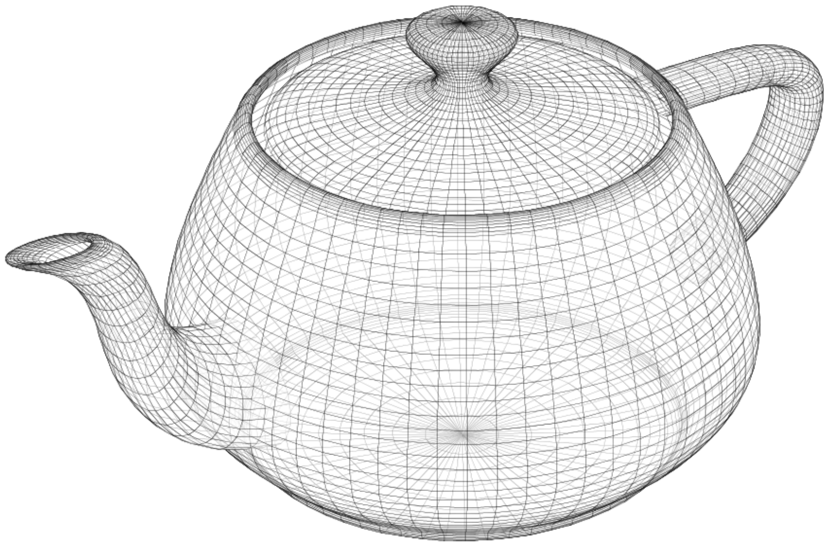
Basic Anti-Aliasing



Production Computer Graphics

Professor Eric Shaffer

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Aliasing

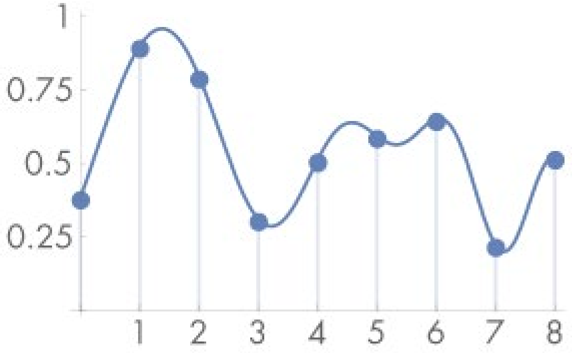
- Aliasing is an effect caused by discrete sampling
- With digital images
  - We have a finite number of pixels
  - We have a finite number of colors
  - …which will not always be able to render a scene accurately
- Some common aliasing phenomena are
  - jaggies
  - moire patterns
  - loss of small details in textures

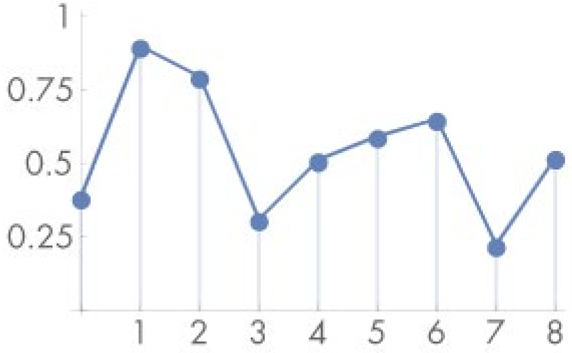
Mario…

Filthy Jagged Original Emulation

Glorious Anti-Aliased PC Emulation

Sampling a Function

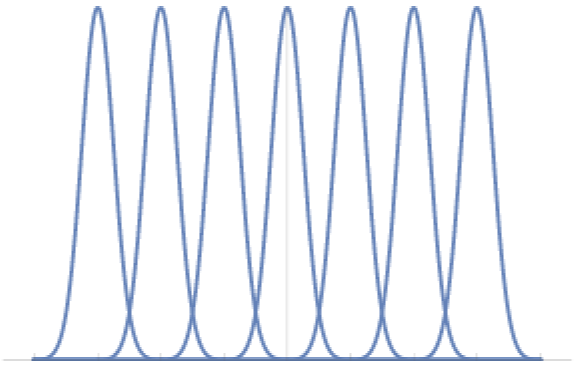




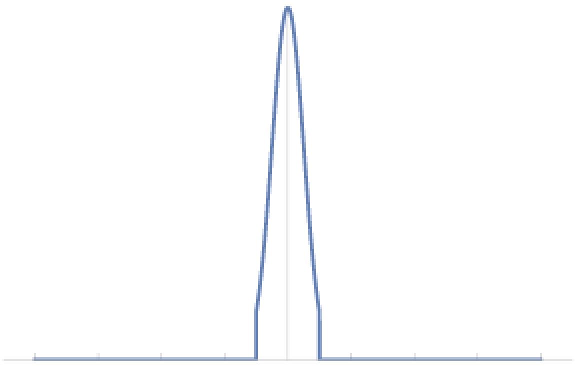
The **Nyquist–Shannon sampling theorem** is a theorem in the field of signal processing which serves as a fundamental bridge between continuous-time signals and discrete-time signals. It establishes a sufficient condition for a sample rate that permits a discrete sequence of *samples* to capture all the information from a continuous-time signal of finite bandwidth.

- Dots indicate point samples
- Can use discrete samples to attempt to reconstruct the original function
- Original function can sometimes be reconstructed exactly from point samples
  - This is remarkable….

Aliasing



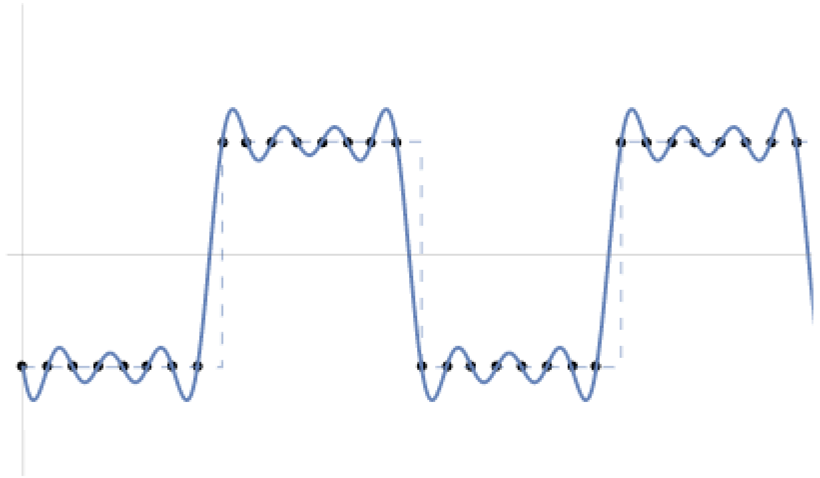
(a)



(b)

- Aliasing: when a function has been reconstructed as a different function
  - Specifically when high frequency details are lost and represented as low frequency

Aliasing in rendering



- Geometry: projected onto the image plane, an object’s boundary introduces a step function
  - the image function’s value instantaneously jumps from one value to another.
- Step functions have infinite frequency content
- The perfect reconstruction filter causes artifacts when applied to aliased samples:
  - ringing artifacts appear in the reconstructed function, an effect known as the Gibbs phenomenon.

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Thinking about Pixels

- When implementing the ray tracer…you should realize we are
  - Dividing an image plane into squares with a given area
  - Reconstructing a function telling use how much light passes through a square

- However…you will read/be told a pixel is not a little square
  - This is also true…if rendering is viewed as a signal processing operation
  - In an image, pixels are point samples of some image function
  - No area associated with point samples

Article | Full-text available

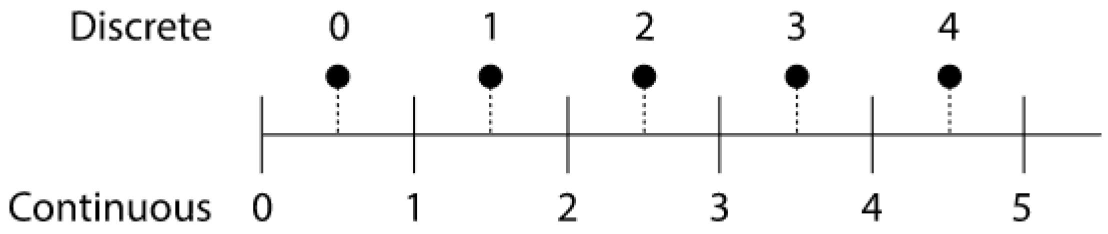
A Pixel Is Not A Little Square, A Pixel Is Not A Little Square, A Pixel Is Not A Little Square

December 1994

Alvy Ray Smith

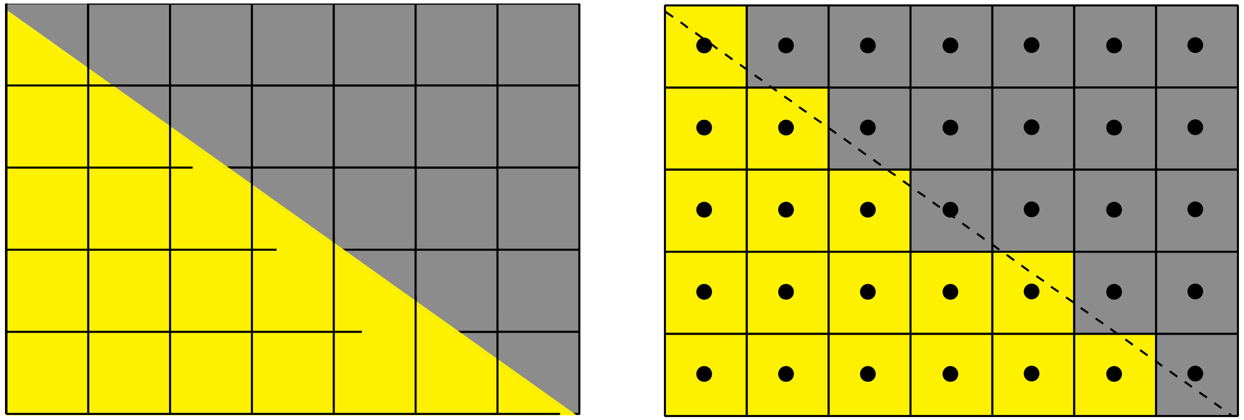
Pixel Addressing

- Pixels have discrete locations in the grid
  - e.g. pixel (3,2) in column 3 and row 2
- Pixels also have a location in a continuous space
  - Continuous conversion from discrete is usually $c = d + \frac{1}{2}$



  - So discrete (3,2) is located at (3.5,2.5)

Aliasing in Ray Tracing



Imagine a yellow polygon in a scene.

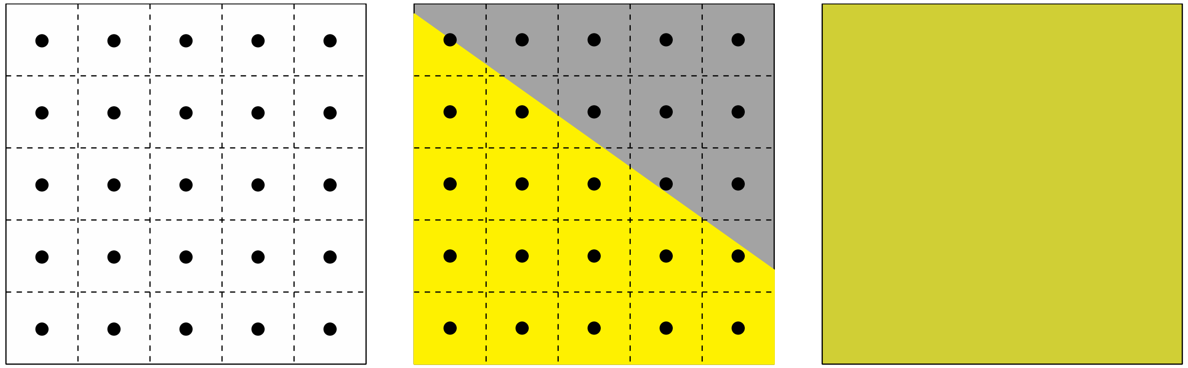
We have a 5x7 array of pixels and shoot rays through the pixel centers

We get a jagged edge….

How could we generate a better approximation?

Anti-aliasing

One remedy to aliasing is to shoot more rays per-pixel

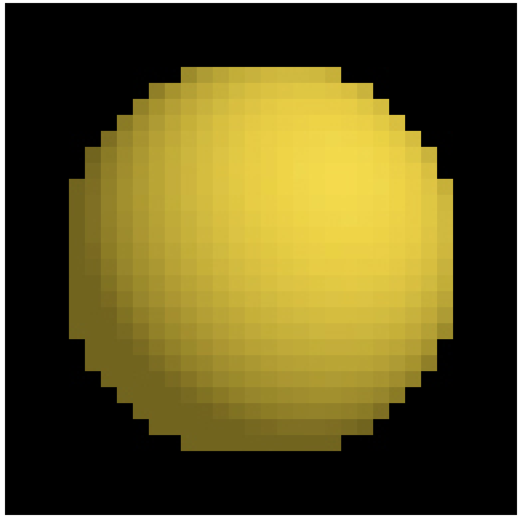


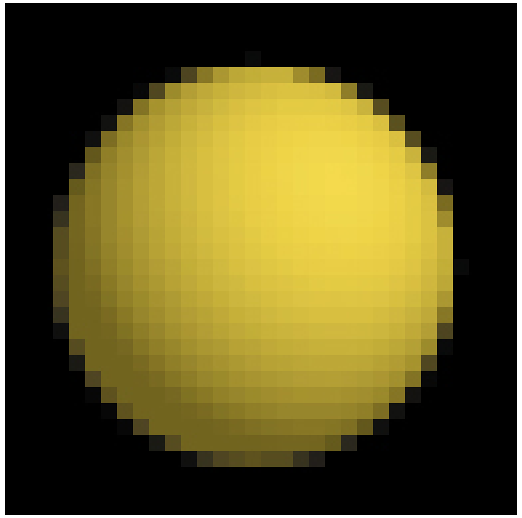
Sampling pattern on a single pixel | Pixel on the edge | Final color

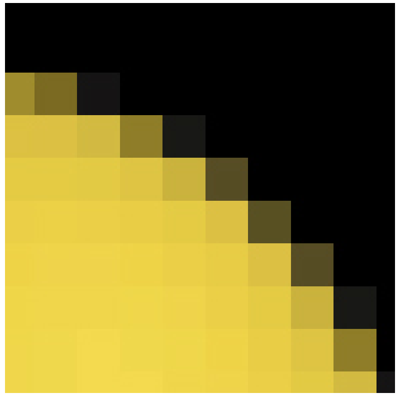
REGULAR SAMPLING

- We use an n by n regular sub-grid and shoot through the sub-pixels
- Color is the average color returned by the sub-samples
- Image shows one pixel with 25 sub-samples

A Ray Tracing Example





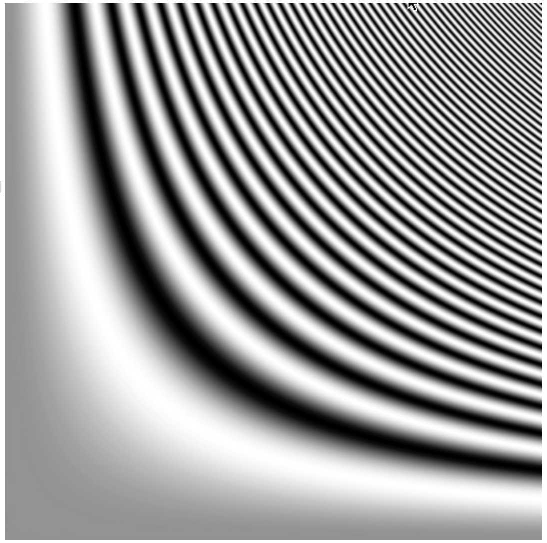


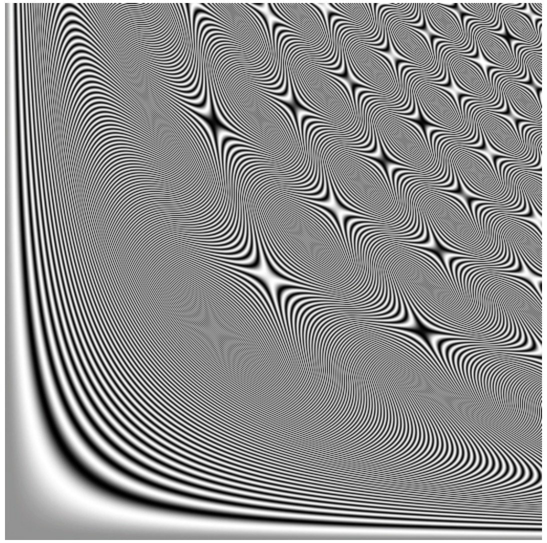
One ray per pixel
Enlarged view

16 rays per pixel

Moire Patterns

$$f(x,y)=\frac{1}{2}(1+\sin(x^2y^2))$$



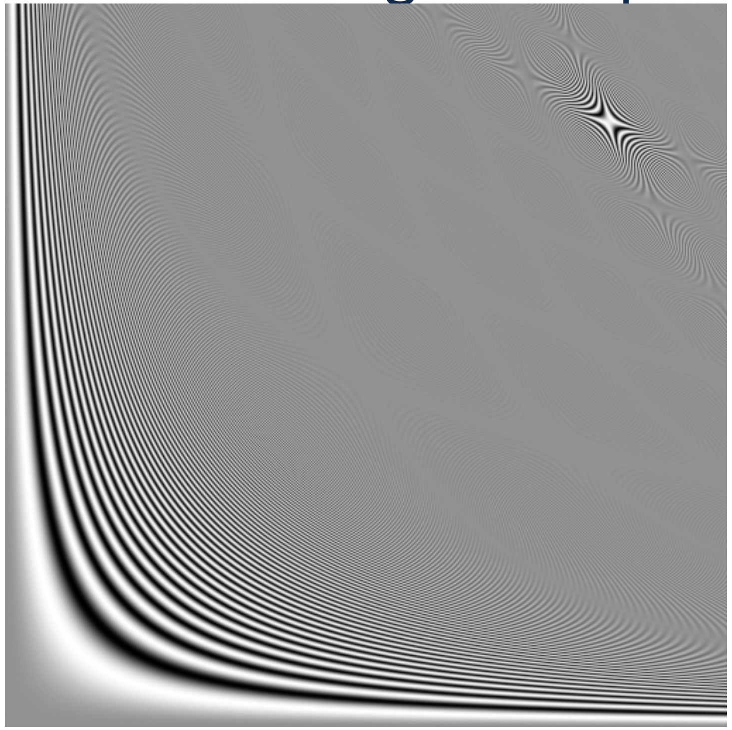


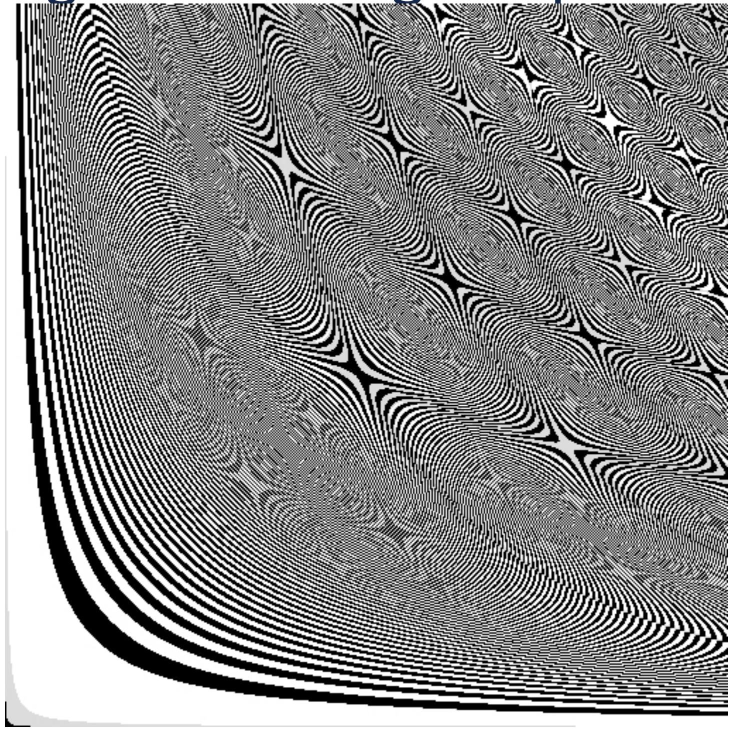
Second images show what happens when we try to squeeze a bigger domain into the same 512x512 pixels

As an aside….how do you ray cast f(x,y) ?

Problems with Regular Sampling

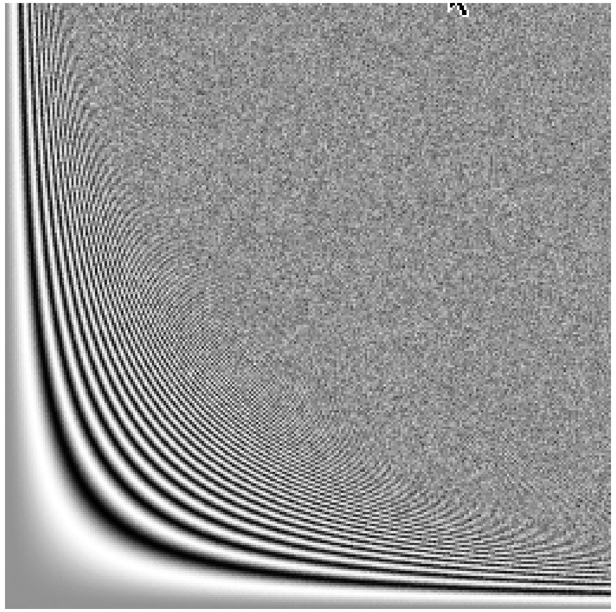
- Still often leads to “regular” artifacts
- Humans are great at perceiving induced regular patterns





Random Sampling

- Could use N random locations in the pixel
- Often makes things look noisy, but…
- …people prefer noise to aliasing visually



2D Sampling

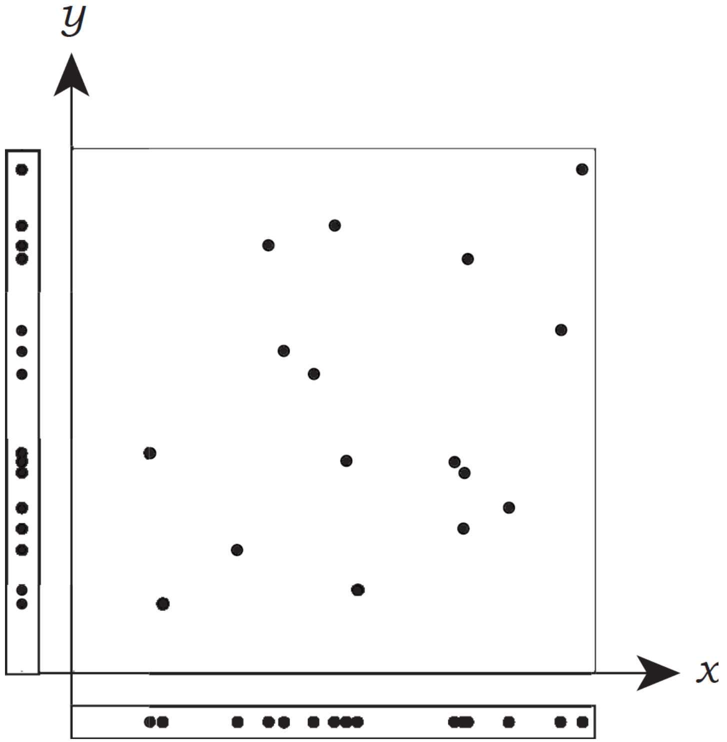
- Assume we are sampling a function on a unit square
- Good sampling
  - Uniform(ish) distribution…avoid gaps and clumps
  - Projections into 1D along x and y are also uniform(ish)
  - There is a non-trivial minimum distance between all sample points
- Such a sample pattern is called *Well-Distributed*

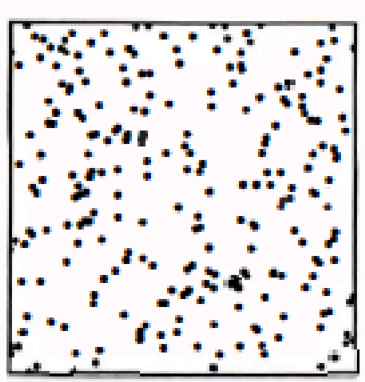
Random

Too irregular

Oversamples some areas…

Undersamples others



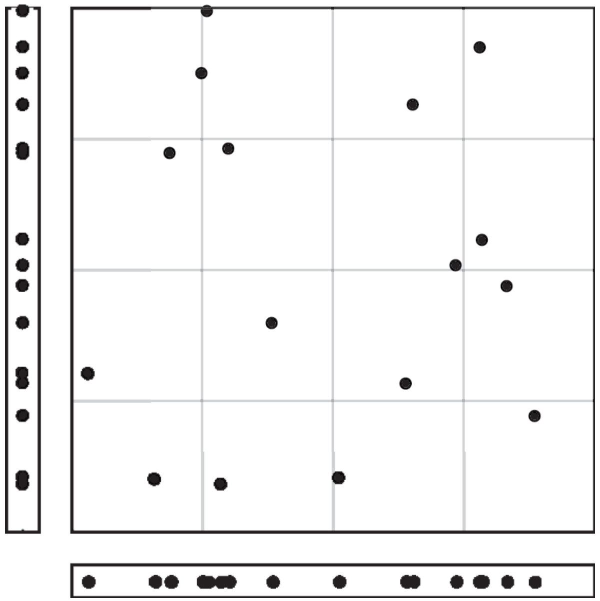


Jittered

- Create a n x n grid covering the domain
  - Or a n^d uniform grid in d dimensional space
- Randomly generate a sample in each cell

- Example of *stratified* sampling
  - Each cell is a strata

- Significantly better than random
- x-y projections can still be poorly distributed



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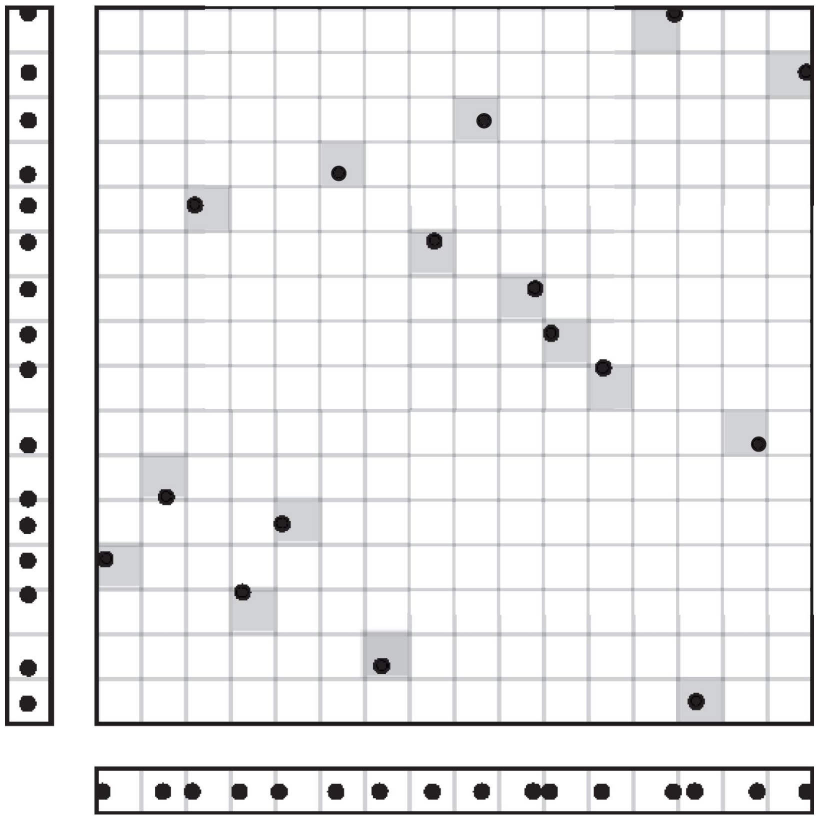
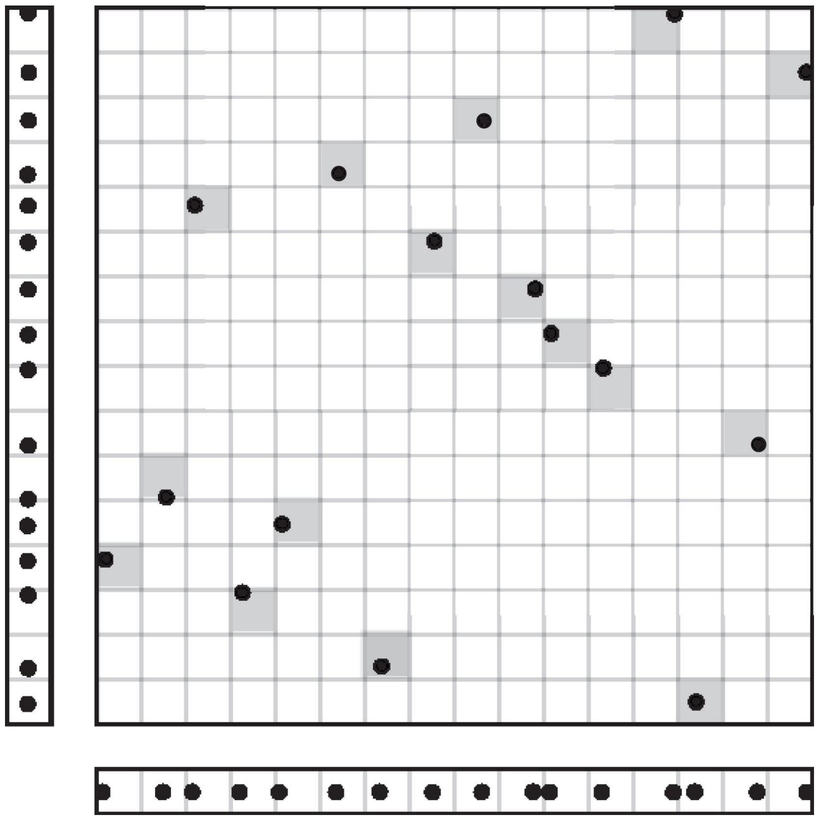
n-rooks

- Also called Latin hypercube sampling
- Use an n x n grid
- One sample exactly in each row and column
    - Again, randomly position a sample within the cell containing it
    - If samples were rooks in chess, no captures can occur



n-Rooks

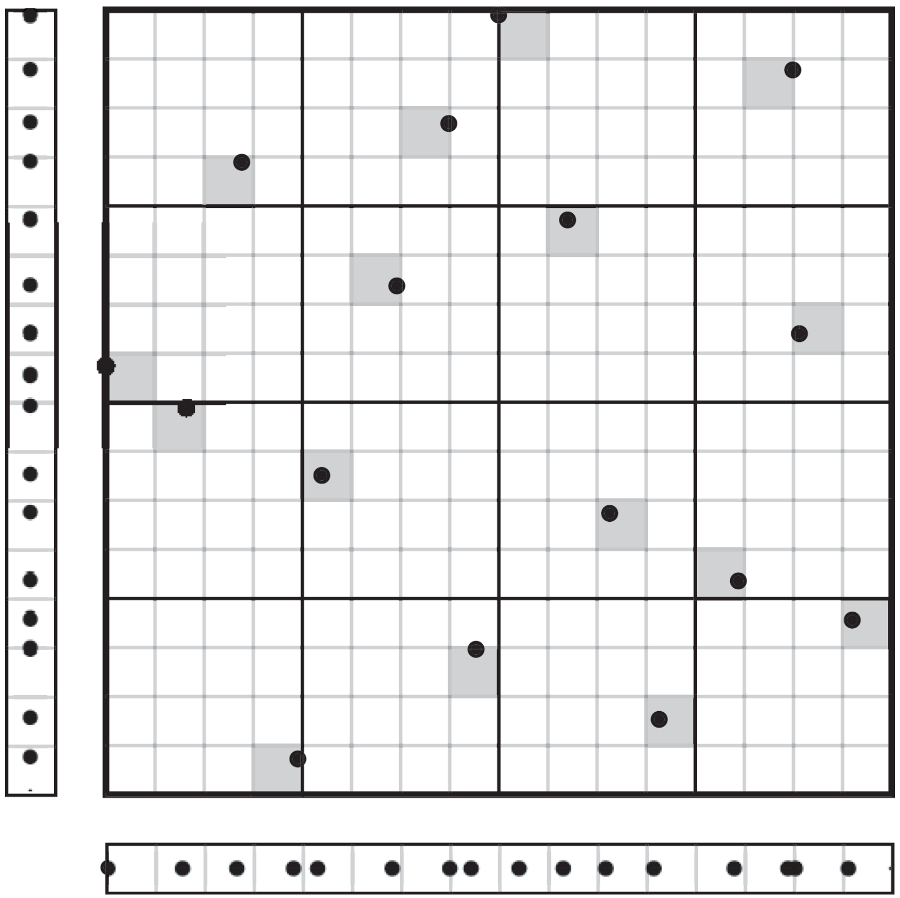
- Produced by random shuffle of diagonal samples
  - Maintains the rook condition
- Use n samples instead of n^2 as in jittered
- 1D distributions are good…better than jittered
- 2D barely better than random…worse than jittered



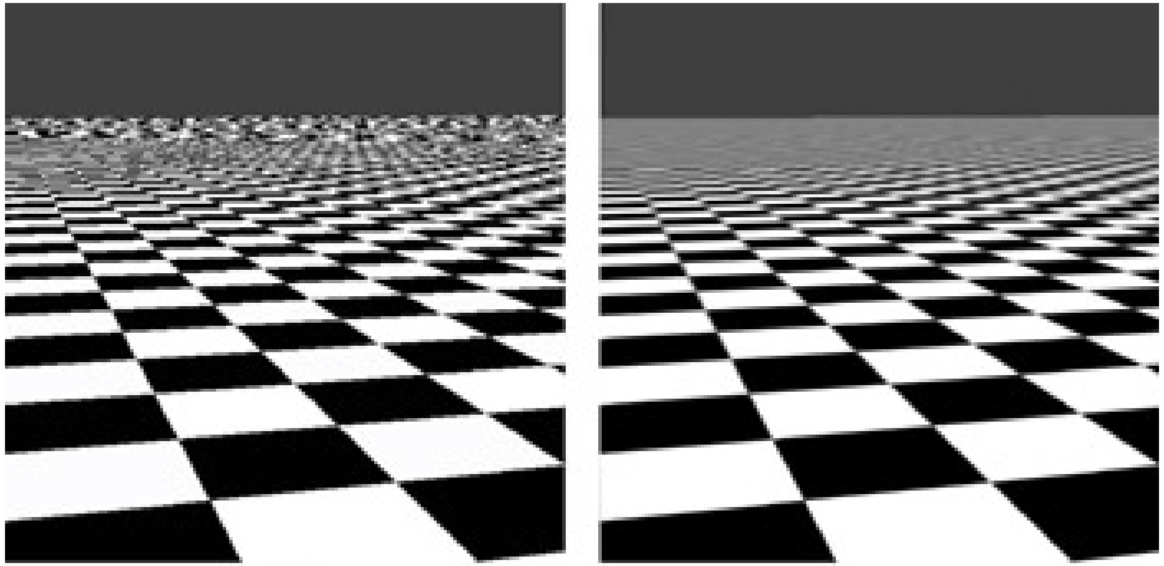
Developed by UIUC alum
Pete Shirley in 1991

Multi-Jittered Sampling

- We use two grids
- For n samples with n a perfect square
  - Coarse grid is $\sqrt{n} \times \sqrt{n}$
  - Fine grid is $n \times n$
  - 1 sample per coarse grid cell
  - For each select unique row & column of fine grid
  - Randomly position sample in fine grid cell
- Good 1D projections from the rook condition
- Good 2D distribution from stratification
- Very good sampling technique



Comparison



(a) (b)

Single ray per pixel Multi-jittering

Pixels on the horizon cover an infinite area
Projected size of a square becomes infinitely small at horizon
We cannot sample enough to preserve infinite detail
But multi-jittered sampling does a reasonable job in (b)

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