Bezier Blossoms

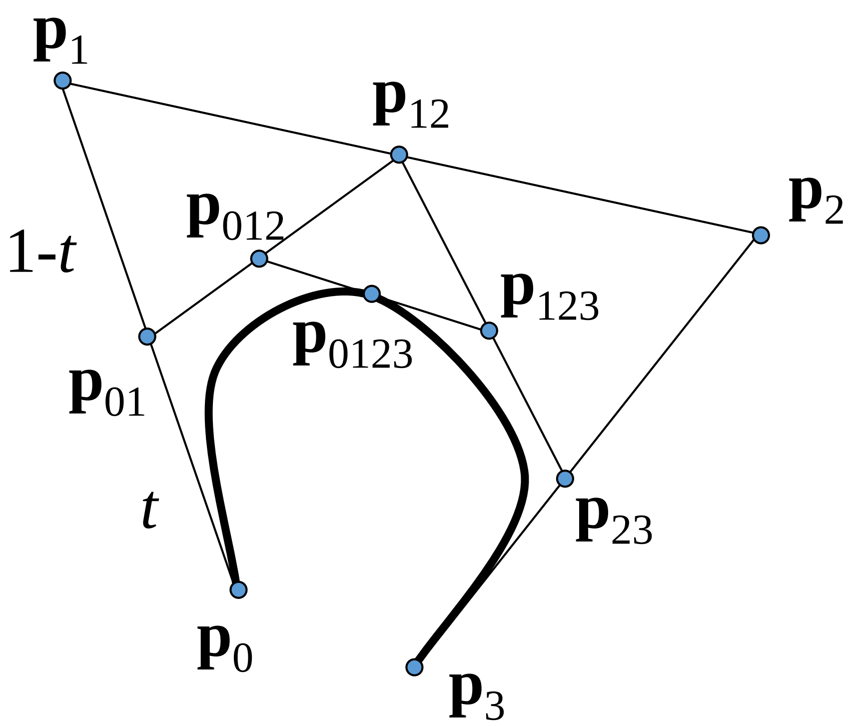
CS 418

Interactive Computer Graphics

John C. Hart

de Casteljau

- de Casteljau algorithm evaluates a point on a Bezier curve by scaffolding lerps
- Blossoming renames the control and intermediate points, like $\mathbf{p}_{12}$, using a polar form, like $\mathbf{p}(0,t,1)$



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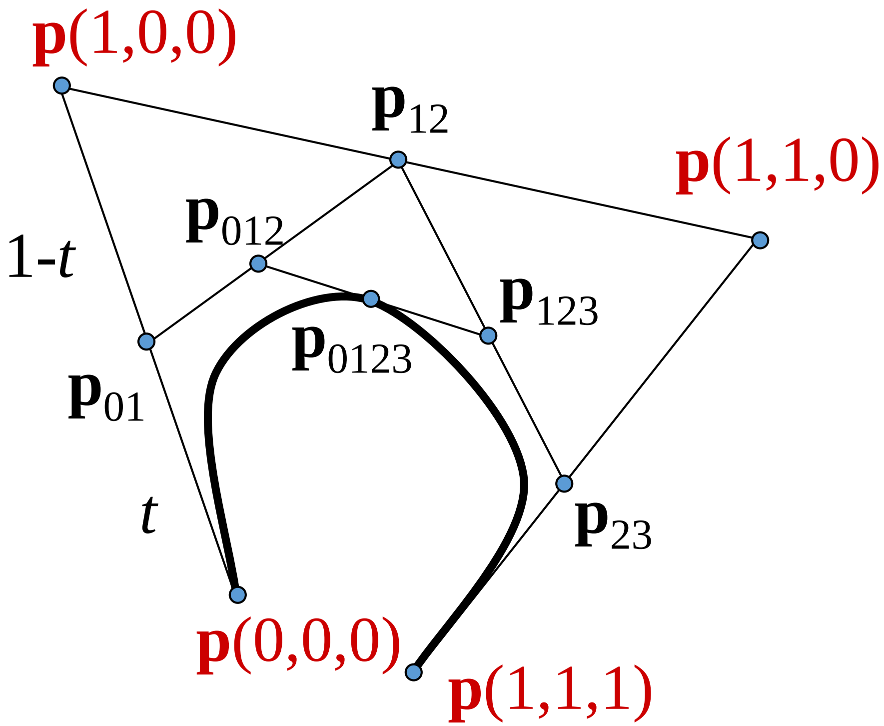

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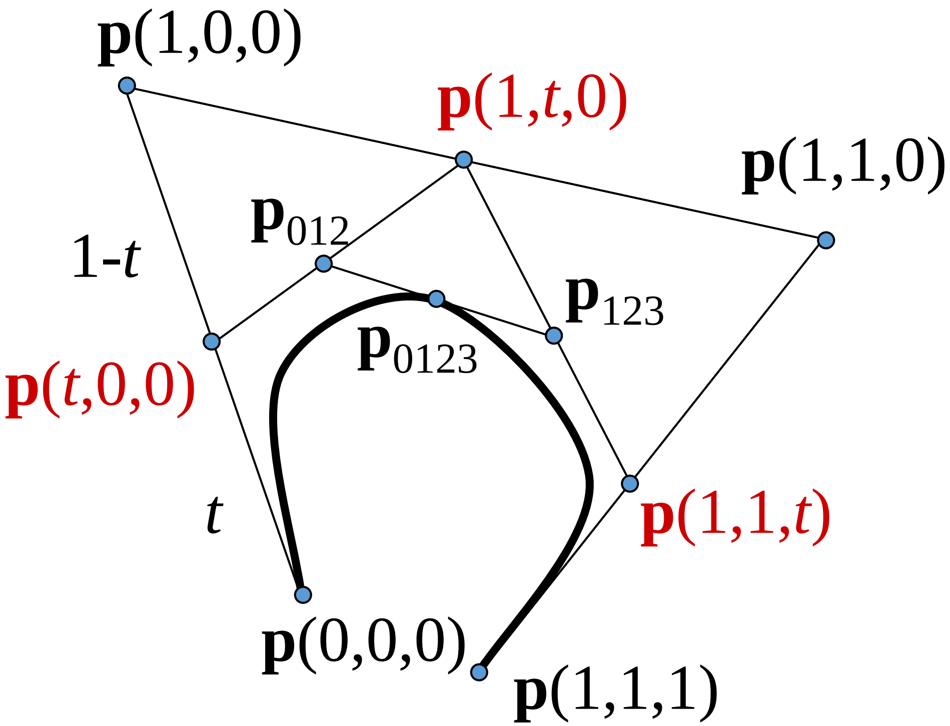

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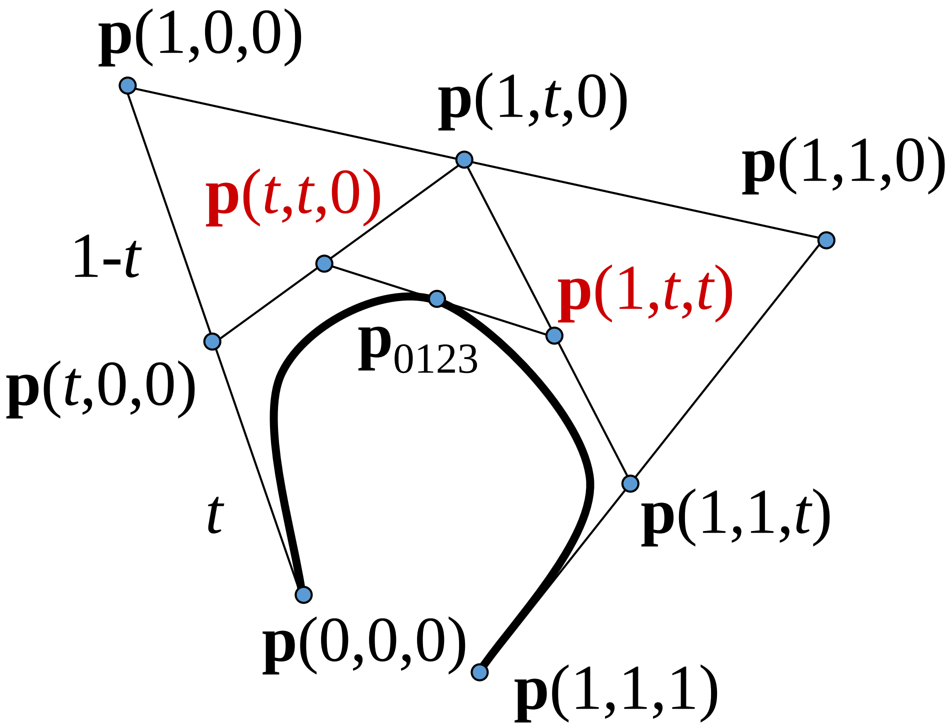

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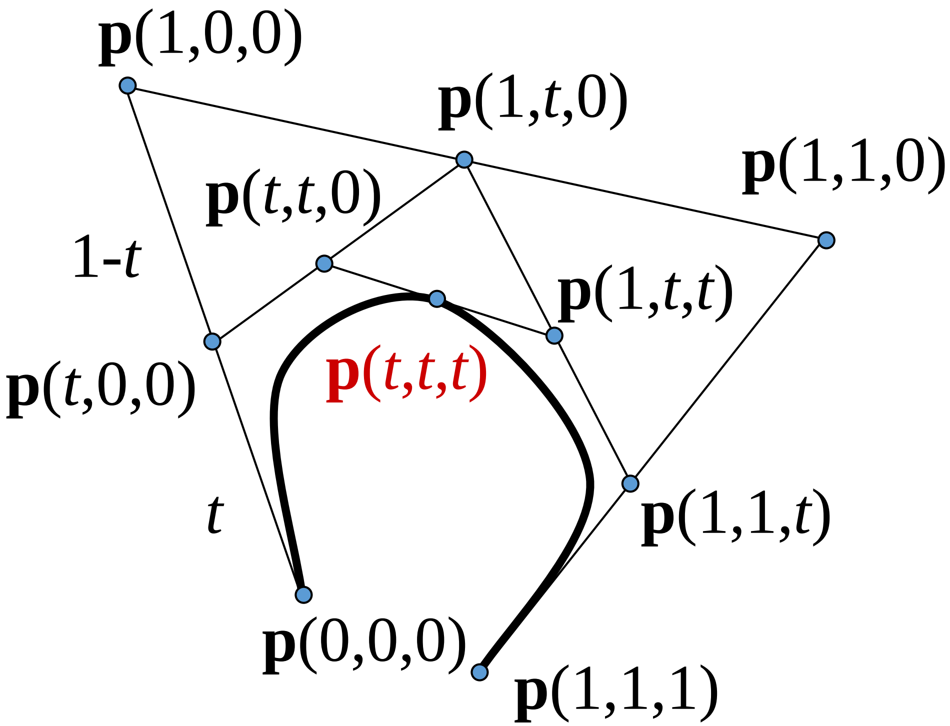
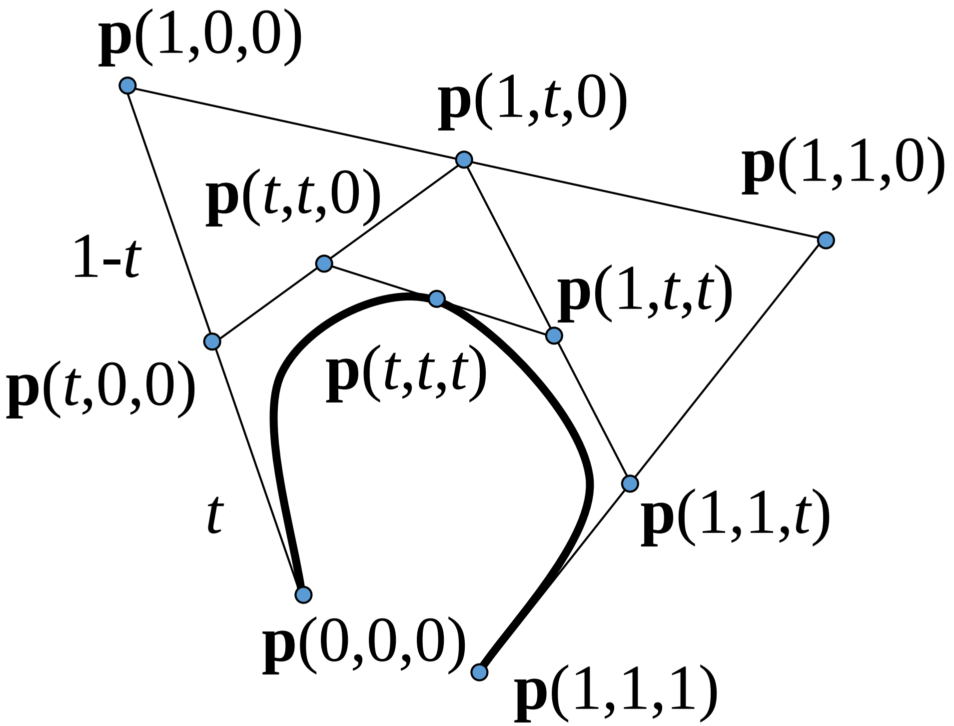


Blossoming Rules

1. # of parameters = degree

Cubic: **p**(_,_,_)



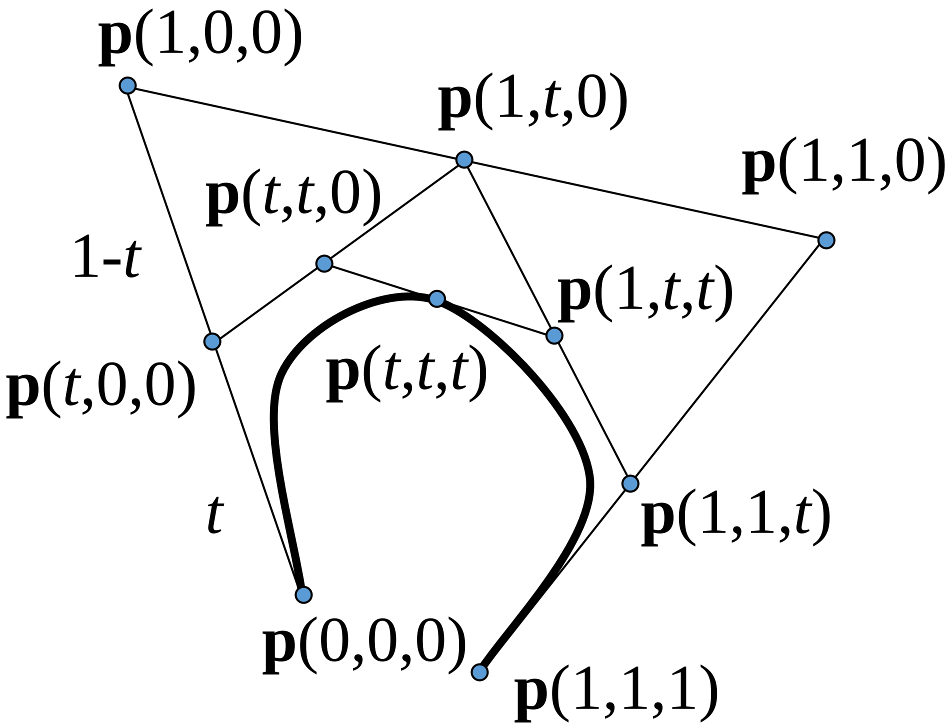
Blossoming Rules

1. # of parameters = degree

Cubic: $\mathbf{p}(\_,\_,\_)$

2. Order doesn't matter

$\mathbf{p}(a,b,c) = \mathbf{p}(b,a,c)$



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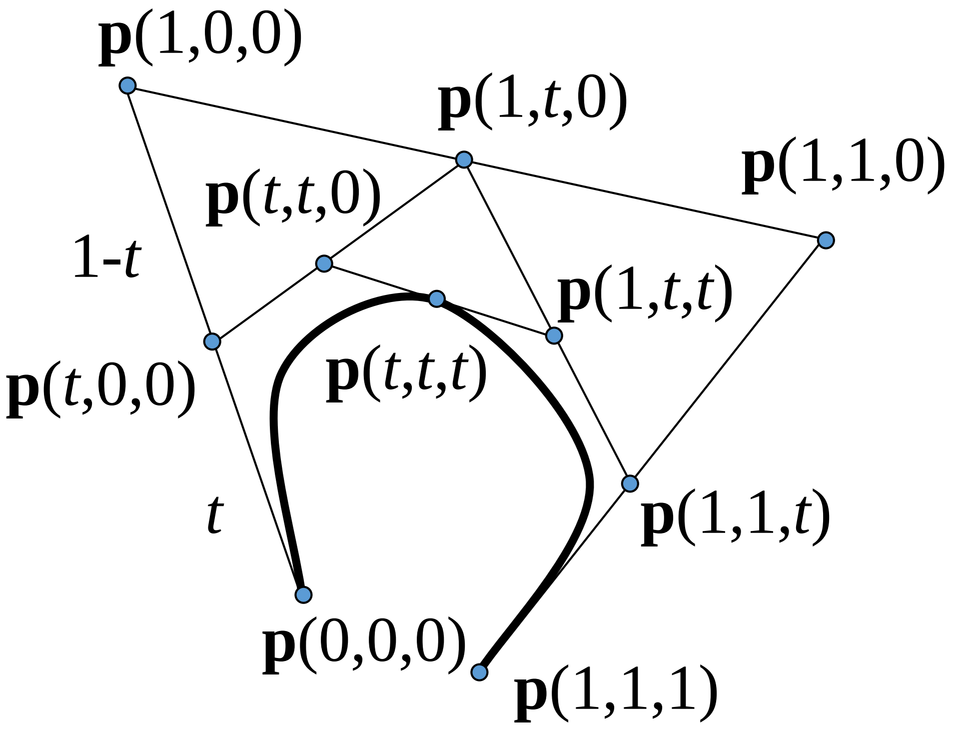
3. Setting up the board

$\mathbf{p}(0,0,0)$, $\mathbf{p}(0,0,1)$,
$\mathbf{p}(0,1,1)$, $\mathbf{p}(1,1,1)$



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3. Setting up the board

$\mathbf{p}(0,0,0)$, $\mathbf{p}(0,0,1)$,
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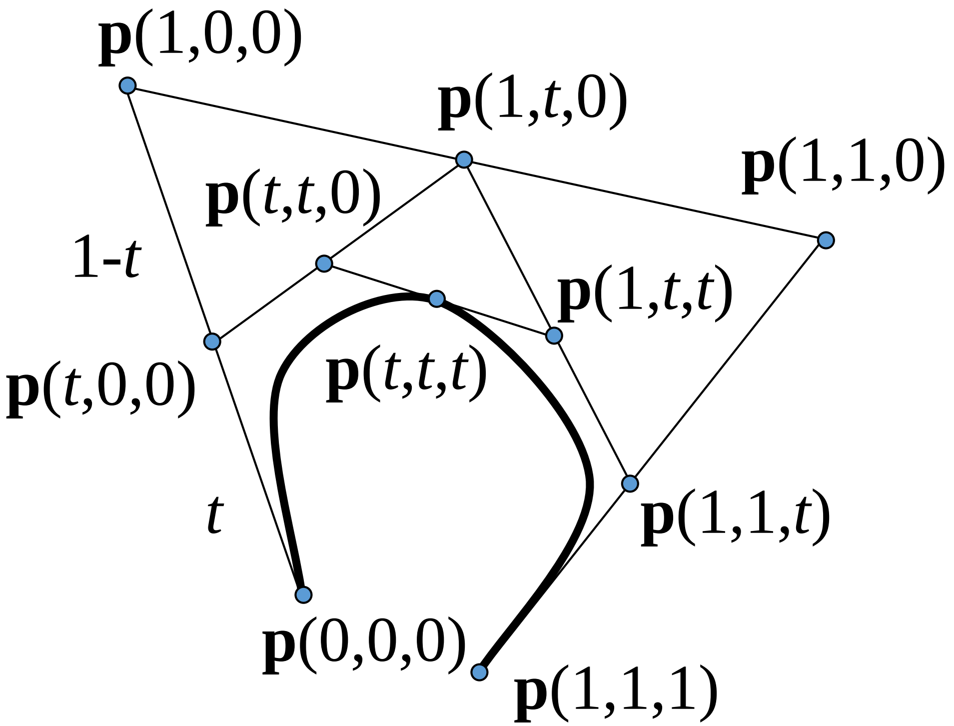
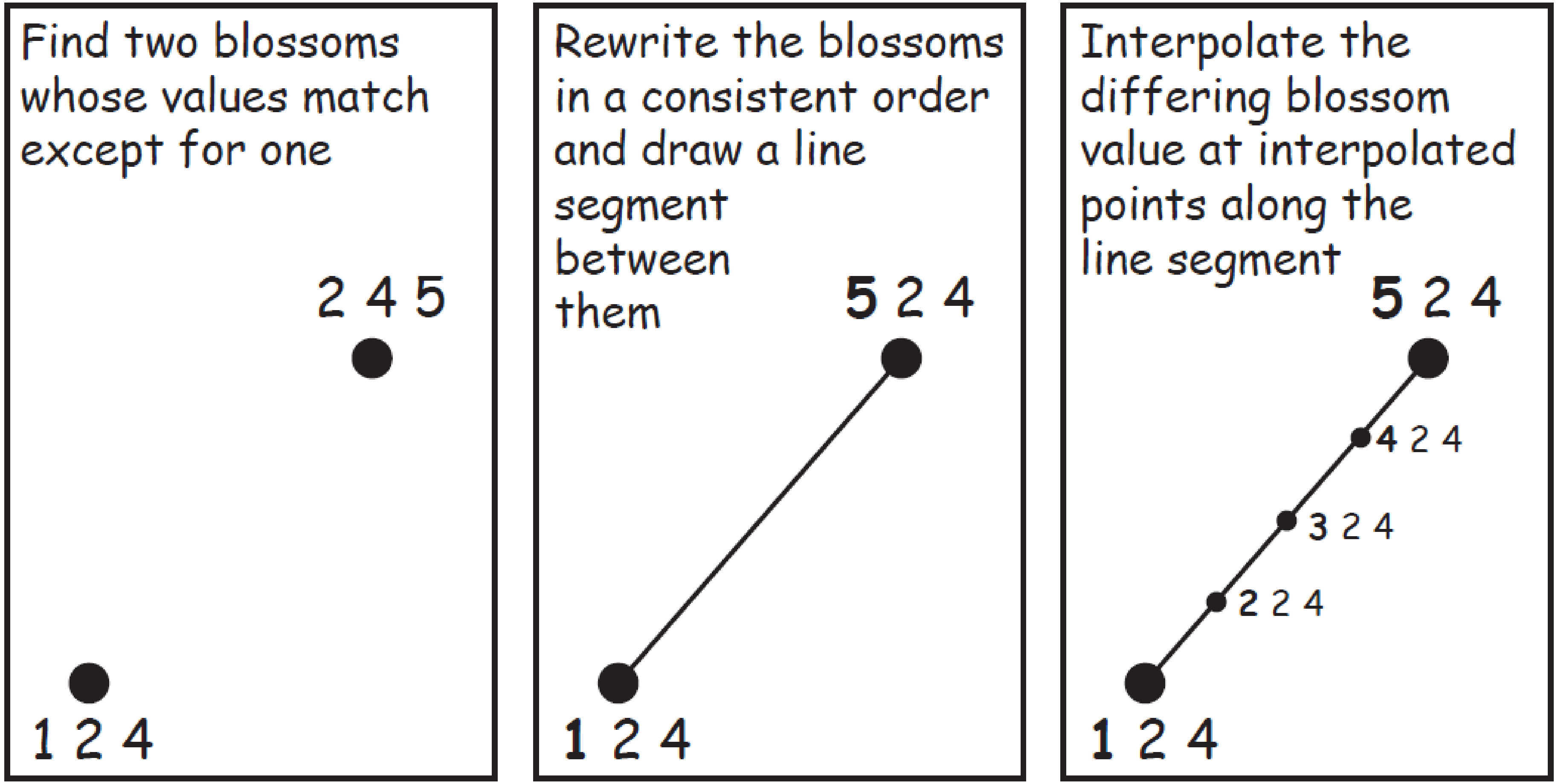
4. Winning the game

$\mathbf{p}(t,t,t)$

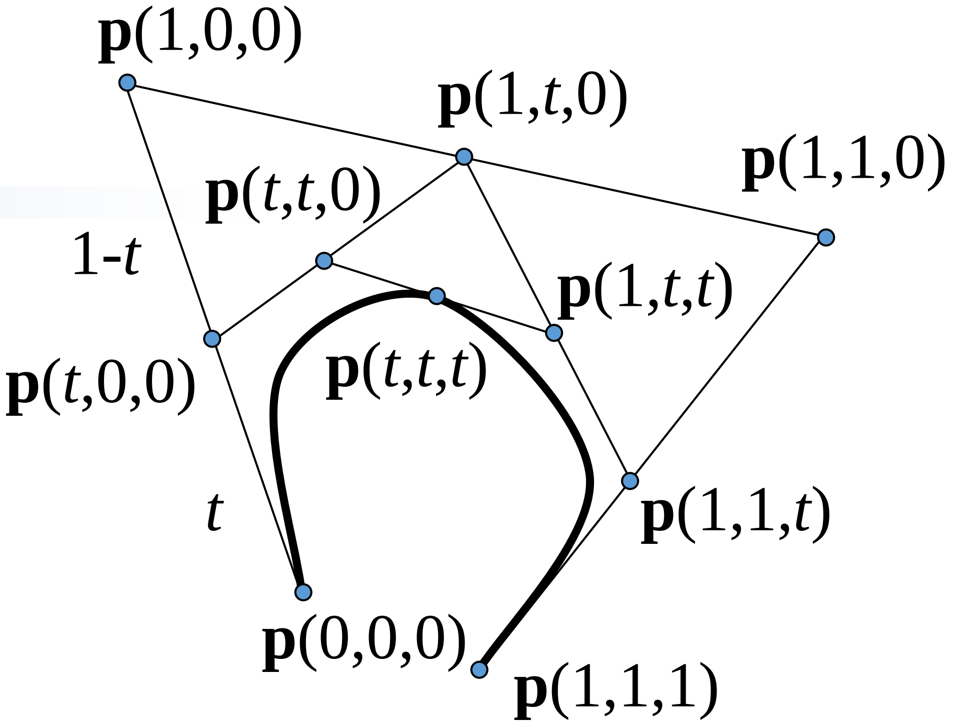


Placing Blossoms



Evaluation

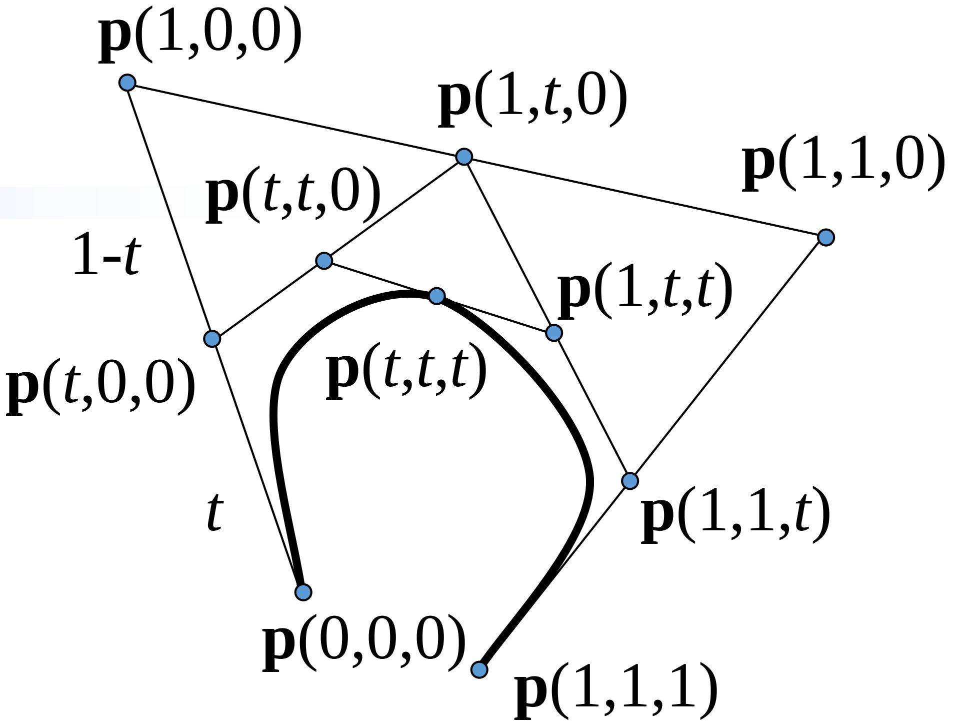
$\mathbf{p}(t) = \mathbf{p}(t,t,t)$



Evaluation

$\mathbf{p}(t) = \mathbf{p}(t,t,t)$

$= (1\text{-}t)\ \mathbf{p}(t,t,0) + t\ \mathbf{p}(t,t,1)$



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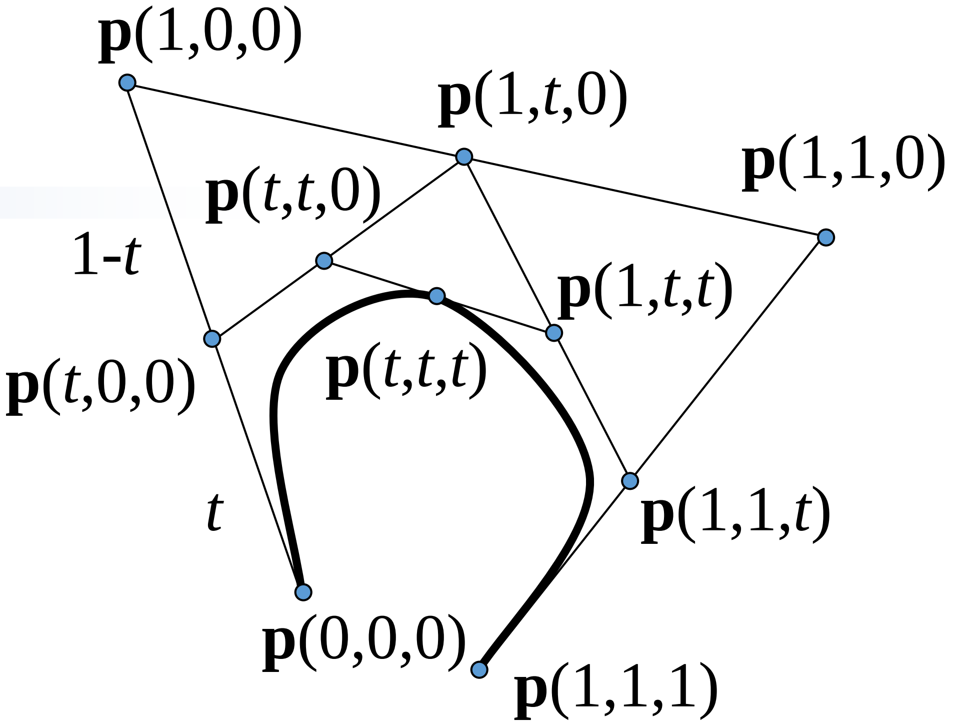

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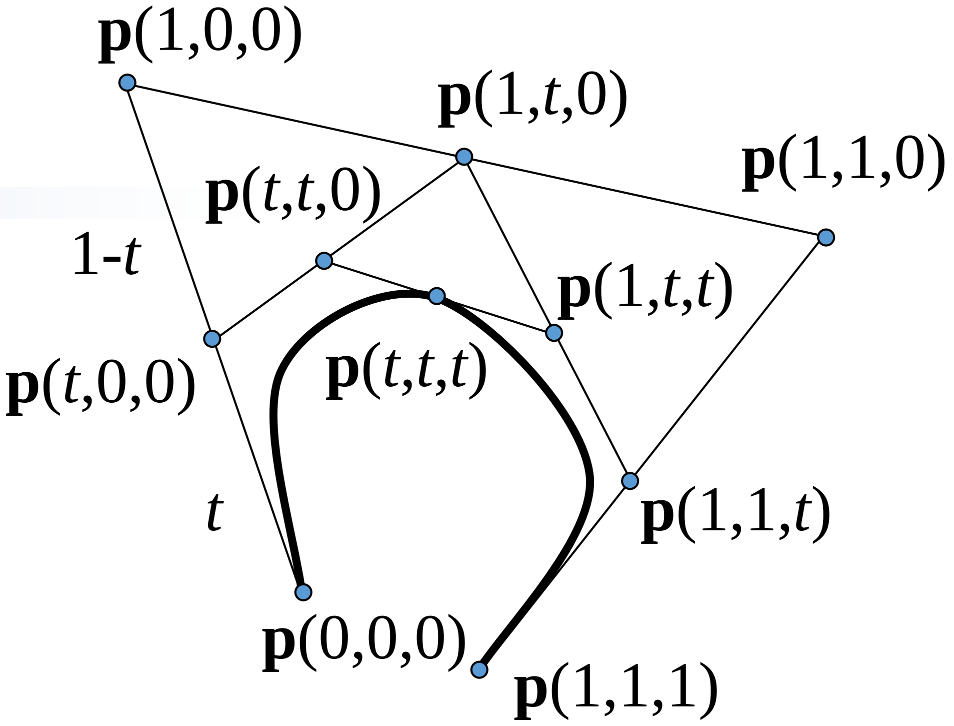

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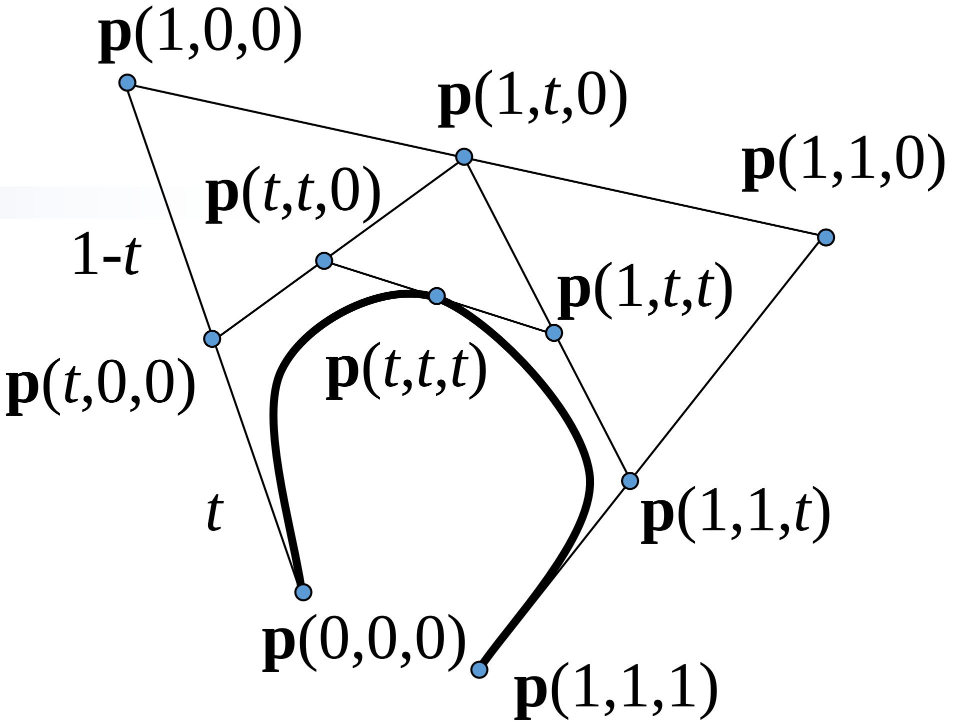

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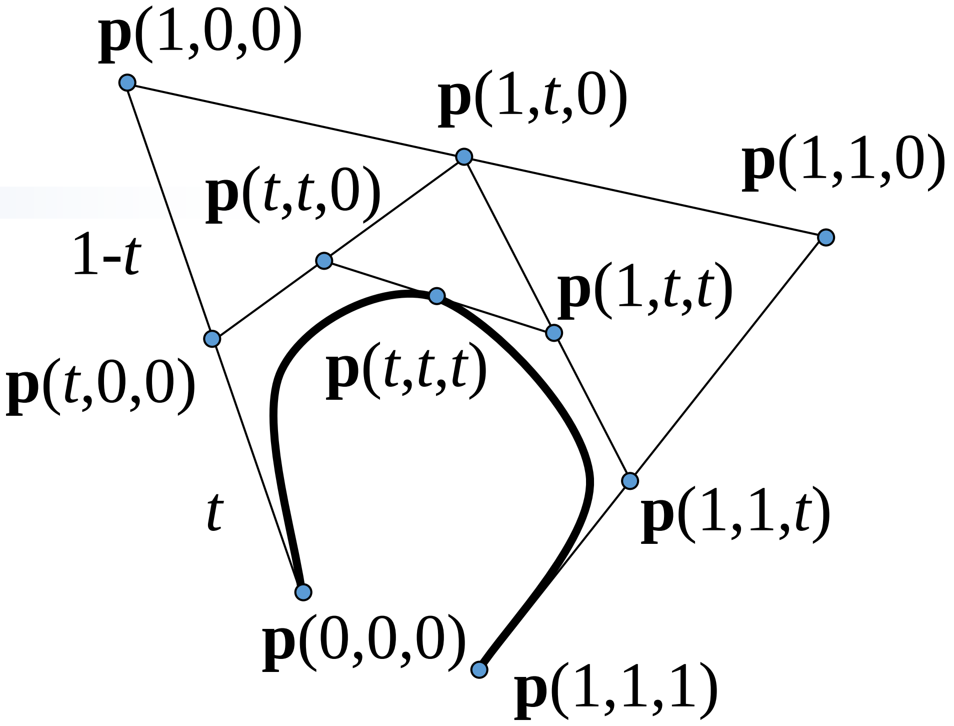

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$= (1\text{-}t)^3\ \mathbf{p}(0,0,0) + 3\ (1\text{-}t)^2\ t\ \mathbf{p}(0,0,1) + 3\ (1\text{-}t)\ t^2\ \mathbf{p}(0,1,1) + t^3\ \mathbf{p}(1,1,1)$