Exercise #2

1) a) Homogeneous coordinates for point $P = (1,2,3)$

- $(1, 2, 3, 1)$
- $(2, 4, 6, 2)$
- $(0.5, 1, 1.5, 0.5)$

b) Homogenenous coordinates for direction vector $\bar{v} = (1,2,3)$

- $(1, 2, 3, 0)$
- $(2, 4, 6, 0)$
- $(0.5, 1, 1.5, 0)$

2) a) Rotate $P = (1, 2, 3)$ 90° around y axis, where R_y is rotation matrix

$P' = R_y \cdot P$

$$= \begin{bmatrix} \overset{0}{\cos 90^\circ} & 0 & \overset{1}{\sin 90^\circ} & 0 \\ 0 & 1 & 0 & 0 \\ -\overset{1}{\sin 90^\circ} & 0 & \overset{0}{\cos 90^\circ} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+0+3+0 \\ 0+2+0+0 \\ -1+0+0+0 \\ 0+0+0\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

$P' = (3, 2, -1)$

b) Scale $P = (1,2,3)$ by two

$P' = S \cdot P$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix}$$

Hence $P' = (2, 4, 6)$

c) Change position of $P = (1,2,3)$ to origin.

$P' = T \cdot P$

new position of the point P will be on the origin

$$= \begin{bmatrix} 1 + t_x \\ 2 + t_y \\ 3 + t_z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(how much it is moved)

Hence, the t_x, t_y, t_z values for this translation is $(-1, -2, -3)$. P', the new point after translation, is $(0,0,0)$.

Hence, translation matrix $T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3) a) Create perspective projection matrix (P)

$$P_m = \begin{bmatrix} \frac{1}{a\tan(\alpha/2)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan(\alpha/2)} & 0 & 0 \\ 0 & 0 & \frac{n}{f-n} & \frac{fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$n = 0.1$

$f = 100$

$\alpha = 45^\circ = \pi/4$

$a = 16:9$

$\tan(45^\circ/2) \cong 0.414$

$1/\tan(\alpha/2) \cong 2.41$

$= 1 + \sqrt{2}$

$\frac{9}{16\tan(\alpha/2)} \cong 1.36$

$= \frac{1}{16}(9 + 9\sqrt{2})$

$$P_m = \begin{bmatrix} \cong 1.36 & 0 & 0 & 0 \\ 0 & \cong 2.41 & 0 & 0 \\ 0 & 0 & 0.\overline{001} & 0.\overline{100} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

b) $P' = P_m P \cong \begin{bmatrix} \cong 1.36 \\ \cong 4.83 \\ 0.\overline{003} + 0.\overline{100} \\ -3 \end{bmatrix} \cong \begin{bmatrix} 1.36 \\ 4.83 \\ 0.\overline{103} \\ -3 \end{bmatrix} \cong \begin{bmatrix} -0.45 \\ -1.61 \\ -0.\overline{03436 7701} \\ 1 \end{bmatrix}$

4) a) Intersection between $P = (-1, -1, 1)$ and ABC.

- Find the ABC plane equation

$\vec{AB} \otimes \vec{AC} = (\vec{B} - \vec{A}) \otimes (\vec{C} - \vec{A}) = \vec{n}$ (normal vector)

$= (-1, 1, 0) \otimes (-1, 0, 1) = (1, 1, 1)$

Hence, the equation of ABC plane is $x + y + z + d = 0$. Use A/B/C points which lie on ABC plane to find d.

Set $A = (1,0,0) \rightarrow 1 + 0 + 0 + d = 0$; $d = -1$

set $B = (0,1,0) \rightarrow 0 + 1 + 0 + d = 0$; $d = -1$

set $C = (0,0,1) \rightarrow 0 + 0 + 1 + d = 0$; $d = -1$

Hence, equation becomes

$x + y + z - 1 = 0$

- check whether P lies in the ABC plane.

$x + y + z + 1 = -1 - \cancel{1} + \cancel{1} - 1 = -2 \neq 0$, meaning that there is no intersection between point P and ABC plane.

b) Angle between r and ABC at the intersection point

$$\theta = \arccos\left(\frac{\vec{n}\cdot\vec{d}}{\|\vec{n}\|\,\|\vec{d}\|}\right)$$ where $\vec{n}$ is the normal vector of plane ABC and $\vec{d}$ is the direction vector of ray.

$\vec{n} = (1,1,1)$
$\vec{d} = (1,2,3)$

$$\theta = \arccos\left(\frac{(1+2+3)}{(\sqrt{1+1+1})(\sqrt{1+4+9})}\right) = \arccos\left(\frac{6}{\sqrt{3}\sqrt{14}}\right) \cong 0.39 \text{ in radians} \cong 22.21^\circ$$

Then to find the angle between r and ABC plane, we do this: $\pi/2 - \theta \cong 67{,}79^\circ$

c) $\bar{n} = (1,1,1)$ found in 4a.

i. $|\bar{n}\cdot\bar{r}| = |(1,1,1)\cdot(1,2,3)| = |(1+2+3)| = 6$

i. $|\bar{n}\times\bar{r}| = |(1,-2,1)| = \sqrt{1+4+1} = \sqrt{6}$

n) • Forward vector ⇒ from camera position(P) to target point (origin)
$\vec{f} = (1,1,1) - (0,0,0) = (1,1,1)$, then normalize it to find $\boxed{\vec{z} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

• Right vector $= (\vec{v} \otimes \vec{f}) = (0,1,0) \otimes \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) =$
$= \left(\frac{1}{\sqrt{3}}, 0, \frac{-1}{\sqrt{3}}\right)$ then normalize it to find $\boxed{\vec{x} = \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)}$

• And up vector, $\vec{y}$ is $(\vec{f} \otimes \vec{z}) = \boxed{\left(\frac{-1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{-1}{\sqrt{6}}\right) = \vec{y}}$

$$V = \text{view matrix} = \begin{array}{c} \begin{array}{cccc} \vec{x} & \vec{y} & \vec{z} & \end{array} \\ \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} & 0 \\ 0 & \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} & 0 \\ -0 & 0 & 0 & 1 \end{bmatrix}^{-1} \end{array}$$

After taking the inverse of the matrix on the left, you will also change the values in the last row which have zeros. After the inverse operation, the last row stays the same.

$\to -\text{dot}(z, (1,1,1)) = 3/\sqrt{3} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\to -\text{dot}(y, (1,1,1)) = \frac{-2}{\sqrt{6}} + \frac{2}{\sqrt{6}} = 0$

$\to -\text{dot}(x, (1,1,1)) = 0$

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$$V = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{6}/6 & \sqrt{3}/3 & 0 \\ 0 & \sqrt{6}/3 & \sqrt{3}/3 & 0 \\ -\sqrt{2}/2 & -\frac{\sqrt{6}}{6} & \sqrt{3}/3 & 0 \\ 0 & 0 & \sqrt{3} & 1 \end{bmatrix}$$