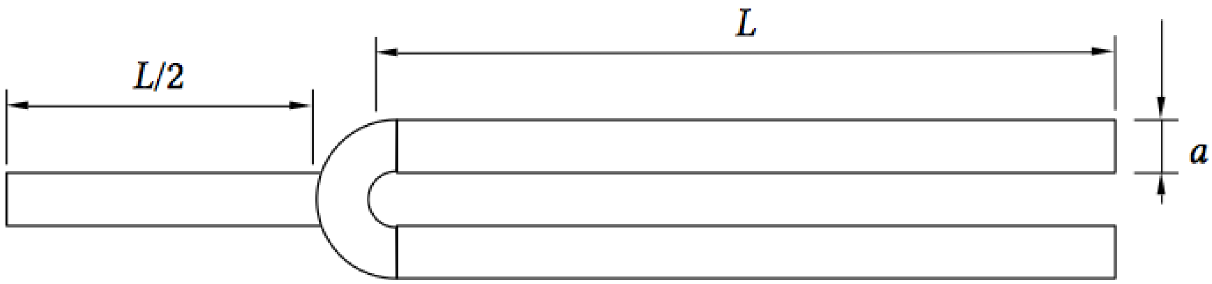
Tuning Fork Frequencies

Tuning forks are steel bars which acoustically resonate at a fixed pitch. They are frequently used to tune musical instruments due to their consistency and ease-of-use. The standard tuning fork sounds A (440 Hz), which is the standard concert pitch. In general, the following formula can be used to determine the frequency of a given tuning fork.

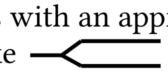
$$f = \frac{1.875^2}{2\pi l^2}\sqrt{\frac{a^2 E}{12\rho}}$$

where f is the frequency the fork vibrates (Hz);

- 1.875 is the smallest positive solution of $\cos x \cosh x = -1$;
- l is the length of the prongs (m) (typically 80–90 mm);
- E is the Young's modulus of the material (Pa);
- a is the edge of the square area A of the prong cross-section (m) (typically 4–5 mm); and
- ρ is the density of the material ($kg \cdot m^{-3}$).



You will create a tuning fork model in two and three dimensions which adhere to the specifications given below. You will then carry out four simulations. In each of these, you will determine the initial vibrational frequency of the model and the first two overtones plus stresses (`MODAL` analysis type with `3` modes and stress values). I recommend using a = 4 mm and starting with L from 80–90 mm.

1. Calculate the characteristics of a tuning fork designed to sound 440 Hz as the dominant pitch. Then simulate this tuning fork as:
    1. A two-dimensional simulation using `LINK180` elements with an appropriate cross-sectional area. (The geometry will be approximately like  .)
    2. A two-dimensional simulation using `BEAM188` elements with thickness the same as the prong width (square cross-section of prong).
    3. A three-dimensional simulation using `SOLID186` elements (full integration).
2. Find dimensions for a middle C (261.26 Hz) tuning fork from the formula above. Construct and model this fork (in three dimensions). Report the modes of your default model, and then tune it such that the dominant mode falls within 5% of the correct tuning frequency (if necessary).

Assume the tuning fork is made of aluminum, $E = 69\times10^{9}$ Pa, $\nu = 0.33$, and $\rho = 2.7\times10^{-3}$ g·mm^{-3}. You should verify whether this unit system is consistent and make certain that you carry out any unit transformations necessary before using the formula above.

You are responsible to show that your mesh converges reasonably and make a case for its sufficient accuracy. (A mesh density of 5% of the overall dimensions is usually a good starting point.) If the handle appears to be causing trouble, you can delete the handle and use a U-shaped geometry for the model; in any case, you need to constrain the base of the Y or U at the junction as it is a frequency node. I found that constraining the lines or points along its edge worked well. You should check the frequency range 30–19,000 Hz (the limits of human hearing).

(Intriguingly, tuning forks can be used to detect bone fractures noninvasively. A low C (128 Hz) tuning fork is toned on the suspected bone, causing the periosteum to vibrate and fire pain receptors, resulting in a local sharp pain. This is then used to recommend an X-ray.)

You will document the simulations in a 8–10 page report (with figures) containing the sections:

- Problem description (tuning fork shape, grid, etc.)
- Numerical values (element parameters, number of nodes, boundary conditions, etc.)
- Computational times (CPU time to solve)
- Observations of numerical behavior (boundary conditions, mesh behavior, etc.)
- Discussion of the physics (modes, etc.)

Include the following plots in your report, with data from each case you will study:

- Mesh of model
- Contour plot of von Mises stress
- Line plot of von Mises stress

The report should be formatted with 1.5 line spacing, 1 inch margins on all sides, and set in 11 point serif font. Figures and tables should be numbered with labels and captions.