Rational Curves

CS 418

Interactive Computer Graphics

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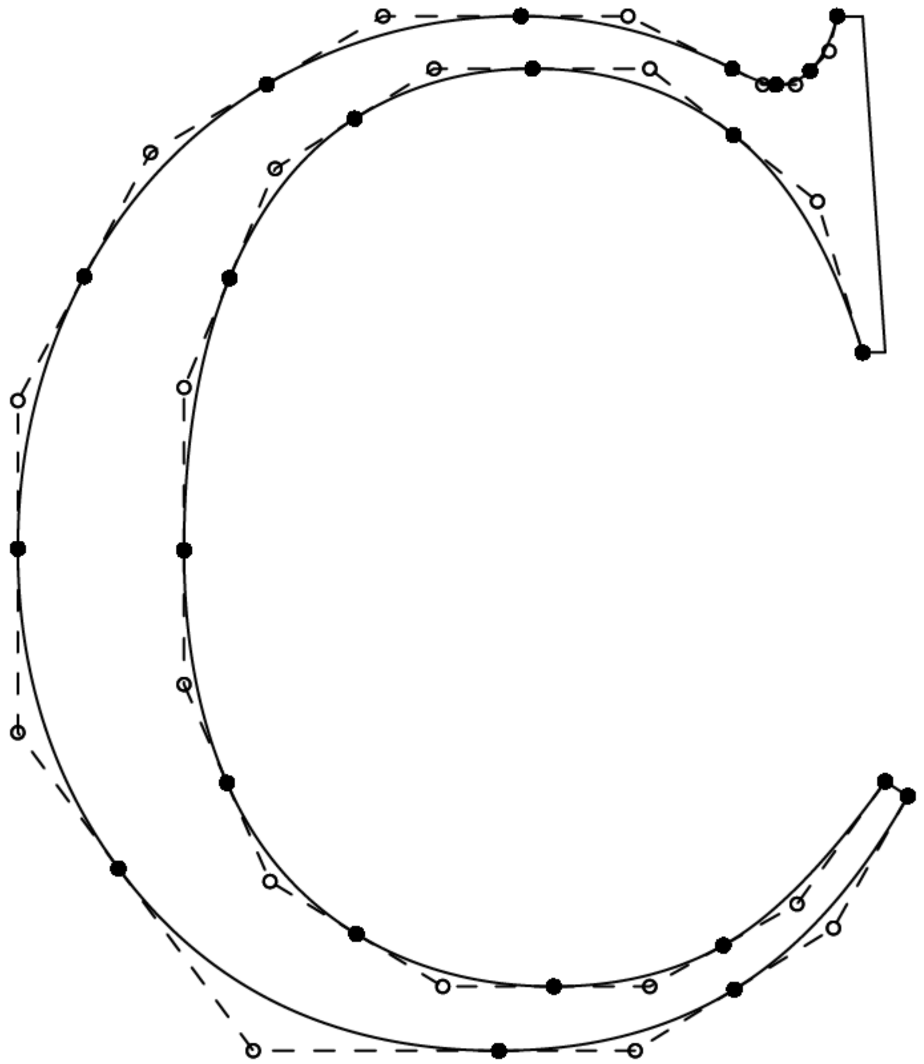
2-D Quadratic Bezier Curves

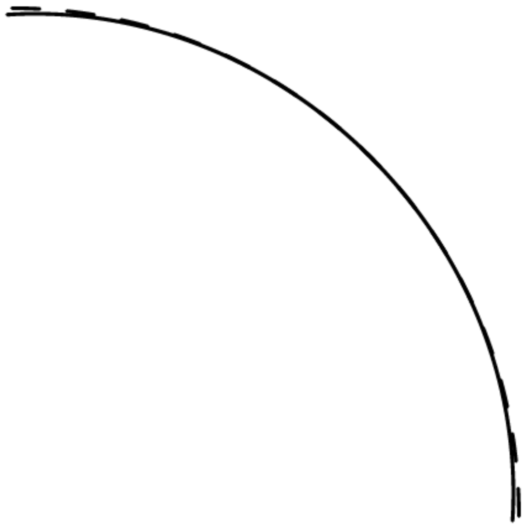
- Three control points

$\mathbf{p}(t) = (1\text{-}t)^2\, \mathbf{p}_0 + 2t(1\text{-}t)\, \mathbf{p}_1 + t^2\, \mathbf{p}_2$

- Always planar because of convex hull property and any three points always lie in some plane
- Used for True-Type fonts

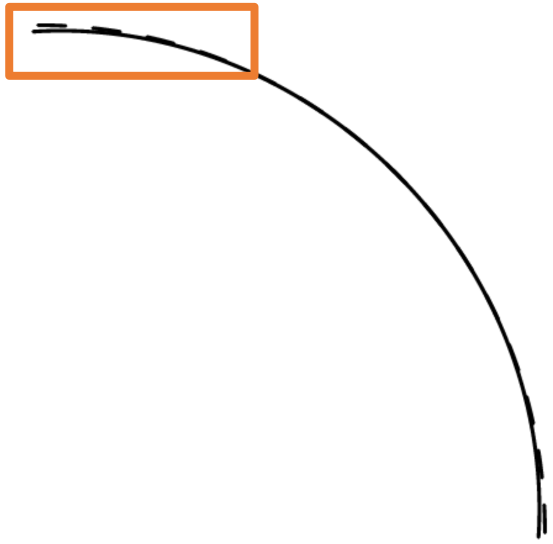


Cubic Arc Approximation

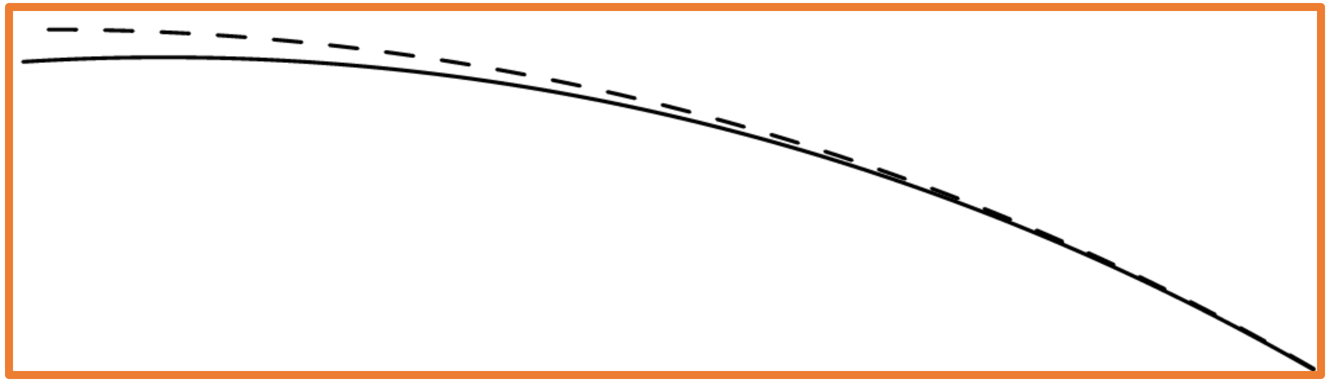


$$
\begin{aligned}
x(t) &= C - S\left(t-\frac{\pi}{4}\right) - C\frac{\left(t-\frac{\pi}{4}\right)^2}{2} + S\frac{\left(t-\frac{\pi}{4}\right)^3}{6},\\
y(t) &= S + C\left(t-\frac{\pi}{4}\right) - S\frac{\left(t-\frac{\pi}{4}\right)^2}{2} - C\frac{\left(t-\frac{\pi}{4}\right)^3}{6},
\end{aligned}
$$

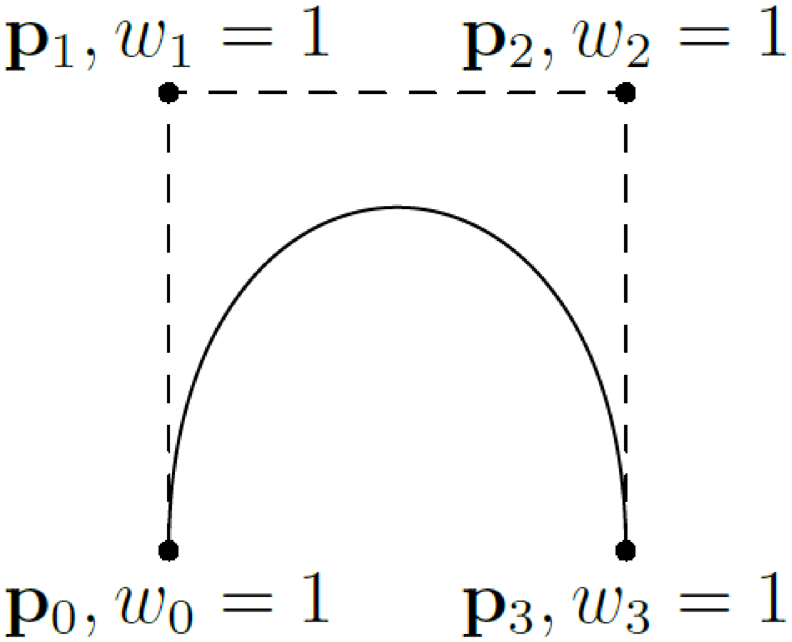
Cubic Arc Approximation



$$\begin{aligned} x(t) &= C - S\left(t - \frac{\pi}{4}\right) - C\frac{\left(t-\frac{\pi}{4}\right)^2}{2} + S\frac{\left(t-\frac{\pi}{4}\right)^3}{6}, \\ y(t) &= S + C\left(t - \frac{\pi}{4}\right) - S\frac{\left(t-\frac{\pi}{4}\right)^2}{2} - C\frac{\left(t-\frac{\pi}{4}\right)^3}{6}, \end{aligned}$$

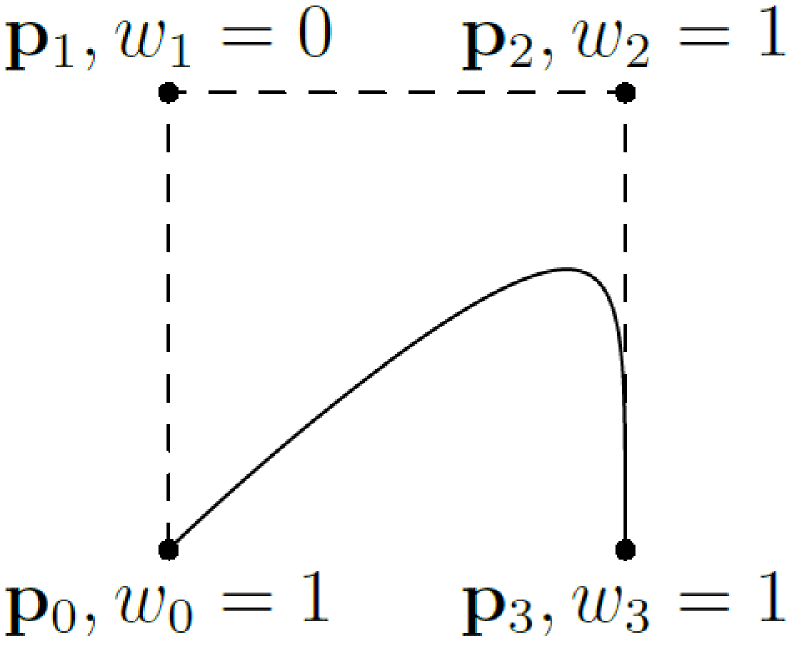


Rational Bezier Curves



$$\mathbf{p}(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) \mathbf{p}_i}{\sum_{i=0}^{n} w_i B_i^n(t)},$$

Rational Bezier Curves



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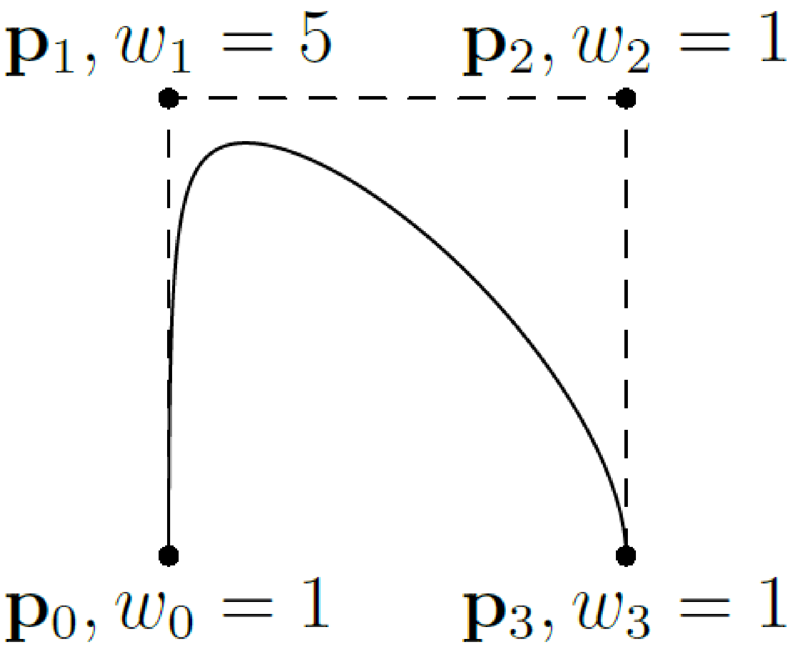
Rational Bezier Curves



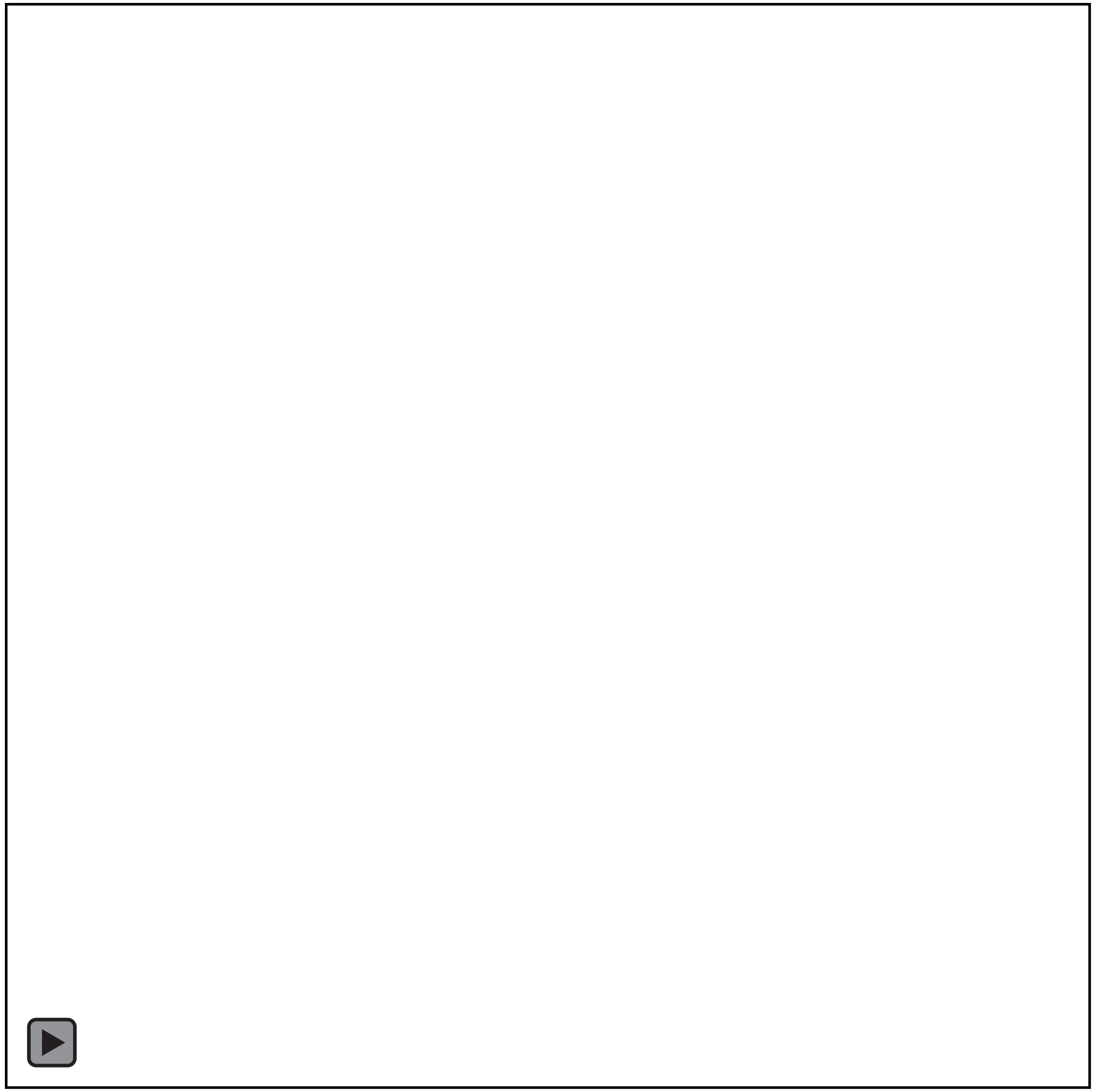
$$\mathbf{p}(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) \mathbf{p}_i}{\sum_{i=0}^{n} w_i B_i^n(t)},$$

Homogeneous Control Points

- Think of the control point $\mathbf{p}_i = (x_i, y_i)$ with weight w_i as a homogeneous control point $\mathbf{P}_i = (w_i\, x_i,\, w_i\, y_i,\, w_i)$
- Then $\mathbf{P}(t) = (w\, x,\, w\, y,\, w)$ and $\mathbf{p}(t) = \mathbf{P}(t)/w(t)$
- $\mathbf{P}(t)$ is an ordinary 3-D B-spline
- $w(t)$ is the denominator

$$\mathbf{p}(t) = \frac{\sum_{i=0}^{n} w_i B_i^n(t) \mathbf{p}_i}{\sum_{i=0}^{n} w_i B_i^n(t)},$$

Homogeneous Control Points



Rational Bezier Arc