

## More on Affine Transformations

1. Suppose we have 2D frame with an origin at (2,2) and basis vectors  $u = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$   $v = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ . Suppose a point  $p=(1,1)$  in  $u,v$  coordinates. What is  $p$  in  $x,y$  coordinates?
2. Construct a matrix that can convert a point in  $u,v$  coordinates to  $x,y$  coordinates for the basis described above.
3. What matrix would perform the window-to-viewport transformation specified by the function call `gl.viewport(0,0,960,640)`? For this question, just ignore the  $z$ -coordinate and imagine the NDC coordinates to be transformed are in the form  $(x, y, 1)$ . Express your answer as a  $3 \times 3$  matrix and express rational numbers as fractions.

In WebGL, the window to viewport transformation is performed by the WebGL library during primitive assembly. You control it using the call `gl.viewport(x, y, w, h)`. It specifies the viewport by giving the lower-left coordinate of the viewport with  $(x, y)$  and the width and height of the viewport with  $w$  and  $h$ .

4. Suppose we have the following set of transformations that map a point in view (or camera) space to world coordinates. What transformations will map world coordinates to camera space?

$$\begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bonus question:

What is the inverse of matrix that performs a scale transformation?

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$