

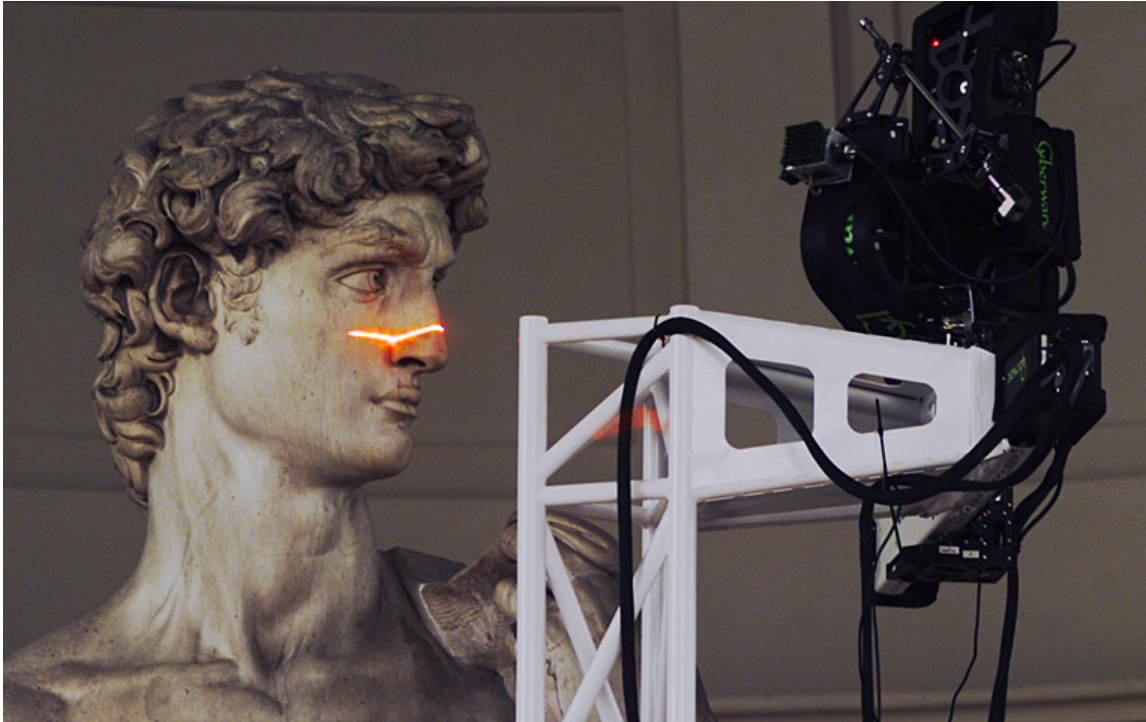
Mesh Simplification

CS418 Interactive Computer Graphics

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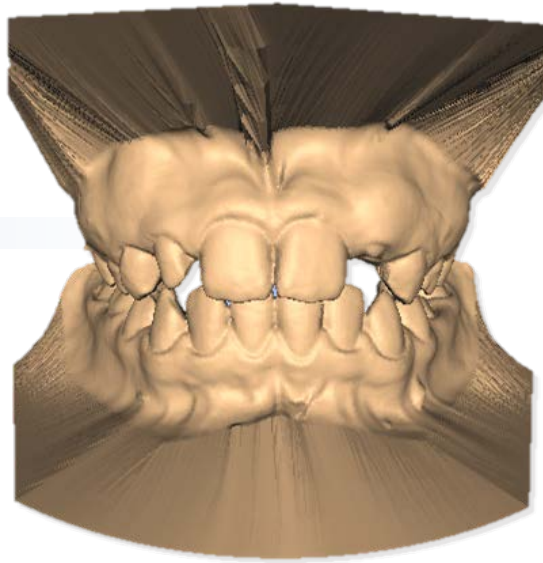
Digital Michelangelo

- In 1998 Marc Levoy takes a sabbatical year in Florence to scan a bunch of Michelangelo sculptures
- David at 1mm resolution
- St. Matthew at 290 μ m resolution

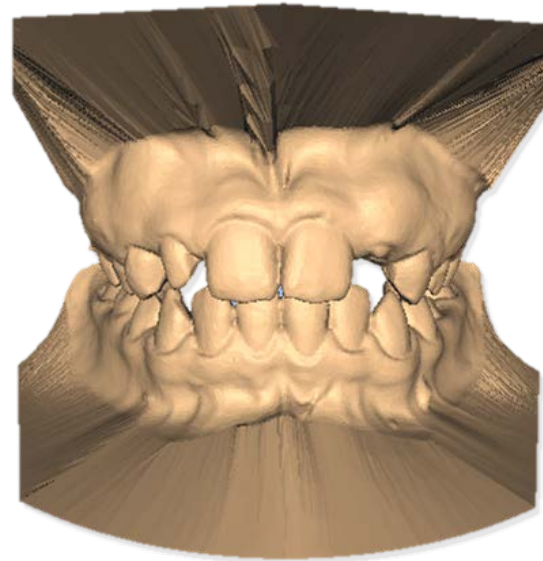


Mesh Simplification

- Meshes often contain more triangles than are necessary for visual fidelity
 - Some surface generation methods run at a fixed resolution regardless of surface detail
 - Meshes might be used on different resolution devices, e.g. cellphone
- Need a method to reduce the number of triangles in a mesh
- Must figure out which triangles to remove while preserving visual shape and appearance



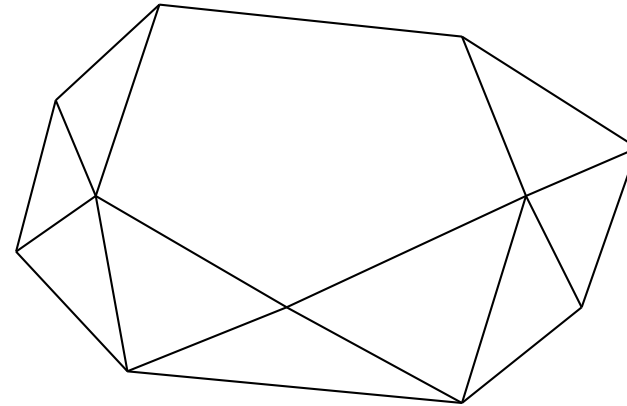
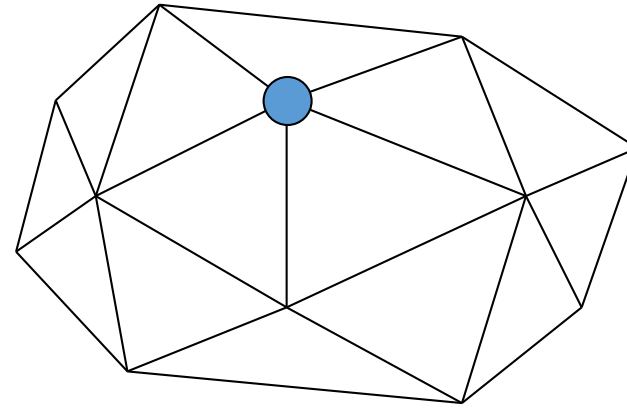
424,376 triangles



60,000 triangles

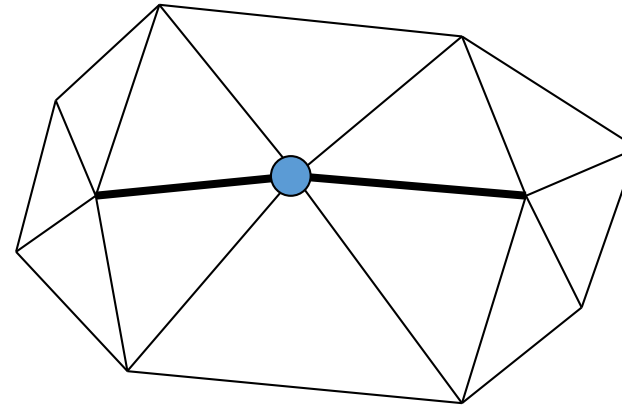
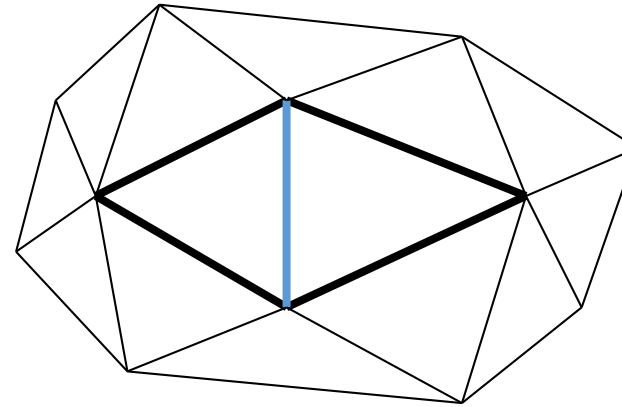
Vertex Decimation

- Removing a vertex turns triangle mesh into polygon mesh



Edge Collapse

- Removing a vertex turns triangle mesh into polygon mesh
- Removing an edge...
 - Merges two vertices into one
 - Removes two faces
 - Mesh still consists of triangles
- Which edges goes first?
- Where should the new vertices go?



Vertex Importance

- Plane equation

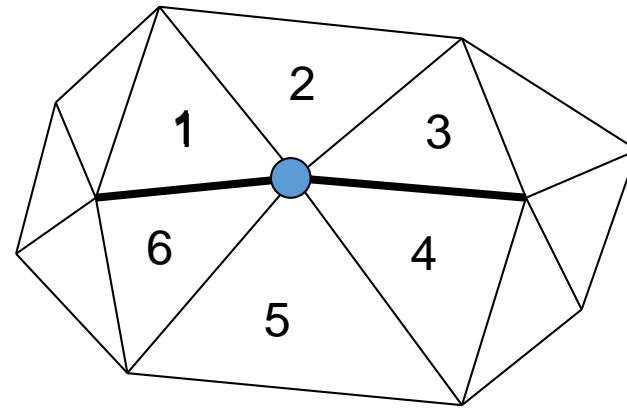
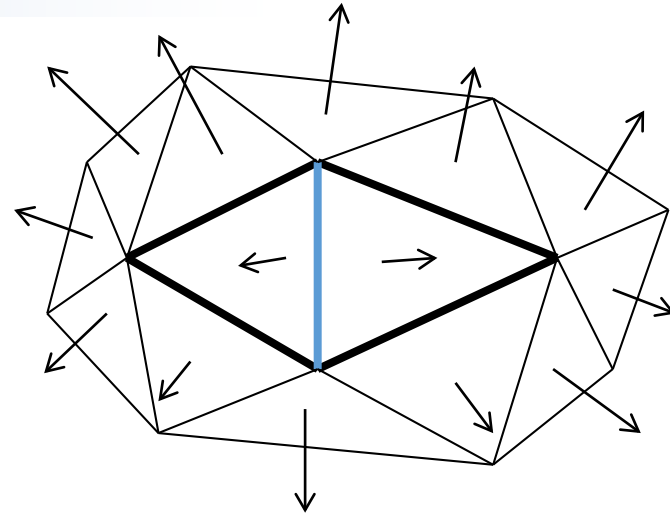
$$P_i(x,y,z) = A_ix + B_iy + C_iz + D$$

$$P_i(\mathbf{x}) = N_i \cdot \mathbf{x} + D$$

- $P_i(\mathbf{x})$ returns signed distance from \mathbf{x} to plane passing through polygon i
- Best position of new vertex position minimizes squared distance to original planes of original polygons

$$QEM(\mathbf{v}) = \sum P_i(\mathbf{v})^2$$

(for adjacent polygons i)



Matrix Representation

- Squared distance from point \mathbf{x} to plane P

$$P^2(\mathbf{x}) = (Ax + By + Cz + D)^2$$

$$= [x \quad y \quad z \quad 1] \begin{bmatrix} A^2 & AB & AC & AD \\ AB & B^2 & BC & BD \\ AC & BC & C^2 & CD \\ AD & BD & CD & D^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \mathbf{x}^T Q \mathbf{x}$$

- Sum of squared distances from point \mathbf{x} to planes P_1 and P_2

$$P_1^2(\mathbf{x}) + P_2^2(\mathbf{x}) = \mathbf{x}^T Q_1 \mathbf{x} + \mathbf{x}^T Q_2 \mathbf{x} = \mathbf{x}^T (Q_1 + Q_2) \mathbf{x}$$

Quadric Error Metric

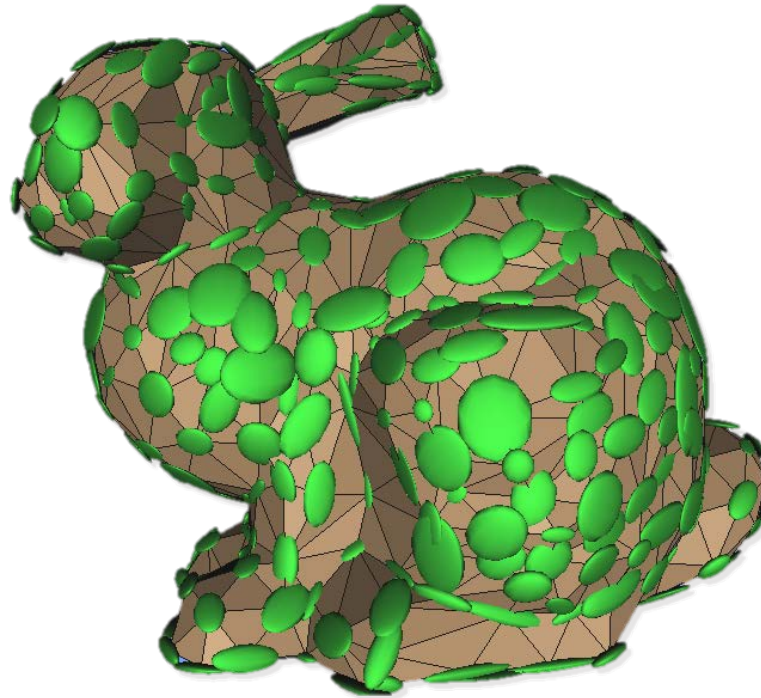
- Find Q for each vertex \mathbf{v}

$$Q(\mathbf{v}) = \sum Q_i$$

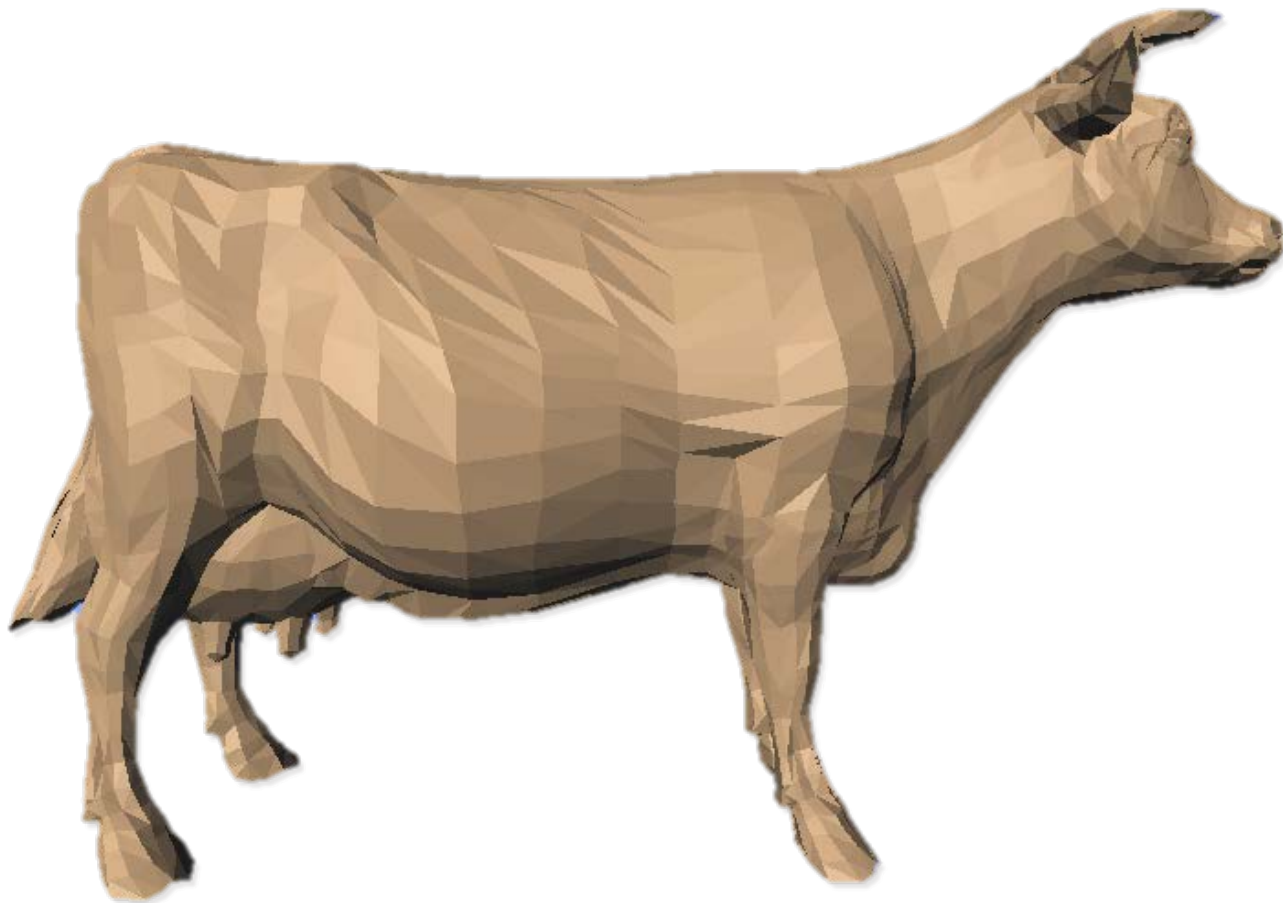
(for adjacent polygons i)

- Edge collapse edges whose vertices $\mathbf{v}_1, \mathbf{v}_2$ have least $Q(\mathbf{v}_1) + Q(\mathbf{v}_2)$
- New vertex \mathbf{v}_{12} can be at \mathbf{v}_1 or \mathbf{v}_2 or optimally at ...

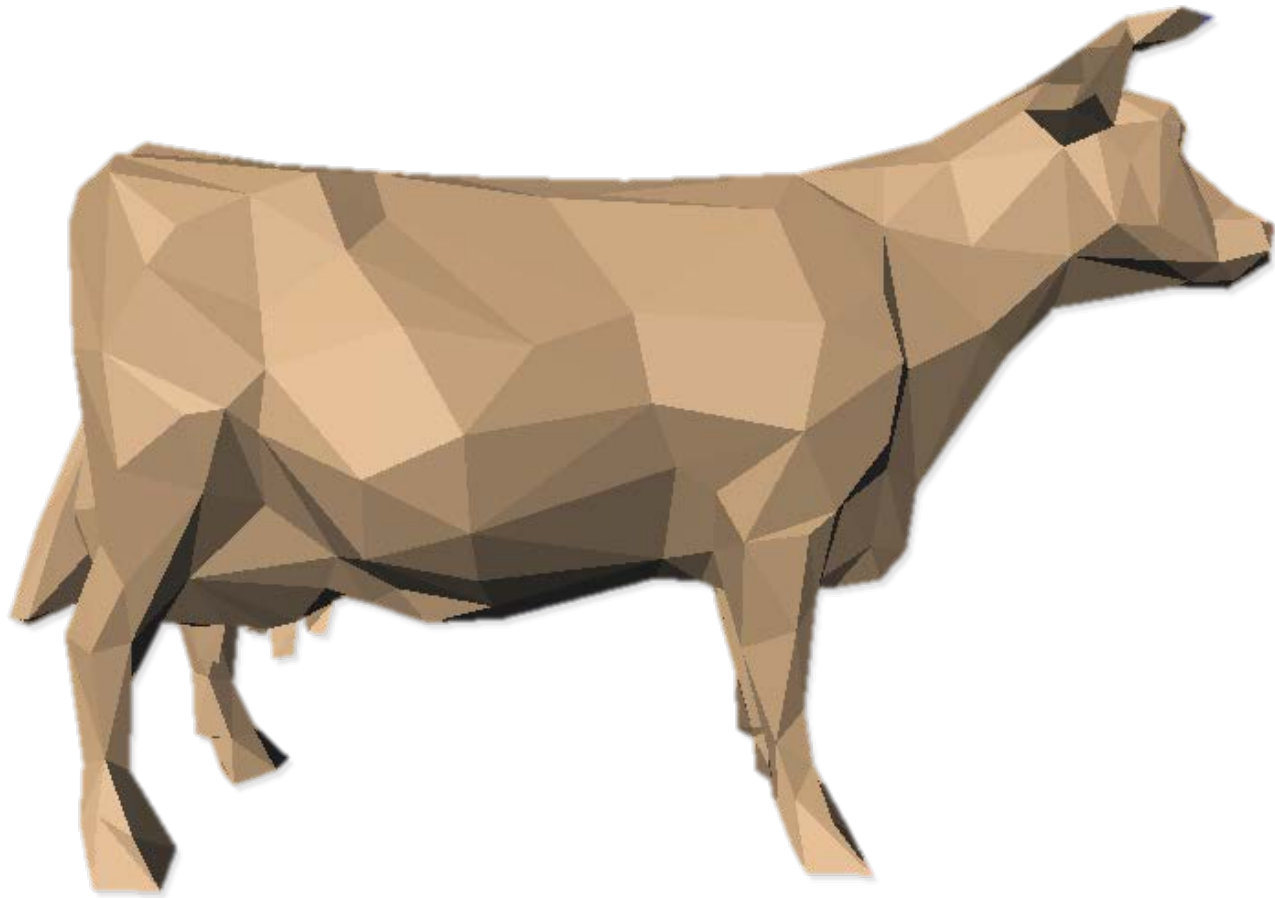
$$\mathbf{v}_{12} = - \begin{bmatrix} A^2 & AB & AC \\ AB & B^2 & BC \\ AC & BC & C^2 \end{bmatrix}^{-1} \begin{bmatrix} AD \\ BD \\ CD \end{bmatrix}$$



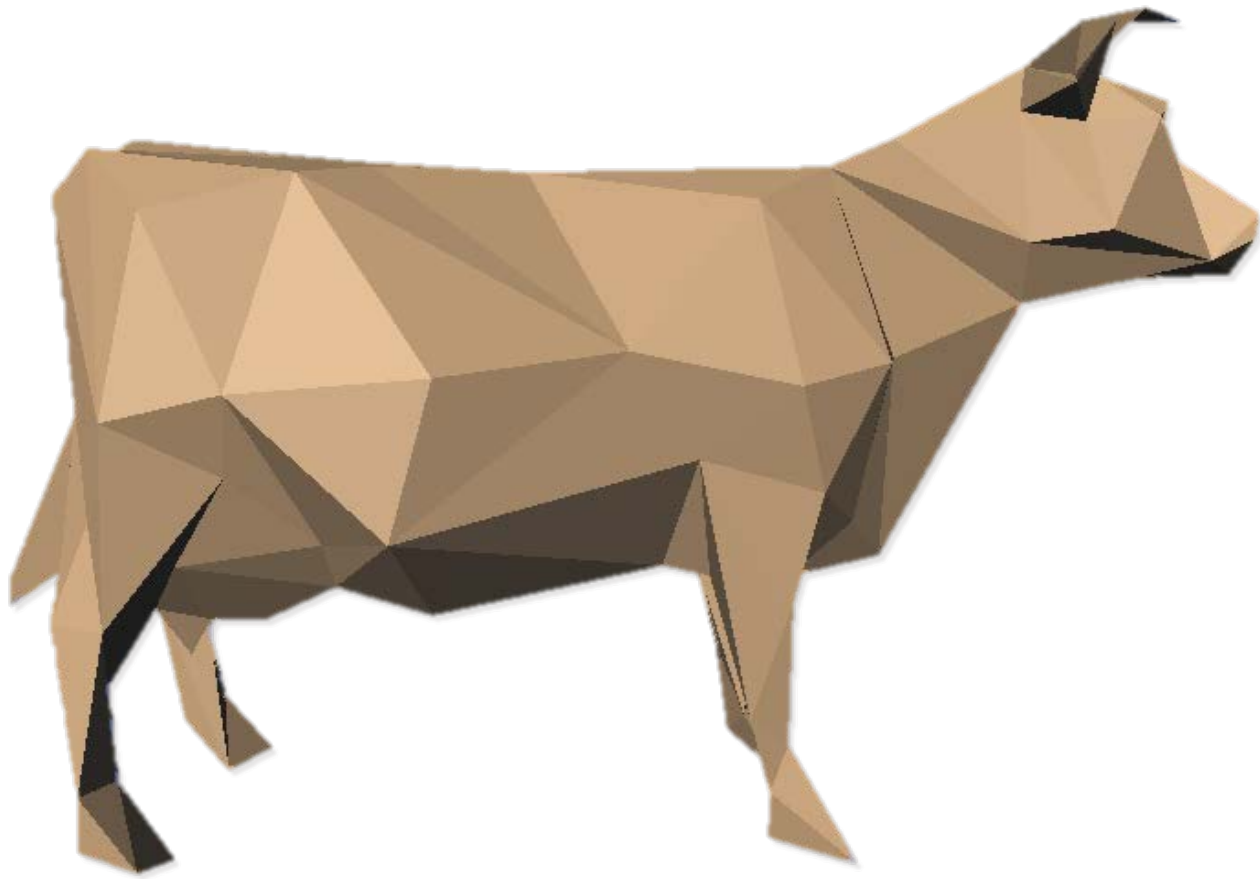
Examples



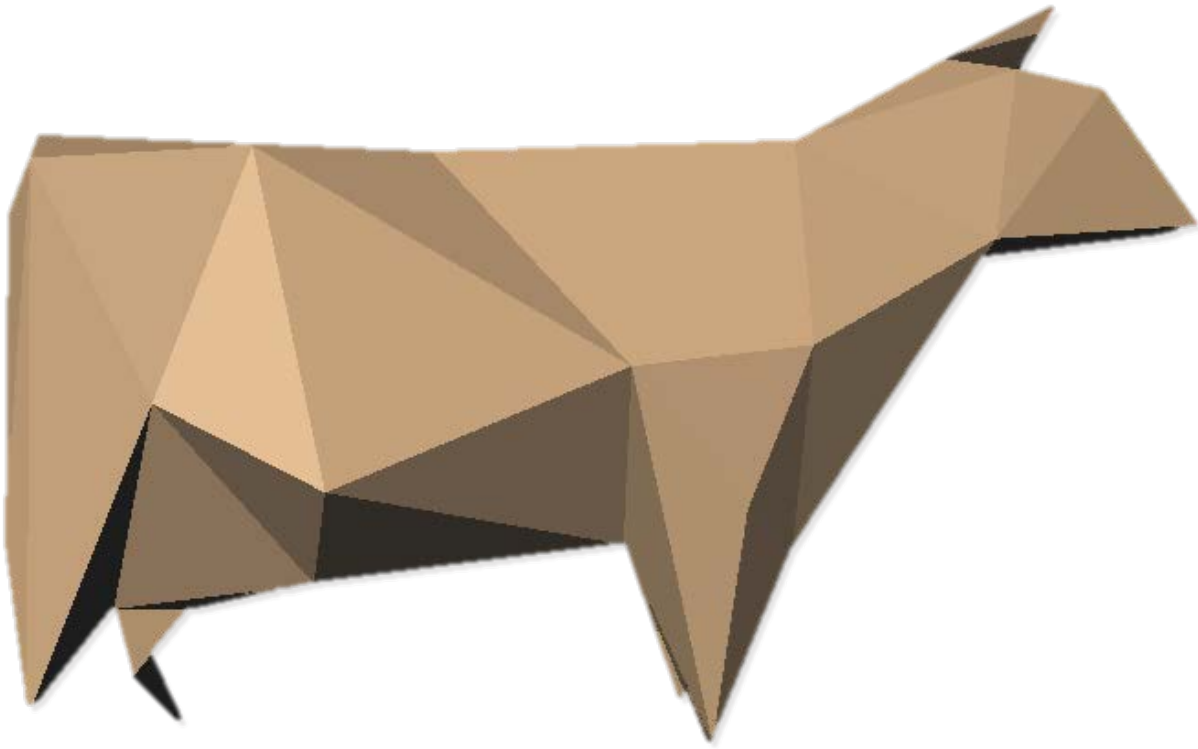
Examples



Examples



Examples



Examples

