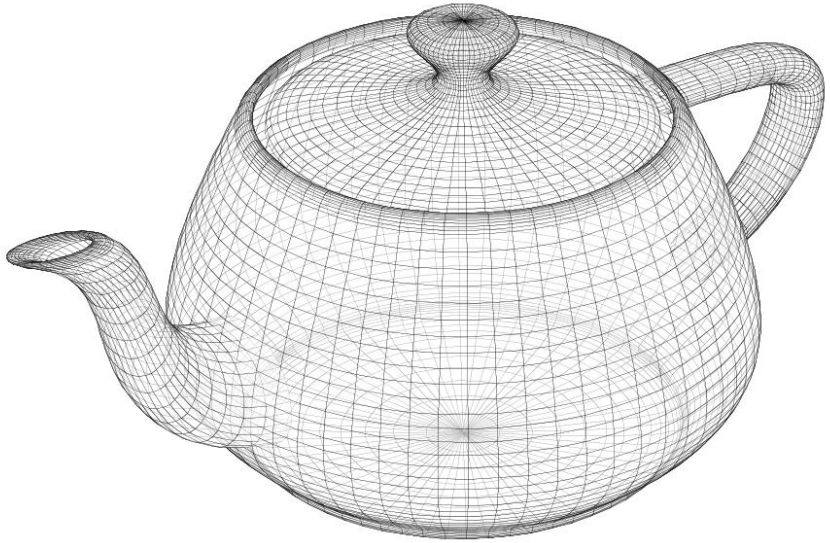

Photon Mapping

Production Computer Graphics

Eric Shaffer

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Light Paths and Regular Expressions

D → a diffuse bounce

S → a specular bounce

V→ volume scattering

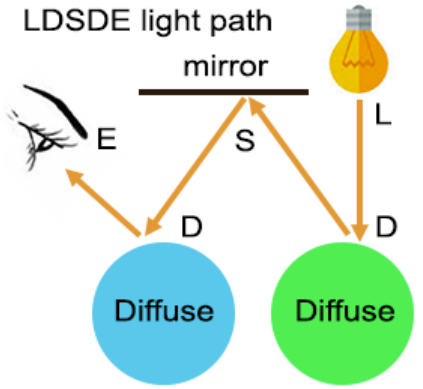
E → the eye

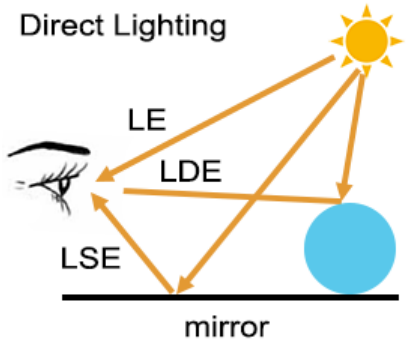
L → a light

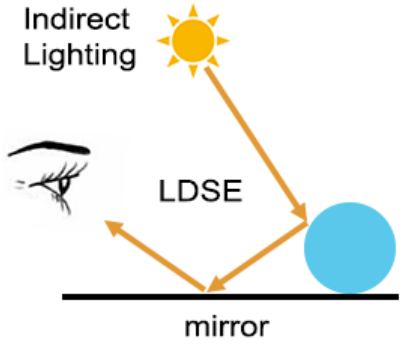
* operator means 0 or more times

+ operator means 1 or more times

| operator means “or”







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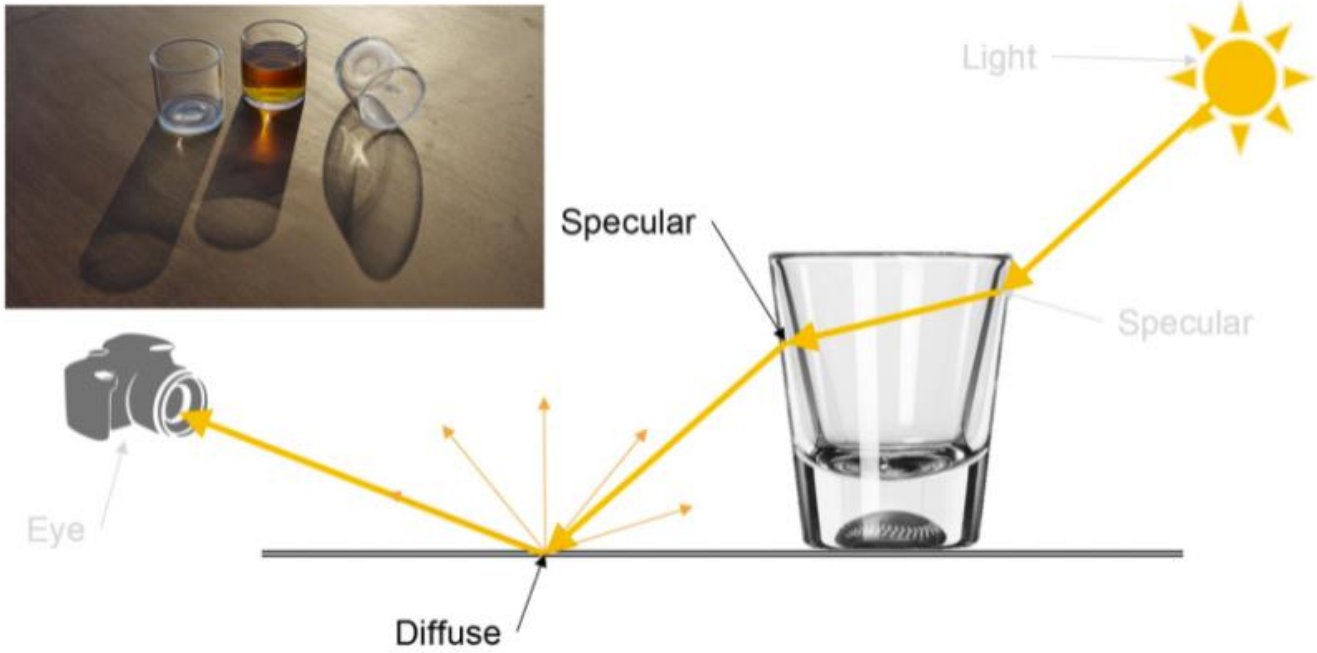
Caustics



Monte Carlo ray tracing handles all paths of light L(D|S)*E

…but not all equally well

Has difficulty sampling LS*DS*E paths

Photon mapping was inspired by this problem

Also more easily simulate participating media (eg fog and smoke)

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Photon Mapping

Created by Henrik Wann Jensen in 1996

Simulates (sort of) the transport of individual photons

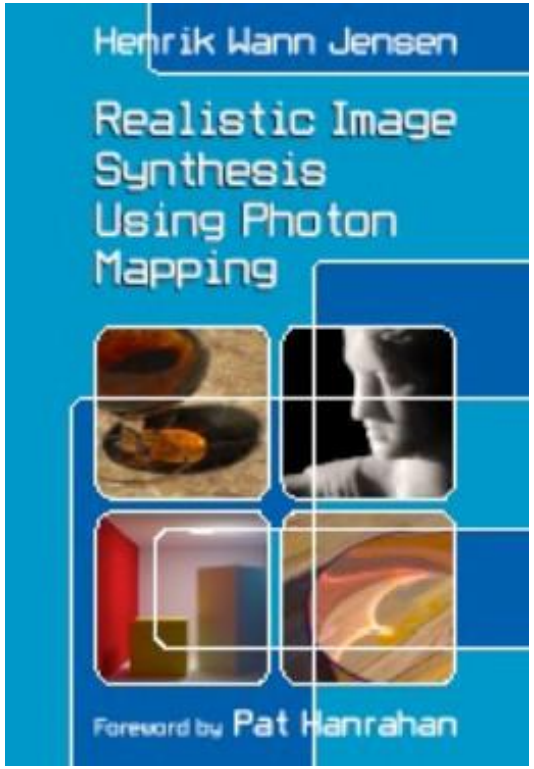
Photons are emitted from light sources

Bounce off specular surfaces

Deposited on diffuse surfaces

- Stored in a k-d tree or similar structure

Photons are then collected by path tracing



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Photon Mapping is Fast(er)

The flux samples in the photon map allow the use of fewer samples than path tracing to generate a converged solution

Photon mapping also parallelizes well

Scene here took 50 minutes with photon mapping

6 hours using Radiance (radiosity+path tracing)

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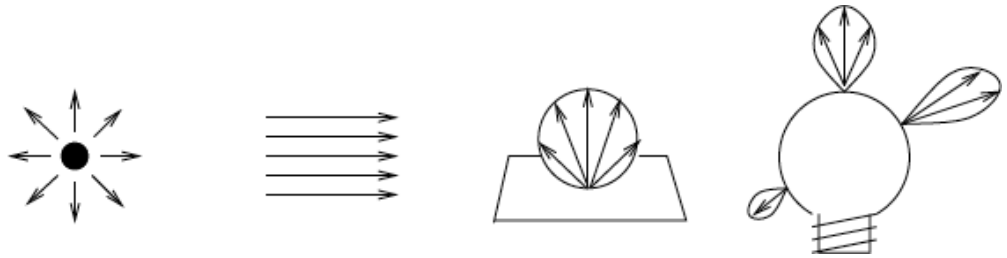
Emission from Light Sources



Diffuse point lights

Emission uniformly in all directions

Directional lights

Emitted in single direction from outside the scene

Area lights

Diffuse square: Emitted in directions sampled from cosine distribution on hemisphere over light

General: Sampling directions more complicated but possible

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Point Light Emission

Power of light is distributed among the photons emitted

$$P_{photon} = \frac{P_{light}}{n_e}$$

N_e is the number of emitted photons
P is "wattage"

To reduce variation in indirect illumination, emitted directions should exhibit low variation

- use low-discrepancy quasi-random sampling
- can you name a quasi-random sequence of numbers?

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Point Light Emission

```
emit_photons_from_diffuse_point_light() {
  n_e = 0                    number of emitted photons
  while (not enough photons) {
    do {                     use simple rejection sampling to find diffuse photon direction
      x = random number between -1 and 1
      y = random number between -1 and 1
      z = random number between -1 and 1
    } while ( x^2 + y^2 + z^2 > 1 )

    d = < x, y, z >
    p = light source position
    trace photon from p in direction d
    n_e = n_e + 1
  }
  scale power of stored photons with 1/n_e
}
```

Multiple Lights

It is possible to use multiple lights

More powerful lights should emit more photons

- Keep the power of the photons relatively even

More lights does not necessarily require more photons

- Total photons required to illuminate a scene is N
- N photons divide up among the lights

Projection Map

Emitting into empty space is a waste

Projection map discretizes the set of directions around a light

- Like a hemicube
- Cells are marked as empty space or not

To emit:

- pick a random non-empty cell and random direction in that cell

Need to scale energy of the photons (why?)

$$P_{photon} = \frac{P_{light}}{n_e} \frac{\text{cells with objects}}{\text{total number of cells}}$$

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Russian Roulette

Recursive path generator needs a stopping condition

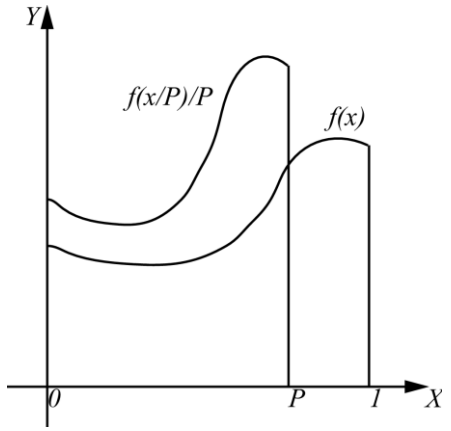
- What is wrong with a simple depth limit?

When a ray hits a light

- Radiance needs to be multiplied by what factors from previous intersection points?
- Suppose we store this factor along a path…
  ….what cutoff criterion is suggested by that possibility?

Russian Roulette

- Imagine evaluating *I* as follows:



$$I = \int_0^1 f(x)\,dx$$

$$I_{rr} = \int_0^1 \frac{1}{P} f\left(\frac{x}{P}\right) dx$$

$$\langle I_{rr} \rangle = \frac{1}{P} f\left(\frac{x}{P}\right) \text{ for } x \leq P$$

$$\langle I_{rr} \rangle = 0 \text{ for } x > P$$

We scale f(x) horizontally and vertically

Expected value remains the same.

Reduce the number of evaluations of f(x).

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Russian Roulette and Bias

A ***biased*** algorithm changes the expected value of the Monte Carlo estimator
It no longer matches the value of the integral.

- Is RR biased? Look at the previous picture….
- What is the benefit of a high absorption factor?
- What is the drawback?

Tracing Photons

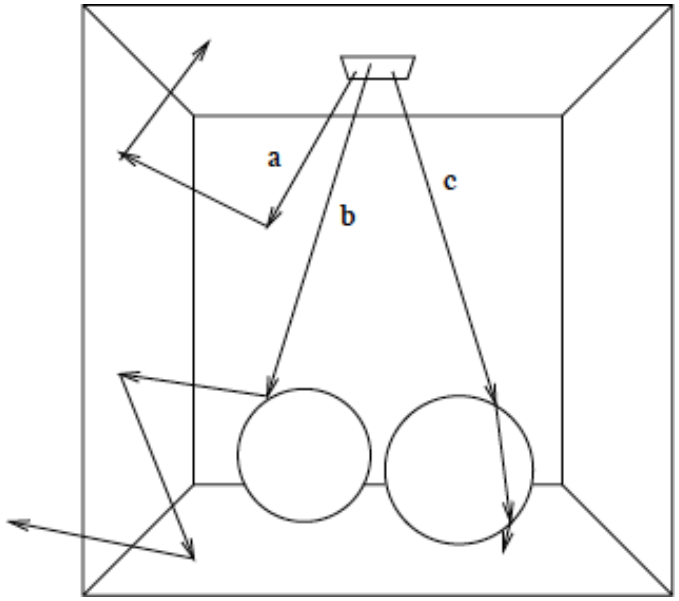
Once emitted, we trace photons into the scene like rays

Photons contact surfaces and can be
- Reflected
- Transmitted
- Absorbed

Material properties and Russian Roulette
- Determine photon fate

Example: Surface with
- Diffuse reflection coefficient d
- Specular reflection coefficient s
- d+s ≤ 1
- Generate random number in [0,1]

$$\xi \in [0, d] \longrightarrow \text{diffuse reflection}$$
$$\xi \in]d, s+d] \longrightarrow \text{specular reflection}$$
$$\xi \in]s+d, 1] \longrightarrow \text{absorption}$$



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Tracing Photons

Need to handle at least 3 color bands

Base decision on max probability of reflection in any band

$$P_r = max(d_r + s_r, d_g + s_g, d_b + s_b)$$

Probability of diffuse reflection

$$P_d = \frac{d_r + d_g + d_b}{d_r + d_g + d_b + s_r + s_g + s_b} P_r$$

Probability of specular reflection

$$P_s = \frac{s_r + s_g + s_b}{d_r + d_g + d_b + s_r + s_g + s_b} P_r$$

$$\begin{aligned} \xi \in [0, P_d] &\longrightarrow \text{diffuse reflection} \\ \xi \in]P_d, P_s + P_d] &\longrightarrow \text{specular reflection} \\ \xi \in]P_s + P_d, 1] &\longrightarrow \text{absorption} \end{aligned}$$

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Tracing Photons

Power of reflected photons should be adjusted

For example, if specular reflection happens:

$$P_{refl,r} = P_{inc,r}\, s_r / P_s$$
$$P_{refl,g} = P_{inc,g}\, s_g / P_s$$
$$P_{refl,b} = P_{inc,b}\, s_b / P_s$$

Can extend the scheme to handle transmission as well

Or more colors…

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Why use Russian Roulette?

We can have a photon impact generate multiple new photons

- e.g. reflection and transmissions
- e.g. 1 photon can become 256 after 8 bounces

Russian Roulette will terminate some of these

- rather than propagating

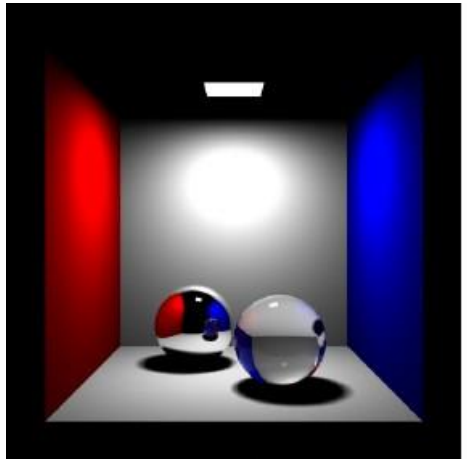
What's the downside to Russian Roulette?

Storing Photons

Photons are not stored on specular surfaces

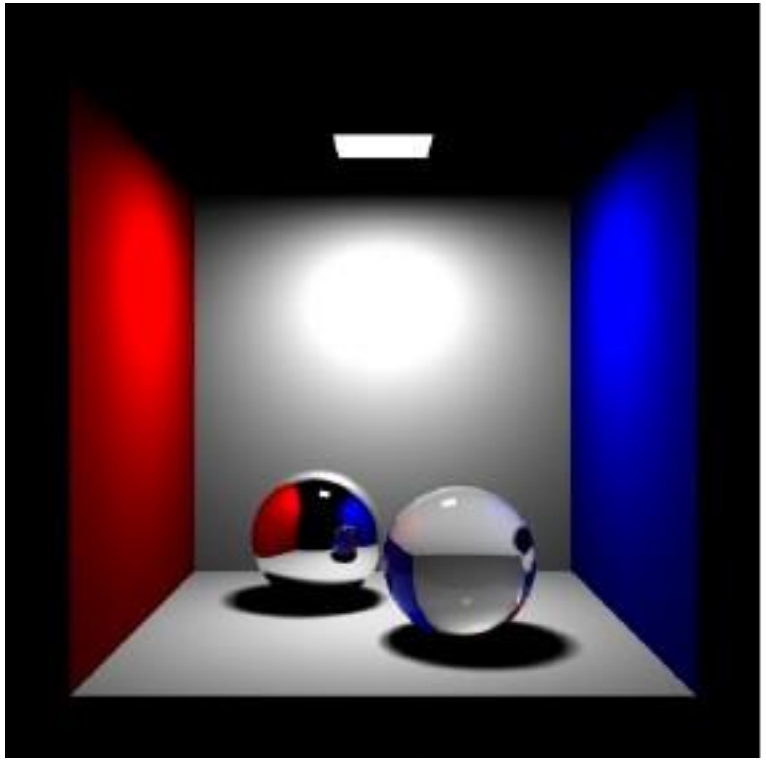
- Can you guess why not?

Are stored on other surfaces



What's the bright spot under the sphere in the photon map?

Example



Ray-traced Image

Photon Map

What is a Photon?

```
struct photon {
  float x,y,z;       // position
  char p[4];         // power packed as 4 chars
  char phi, theta;   // compressed incident direction
  short flag;        // flag used in kdtree
}
```

Power can also be a float in 3 color channels instead of packed char representation.

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Encoding Directions

Incident direction needed to compute

- Contribution for non-Lambertian surfaces
- Front or backfacing arrival on Lambertian surfaces

Map spherical coordinates of photon to 65536 directions

```
phi = 255 * (atan2(dy,dx)+PI) / (2*PI)
theta = 255 * acos(dx) / PI
```

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Multiple Photon Maps

Caustic photon map: contains photons that have been through at least one specular reflection before hitting a diffuse surface: LS^+D.

Global photon map: an approximate representation of the global illumination solution for the scene for all diffuse surfaces: $L\{S|D|V\}^*D$

Volume photon map: indirect illumination of a participating medium: $L\{S|D|V\}^+V$.

Usually constructed in 2 passes

- One for caustic (needs more samples)
- One for global illumination (including volume)

Storing Photons

Photons emitted only once during photon tracing…but:

> specular reflections the best way is to trace a ray in the mirror direction using standard ray tracing. For all other photon-surface interactions, data is stored in a global data structure, the *photon map*. Note that each emitted photon can be stored several times along its path. Also, information about a photon is stored at the surface where it is absorbed if that surface is diffuse.

Stored in a static data structure for the rendering phase

Structure must support:

- Query: Closest photon(s) to a point in space
- Efficiently handle non-uniform distributions of photons

Jensen used a balanced kd-tree

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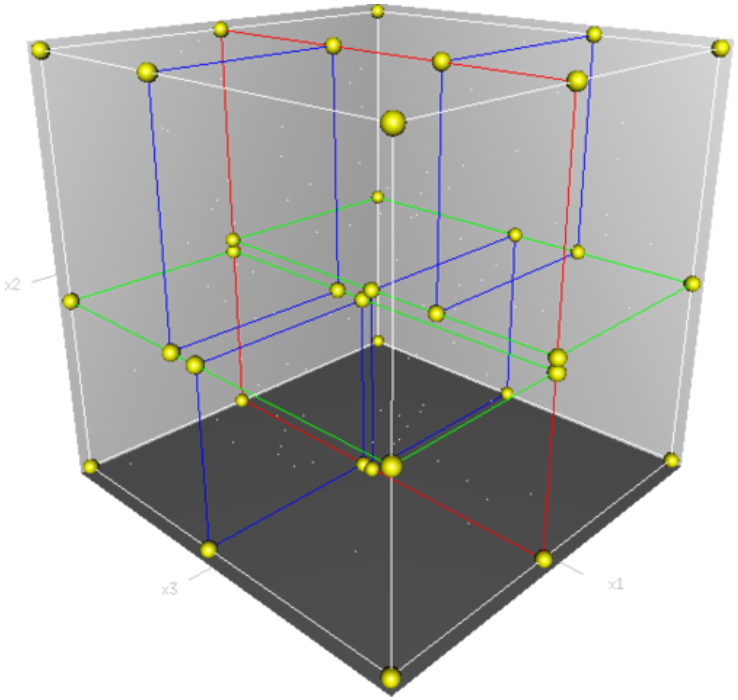
Balanced kd-tree



Assume N photons in map

O(log N) to find closest photon

O(N log N) construction

Represented by a pointerless heap-like structure

Photons stored in leaves

Balanced by median splitting along dimension of greatest coordinate spread

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Balanced kd-tree

```
kdtree *balance( points ) {
  Find the cube surrounding the points
  Select dimension dim in which the cube is largest
  Find median of the points in dim
  s1 = all points below median
  s2 = all points above median
  node = median
  node.left = balance( s1 )
  node.right = balance( s2 )
  return node
}
```

Computing Radiance

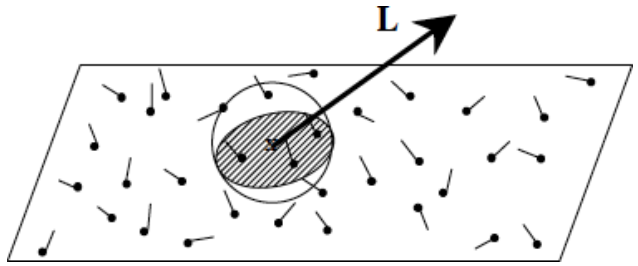
To compute radiance we need to integrate incoming flux

Reflected radiance $$L_r(x,\vec{\omega}) = \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_i(x,\vec{\omega}') |\vec{n}_x \cdot \vec{\omega}'| \, d\omega_i'$$

Relationship between flux and radiance $$L_i(x,\vec{\omega}') = \frac{d^2\Phi_i(x,\vec{\omega}')}{\cos\theta_i \, d\omega_i' \, dA_i}$$

Rewrite the integral

$$\begin{aligned} L_r(x,\vec{\omega}) &= \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d^2\Phi_i(x,\vec{\omega}')}{\cos\theta_i \, d\omega_i' \, dA_i} |\vec{n}_x \cdot \vec{\omega}'| \, d\omega_i' \\ &= \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d^2\Phi_i(x,\vec{\omega}')}{dA_i} . \end{aligned}$$



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Computing Radiance

Approximate integral using n photons closest to x

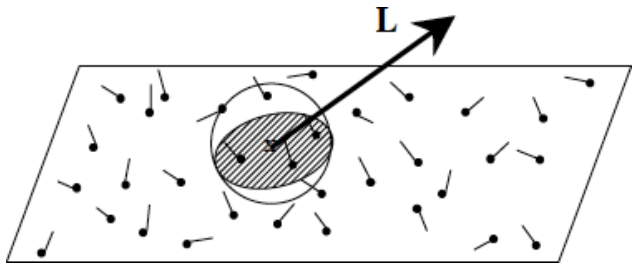
$$L_r(x,\vec{\omega}) \approx \sum_{p=1}^{n} f_r(x,\vec{\omega}_p,\vec{\omega})\frac{\Delta\Phi_p(x,\vec{\omega}_p)}{\Delta A}$$

Denominator related to density ot photons $\Delta A = \pi r^2$

Power of a photon is $\Delta\Phi_p(\vec{\omega}_p)$

Finally we have:

$$L_r(x,\vec{\omega}) \approx \frac{1}{\pi r^2}\sum_{p=1}^{N} f_r(x,\vec{\omega}_p,\vec{\omega})\Delta\Phi_p(x,\vec{\omega}_p)$$



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Error in Radiance Estimate

- Wrong photons can be included at corners and sharp edges
  - Result of sampling using a sphere
- Edges and corners also cause the area estimate to be wrong
- Accuracy increases with more photons
- Can show mathematically that arbitrarily good estimates can be obtained using fewer than an infinite number of photons
  - Does not apply to purely specular BRDFs

Alternative Sampling Volumes

One could use a cube or disc instead of sphere

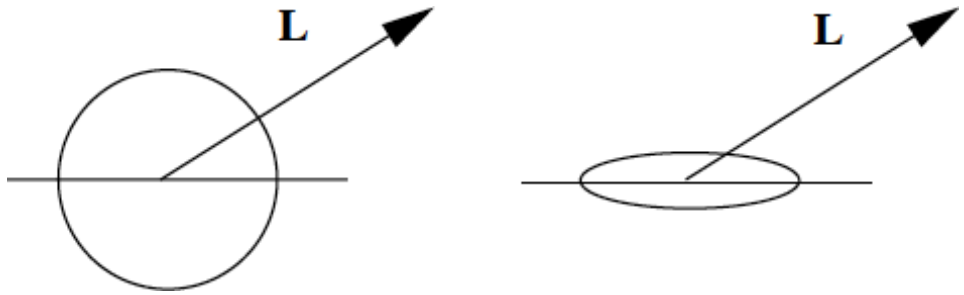
- Projected area and distance computations are simple for sphere

Projected area will change for different volumes

- Intersection of volume and tangent plane at x

Disc is attractive alternative

- What problem would it help solve?



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Reducing Error: Filtering

Visually, error can be seen as blurring at sharp edges

- eg, lose sharp edge of caustic

Filtering increases weight of photons closest to x

Use a radially symmetric 2d filter

- Cone filter
- d is distance between x and a photon
- r is max distance between x and any photon
- k is a constant <= 1

$$L_r(x,\vec{\omega}) \approx \frac{\sum_{p=1}^{N} f_r(x,\vec{\omega}_p,\vec{\omega})\Delta\Phi_p(x,\vec{\omega}_p)w_{pc}}{(1-\frac{2}{3k})\pi r^2} \qquad w_{pc} = 1 - \frac{d_p}{k\,r}$$

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Gaussian Filter

r is maximum distance between sample and x

d is distance between x and a sample

$$\alpha = 0.918 \text{ and } \beta = 1.953$$

$$w_{pg} = \alpha \left[1 - \frac{1 - e^{-\beta \frac{d_p^2}{2r^2}}}{1 - e^{-\beta}} \right]$$

$$L_r(x, \vec{\omega}) \approx \sum_{p=1}^{N} f_r(x, \vec{\omega}_p, \vec{\omega}) \Delta\Phi_p(x, \vec{\omega}_p) w_{pg}$$

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Estimating Radiance in Participating Media

$$
\begin{aligned}
L_i(x,\vec{\omega}) &= \int_\Omega f(x,\vec{\omega}',\vec{\omega})\, L(x,\vec{\omega}')\, d\omega' \\
&= \int_\Omega f(x,\vec{\omega}',\vec{\omega})\, \frac{d^2\Phi(x,\vec{\omega}')}{\sigma(x)\, d\omega'\, dV}\, d\omega' \\
&= \frac{1}{\sigma(x)} \int_\Omega f(x,\vec{\omega}',\vec{\omega})\, \frac{d^2\Phi(x,\vec{\omega}')}{dV} \\
&\approx \frac{1}{\sigma(x)} \sum_{p=1}^{n} f(x,\vec{\omega}'_p,\vec{\omega})\, \frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\frac{4}{3}\pi r^3},
\end{aligned}
$$

Scattering coefficient $\sigma(x)$

f is the phase function

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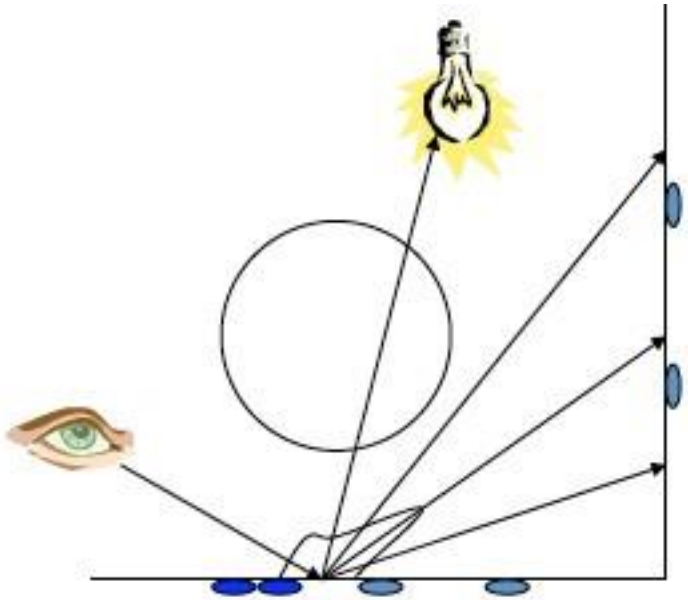
Rendering

- Rendered by glossy-surface distributed ray tracing
- When ray hits first diffuse surface…
  - Compute direct illumination
  - Compute reflected radiance of caustic map photons
  - Ignore global map photons
  - Importance sample BRDF f_r as usual
  - Use global photon map to importance sample incident radiance function L_i
  - Evaluate reflectance integral by casting rays and accumulating radiances from global photon map

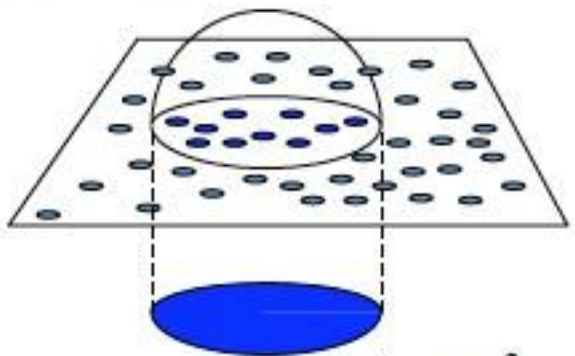


First diffuse intersection. Return radiance of caustic map photons here, but ignore global map photons

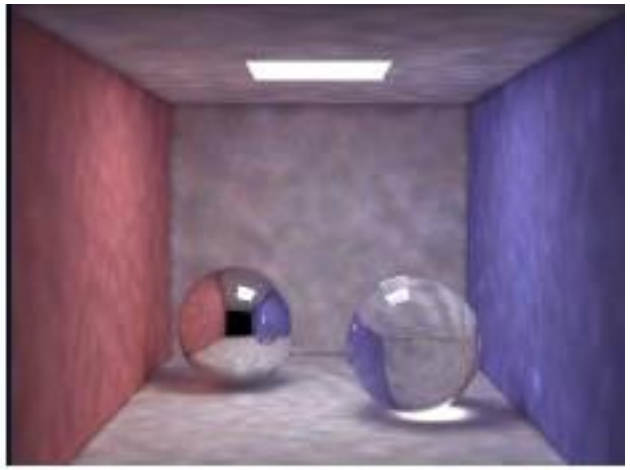
Use global map photons to return radiance when evaluating Li at first diffuse intersection.

How Many Photons?

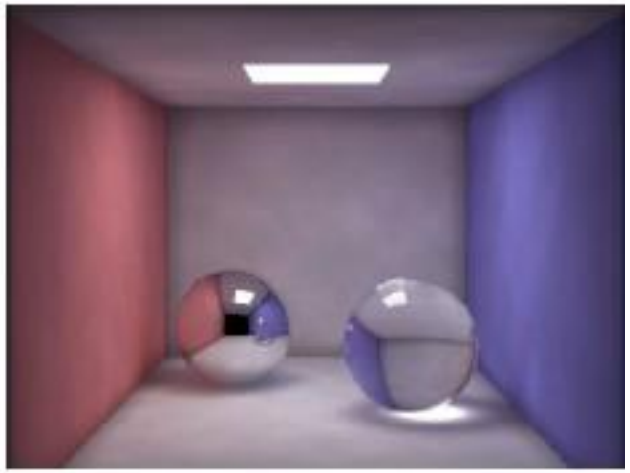
- How big is the disk radius *r*?
- Large enough that the disk surrounds the *n* nearest photons.
- The number of photons used for a radiance estimate *n* is usually between 50 and 500.



$$\Delta A = \pi r^2$$



Radiance estimate using 50 photons



Radiance estimate using 500 photons