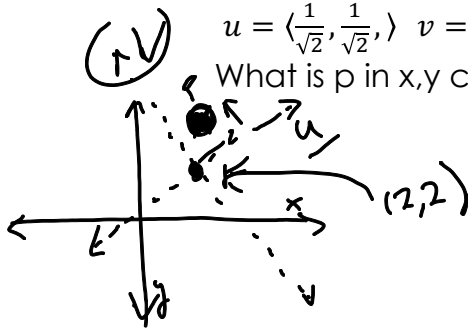


More on Affine Transformations

1. Suppose we have 2D frame with an origin at (2,2) and basis vectors $u = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ $v = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$. Suppose a point $p = (1,1)$ in u,v coordinates.



What is p in x,y coordinates?

$$\begin{aligned} p_x &= 1(\vec{u}) + 1(\vec{v}) + (2,2) \\ &= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle + \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle + (2,2) \\ &= \langle 0, \frac{2}{\sqrt{2}} \rangle + (2,2) \\ &= (2, 2 + \frac{2}{\sqrt{2}}) \end{aligned}$$

2. Construct a matrix that can convert a point in u,v coordinates to x,y coordinates for the basis described above.

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{e} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 2 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

3. What matrix would perform the window-to-viewport transformation specified by the function call `gl.viewport(0,0,960,640)`? For this question, just ignore the z-coordinate and imagine the NDC coordinates to be transformed are in the form $(x, y, 1)$. Express your answer as a 3×3 matrix and express rational numbers as fractions.

In WebGL, the window to viewport transformation is performed by the WebGL library during primitive assembly. You control it using the call `gl.viewport(x, y, w, h)`. It specifies the viewport by giving the lower-left coordinate of the viewport with (x, y) and the width and height of the viewport with w and h .

$$\begin{bmatrix} 959/2 & 0 & 0 \\ 0 & 639/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 959/2 & 0 & 959/2 \\ 0 & 639/2 & 639/2 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Suppose we have the following set of transformations that map a point in view (or camera) space to world coordinates. What transformations will map world coordinates to camera space?

$$\left(\begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} =$$

$$\left(M_1 M_2 \dots M_{N-1} M_N \right)^{-1}$$

$$M_N^{-1} M_{N-1}^{-1} \dots M_2^{-1} M_1^{-1}$$

$$\begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bonus question:

What is the inverse of matrix that performs a scale transformation?

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \rightarrow \begin{bmatrix} 1/s_x & & & \\ & 1/s_y & & \\ & & 1/s_z & \\ & & & 1 \end{bmatrix}$$