2D Geometry

For the following questions, assume we are working in 2D using homogenous coordinates.

1. Which of the following sets of transformations commute?
   a. A rotation and a uniform scaling
   b. A rotation and a squash (non-uniform scaling)
   c. A rotation and a translation
   d. Two translations

2. Create a single matrix that encodes the following transformations:
   a. Translate by +2 in X
   b. Scale by a factor of 4 uniformly

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 8 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Imagine you had a line segment with endpoints (2,1) and (4,1). Can you construct a transformation matrix that will rotate the segment by 90 degrees around its midpoint?

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Suppose a line is defined by the points (2,4) and (10,6). Write the equation in a parametric form.

$$x(t) = (1 - t)2 + (t)10$$
$$y(t) = (1 - t)4 + (t)6$$