Mesh Simplification and Edge Collapses

Assume you have a closed triangulated mesh surface mesh with 3339 triangles. After you perform 834 edge collapses how many triangles remain in the mesh?

Answer = integer

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$3339 - 2(834) = ?$

Each edge collapse
removes 2 triangles

Points

A cubic Bézier curve is a parametric polynomial curve given by:

$$X(t) = (1-t)^3 b_0 + 3(1-t)^2 t b_1 + 3(1-t)t^2 b_2 + t^3 b_3$$

where b_i are the control points. Suppose the control points are $b_0 = (-1, 0)$, $b_1 = (0, 1)$, $b_2 = (0, -1)$, and $b_3 = (1, 0)$. Use de Casteljau's algorithm to find the coordinates of $X(0.25)$, and check it with the polynomial equation above.

$X(0.25) =$ [ ] ?

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$b_0^1 = (1-t) b_0 + t b_1$

Arithmetic not guaranteed!

$b_1^1 = (1-t) b_1 + t b_2$

$b_2^1 = (1-t) b_2 + t b_3$

$b_0^1 = 3/4 \, (-1, 0) + 1/4 \, (0, 1) = (-3/4, 1/4)$

$b_1^1 = 3/4 \, (0, 1) + 1/4 \, (0, -1) = (0, 1/2)$

$b_2^1 = 3/4 \, (0, 1) + 1/4 \, (1, 0) = (1/4, 3/4)$

$b_0^2 = (1-t) b_0^1 + t b_1^1 = 3/4 \, (-3/4, 1/4) + 1/4 \, (0, 1/2)$

$= (-9/16, 11/16)$

$b_1^2 = (1-t) b_1^1 + t b_2^1 = 3/4 \, (0, 1/2) + 1/4 \, (1/4, 3/4)$

$= (0, 3/8) + (1/16, 3/16) = (1/16, 9/16)$

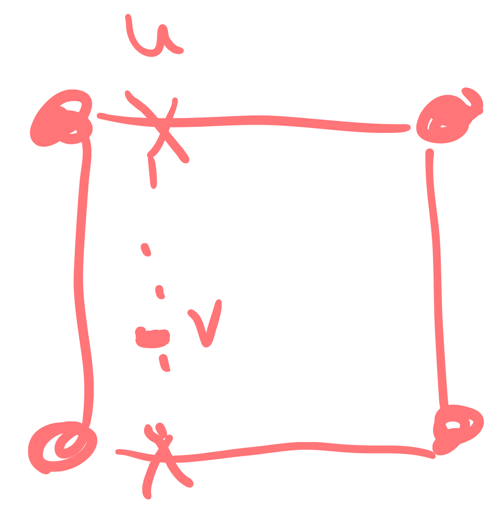
$b_0^3 = (1-t) b_0^2 + t b_1^2 = 3/4 (-9/16, 11/16) + 1/4 (1/16, 9/16) = \left(\frac{-26}{64}, 42/64\right)$

Bilinear Patch

Suppose our control points are

$b_{0,0} = (0,0,0), b_{1,0} = (1,0,0), b_{0,1} = (0,1,0), b_{1,1} = (1,1,1)$

What point on the patch is found by evaluating X(0.3, 0.6)



[ ] ?

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$$\begin{pmatrix}1-u & u\end{pmatrix}\begin{pmatrix}b_{00} & b_{01}\\ b_{10} & b_{11}\end{pmatrix}\begin{pmatrix}1-v\\ v\end{pmatrix} =$$

$$\begin{pmatrix}0.7 & 0.3\end{pmatrix}\begin{pmatrix}(0,0,0) & (0,1,0)\\ (1,0,0) & (1,1,1)\end{pmatrix}\begin{pmatrix}0.4\\ 0.6\end{pmatrix} =$$

$$\begin{pmatrix}(0,0,0)+(0.3,0,0) & (0,0.7,0)+(0.3,0.3,0.3)\end{pmatrix}\begin{pmatrix}0.4\\ 0.6\end{pmatrix} =$$

$$0.4 \times (0.3,0,0) + 0.6 \times (0.3, 1.0, 0.3) =$$

$$(0.12, 0, 0) + (0.18, 0.6, 0.18) =$$

$$(0.3, 0.6, 0.18)$$

Indexed Face Format for Meshes

Imagine we generate a rectangular mesh with a boundary using an $n+1$ by $n+1$ grid of vertices.
In the example above, it is using $n = 5$. We have $(5+1)*(5+1)$ grid of vertices and $5*5*2$ triangles.
Triangles are generated as shown, cutting each grid square into 2 triangles.

The mesh is stored using an **indexed face format** (like an OBJ file, for example).
Assuming:

1. each vertex is specified by **3 floating point coordinates** (the image doesn't show z-axis)
2. floating point numbers use **4 bytes** of space
3. integer indices use **2 bytes** of space.

Suppose you need to store a mesh with $n = 6$. How much space is used by the mesh?

bytes = integer

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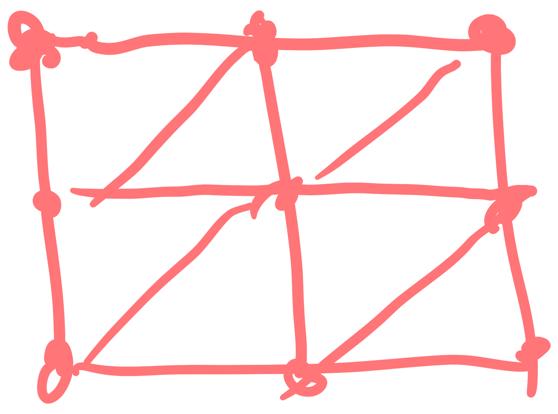

$7 \times 7 \times 3 \times 4 +$

$6 \times 6 \times 2 \times 3 \times 2 = ?$

vertex buffer of $(n+1)^2$
vertices w/ 3 coords
4 bytes per number

triangle buffer of $2n^2$
triangles with 3 indices
2 bytes per number

$103 \times 6 = ?$

For cubics, algorithm will require 6 lerps per point

Evaluating Bezier Curves

Suppose we have a 2D quadratic Bezier curve with control points

$b_0 = (-4, -3)$
$b_1 = (-1, -2)$
$b_2 = (3, 2)$

Recall that the Berstein polynomials for a quadratic Bezier curve are:

$$B_0^2(t) = (1-t)^2$$

$$B_1^2(t) = (1-t)2t$$

$$B_2^2(t) = t^2$$

What is the y coordinate of the point on the curve at $t = 0.2$
Your answer is expected to be correct to 2 significant digits.

$y =$ number (2 significant figures) ?

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$p(t) = (1-t)^2 b_0 + (1-t)2t b_1 + t^2 b_2$

$= (0.8)^2 (-4,-3) + (0.8)2(0.2)(-1,-2) + (0.2)^2 (3,2)$

Only need y coord

$= 0.64(-3) + 0.256(-2) + 0.4(2)$

$= \ldots\ldots$