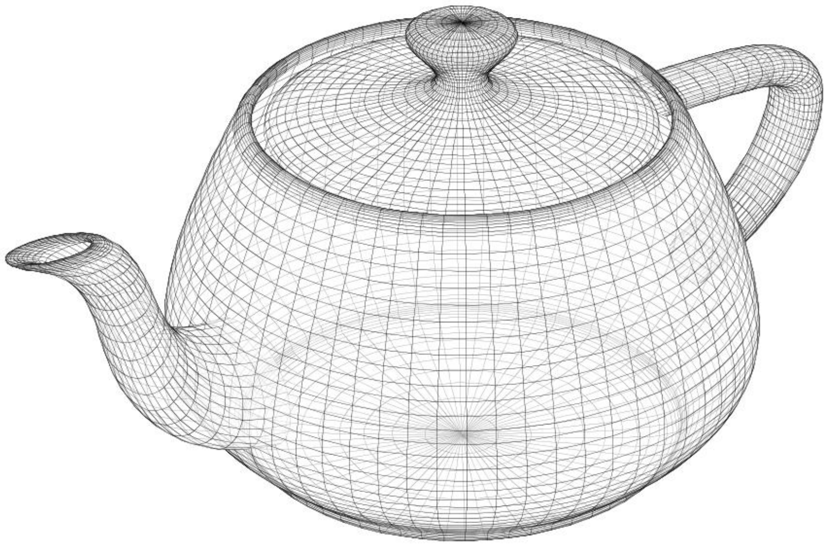


Radiosity

Production Computer Graphics

Eric Shaffer

Radiosity

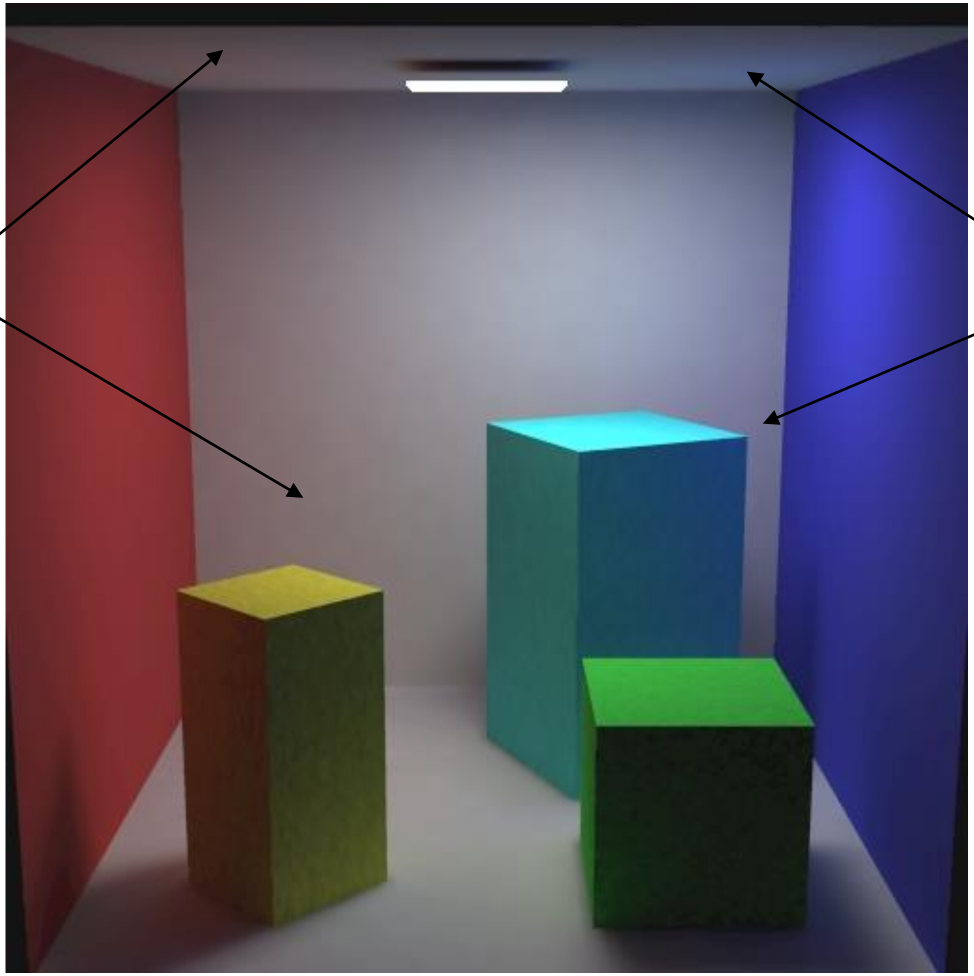
Radiosity solves the rendering equation very efficiently

Can generate global illumination effects like color-bleeding

Can render a reasonably realistic scene in real-time

But only for diffuse surfaces and diffuse lighting

Cornell Box



Example

Diffuse Interreflections

Use only Lambertian surfaces and emitters

- Radiance independent of viewing direction
- Consider total power leaving per unit area of a surface

Can generate soft shadows

Initially used methods from heat transfer literature

Thermal Transfer Literature

Perry, R.L., and Speck, E.P., 1962, "Geometric factors for thermal radiation exchange between cows and their surroundings," Trans. Am. Soc. Ag. Engnrs., General Ed., vol. 5, no. 1, pp. 31-37.

Used mechanical integrator to measure factors from various wall elements to a cow, and presents some results for size of equivalent sphere that gives same factor as cow. It is found that the sphere origin should be placed at one-fourth of the withers to pin-bone distance behind the withers, at a height above the floor of two-thirds of the height at the withers, and that the equivalent sphere radius should be 1.8, 2.08, or 1.78 times the heart girth for exchange with the floor and ceiling, sidewalls, or front and back walls, respectively. Also discusses exchange between cows and entire bounding walls, floor and ceiling, and between parallel cows.

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Physics Review: Irradiance & Radiosity

Irradiance E

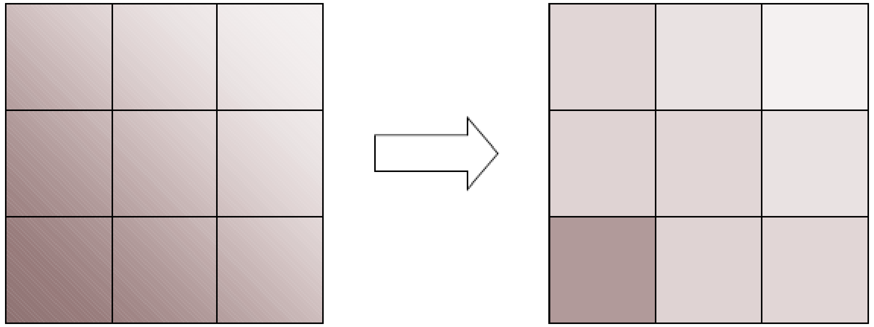
- is the power **received** per unit surface area
- Units: W/m^2

Radiosity

- Power per unit area **leaving** the surface
- Like irradiance

Planar Piecewise Constancy

Subdivide scene into small uniform polygons



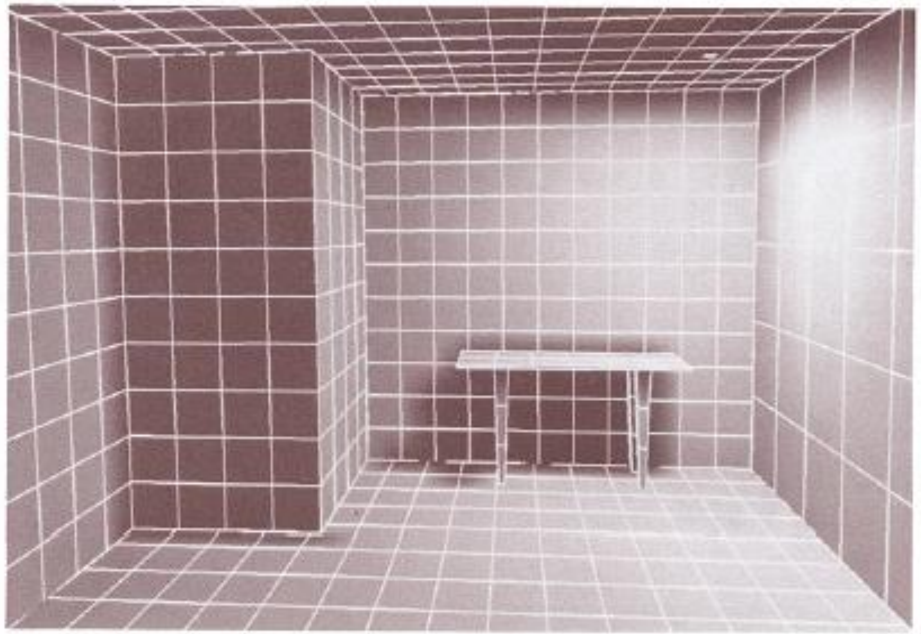


Table in room sequence from Cohen and Wallace

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Power Equation

Power from each polygon:

What is the typo in the equation?

$$\forall i : \Phi_i = \Phi_{ei} + \rho_i \sum_{j=1}^{N} \Phi_j \, F(i \to j)$$

Linear System of Equations:

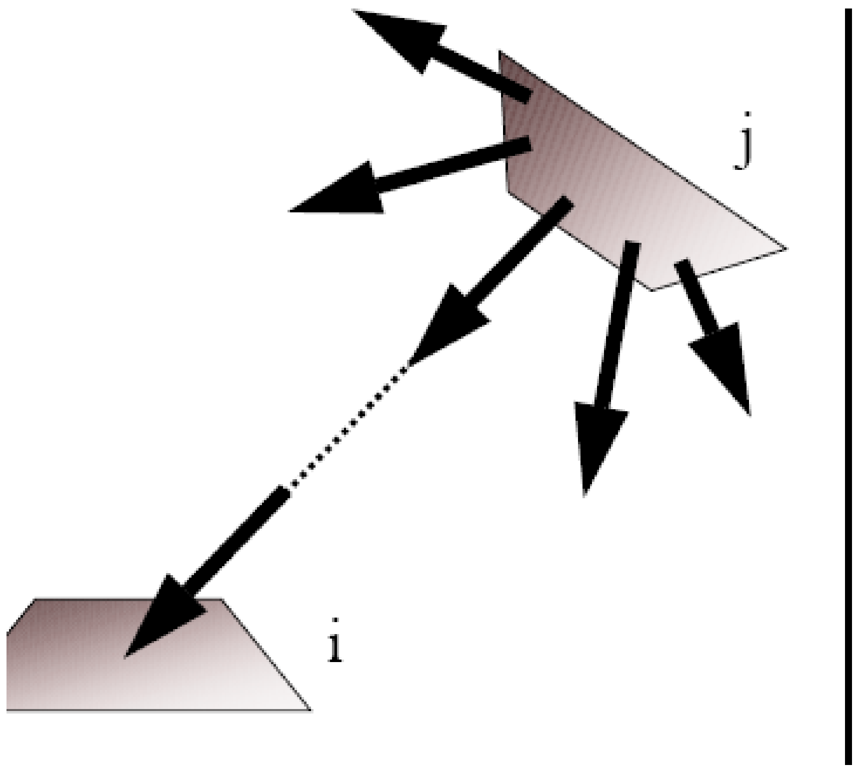
- Φ_i : power of patch i (unknown)
- $\Phi_{e,i}$: emission of patch i (known)
- ρ_I : reflectivity of patch i (known)
- $F(j \to i)$: form-factor (coefficients of matrix)

Form Factor

$F_{j \rightarrow i}$= fraction of power emitted by j received by i

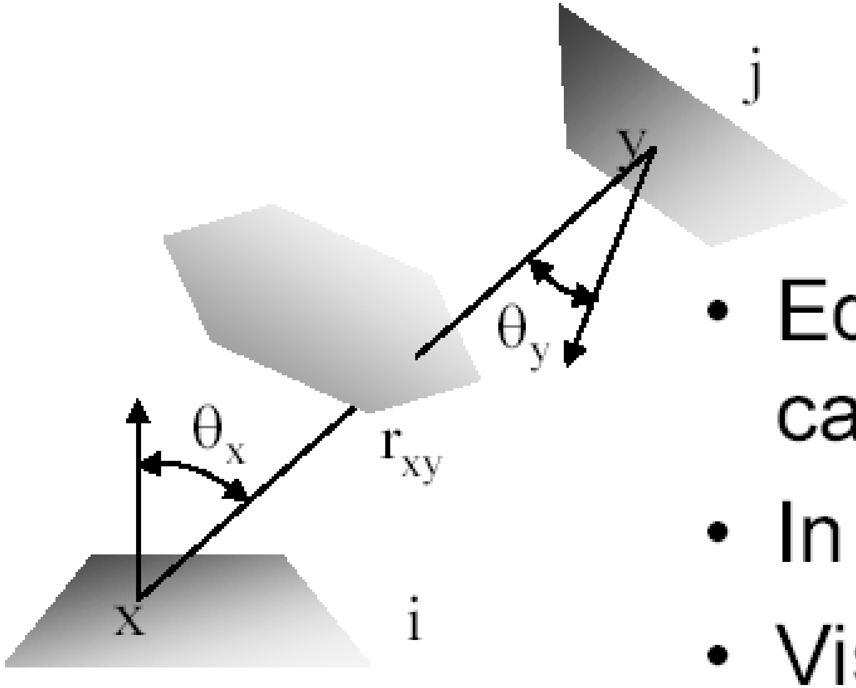
Elements of the Form Factor

- Area
  - If i is smaller it receives less power
- Orientation
  - If i faces j it receives more
- Distance
  - if i is farther away it receives less power



Form Factors

$$F(j \to i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y)\, dA_y\, dA_x$$



- Equations for special cases (polygons)
- In general hard problem
- Visibility makes it harder

Form Factors

$$F(j \to i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y)\, dA_y\, dA_x$$

$$F(i \to j) = \frac{1}{A_i} \int_{A_j} \int_{A_i} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y)\, dA_x\, dA_y$$

$$F(i \to j) A_i = F(j \to i) A_j$$

Form Factor Computation

$$F(j \to i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r_{xy}^2} V(x,y)\, dA_y\, dA_y$$

- Schroeder and Hanrahan derived an analytic expression for polygonal surfaces.

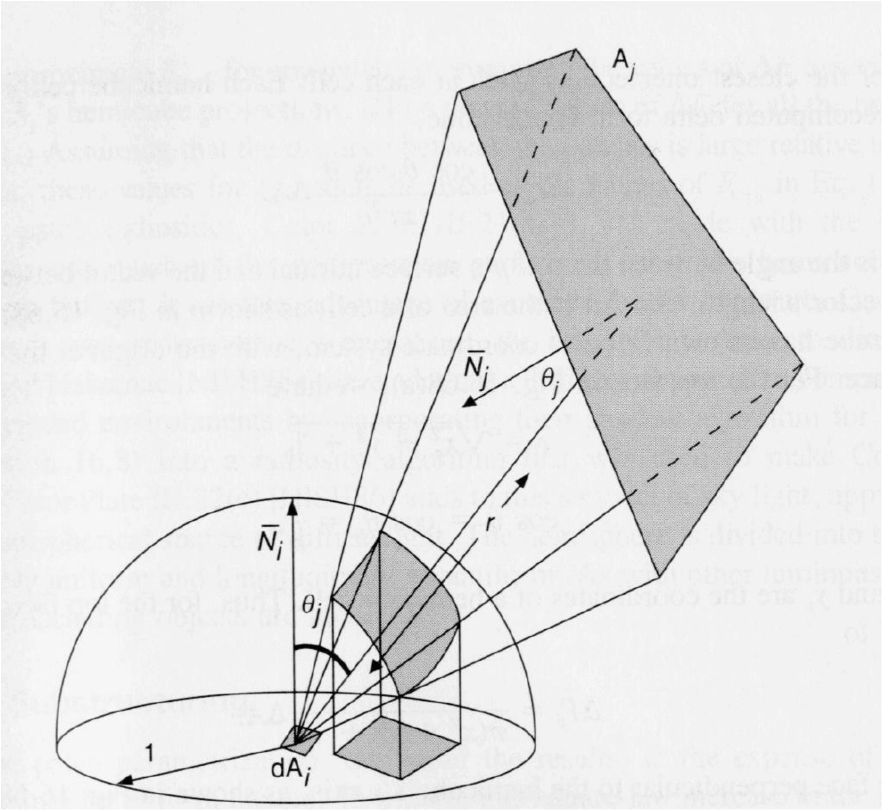
- In general, computing double integral is hard.

- Use Monte Carlo Integration.
    - **Most modern (21st century) implementations use sampling**

Form Factor – Computation

Spherical projections to model form factor

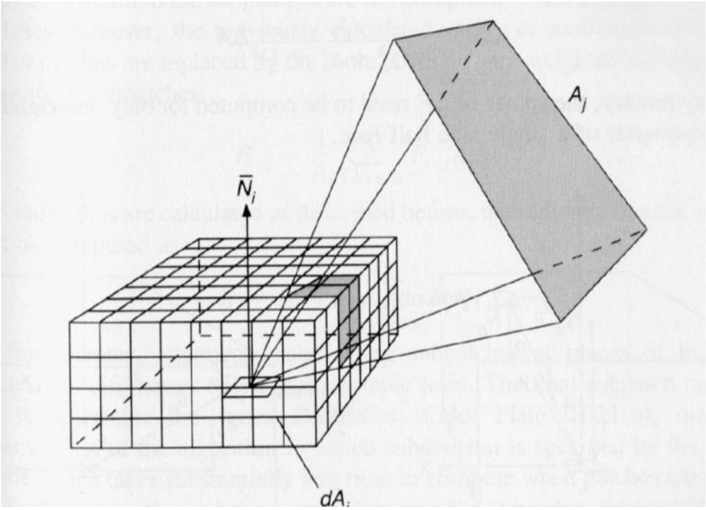
- Project polygon A_j on unit hemisphere centered at A_i
- Project this projection to base of hemisphere



Form Factor –Computation

Hemicube allows faster computations

- Analytic solution of hemisphere is expensive
- Use rectangular approximation, hemicube
- cosine terms for top and sides are simplified
- Dimension of 50 – 200 squares is good



Computing Radiosity

$$\Phi_i = \Phi_{e,i} + \rho_i \sum_{j=1}^{N} \Phi_j F(j \rightarrow i)$$

$$\Big\downarrow \quad \text{Divide by } A_i$$

$$\frac{\Phi_i}{A_i} = \frac{\Phi_{e,i}}{A_i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(j \rightarrow i)}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j \dfrac{F(i \rightarrow j) A_i}{A_j}}{A_i}$$

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(i \rightarrow j)}{A_j}$$

$$B_i = B_{e,i} + \rho_i \sum_{i=1}^{N} B_j F(i \rightarrow j)$$

Form Linear System of Radiosity Equations

$$\forall \text{patches } i: \qquad B_i = B_{ei} + \rho_i \sum_j F_{i \to j} B_j$$

$$\begin{bmatrix} 1 - \rho_1 F_{1 \to 1} & -\rho_1 F_{1 \to 2} & \dots & -\rho_1 F_{1 \to n} \\ -\rho_2 F_{2 \to 1} & 1 - \rho_2 F_{2 \to 2} & \dots & -\rho_2 F_{2 \to n} \\ \dots & \dots & \dots & \dots \\ -\rho_n F_{n \to 1} & -\rho_n F_{n \to 2} & \dots & 1 - \rho_n F_{n \to n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \dots \\ B_n \end{bmatrix} = \begin{bmatrix} B_{e1} \\ B_{e2} \\ \dots \\ B_{en} \end{bmatrix}$$

Known

Unknown

Known

Solving the System

- Jacobi iteration
- Start with initial guess for energy distribution (light sources)
- Update radiosity/power of all patches based on the previous guess

$$B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \to j)$$

new value

old values

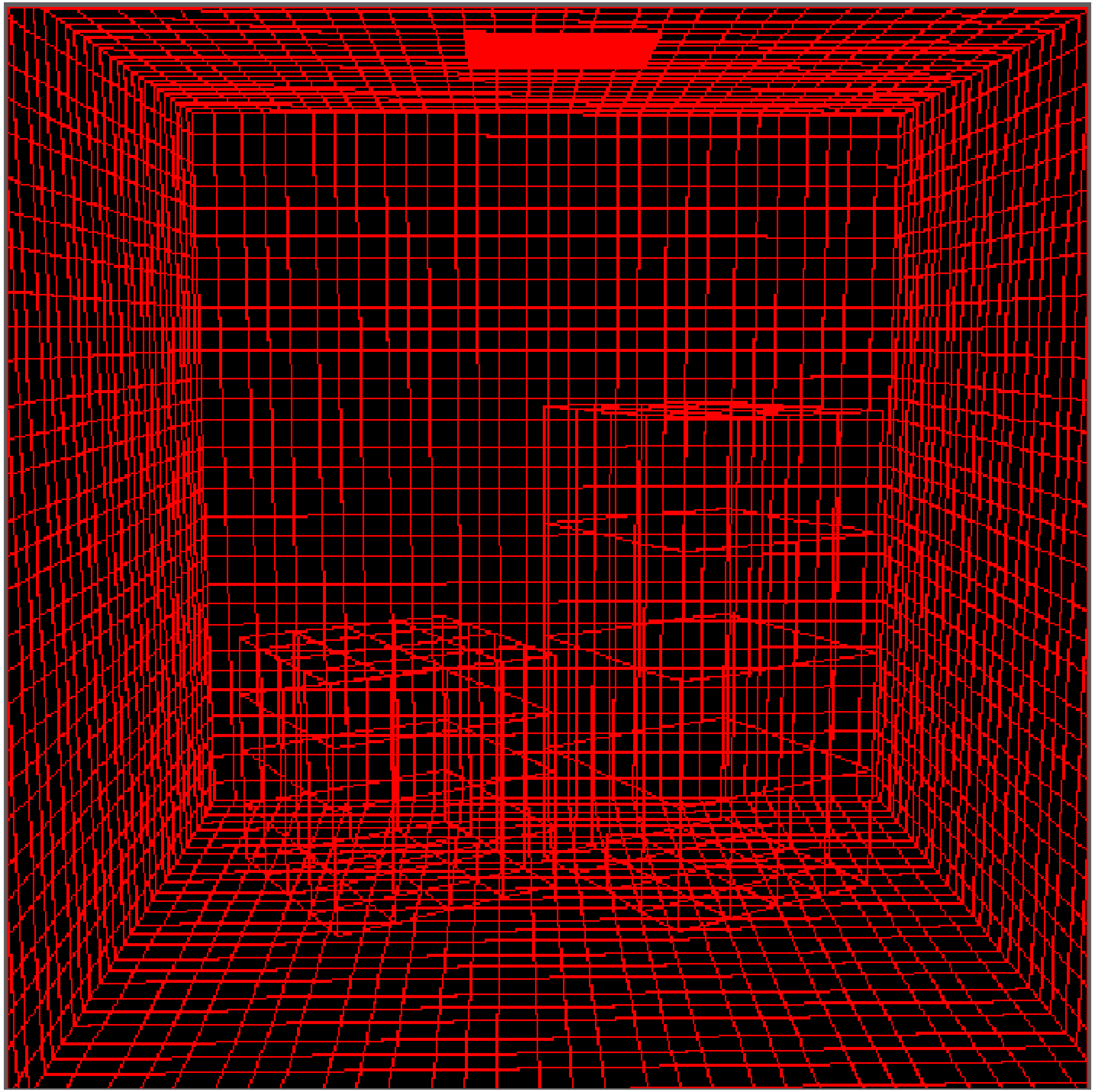
- Repeat until converged

Solving the System

In general, Jacobi iteration is very slow to converge

BUT

- It is highly parallelizable (how?)
- In the case of radiosity, the reflectivities are in [0,1]
  ....which greatly speeds convergence
- Should converge in only a few iterations



Wireframe

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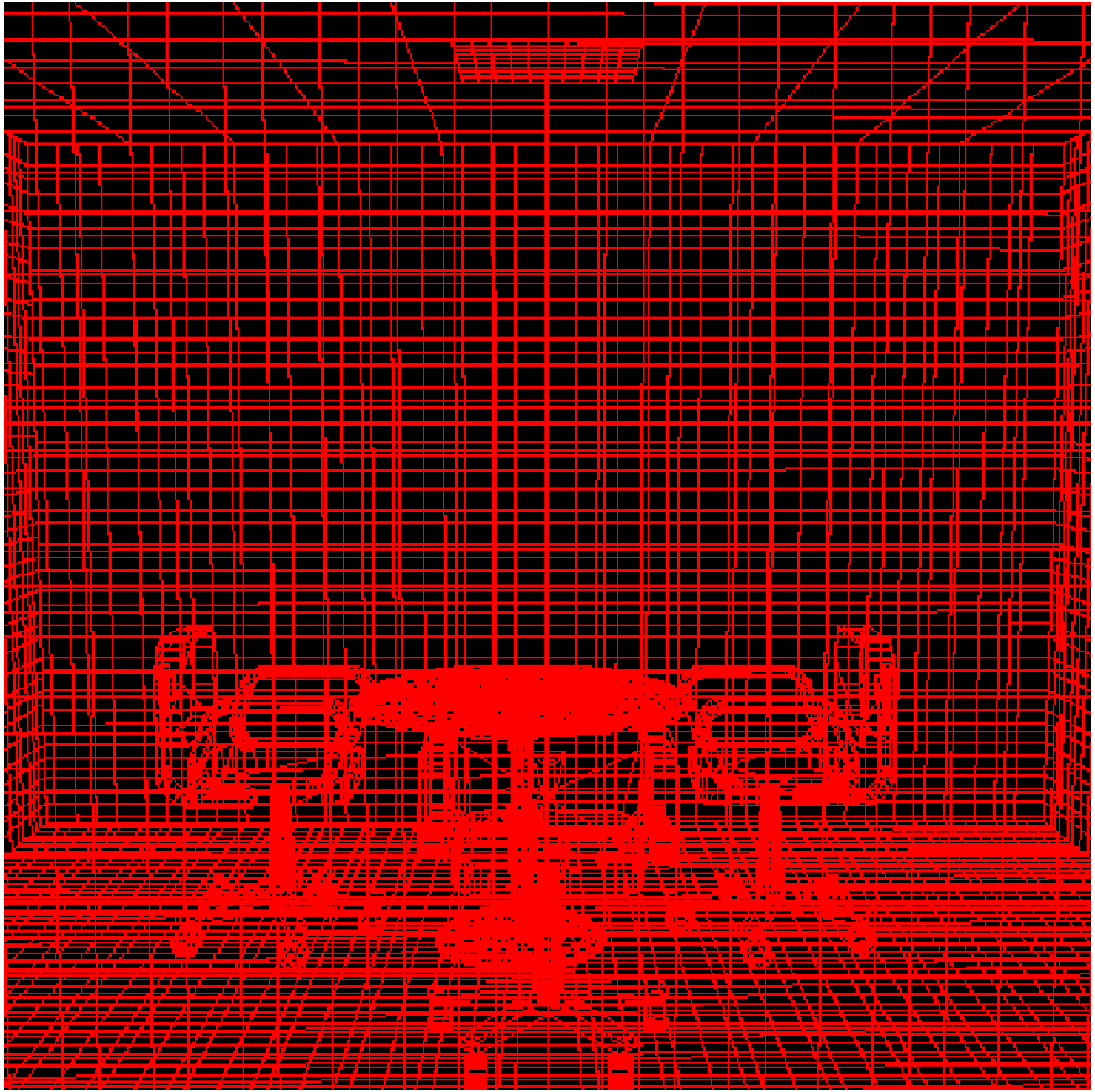

Wireframe

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- Classical Approach
- Low Res

- Classical Approach
- High Res
- More accurate

- Classical Approach
- High Res
- Interpolated

Sample Scenes

From Cohen, Chen, Wallace and Greenberg 1988

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Sample Scenes

Sample Scenes

Sample Scenes

Radiosity: Summary

Classic radiosity = finite element method

Assumes

- Diffuse reflectance
- Polygonal surfaces

Advantages

- View independent solution
- Can pre-compute for a set of light sources
- Excellent for walkthroughs

Combining Ray-Tracing and Radiosity

First Pass: Diffuse Inter-reflections

View independent, global diffuse illumination computed with radiosity.

Second Pass: Specular Inter-reflections

View dependent, global specular illumination computed with ray-tracing.

Combine strengths of radiosity and ray-tracing.