More B-Spline Blossoms

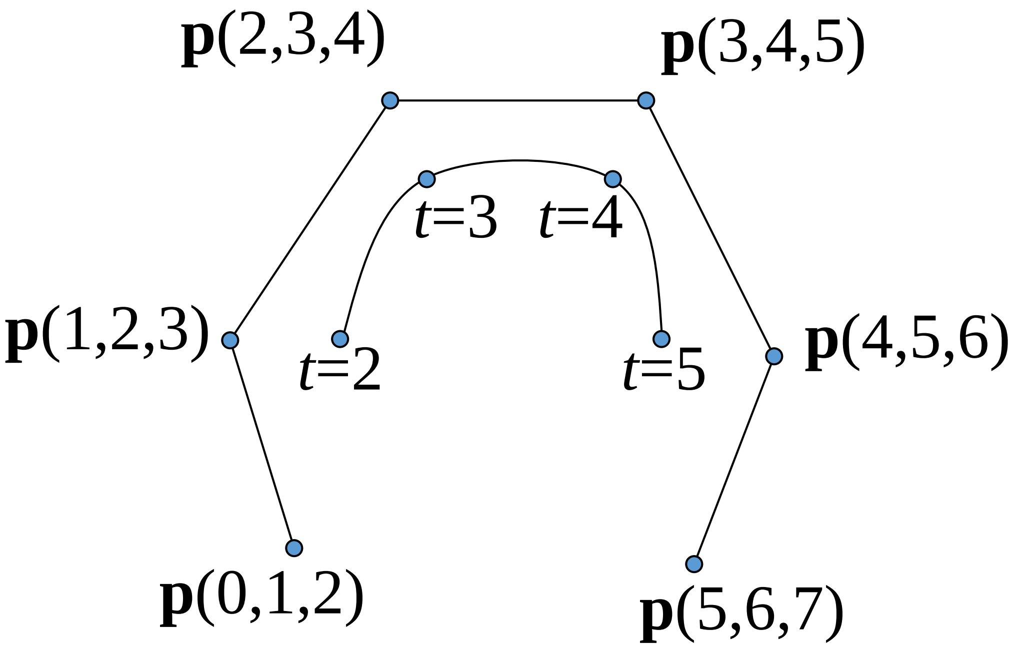
CS 418

Interactive Computer Graphics

John C. Hart

Knot Insertion

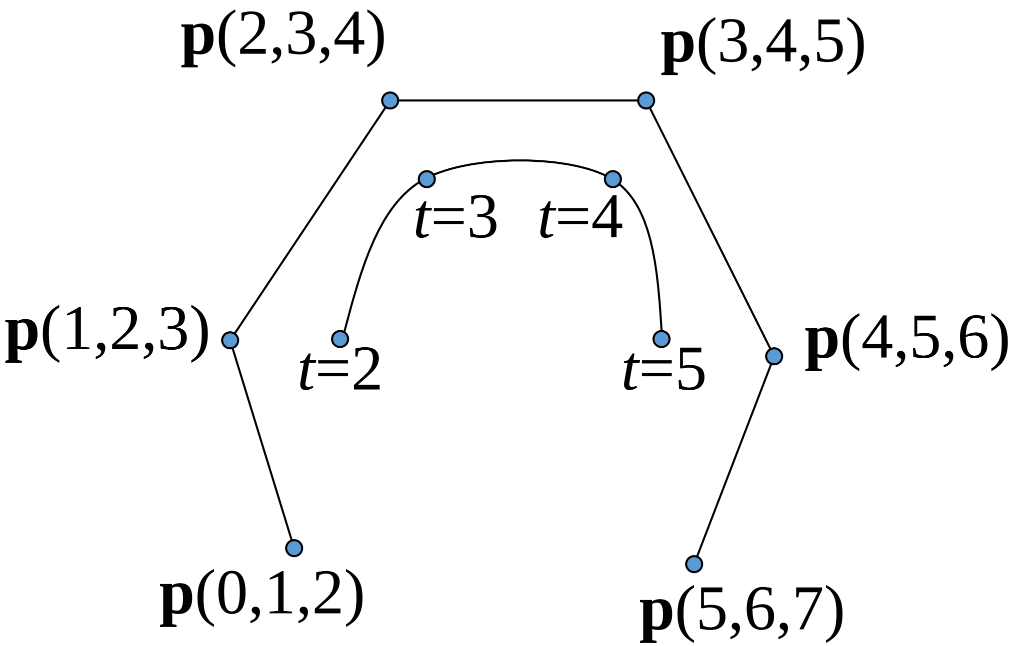
- Suppose we want to add a knot at t = 3.5



knot vector: [0 1 2 3 4 5 6 7]

Knot Insertion

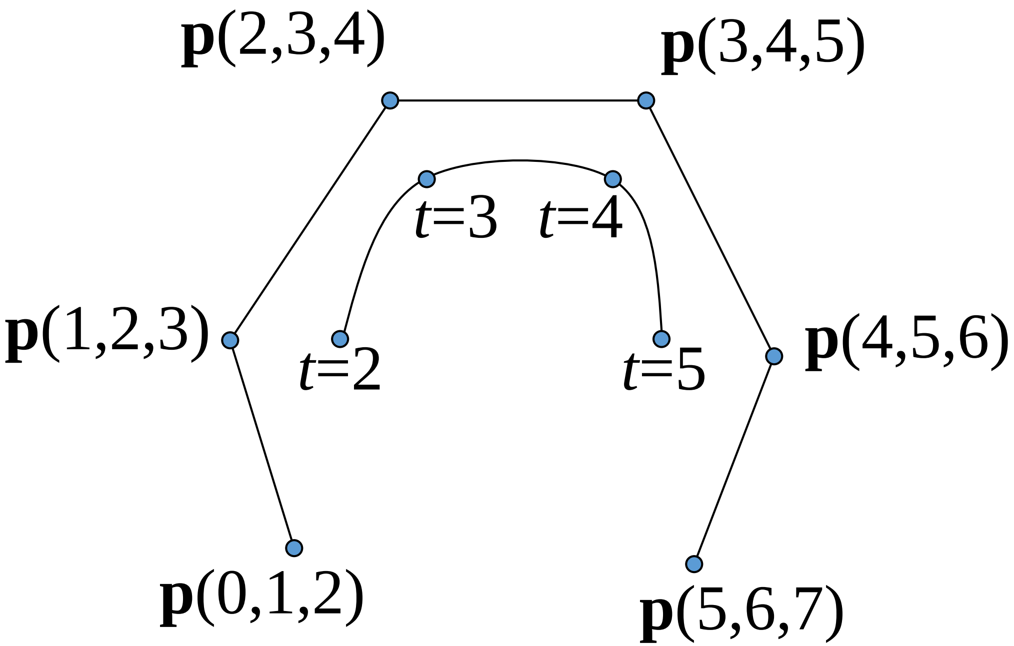
- Suppose we want to add a knot at t = 3.5



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

Knot Insertion

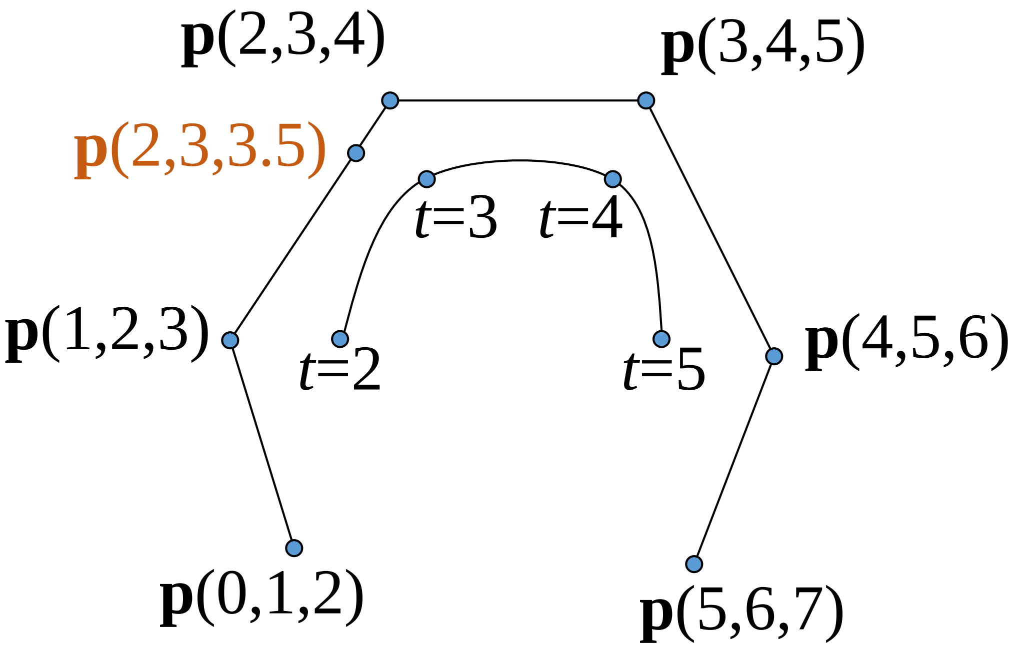
- Suppose we want to add a knot at t = 3.5
- Then we need new cp's



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

Knot Insertion

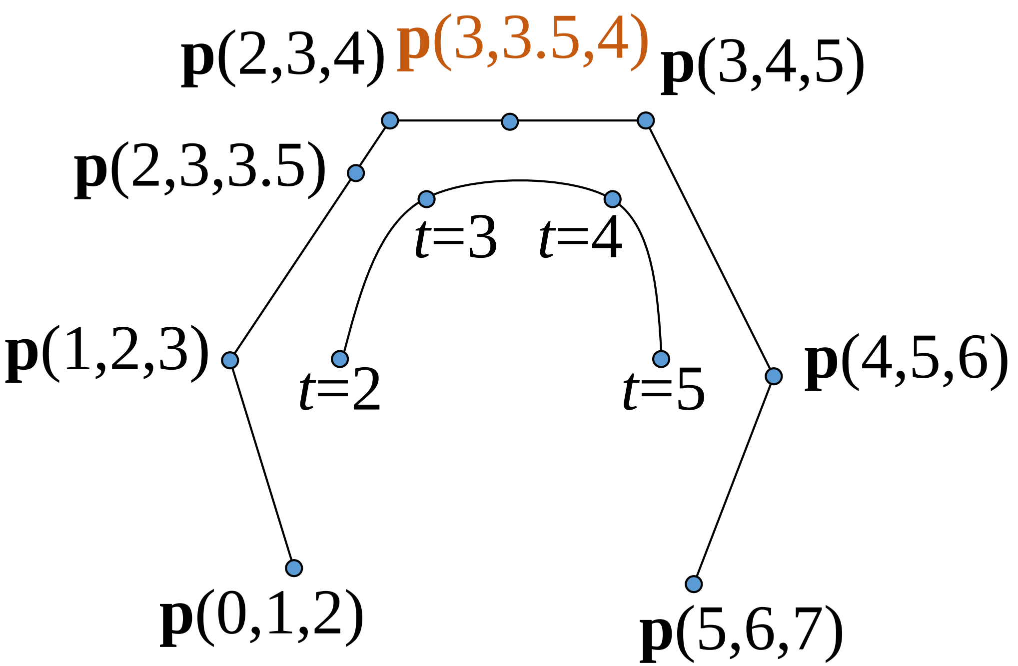
- Suppose we want to add a knot at t = 3.5
- Then we need new cp's **p**(2,3,3.5)



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

Knot Insertion

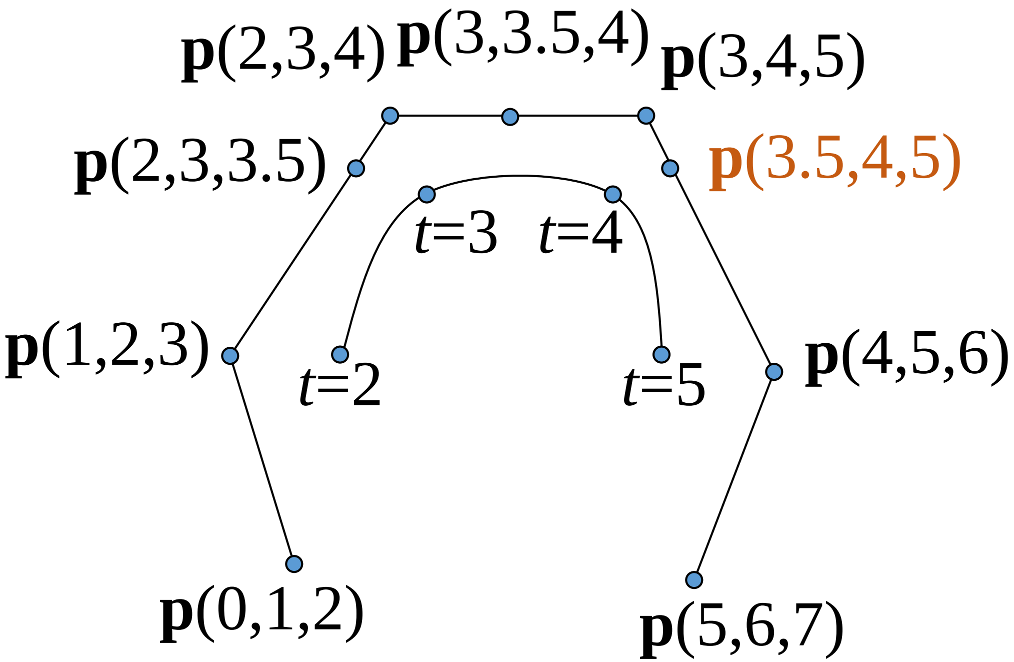
- Suppose we want to add a knot at t = 3.5
- Then we need new cp's **p**(2,3,3.5), **p**(3,3.5,4)



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

Knot Insertion

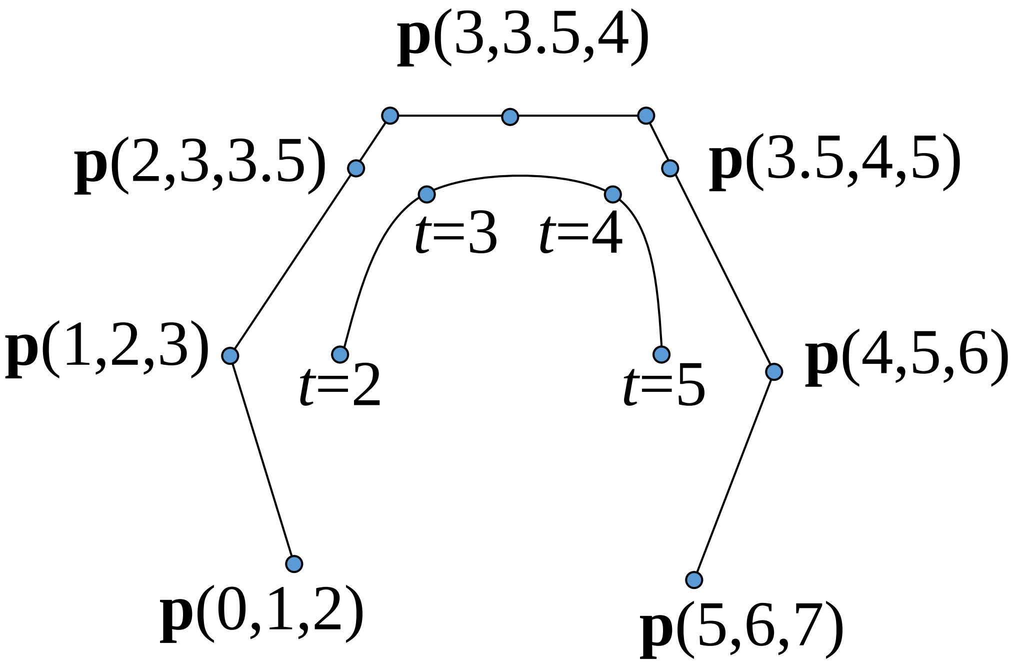
- Suppose we want to add a knot at t = 3.5
- Then we need new cp's **p**(2,3,3.5), **p**(3,3.5,4) and **p**(3.5,4,5)



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

Knot Insertion

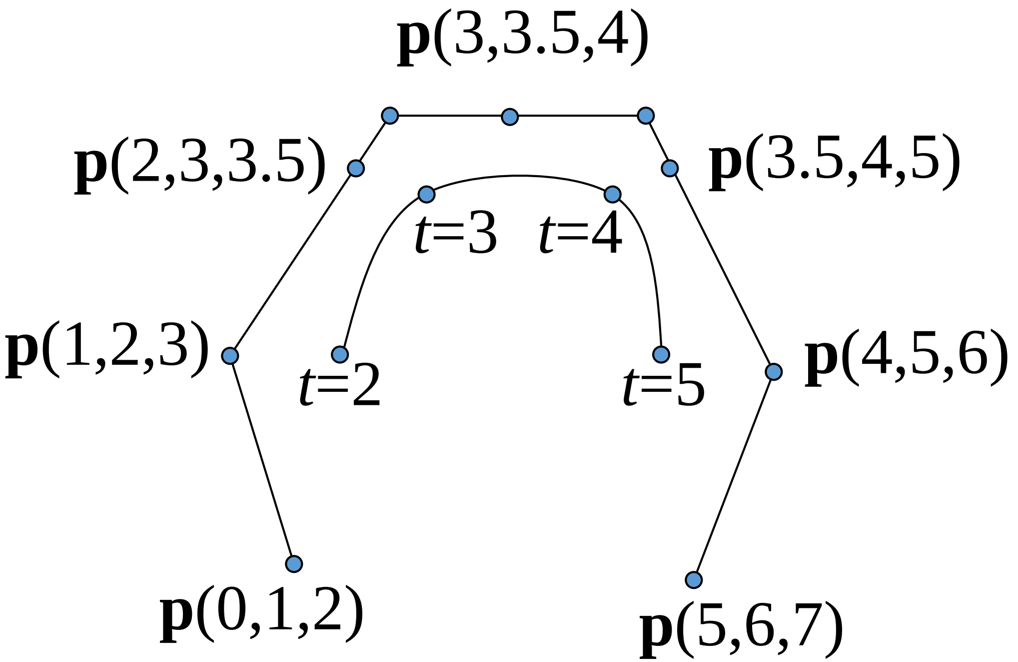
- Suppose we want to add a knot at t = 3.5
- Then we need new cp's **p**(2,3,3.5), **p**(3,3.5,4) and **p**(3.5,4,5) and can get rid of **p**(2,3,4) and **p**(3,4,5)



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

Knot Insertion

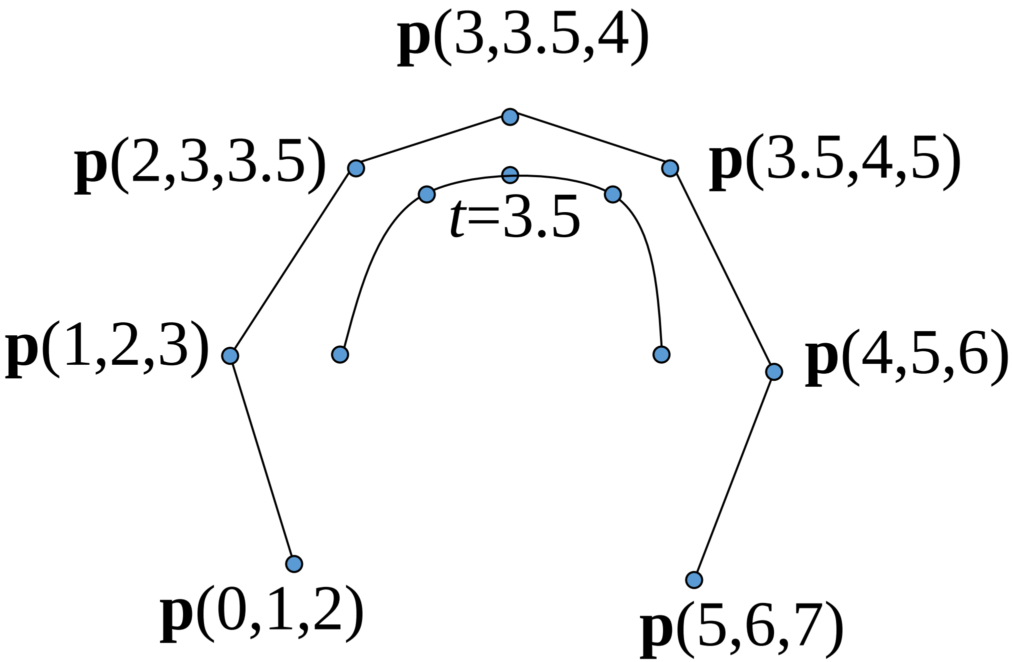
- Suppose we want to add a knot at t = 3.5
- Then we need new cp's **p**(2,3,3.5), **p**(3,3.5,4) and **p**(3.5,4,5) and can get rid of **p**(2,3,4) and **p**(3,4,5)



knot vector: [0 1 2 3 **3.5** 4 5 6 7]

de Boor Algorithm

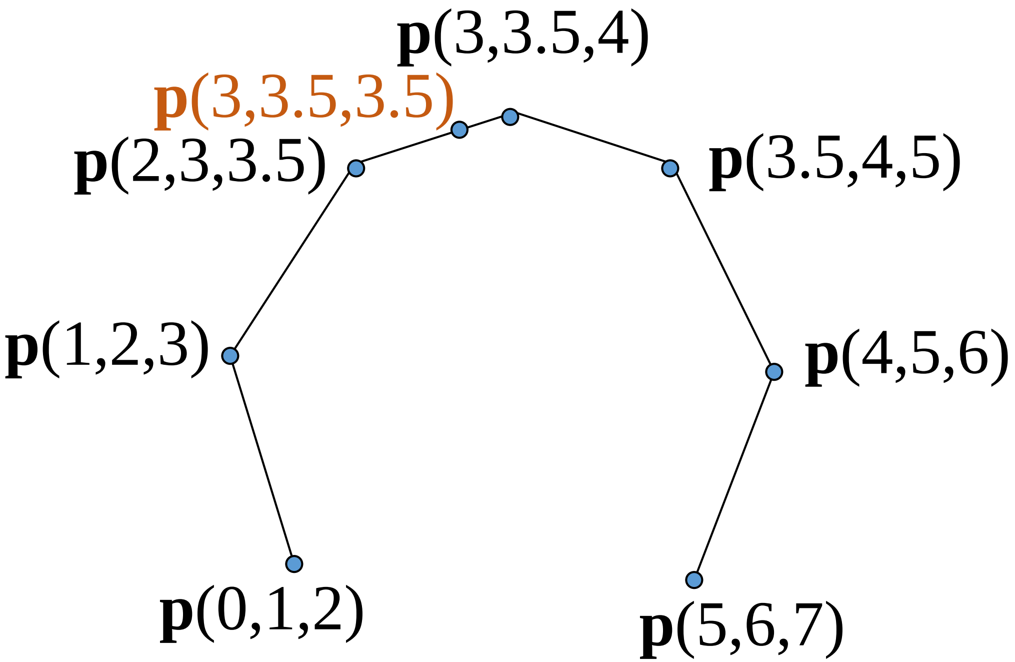
- What if we want to evaluate **p**(3.5)?
- Then create a triple knot at t = 3.5 and figure out where to put the control point **p**(3.5,3.5,3.5)



knot vector: [0 1 2 3 3.5 4 5 6 7]

de Boor Algorithm

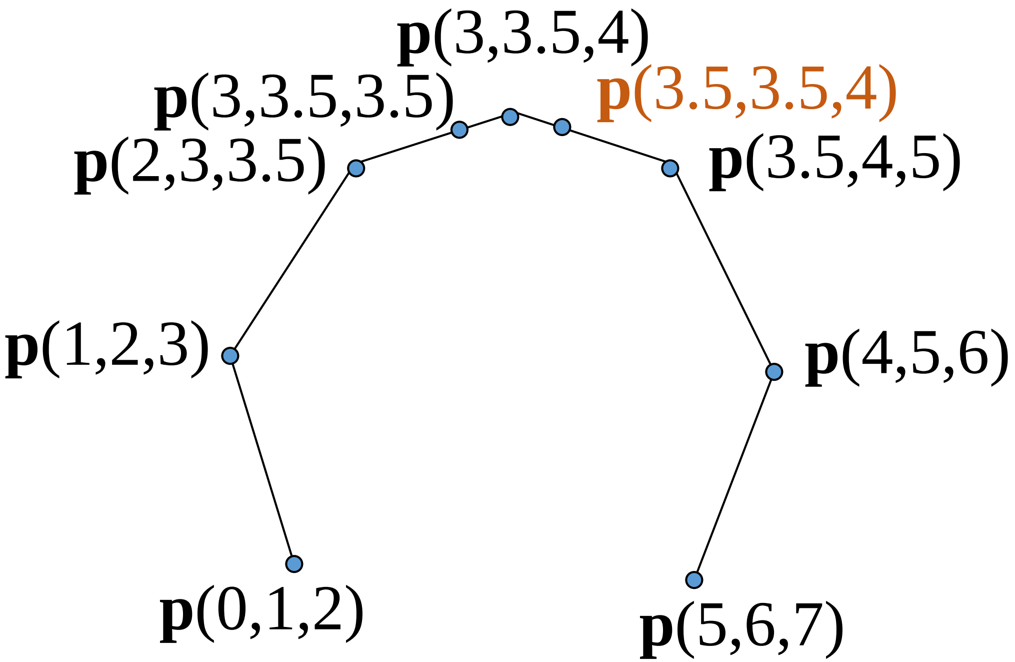
- What if we want to evaluate **p**(3.5)?
- Then create a triple knot at t = 3.5 and figure out where to put the control point **p**(3.5,3.5,3.5)
- Need **p**(3,3.5,3.5)



knot vector: [0 1 2 3 3.5 4 5 6 7]

de Boor Algorithm

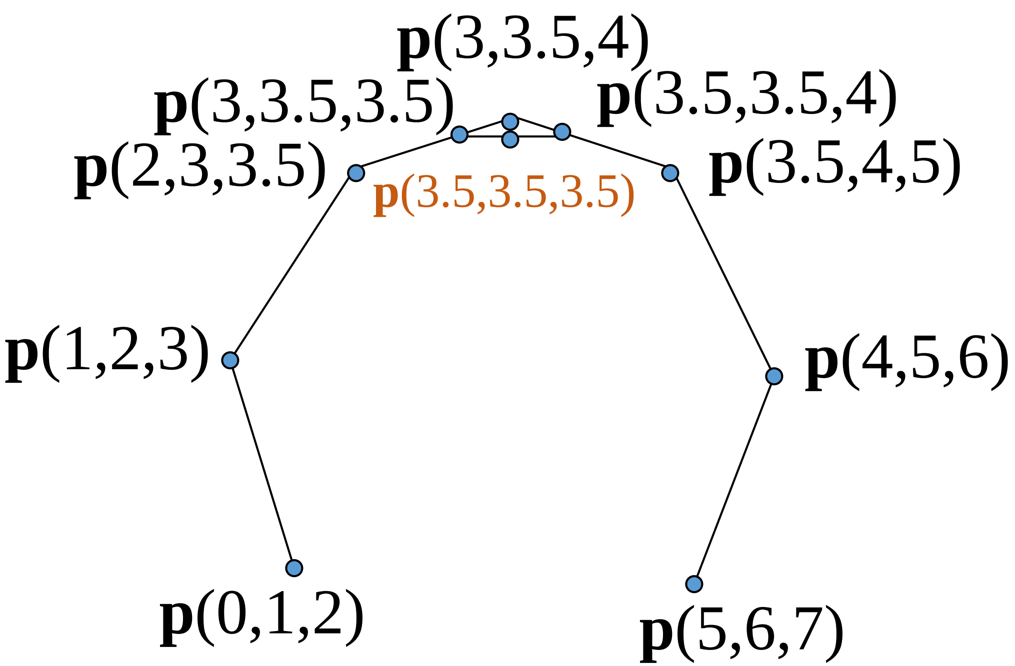
- What if we want to evaluate **p**(3.5)?
- Then create a triple knot at t = 3.5 and figure out where to put the control point **p**(3.5,3.5,3.5)
- Need **p**(3,3.5,3.5) and **p**(3.5,3.5,4)



knot vector: [0 1 2 3 3.5 4 5 6 7]

de Boor Algorithm

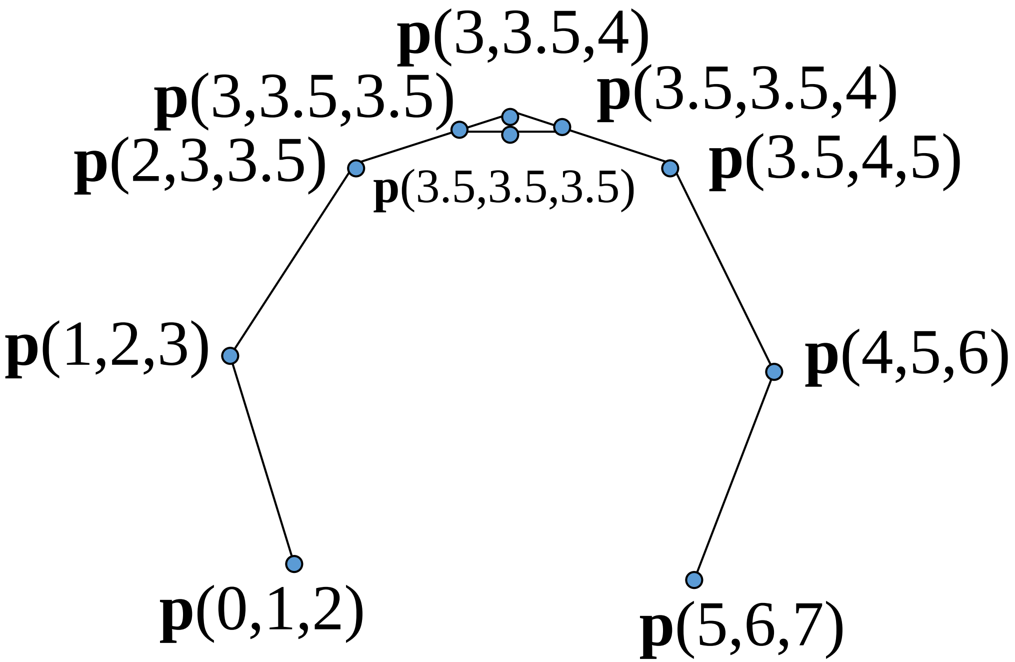
- What if we want to evaluate **p**(3.5)?
- Then create a triple knot at t = 3.5 and figure out where to put the control point **p**(3.5,3.5,3.5)
- Need **p**(3,3.5,3.5) and **p**(3.5,3.5,4)



knot vector: [0 1 2 3 3.5 4 5 6 7]

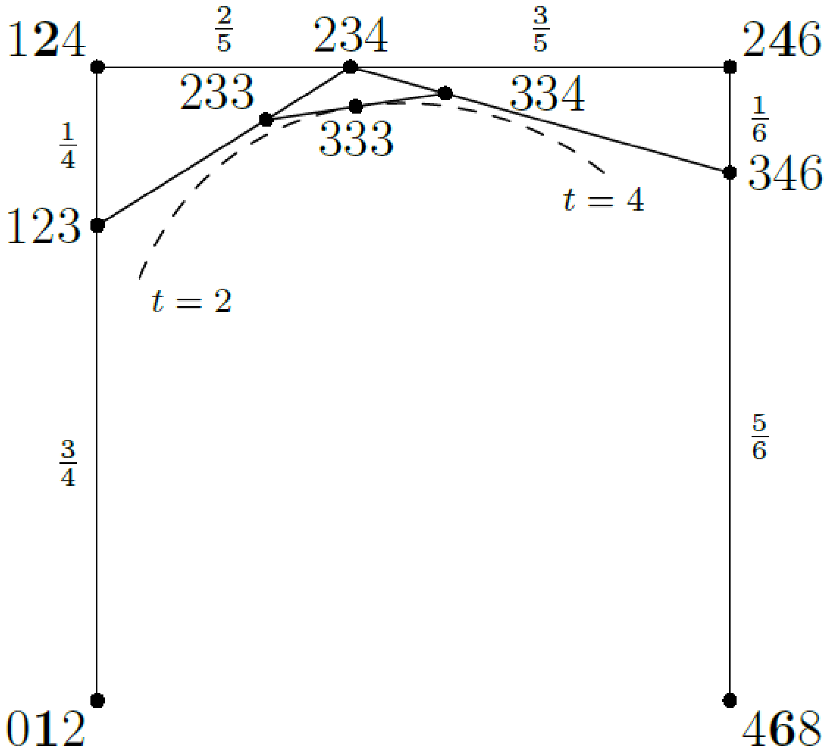
de Boor Algorithm

- What if we want to evaluate **p**(3.5)?
- Then create a triple knot at t = 3.5 and figure out where to put the control point **p**(3.5,3.5,3.5)
- Need **p**(3,3.5,3.5) and **p**(3.5,3.5,4)
- Also subdivides B-spline into [0,1,2,3,3.5,3.5,3.5] and [3.5,3.5,3.5,4,5,6,7]



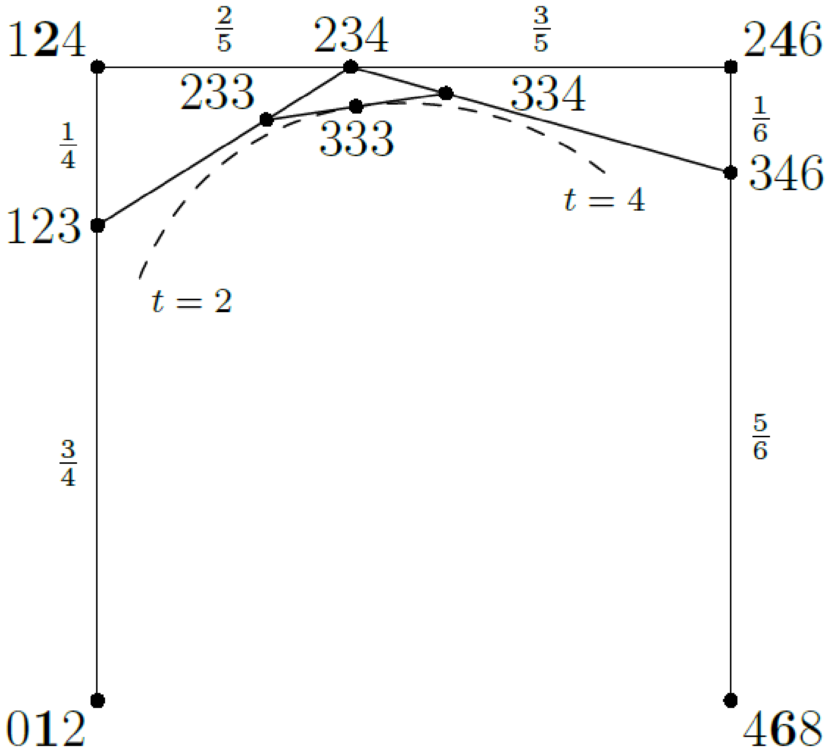
knot vector: [0 1 2 3 3.5 3.5 3.5 4 5 6 7]

de Boor Algorithm

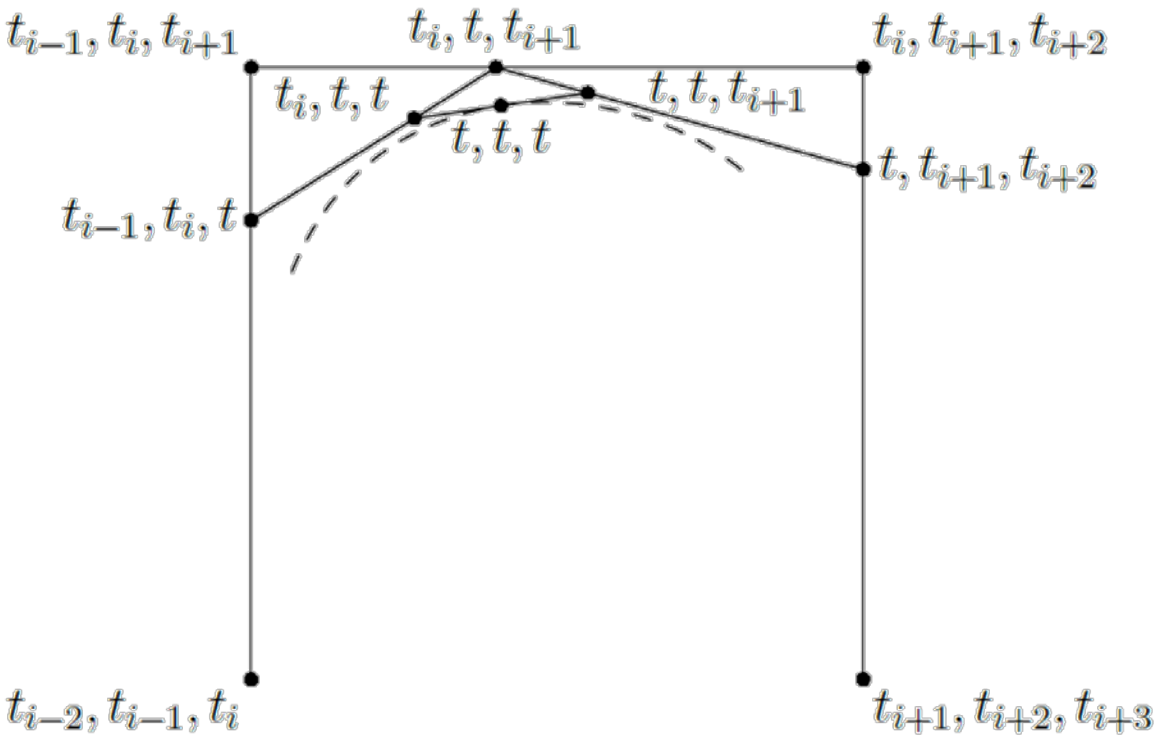


$[0, 1, 2, 4, 6, 8]$

de Boor Algorithm



$[0, 1, 2, 4, 6, 8]$



$[\ldots, t_{i-2}, t_{i-1}, t_i, t_{i+1}, t_{i+2}, t_{i+3}, \ldots]$

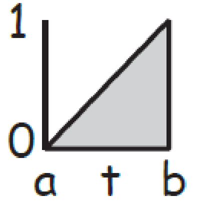
de Boor Algorithm

The ratio $\frac{t-a}{b-a}$ ranges from a to b as t grows from 0 to 1

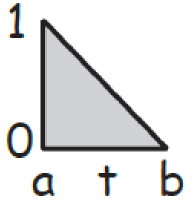
$t=a \rightarrow \frac{t-a}{b-a} = 0$

$t=b \rightarrow \frac{t-a}{b-a} = 1$

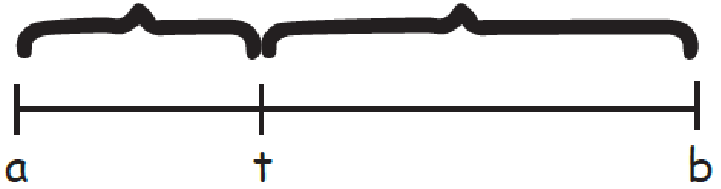
$\frac{t-a}{b-a}$



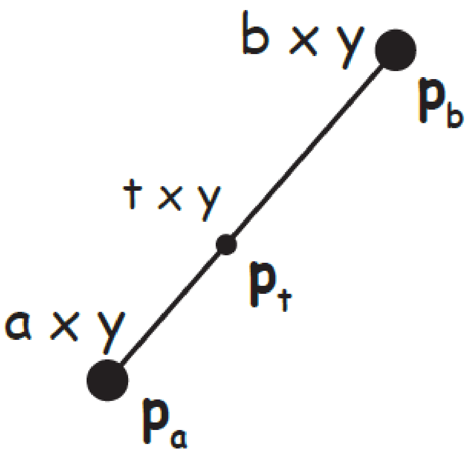
$1 - \frac{t-a}{b-a} = \frac{b-t}{b-a}$



Proportions: $\frac{t-a}{b-a}$ $\frac{b-t}{b-a}$

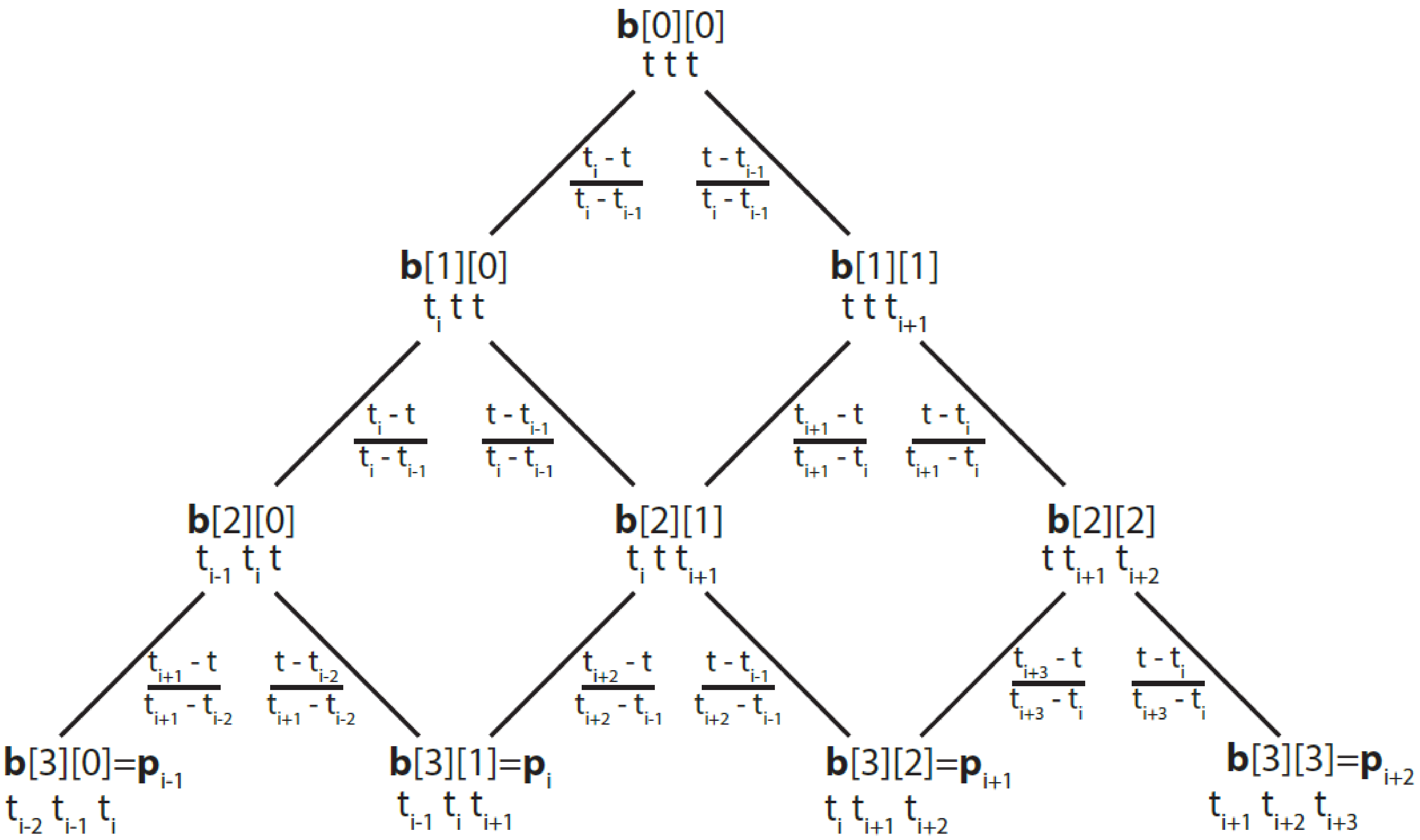


These ratios find the positions of new blossoms

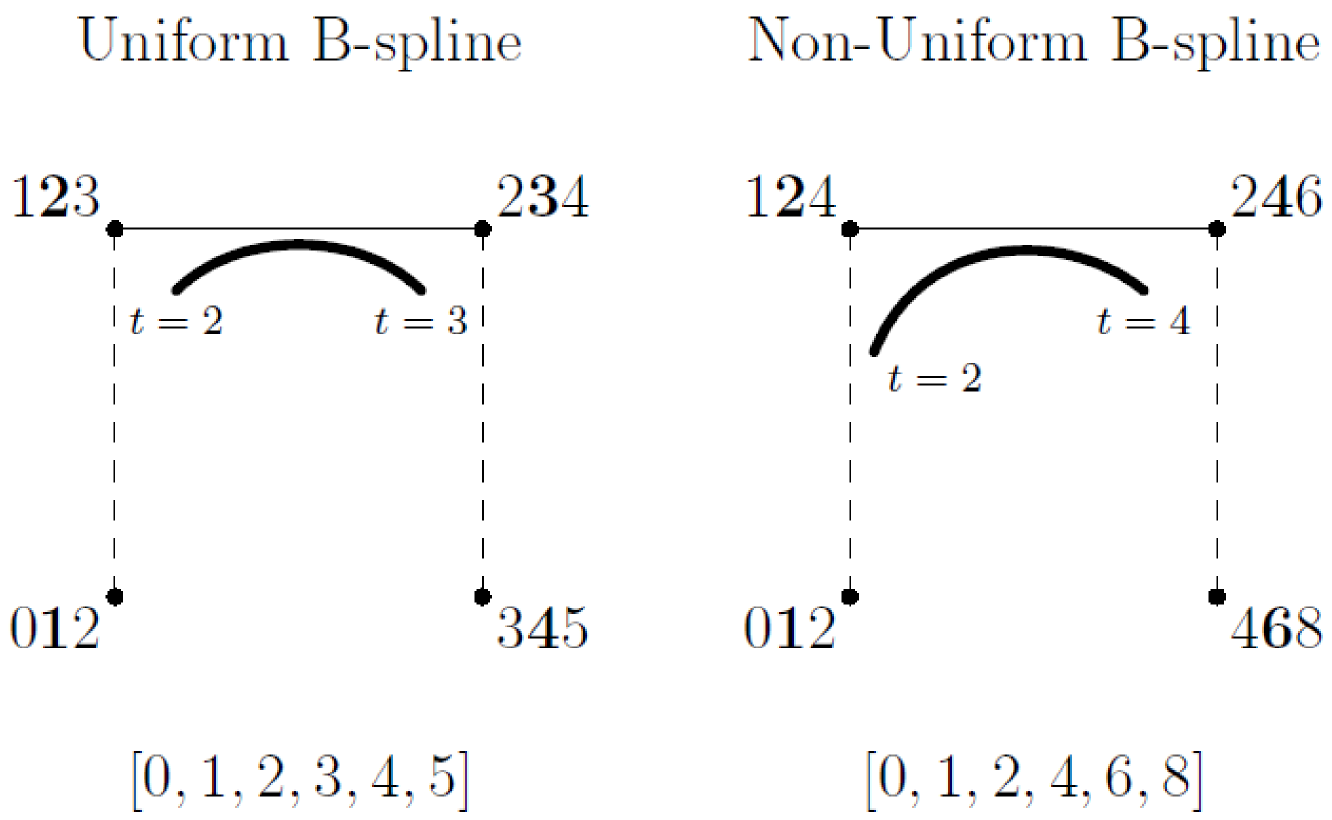


$$p_t = \frac{b-t}{b-a} p_a + \frac{t-a}{b-a} p_b$$

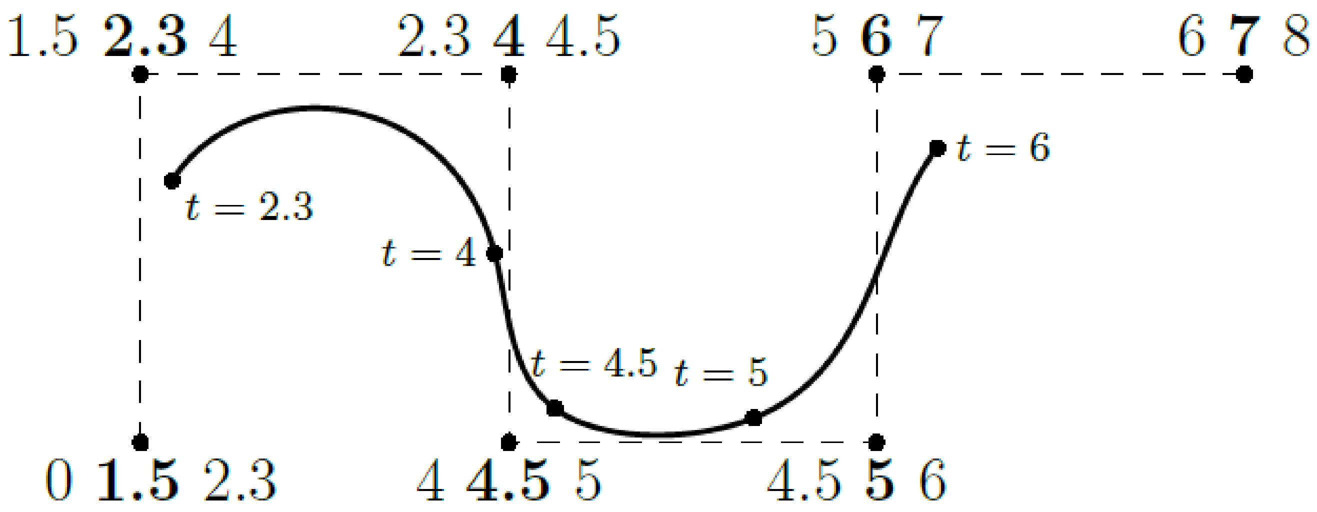
de Boor Algorithm



Continuity

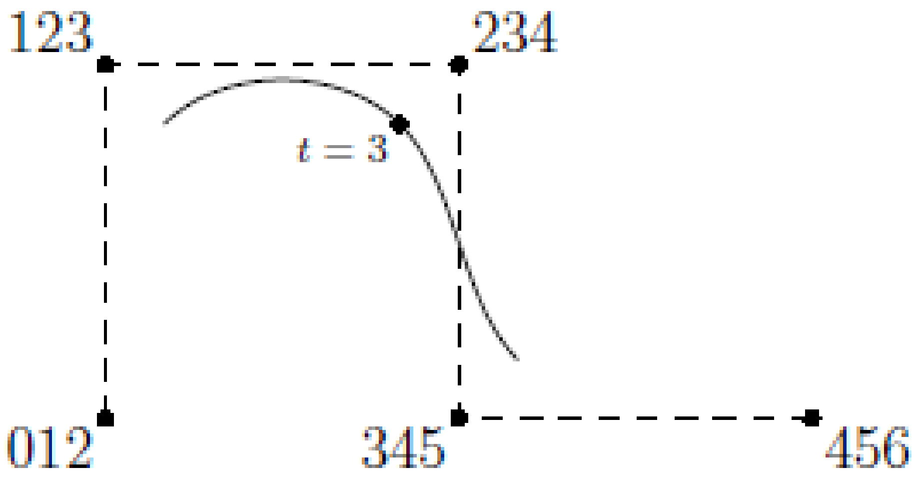

Uniform B-spline
Non-Uniform B-spline
123
234
t = 2
t = 3
012
345
124
246
t = 4
t = 2
012
468
[0, 1, 2, 3, 4, 5]
[0, 1, 2, 4, 6, 8]

Continuity



[0,1.5,2.3,4,4.5,5,6,7,8]

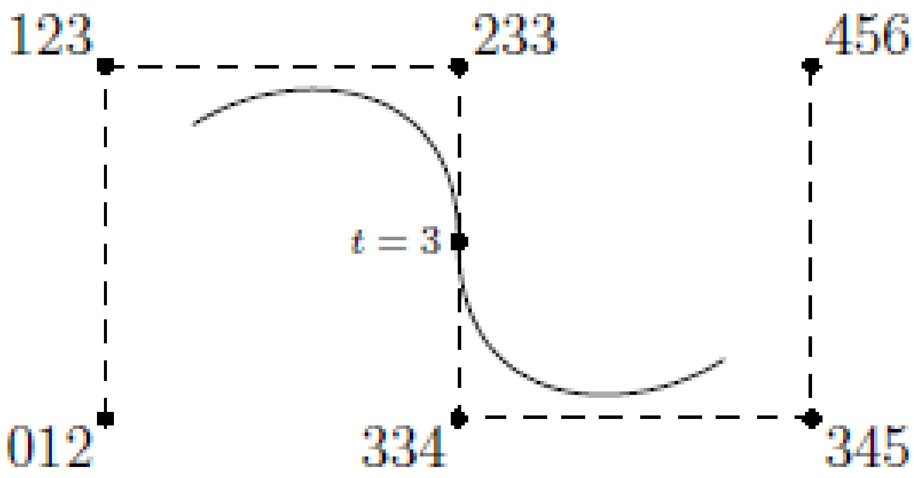
Continuity



$[0, 1, 2, 3, 4, 5]$

2nd derivative continuity

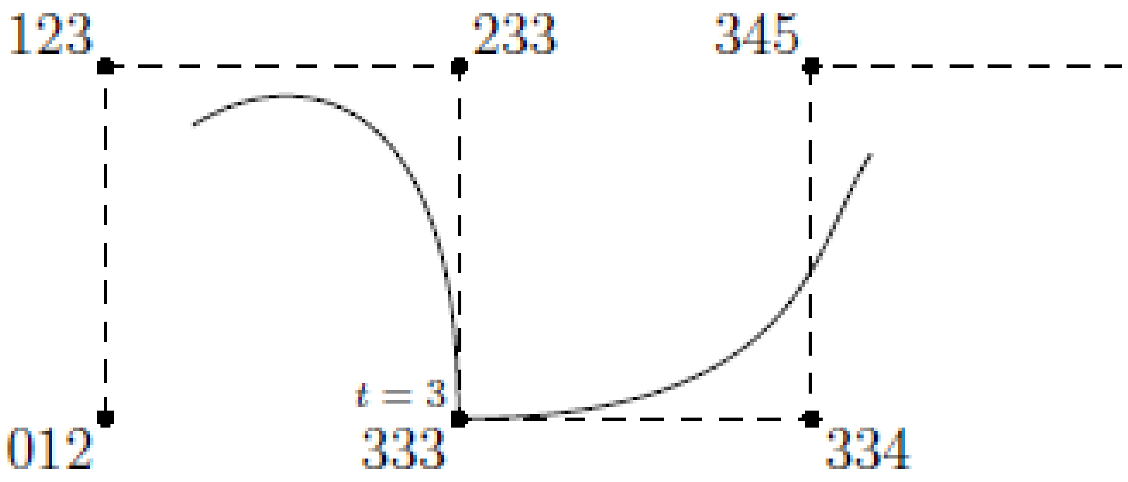
Continuity



$[0, 1, 2, 3, 3, 4, 5, 6]$

1^{st} derivative continuity

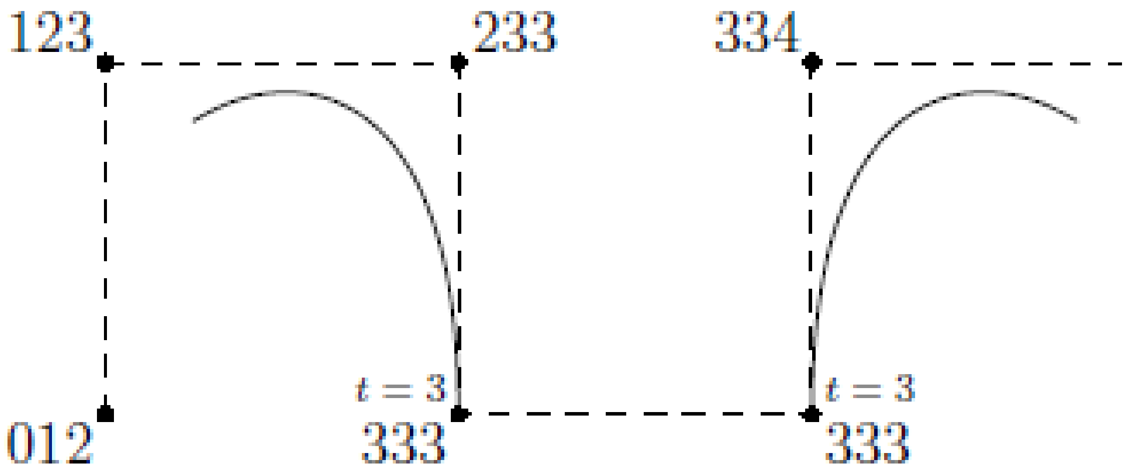
Continuity



$[0, 1, 2, 3, 3, 3, 4, 5, 6]$

continuity

Continuity



$[0, 1, 2, 3, 3, 3, 3, 4, 5, 6]$

discontinuity