

# **Permutations & Combinations**

**Extension 1 Mathematics**

**HSC Revision**

# Multiplication Rule

If one event can occur in **m** ways, a second event in **n** ways and a third event in **r**, then the three events can occur in **m × n × r** ways.

**Example** Erin has 5 tops, 6 skirts and 4 caps from which to choose an outfit.

In how many ways can she select one top, one skirt and one cap?

**Solution:** **Ways = 5 × 6 × 4**

# Repetition of an Event

If one event with **n** outcomes occurs **r** times with repetition allowed, then the number of ordered arrangements is  **$n^r$**

**Example 1** What is the number of arrangements if a die is rolled

(a) 2 times ?  $6 \times 6 = 6^2$

(b) 3 times ?  $6 \times 6 \times 6 = 6^3$

(b) r times ?  $6 \times 6 \times 6 \times \dots = 6^r$

# Repetition of an Event

## Example 2

(a) How many different car number plates are possible

with 3 letters followed by 3 digits?

**Solution:**  $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 26^3 \times 10^3$

(b) How many of these number plates begin with ABC?  
?

**Solution:**  $1 \times 1 \times 1 \times 10 \times 10 \times 10 = 10^3$

(c) If a plate is chosen at random, what is the probability that it begins with ABC?

**Solution:**  $\frac{10^3}{26^3 \times 10^3} = \frac{1}{26^3}$

# Factorial Representation

$$n! = n(n - 1)(n - 2).....3 \times 2 \times 1$$

For example  $5! = 5.4.3.2.1$

**Note**  $0! = 1$

## Example

a) In how many ways can 6 people be arranged in a row?

**Solution :**  $6.5.4.3.2.1 = 6!$

b) How many arrangements are possible if only 3 of them are chosen?

**Solution:**  $6.5.4 = 120$

# Arrangements or Permutations

Distinctly ordered sets are called **arrangements** or **permutations**.

The number of permutations of **n** objects taken **r** at a time is given by:

$${}^n\mathbf{P}_r = \frac{\mathbf{n}!}{(\mathbf{n} - \mathbf{r})!}$$

where      n = number of objects  
              r = number of positions

# Arrangements or Permutations

Eg 1. A maths debating team consists of 4 speakers.

a) In how many ways can all 4 speakers be arranged in a row for a photo?

**Solution :**  $4.3.2.1 = 4!$  or  ${}^4P_4$

b) How many ways can the captain and vice-captain be chosen?

**Solution :**  $4.3 = 12$  or  ${}^4P_2$



# Arrangements or Permutations

Eg 2. A flutter on the horses  
There are 7 horses in a race.



a) In how many different orders can the horses finish?

**Solution :**  $7.6.5.4.3.2.1 = 7!$  or  ${}^7P_7$

b) How many trifectas (1<sup>st</sup> , 2<sup>nd</sup> and 3<sup>rd</sup>) are possible?

**Solution :**  $7.6.5 = 210$  or  ${}^7P_3$





# Permutations with Restrictions

Eg. In how many ways can 5 boys and 4 girls be arranged on a bench if

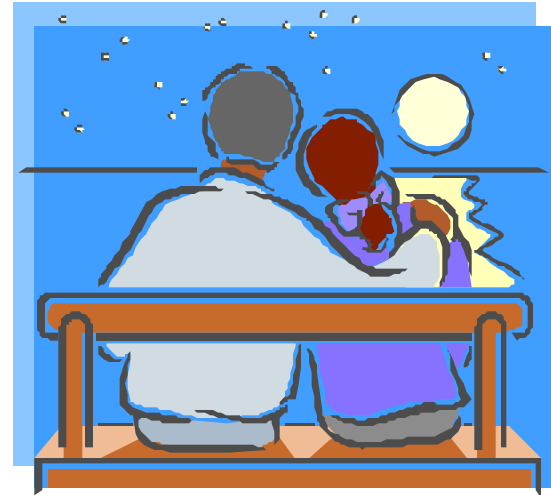
a) there are no restrictions?

**Solution :** 9! or  ${}^9P_9$

c) boys and girls alternate?

**Solution :** A boy will be on each end

$$\begin{aligned} \text{BGBGBGBGB} &= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 \\ &= 5! \times 4! \text{ or } {}^5P_5 \times {}^4P_4 \end{aligned}$$



# Permutations with Restrictions

Eg. In how many ways can 5 boys and 4 girls be arranged on a bench if

c) boys and girls are in separate groups?

**Solution :** Boys & Girls or Girls & Boys

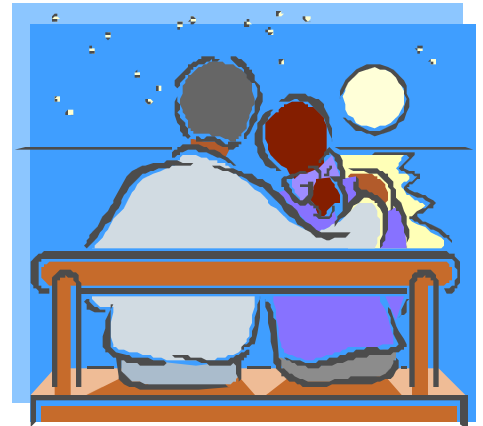
$$= 5! \times 4! + 4! \times 5! = 5! \times 4! \times 2$$

$$\text{or } {}^5P_5 \times {}^4P_4 \times 2$$

d) Anne and Jim wish to stay together?

**Solution :** (AJ) \_ \_ \_ \_ \_

$$= 2 \times 8! \text{ or } 2 \times {}^8P_8$$



# Arrangements with Repetitions

If we have  $n$  elements of which  $x$  are alike of one kind,  $y$  are alike of another kind,  $z$  are alike of another kind,

..... then the number of ordered selections or permutations is given by:

$$\frac{n!}{x! y! z!}$$

# Arrangements with Repetitions

Eg.1 How many different arrangements of the word **PARRAMATTA** are possible?

**Solution :** 10 letters but note repetition  
(4 A's, 2 R's, 2 T's)

P

A A A A

R R

M

T T

$$\begin{aligned}\text{No. of} \\ \text{arrangements} &= \frac{10!}{4! 2! 2!} \\ &= 37\,800\end{aligned}$$



# Arrangements with Restrictions

Eg 1. How many arrangements of the letters of the word REMAND are possible if:

a) there are no restrictions?

**Solution :**  ${}^6P_6 = 720$  or  $6!$

b) they begin with RE?

**Solution :**  $\underline{R} \underline{E} \_ \_ \_ \_ = {}^4P_4 = 24$  or  $4!$

c) they do not begin with RE?

**Solution :**  $\text{Total} - (b) = 6! - 4! = 696$

# Arrangements with Restrictions

Eg 1. How many arrangements of the letters of the word REMAND are possible if:

d) they have RE together in order?

**Solution :** **(RE)** \_ \_ \_ \_ =  ${}^5P_5 = 120$  or  $5!$

e) they have REM together in any order?

**Solution :** **(REM)** \_ \_ \_ =  ${}^3P_3 \times {}^4P_4 = 144$

f) R, E and M are not to be together?

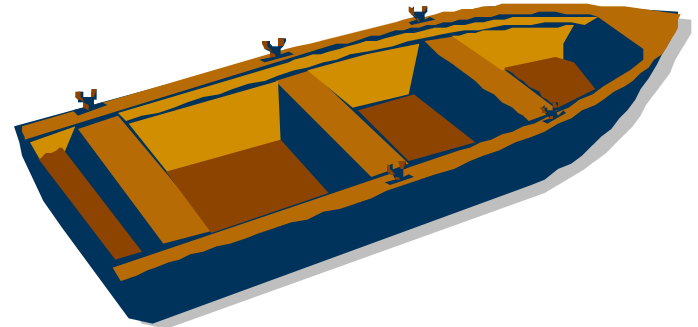
**Solution :** **Total – (e) =  $6! - 144 = 576$**

# Arrangements with Restrictions

Eg 2. There are 6 boys who enter a boat with 8 seats, 4 on each side. In how many ways can

a) they sit anywhere?

**Solution :**  ${}^8P_6$



b) two boys A and B sit on the port side and another boy W sit on the starboard side?

**Solution :**  $A \ \& \ B = {}^4P_2$

$W = {}^4P_1$

$\text{Others} = {}^5P_3$

**Total**  $= {}^4P_2 \times {}^4P_1 \times {}^5P_3$



# Arrangements with Restrictions

Eg 3. From the digits 2, 3, 4, 5, 6

a) how many numbers greater than 4 000 can be formed?

**Solution :** 5 digits (any) =  ${}^5P_5$

4 digits (must start with digit  $\geq 4$ ) =  ${}^3P_1 \times {}^4P_3$

**Total** =  ${}^5P_5 + {}^3P_1 \times {}^4P_3$

b) how many 4 digit numbers would be even?

**Even (ends with 2, 4 or 6)** =  $\_ \_ \_ {}^3P_1$

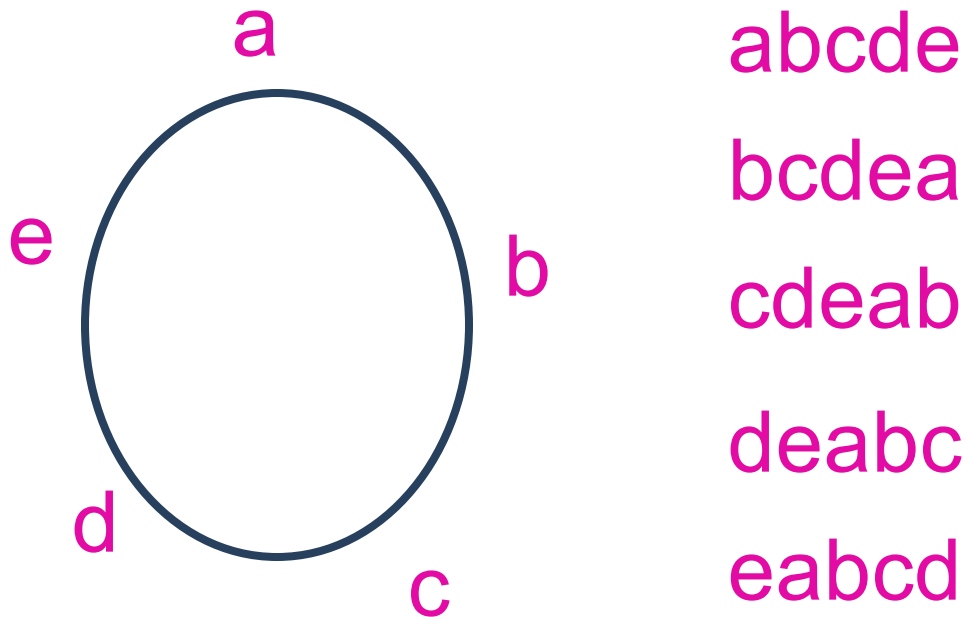
$= {}^4P_3 \times {}^3P_1$



# Circular Arrangements

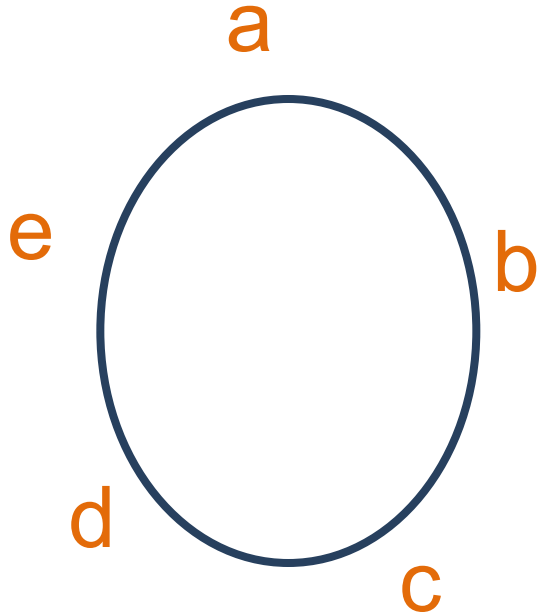
Circular arrangements are permutations in which objects are arranged in a circle.

Consider arranging 5 objects (a, b, c, d, e) around a circular table. The arrangements



are different in a line, but are **identical** around a circle.

# Circular Arrangements



To calculate the number of ways in which  $n$  objects can be arranged in a circle, we arbitrarily fix the position of one object, so the remaining  $(n-1)$  objects can be arranged as if they were on a straight line in  $(n-1)!$  ways.

**i.e. the number of arrangements =  $(n - 1) !$   
in a circle**

# Circular Arrangements

Eg 1. At a dinner party 6 men and 6 women sit at a round table. In how many ways can they sit if:

a) there are no restrictions

**Solution :**

$$(12 - 1)! = 11!$$



b) men and women alternate

**Solution :**  $(6 - 1)! \times 6! = 5! \times 6!$

# Circular Arrangements

Eg 1. At a dinner party 6 men and 6 women sit at a round table. In how many ways can they sit if:

c) Ted and Carol must sit together

**Solution :** (TC) & other 10 =  $2! \times 10!$

d) Bob, Ted and Carol must sit together

**Solution :** (BTC) & other 9 =  $3! \times 9!$

# Circular Arrangements

Eg 1. At a dinner party 6 men and 6 women sit at a round table. In how many ways can they sit if:

d) Neither Bob nor Carol can sit next to Ted.

**Solution :** Seat 2 of the other 9 people next to Ted in  $(9 \times 8)$  ways or  ${}^9P_2$

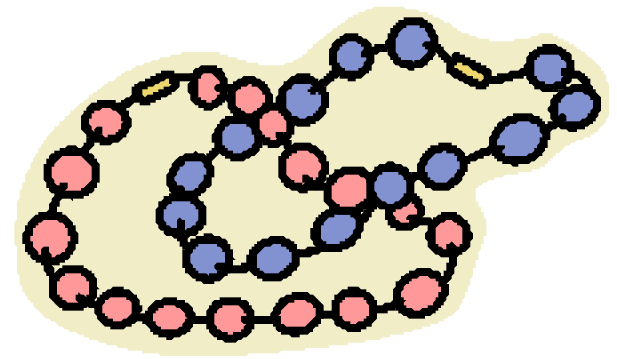
Then sit the remaining 9 people (including Bob and Carol) in  $9!$  ways

**Ways** =  $(9 \times 8) \times 9!$  or  ${}^9P_2 \times 9!$

# Circular Arrangements

Eg 2. In how many ways can 8 differently coloured beads be threaded on a string?

**Solution :**



**As necklace can be turned over, clockwise and anti-clockwise arrangements are the same**

$$= (8-1)! \div 2 = 7! \div 2$$

# Unordered Selections

The number of different **combinations** (i.e. unordered sets) of **r** objects from **n** distinct objects is represented by :

No. of Combinations	=	$\frac{\text{number of permutations}}{\text{arrangements of } r \text{ objects}}$
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and is denoted by

${}^n\mathbf{C}_r = \frac{{}^n\mathbf{P}_r}{r!} = \frac{n!}{r! (n - r)!}$
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# Combinations

Eg 1. How many ways can a basketball team of 5 players be chosen from 8 players?

**Solution :**

$${}^8C_5$$



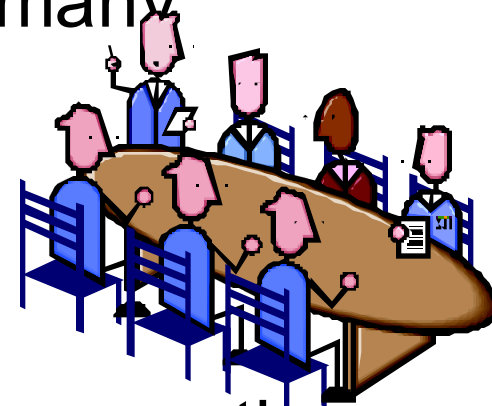


# Combinations

Eg 2. A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if

a) there are no restrictions?

**Solution :**  $^{10}C_5$



b) one particular person must be chosen on the committee?

**Solution :**  $\underline{1} \times {}^9C_4$

c) one particular woman must be excluded from the committee?

**Solution :**  ${}^9C_5$

# Combinations

Eg 2. A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if:

d) there are to be 3 men and 2 women?

**Solution :**      **Men & Women =  ${}^6C_3 \times {}^4C_2$**

e) there are to be men only?

**Solution :**       **${}^6C_5$**

f) there is to be a majority of women?

**Solution :**

**3 Women & 2 men Or 4 Women & 1 man**

$$= {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

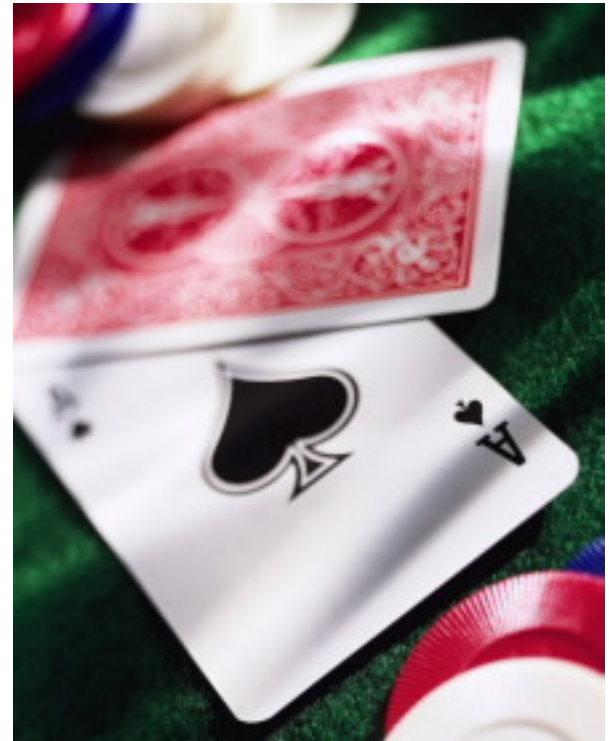
# Combinations

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

(i) What is the total possible number of hands if there are no restrictions?

**Solution :**

$${}^{52}C_5$$



# Combinations

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

ii) In how many of these hands are there:

a) 4 Kings?

**Solution :**  ${}^4C_4 \times {}^{48}C_1$  or  $1 \times 48$

b) 2 Clubs and 3 Hearts?

**Solution :**  ${}^{13}C_2 \times {}^{13}C_3$

# Combinations

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

- ii) In how many of these hands are there:
- c) all Hearts?

**Solution :**  $^{13}C_5$



- d) all the same colour?

**Solution :** **Red or Black**  $^{26}C_5 + ^{26}C_5 = 2 \times ^{26}C_5$   
**=**

# Combinations

Eg 3. In a hand of poker, 5 cards are dealt from a regular pack of 52 cards.

ii) In how many of these hands are there

e) four of the same kind?

**Solution :**

$${}^4C_4 \times {}^{48}C_1 \times 13 = 1 \times 48 \times 13$$

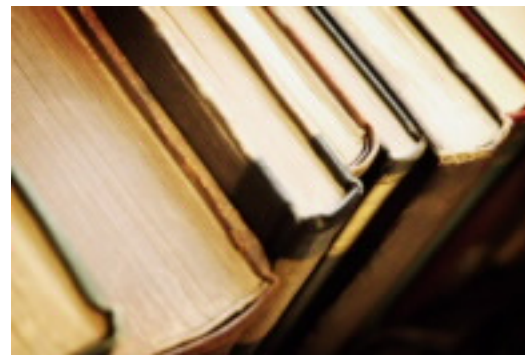
f) 3 Aces and two Kings?

**Solution :**  ${}^4C_3 \times {}^4C_2$



# Further Permutations and Combinations

Eg.1 If 4 Maths books are selected from 6 different Maths books and 3 English books are chosen from 5 different English books, how many ways can the seven books be arranged on a shelf:



a) If there are no restrictions?

**Solution :**  ${}^6C_4 \times {}^5C_3 \times 7!$

c) If the 4 Maths books remain together?

**Solution :**  $= (MMMM) \_ \_ \_$

$$= {}^6P_4 \times {}^5C_3 \times 4! \text{ or } ({}^6C_4 \times 4!) \times {}^5C_3 \times 4!$$

# Further Permutations and Combinations

Eg.1 If 4 Maths books are selected from 6 different Maths books and 3 English books are chosen from 5 different English books, how many ways can the seven books be arranged on a shelf if:

c) a Maths book is at the beginning of the shelf?



**Solution :**      **= M \_ \_ \_ \_ \_**

$$= 6 \times {}^5C_3 \times {}^5C_3 \times 6!$$



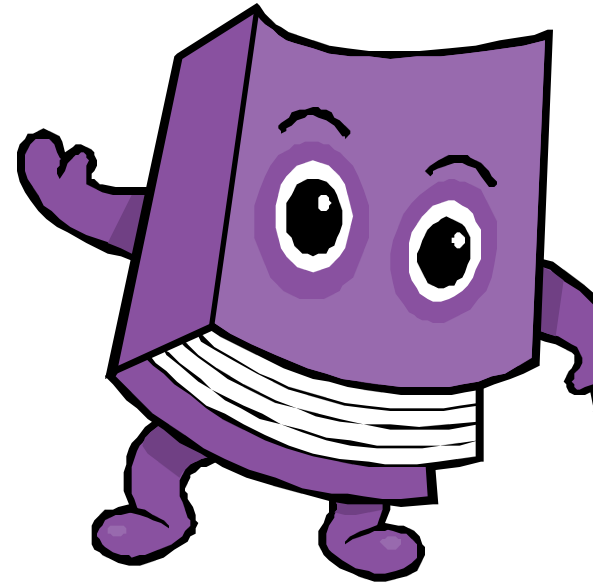
# Further Permutations and Combinations

Eg.1 If 4 Maths books are selected from 6 different Maths books and 3 English books are chosen from 5 different English books, how many ways can the seven books be arranged on a shelf if:

d) Maths and English books alternate

**Solution :**      **= M E M E M E M**

$$= {}^6P_4 \times {}^5P_3$$



# Further Permutations and Combinations

Eg.1 If 4 Maths books are selected from 6 different Maths books and 3 English books are chosen from 5 different English books, how many ways can the seven books be arranged on a shelf if:

e) A Maths is at the beginning and an English book is in the middle of the shelf.

**Solution :**

$$\text{M} \_ \_ \text{E} \_ \_ \_$$
$$= 6 \times 5 \times {}^5\text{C}_3 \times {}^4\text{C}_2 \times 5!$$

# Further Permutations and Combinations

Eg 2. (i) How many different 8 letter words are possible using the letters of the word SYLLABUS ?

**Solution :**    **2 S's & 2 L's**

$$\begin{aligned}\text{Words} &= \frac{8!}{2! \times 2!} \\ &= 10\,080\end{aligned}$$

# Further Permutations and Combinations

**SYLLABUS = 10 080 permutations**

(ii) If a word is chosen at random, find the probability that the word:

a) contains the two S's together

**Solution :** **(SS)** \_ \_ \_ \_ \_ **(Two L's)**

$$\text{Words} = \frac{7!}{2!} = 2520 \qquad \text{Prob} = \frac{2520}{10080} = \frac{1}{4}$$

b) begins and ends with L

**Solution :** **L** \_ \_ \_ \_ \_ **L** **(Two S's)**

$$\text{Words} = \frac{6!}{2!} = 360 \qquad \text{Prob} = \frac{360}{10080} = \frac{1}{28}$$