Solving the 1D Heat Equation using the Forward Time - Centered Space Finite Difference Method

The heat equation in 1D is given by:

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2}$$

Where, u => Temperature, t => time, c => coefficient of heat conduction

Given a rod of length L, we will try to solve for the temperature profile along the x direction. Subdividing L into nx-1 equal subintervals, we get the length of each sub-interval as:

$$dx = L/(nx - 1)$$

Now, the second independent variable (apart from x) is time t, and assuming we are solving for the Temperature profile on an interval of time [0,tmax], so for nt points (or nt -1 time steps), each time step is given as:

$$dt = tmax/(nt - 1)$$

We will then denote the approximate solution at the grid points by:

$$u_{i,j} \approx u(x_i, t_j)$$

The heat equation can then be replaced by the difference equation:

$$\frac{u_{i,j+1} - u_{i,j}}{dt} = \frac{c}{dx^2}\left(u_{i-1,j} - 2u_{i,j} + u_{i+1,j} \right)$$

This equation can be solved for $u_{i,j+1}$:

$$u_{i,j+1} = ru_{i-1,j} + (1 - 2r)u_{i,j} + ru_{i+1,j}$$

Where r = (cdt) / (dx)^2 .

Initial conditions: for all i, at first time step, i.e, j = 1 :

$$\boldsymbol{u_{i,1} = \sin(\frac{\pi x}{L})}$$

Boundary conditions : for all j, at the rod edges:

$$\boldsymbol{u_{1,j} = u_{nx,j} = 0}$$

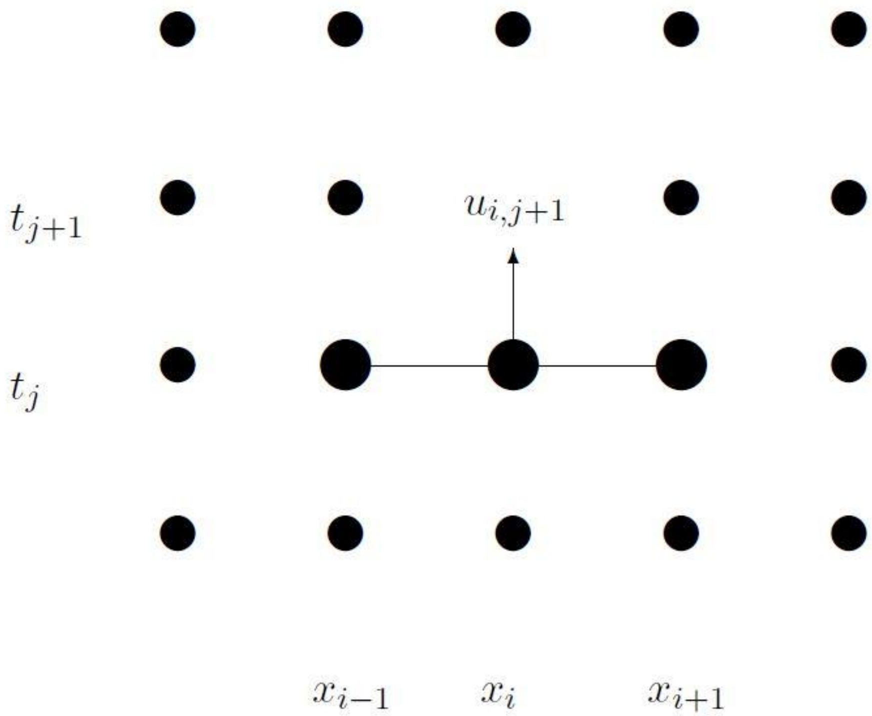

Figure 1: Marching forward in time