Physics Engine with Euler Integration

Imagine you implement a physics engine and are animating a particle.

At time $t = 0$, a particle begins at position $\begin{bmatrix}1\\2\\3\end{bmatrix}$

It is moving with velocity $\begin{bmatrix}1\\-3\\0\end{bmatrix}$ per second. It has acceleration $\begin{bmatrix}2\\-3\\-3\end{bmatrix}$ per second per second.

Using Euler integration, with a timestep of 1 second, what is the position of the particle at time $t = 2$ seconds (i.e. after 2 time steps)?

Assume that acceleration is constant.

Assume that in each time step, updating position happens before updating velocity.

Enter your answer as integer values (NO decimal point)!

2

5 -7 0

pos 1 2 3 → 2 -1 3

vel 1 -3 0 → 3 -6 -3

t 0 1 2

acc 2 -3 -3

Euler Integration 1

Suppose Euler integration is used to generate particle positions in a simple Newtonian physics particle simulation. Which of the following would be associated with larger error in the computed positions?

- (a) Both a smaller timestep and large acceleration
- (b) Both larger acceleration particles and a larger timestep
- (c) A smaller timestep
- (d) A larger timestep
- (e) Particles with large accelerations

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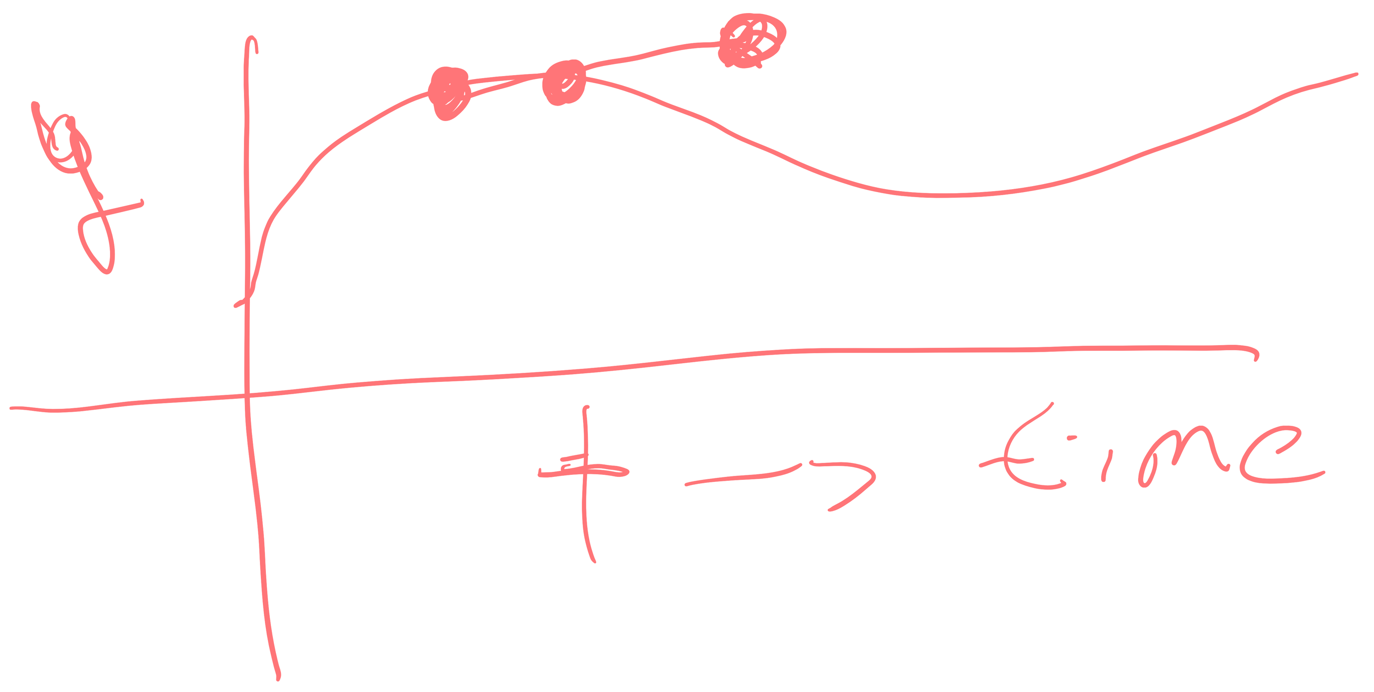
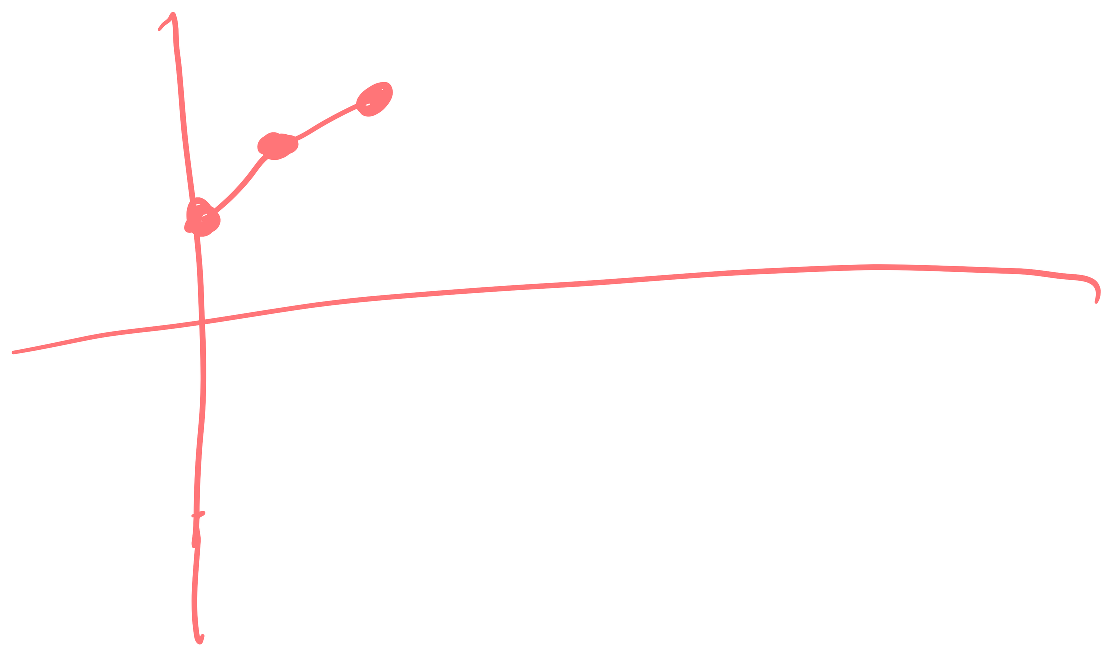




Sphere Sphere Collision Detection

distance between C_i

$$\mathbf{d}(t) = (C_0 + t\mathbf{v}_0) - (C_1 + t\mathbf{v}_1) = \underbrace{(C_0 - C_1)}_{s} + t\underbrace{(\mathbf{v}_0 - \mathbf{v}_1)}_{v}$$

$C_i \rightarrow$ centers

$r_i \rightarrow$ radii

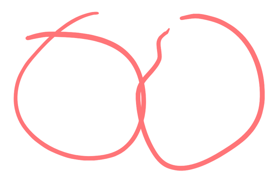
$v_i \rightarrow$ velocity

$$\mathbf{d}(t)\cdot\mathbf{d}(t) = (r_0 + r_1)^2 \Leftrightarrow \qquad \textit{(original expression)}$$

$$(\mathbf{s} + t\,\mathbf{v})\cdot(\mathbf{s} + t\,\mathbf{v}) = r^2 \Leftrightarrow \qquad \textit{(substituting } \mathbf{d}(t) = \mathbf{s} + t\,\mathbf{v}\textit{)}$$

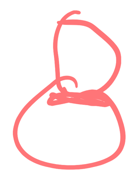
$$(\mathbf{s}\cdot\mathbf{s}) + 2(\mathbf{v}\cdot\mathbf{s})t + (\mathbf{v}\cdot\mathbf{v})t^2 = r^2 \Leftrightarrow \qquad \textit{(expanding dot product)}$$

$$(\mathbf{v}\cdot\mathbf{v})t^2 + 2(\mathbf{v}\cdot\mathbf{s})t + (\mathbf{s}\cdot\mathbf{s} - r^2) = 0 \qquad \textit{(canonic form for quadratic equation)}$$



The parametric formula derived here shows how one can determine at what future time t two spheres will collide. Which of the following conditions below would indicate that the spheres will not collide in the future? Check all that apply.

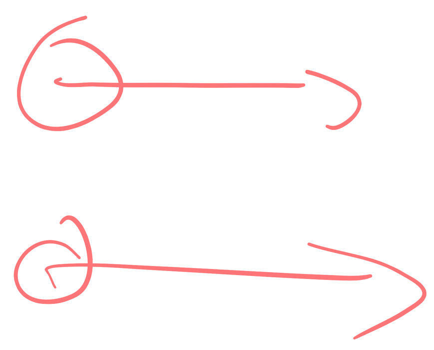
- [ ] (a) Solving for t yields only a single real root with a multuplicity of 2.



- [x] (b) All solutions for t are complex numbers.
- [ ] (c) A divsion by 0 when solving for t.
- [x] (d) All solutions for t are negative.

if $v_0 = v_1$ can happen but does not imply spheres will not be in collision in future

Select all possible options that apply.



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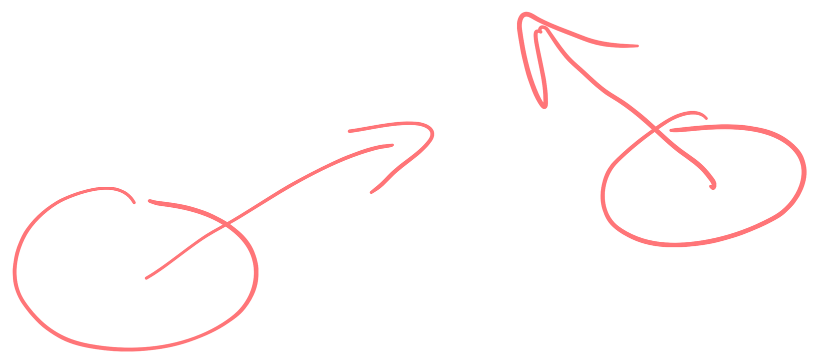
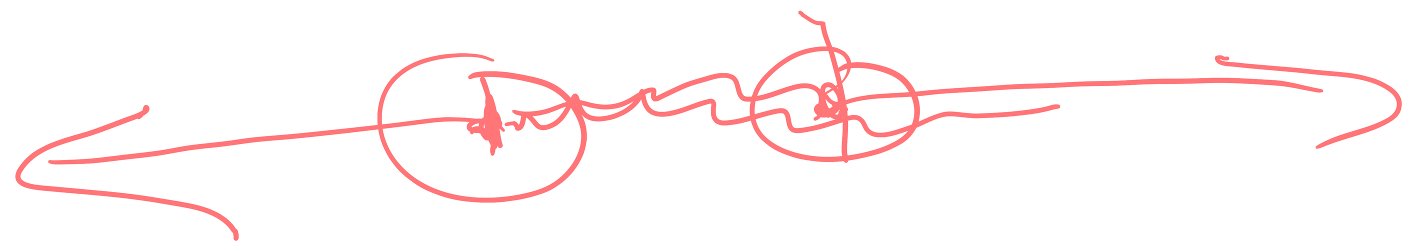
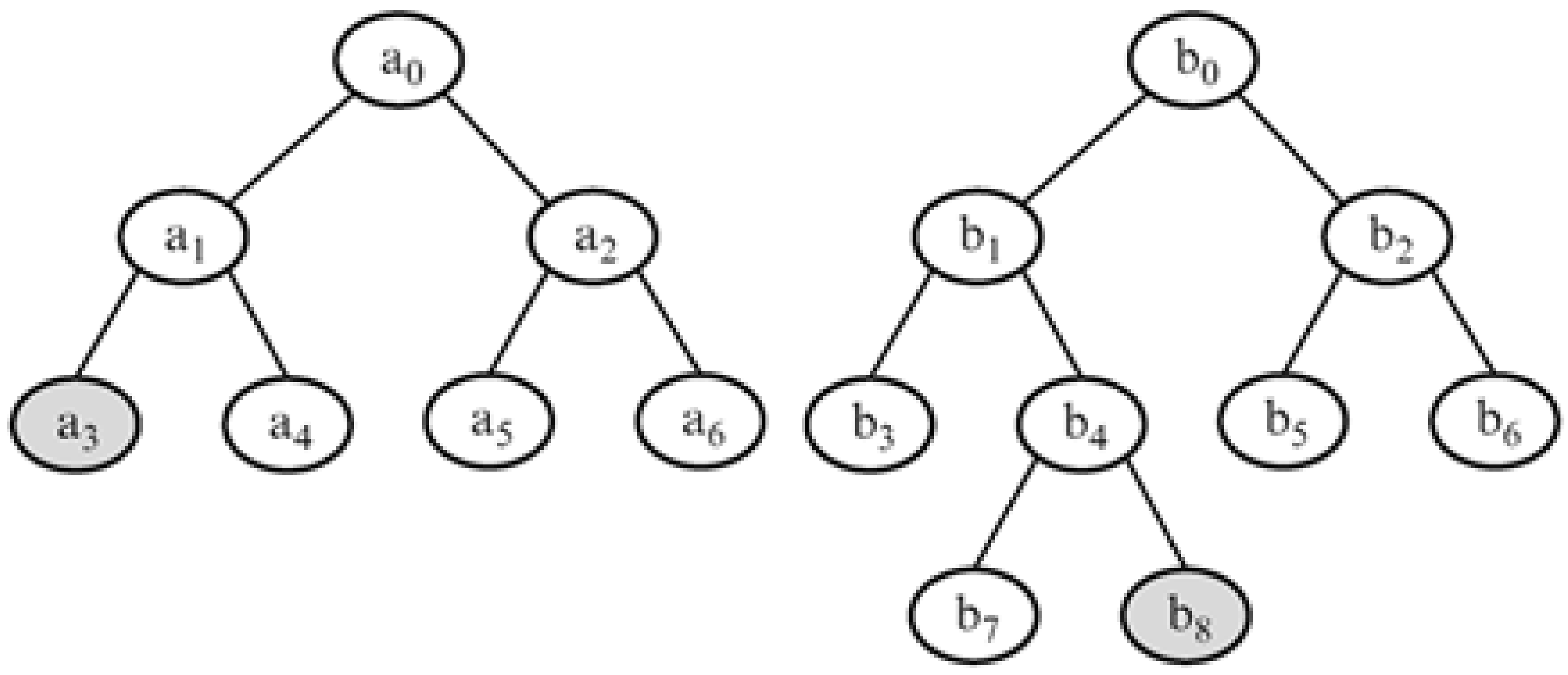




Collision Detection Between BVHs

For the following questions, refer to the image below:



1
4
16
8

29

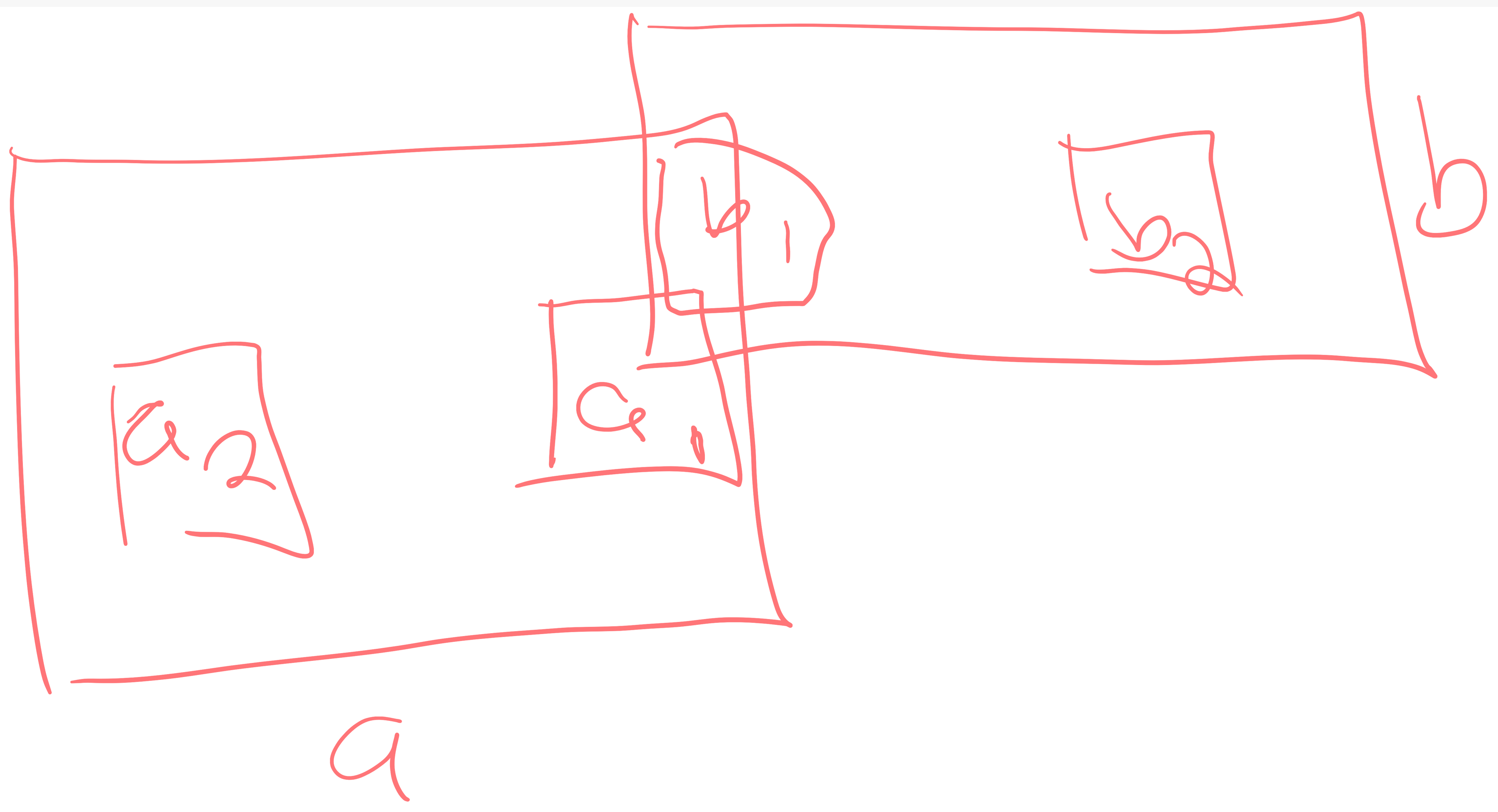
We have two objects **a** and **b**, each with their own BVH as shown below. Each node has a bounding volume associated with it. Suppose only the geometry in nodes a_3 and b_8 are in collision. What is the minimal number of bounding volume intersection tests that might be needed to determine this? This means you should assume there are no false positive bounding box intersection tests.

a_3 b_8

integer 11

Suppose instead that the only geometry in collision is in nodes a_4 and b_5 **but all the bounding volumes of all the nodes are in collision** (so you get the maximum number of false positives). How many bounding volume intersection tests are performed in that case?

integer 29



$a_0 b_0$

$a_1 b_1$ $a_1 b_2$ $a_2 b_1$ $a_2 b_2$

$a_3 b_3$ $a_3 b_4$ $a_4 b_3$ $a_4 b_4$

$a_3 b_7$ $a_3 b_8$

Euler's Method and Error

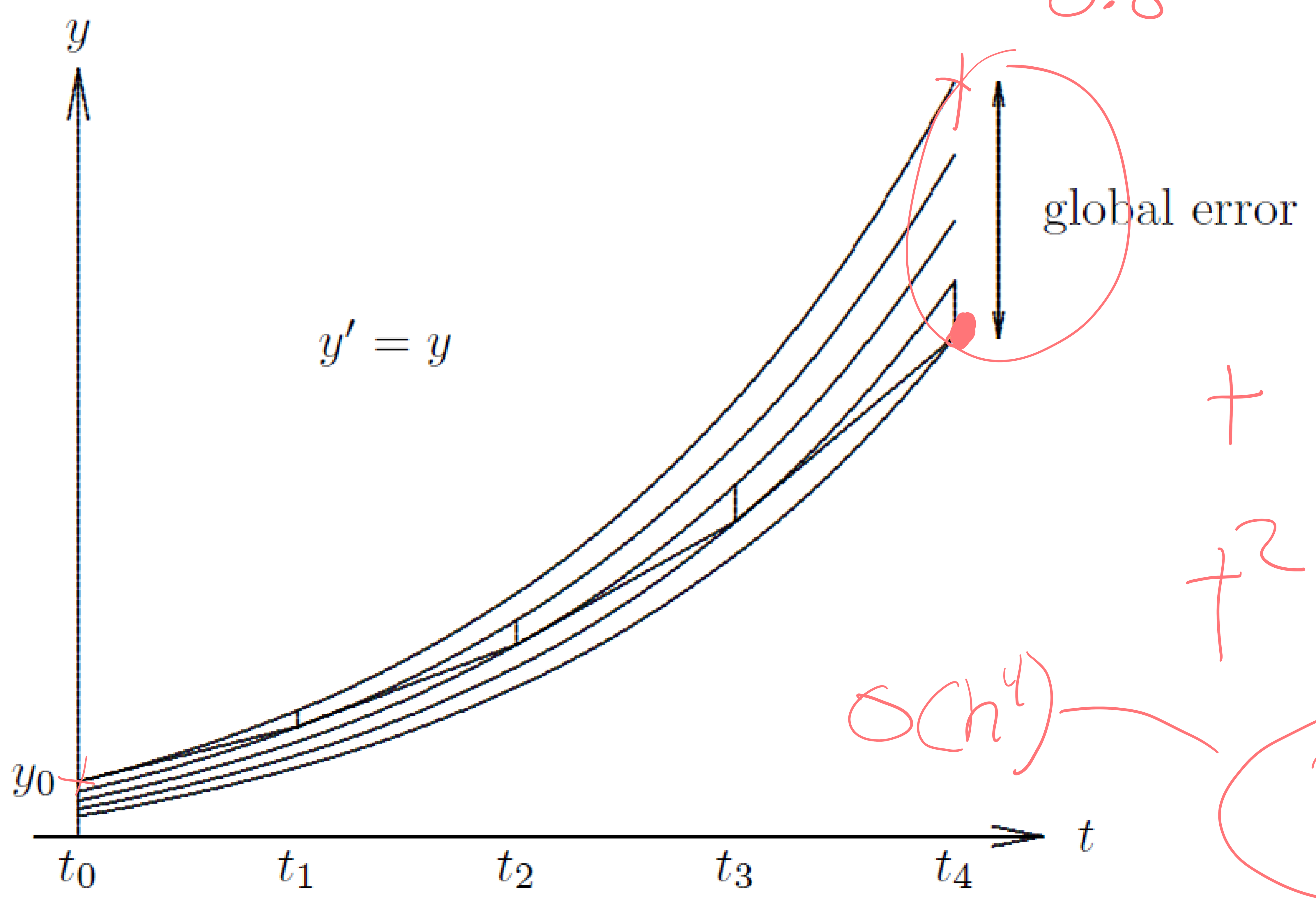
Suppose you are using Euler's method and reduce your step-size t by multiplying it by 0.8.
The error E will be reduced by some amount and will now be kE with $k \in [0, 1]$.
What is this value k for the local error? What is this value k for the global error?

local error reduction $k_{local} =$	number (rtol=0.0001, atol=0.0001)	❓
global error reduction $k_{global} =$	number (rtol=0.0001, atol=0.0001)	❓

$.8^2 = .64$ $O(t^2)$

$O(t)$

0.8



$t \to .8t$

$t^2 \to .64 t^2$

$O(h^4)$ — RK4

Figure 9.6: Local and global errors in Euler's method for $y' = y$.