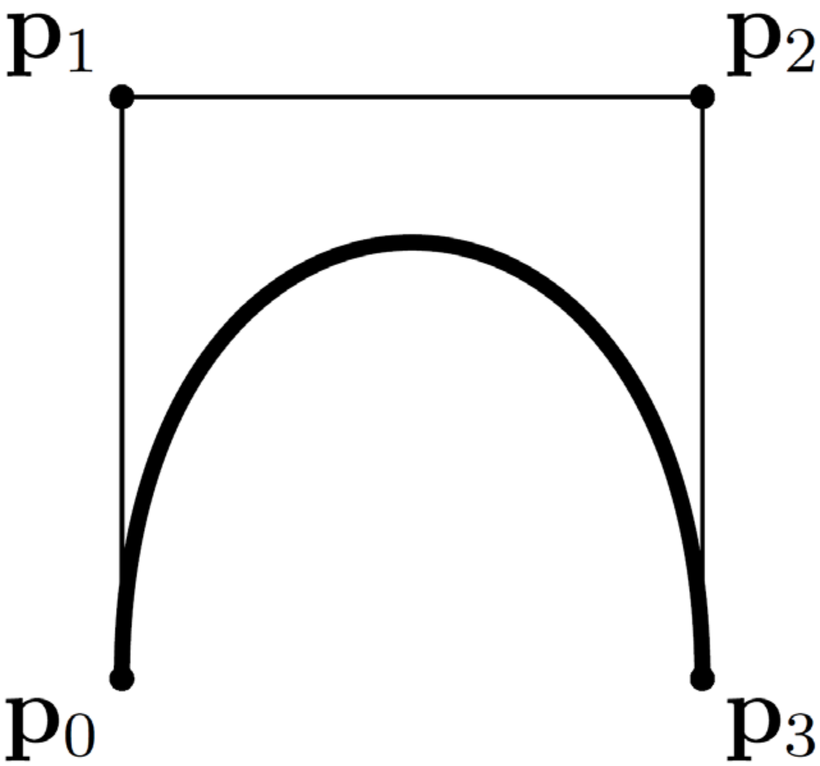
B-Splines

CS 418

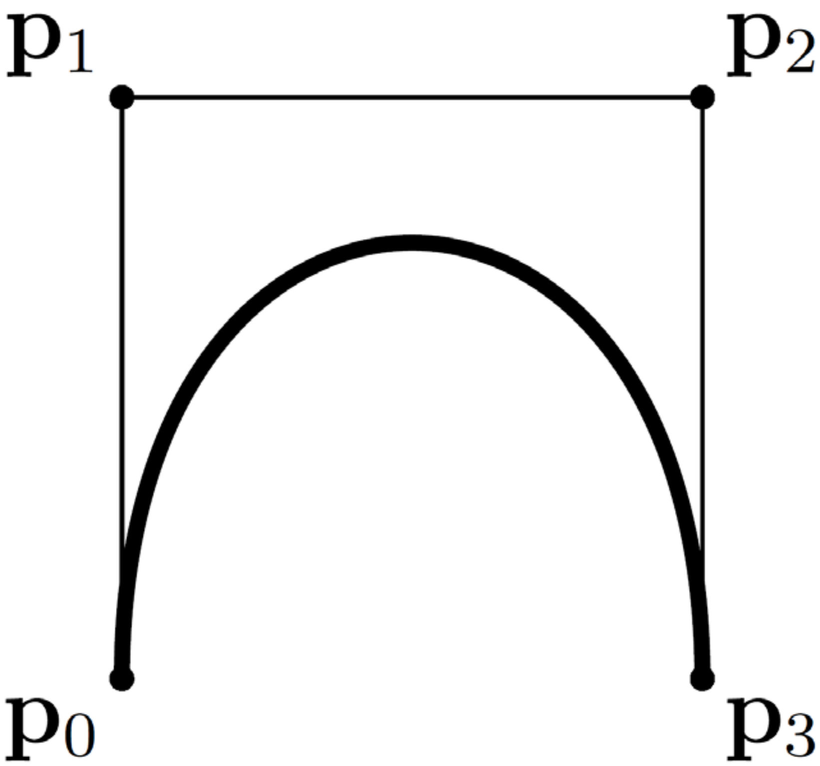
Interactive Computer Graphics

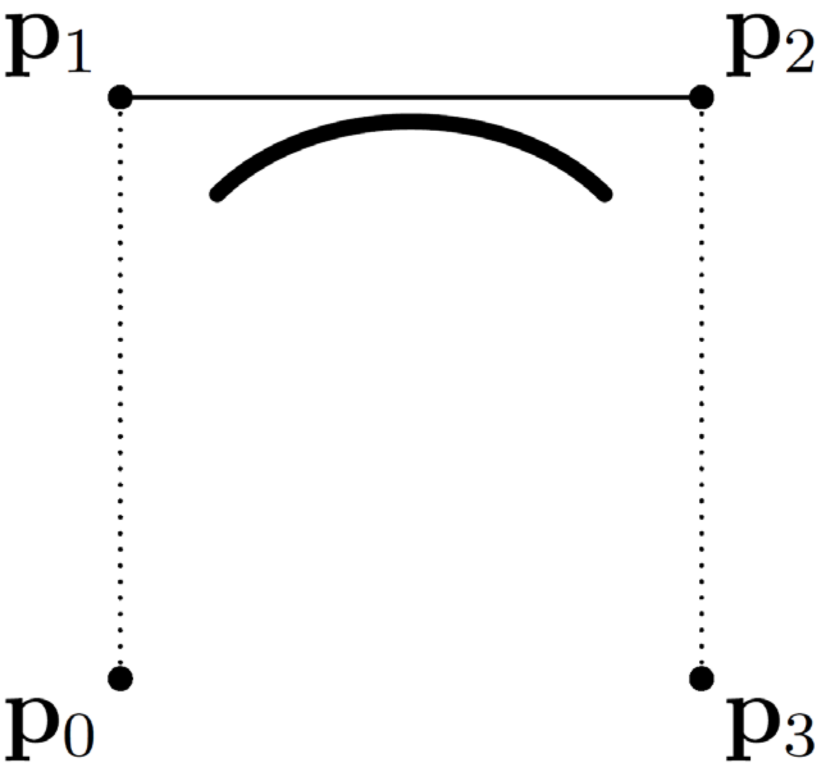
John C. Hart

Bezier Curve

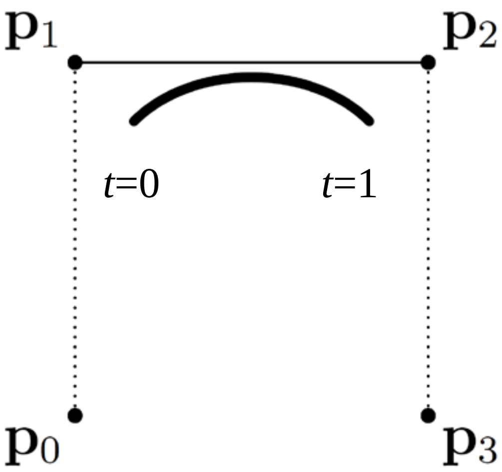


Bezier Curve v. B-Spline



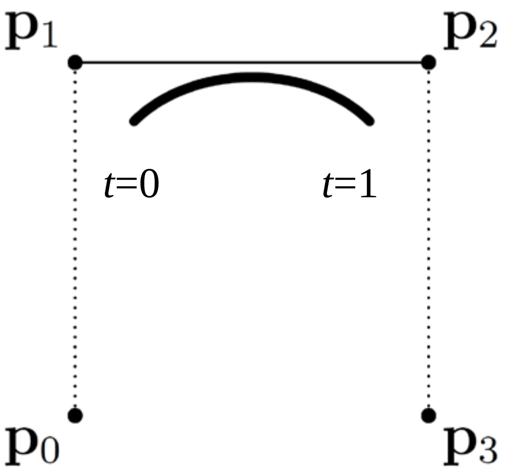


B-Spline Segment



$$
\begin{aligned}
\mathbf{p}(t) = {} & (-1/6\,\mathbf{p}_0 + 1/2\,\mathbf{p}_1 - 1/2\,\mathbf{p}_2 + 1/6\,\mathbf{p}_3)\,t^3 + \\
& (\;1/2\,\mathbf{p}_0 - \mathbf{p}_1 + 1/2\,\mathbf{p}_2)\,t^2 + \\
& (-1/2\,\mathbf{p}_0 + 1/2\,\mathbf{p}_2)\,t + \\
& \;1/6\,\mathbf{p}_0 + 2/3\,\mathbf{p}_1 + 1/6\,\mathbf{p}_2
\end{aligned}
$$

B-Spline Segment

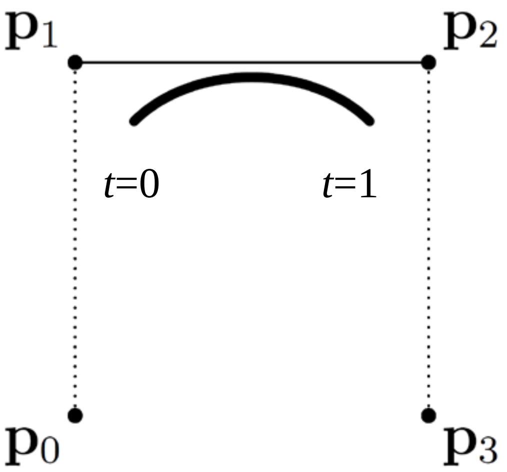


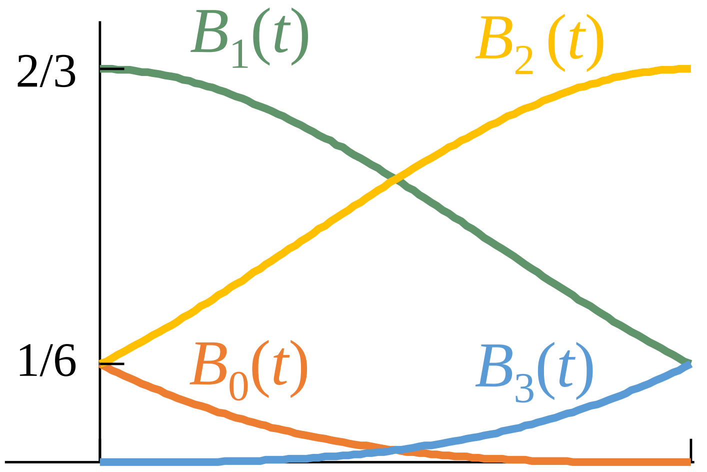
$$\begin{aligned}\mathbf{p}(t) = &(-1/6\,\mathbf{p}_0 + 1/2\,\mathbf{p}_1 - 1/2\,\mathbf{p}_2 + 1/6\,\mathbf{p}_3)\,t^3 + \\ &(\;\;1/2\,\mathbf{p}_0 - \mathbf{p}_1 + 1/2\,\mathbf{p}_2)\,t^2 + \\ &(-1/2\,\mathbf{p}_0 + 1/2\,\mathbf{p}_2)\,t + \\ &\;\;1/6\,\mathbf{p}_0 + 2/3\,\mathbf{p}_1 + 1/6\,\mathbf{p}_2\end{aligned}$$

$$\begin{aligned}\mathbf{p}(t) = &(-1/6t^3 + 1/2t^2 - 1/2t + 1/6)\mathbf{p}_0 + \\ &(\;\;1/2t^3 - t^2 + 2/3)\mathbf{p}_1 + \\ &(-1/2t^3 + 1/2t^2 + 1/2t + 1/6)\mathbf{p}_2 + \\ &(\;\;1/6t^3)\mathbf{p}_3\end{aligned}$$

$$= B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

B-Spline Basis

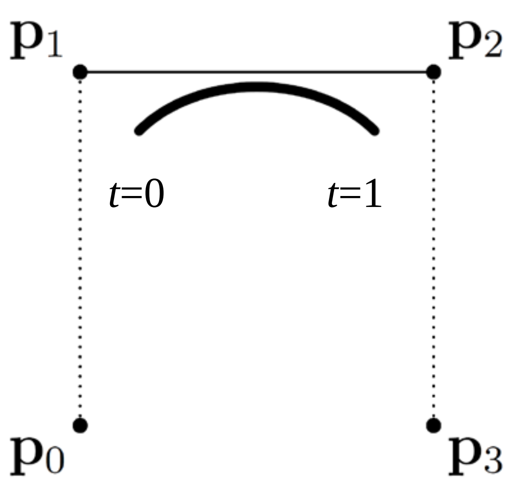


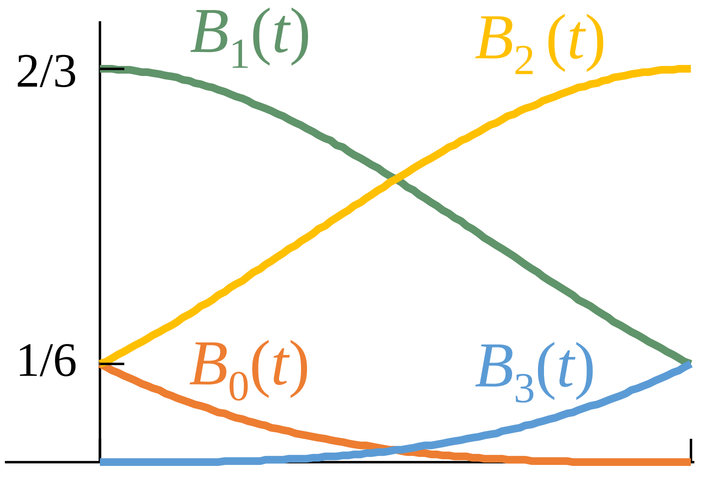


$$
\begin{aligned}
\mathbf{p}(t) = {} & (-1/6t^3 + 1/2t^2 - 1/2t + 1/6)\mathbf{p}_0 + \\
& (\ 1/2t^3 - t^2 + 2/3)\mathbf{p}_1 + \\
& (-1/2t^3 + 1/2t^2 + 1/2t + 1/6)\mathbf{p}_2 + \\
& (\ 1/6t^3)\mathbf{p}_3
\end{aligned}
$$

$$= B_0(t)\mathbf{p}_0+B_1(t)\mathbf{p}_1+B_2(t)\mathbf{p}_2+B_3(t)\mathbf{p}_3$$

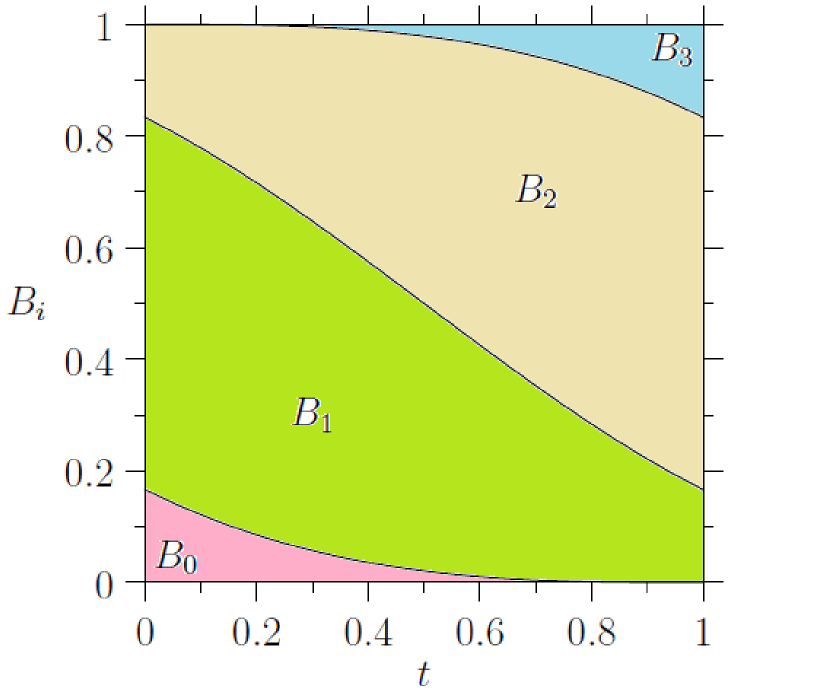
Partition of Unity





$$
\begin{aligned}
\mathbf{p}(t) = {} & (-1/6t^3 + 1/2t^2 - 1/2t + 1/6)\mathbf{p}_0 + \\
& (1/2t^3 - t^2 + 2/3)\mathbf{p}_1 + \\
& (-1/2t^3 + 1/2t^2 + 1/2t + 1/6)\mathbf{p}_2 + \\
& (1/6t^3)\mathbf{p}_3
\end{aligned}
$$

$$= B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

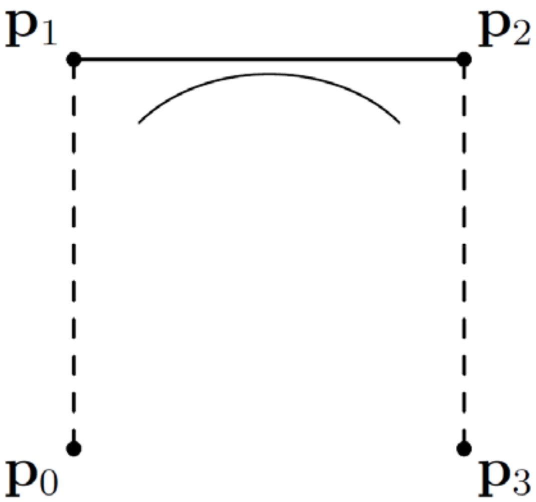


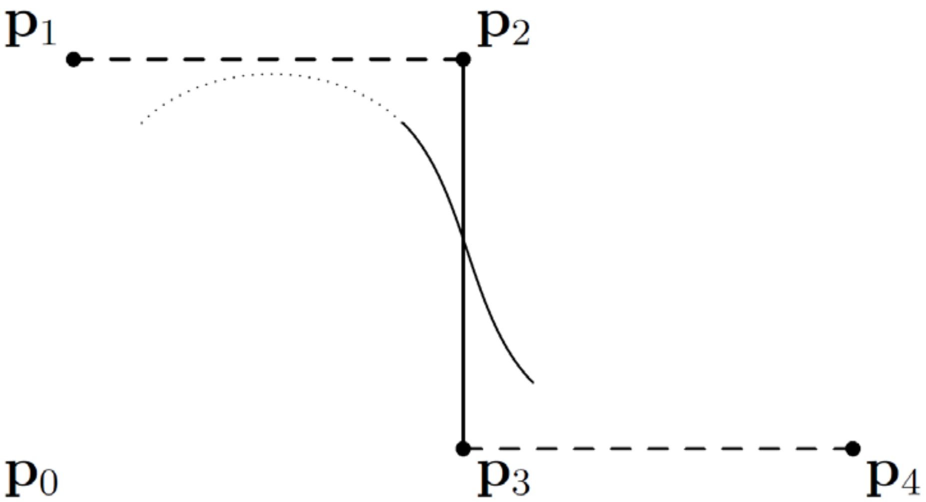
$$B_0(t) + B_1(t) + B_2(t) + B_3(t) = 1$$

$$B_i(t) \geq 0$$

B-spline curve always in convex hull of control points

Continuity

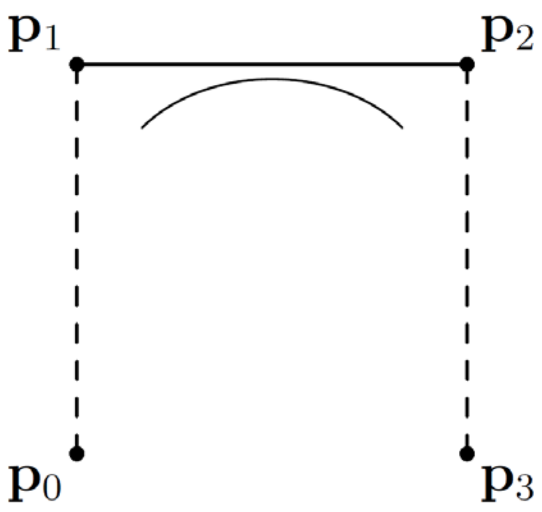


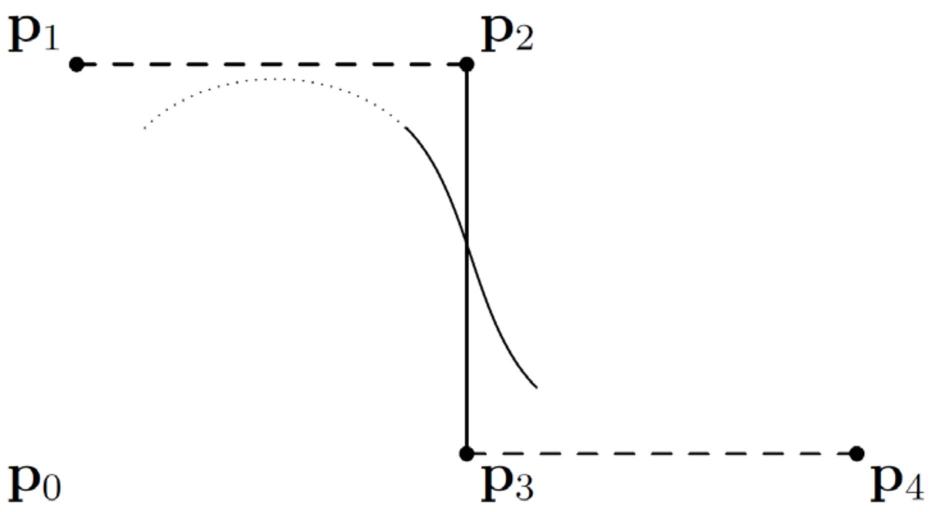


$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

Continuity



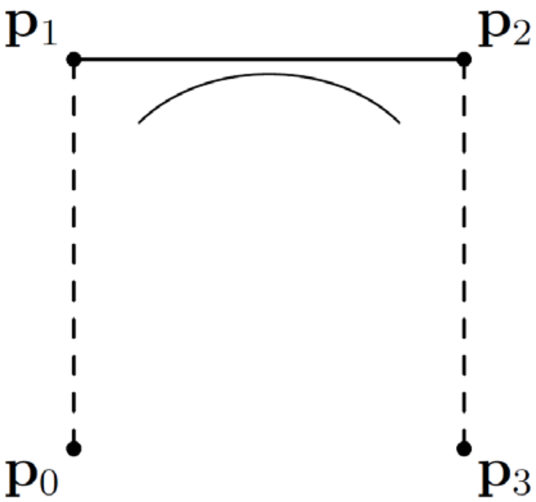


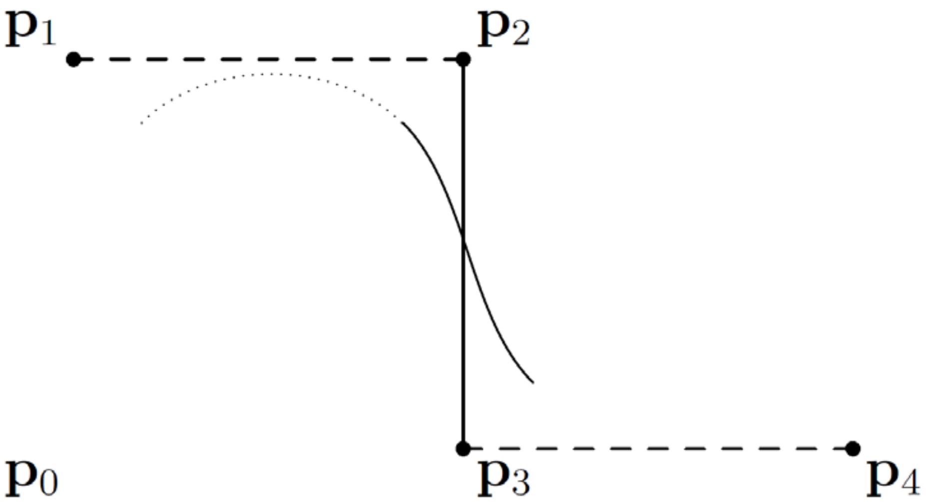
$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

$$\mathbf{p}_{0123}(1) = \mathbf{p}_{0123}(0)$$

Continuity



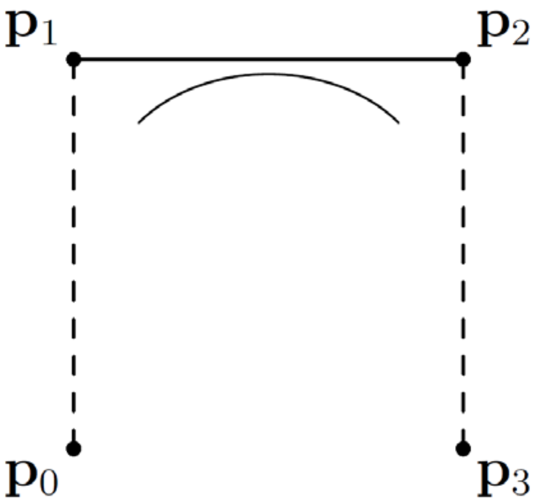


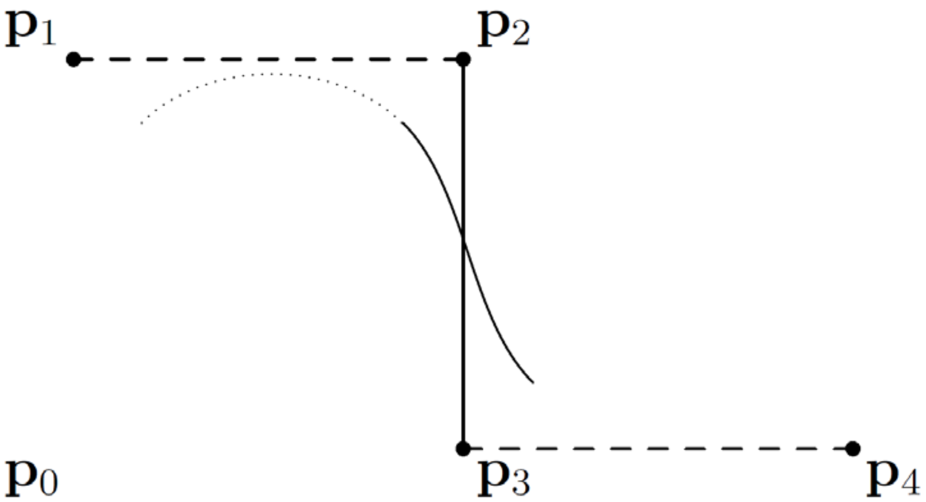
$$\begin{aligned}\mathbf{p}_{0123}(t) = {} & B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + \\ & B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3\end{aligned}$$

$$\begin{aligned}\mathbf{p}_{1234}(t) = {} & B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + \\ & B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4\end{aligned}$$

$$\mathbf{p}_{0123}(1) = \mathbf{p}_{0123}(0)$$
$$\mathbf{p'}_{0123}(1) = \mathbf{p'}_{0123}(0)$$

Continuity



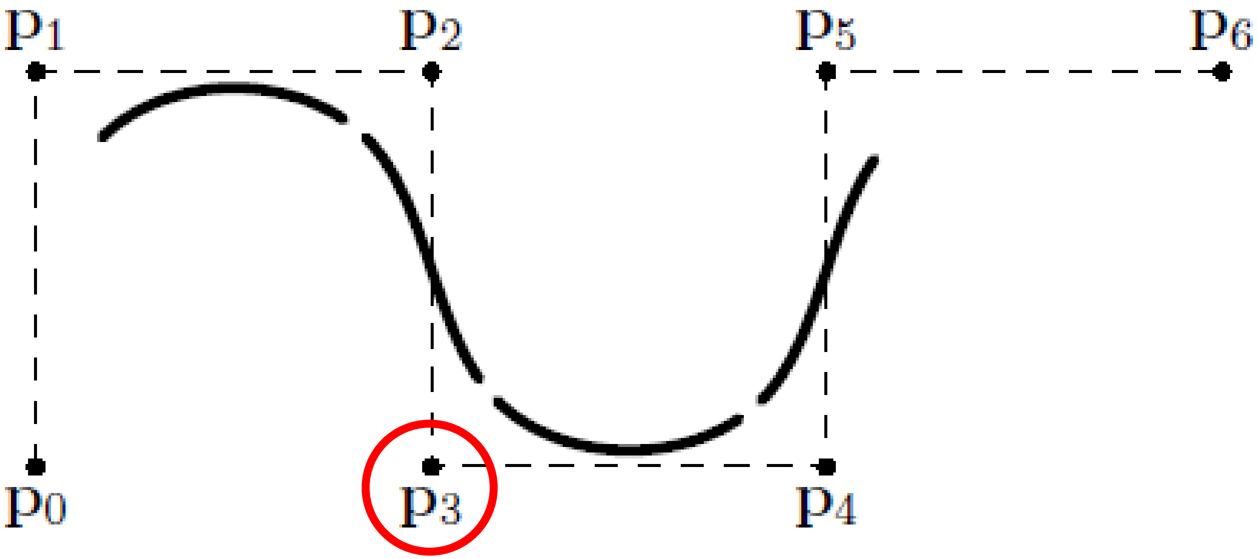


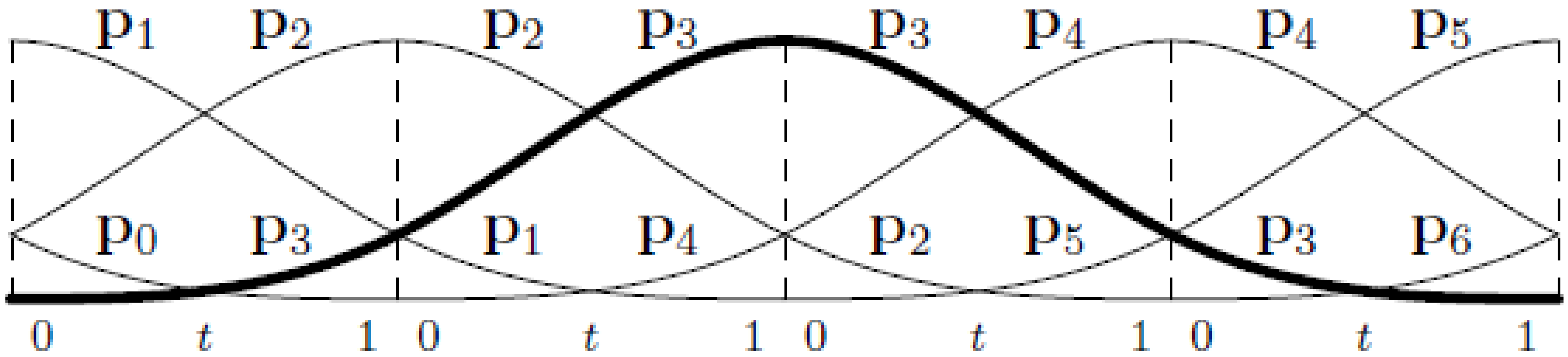
$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

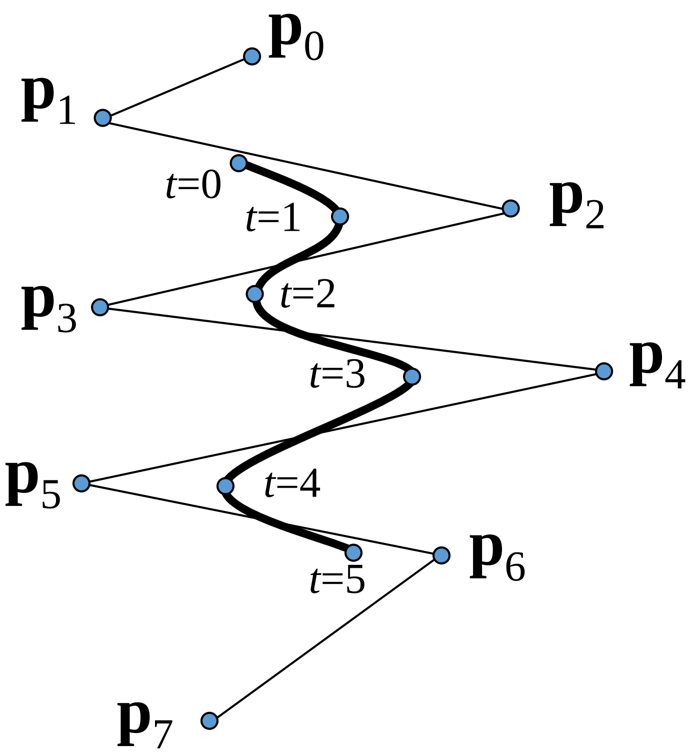
$$\mathbf{p}_{0123}(1) = \mathbf{p}_{0123}(0)$$
$$\mathbf{p}'_{0123}(1) = \mathbf{p}'_{0123}(0)$$
$$\mathbf{p}''_{0123}(1) = \mathbf{p}''_{0123}(0)$$

Joining B-Splines

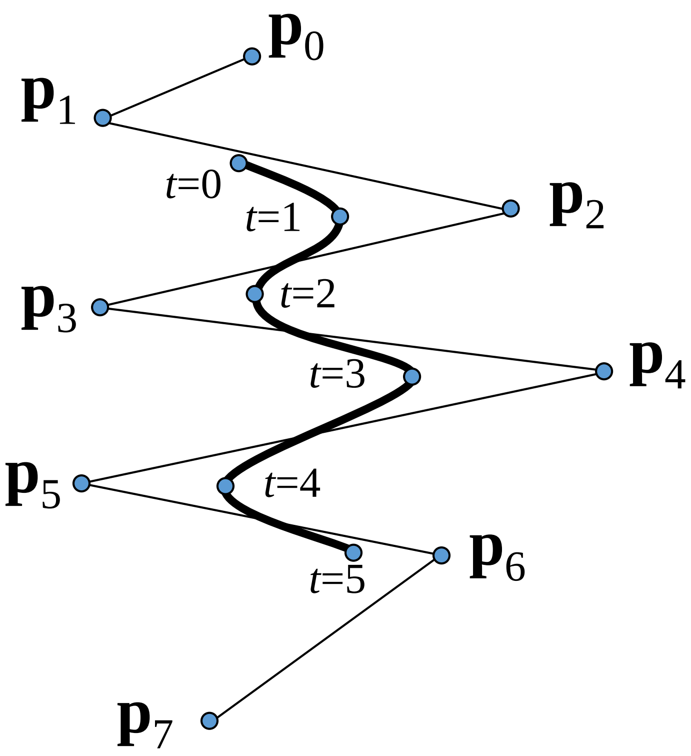


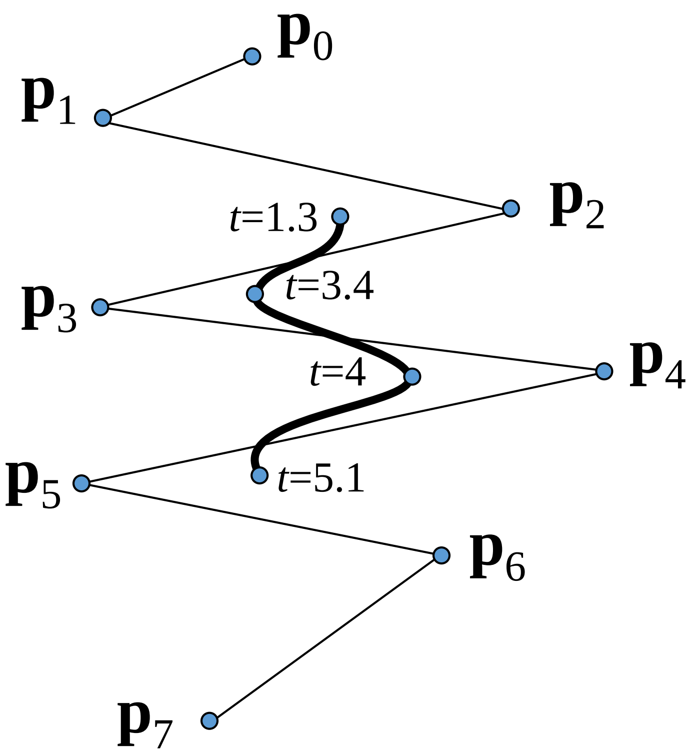


Uniform B-Splines



Uniform v. Non-Uniform B-Splines





knot vector: [0, .5, 1.3, 3.4, 4, 5.1, 6, 7]