Subdivision Surfaces

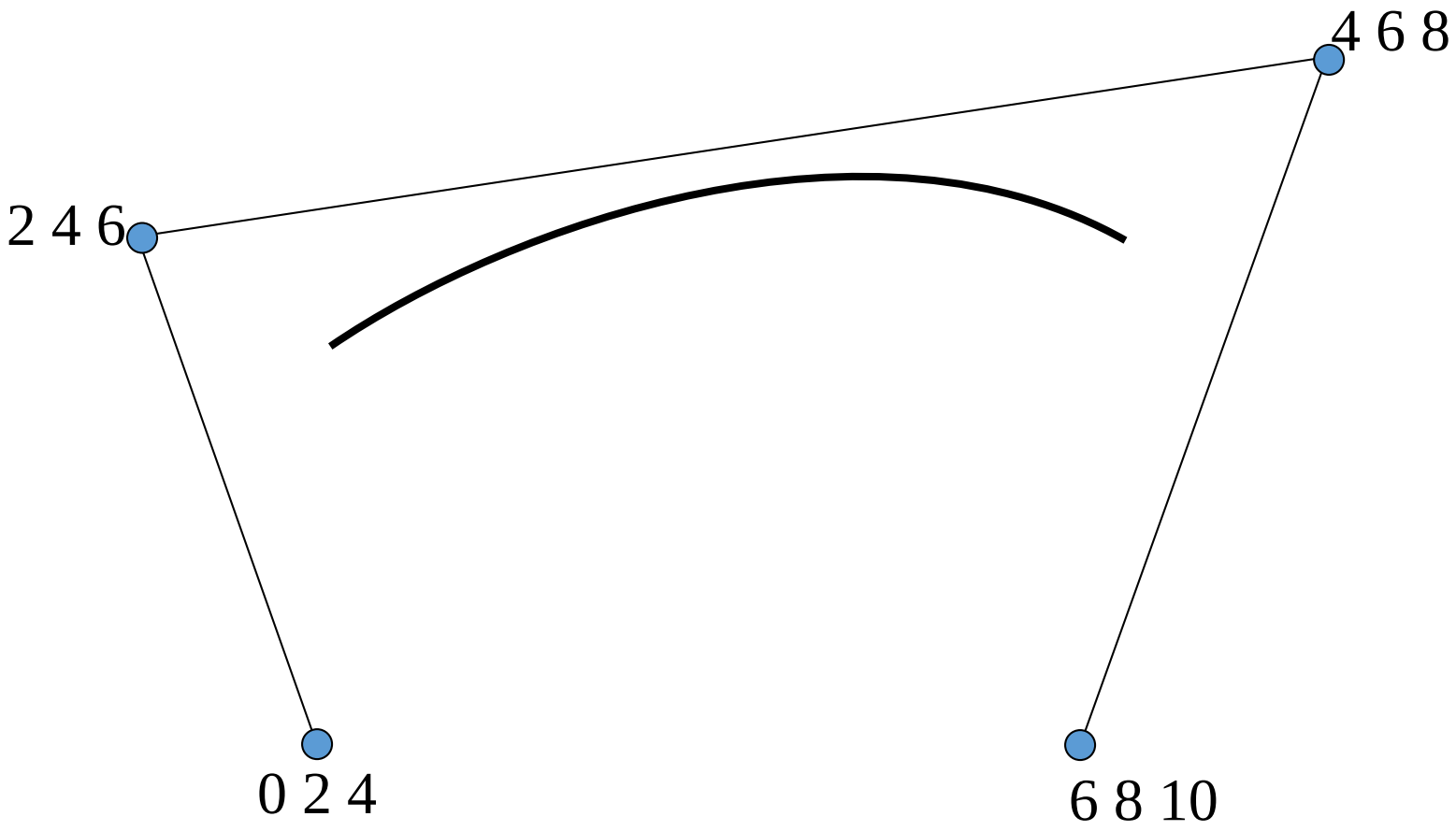
CS 418

Intro to Computer Graphics

John C. Hart

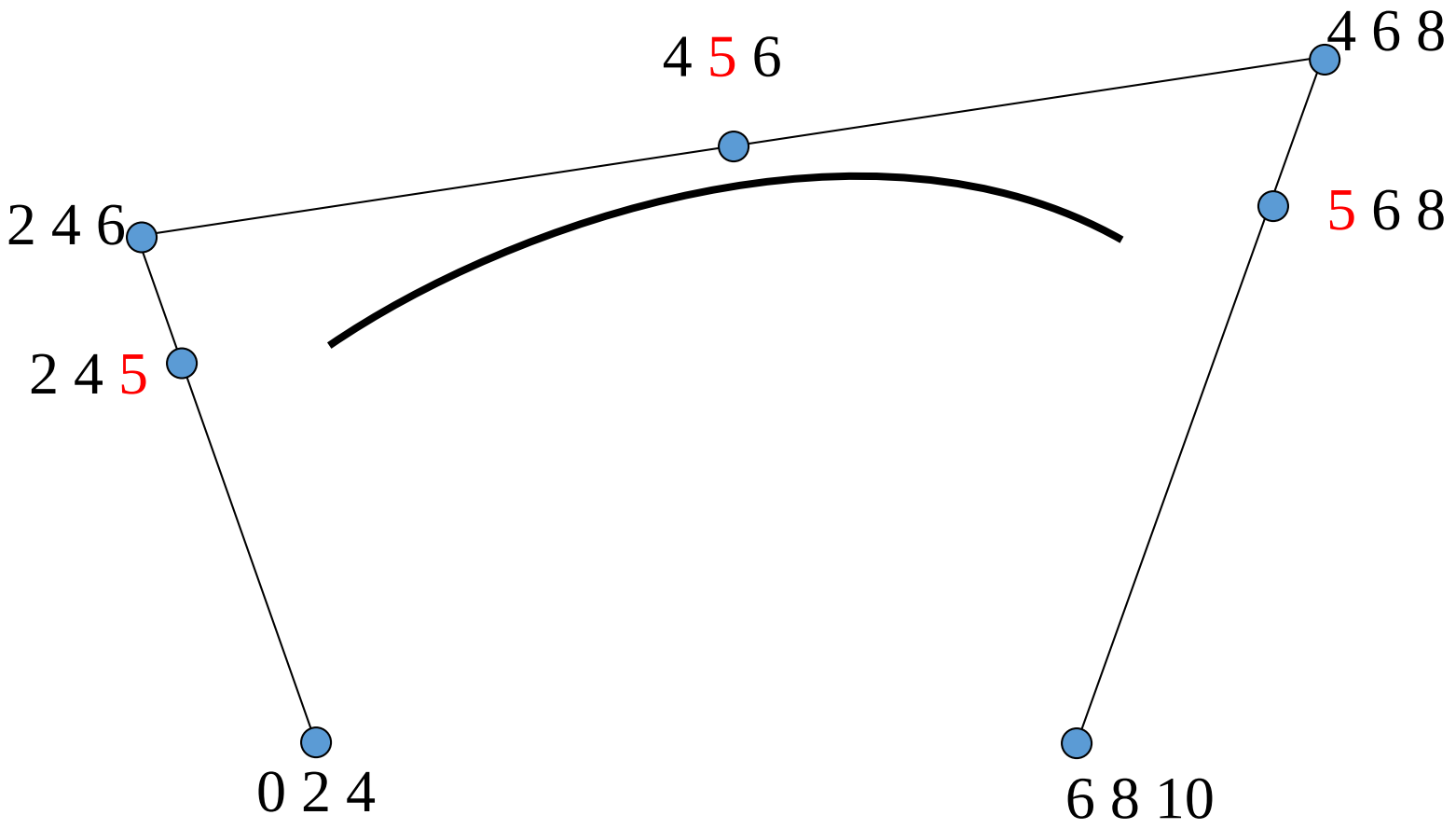
Knot Insertion

[0 2 4 6 8 10]



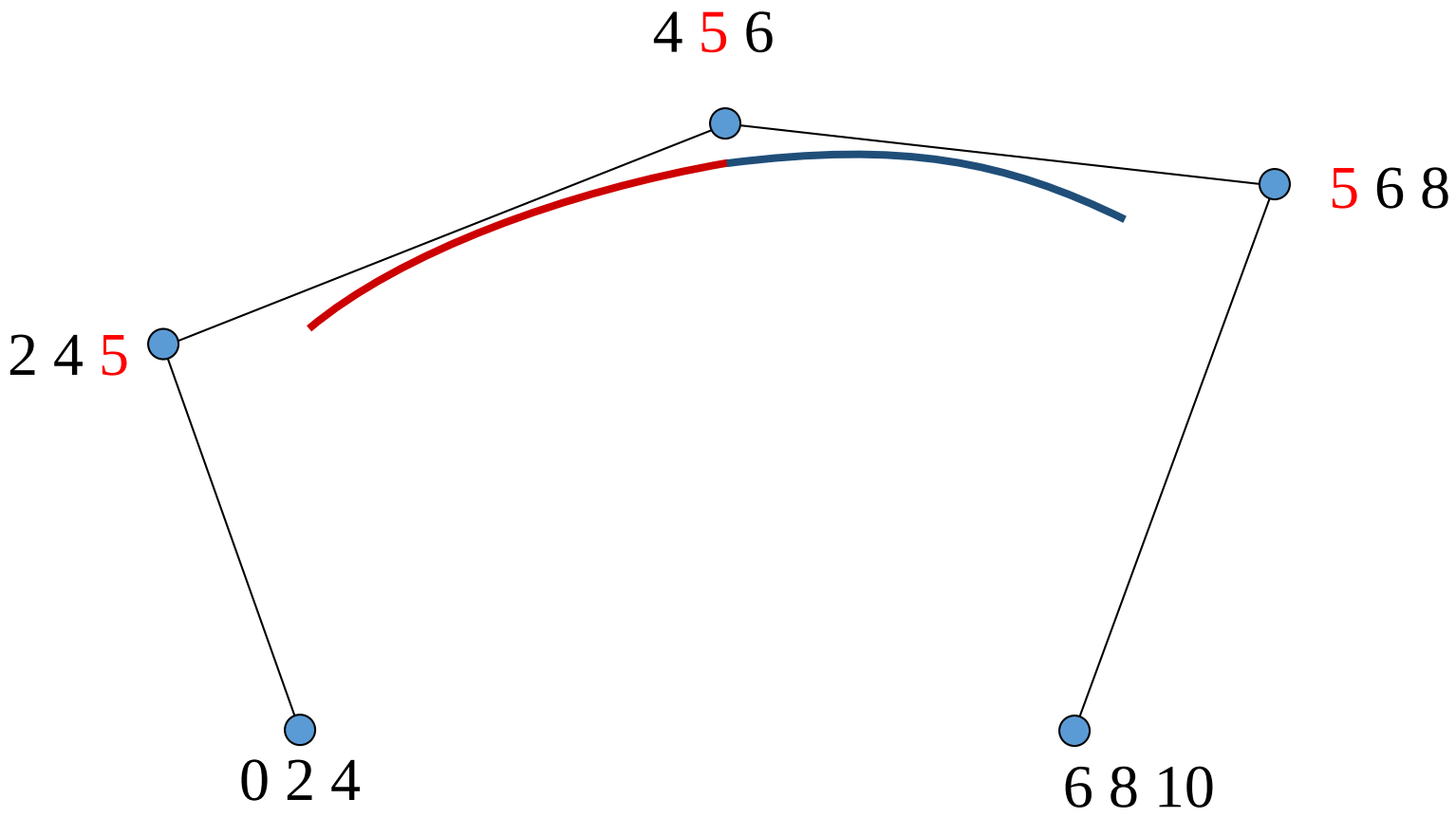
Knot Insertion

[0 2 4 5 6 8 10]

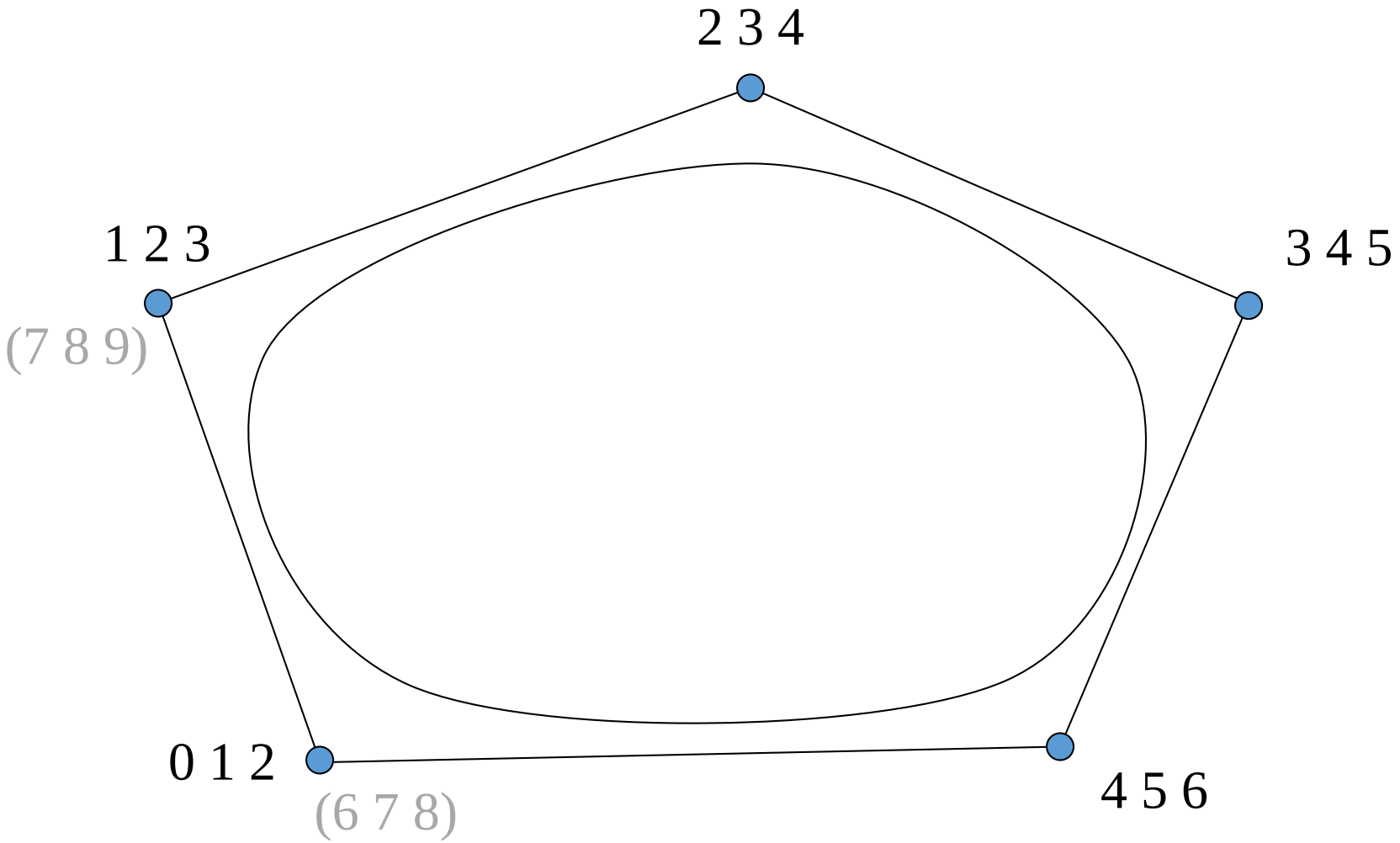


Knot Insertion

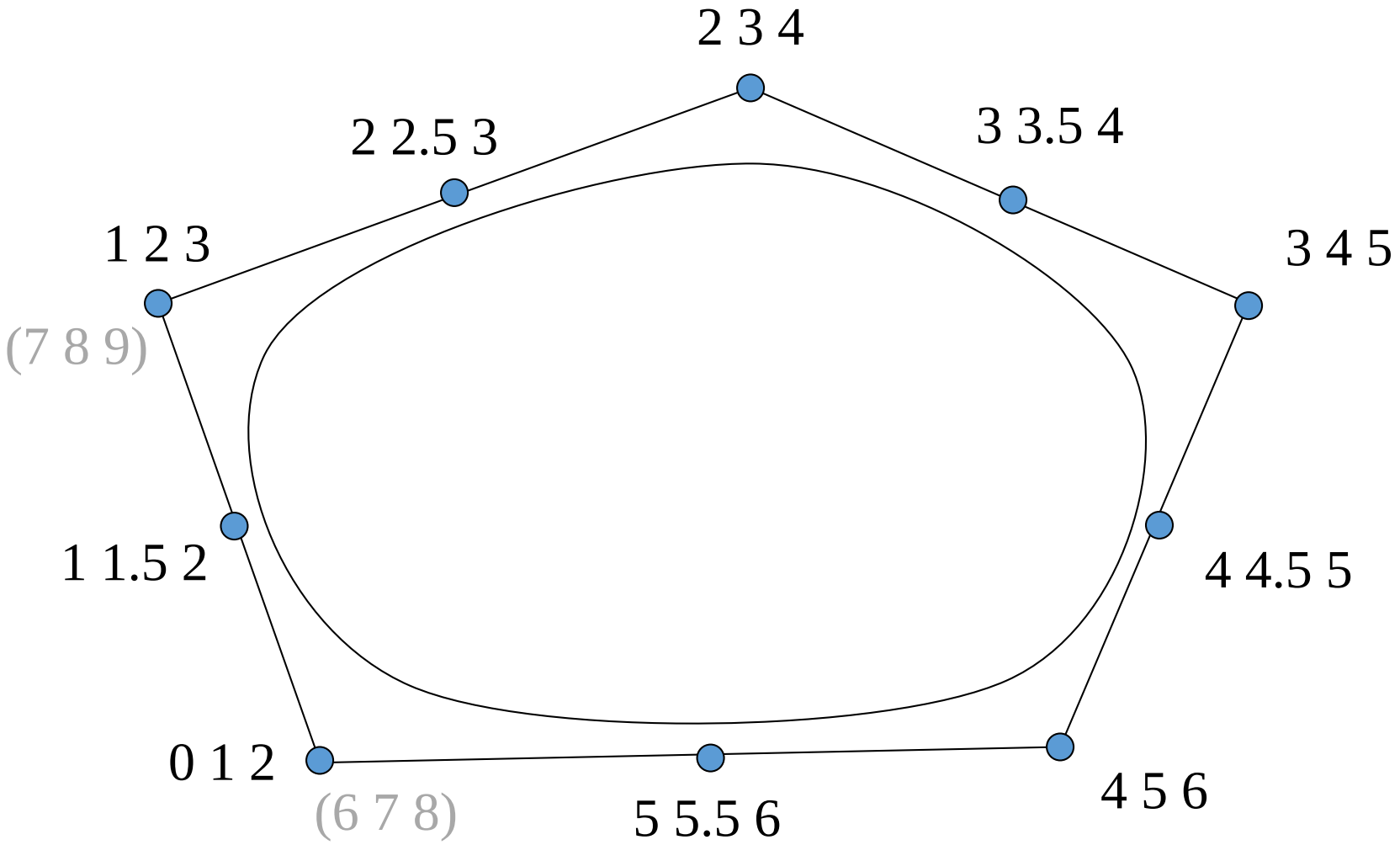
[0 2 4 5 6 8 10]



Loop Knot Insertion



Loop Knot Insertion



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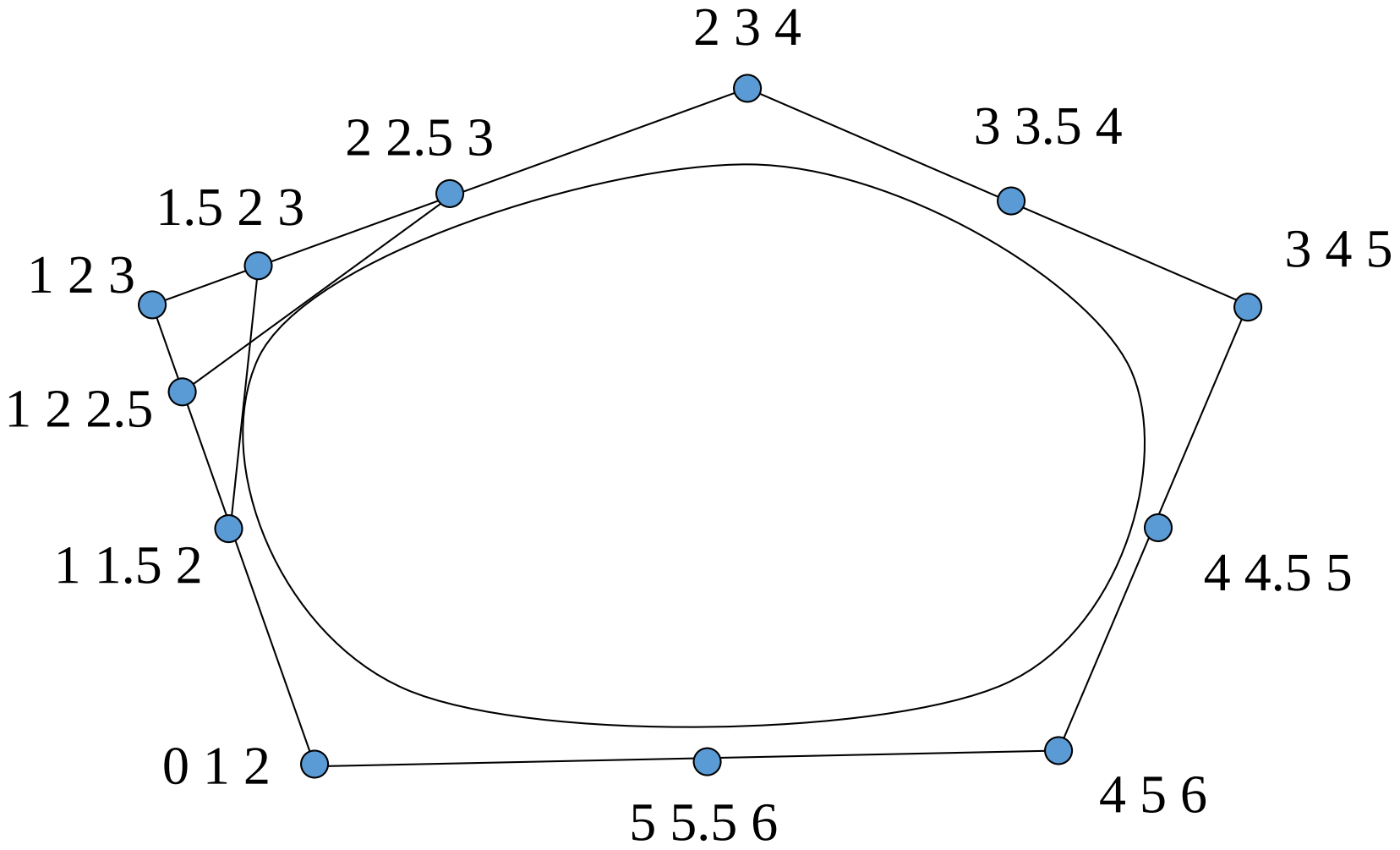

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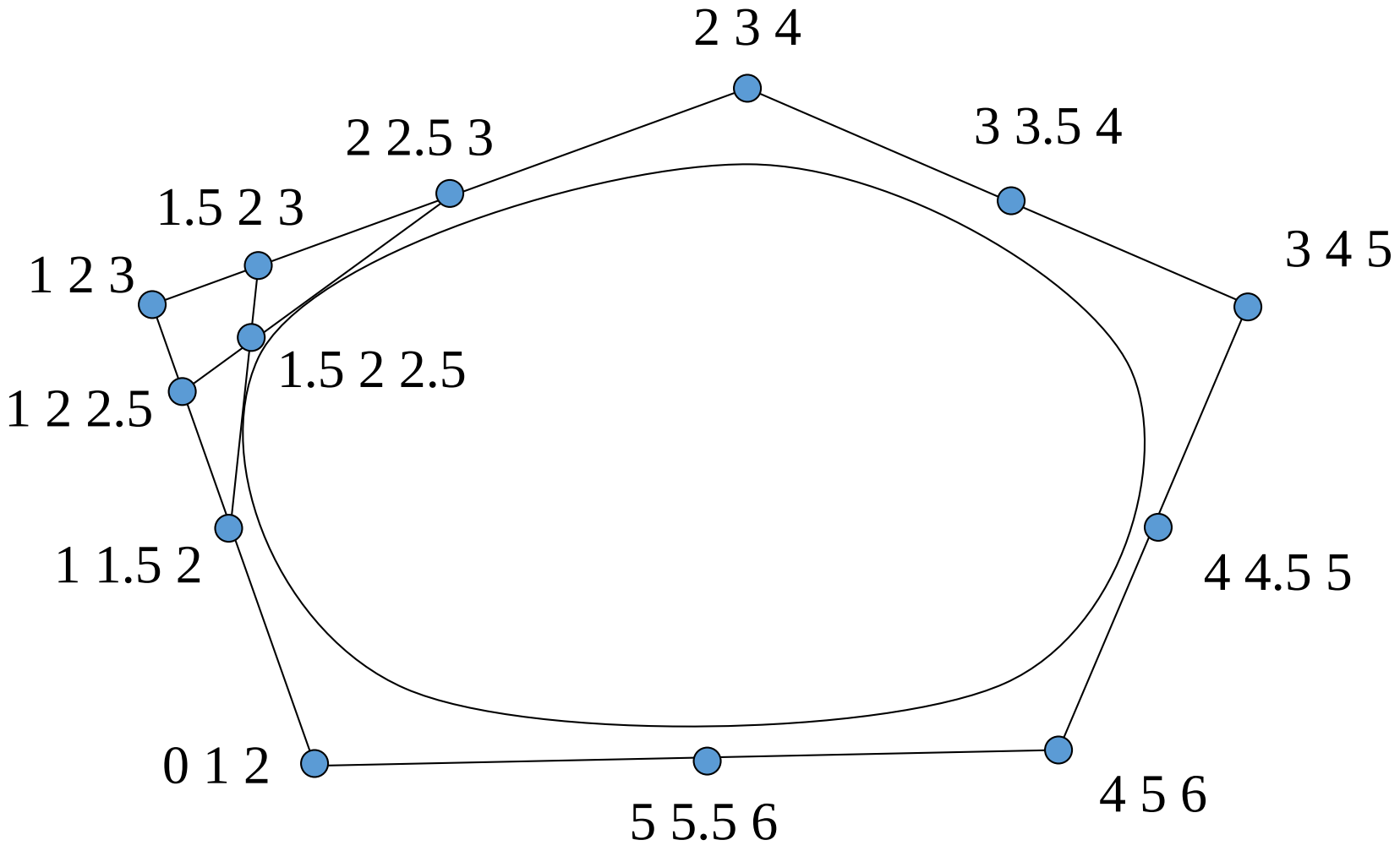
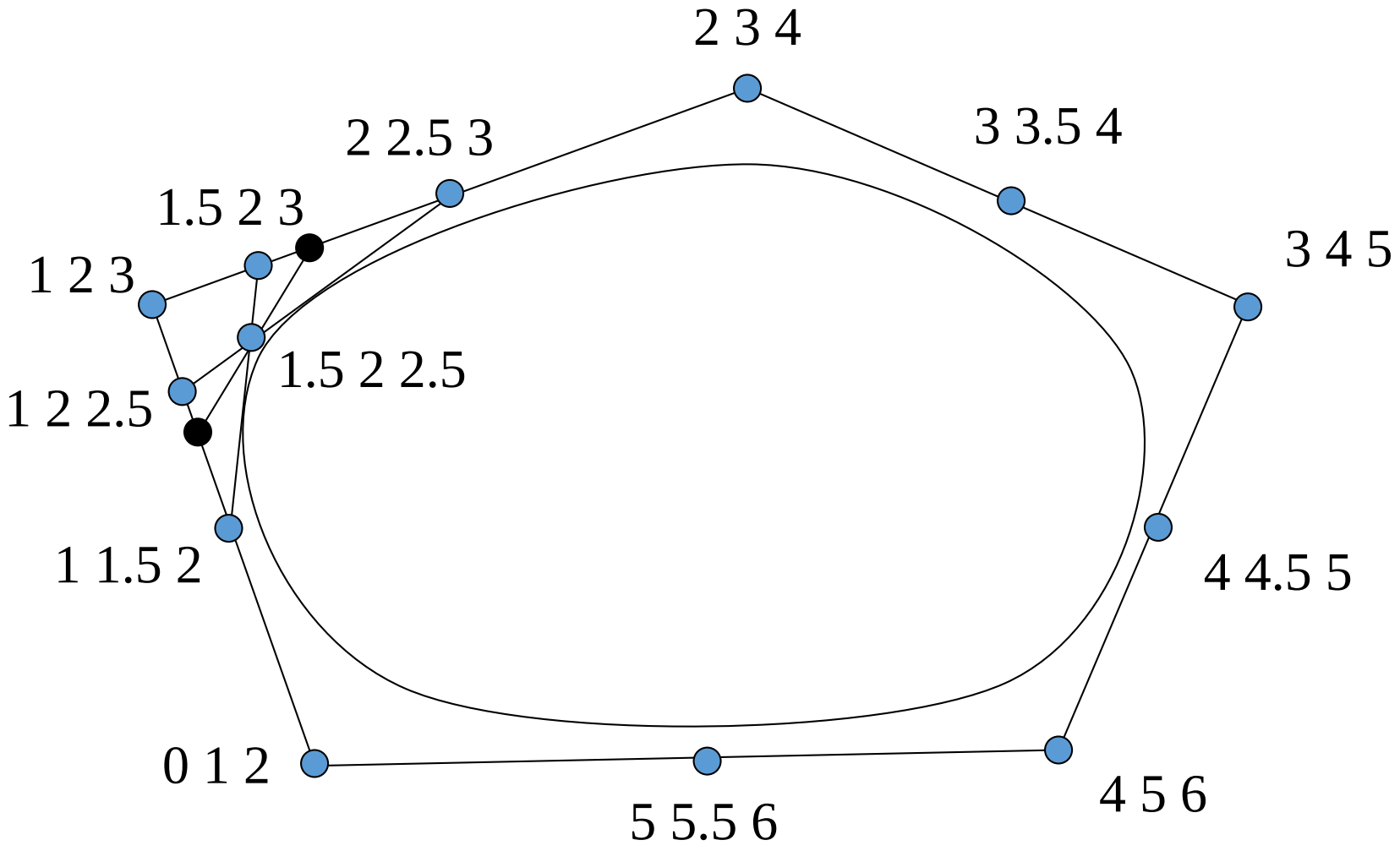
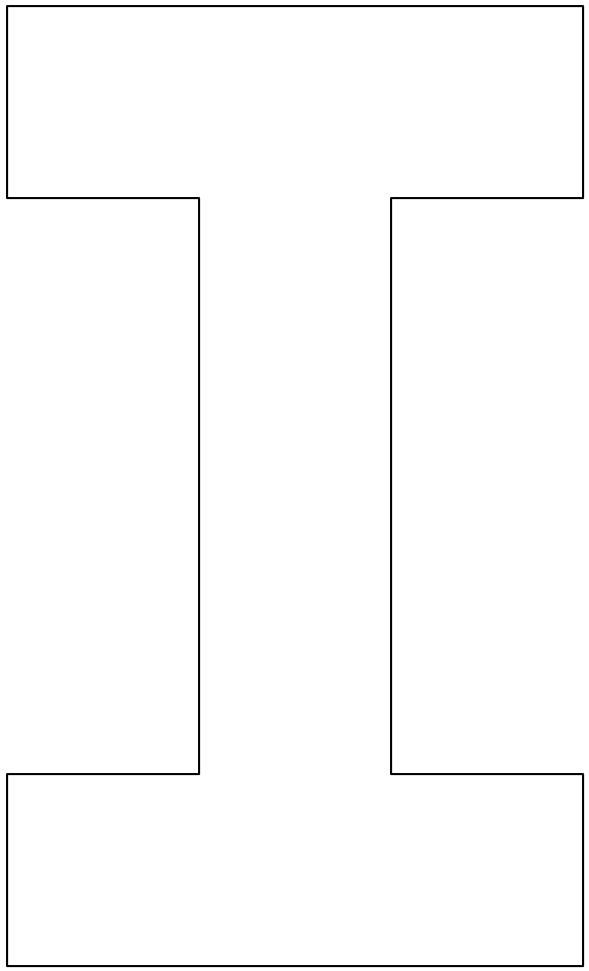


Loop Knot Insertion

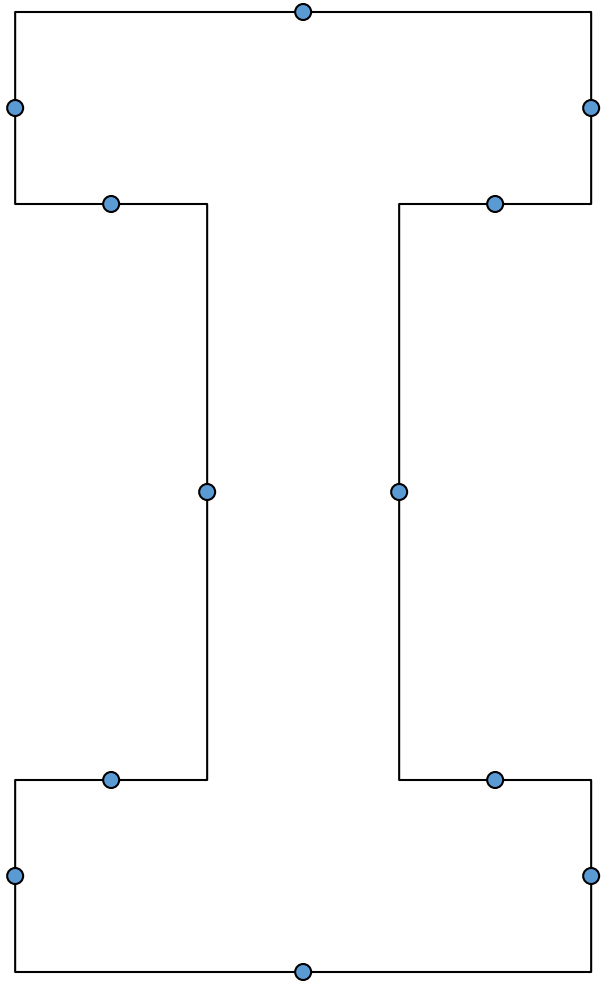


Smoothing a Polygon



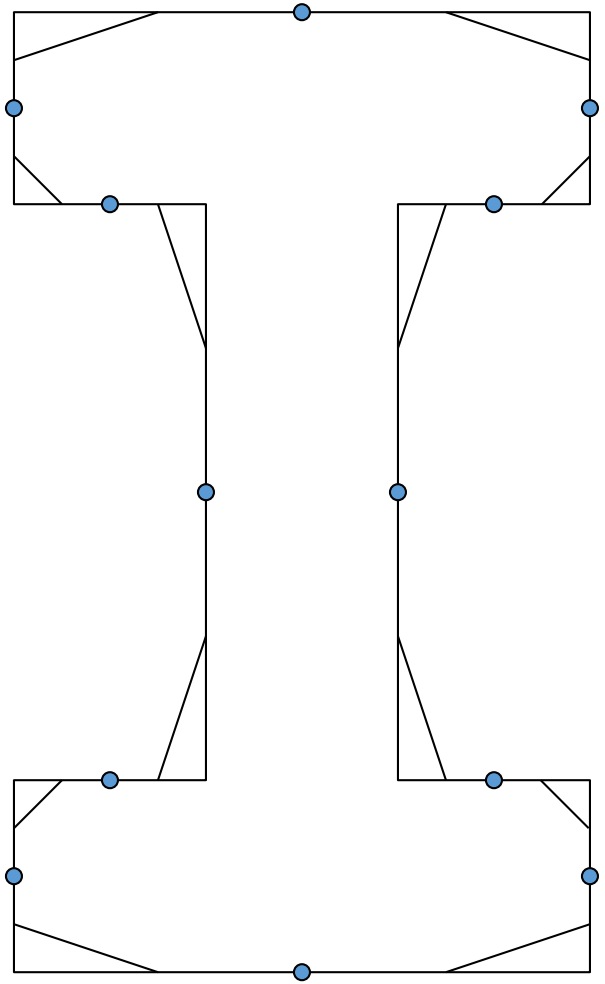
Smoothing a Polygon

1. Add edge midpoints



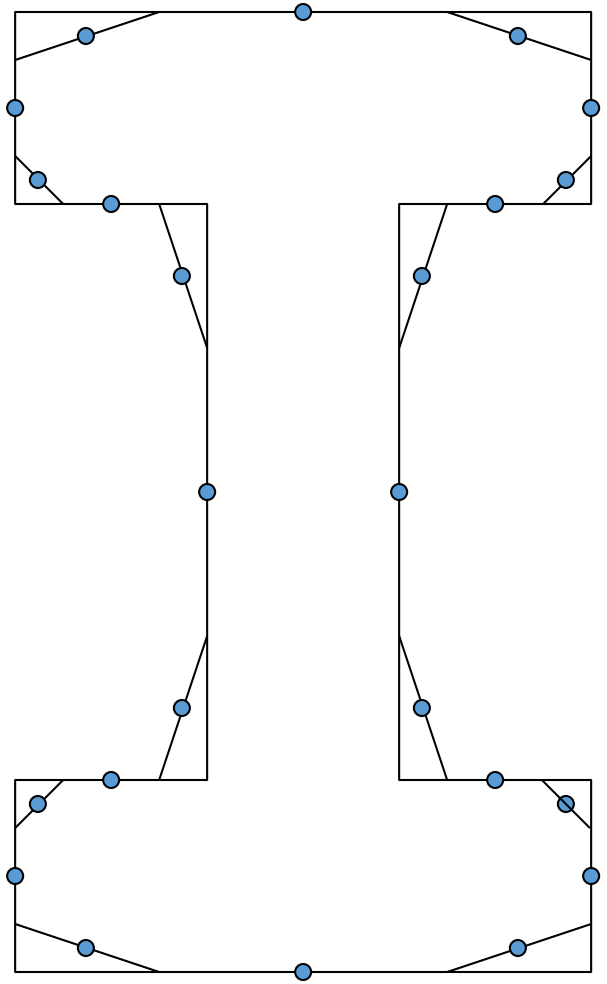
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex



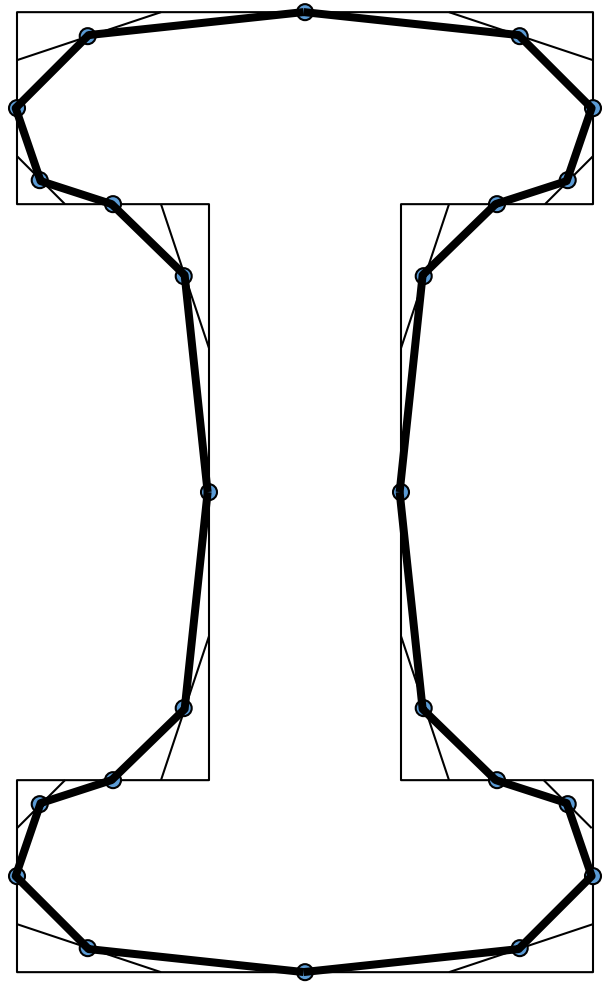
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
3. Add strut midpoints



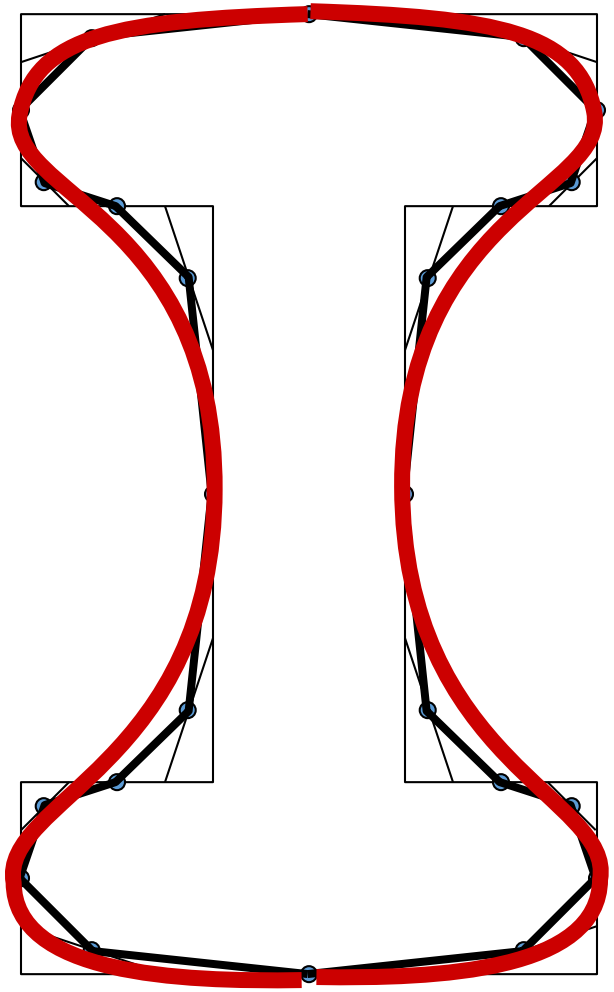
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
3. Add strut midpoints
4. Connect



Smoothing a Polygon

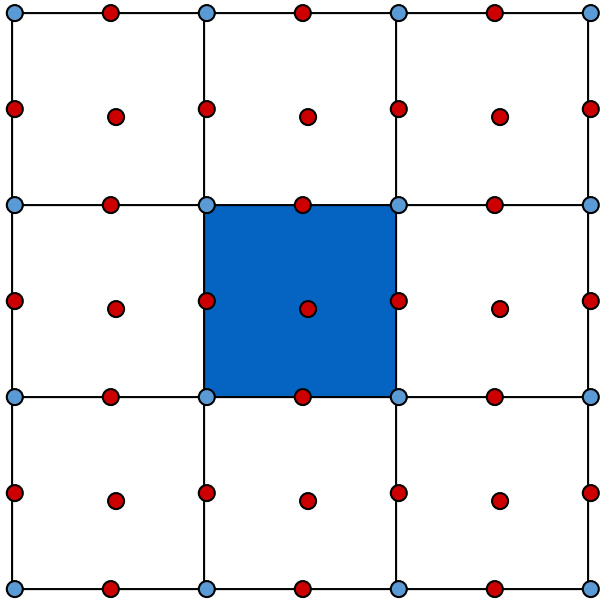
1. Add edge midpoints
2. Add struts
   - Struts connect midpoints of segments from vertices to edge midpoints
   - One strut per vertex
3. Add strut midpoints
4. Connect
5. Repeat



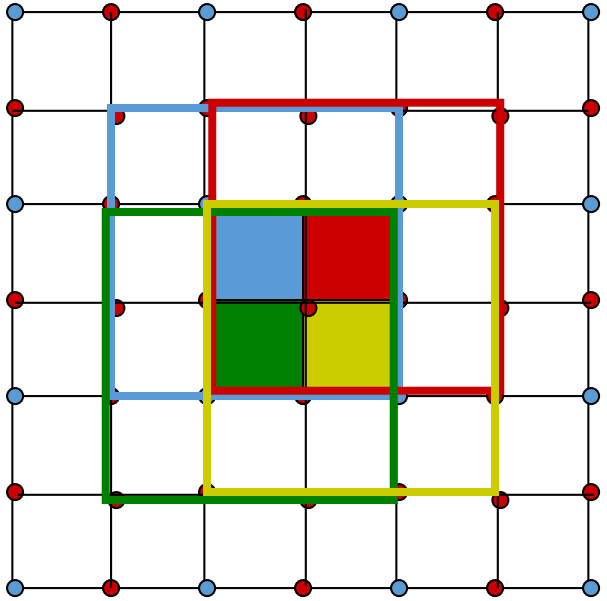
B-Spline Patches

- Tensor product of two curves

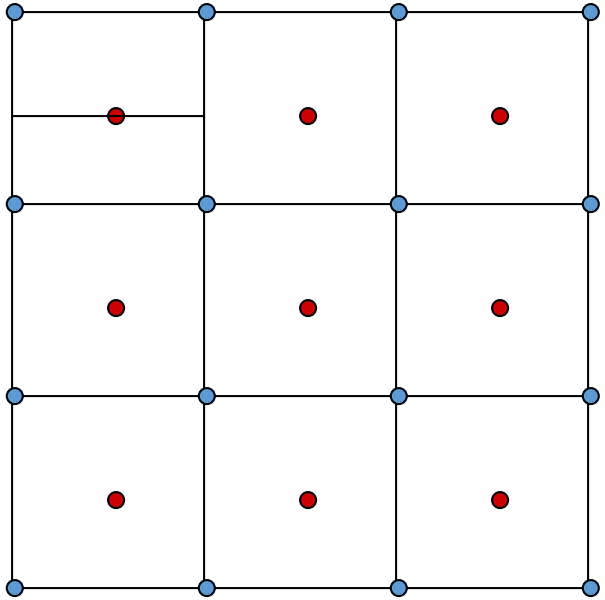
$$\mathbf{p}(s,t) = \sum_{j=0}^{n} \sum_{i=0}^{n} N_j^n(s) N_i^n(t) \mathbf{p}_{ij}$$



- Need to subdivide control points to create four sub-patches
- Need to generate new control points
  - vertex points (replacing control points)
  - edge points
  - face points



Face Points



- Approximate edge points as midpoint of control points

  E = 1/2 **p** + 1/2 **p**

- Face point is midpoint of approximate edge points

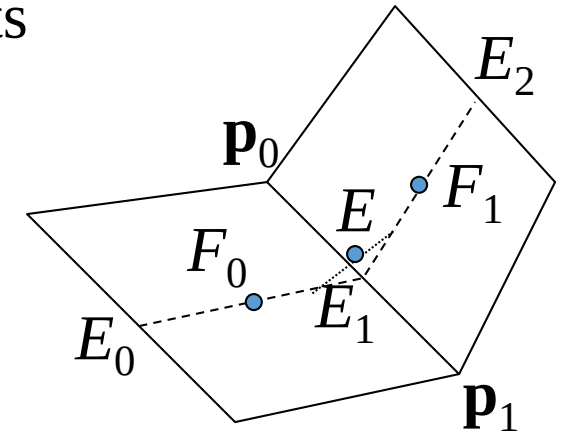
  F = 1/2 E + 1/2 E

  = 1/4 **p** + 1/4 **p** + 1/4 **p** + 1/4 **p**

Edge Points

- Face points are midpoints between approx. edge points
- Approx. edge point is midpoint between control points
- Actual edge point is midpoint between midpoints between approx edge point and face points

$$
\begin{aligned}
E = \;& 1/2\,(1/2\,(1/2\,E_0 + 1/2\,E_1) + 1/2\,E_1) + \\
& 1/2\,(1/2\,E_1 + 1/2\,(1/2\,E_1 + 1/2\,E_2)) \\
= \;& 1/2\,(1/2\,F_0 + 1/2\,(1/2\,\mathbf{p}_0 + 1/2\,\mathbf{p}_1)) + \\
& 1/2\,(1/2\,(1/2\,\mathbf{p}_0 + 1/2\,\mathbf{p}_1) + 1/2\,F_1) \\
= \;& 1/4\,(F_0 + \mathbf{p}_0 + \mathbf{p}_1 + F_1)
\end{aligned}
$$



Vertex Points

$V_0 = 1/4\ E_0 + 1/2\ \mathbf{p}_0 + 1/4\ E_1$

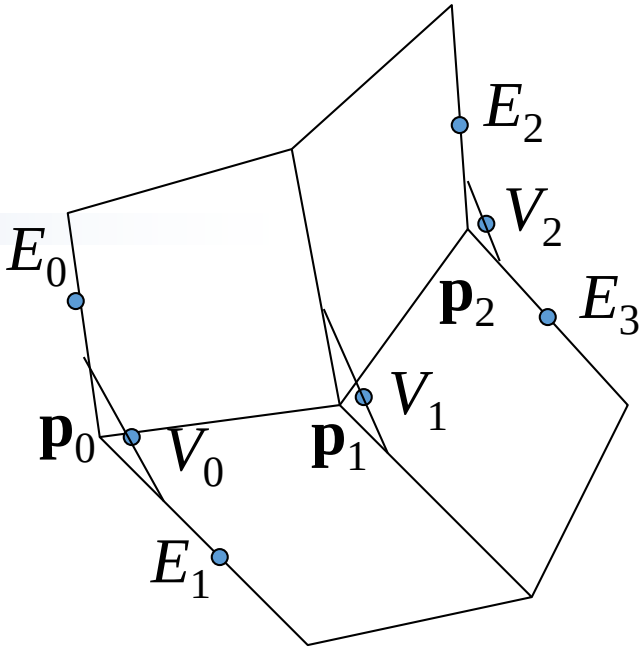
$V_2 = 1/4\ E_2 + 1/2\ \mathbf{p}_2 + 1/4\ E_3$

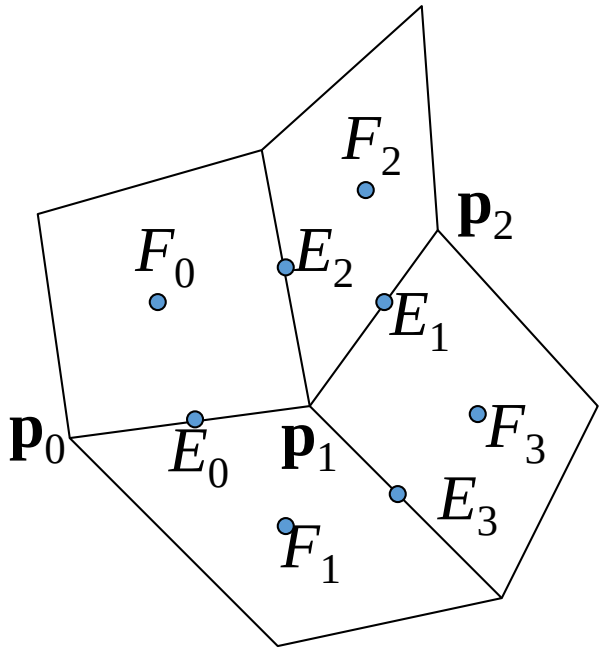
$V = 1/2\ (1/2\ (1/2\ V_0 + 1/2\ V_1) + 1/2\ V_1) +$
$1/2\ (1/2\ V_1 + 1/2\ (1/2\ V_1 + 1/2\ V_2)$

$= 1/4\ (1/4\ (F_0 + F_1 + \mathbf{p}_0 + \mathbf{p}_1) +$
$1/4\ (F_2 + F_3 + \mathbf{p}_1 + \mathbf{p}_2) + 2\ V_1)$

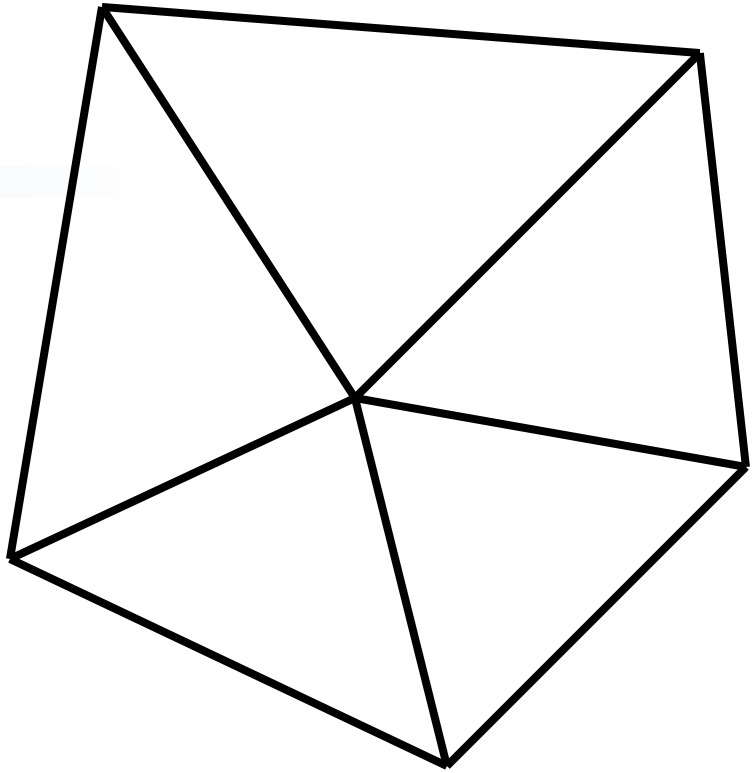
$= 1/4\ (1/4\ (F_0 + F_1 + F_2 + F_3) +$
$1/4\ (\mathbf{p}_0 + 2\ \mathbf{p}_1 + \mathbf{p}_2) +$
$2/4\ (E_2 + E_3 + 2\ \mathbf{p}_1))$

$= 1/16(F_0 + F_1 + F_2 + F_3 +$
$2E_0 + 2E_1 + 2E_2 + 2E_3 + 4\mathbf{p}_1)$





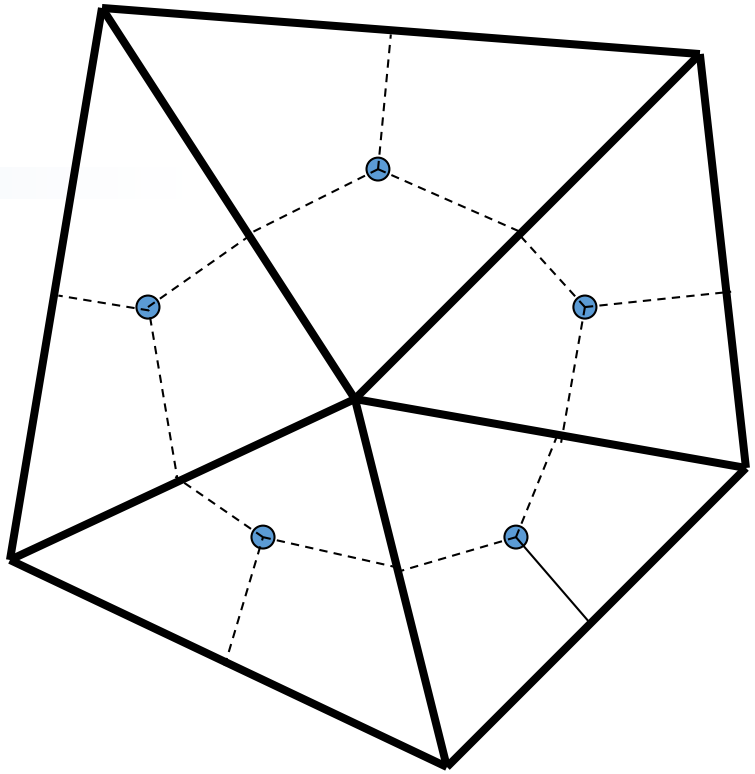
Catmull-Clark Subdiv



Catmull-Clark Subdiv

- Add new “face” vertex at each face centroid

  centroid = average of face’s vertices

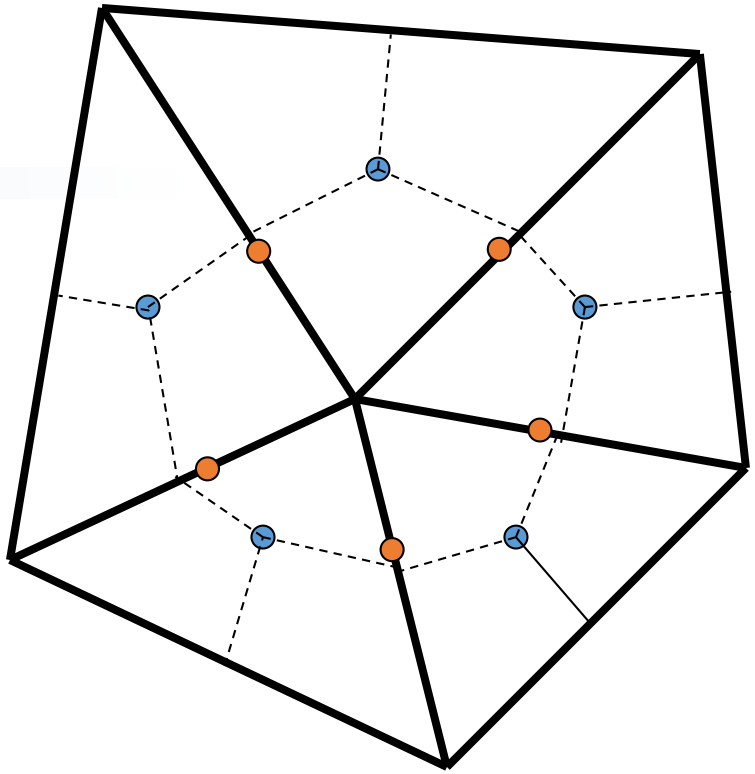


Catmull-Clark Subdiv

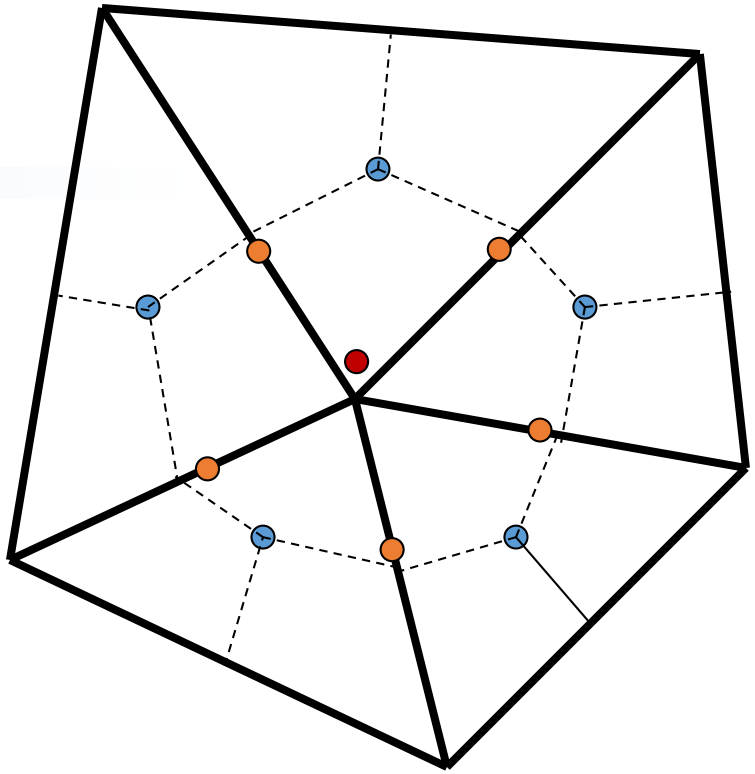
- Add new “face” vertex at each face centroid

  centroid = average of face’s vertices

- Add new “edge” vertex at the average of each edge’s endpoints and adjacent face centroids



Catmull-Clark Subdiv



- Add new “face” vertex at each face centroid

  centroid = average of face’s vertices

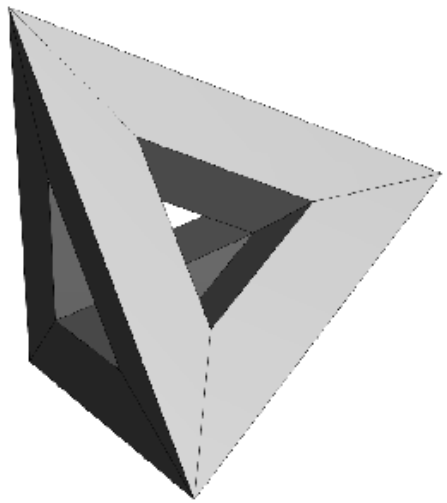
- Add new “edge” vertex at the average of each edge’s endpoints and adjacent face centroids

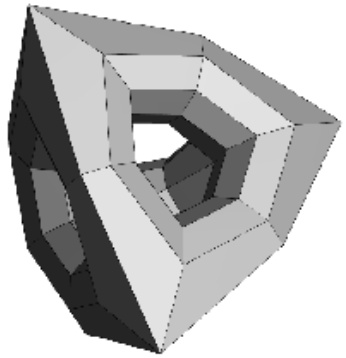
- Move each vertex to a new position that is…

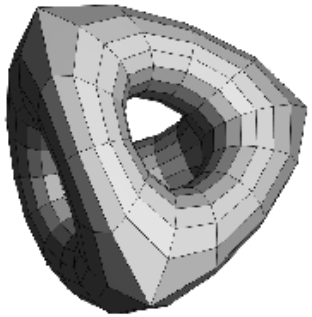
$$\frac{1 \times \text{ave. adjacent face centroids} + 2 \times \text{ave. adjacent edge midpoints} + (n\text{-}3) \times \text{current vertex position}}{n}$$

where n is the valence of the vertex (# of neighboring edges, also # of adjacent faces)

Example







Creases

$$f^{i+1}_{j} = \text{Centroid of polygon}$$

$$e^{i+1}_{j} = (v^i + e^i_j)/2$$

- Dart vertex (one sharp edge):

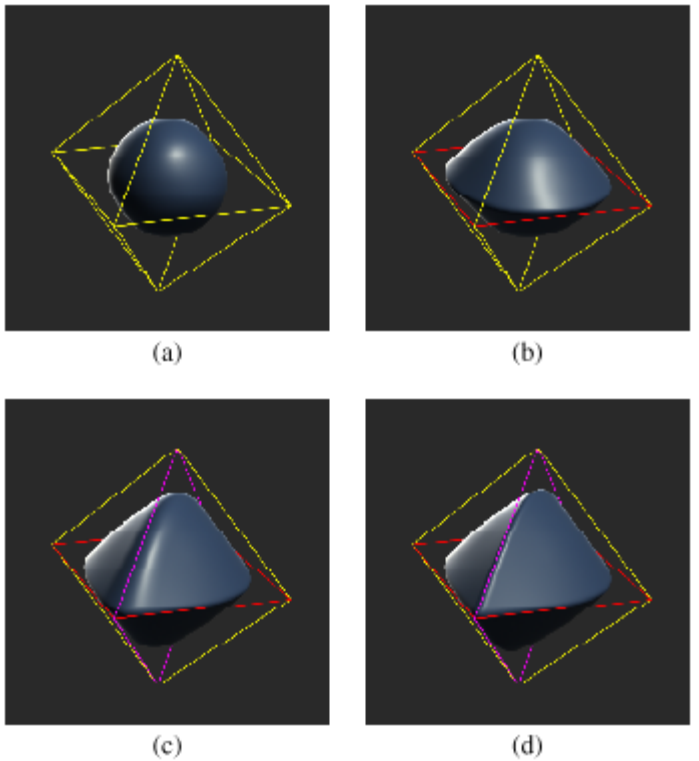
$$v^{i+1} = (n\text{-}2)/n\ v^i + 1/n^2\ \Sigma_j\ e^i_j + 1/n^2\ \Sigma_j\ f^{i+1}_{j}$$

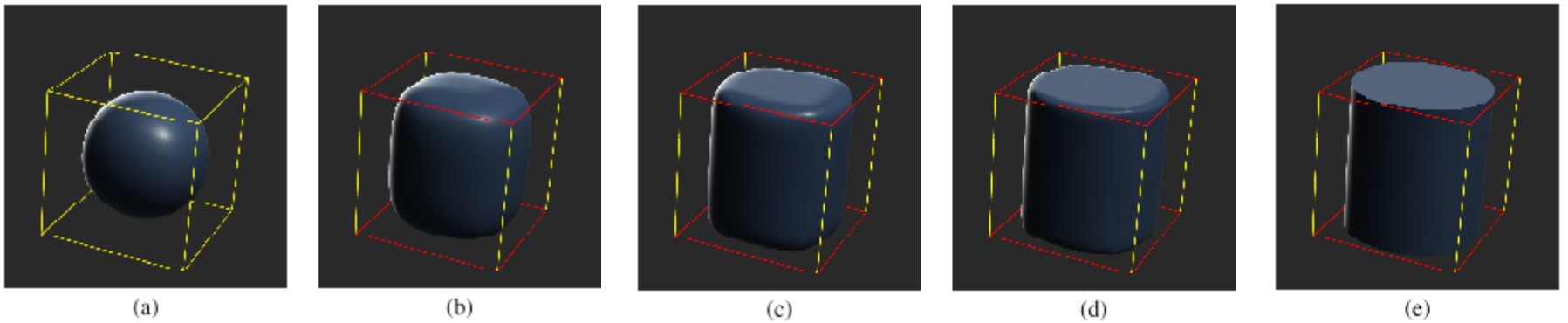
- Crease vertex (two sharp edges):

$$v^{i+1} = (e^i_j + 6v^i + e^i_k)/8$$

- Corner vertex (three or more sharp edges)

$$v^{i+1} = v^i$$





Another Example

Success?