Lecture 6

January 25, 2016

0.1 Review of Last Friday

When you row reduce a matrix you get pivot and non-pivot columns. Pivot columns are those whose bottom halfs follow the identity matrix pattern.

$$\text{Nullity}(A) = \text{Number of non-pivot columns}$$

$$\text{Rank}(A) = \text{Number of pivot columns}$$

$$\text{Rank}(A) + \text{Nullity}(A) = \text{Total number of columns}$$

1 Inverse Matrices

Given a matrix A, the inverse of A is a matrix B such that (where I is the identity matrix):

$$A * B = I = B * A$$

For a 2×2 matrix we can find the inverse like:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For bigger matrices the process becomes harder and its better to not have an equation but an algorithm. To find the inverse of an $n \times n$ matrix augment A with I and perform a row reduction:

$$[A \mid I] \sim [I \mid B] \rightarrow B = A^{-1}$$

2 Basis & Linear Mapping

2.1 Coordinates in a non-standard Basis

Let a basis S be $\{\vec{v_1}, \dots, \vec{v_n}\}$. Any vector $\vec{x}$ in S is written uniquely as (where $c_k \in R$):

$$\vec{x} = c_1\vec{v_1} + \cdots + c_n\vec{v_n}$$

The coordinates of $\vec{x}$ in the basis of $\vec{v}$ is written in the following way:

$$\bar{\psi}_{\{\vec{v}\}}(\vec{x}) = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

Example $\{3, 2x-1, 2x^2+x, x^3-2x^2+1\}$ is a basis of polynomial space of order 3. Find the coordinates of $3+2x^2+x^3$ in this basis.

Doing this is equivalent to solving this equation:

$$3+2x^2+x^3 = c_1(3)+c_2(2x-1)+c_3(2x^2+x)+c_4(x^3-2x^2+1)$$

What are the coefficients of x^3?

$$1x^3 = c_4x^3 \to c_4 = 1$$

What about x^2?

$$2x^2 = 2c_3x^2 - 2x^2 \to c_3 = 2$$

No x term, so:

$$0x = 2c_2x + 2x \to c_2 = -1$$

What about 3?

$$3 = 3c_1 + 1 + 1 \to c_1 = \frac{1}{3}$$

therefore the coordinates are: $(1, 2, -1, \frac{1}{3})$.

3 Linear Mapping

$T : V \to W$ is a linear mapping if $T(\vec{0}) = \vec{0}$ and $T(\alpha\vec{v_1}+\beta\vec{v_2}) = \alpha T(\vec{v_1})+\beta T(\vec{v_2})$

A linear mapping between $V = \{1, x, x^2\}$ and $W = \{1, x\}$ can be seen as the derivative from an order 2 to an order 1 polynomial.

This will be explained further next lecture ...