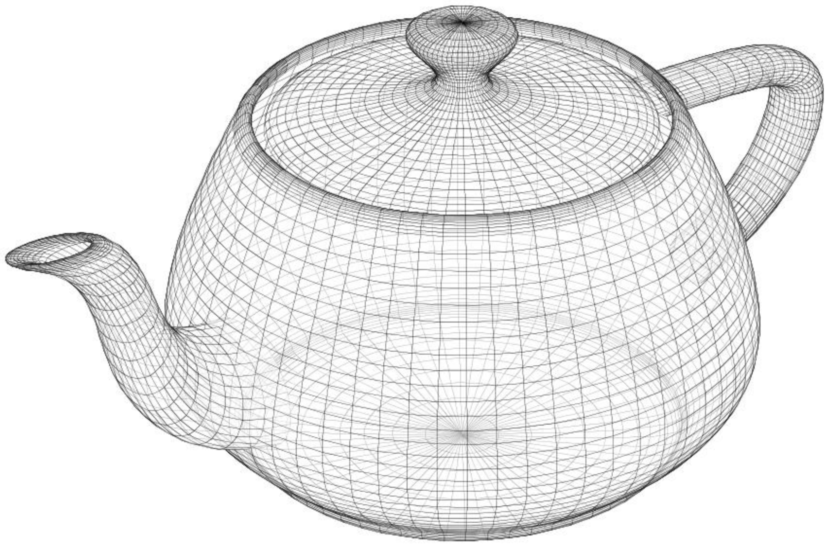

Modeling with Implicit Surfaces

Production Computer Graphics

Eric Shaffer

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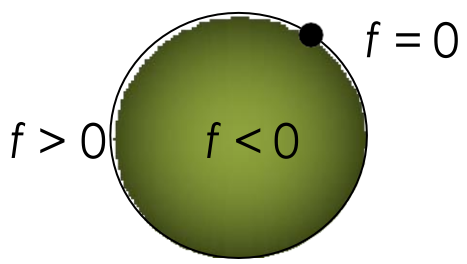
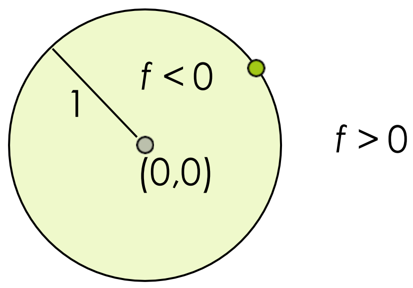
Implicit Surfaces

Real function $f\ (x,y,z)$

- Classifies points in space
- CAGD: inside $f < 0$, outside $f > 0$
- Surface $f^{-1}(0)$: Manifold if zero is a regular value of f



$z = f(x,y)$



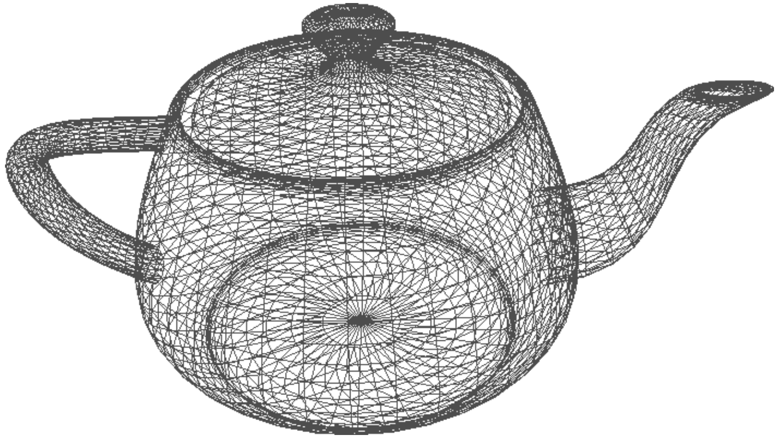
Circle example

$f(x,y) = x^2 + y^2 - 1$

Why Use Implicits?

Versus polygons…Implicits have upsides and downsides

- Upside: smoother
- Upside: compact, fewer higher-level primitives
- Downside: harder to real-time render
- Downside: can’t reproduce sharp edges



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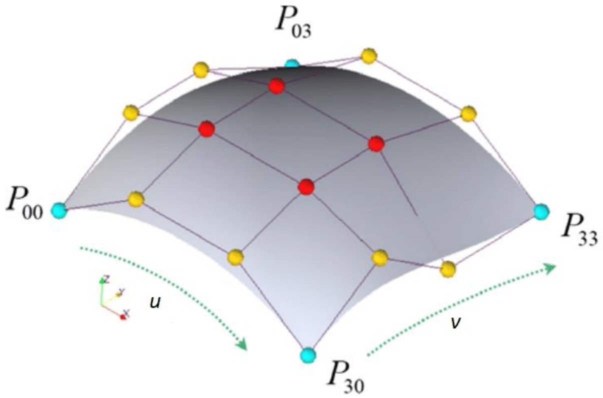
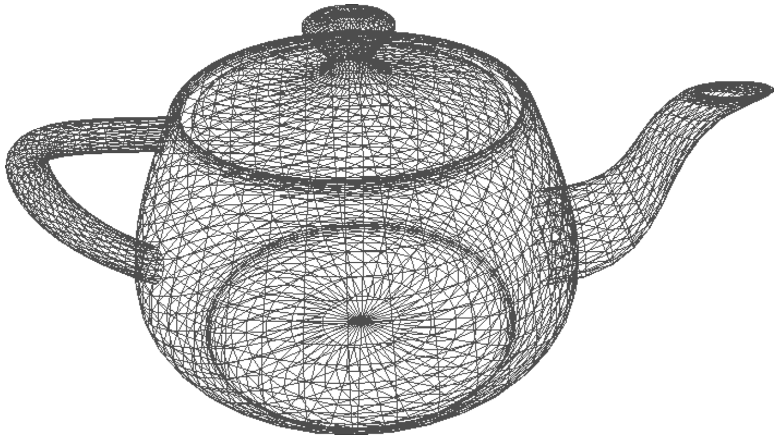
Why Use Implicits?

Versus parametric patches...implicits are

- easier to blend
- no topology problems
- lower degree
- easier to ray trace
- well defined interior
- downside: harder to parameterize
- **What's an example of a parametric patch?**





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CSG: Constructive Solid Geometry

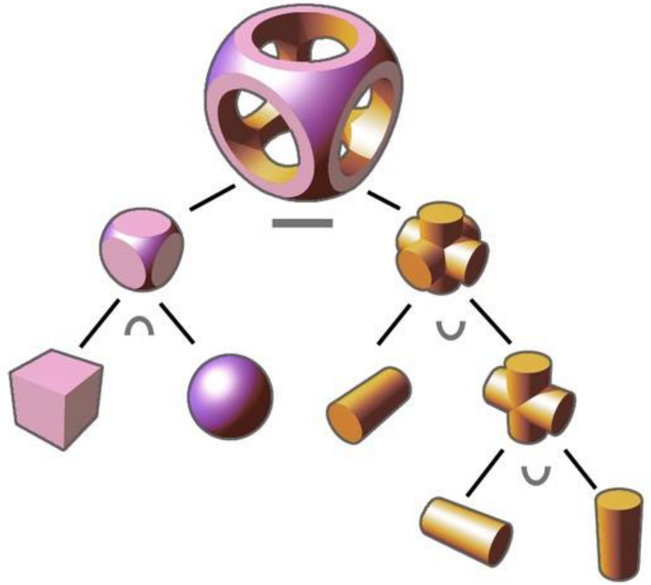
Assume $f < 0$ inside

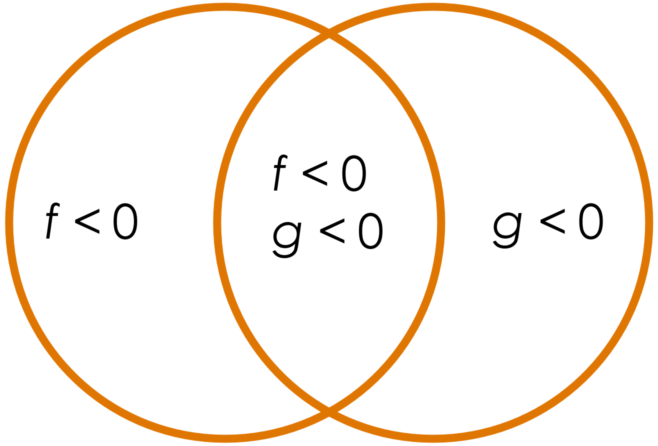
CSG ops by min/max ops

- Union: min(f,g)
- Intersection: max(f,g)
- Complement: $-f$
- Subtraction: max(f,$-g$)

Problem: C^1 discontinuity

Can we smooth the blend crease?





Implicit surface for a unit cube max(|x|,|y|,|z|)-1

Ray Tracing CSG Objects

Intersect the ray with each basic object

Where are these object located in the tree?

Each intersection generates an interval on the ray

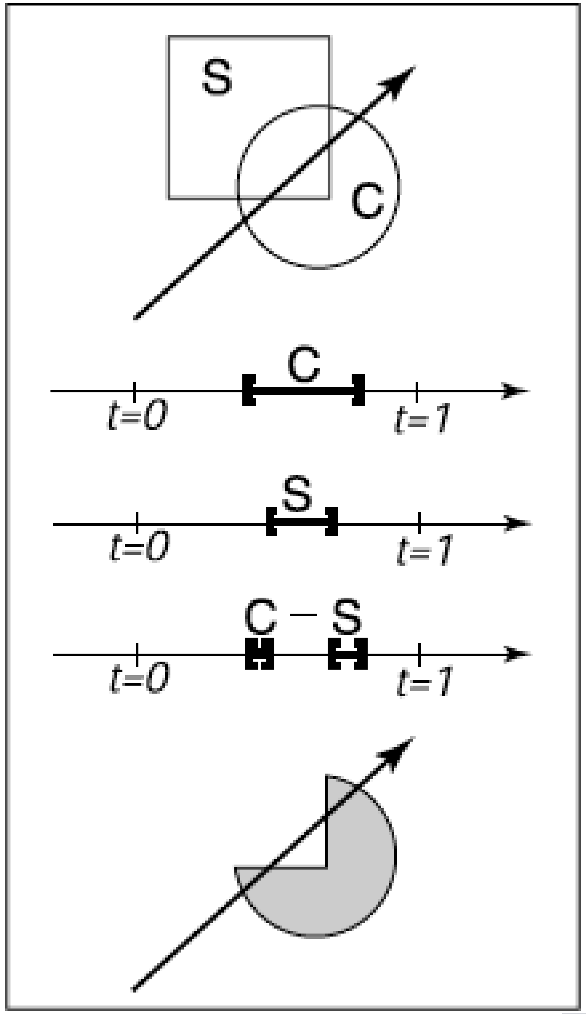
Perform the boolean operations on the intervals

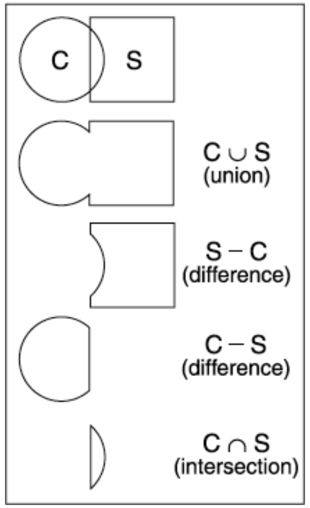
Pro-tip: .Work in parametric space, not world coordinates

How do you perform ray intersections?

For ray $r(t) = o + t\vec{d}$ find t such that $f\big(r(t)\big) = 0$

Can use numerical root-finding like Newton's Method if necessary





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Surface Normals

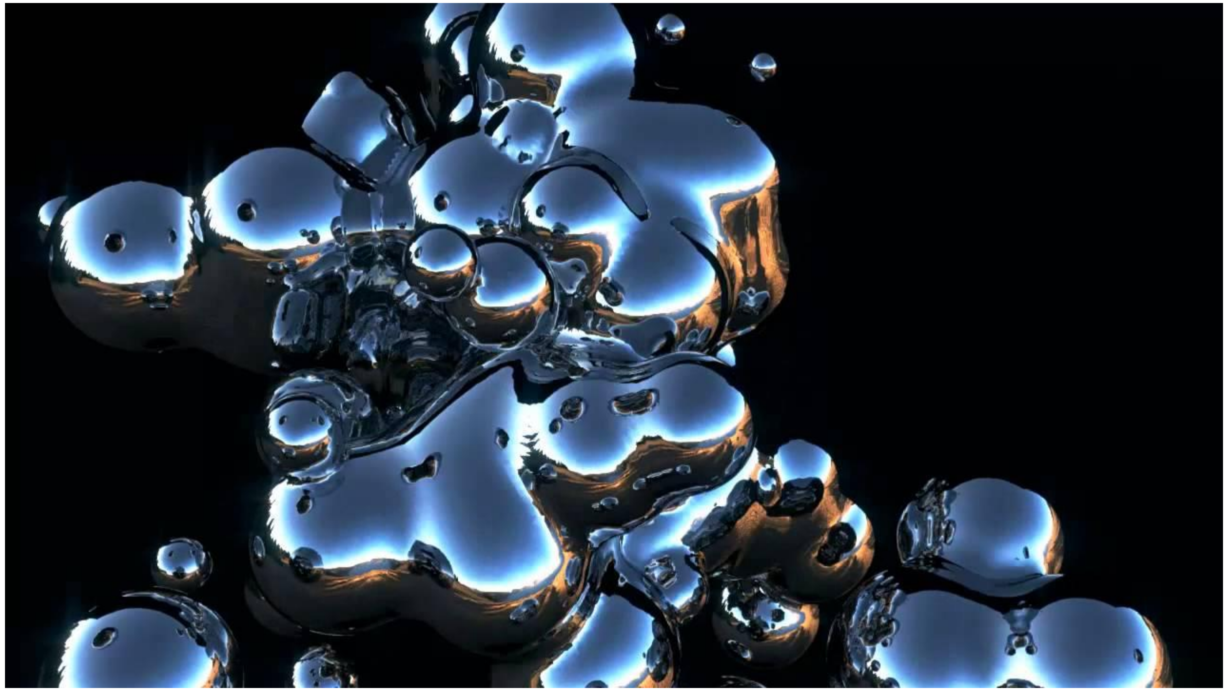
Surface normal usually gradient of function

$$\nabla f(x, y, z) = \left({}^{\partial f}/_{\partial x}, {}^{\partial f}/_{\partial y}, {}^{\partial f}/_{\partial z}\right)$$

- Gradient not necessarily unit length
- Gradient points in direction of increasing *f*
- Outward when *f* < 0 denotes interior
- Inward when *f* > 0 denotes interior

Uses of Blobby Modelling

- Organic forms and nonlinear shapes
- Scientific modelling (electron orbitals, some medical imaging)
- Muscles and joints with skin
- Rapid prototyping
- CAD/CAM solid geometry



Blending Implicits

Metaballs is one approach

- Each ball is defined by
  - A point s
  - Function that falls off with distance from s
  - Example

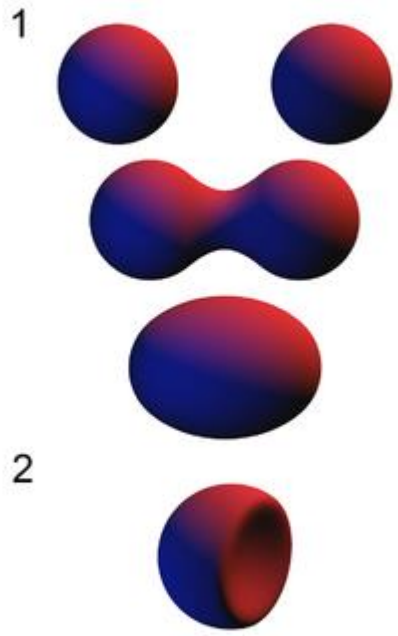
$$f(x,y,z) = 1/((x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2)$$

- We can sum up the functions for all s_i
- Solid is everywhere that sum is greater than a threshold T

$$\sum_{i=0}^{m} metaball_i(x,y,z) \geq T$$

1 shows two positive metaballs with different falloff

2 shows a positive and negative metaball



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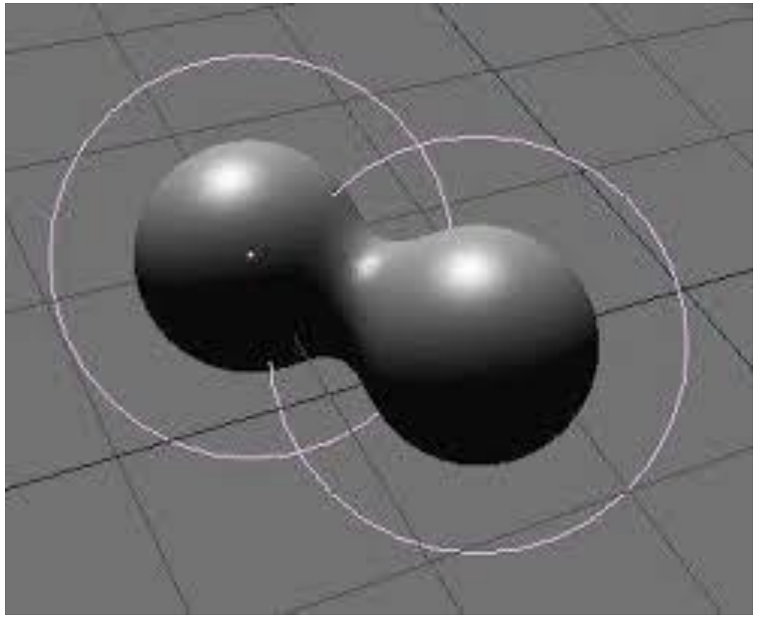
What is a Good Blobby Function?

Several force functions work well. Examples:

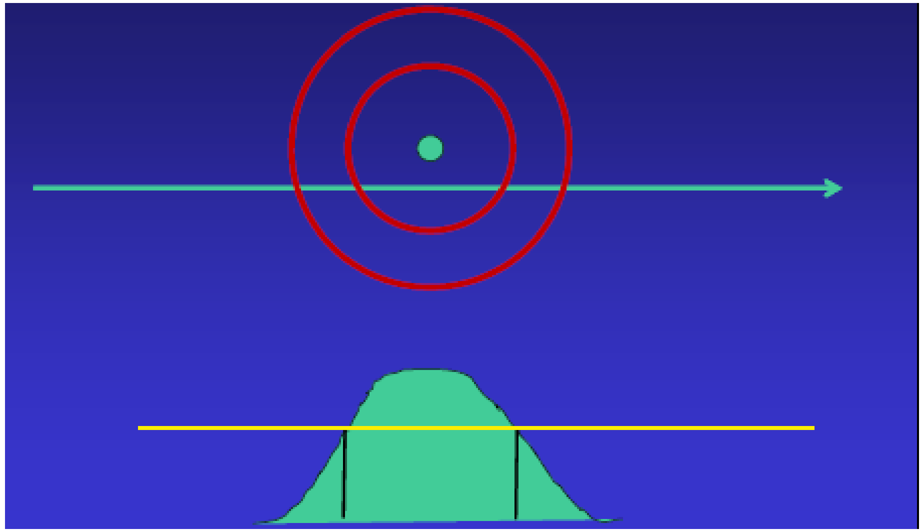
- “Blobby Molecules” - Jim Blinn
  - $F(r) = a\, e^{-br^2}$
  - ‘b’ is related to the standard deviation of the curve
  - ‘a’ to the height.

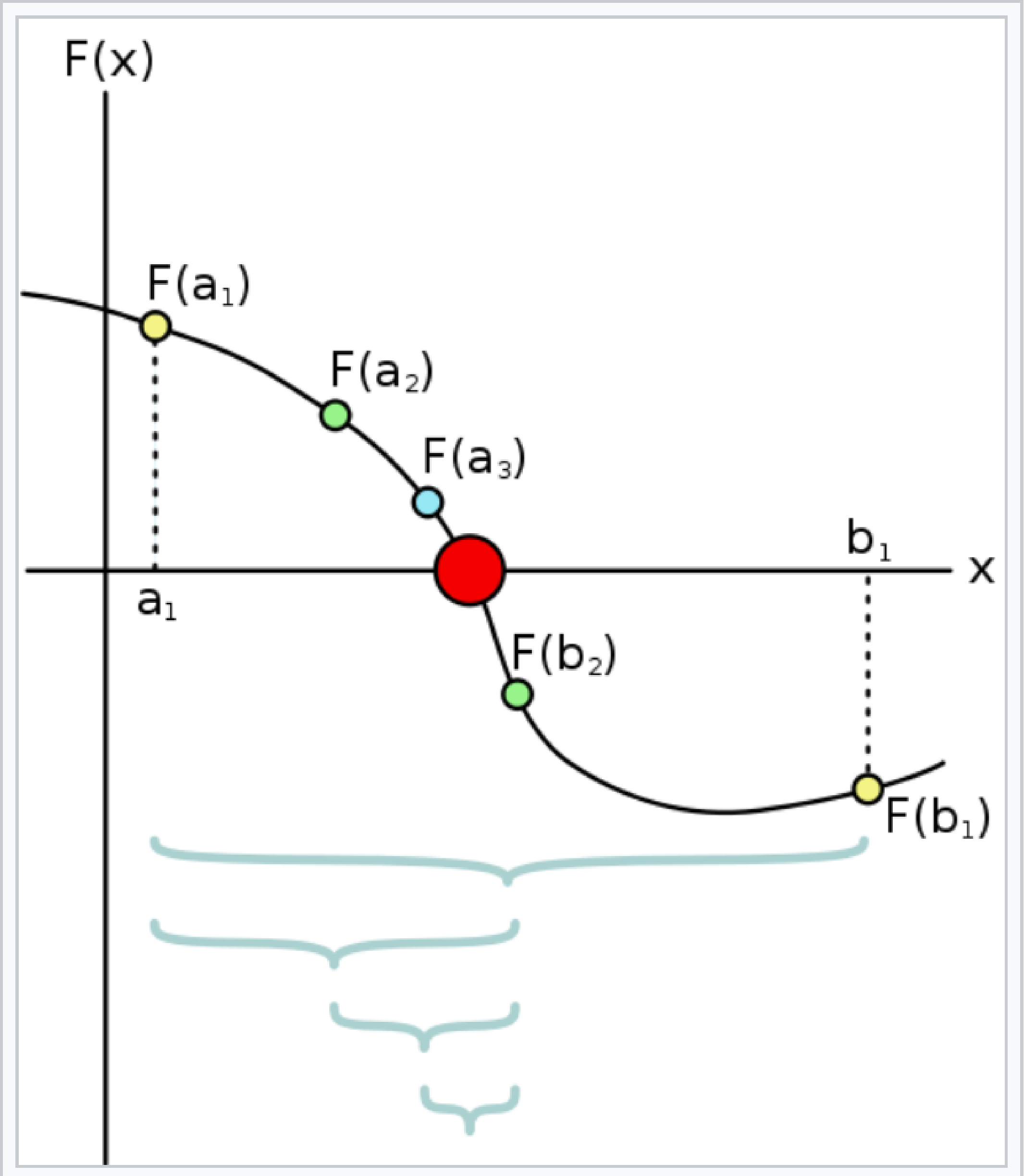


Ray Intersections

Find smallest t along ray at function equals T
....T is the threshold value

Again, root-finding $\sum metaball_i \; - T = 0$
Can use Newton...or Bisection....or Secant....





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