Cubic Curves

CS 418

Interactive Computer Graphics

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Why Cubic Curves

- Polynomials (like degree 3 cubics) are well understood with beautiful mathematical formulations

$$At^3 + Bt^2 + Ct + D$$

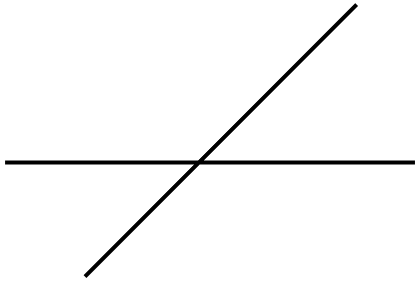
- Cubics provide enough flexibility to space curves, whereas quadratics are limited to planar curves

$$\begin{array}{ll} A_x t^3 + B_x t^2 + C_x t + D_x & \qquad B_x t^2 + C_x t + D_x \\ A_y t^3 + B_y t^2 + C_y t + D_y & \qquad B_y t^2 + C_y t + D_y \\ A_z t^3 + B_z t^2 + C_z t + D_z & \qquad B_z t^2 + C_z t + D_z \end{array}$$

Wiggle Theorem

(Bezout's Theorem)

$y = x$



Wiggle Theorem (Bezout's Theorem)

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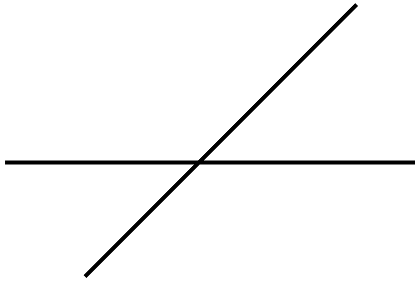
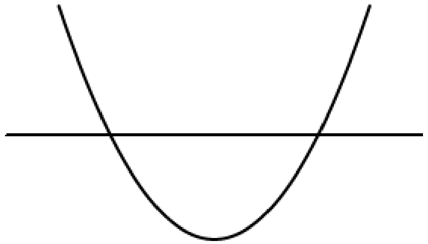


$y = x^2 - 1$



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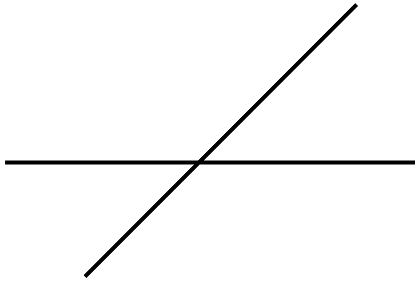

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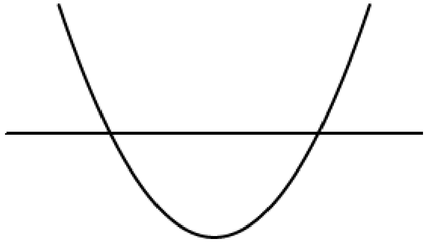

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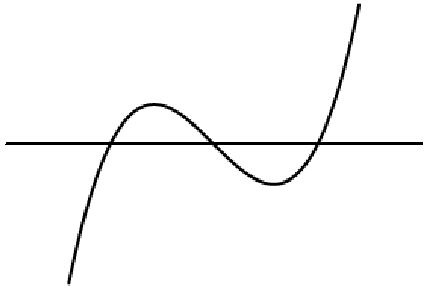

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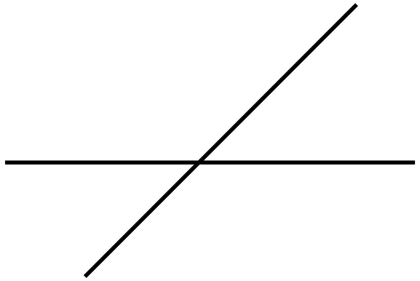

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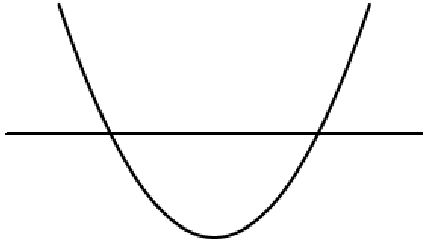

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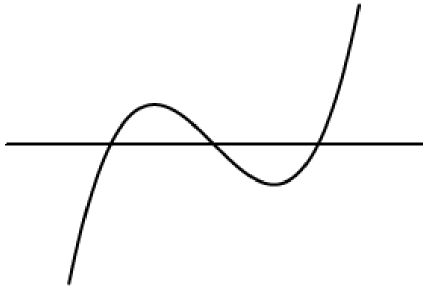
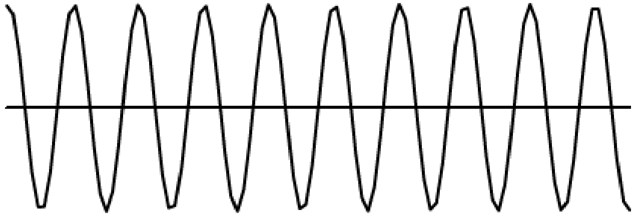


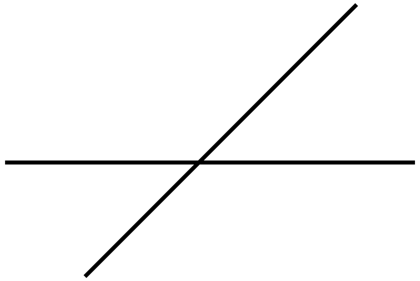
$y = \sin x$



Wiggle Theorem

(Bezout's Theorem)

$y = x$



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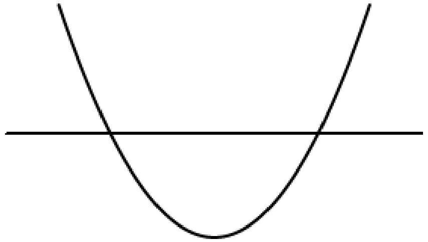

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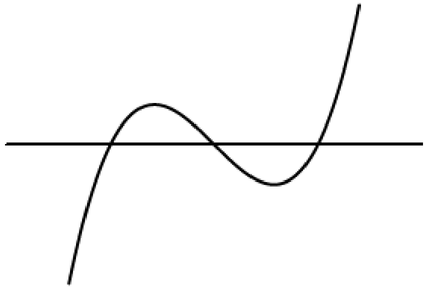
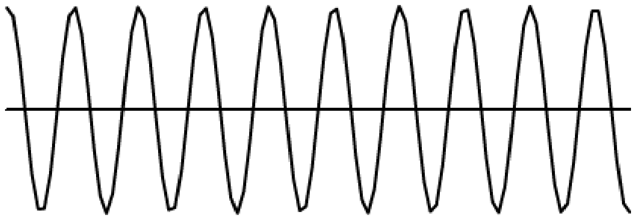


$y = \sin x = x - (1/3!)\, x^3 + (1/5!)\, x^5 - \ldots$



Lagrangian Interpolation

- Cubic polynomial

$$y = Ax^3 + Bx^2 + Cx + D$$

- Solve for

$$D = 0$$

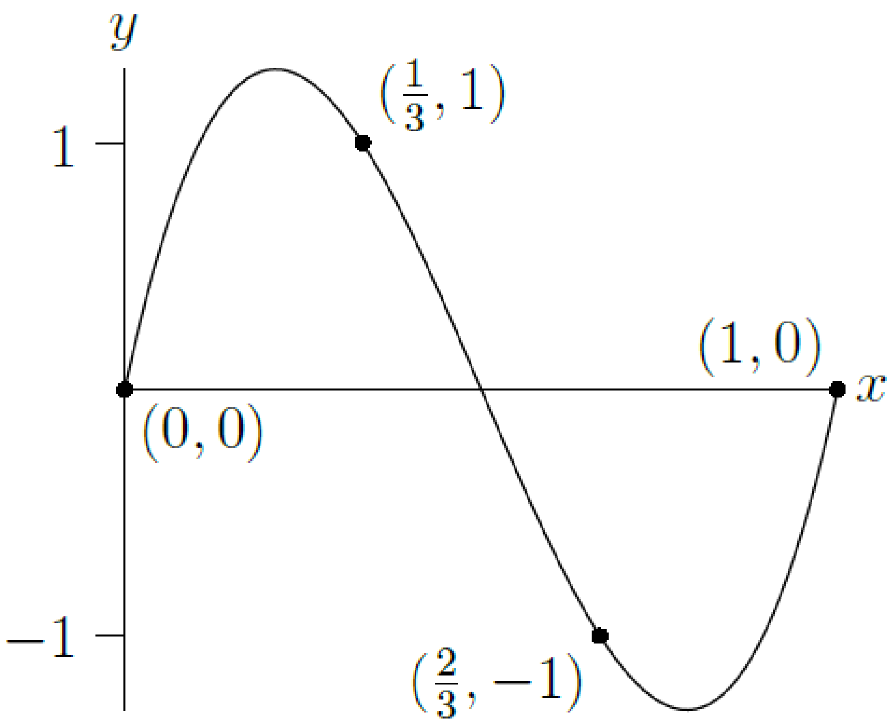
$$1 = A(1/3)^3 + B(1/3)^2 + C(1/3)$$

$$-1 = A(2/3)^3 + B(2/3)^2 + C(2/3)$$

$$0 = A + B + C$$

- Result

$$y = 27\,x^3 - 40\tfrac{1}{2}\,x^2 + 13\tfrac{1}{2}\,x$$



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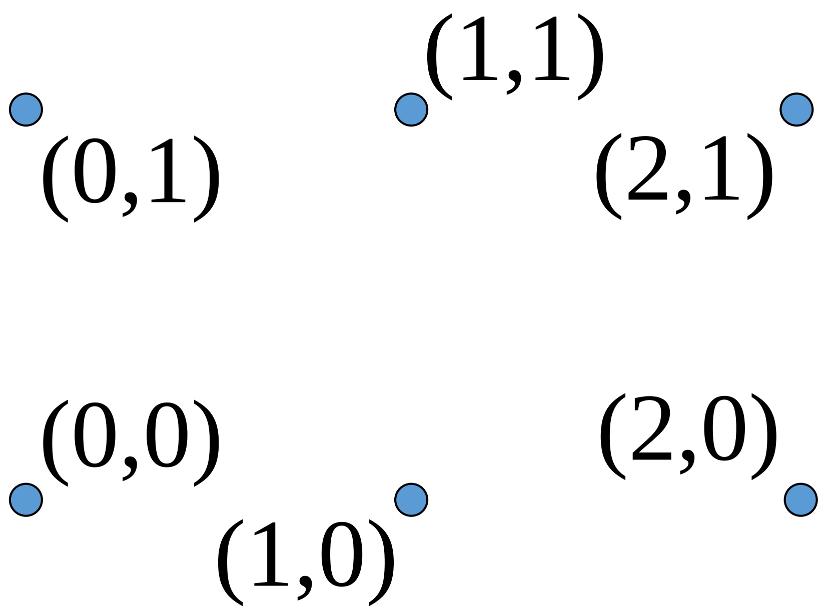

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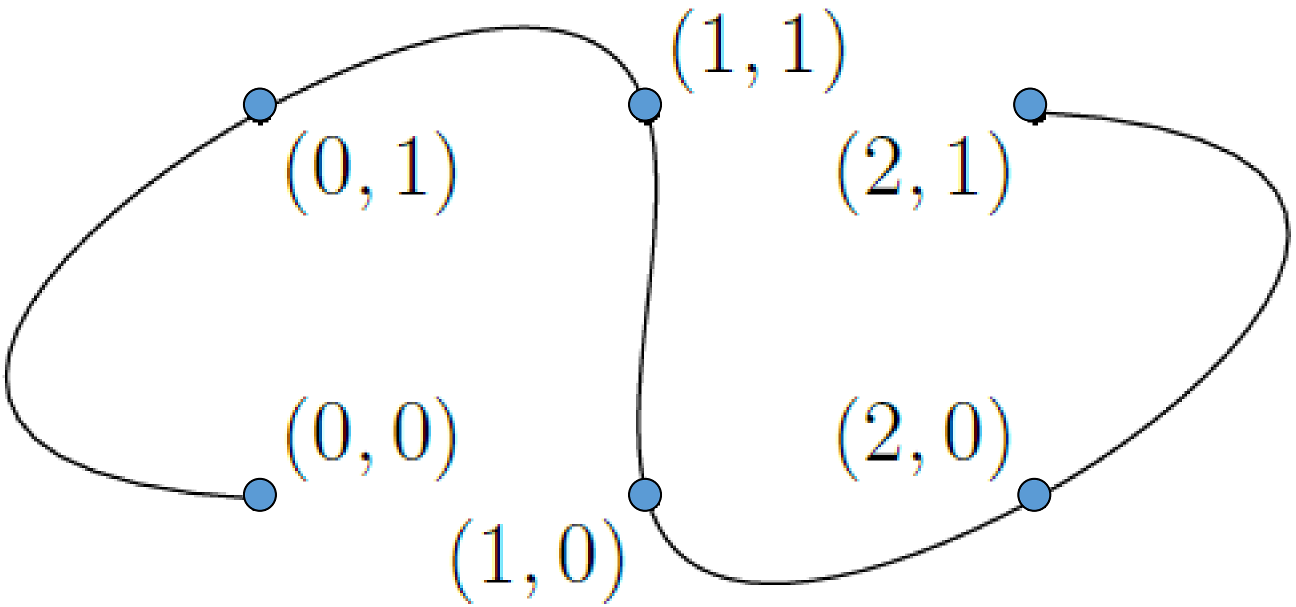

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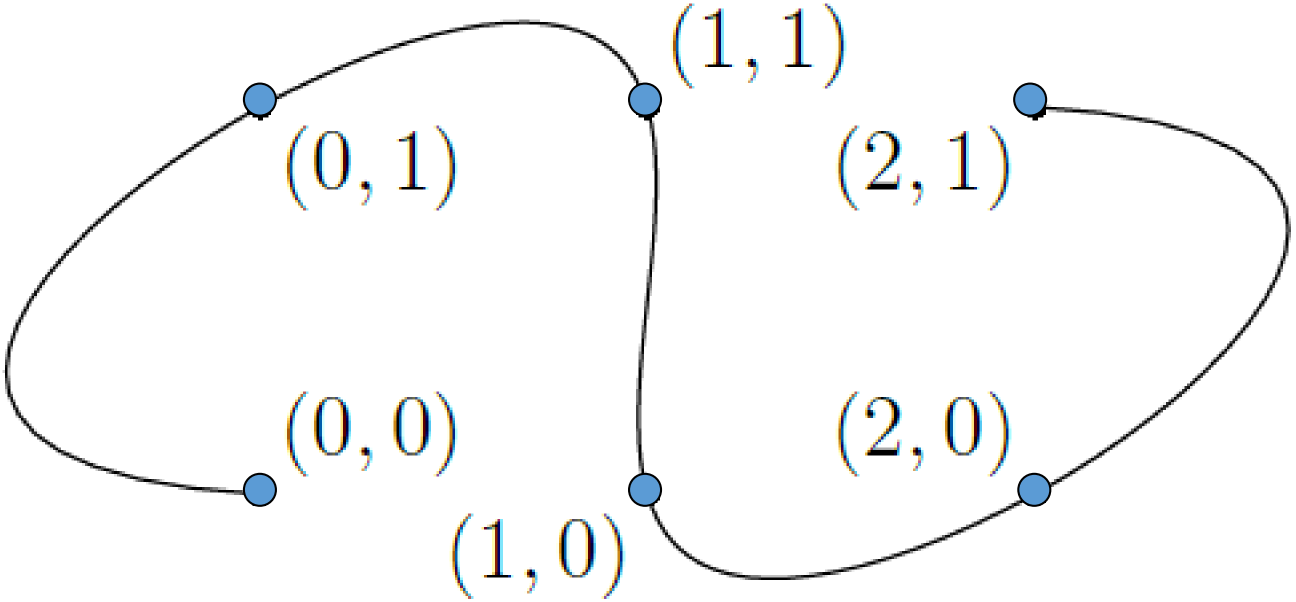

$$At^5 + Bt^4 + Ct^3 + Dt^2 + E^t + F$$

Lagrangian Interpolation