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Design and Modeling of a Hydrogen Fuel Cell Powered VTOL UAV

Energy System Sizing, Aerodynamic Modeling, and Analysis

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Introduction

This document explores the physics model of a 5 motor configuration of a VTOL UAV, powered by a hydrogen fuel cell.

Physics Model - 5 Motor Configuration

0. Constants

g	gravitational acceleration (m/s^2)
ρ_{air}	air density (kg/m^3)

1. Geometry and Mass

$m \equiv m_{\text{plane}}$	total aircraft mass including fuel (kg)
x_G, y_G, z_G	center-of-gravity coordinates (m) in body frame
$\mathbf{r}_i = (x_i, y_i, z_i)$	position vector of rotor i relative to CG (m)
$N = 5$	number of rotors (motors)
Rotors $i = 1, 2, 3$	tilt rotors (variable tilt ϕ_i)
Rotors $i = 4, 5$	fixed vertical wing rotors (no tilt)

2. Rotor thrust and decomposition

Let T_i be the thrust magnitude produced by rotor i (N). For a tilt rotor i with tilt angle ϕ_i measured from vertical towards the aircraft nose:

$$T_i^{(V)} = T_i \cos\phi_i \quad \text{(vertical component of thrust, N)}$$
$$T_i^{(H)} = T_i \sin\phi_i \quad \text{(forward/horizontal component, N)}$$

For fixed vertical rotors (4,5), $\phi_i = 0$ so $T_i^{(V)} = T_i$ and $T_i^{(H)} = 0$.

3. Force balance (translational)

Define body-aligned axes: $+\hat{z}$ upward, $+\hat{x}$ forward, $+\hat{y}$ right wing.

Vertical (hover) equilibrium:

$$\sum_{i=1}^{N} T_i^{(V)} + L_{\text{wing}} - mg = m\,a_z$$

where L_{wing} is aerodynamic wing lift (positive upward) and a_z is vertical acceleration (m/s^2). For steady hover level flight set $a_z = 0$.

Longitudinal (forward) equation:

$$\sum_{i=1}^{N} T_i^{(H)} - D - mg\sin\theta = m\,a_x$$

where D is total aerodynamic drag, θ is body pitch angle (positive nose-up), and a_x is forward acceleration. For steady cruise $a_x = 0$.

Lateral (side) equation:

$$\sum_{i=1}^{N} T_i^{(y)} - Y = m\,a_y$$

(Usually small; $T_i^{(y)}$ are lateral components if rotor axis misaligned; Y is side aerodynamic force.)

4. Wing Aerodynamics (Cruise)

Let S be wing area, C_L lift coefficient, C_D drag coefficient, and V the flight speed (m/s). Then:

$$\begin{aligned} L_{\text{wing}} &= \tfrac{1}{2}\rho_{\text{air}} V^2 S C_L \\ D &= \tfrac{1}{2}\rho_{\text{air}} V^2 S C_D \end{aligned}$$

For level cruise (no vertical acceleration) the total vertical force balance becomes:

$$\sum_{i=1}^{N} T_i \cos\phi_i + \tfrac{1}{2}\rho V^2 S C_L = mg$$

5. Rotor Momentum Theory and Rotor Power

For rotor i with rotor disk area A_i (m^2), momentum theory gives the induced velocity in hover (vertical-only thrust component):

$$v_{i,\mathrm{ind}} \approx \sqrt{\frac{T_i^{(V)}}{2\rho_{\mathrm{air}} A_i}}.$$

(For heavily loaded rotors, more detailed rotor aerodynamics may be used.)

Mechanical power required by rotor i (ideal induced power approximation) is:

$$P_{i,\mathrm{ind}} = T_i^{(V)}\, v_{i,\mathrm{ind}} = T_i^{(V)} \sqrt{\frac{T_i^{(V)}}{2\rho_{\mathrm{air}} A_i}}.$$

Include profile and parasitic losses by adding a profile power $P_{i,\mathrm{profile}}$ (empirical) or using an overall rotor efficiency $\eta_{i,\mathrm{rot}}$. The electrical power drawn from the motor+ESC for rotor i is:

$$P_{i,\mathrm{elec}} = \frac{P_{i,\mathrm{ind}} + P_{i,\mathrm{profile}}}{\eta_{i,\mathrm{prop}}}$$

A common compact form (using an effective propeller efficiency η_{prop}):

$$P_{i,\mathrm{elec}} \approx \frac{T_i^{(V)}\, v_{i,\mathrm{ind}}}{\eta_{\mathrm{prop}}}$$

For the horizontal component induced by tilting the rotor, the incremental power to produce horizontal thrust is treated either by tilt-axis motor power model or approximated by using motor power to accelerate airflow; for design-level work we often compute total aerodynamic power required for forward thrust separately (windmill/propulsive efficiency models).

6. Total Electrical Power Required

Total electrical power demand for propulsion:

$$P_{\mathrm{elec,prop}} = \sum_{i=1}^{N} P_{i,\mathrm{elec}}$$

If avionics, autopilot, and fuel cell balance-of-plant draw P_{av}, then total electrical load is:

$$P_{\text{elec,total}} = P_{\text{elec,prop}} + P_{\text{av}}$$

7. Moments and Control (Static Equations)

Compute moments about the center of gravity due to rotor thrusts (for roll, pitch, yaw):

Roll moment (about x-axis):

$$M_x = \sum_{i=1}^{N} \left(y_i \, T_i^{(V)} \right) + M_{x,\text{aero}}$$

where y_i is lateral offset of rotor i (positive to the right). $M_{x,\text{aero}}$ includes aerodynamic rolling moments.

Pitch moment (about y-axis):

$$M_y = \sum_{i=1}^{N} \left(z_i \, T_i^{(H)} - x_i \, T_i^{(V)} \right) + M_{y,\text{aero}}$$

Yaw moment (about z-axis):

$$M_z = \sum_{i=1}^{N} \tau_{i,\text{rotor}} + M_{z,\text{aero}}$$

where $\tau_{i,\text{rotor}}$ are rotor torque/anti-torque contributions (including propeller torques and control differential).

For small-angle linearization, the rotational equations of motion are:

$$\mathbf{I}\,\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}\,\boldsymbol{\omega}) = \mathbf{M}$$

where $\mathbf{I}$ is inertia tensor, $\boldsymbol{\omega}$ is angular velocity vector, and $\mathbf{M} = (M_x, M_y, M_z)^T$.

8. Flight Phases

Hover: $\phi_i \approx 0$ for tilt rotors.

$$\sum_{i=1}^{5} T_i = mg \quad \text{(steady hover)}$$

Transition: tilt angles $\phi_i(t)$ change from 0 to a cruise tilt ϕ_{cruise}. Equations of motion (longitudinal + rotational) must be integrated:

$$\begin{aligned} m\dot{V} &= \sum_i T_i \sin\phi_i - D - mg\sin\theta \\ m\dot{w} &= \sum_i T_i \cos\phi_i + L_{\text{wing}} - mg\cos\theta \end{aligned}$$

with attitude dynamics from rotational equations.

Cruise: tilt rotors are near horizontal (large ϕ_i), wings supply lift:

$$\tfrac{1}{2}\rho V^2 S C_L \approx mg - \sum_i T_i \cos\phi_i$$

and thrust requirement forward:

$$\sum_i T_i \sin\phi_i \approx D$$

9. Fuel Cell Energy and Hydrogen Consumption

Define:

η_{fc}	fuel cell electrical efficiency (fraction)
LHV_{H_2}	hydrogen lower heating value (kWh/kg)
$P_{\text{elec,total}}(t)$	total electrical power demand (W) as a function of time

Total electrical energy required for a mission of duration T_{mission}:

$$E_{\text{req}} = \int_0^{T_{\text{mission}}} P_{\text{elec,total}}(t)\, dt \quad \text{(Wh)}$$

(If using hours as time units, ensure consistency; convert W to kW or Wh appropriately.)

Usable energy per kg of hydrogen:

$$E_{\mathrm{H_2,usable}}\,[\mathrm{kWh/kg}] \;=\; \mathrm{LHV}_{\mathrm{H_2}} \cdot \eta_{\mathrm{fc}}$$

Hydrogen mass required (kg):

$$m_{\mathrm{H_2}} \;=\; \frac{E_{\mathrm{req}}}{E_{\mathrm{H_2,usable}}}$$

Include a safety factor s (e.g., $s = 1.5$–2.0):

$$m_{\mathrm{H_2,onboard}} = s \cdot m_{\mathrm{H_2}}$$

Update aircraft mass:

$$m = m_{\mathrm{dry}} + m_{\mathrm{fuelcell}} + m_{\mathrm{H_2,onboard}} + m_{\mathrm{payload}} + m_{\mathrm{batt}}$$

Because m appears in thrust and power calculations, this is a coupled problem $\rightarrow$ solve iteratively.

10. Iterative Solution Procedure

1. Initialize $m^{(0)} = m_{\mathrm{dry}} +$ initial guess $m^{(0)}_{\mathrm{H_2,onboard}}$.
2. For iteration k:
   (a) Compute rotor thrust requirements $T_i^{(k)}$ to satisfy hover/mission constraints using current $m^{(k)}$.
   (b) Compute induced velocities $v^{(k)}_{i,\mathrm{ind}}$ and rotor electrical powers $P^{(k)}_{i,\mathrm{elec}}$.
   (c) Compute total electrical power $P^{(k)}_{\mathrm{elec,total}}(t)$ and integrate to get $E^{(k)}_{\mathrm{req}}$.
   (d) Compute required hydrogen mass:

   $$m^{(k+1)}_{\mathrm{H_2}} = \frac{E^{(k)}_{\mathrm{req}}}{E_{\mathrm{H_2,usable}}}$$

   (e) Apply safety factor s:

   $$m^{(k+1)}_{\mathrm{H_2,onboard}} = s \cdot m^{(k+1)}_{\mathrm{H_2}}.$$

(f) Update mass:

$$m^{(k+1)} = m_{\text{dry}} + m_{\text{fuelcell}} + m_{\text{H}_2,\text{onboard}}^{(k+1)} + m_{\text{payload}} + m_{\text{batt}}.$$

3. Check convergence: if $|m^{(k+1)} - m^{(k)}| < \epsilon$ stop; else $k \leftarrow k + 1$.

11. Control Allocation and Roll Assist

Roll control (about the longitudinal body axis) is provided by differential vertical thrust of wing rotors and differential thrust of tilt rotors. The roll moment available is:

$$M_x = \sum_{i=1}^{5} y_i \, T_i^{(V)}$$

To achieve commanded roll moment $M_{x,\text{cmd}}$, solve for rotor thrust set-points T_i subject to:

$$\begin{aligned}
&\sum_i T_i^{(V)} = mg - L_{\text{wing}} \quad \text{(vertical equilibrium)} \\
&\sum_i y_i T_i^{(V)} = M_{x,\text{cmd}} \\
&0 \le T_i \le T_{i,\max}
\end{aligned}$$

This is a small linear program / constrained least-squares problem used in control allocation.