

# Bezier Blossoms

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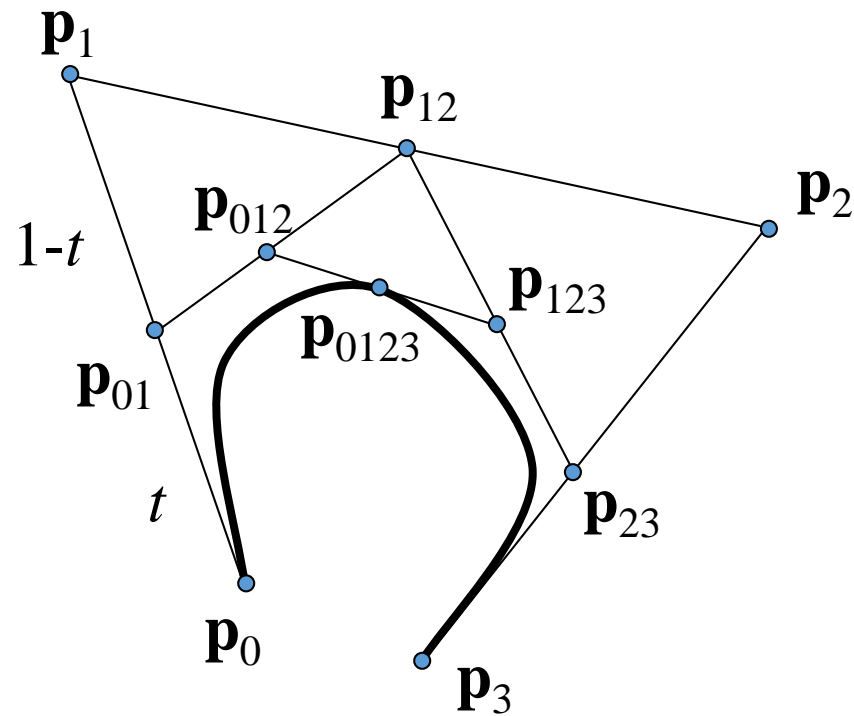
CS 418

Interactive Computer Graphics

John C. Hart

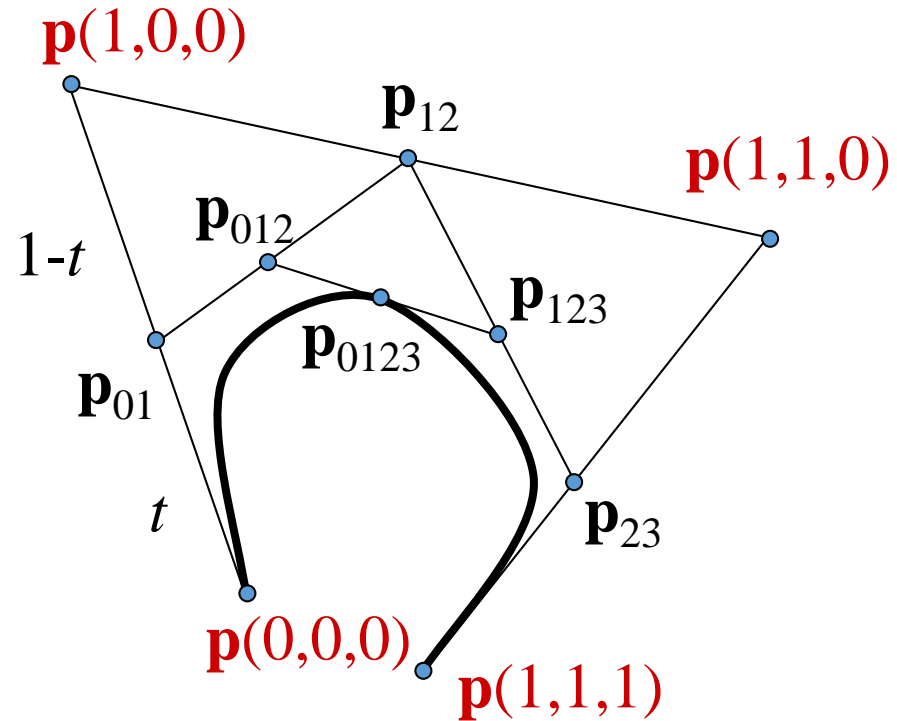
# de Casteljau

- de Casteljau algorithm evaluates a point on a Bezier curve by scaffolding lerps
- Blossoming renames the control and intermediate points, like  $\mathbf{p}_{12}$ , using a polar form, like  $\mathbf{p}(0,t,1)$



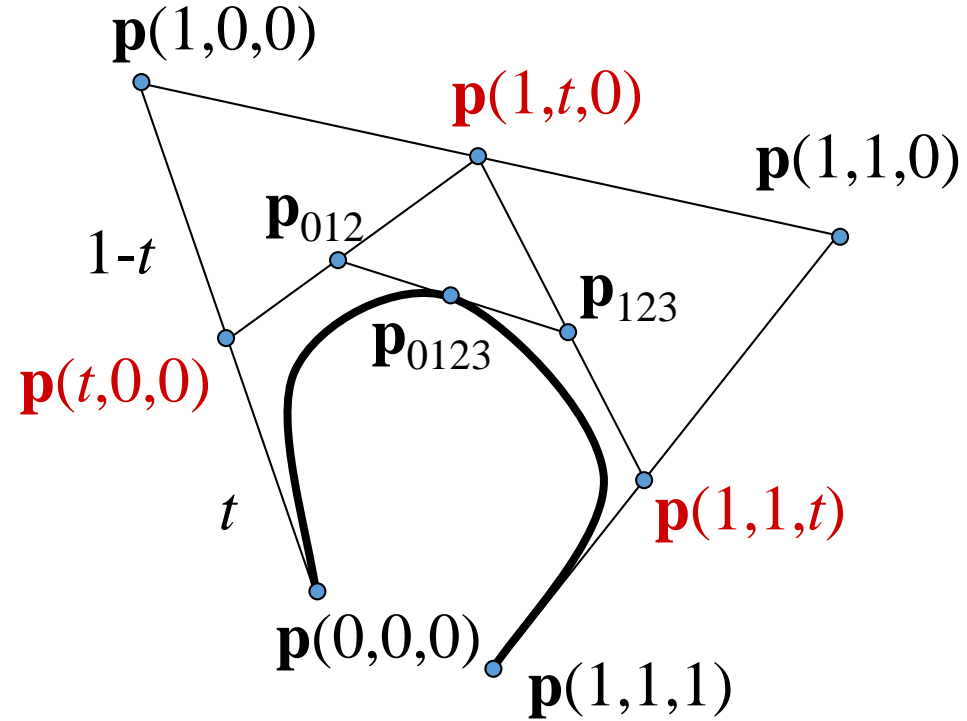
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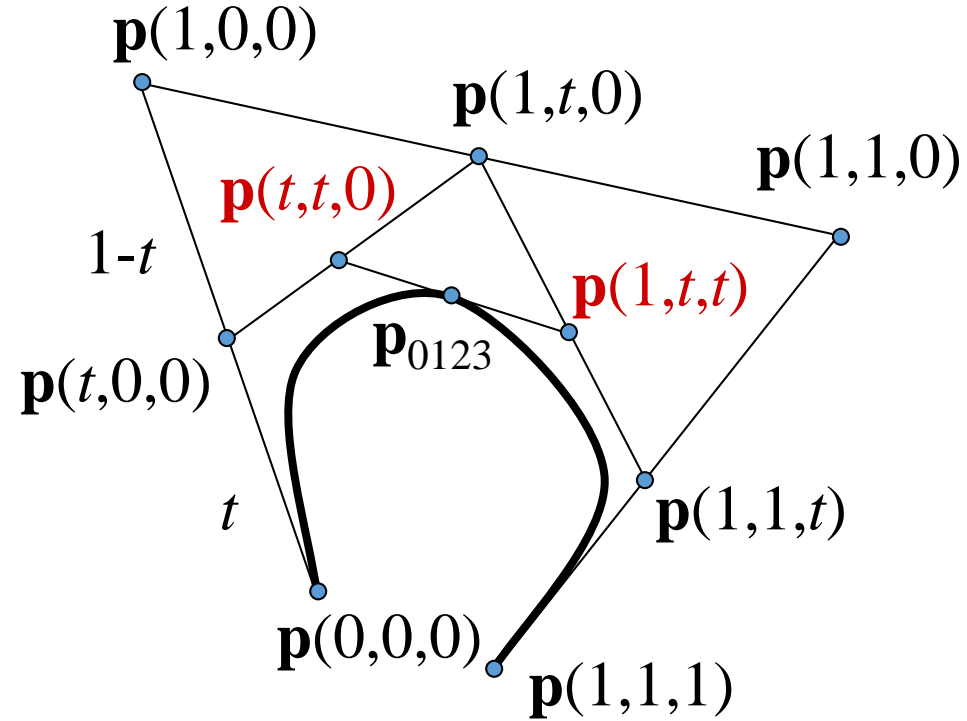
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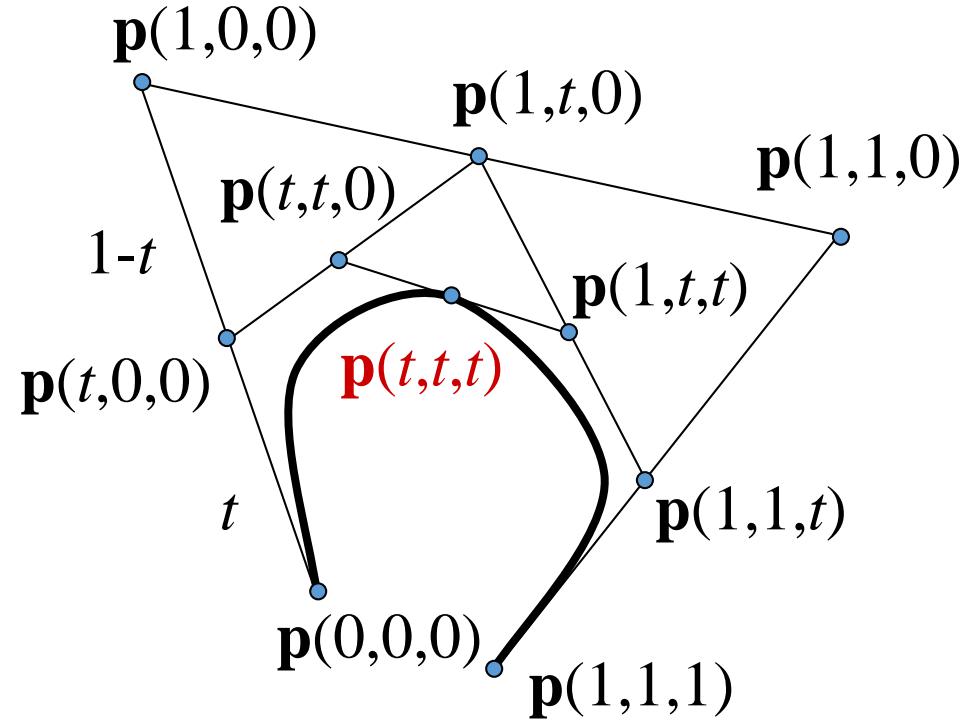
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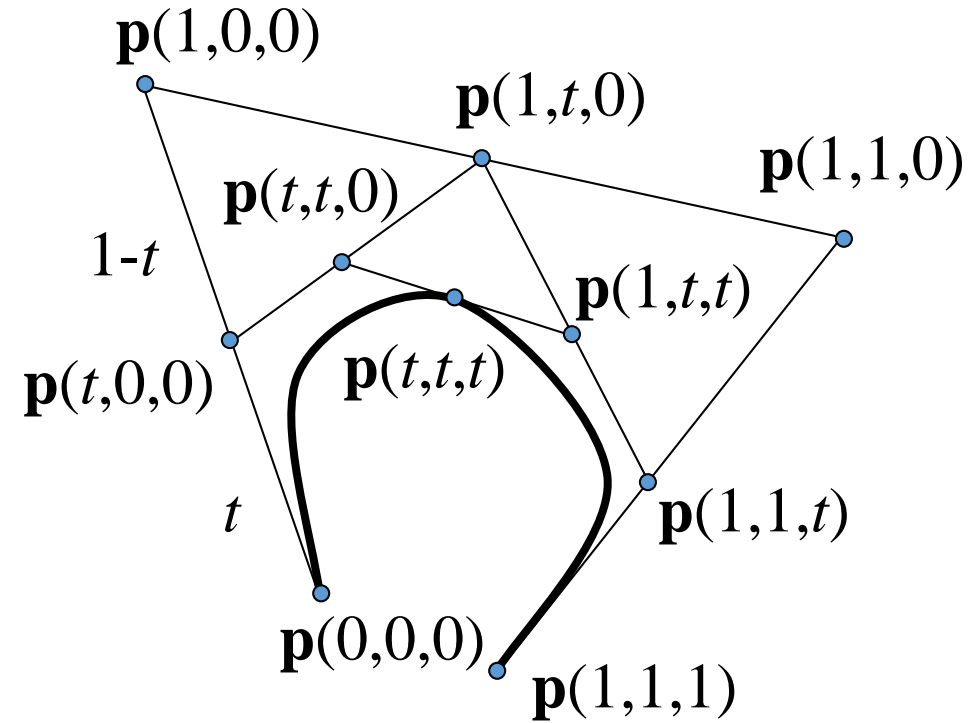
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# Blossoming Rules

1. # of parameters = degree

Cubic:  $\mathbf{p}(\_,\_,\_)$



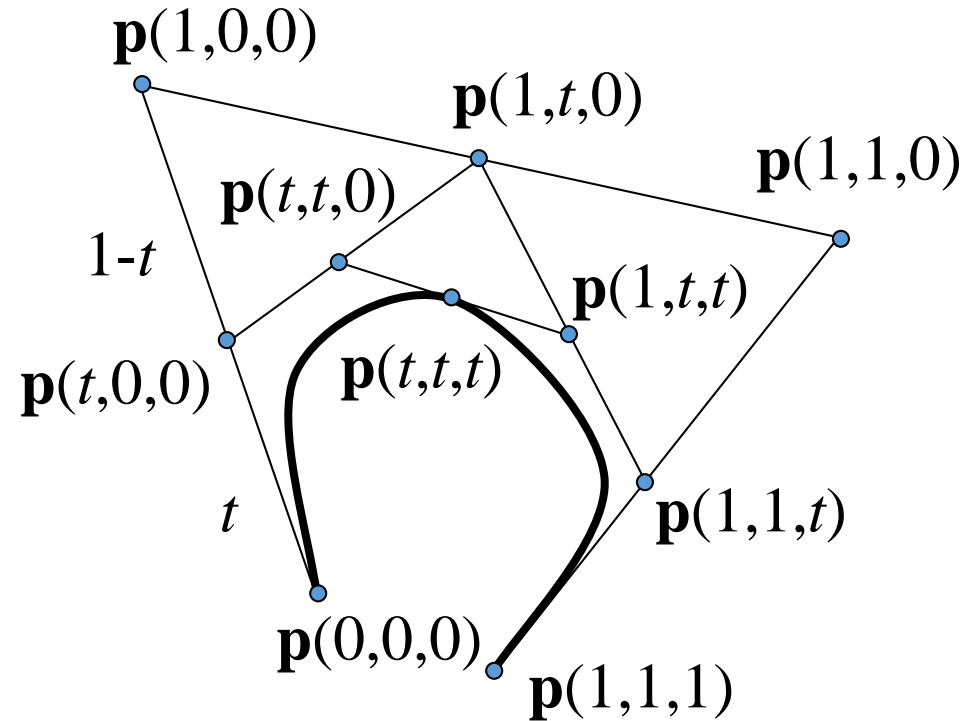
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Cubic:  $\mathbf{p}(\_,\_,\_)$

2. Order doesn't matter

$$\mathbf{p}(a,b,c) = \mathbf{p}(b,a,c)$$





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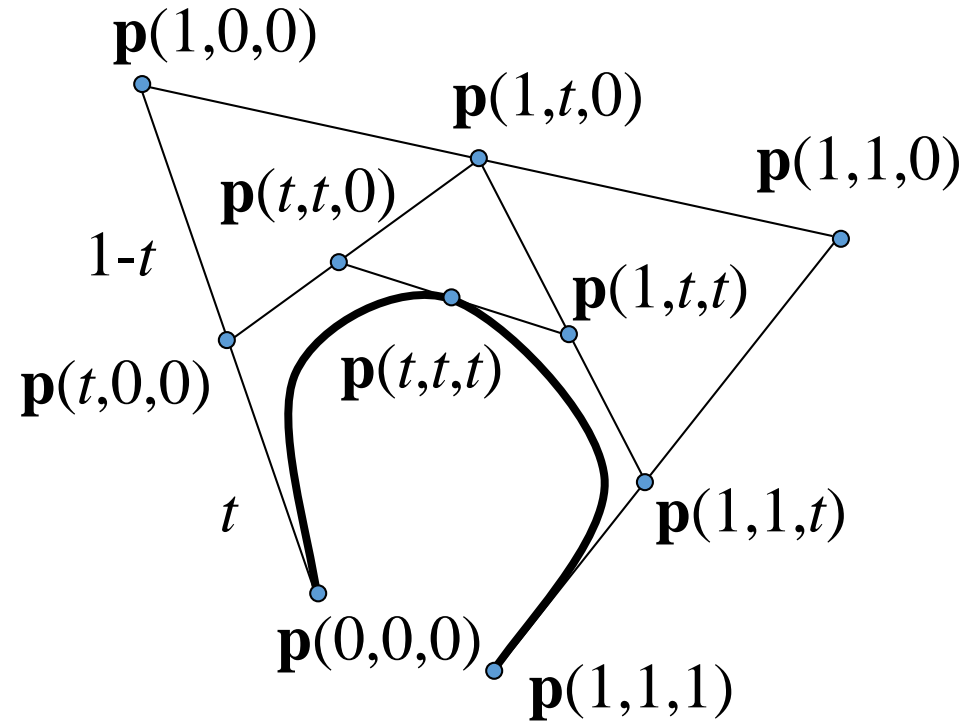
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3. Setting up the board

$$\mathbf{p}(0,0,0), \mathbf{p}(0,0,1), \\ \mathbf{p}(0,1,1), \mathbf{p}(1,1,1)$$



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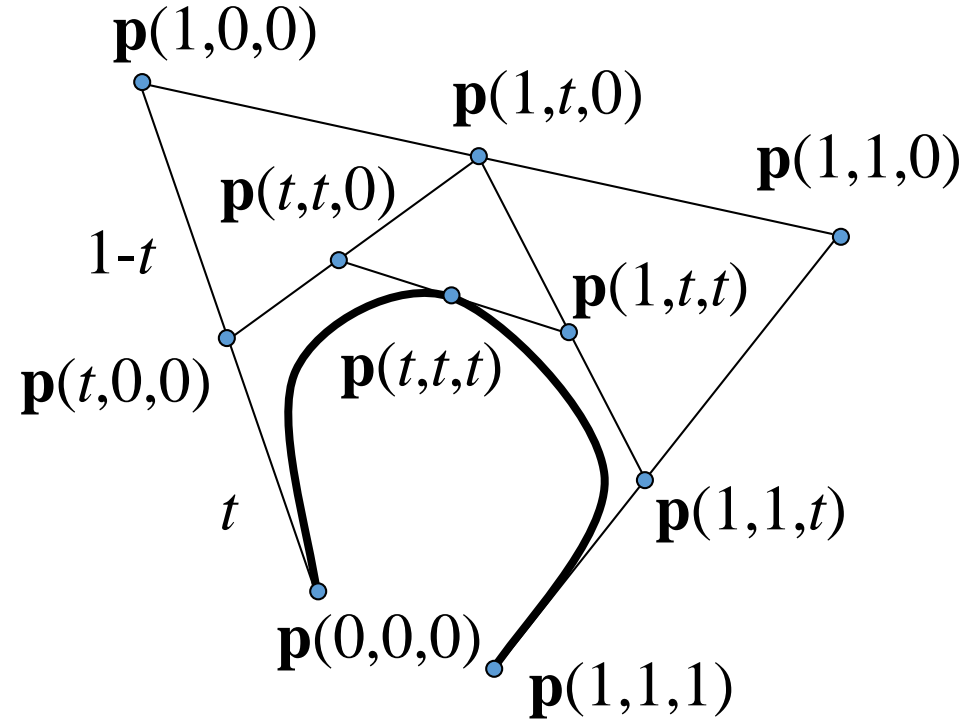
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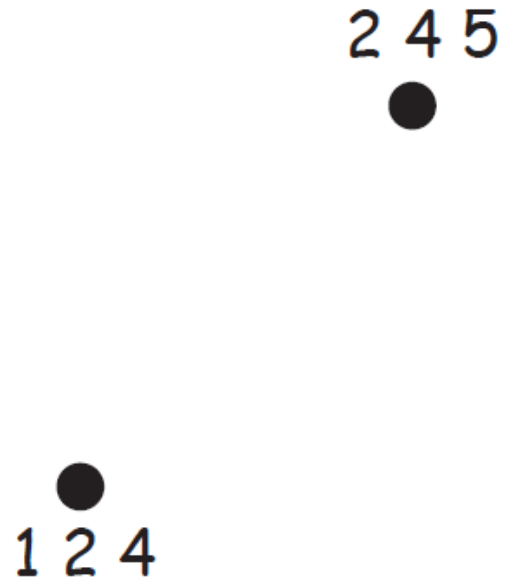
4. Winning the game

$$\mathbf{p}(t,t,t)$$

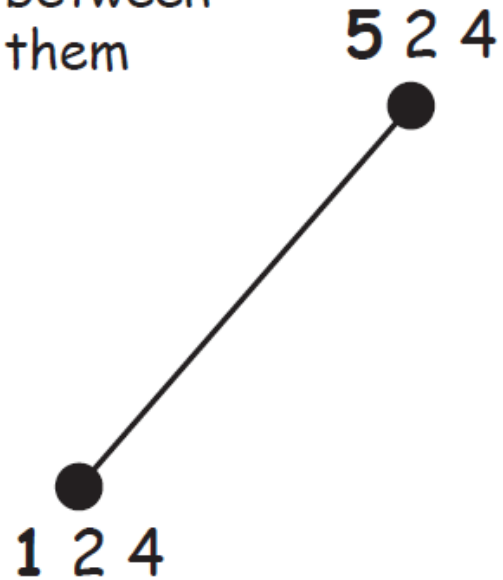


# Placing Blossoms

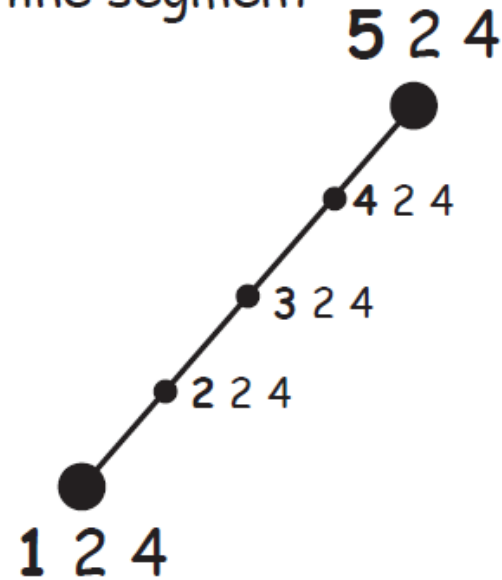
Find two blossoms whose values match except for one



Rewrite the blossoms in a consistent order and draw a line segment between them

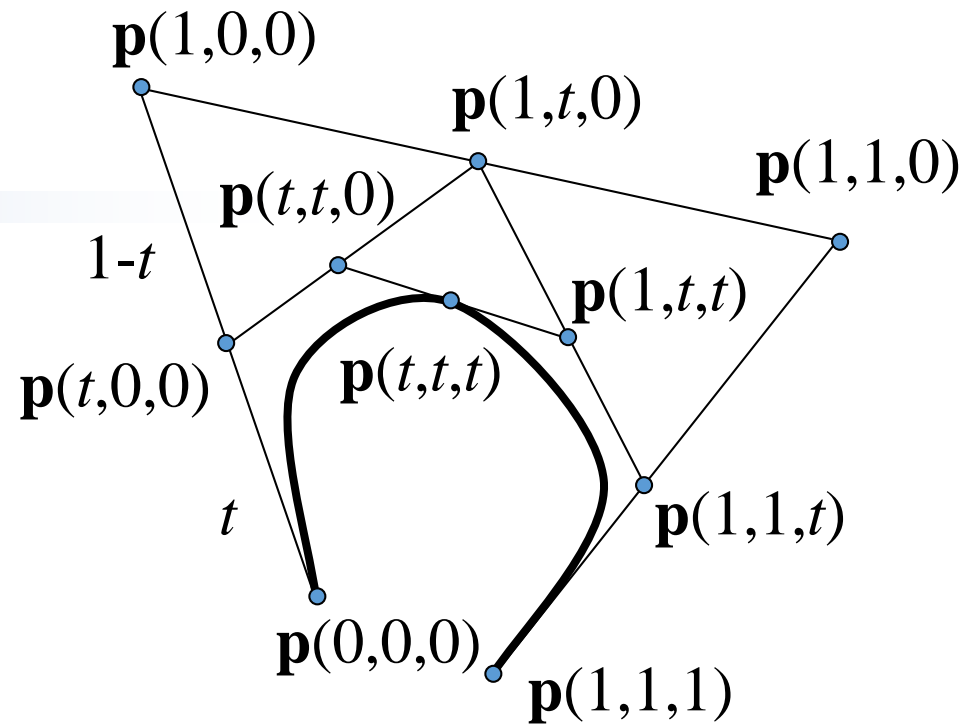


Interpolate the differing blossom value at interpolated points along the line segment



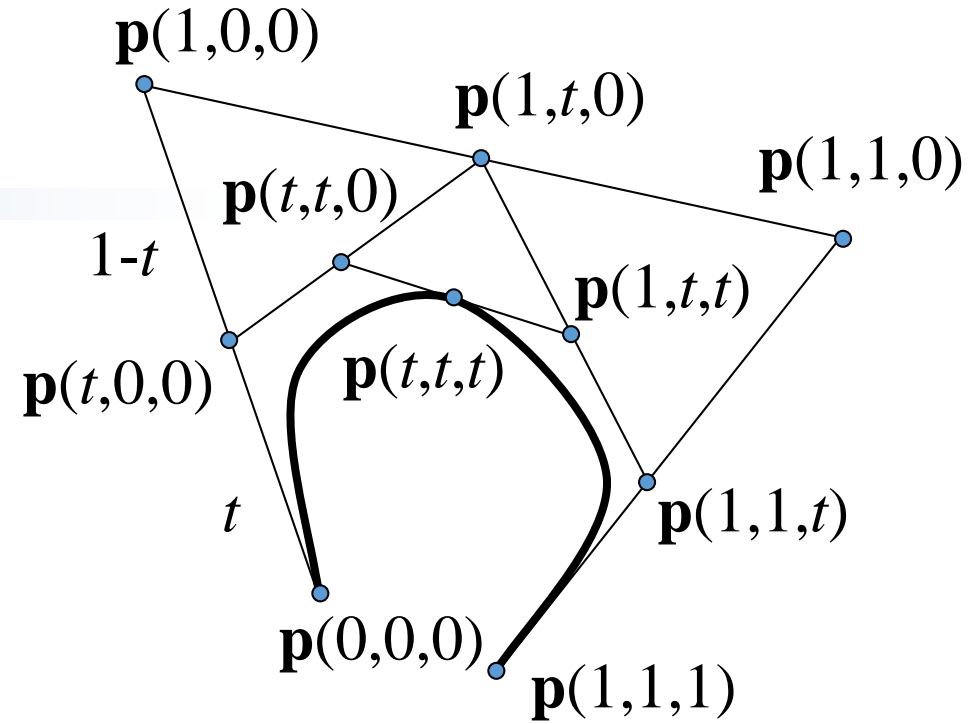
# Evaluation

$$\mathbf{p}(t) = \mathbf{p}(t, t, t)$$



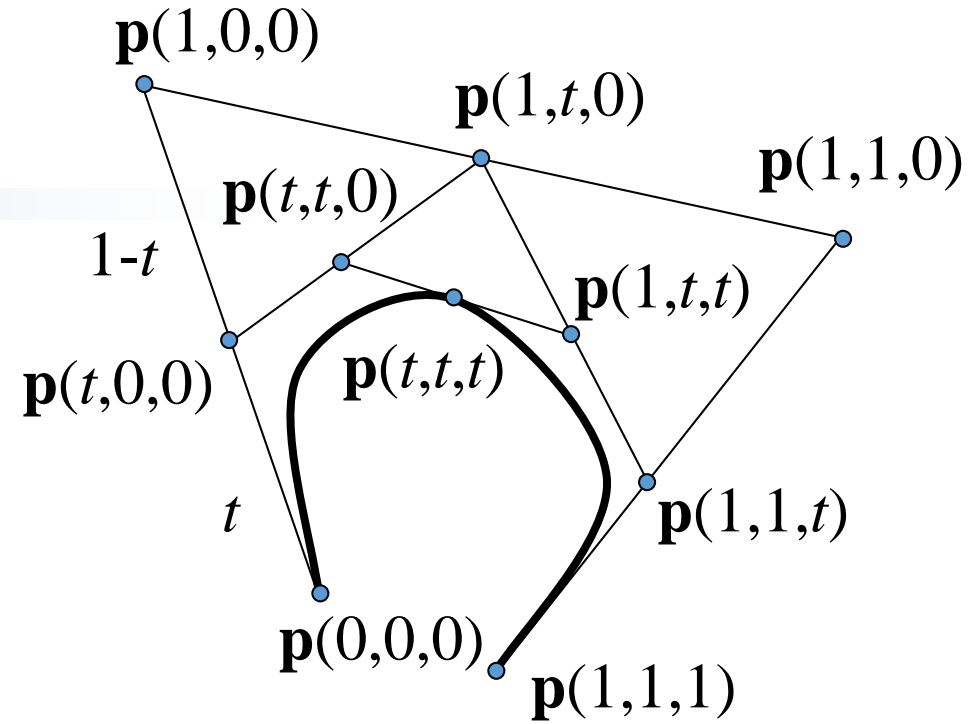
# Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1)\end{aligned}$$



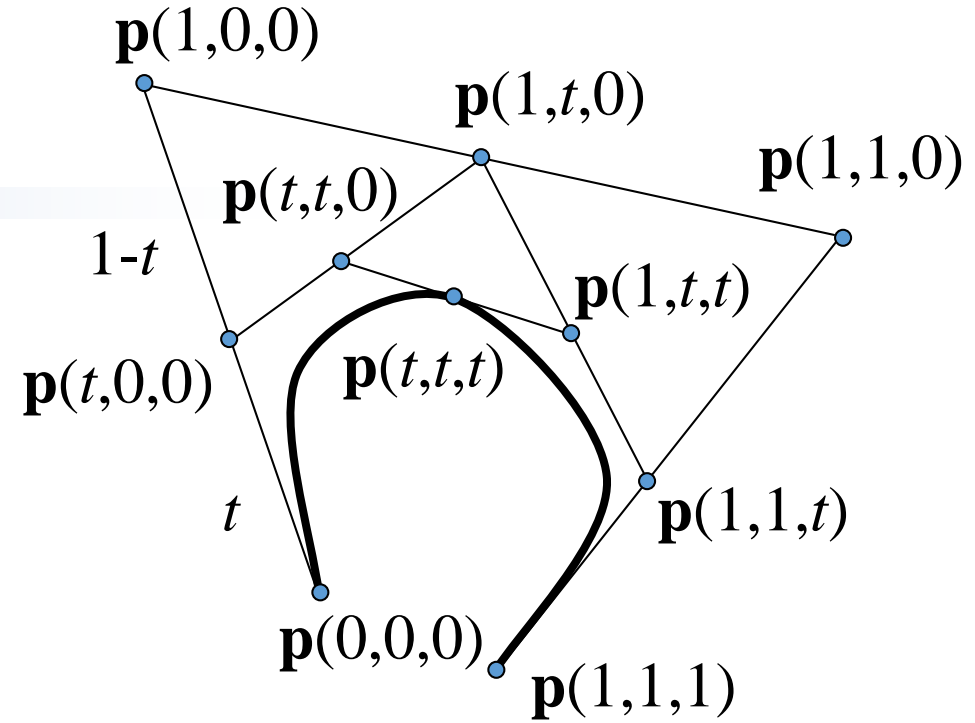
# Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1) \\ &= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ &\quad + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)]\end{aligned}$$



# Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1) \\ &= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ &\quad + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)] \\ &= (1-t)^2 \mathbf{p}(t,0,0) + 2 (1-t) t \mathbf{p}(t,0,1) + t^2 \mathbf{p}(t,1,1)\end{aligned}$$



# Evaluation

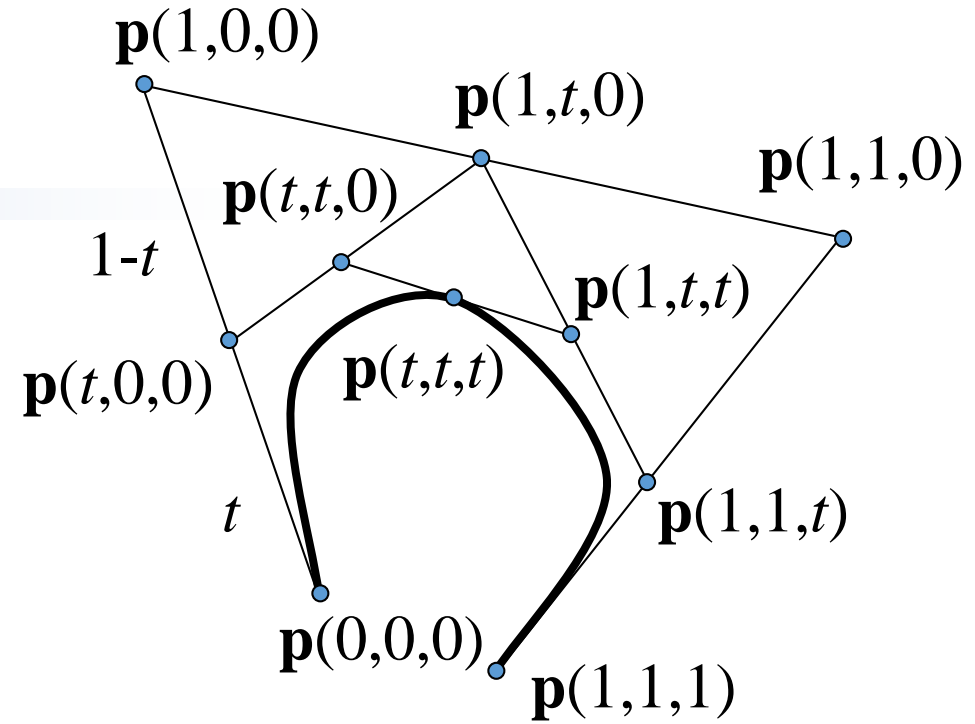
$$\mathbf{p}(t) = \mathbf{p}(t,t,t)$$

$$= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1)$$

$$= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)]$$

$$= (1-t)^2 \mathbf{p}(t,0,0) + 2 (1-t) t \mathbf{p}(t,0,1) + t^2 \mathbf{p}(t,1,1)$$

$$= (1-t)^2[(1-t)\mathbf{p}(0,0,0)+t\mathbf{p}(1,0,0)]+2(1-t)t[(1-t)\mathbf{p}(0,0,1)+t\mathbf{p}(1,0,1)]+t^2[(1-t)\mathbf{p}(0,1,1)+t\mathbf{p}(1,1,1)]$$





# Evaluation

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}(t,t,t) \\ &= (1-t) \mathbf{p}(t,t,0) + t \mathbf{p}(t,t,1) \\ &= (1-t)[(1-t) \mathbf{p}(t,0,0) + t \mathbf{p}(t,0,1)] \\ &\quad + t [(1-t) \mathbf{p}(t,0,1) + t \mathbf{p}(t,1,1)] \\ &= (1-t)^2 \mathbf{p}(t,0,0) + 2 (1-t) t \mathbf{p}(t,0,1) + t^2 \mathbf{p}(t,1,1) \\ &= (1-t)^2 [(1-t) \mathbf{p}(0,0,0) + t \mathbf{p}(1,0,0)] + 2(1-t)t[(1-t) \mathbf{p}(0,0,1) + t \mathbf{p}(1,0,1)] + t^2 [(1-t) \mathbf{p}(0,1,1) + t \mathbf{p}(1,1,1)] \\ &= (1-t)^3 \mathbf{p}(0,0,0) + 3 (1-t)^2 t \mathbf{p}(0,0,1) + 3 (1-t) t^2 \mathbf{p}(0,1,1) + t^3 \mathbf{p}(1,1,1)\end{aligned}$$

