



Animation

Basic Concepts and Kinematics

CS 415: Game Development

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Animation...



Imaged by Heritage Auctions, HA.com

Animation is derived from the Latin *anima*:

“the act, process, or result of imparting life, interest, spirit, motion, or activity”

“Motion is a defining property of life and much of the true art of animation is about how to tell a story, show emotion, or even express subtle details of human character through motion.”

- Michael Ashikhmin

Approaches to Computer Animation

Keyframing

gives the most direct control to the animator who provides necessary data at some moments in time and the computer fills in the rest.

Procedural

involves specially designed, often empirical, mathematical functions and procedures whose output resembles some particular motion.

Physics-based

solve differential equation of motion.

Motion capture

uses special equipment or techniques to record real-world motion and then transfers this motion into that of computer models.

Character Animation

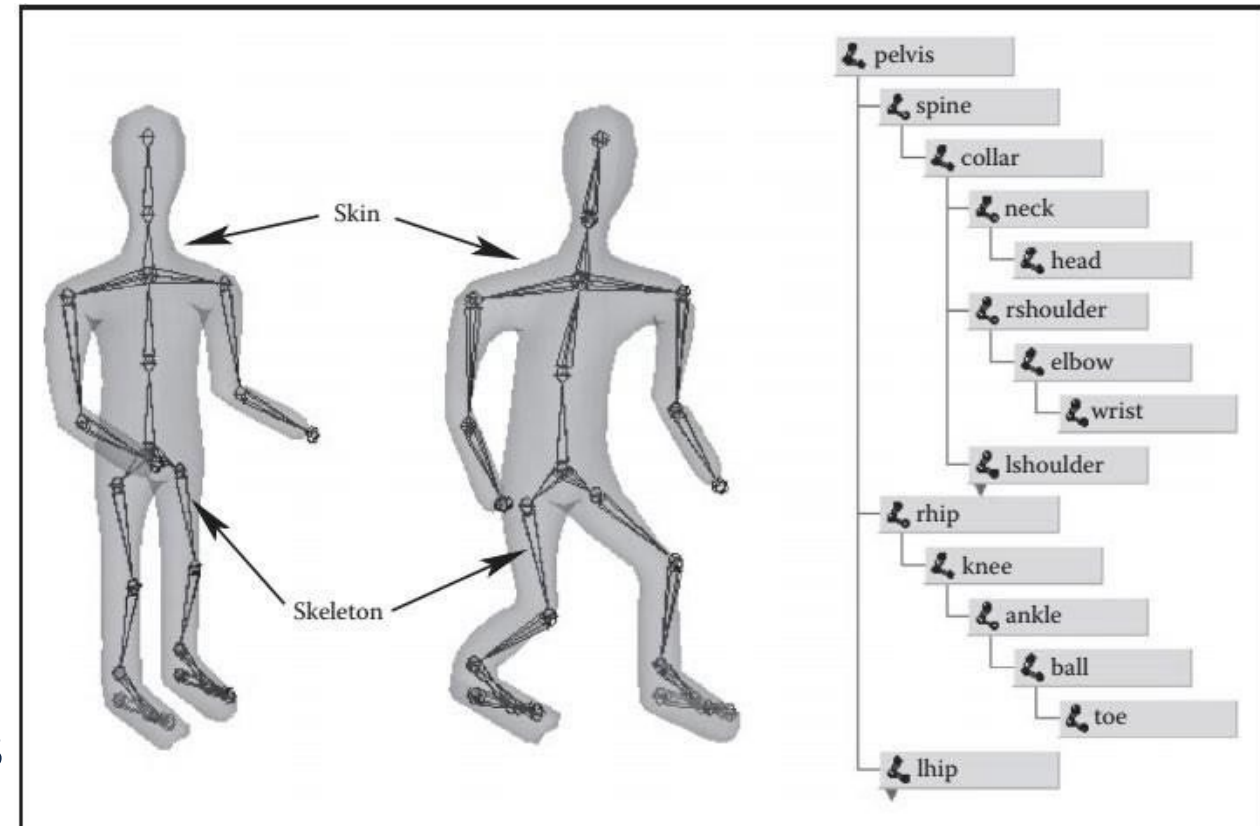
Animation of articulated figures uses keyframing and specialized deformation techniques.

The character model has two layers

A highly detailed surface the viewer will see

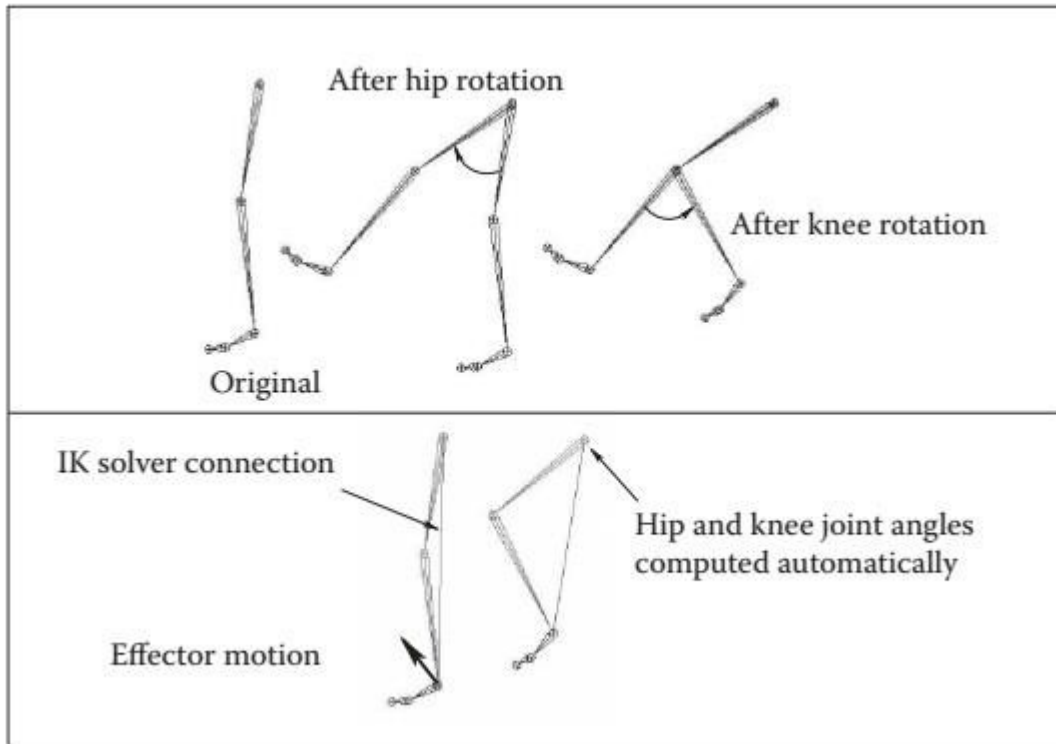
A skeleton underneath

- This is a hierarchical structure (a tree) of joints
- Provides a kinematic model of the figure
- Used exclusively for animation.



CG animation can involve *forward kinematics* or *inverse kinematics*

What is Kinematics?



kin·e·mat·ics

/ˌkɪnəˈmædɪks/

noun

the branch of mechanics concerned with the motion of objects without reference to the forces which cause the motion.

- the features or properties of motion in an object.

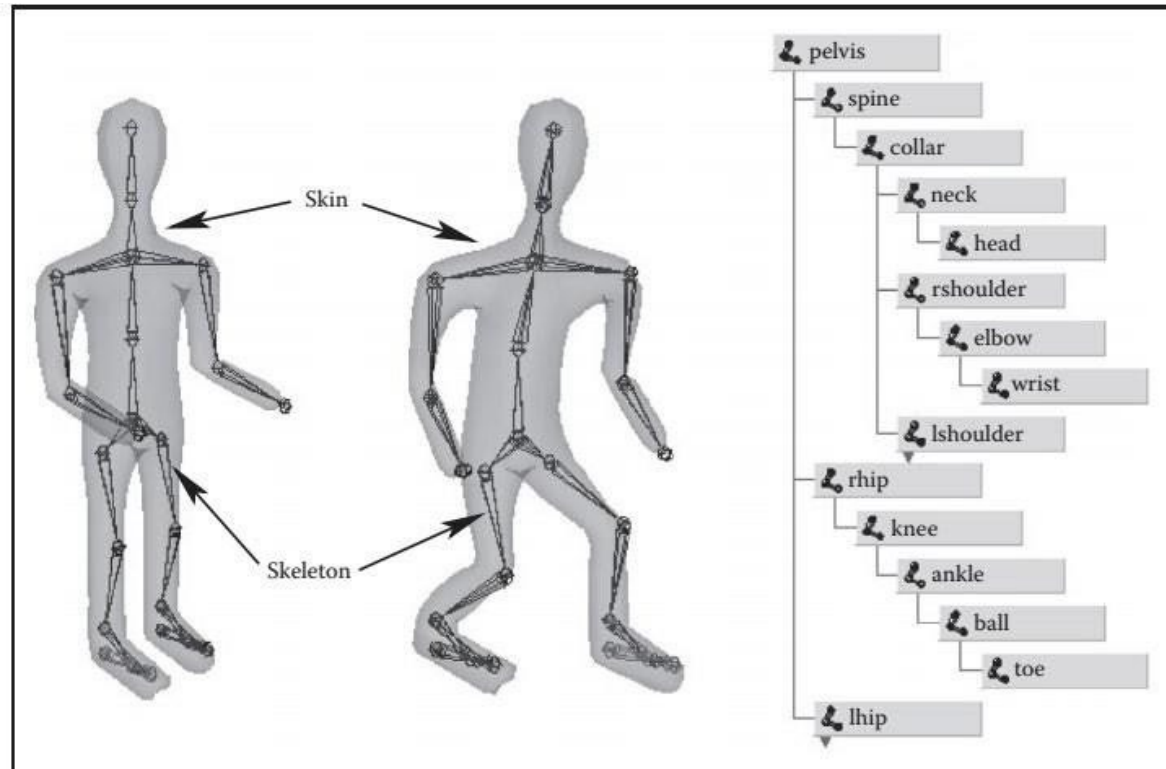
plural noun: **kinematics**

Forward Kinematics

Each of the skeleton's joints acts as a parent for the hierarchy below it.

The root represents the whole character, is positioned directly in the world coordinate system.

One can obtain a transformation which relates local space of any joint to the world system by simply concatenating transformations along the path from the root to the joint.



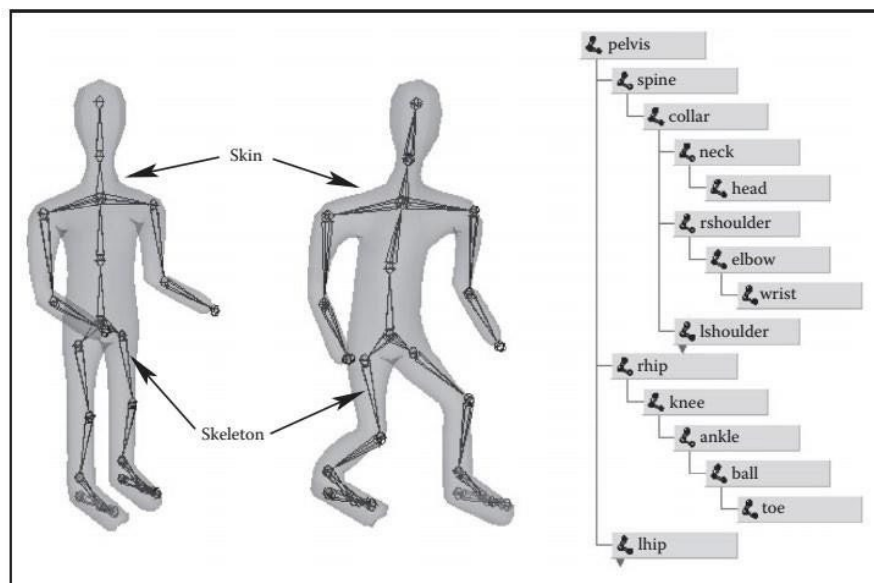
Forward Kinematics

To evaluate the whole skeleton (i.e., find position and orientation of all joints), a depth-first traversal of the complete tree of joints is performed.

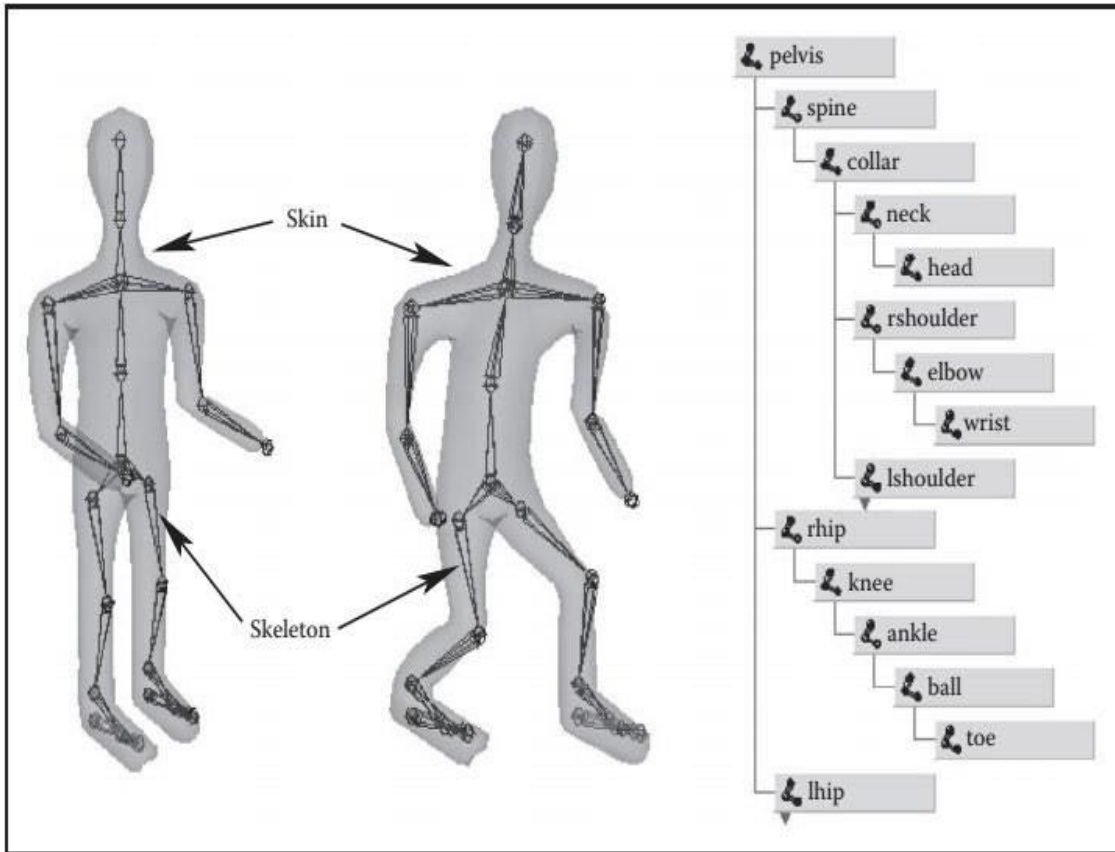
A transformation stack is a natural data structure to help with this task.

While traversing down the tree, the current composite matrix is pushed on the stack and a new one is created by multiplying the current matrix with the one stored at the joint.

When backtracking to the parent, this extra transformation should be undone before another branch is visited; this is easily done by simply popping the stack

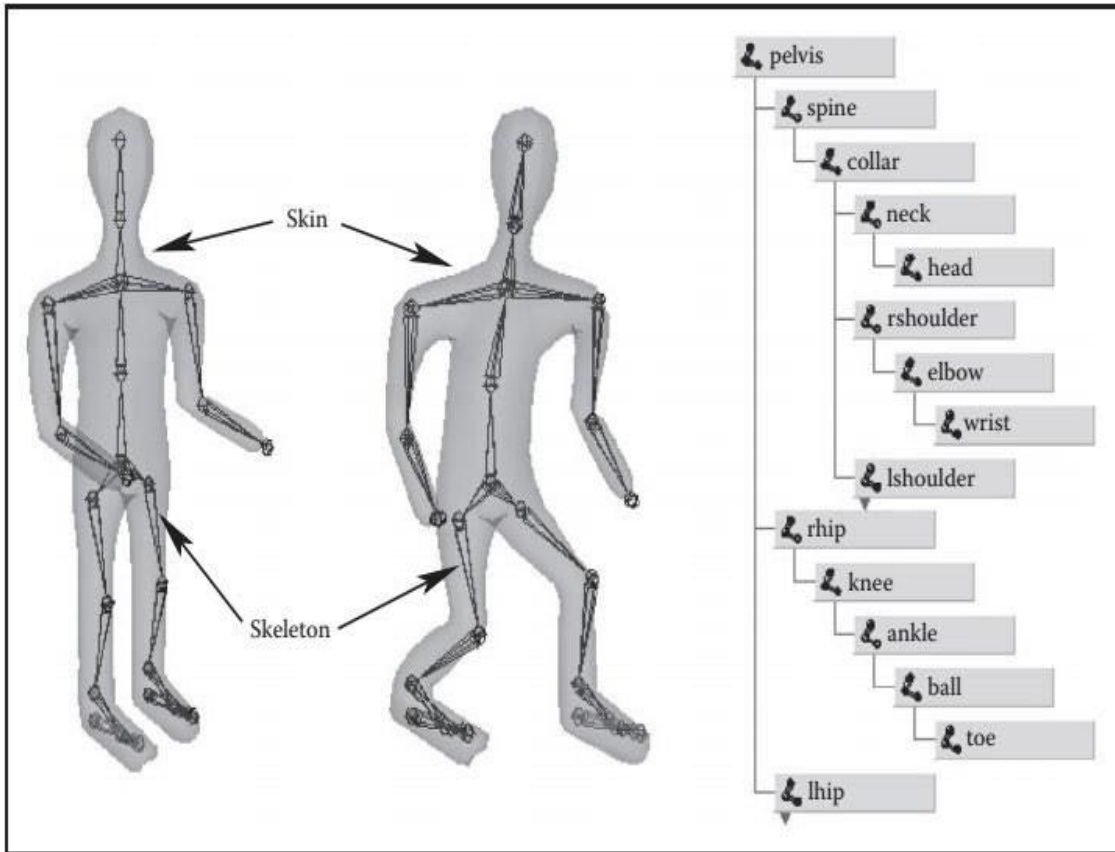


Forward Kinematics Example



Generating the position of the head

Forward Kinematics Example



Generating the position of the wrist

Problems With Forward Kinematics

To animate with forward kinematics, rotational parameters of all joints are manipulated directly.

- In most situations, the animator just wants them to move naturally “on their own”
- Easier to specify the behavior of the endpoint of a joint chain
- Typically corresponds to an ankle or a tip of a finger

Animator would rather have parameters internal joints be determined from the motion of the end effector automatically by the system.

Inverse kinematics (IK) allows us to do just that

Inverse Kinematics

Let \mathbf{x} be the position of the end effector and α be the vector of parameters needed to specify all internal joints along the chain from the root to the final joint.

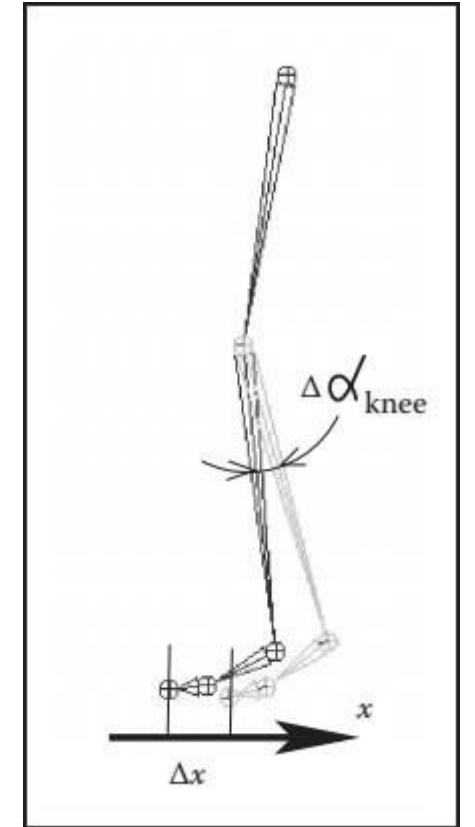
$$\mathbf{x} = (x_1, x_2, x_3)^T$$

Each of the variables in \mathbf{x} is a function of α , it can be written as a vector equation $\mathbf{x} = \mathbf{F}(\alpha)$. If we change the internal joint parameters by a small amount $\delta\alpha$, a resulting change $\delta\mathbf{x}$ in the position of the end effector can be written as:

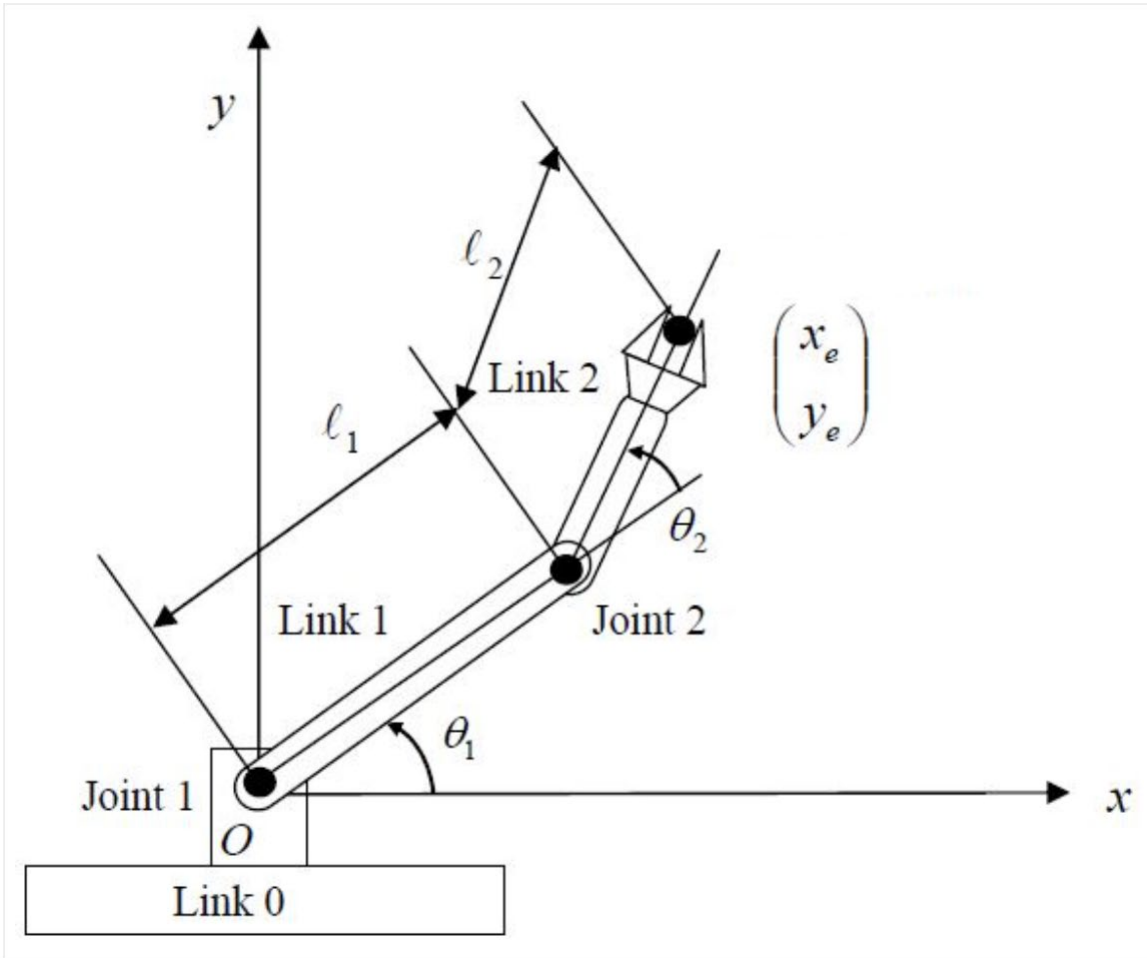
$$\delta\mathbf{x} = \frac{\partial \mathbf{F}}{\partial \alpha} \delta\alpha \quad \frac{\partial \mathbf{F}}{\partial \alpha} = \begin{bmatrix} \frac{\partial f_1}{\partial \alpha_1} & \frac{\partial f_1}{\partial \alpha_2} & \cdots & \frac{\partial f_1}{\partial \alpha_n} \\ \frac{\partial f_2}{\partial \alpha_1} & \frac{\partial f_2}{\partial \alpha_2} & \cdots & \frac{\partial f_2}{\partial \alpha_n} \\ \frac{\partial f_3}{\partial \alpha_1} & \frac{\partial f_3}{\partial \alpha_2} & \cdots & \frac{\partial f_3}{\partial \alpha_n} \end{bmatrix}$$

Inverse Kinematics

- At each moment in time, we know the desired position of the end effector (set by the animator) and the effector's current position
- Subtracting the two, we will get the desired adjustment δx .
- Elements of the Jacobian matrix are related to changes in a coordinate of the end effector when a particular internal parameter is changed while others remain fixed
- These elements can be computed for any given skeleton configuration using geometric relationships
- The only remaining unknowns in the system of equations are the changes in internal parameters α .
- Once we solve for them, we update $\alpha = \alpha + \delta\alpha$ which gives all the necessary information for the FK procedure to reposition the skeleton.



IK Example (2D)



$\mathbf{x} = \mathbf{F}(\boldsymbol{\alpha})$ for this example is:

$$x_e(\theta_1, \theta_2) = \ell_1 \cos(\theta_1) + \ell_2 \cos(\theta_1 + \theta_2)$$

$$y_e(\theta_1, \theta_2) = \ell_1 \sin(\theta_1) + \ell_2 \sin(\theta_1 + \theta_2)$$

$$d\mathbf{x} = \mathbf{J} \cdot d\mathbf{q}$$

Note that:

$$d\mathbf{x} = \begin{pmatrix} dx_e \\ dy_e \end{pmatrix}, d\mathbf{q} = \begin{pmatrix} d\theta_1 \\ d\theta_2 \end{pmatrix}$$

$$\mathbf{J} = \begin{bmatrix} -\ell_1 \sin(\theta_1) - \ell_2 \sin(\theta_1 + \theta_2) & -\ell_2 \sin(\theta_1 + \theta_2) \\ \ell_1 \cos(\theta_1) - \ell_2 \cos(\theta_1 + \theta_2) & -\ell_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

IK – Numerical Derivatives

$$\frac{\partial \mathbf{F}}{\partial \alpha} = \begin{bmatrix} \frac{\partial f_1}{\partial \alpha_1} & \frac{\partial f_1}{\partial \alpha_2} & \cdots & \frac{\partial f_1}{\partial \alpha_n} \\ \frac{\partial f_2}{\partial \alpha_1} & \frac{\partial f_2}{\partial \alpha_2} & \cdots & \frac{\partial f_2}{\partial \alpha_n} \\ \frac{\partial f_3}{\partial \alpha_1} & \frac{\partial f_3}{\partial \alpha_2} & \cdots & \frac{\partial f_3}{\partial \alpha_n} \\ \frac{\partial f_4}{\partial \alpha_1} & \frac{\partial f_4}{\partial \alpha_2} & \cdots & \frac{\partial f_4}{\partial \alpha_n} \end{bmatrix}$$

$$\mathbf{J} = \begin{pmatrix} -l_1 \sin(\theta_1) - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos(\theta_1) - l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{pmatrix}$$

This is using analytical partial derivatives but those can be difficult to compute for more complicated problems...instead we can approximate them numerically.

IK – Numerical Derivatives

$$\frac{\partial \mathbf{F}}{\partial \alpha} = \begin{bmatrix} \frac{\partial f_1}{\partial \alpha_1} & \frac{\partial f_1}{\partial \alpha_2} & \cdots & \frac{\partial f_1}{\partial \alpha_n} \\ \frac{\partial f_2}{\partial \alpha_1} & \frac{\partial f_2}{\partial \alpha_2} & \cdots & \frac{\partial f_2}{\partial \alpha_n} \\ \frac{\partial f_3}{\partial \alpha_1} & \frac{\partial f_3}{\partial \alpha_2} & \cdots & \frac{\partial f_3}{\partial \alpha_n} \\ \frac{\partial f_4}{\partial \alpha_1} & \frac{\partial f_4}{\partial \alpha_2} & \cdots & \frac{\partial f_4}{\partial \alpha_n} \end{bmatrix}$$

$$f'(x) \approx \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

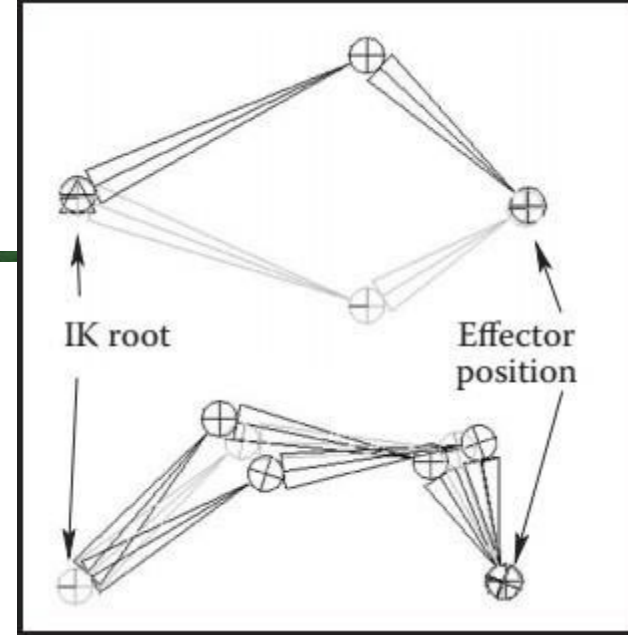
Can approximate a partial derivative using the central difference formula...we just want to know the rate of change of a coordinate with respect to a change in a joint, with that rate expressed as a number

Can use Central Difference approximation of the derivative

Here h will be a change in an internal joint parameter α

$f(x+h)$ will give us a coordinate (e.g. f_1 gives x) of the end effector that results from the change in the joint parameter

Inverse Kinematics



- Unfortunately, the system cannot usually be solved analytically
- Moreover, it is in most cases underconstrained, i.e.,
 - number of unknown internal joint parameters α exceeds number of variables in vector x
- This means that different motions of the skeleton can result in the same motion of the end effector

Inverse Kinematics

- A combination of FK and IK approaches is typically used to animate the skeleton
- Many common motions (walking or running cycles, grasping, reaching, etc.) exhibit well-known patterns of mutual joint motion making it possible to quickly create natural looking motion or even use a library of such “clips.”
- The animator then adjusts this generic result to give it more individuality.

