Exercise 2

Computer Graphics - COMP.CE.430

Computer graphics mathematics

This week we will be learning computer graphics math. The goal of this week is to understand homogenous coordinate system, vectors and matrix transformations. Return your solutions to Moodle as a part of Project Work 1.

Tasks

1. A point in a 3D-space can have many different homogeneous coordinates. **(1p)**
    a. Give three unique homogeneous coordinates for a point $P = (1, 2, 3)$.
    b. Give three unique homogeneous coordinates for a direction vector $\bar{v} = (1, 2, 3)$.

2. Calculate following transforms for a point $P = (1, 2, 3)$ by using matrix transforms. **(1p)**
    a. Rotate P *90* ° around y axis.
    b. Scale P by two.
    c. Change the position of P to origin.

3. Objects that are farther away appear much smaller in the real life. A perspective is the reason for this phenomenon. In the computer graphics, the perspective can be mimicked by using a perspective projection matrix for transforming 3D-space points. **(1p)**
    a. Create the perspective projection matrix when the near distance $n = 0.1$, the far distance $f = 100.0$, the field of view $\alpha = 45°$. Use aspect ratio 16:9.
    b. Calculate the perspective projection for a point $P = (1, 2, 3)$

BONUS tasks:

4. 3D-space has a point $P = (-1, -1, 1)$ and a ray $\bar{r} = (1, 2, 3)$ which starts from point P. There is also a plane ABC which includes points $A = (1, 0, 0)$, $B = (0, 1, 0)$ and $C = (0, 0, 1)$. **(1p)**
    a. Find the intersection point between P and ABC.
    b. Calculate the angle between r and ABC at the intersection point.
    c. The plane ABC has a normal $\bar{n}$.
        i. Calculate $|\bar{n} \cdot \bar{r}|$.
        ii. Calculate $|\bar{n} \times \bar{r}|$.

5. 3D-space has a camera at point $P = (1, 1, 1)$. The camera is looking at origin. Calculate a view space matrix by using up vector $\bar{u} = (0, 1, 0)$.
   (1p)

   Hint. You have to find three vectors which create the orthonormal basis of the view space. Up vector helps you to get another basis component of the view space. The calculation involves a matrix inversion which you are allowed to skip and just mark with –1 exponent.