# 1 Problem Target

Description of the problem we are trying to solve (experiment)

# 2 Governing Equations

We solve the compressible reactive flow equations

$$\frac{\partial \boldsymbol{Q}}{\partial t} + \frac{\partial \boldsymbol{F}_i^I}{\partial x_i} - \frac{\partial \boldsymbol{F}_i^V}{\partial x_i} - \boldsymbol{S} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{Q} = [\rho,\, \rho u_i,\, \rho E,\, \rho Y_k]$ is the vector of conserved variables, with $\rho$ the density, $\rho u_i$ the momentum in the $i^{\text{th}}$ direction, $\rho E$ the total energy density

$$\rho E = \frac{1}{\rho} u_i u_i + \sum_{i=1}^{N} \rho Y_k e_k, \tag{2}$$

where $e_k$ and $\rho Y_k$ are the internal energy and density of the $k^{\text{th}}$ species for $k = 1, \ldots, N-1$. The $N^{\text{th}}$ species is $N_2$ and is treated as abundant, so its density is computed from

$$\rho Y_N = \rho - \sum_{k=1}^{N-1} \rho Y_k. \tag{3}$$

The inviscid $\boldsymbol{F}_i^I$ and viscous $\boldsymbol{F}_i^V$ fluxes, and source terms $\boldsymbol{S}$ are

$$\boldsymbol{F}_i^I = \begin{bmatrix} \rho u_i \\ \rho u_1 u_i + p\delta_{i1} \\ \rho u_2 u_i + p\delta_{i2} \\ \rho u_3 u_i + p\delta_{i3} \\ u_i(\rho E + p) \\ \rho Y_k u_i \end{bmatrix}, \quad \boldsymbol{F}_i^V = \begin{bmatrix} 0 \\ \tau_{1i} \\ \tau_{2i} \\ \tau_{3i} \\ u_j \tau_{ij} - q_i \\ \varphi_{ki} \end{bmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ W_k \dot{\omega}_k \end{bmatrix}, \tag{4}$$

where $p$ is the pressure, $\delta_{ij}$ is the Kronecker delta, $\tau_{ij}$ the viscous stress tensor, $q_i$ the heat flux, $\varphi_{k,i}$ the diffusion flux of species $k$, $W_k$ its molecular weight and $\dot{\omega}_k$ its net chemical production rate.

Models for the viscous stress tensor $\tau_{ij}$, heat flux $q_i$, diffusion fluxes $\varphi_{ki}$, and net production rates $\dot{\omega}_i$ must be specified to close (1). The viscous stress tensor is that of a Newtonian fluid,

$$\tau_{ij} = 2\mu \left[ S_{ij} - \frac{1}{3} \frac{\partial u_m}{\partial x_m} \right], \tag{5}$$

where $\mu$ is the dynamic viscosity and

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{6}$$

is the strain-rate tensor. The heat flux is

$$q_i = -\frac{\partial(\lambda T)}{\partial x_i} + h_k \varphi_{ki}, \tag{7}$$

where $\lambda$ is the thermal conductivity and $h_k$ is enthalpy of species $k$. The diffusion fluxes are

$$\varphi_{ki} = \varphi_{k,i}^* + \varphi_{ki}^c, \tag{8}$$

$$\varphi_{ki}^* = -\rho D_{k,m} \frac{W_k}{W} \frac{\partial X_k}{\partial x_i}, \tag{9}$$

$$\varphi_{ki}^c = -Y_k \sum_{n=1}^{N} \varphi_{ni}^*, \tag{10}$$

where $\varphi^*_{k,i}$ is its mixture-average approximation, $\varphi^c_{ki}$ a correction flux to ensure mass conservation, $D_{k,m}$ the mixture-averaged diffusivity of species $k$, $X_k = W_{(k)}Y_k/W$ its mole fraction (where the parenthesis precludes summation over repeated indices), and $W$ the mean molecular weight. The net production rates are given by

$$\dot{\omega}_k = \nu_{kr} R_r \tag{11}$$

where $\nu_{kr}$ is the net stoichiometric coefficient of species $k$ in reaction $r$, and $R_r$ is the rate of progress of reaction $r$, the specific form of which is readily available [1, 2].

## 2.1 Boundary Conditions

Navier–Stokes characteristic boundary conditions [3] are enforced at the combustor inlet and (supersonic inflow and non-reflective outflow), injector inlet (supersonic inflow), and the walls (no-slip). Walls are taken to be impermeable [4, 5],

$$n_i \varphi_{ki} = 0, \tag{12}$$

for $i = 1, \ldots, N$, where $n_i$ is the wall-normal vector.

To absorb outgoing disturbances and minimize reflections from computational boundaries, absorbing buffer zones are used near the boundaries [6, 7]. In these zones, a source term

$$S_b = -\Upsilon(\boldsymbol{x}) \left( \frac{\boldsymbol{Q} - \boldsymbol{Q}_{\text{ref}}}{\tau_b} \right) \tag{13}$$

is added to (1) to penalize deviations from a prescribed reference state at the boundary $\boldsymbol{Q}_{\text{ref}}$, where $\Upsilon$ is the buffer's support, $\tau_b$ is the time scale over which the penalty acts. Here, we use

$$\tau_b = \frac{|\boldsymbol{x}_b - \boldsymbol{x}_0|}{1.25\, c_{\text{ref}}}, \tag{14}$$

$$\Upsilon(\boldsymbol{x}) = \left( \frac{|\boldsymbol{x} - \boldsymbol{x}_0|}{|\boldsymbol{x}_b - \boldsymbol{x}_0|} \right)^2, \tag{15}$$

where $\boldsymbol{x}_b$ and $\boldsymbol{x}_0$ denotes the boundary and the end of the buffer zone, and $c_{\text{ref}}$ is the reference speed of sound at the boundary.

## 2.2 Equation of State

# 3 Numerical Model

DG stuff goes here

# 4 Numerical Setup

mesh stuff initialization stuff

# 5 Example Results

## References

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