CS 418: Interactive Computer Graphics
In-class Worksheet

Bezier Curves

Cubic Bezier Curves

A Bezier curve is a parametric polynomial curve given by:

$X(t) = (1-t)^3 b_0 + 3(1-t)^2 t b_1 + 3(1-t) t^2 b_2 + t^3 b_3$

where b_i are the control points.

The tangent vector of the curve can be found by
$X(t) = 3(b_1 - b_0)(1-t)^2 + 6(b_2 - b_1)(1-t)t + 3(b_3 - b_2)t^2$

1. **The de Casteljau Algorithm**
   Suppose our control points are

   b_0=(-1,0) b_1=(0,1) b_2=(0,-1) b_3=(1,0)

   Use the de Casteljau algorithm to find the coordinates of X(1/4). Check that you get the same answer from using the parametric expression given above.

2. **Tangents to a Bezier Curve**
   - **a.** What are the tangents at the controls b_0 and b_3? Give the answer as a pair of functions.

   - **b.** Suppose you wish to join two Bezier curves A and B. Curve A control points a0, a1, a2, and a3 are already set. You set the control point b0=a3 to join the curves.

     Where does control point b1 need to be placed to make the tangents match at b0? Write your answer as function of the control points for A.