Lecture 3

January 15, 2016

12.4 Cont.

Recall

- if $\vec{u} \cdot \vec{v} = 0$ then $\vec{u}$ and $\vec{v}$ are perpendicular to each other.
- if $\vec{u} \times \vec{v} = \vec{0}$ then $\vec{u}$ and $\vec{v}$ are parallel.

More Vector Properties

- $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
- $\mid \vec{u} \times \vec{v} \mid$ is the area of the parallelogram whose sides are these two vectors.
- $\vec{u} \times \vec{v} = \vec{n} \mid \vec{u} \mid\mid \vec{v} \mid \sin\theta$ where $\vec{n}$ is the normal of the two vectors.
- $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$

Triple Scalar Product

The triple scalar product of $\vec{u}$, $\vec{v}$, and $\vec{w}$:

$$(\vec{u} \times \vec{v}) \cdot \vec{w}$$

It is equivalent to the determinant of the matrix:

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

These vectors form a corner of a parallelepiped in space. The absolute value of the triple scalar is the volume of this shape:

$$\mid (\vec{u} \times \vec{v}) \cdot \vec{w} \mid = \text{volume of parallelepiped}$$

12.5 Lines & Planes in space

Equation of a Line

Suppose we have a line L that has a point which passes through a point $\vec{P_0}$ and this line is parallel to $\vec{v} = \langle v_1, v_2, v_3 \rangle$. Then the equation for a line is as follows:

$$L(t) = t * (\vec{v} - \vec{P_0})$$

Equation of a Plane

Let $\vec{P}$ be any point on the plane and $\vec{P_0}$ be a point on the plane, also let $\vec{n}$ be a vector perpendicular to the plane. From the dot product we get:

$$(\vec{P} - \vec{P_0}) \cdot \vec{n} = 0$$

If $\vec{n} = \langle A, B, C \rangle$ then the equation becomes:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

Moving the constants to one side and assigning them to D creates the standard equation of a plane:

$$Ax + By + Cz = Ax_0 + By_0 + Cz_0 = D$$

Example The points $P(1, 1, 1)$, $Q(2, 1, 3)$, and $R(3, -1, 1)$ on the plane E. Find the equation of the plane E.

We need to find the normal vector, just cross two vectors parallel to the plane:

$$\vec{PQ} = \langle 1, 0, 2 \rangle$$
$$\vec{PR} = \langle 2, -2, 0 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ 2 & -2 & 0 \end{vmatrix} = \langle 4, 4, -2 \rangle = \langle 2, 2, -1 \rangle = \vec{n}$$

Then we pick a point on the vector to act as $\vec{P_0}$, lets do P

The equation is then:

$$4(x - 1) + 4(y - 1) - 2(z - 1) = 0$$

Or:

$$2(x - 1) + 2(y - 1) - (z - 1) = 0$$

Or (most commonly):

$$2x + 2y - z = 3$$

Distance from point to line

To find the distance form a point S to a line L, then this distance is

$$d = \frac{\mid \vec{PS} \times \vec{v} \mid}{\mid \vec{v} \mid}$$

Where $\vec{v}$ is a vecotr parallel to the line and P is a point on the line.

Distance from point to plane

To find the distance between a point S to ao plane E:

$$d = \frac{\vec{PS} \cdot \vec{n}}{\mid \vec{n} \mid}$$

where P is a point on the plane and $\vec{n}$ is the normal vector for that plane.