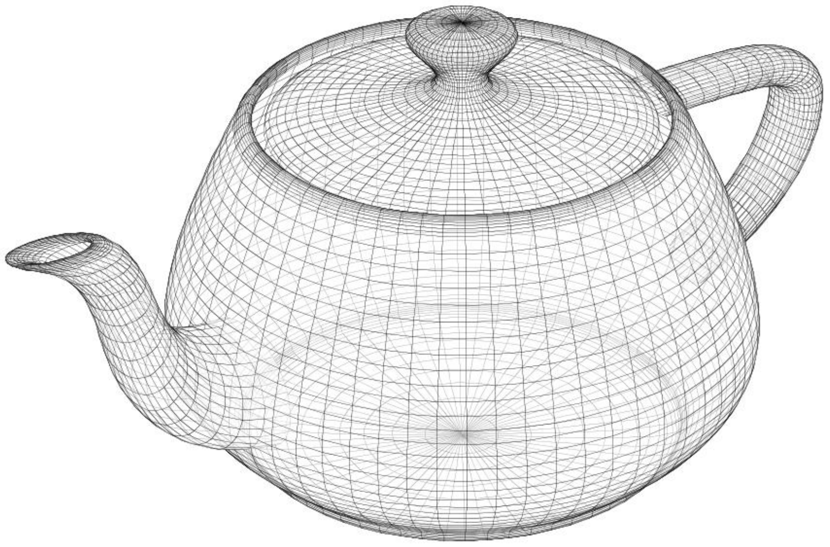


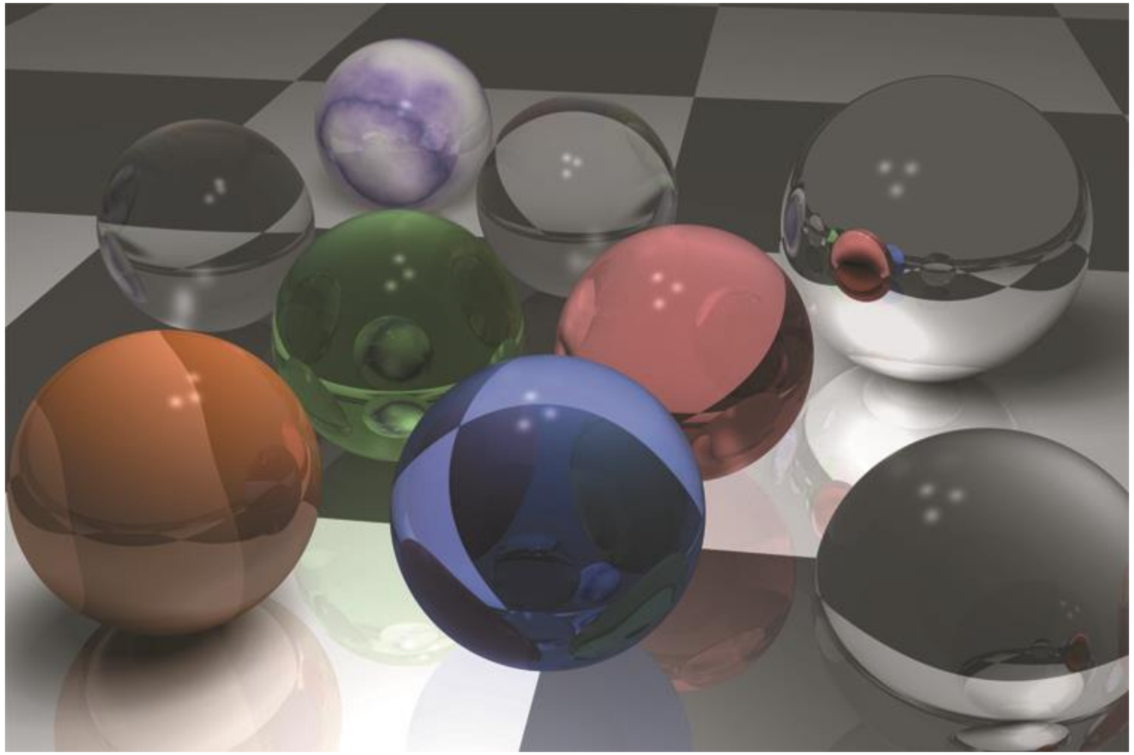
Transparency

Production Computer Graphics

Eric Shaffer

ILLINOIS

Transparency



Transparent media allow light to pass through them

Glass, clear plastic, air, water are all transparent media

A subset of materials know *dielectrics* which are also insulators

Any guesses as to how water can be on that list?

Index of Refraction

- Speed of light is c = 2.99 x 10^7 in a vacuum
  - Speed v is lower through a medium like air
- Absolute index of refraction $\eta = c/v$

Index of Refraction for various media

Media	Index of Refraction
Vacuum	1.00
Air	1.0003
Carbon dioxide gas	1.0005
Ice	1.31
Pure water	1.33
Ethyl alcohol	1.36
Quartz	1.46
Vegetable oil	1.47
Olive oil	1.48
Acrylic	1.49
Table salt	1.51
Glass	1.52
Sapphire	1.77
Zircon	1.92
Cubic zirconia	2.16
Diamond	2.42
Gallium phosphide	3.50

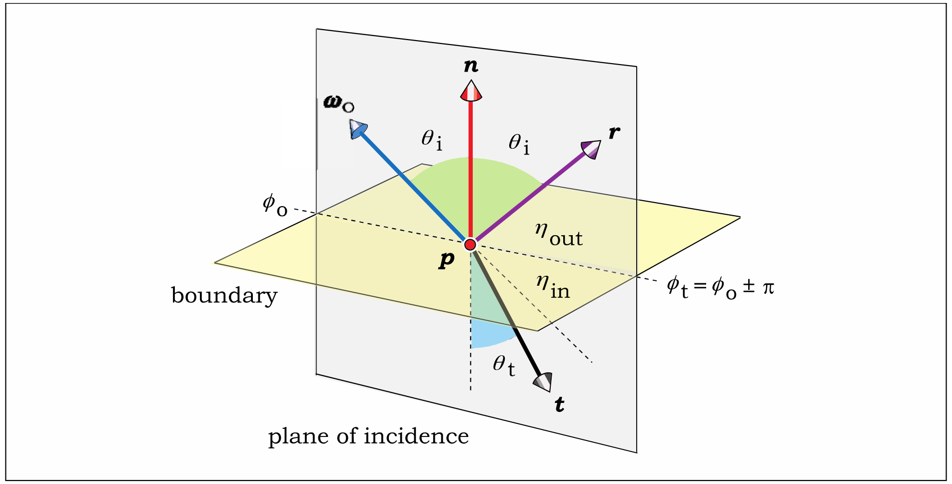
Surface Physics

When a ray hits the surface of a transparent medium

- A reflection ray r is generated
- A transmission ray t is generated

If the boundary is optically smooth

- ω_o, t, and r are in the plane of incidence
- Transmission will be perfectly specular
- Optically smooth → rough features much smaller than light λ



Surface Physics

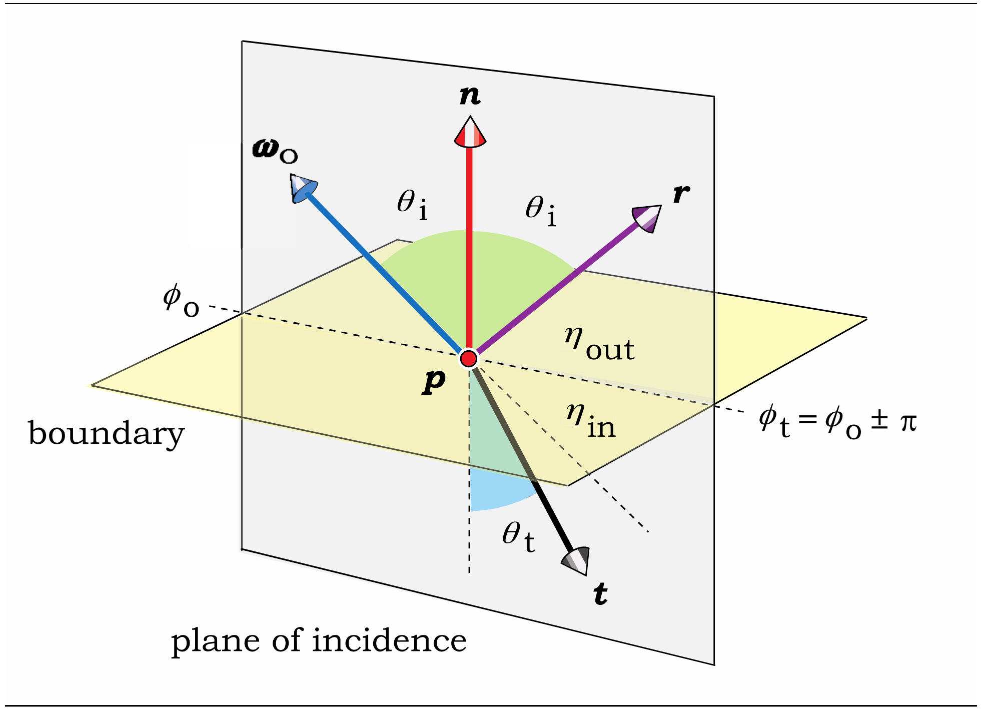
- Relative Index of Refraction is $\eta = \eta_{in}/\eta_{out}$
- Need to find a direction for the ray t using Snell's law

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_{in}}{\eta_{out}} = \eta$$

$$\boldsymbol{t} = \frac{1}{\eta}\boldsymbol{\omega}_o - \left(\cos\theta_t - \frac{1}{\eta}\cos\theta_i\right)\boldsymbol{n},$$

$$\cos\theta_i = \boldsymbol{n} \bullet \boldsymbol{\omega}_o$$

$$\cos\theta_t = \left[1 - \frac{1}{\eta^2}(1 - \cos^2\theta_i)\right]^{1/2}$$



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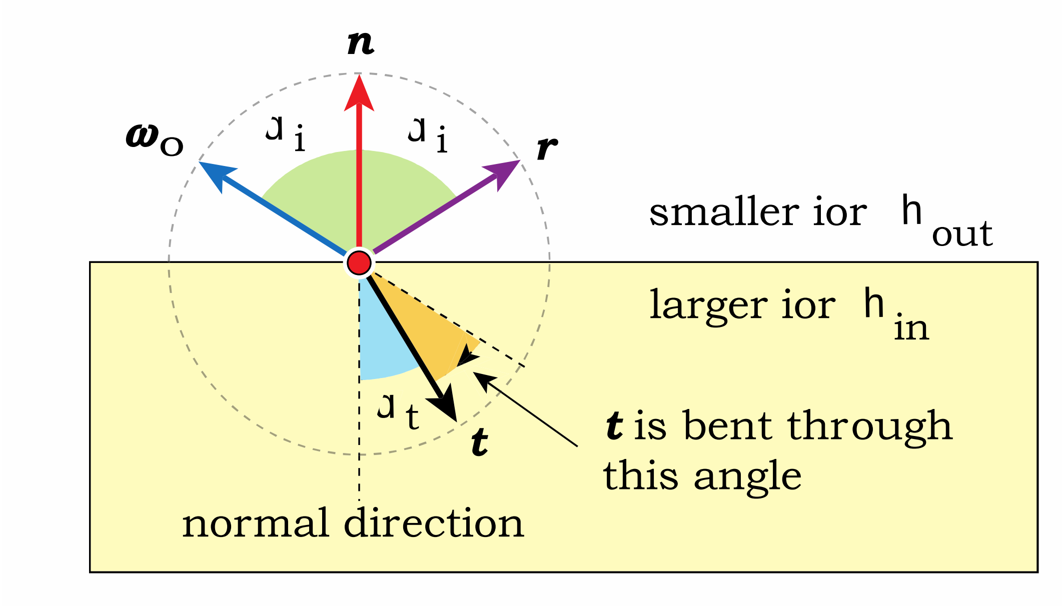
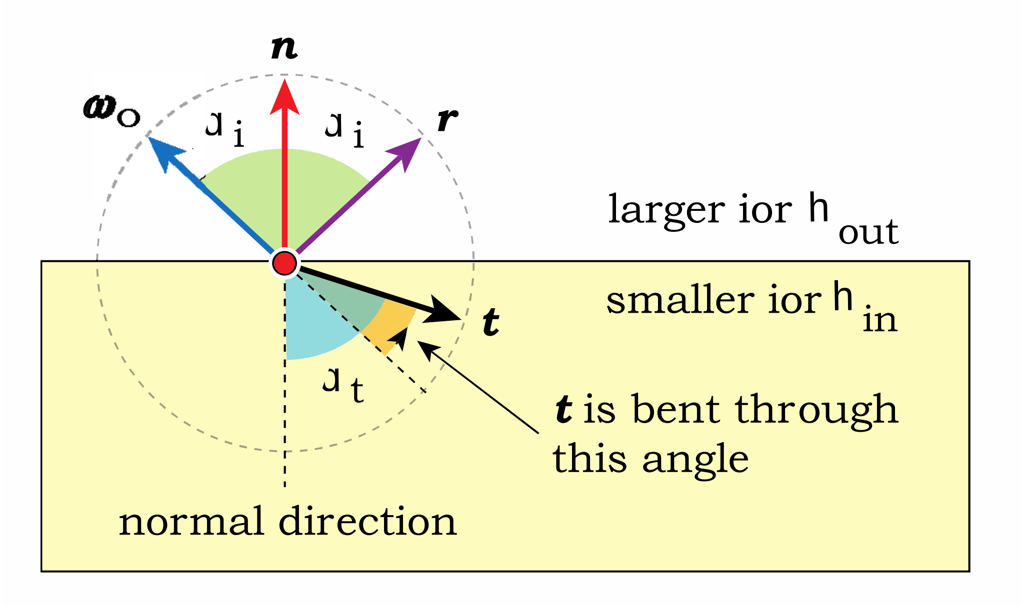
Surface Physics

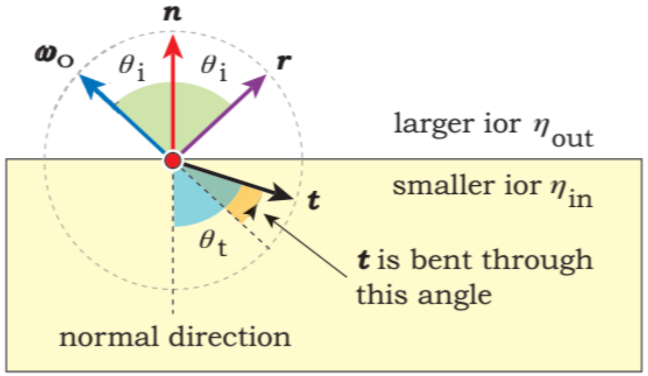
t is bent away from ω_o

- $\eta_{out} < \eta_{in}$ it bends towards the normal
- $\eta_{out} > \eta_{in}$ it bends away from the normal





Total Internal Reflection



As the transmission direction approaches the surface

- The energy in the transmission ray decreases
- The energy in the reflection ray increases

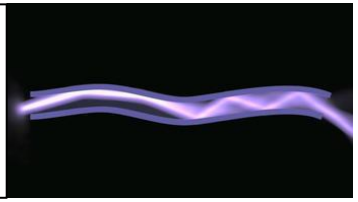
When θ_i exceeds the critical angle

- The transmission ray ceases to exist
- All the energy is contained in the reflection ray

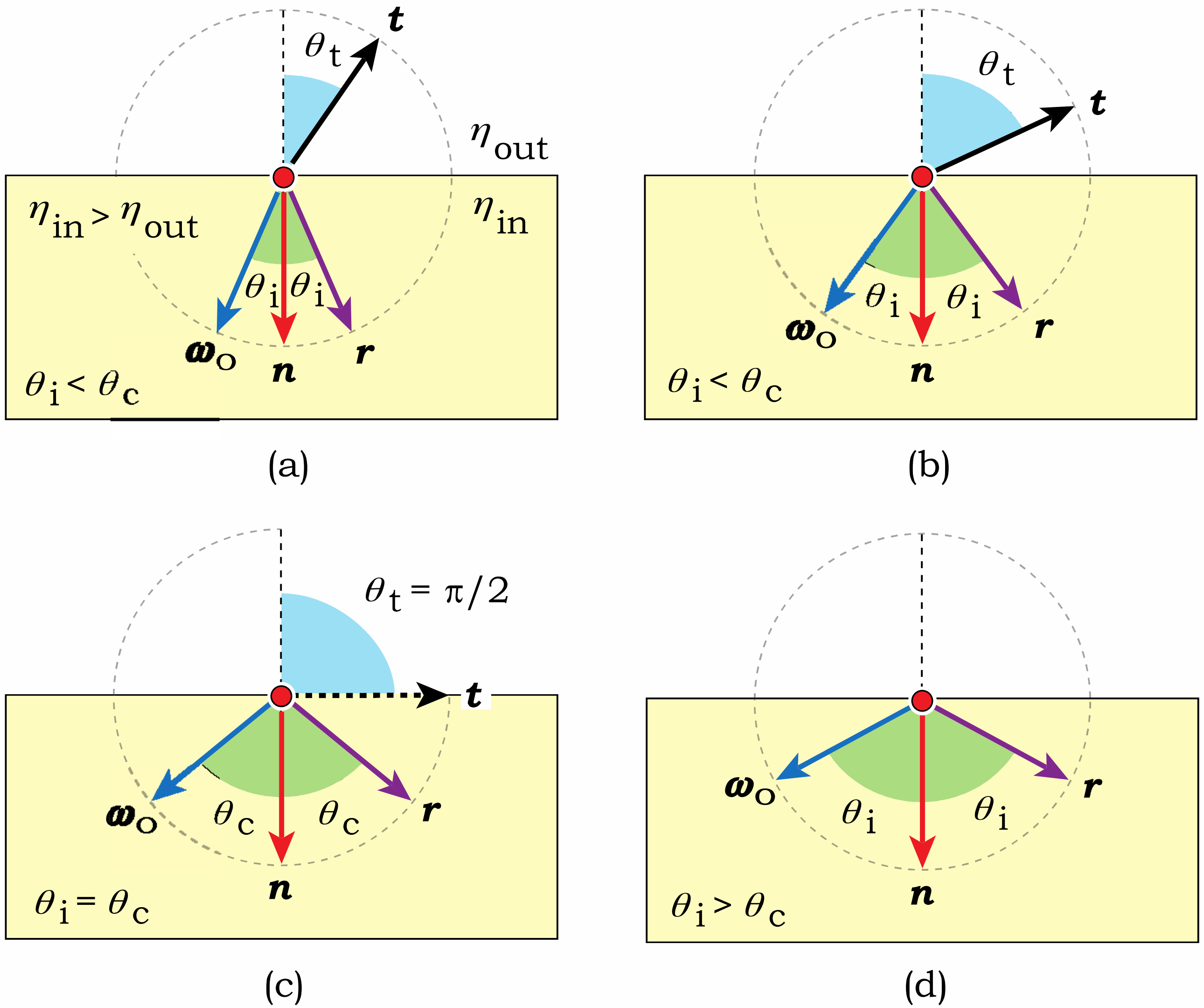
You can test for that condition

$$1 - \frac{1}{\eta^2}\left(1 - \cos^2 \theta_i\right) < 0$$

It's how fiber-optic cable works



Total Internal Reflection



The Illumination Model

Illumination model is similar to that for reflection

- But indirect component includes transmission plus reflection

Instead of a BRDF we have a BTDF

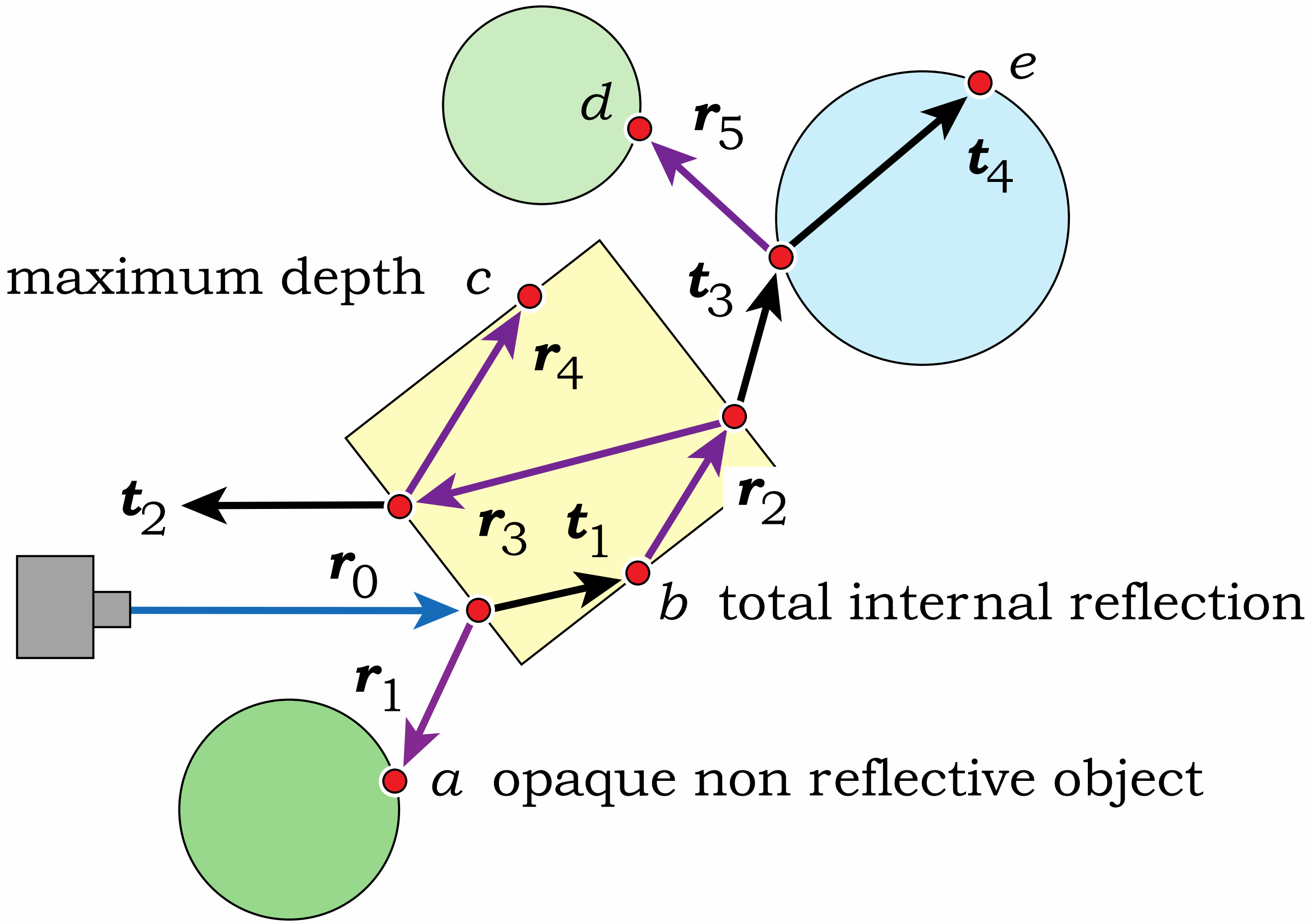
- bidirectional transmission distribution function

$$L_r(p,\omega_o) = L_{direct}(p,\omega_o) + L_{indirect}(p,\omega_o)$$

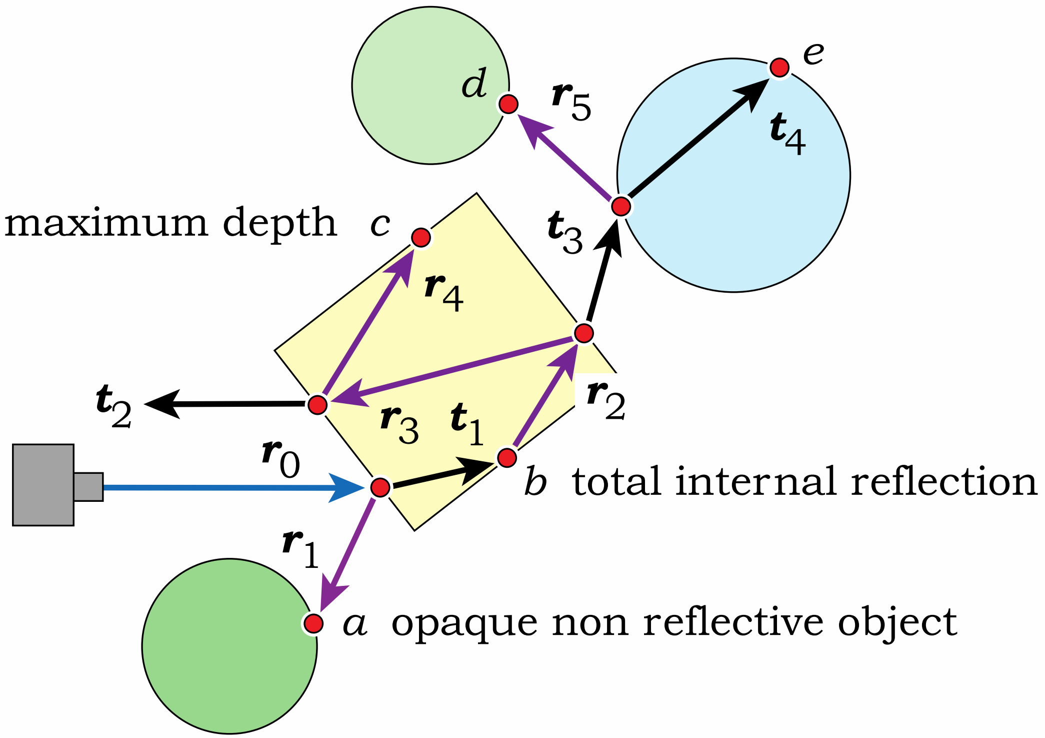
$$L_{indirect}(p,\omega_o) = L_r(p,\omega_o) + L_t(p,\omega_o)$$

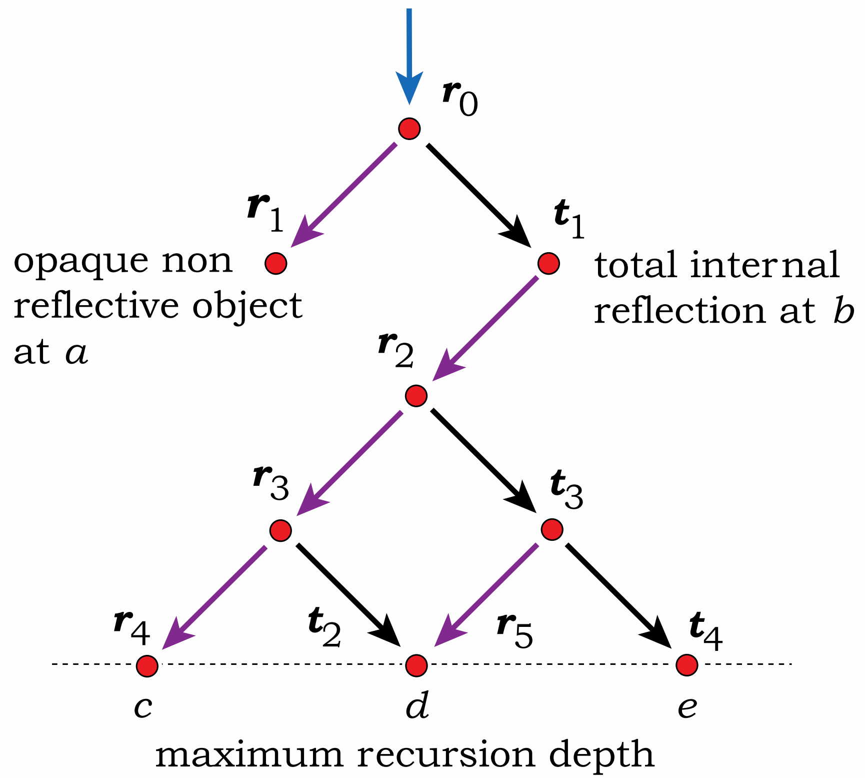
$$L_t(p,\omega_o) = \int_{2\pi^+} f_{t,s}(p,\omega_i,\omega_o) L_o(r_c(p,\omega_i),-\omega_i) \left|\cos\theta_i\right| d\omega_i$$

The Illumination Model



The Illumination Model





Illumination Model

Exitant radiance at point p is:

$$L_r(p, \omega_o) = L_{direct}(p, \omega_o) + L_{indirect}(p, \omega_o),$$

$$L_{indirect}(p, \omega_o) = L_r(p, \omega_o) + L_t(p, \omega_o)$$

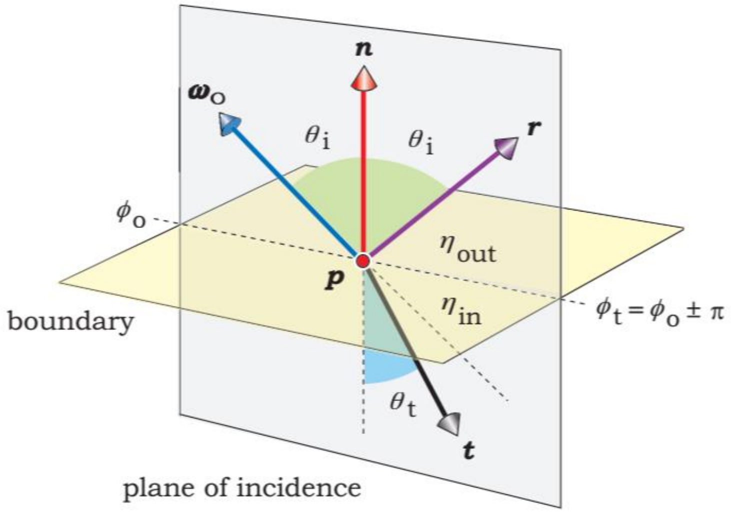
$$L_t(p,\omega_o) = \int_{2\pi^-} f_{t,s}(p,\omega_i,\omega_o) L_o(r_c(p,\omega_i), -\omega_i) |\cos\theta_i| d\omega_i$$

$$f_{t,s}(p,\omega_i,\omega_o) = k_t \left(\frac{\eta_t^2}{\eta_i^2}\right) \frac{\delta(\omega_i - t(n,\omega_o))}{|\cos\theta_i|}$$

$k_t \in [0, 1]$ is the *transmission coefficient*.

$k_r + k_t = 1$

$$L_t(p,\omega_o) = k_t \left(\frac{\eta_t^2}{\eta_i^2}\right) L_i(p,\omega_i)$$



$$\delta(\omega_i - t(n, \omega_o))$$

selects a single direction along which radiance is transmitted

The Illumination Model

Since we model perfect refraction

- We do not need to compute an integral

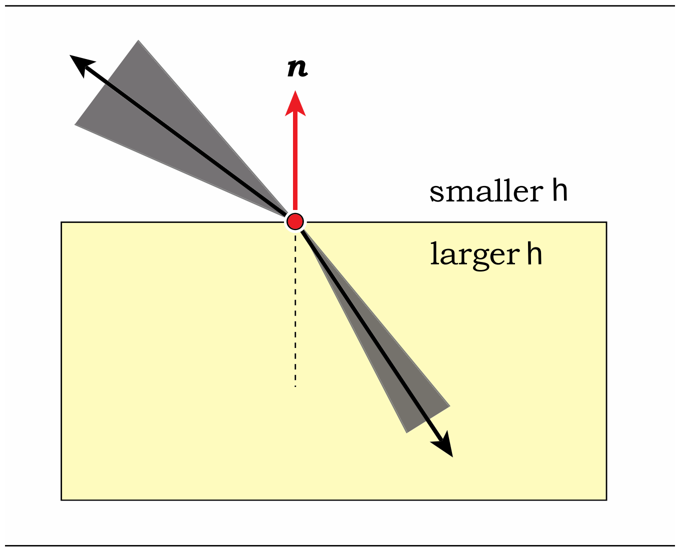
- Radiance changes as light crosses the boundary

$$L_t = k_t \frac{\eta_i^2}{\eta_t^2} L_i$$

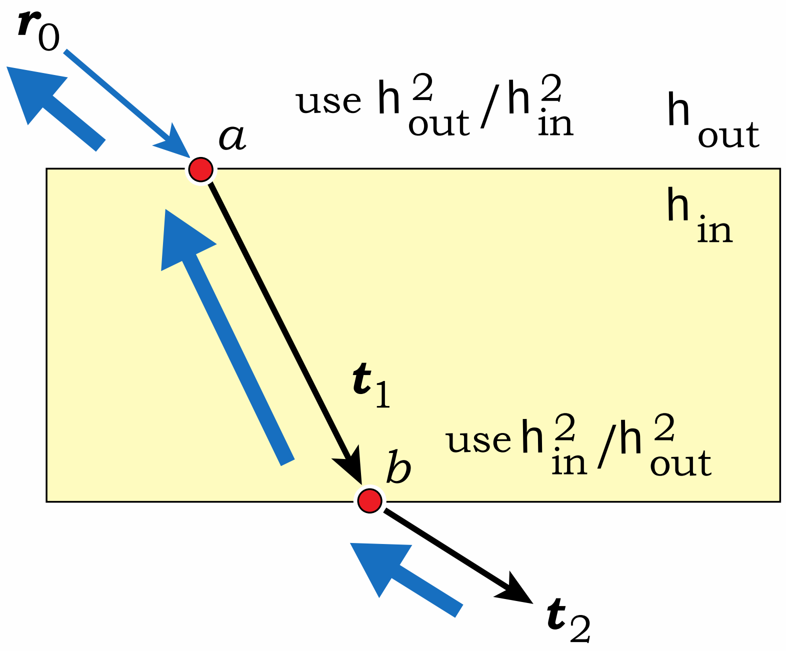
- k_t is the transmission coefficient [0,1]

$$f_{t,s}(p,\omega_i,\omega_o) = k_t \left(\frac{\eta_i^2}{\eta_t^2}\right) \frac{\delta(\omega_i - t(n,\omega_o))}{|\cos(\theta_i)|}$$

$$L_t(p,\omega_o) = k_t \left(\frac{\eta_i^2}{\eta_t^2}\right) L_i(p,\omega_i)$$



The Illumination Model



Need to keep track of the direction of radiance transfer

- Choose of refraction terms correctly

At point *a* radiance transfer is from inside to outside

At point *b* the transfer is from outside to inside

Implementation Considerations

Total internal reflection

- Can happen when a ray strikes a transparent surface from the inside or outside
- When this happens, reflection coefficient should be set to 1.0

On a ray hit from inside an object, to compute **t**

- reverse the normal direction
- invert the relative index of refraction
- change sign of cos θ_i

Example Code

```
RGBColor
Whitted::trace_ray(const Ray ray, const int depth) const {
     if (depth > world_ptr->vp.max_depth)
          return (black);
     else {
          ShadeRec sr(world_ptr->hit_objects(ray));

          if (sr.hit_an_object) {
               sr.depth = depth;
               sr.ray = ray;

               return (sr.material_ptr->shade(sr));
          }
          else
               return (world_ptr->background_color);
     }
}
```

Tracing a primary ray

Example Code

```
RGBColor
Transparent::shade(ShadeRec& sr) {
	RGBColor L(Phong::shade(sr));

	Vector3D wo = -sr.ray.d;
	Vector3D wi;
	RGBColor fr = reflective_brdf->sample_f(sr, wo, wi);	// computes wi
	Ray reflected_ray(sr.hit_point, wi);

	if(specular_btdf->tir(sr))
		L += sr.w.tracer_ptr->trace_ray(reflected_ray, sr.depth + 1);
		// kr = 1.0
	else {
		Vector3D wt;
		RGBColor ft = specular_btdf->sample_f(sr, wo, wt);	// computes wt
		Ray transmitted_ray(sr.hit_point, wt);

		L += fr * sr.w.tracer_ptr->trace_ray(reflected_ray, sr.depth + 1)
			* fabs(sr.normal * wi);
		L += ft * sr.w.tracer_ptr->trace_ray(transmitted_ray, sr.depth + 1)
			* fabs(sr.normal * wt);
	}

	return (L);
}
```

Generates RGB color returned by a ray hitting a transparent object....

Example Code

```
RGBColor
PerfectTransmitter::sample_f(const ShadeRec& sr,
        const Vector3D& wo,
        Vector3D& wt) const {
    Normal n(sr.normal);
    float cos_thetai = n * wo;
    float eta = ior;

    if (cos_theta_i < 0.0) {
        cos_theta_i = -cos_theta_i;
        n = -n;
        eta = 1.0 / eta;
    }

    float temp = 1.0 - (1.0 - cos_theta_i * cos_theta_i) / (eta * eta);
    float cos_theta2 = sqrt(temp);
    wt = -wo / eta - (cos_theta2 - cos_theta_i / eta) * n;

    return (kt / (eta * eta) * white / fabs(sr.normal * wt));
}
```

Generates transmission ray direction wt and returns

$$f_{t,s}(p,\omega_i,\omega_o) = k_t \left(\frac{\eta_t^2}{\eta_i^2}\right) \frac{\delta(\omega_i - t(n,\omega_o))}{|\cos\theta_i|}$$

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Example Code

```
RGBColor
PerfectSpecular::sample_f(const ShadeRec& sr, const Vector3D& wo,
Vector3D& wi) const {
     float ndotwo = sr.normal * wo;
     wi = -wo + 2.0 * sr.normal * ndotwo;

     return (kr * cr / (sr.normal * wi));
}
```

BRDF function for a perfect mirror

```
bool
PerfectTransmitter::tir(const ShadeRec& sr) const {
     Vector3D wo(-sr.ray.d);
     float cos_thetai = sr.normal * wo;
     float eta = ior;

     if (cos_thetai < 0.0)
          eta = 1.0 / eta;

     return (1.0 - (1.0 - cos_thetai * cos_thetai) / (eta * eta) < 0.0);
}
```

Test for total internal reflection

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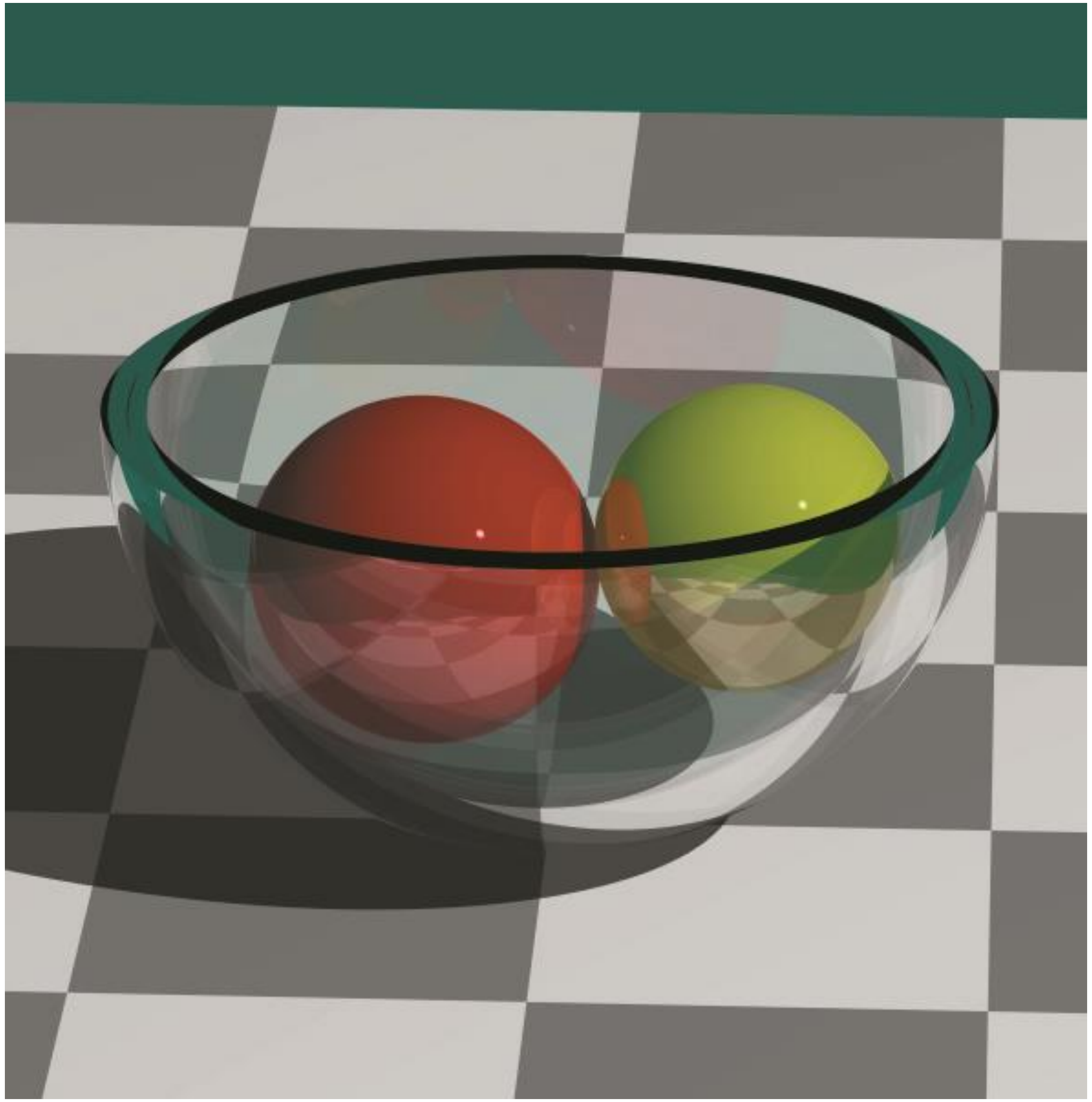
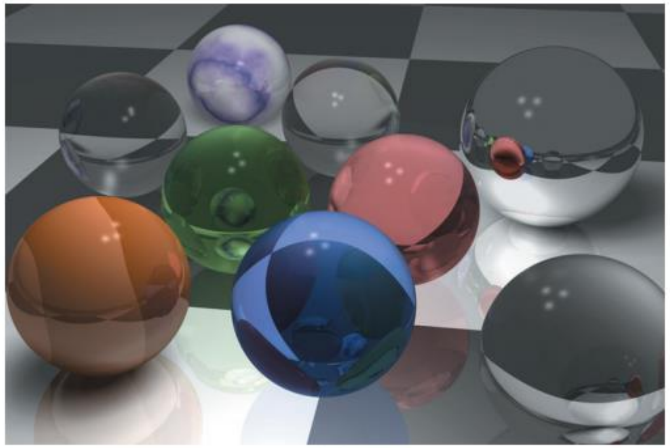
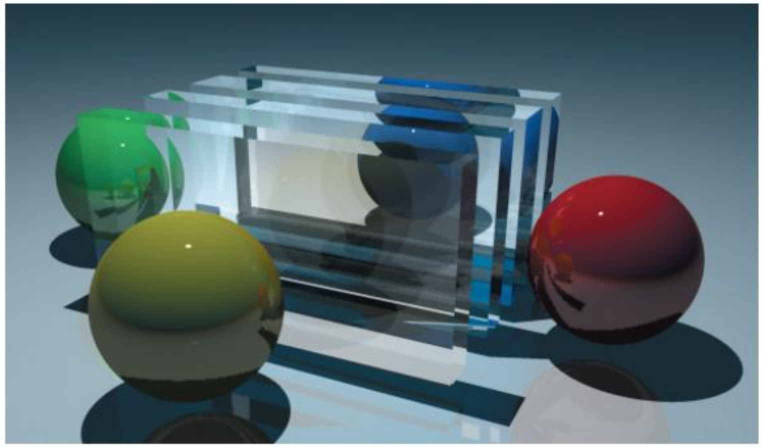
Examples





(a)

(b)



(c)