A and k, that actually $C(A) = \mathbb{R}^5$? If you think it is possible, provide an example. If not, explain why not.