Hermite Curves

CS 418

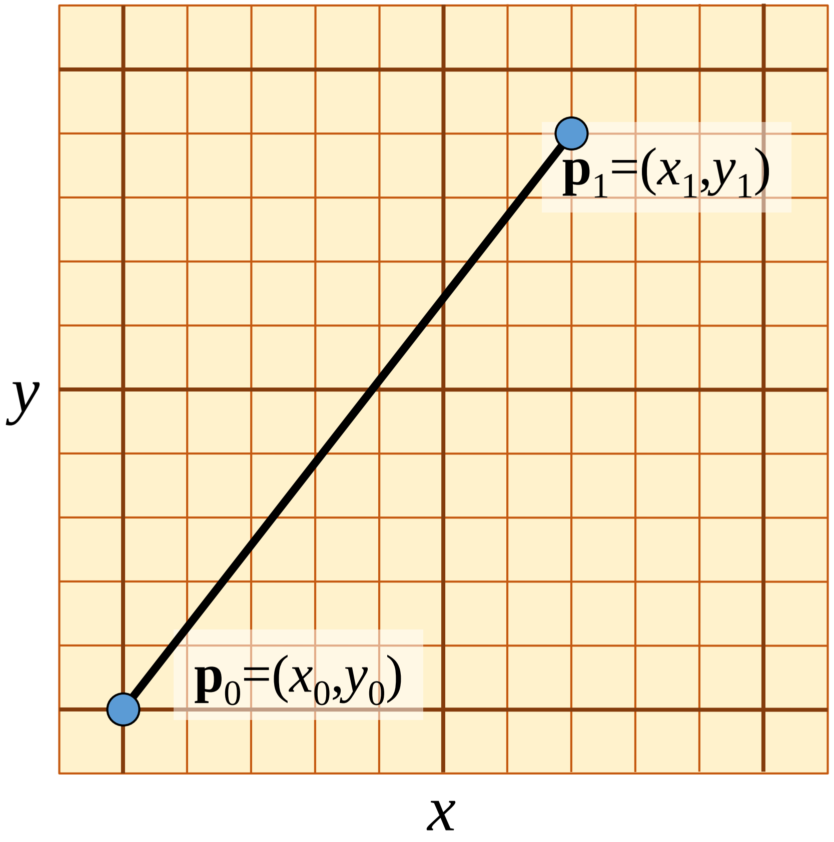
Interactive Computer Graphics

John C. Hart

Linear Interpolation

- Define a parametric function $\mathbf{p}(t)$

$$\mathbf{p}(0) = \mathbf{p}_0,\ \mathbf{p}(1) = \mathbf{p}_1$$



Linear Interpolation

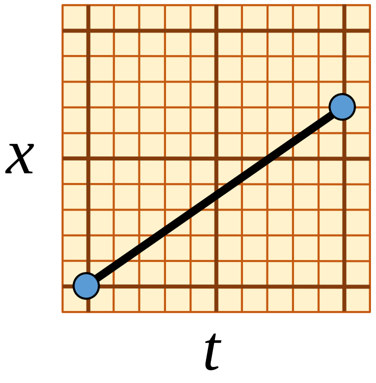
- Define a parametric function $\mathbf{p}(t)$

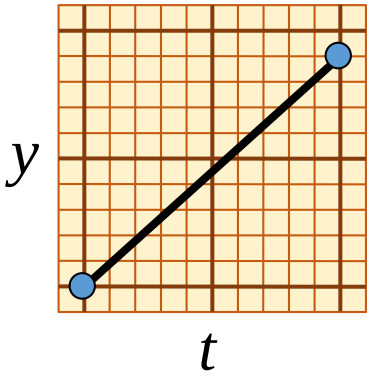
$$\mathbf{p}(0) = \mathbf{p}_0,\ \mathbf{p}(1) = \mathbf{p}_1$$

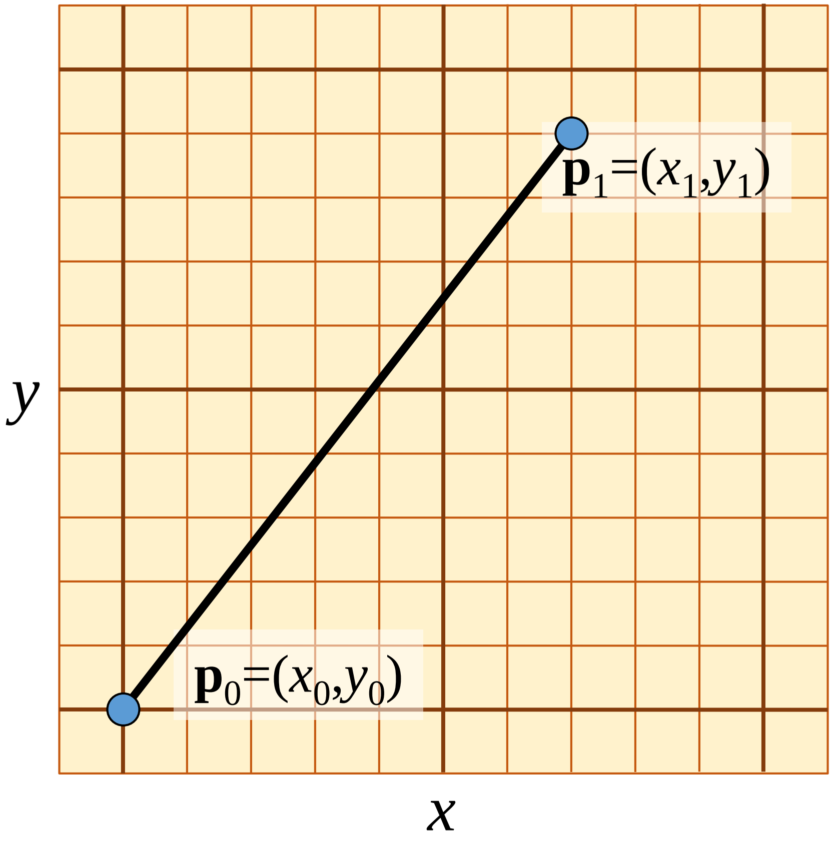
- Separate into coordinate functions

$$\mathbf{p}(t) = (x(t), y(t))$$

$$x(0) = x_0 \qquad y(0) = y_0,$$
$$x(1) = x_1 \qquad y(1) = y_1$$







Linear Interpolation

- Define a parametric function $\mathbf{p}(t)$

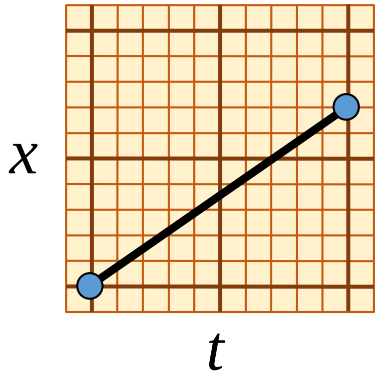
$$\mathbf{p}(0) = \mathbf{p}_0,\ \mathbf{p}(1) = \mathbf{p}_1$$

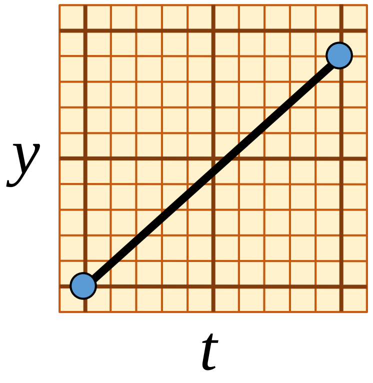
- Interpolate

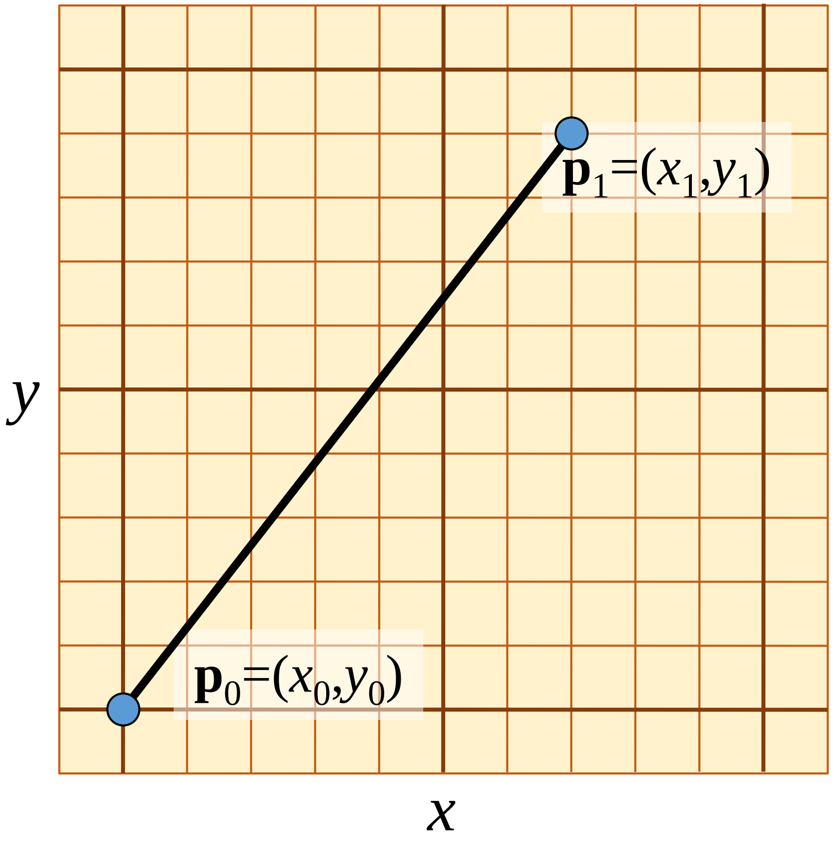
$$\mathbf{p}(t) = \mathbf{p}_0 + t\,(\mathbf{p}_1 - \mathbf{p}_0) = (1\text{-}t)\mathbf{p}_0 + t\,\mathbf{p}_1$$

$$x(t) = x_0 + t(x_1 - x_0) = (1\text{-}t)x_0 + t\,x_1$$

$$y(t) = y_0 + t(y_1 - y_0) = (1\text{-}t)y_0 + t\,y_1$$

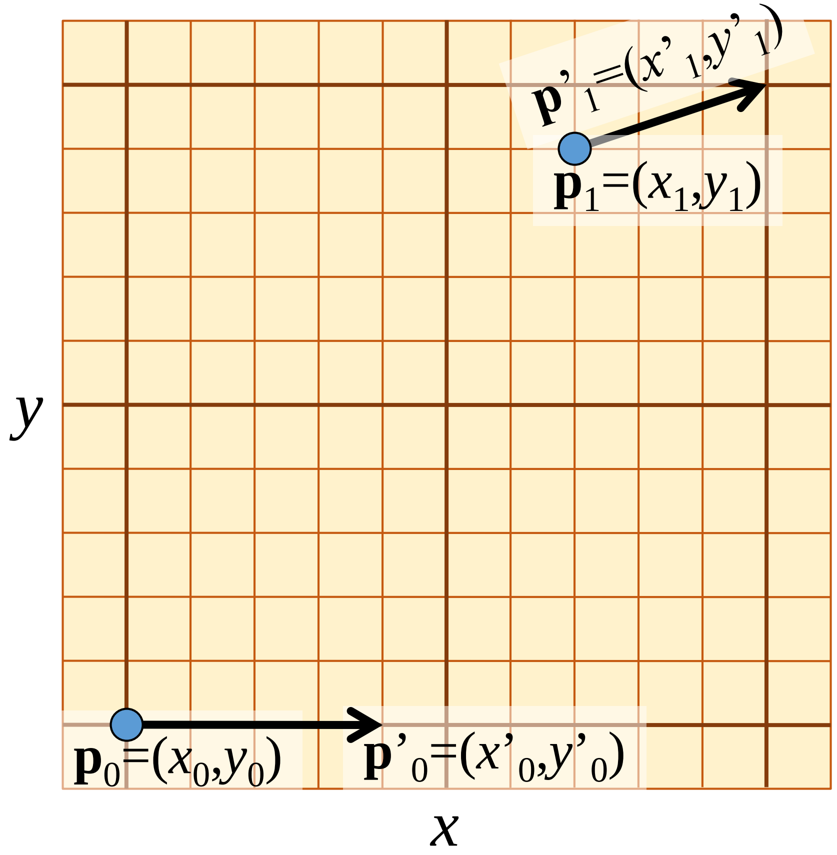






Hermite Interpolation

- From point $\mathbf{p}_0$ along $\mathbf{p}'_0$ to point $\mathbf{p}_1$ toward $\mathbf{p}'_1$



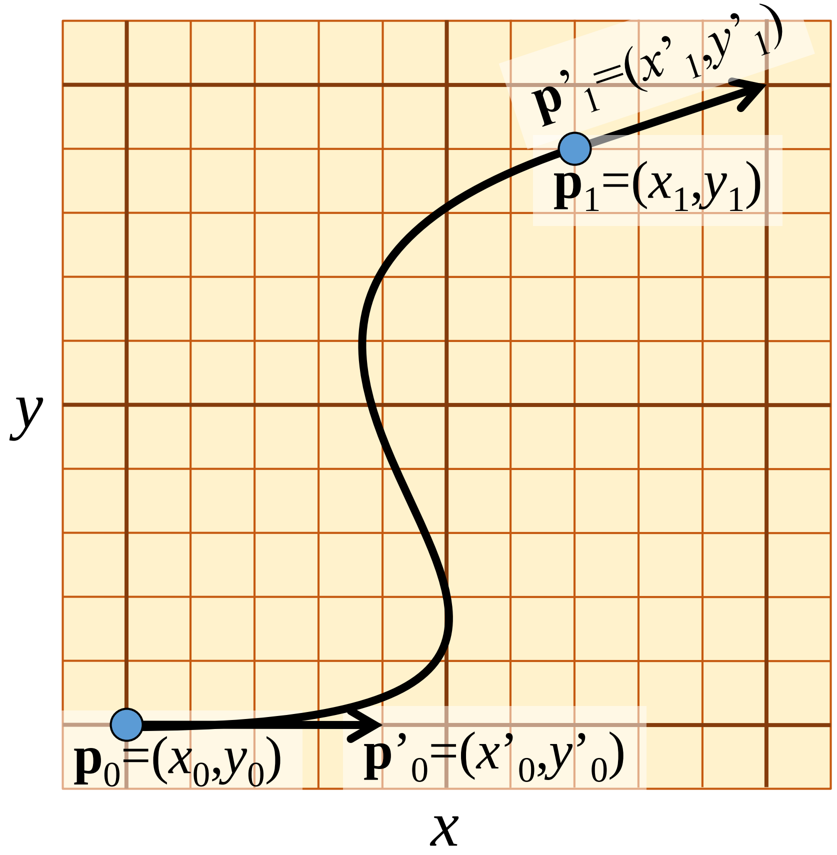
Hermite Interpolation

- From point $\mathbf{p}_0$ along $\mathbf{p}'_0$ to point $\mathbf{p}_1$ toward $\mathbf{p}'_1$

- Define a parametric function $\mathbf{p}(t)$

$$\mathbf{p}(0) = \mathbf{p}_0,\ \mathbf{p}(1) = \mathbf{p}_1$$

$$\mathbf{p}'(0) = \mathbf{p}'_0,\ \mathbf{p}'(1) = \mathbf{p}'_1$$



Hermite Interpolation

- Define a parametric function $\mathbf{p}(t)$

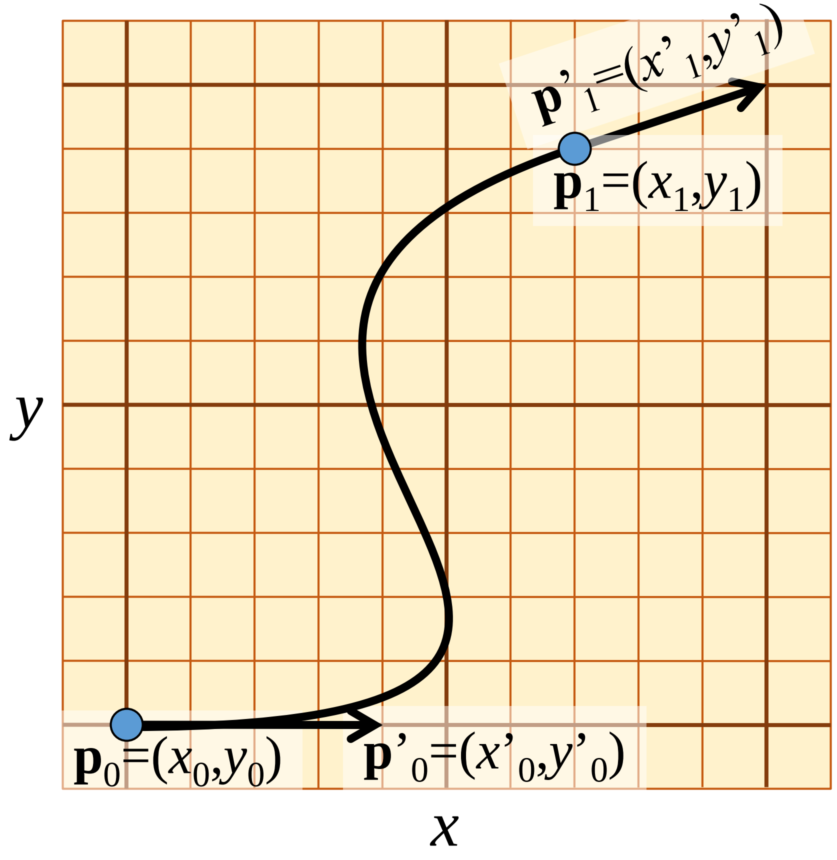
$$\mathbf{p}(0) = \mathbf{p}_0,\ \mathbf{p}(1) = \mathbf{p}_1$$

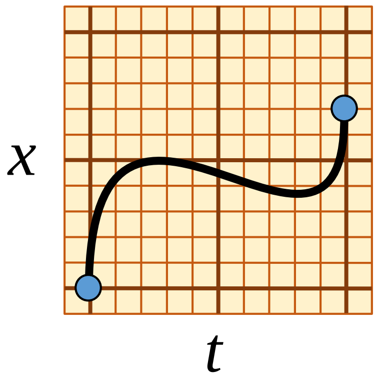
$$\mathbf{p'}(0) = \mathbf{p'}_0,\ \mathbf{p'}(1) = \mathbf{p'}_1$$

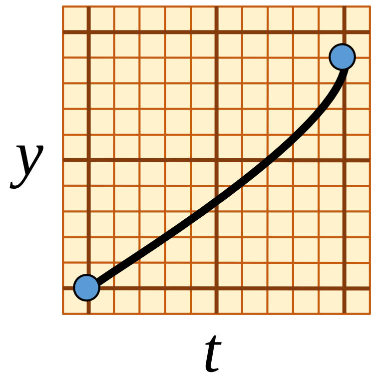
- Separate into coordinate functions

$$x(0) = x_0,\ x(1) = x_1$$

$$x'(0) = x'_0,\ x'(1) = x'_1$$





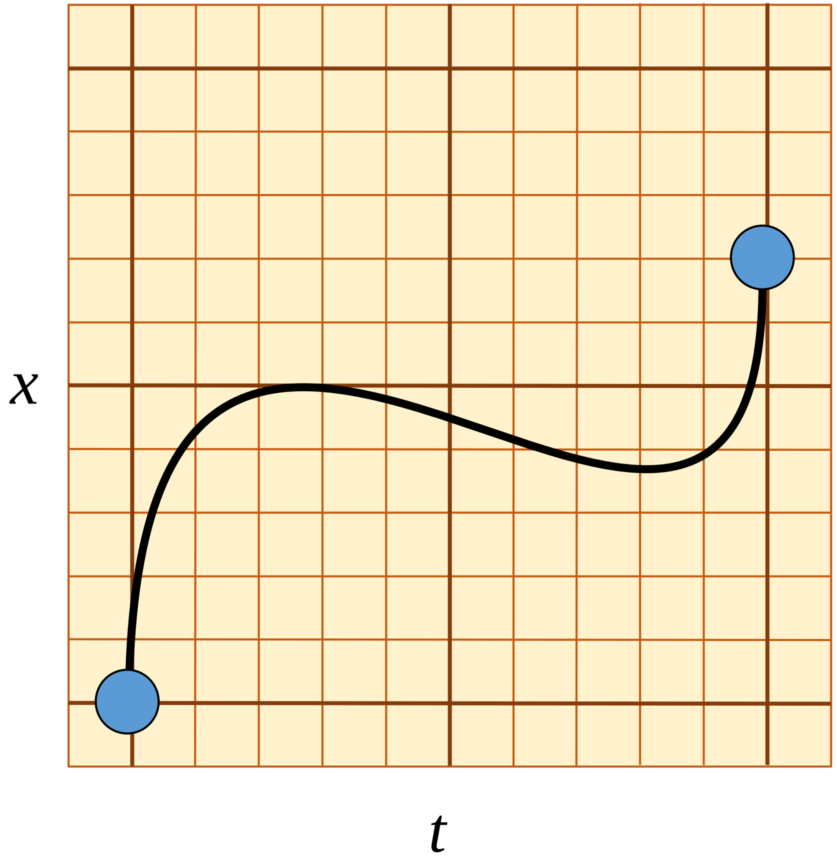


Hermite Interpolation

- Separate into coordinate functions

$$x(0) = x_0,\ x(1) = x_1$$

$$x'(0) = x'_0,\ x'(1) = x'_1$$



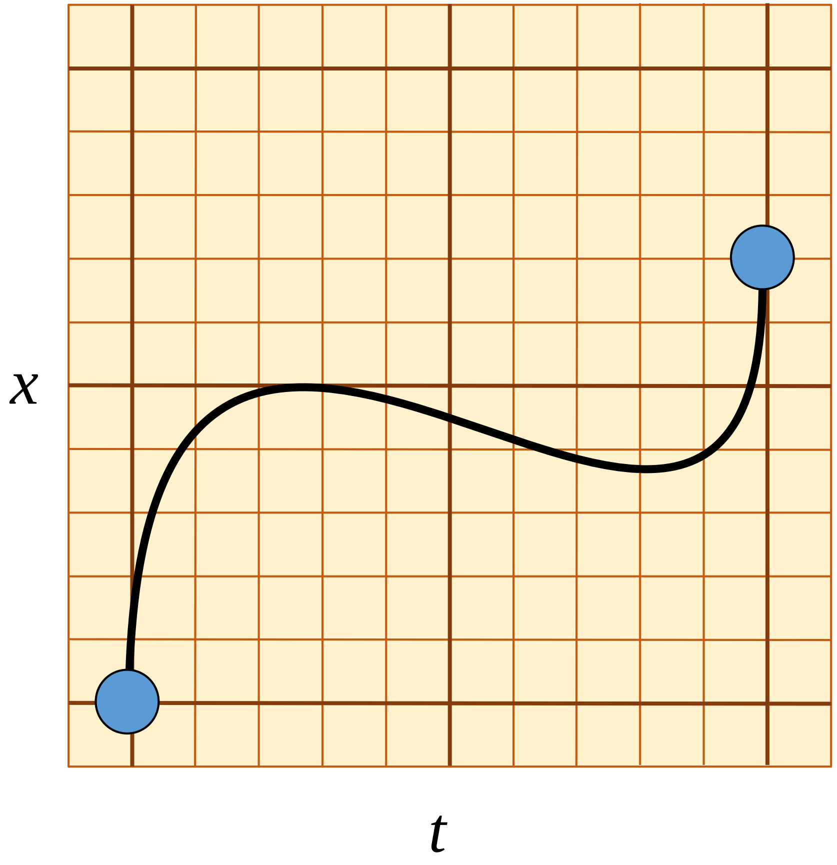
Hermite Interpolation

- Separate into coordinate functions

$$x(0) = x_0,\ x(1) = x_1$$
$$x'(0) = x'_0,\ x'(1) = x'_1$$

- Need cubic function

$$x(t) = At^3 + Bt^2 + Ct + D$$
$$x'(t) = 3At^2 + 2Bt + C$$



Hermite Interpolation

- Separate into coordinate functions

$$x(0) = x_0,\ x(1) = x_1$$

$$x'(0) = x'_0,\ x'(1) = x'_1$$

- Need cubic function

$$x(t) = At^3 + Bt^2 + Ct + D$$

$$x'(t) = 3At^2 + 2Bt + C$$

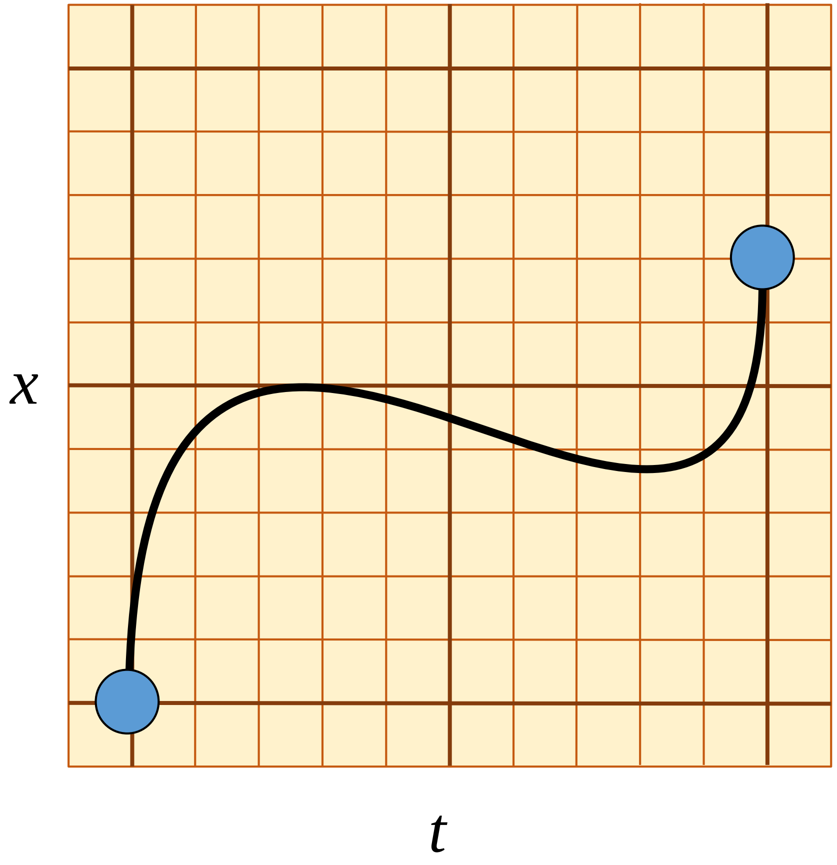
- Solve

$$A = 2x_0 - 2x_1 + x'_0 + x'_1$$

$$B = -3x_0 + 3x_1 - 2x'_0 - x'_1$$

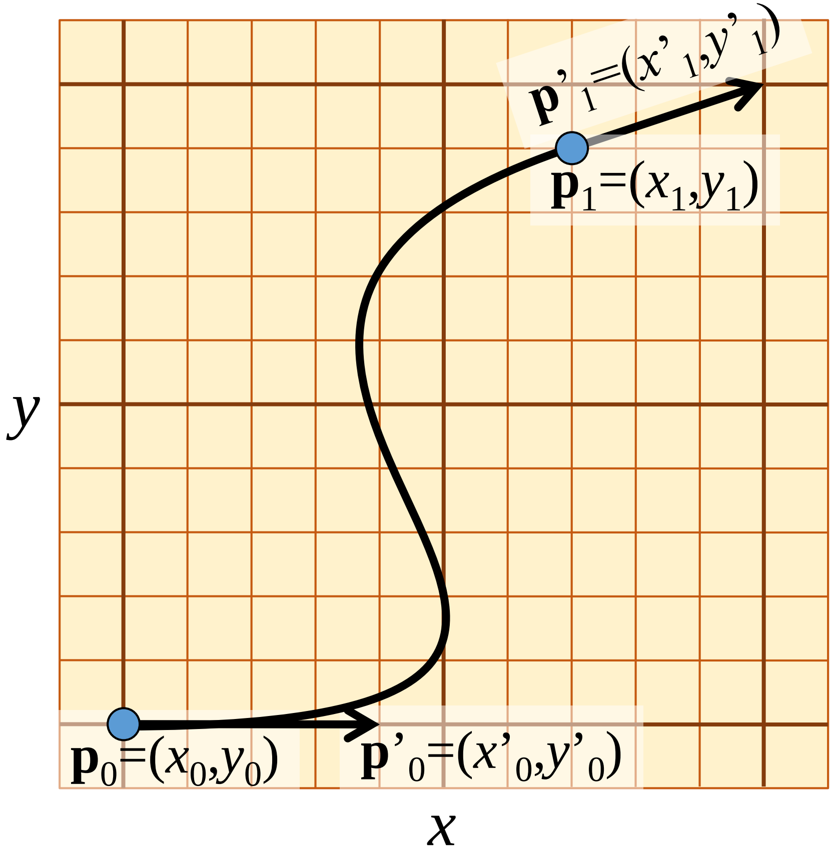
$$C = x'_0$$

$$D = x_0$$

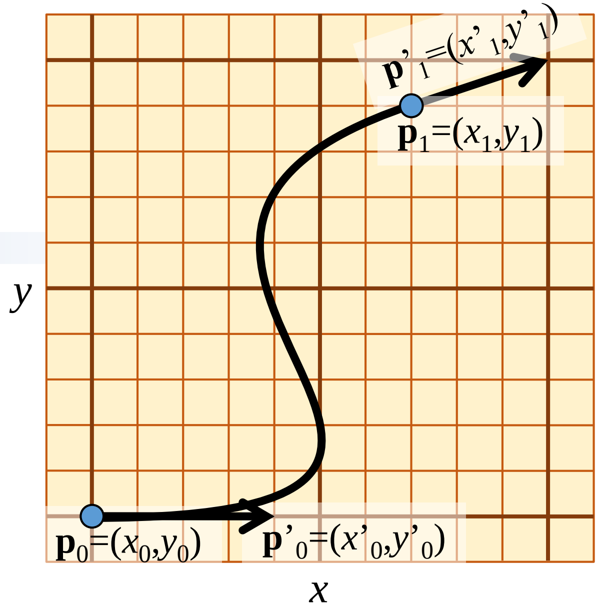


Hermite Interpolation

$$
\begin{aligned}
\mathbf{p}(t) = \quad & (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) && t^3 + \\
& (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) && t^2 + \\
& \qquad\qquad\qquad \mathbf{p}'_0 && t + \\
& \mathbf{p}_0 && (1)
\end{aligned}
$$



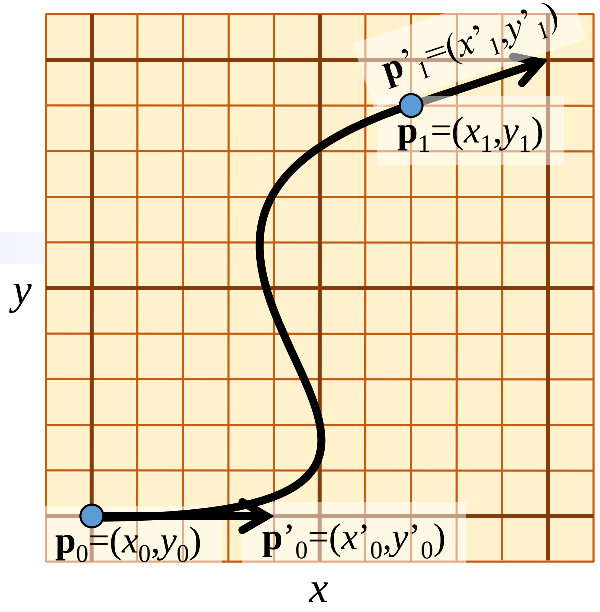
Hermite Interpolation



$$
\begin{aligned}
\mathbf{p}(t) = \quad & (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) && t^3 + \\
& (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) && t^2 + \\
& \mathbf{p}'_0 && t + \\
& \mathbf{p}_0 && (1)
\end{aligned}
$$

$$
\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}'_0 \\ \mathbf{p}'_1 \end{bmatrix}
$$

Hermite Interpolation



$$
\begin{aligned}
\mathbf{p}(t) = \; & (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) && t^3 + \\
& (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) && t^2 + \\
& \mathbf{p}'_0 && t + \\
& \mathbf{p}_0 && (1)
\end{aligned}
$$

$$
\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}'_0 \\ \mathbf{p}'_1 \end{bmatrix}
$$

$$
\begin{aligned}
\mathbf{p}(\mathrm{t}) = \; & (2t^3 - 3t^2 + 1) && \mathbf{p}_0 + \\
& (-2t^3 + 3t^2) && \mathbf{p}_1 + \\
& (t^3 - 2t^2 + t) && \mathbf{p}'_0 + \\
& (t^3 - t^2) && \mathbf{p}'_1
\end{aligned}
$$

Hermite Interpolation

$$\begin{aligned}\mathbf{p}(t) = \quad & (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) && t^3 + \\ & (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) && t^2 + \\ & \qquad\qquad \mathbf{p}'_0 && t + \\ & \mathbf{p}_0 && (1)\end{aligned}$$

$$\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}'_0 \\ \mathbf{p}'_1 \end{bmatrix}$$

$$\begin{aligned}\mathbf{p}(\mathrm{t}) = \quad & (2t^3 - 3t^2 + 1) && \mathbf{p}_0 + \\ & (-2t^3 + 3t^2) && \mathbf{p}_1 + \\ & (t^3 - 2t^2 + t) && \mathbf{p}'_0 + \\ & (t^3 - t^2) && \mathbf{p}'_1\end{aligned}$$