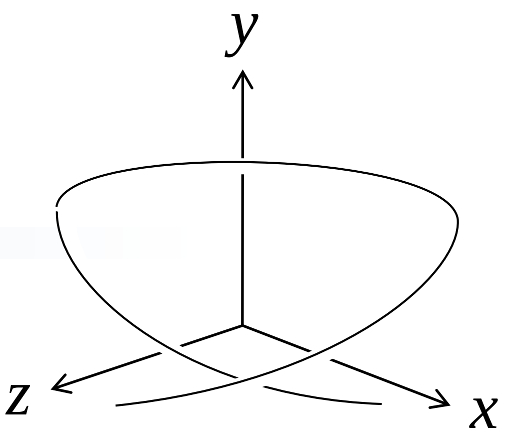
Parametric Surfaces

CS 418

Intro to Computer Graphics

John C. Hart

Space Curves



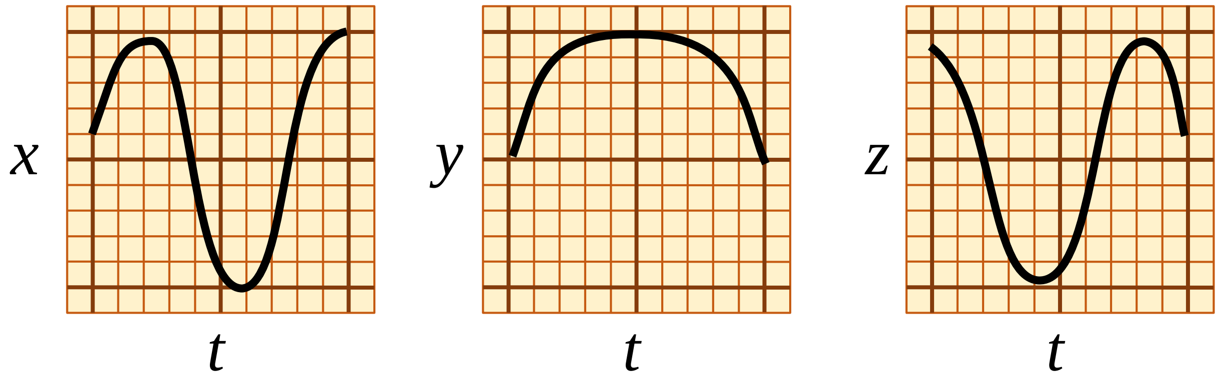
Separate into three coordinate functions

$$\mathbf{p}(t) = (x(t), y(t), z(t))$$

$$x(t) = (1-t)^3\, x_0 + 3t(1-t)^2\, x_1 + 3t^2(1-t)\, x_2 + t^3\, x_3$$

$$y(t) = (1-t)^3\, y_0 + 3t(1-t)^2\, y_1 + 3t^2(1-t)\, y_2 + t^3\, y_3$$

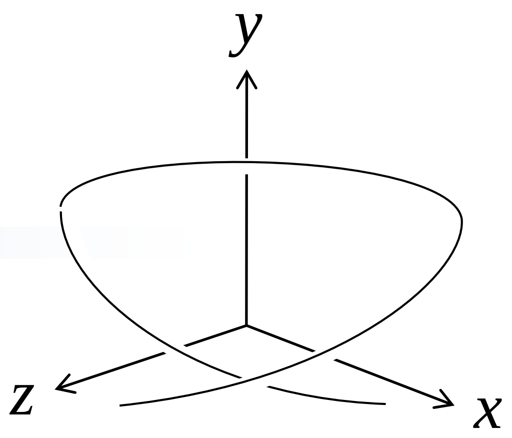
$$z(t) = (1-t)^3\, z_0 + 3t(1-t)^2\, z_1 + 3t^2(1-t)\, z_2 + t^3\, z_3$$

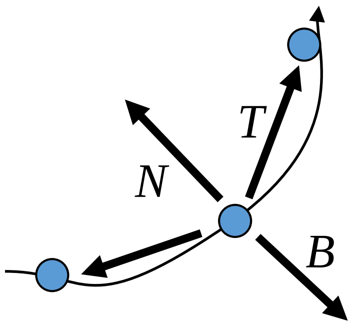


Space Curves

Make your own roller-coaster ride

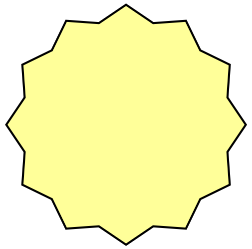
- Camera position along space curve
- Look at point is next position along space curve (tangent)
- Binormal is cross product of vector to next position with vector to previous position
- Up direction (normal) is cross product of binormal with tangent



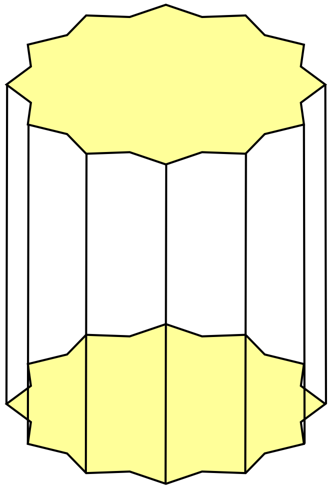


Extrusion

- Two 3-D copies of each 2-D curve



$\mathbf{p}(t) = (x(t), y(t))$



$\mathbf{p}_1(t) = (x(t), y(t), 1)$

$\mathbf{p}_0(t) = (x(t), y(t), 0)$

- Create a mesh of quads (or tri-strip)

  $\mathbf{p}_0(t)$, $\mathbf{p}_1(t)$, $\mathbf{p}_0(t+\Delta t)$, $\mathbf{p}_1(t+\Delta t)$

Generalized Cylinder

- Construct a 2-D profile curve

$$\mathbf{q}(s) = (a(s), b(s))$$

- Construct a space curve

$$\mathbf{p}(t) = (x(t), y(t), z(t))$$

- Construct a Frenet frame at each point along space curve

$$T(t) = \mathbf{p}(t+\Delta t) - \mathbf{p}(t)$$

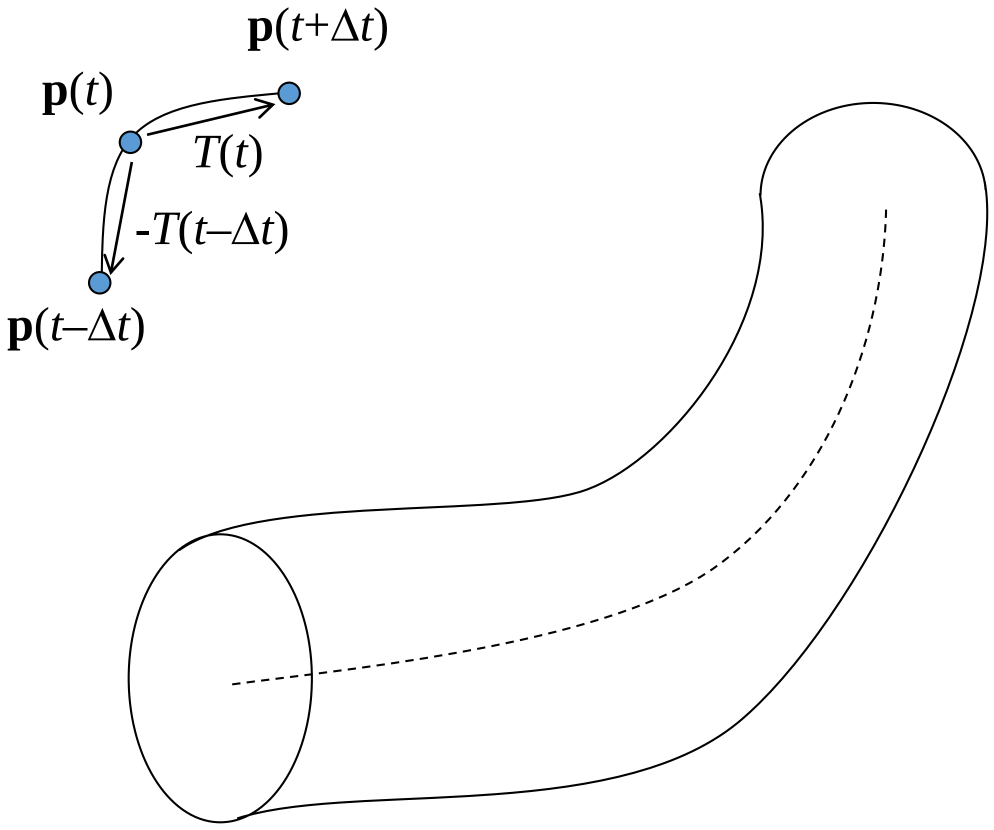
$$B(t) = T(t) \times -T(t - \Delta t)$$

$$N(t) = B(t) \times T(t)$$

(all normalized)

- Plot 2-D curve in (N,B) space

$$\mathbf{gc}(s,t) = \mathbf{p}(t) + a(s)\, N(t) + b(s)\, B(t)$$



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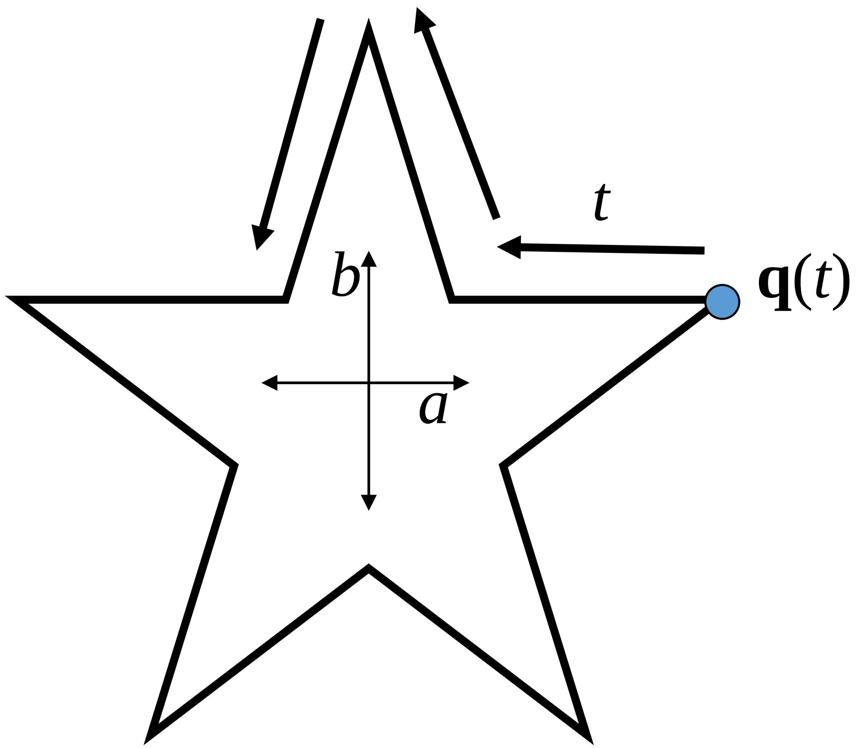

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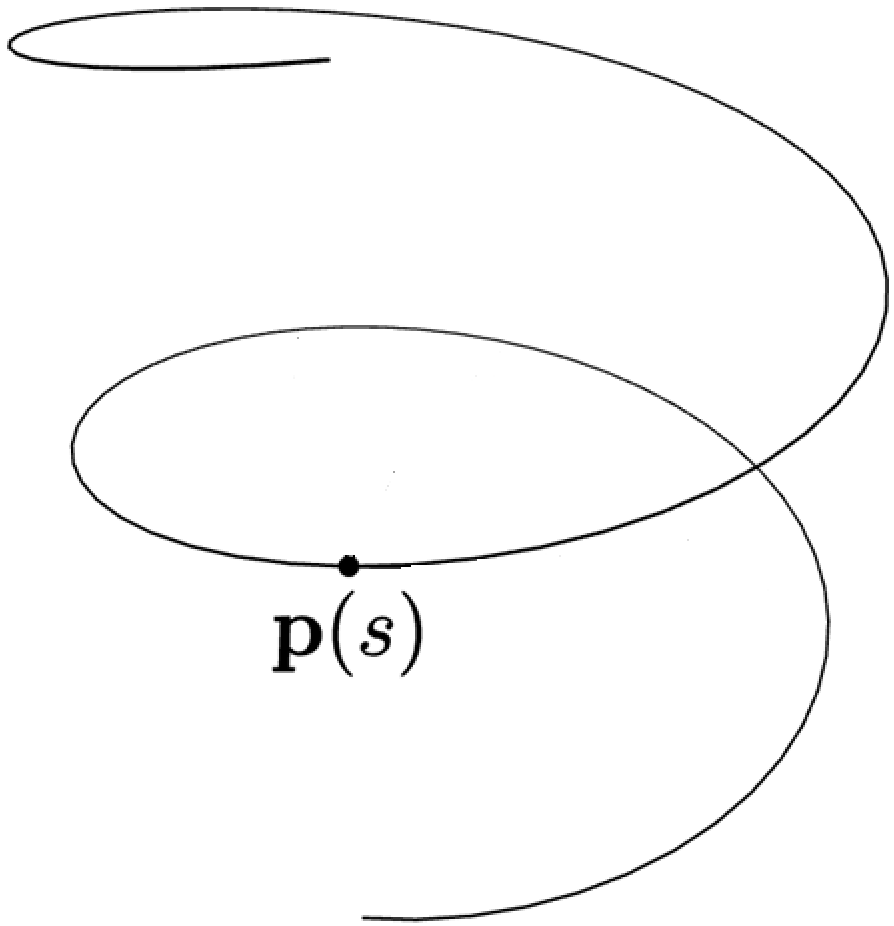

Generalized Cylinder

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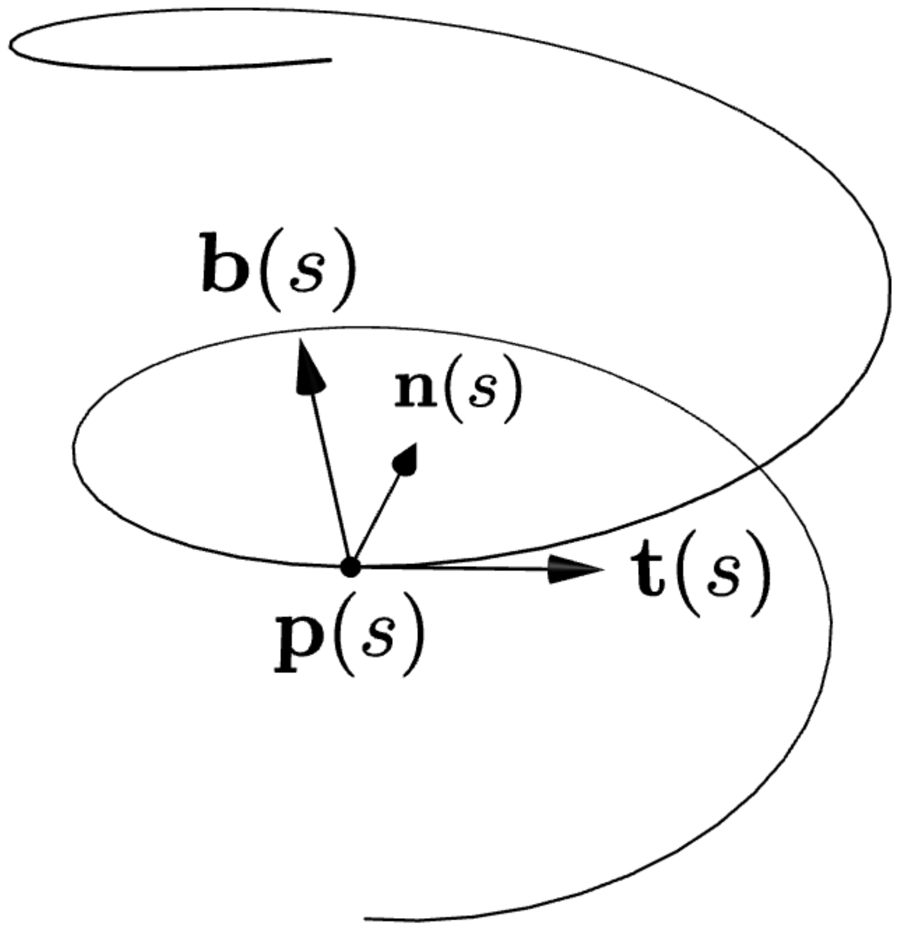
$\mathbf{t}(s) = \mathbf{p}(s + \Delta s) - \mathbf{p}(s)$

$\mathbf{b}(s) = \mathbf{t}(s) \times -\mathbf{t}(s - \Delta s)$

$\mathbf{n}(s) = \mathbf{b}(s) \times \mathbf{t}(s)$

(all normalized)



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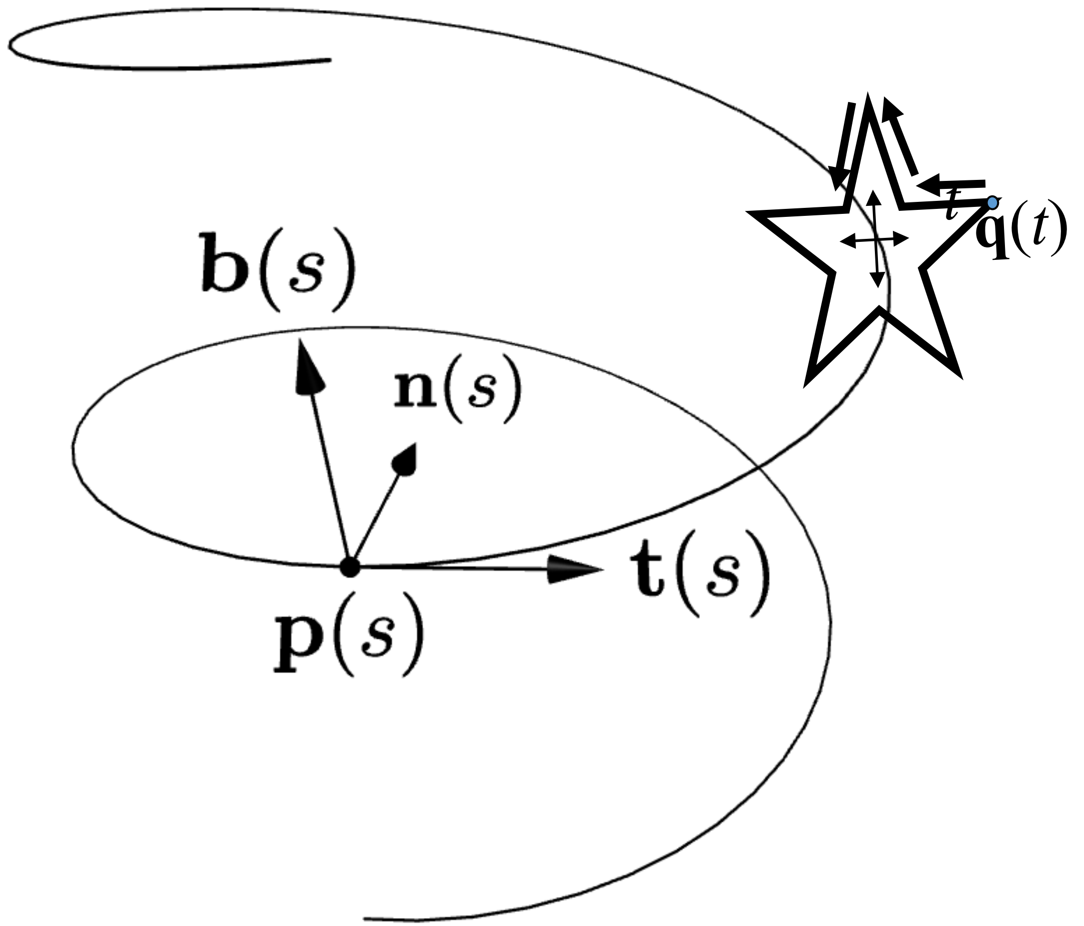

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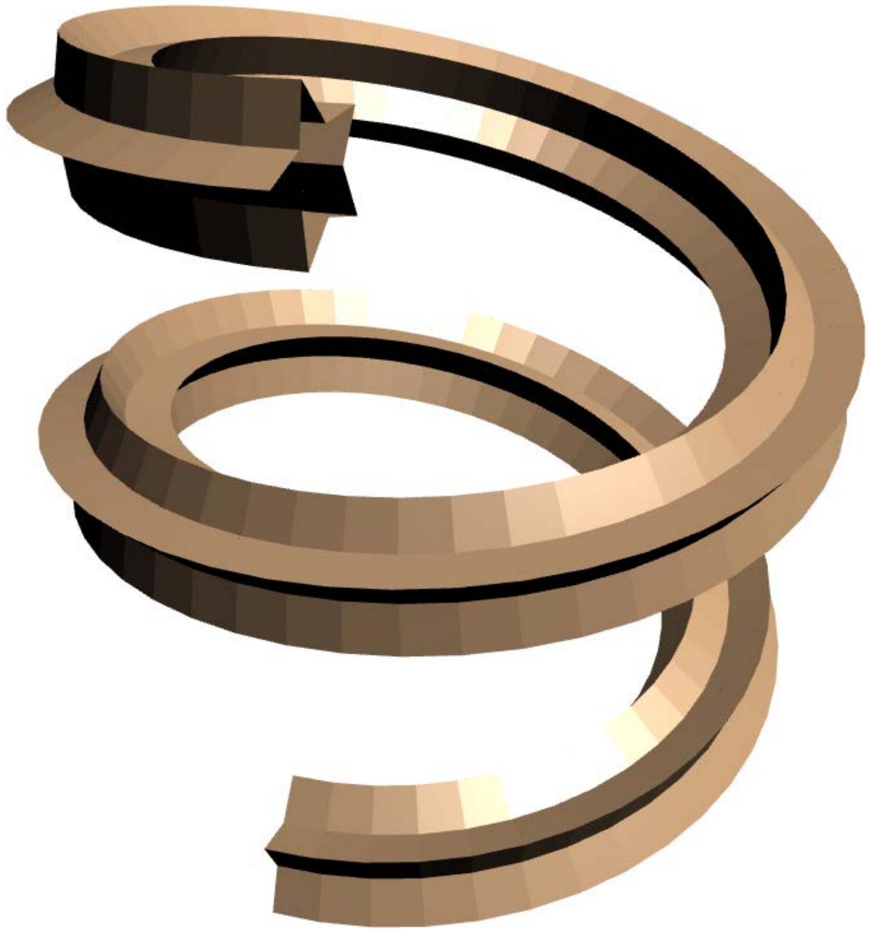


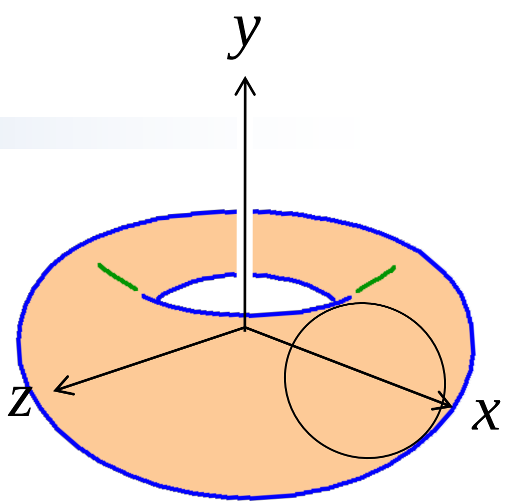
Revolution

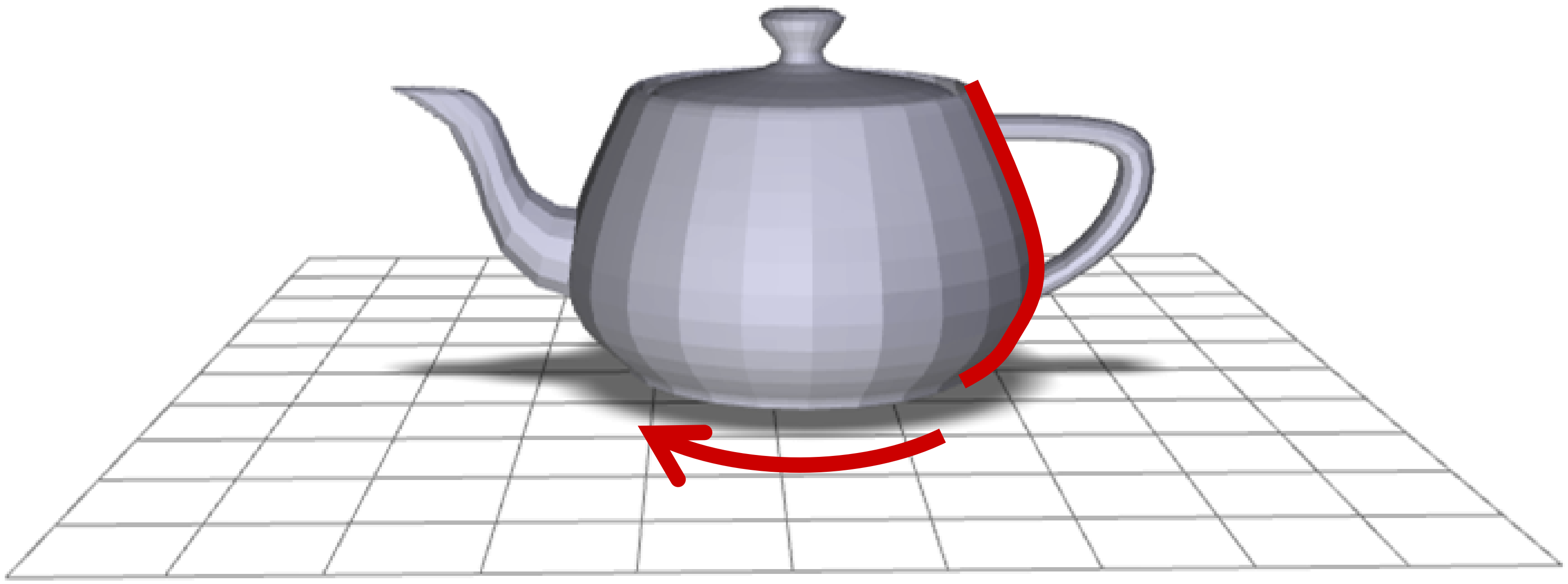
- Construct a 2-D profile curve

$$\mathbf{q}(t) = (a(t), b(t))$$

- Rotate about y axis

$$\mathbf{p}(s,t) = (a(t)\cos 2\pi s,\ b(t),\ a(t)\sin 2\pi s)$$

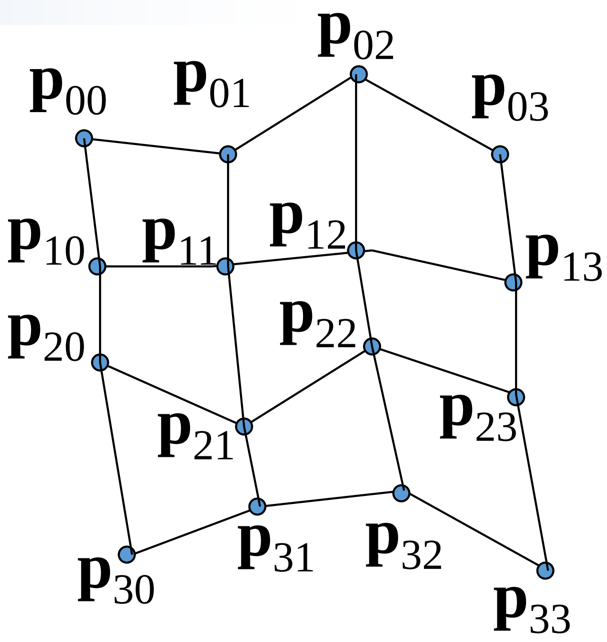




Bezier Patches

- Bezier patch
  - Tensor product of two Bezier curves

$$p(s,t) = \sum_{j=1}^{n}\sum_{i=1}^{n} B_j^n(s) B_i^n(t)\mathbf{p}_{ij}$$



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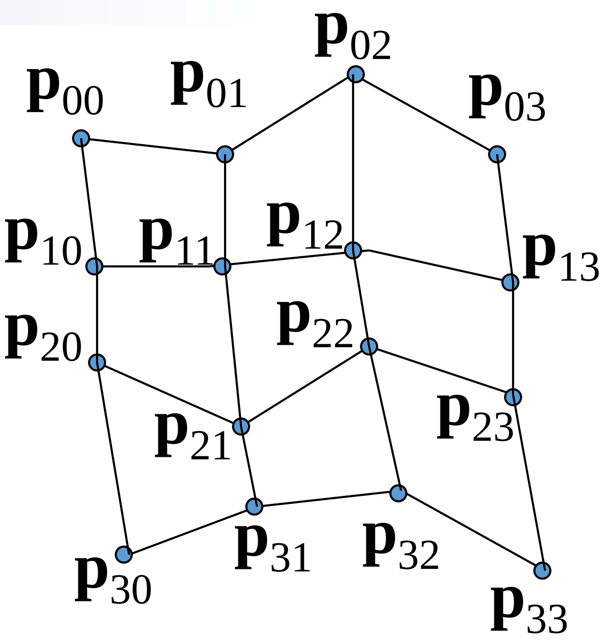

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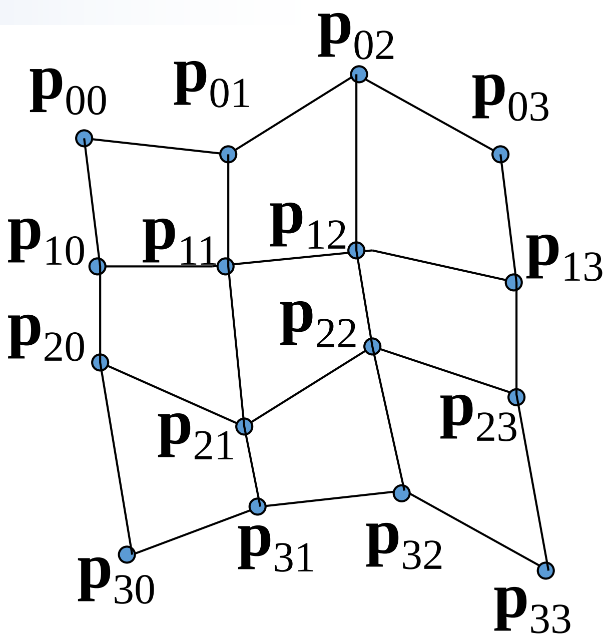

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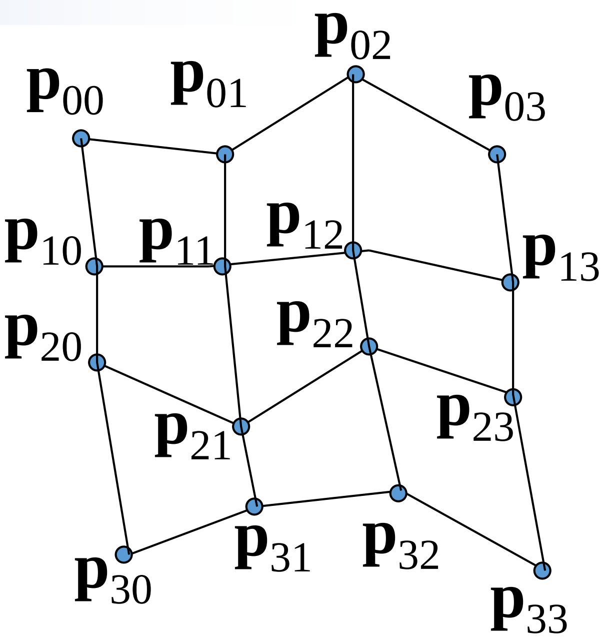
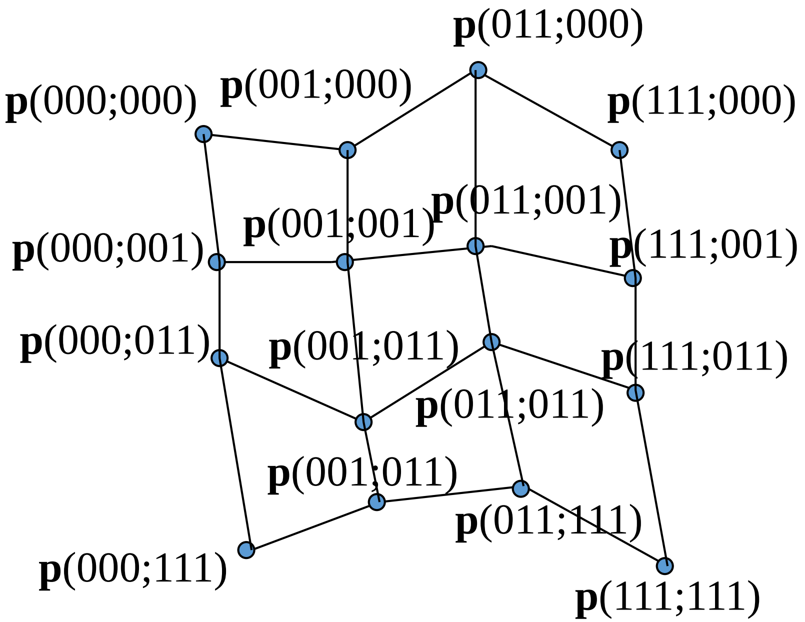
- Works same way for B-splines



Blossoming Patches

- Curves: p(t) → p(t,t,t)
- Patches: p(s,t) → p(s,s,s;t,t,t)
- Variables not allowed to cross the semicolon
- In patches, bilinear interpolation replaces linear interpolation in curves



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