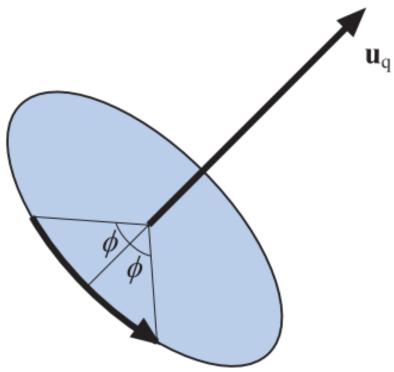
CS 418: Interactive Computer Graphics
In-class Worksheet: Quaternions

Quaternions



A rotation transform represented by a unit quaternion,
$q = (\cos \varphi , \sin \varphi\, u_q)$.
The transform rotates 2φ radians around the axis u_q.
from *Real-Time Rendering*, 4th Edition

1. What rotations are performed by the following quaternions:

   a. (0, (1,0,0))

   b. (0, (0,1,0))

   c. (0, (0,0,1))

2. Compute a quaternion that performs twice the rotation of the quaternion (0.965, (0.149, -0.149, 0.149)). You can use a calculator….

3. Let v_1 and v_2 be nonparallel 3D unit vectors with an angle of θ between them. Find the unit quaternion $(c, (s\ \mathrm{a}))$ where $\mathrm{a} = \frac{v_1 \times v_2}{\sin\theta}$ that rotates v_1 onto v_2

4. What are the comparative computational costs of generating a rotation matrix from Euler Angles versus a quaternion? Let's assume that multiplication and addition each are 1 FLOP and that evaluating a sine or cosine is 5 FLOPs. *Note: the only way to really know the comparative cost of a trig function on a system is to profile it…but the weighting in this question should be approximately correct.*

For reference, here is a rotation matrix constructed from a quaternion (q0, (q1,q2,q3)):

$$\begin{bmatrix} 1-2q_2^2-2q_3^2 & 2q_1q_2+2q_0q_3 & 2q_1q_3-2q_0q_2 \\ 2q_1q_2-2q_0q_3 & 1-2q_1^2-2q_3^2 & 2q_2q_3+2q_0q_1 \\ 2q_1q_3+2q_0q_2 & 2q_2q_3-2q_0q_1 & 1-2q_1^2-2q_2^2 \end{bmatrix}$$