Elongation of a Bar under Weight (5 points)

You will first consider a simple steel bar subject to the body force of gravity—in other words, what is the deformation of a suspended steel bar under its own weight?

The bar is made of steel, with $E = 3×10^7$ psi, $v = 0.3$, and $\rho = 7.36×10^{-4}$ lb$_f$·s·in^{-4} [Madenci2015]. (Always check your units for consistency; document any necessary dimensional conversion. As an aside, to convert from lb$_m$ to lb$_f$·s·in^{-1} one divides by 386.)

Create a linear vertical two-dimensional model of a beam using `BEAM188`. The model geometry should describe a beam 20 ″ in length with a ⌀ 4 ″ circular cross section (set in the `Sections→Beam→Common Sections` dialog). Use 20 divisions when meshing.

The top end of the bar is secured (no degrees of freedom of displacement are allowed), while the bottom end is free. The gravitational body force is applied as a structural inertial gravity load. The constant for gravity may be found by converting from 9.80665 m·s^{-2} to in·s^{-2}. Make sure that you apply this force in the proper direction (down).

Write a brief report (a paragraph or so plus figures) to report the maximum displacement of any node (the bottom end) and a graphic of the displaced and original models.

References

Madenci2015: E. Madenci, I. Guven. (2015) *The Finite Element Method and Applications in Engineering Using ANSYS*. Heidelberg: Springer-Verlag, pp. 314–317. DOI:10.1007/978-1-4899-7550-8

Analysis of Bicycle Frames (20 points)

When bicycles (two-wheeled velocipedes) were first invented around two hundred years ago, a variety of configurations for both the wheels and the frames were invented by industrious engineers. By the 1900s, however, almost all manufacturers of bicycles had settled on the ubiquitous diamond frame we use today. What advantages does this configuration offer to both manufacturer and rider?

You will document your simulations in a 7–10 page report (with figures) containing sections on:
- Problem description (specimen shape, grid, etc.)
- Numerical values (element parameters, number of nodes, boundary conditions, etc.)
- Computational times (CPU time to solve)
- Observations of numerical behavior (mesh behavior, etc.)
- Discussion of the physics (von Mises stress maxima, largest x and y components of stress and displacement, largest displacement, etc.)

Include the following plots in your report, with data from each case you will study:
- Mesh of model
- Deformed result with outline of undeformed original (with scale factor noted)
- Contour plot of von Mises stress
- Line plot of von Mises stress

Draisine Frame

The draisine is the earliest documented two-wheeled velocipede, or bicycle. The draisine and its immediate successors suffered from two flaws, from a modern perspective: there was no shock absorbance (tires were of wood and iron) and there was no means of propulsion except pushing on the ground with the feet. Nevertheless, the "bike craze" grew over the next few decades, aided by the subsequent introduction of pedals, chain, and crank as well as the pneumatic tire.

Draisine frames were originally made of wood. Define a new wood laminate material with the following properties: `Structural→Linear→Elastic→Isotropic:` E = 49.7 GPa, ν = 0.35, and ρ = 624 kg·m^{-3} [Ferreira2014] (again, check the units for consistency!).

> You will use wood laminate instead of wood in this case because wood properties are best expressed in a cylindrical coordinate system, rather than the cartesian coordinate system used by ANSYS for defining orthotropic material properties. Since natural and laminate wood are both anisotropic, we will also make the simplifying assumption that wood laminate is isotropic.

Model the frame with a fixed rear wheel point (keypoint 1) and a   *y*-fixed front wheel point (keypoint 6); this allows the frame to relax but not sink into the ground. The load of the rider is concentrated at the seat (keypoint 3). Define the following keypoints for the frame (check your units!):

	x/cm	*y*/cm
1	0.0	0.0
2	15.4	23.1
3	31.7	23.1
4	50.0	30.8
5	100.0	30.8
6	84.6	0.0
7	63.5	30.8
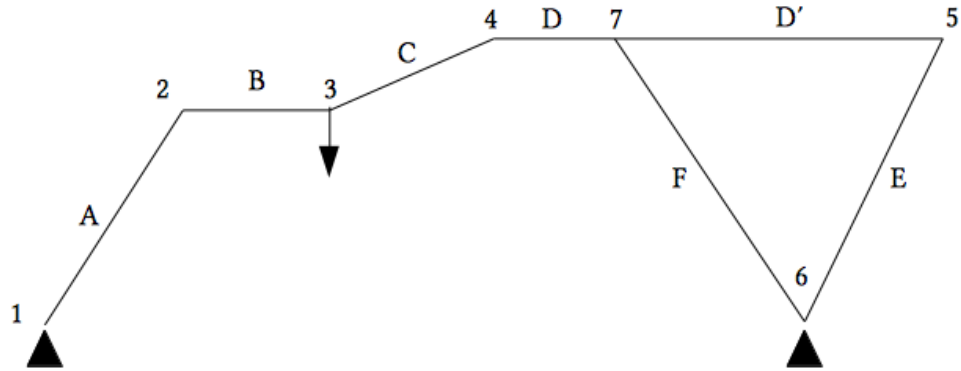
Mesh with one division per line using the `BEAM188` element with cross-section ● of radius 1 cm. Note that D and D′ are two separate elements in this case. (Remember, you can list the features you've created to double-check your input work. Also, save often.)

☞Aside: Overthinking the Force

A stationary adult human of mass 75 kg (165 lb) applies a force of 735 N of force to the bicycle seat (assuming no mass distribution across the pedals and handlebars).

Now, that's for a *stationary* adult—if you hit a speed bump at 16 km·h^{-1} and your entire body flies up and lands again on the seat, what happens?

Assuming the small angle of this speed bump to be approximable as a wedge, we observe, for angle

$$\theta = \arctan\left(\frac{7.5\,cm}{15\,cm}\right) = 26.6°$$

that the vertical velocity change is

$$v_{vert} = \left(4.44\,m{\cdot}s^{-1}\right)\sin\left(26.6°\right) = 2.00\,m{\cdot}s^{-1}$$

This velocity change takes place over a time span of

$$\Delta t = \frac{1.5\times10^{-2}\,m}{4.44\,m{\cdot}s^{-1}} = 3.4\times10^{-2}\,s$$

Combining these, we have an acceleration

$$\frac{dv}{dt} = \frac{2.00\,m{\cdot}s^{-1}}{3.4\times10^{-2}\,s} = 58.8\,m{\cdot}s^{-2} = 5.99\,g$$

which is quite a solid jolt. This becomes

$$F = ma = \left(75\,kg\right)\left(6\,g\right) = 4410\,N$$
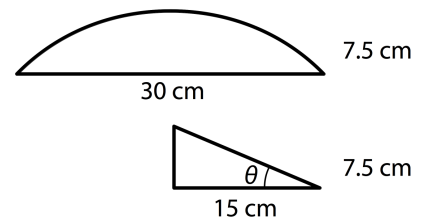
which clearly stresses both the bicycle frame and the pelvis.

For more information on biomechanics and forces on the human body, check out Jason Lake's article on biomechanics and Newton's laws[Lake2013].

Define boundary conditions as given in the geometry diagram above. Add the load on the basis of a stationary adult human, 735 N, and solve the system.

Answer the following questions (and include plots of the deformed system and von Mises stress):

- What is the maximum displacement? Report your nodal displacement (vector sum).

- How close is the material to failure?

- Plot the von Mises stress by element. Where is it at a maximum, and what is that value? Report the tabulated stress.

Serpentine Frame

Eventually, a serpentine frame was introduced, which along with other alterations reduced the jarring experienced by the rider on the "boneshakers".

Follow the steps for the draisine frame except using the following geometry, material, and boundary conditions.

Serpentine frames were made out of cast iron. Find the Young's modulus, Poisson ratio, and density for grey cast iron (and document your values in your report!) Use elements of the same dimensions as you did with the draisine frame.

Model the frame with a fixed rear wheel point (keypoint 1) and a *y*-fixed front wheel point (keypoint 6); this allows the frame to relax but not sink into the ground. The load of the rider is concentrated at the seat (keypoint 3). Define the following keypoints for the frame (check your units!):

|	*x*/cm	*y*/cm
1	0.0	3.9
2	0.0	59.2
3	42.1	29.0
4	100.0	31.6
5	77.6	39.5
6	57.9	34.2
7	77.6	0.0
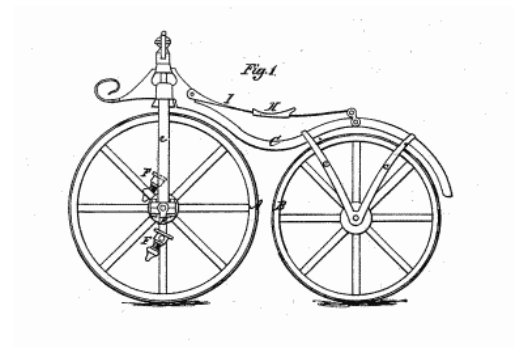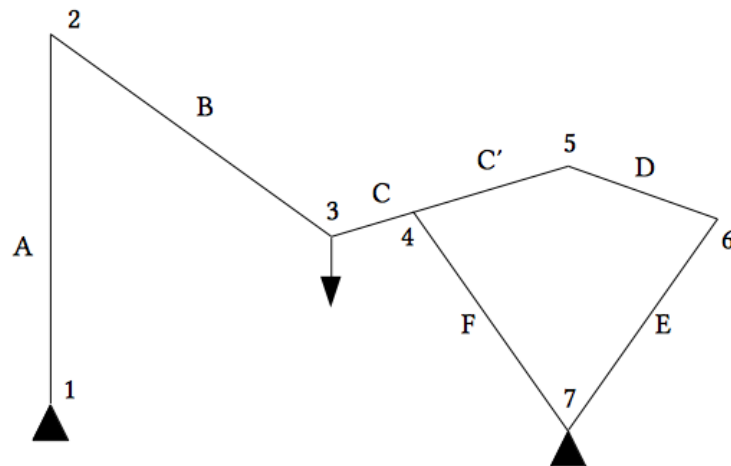
Mesh with one division per line.   Note that C and C′ are two separate elements in this case. (Remember, you can list the features you've created to double-check your input work. Also, save often.)

Add the load on the basis of a stationary adult human, 735 N, and solve the system.

Answer the following questions (and include plots of the deformed system and von Mises stress):

- What is the maximum displacement? Report your nodal displacement (vector sum).

- How close is the material to failure?

- Plot the von Mises stress by element. Where is it at a maximum, and what is that value? Report the tabulated stress.
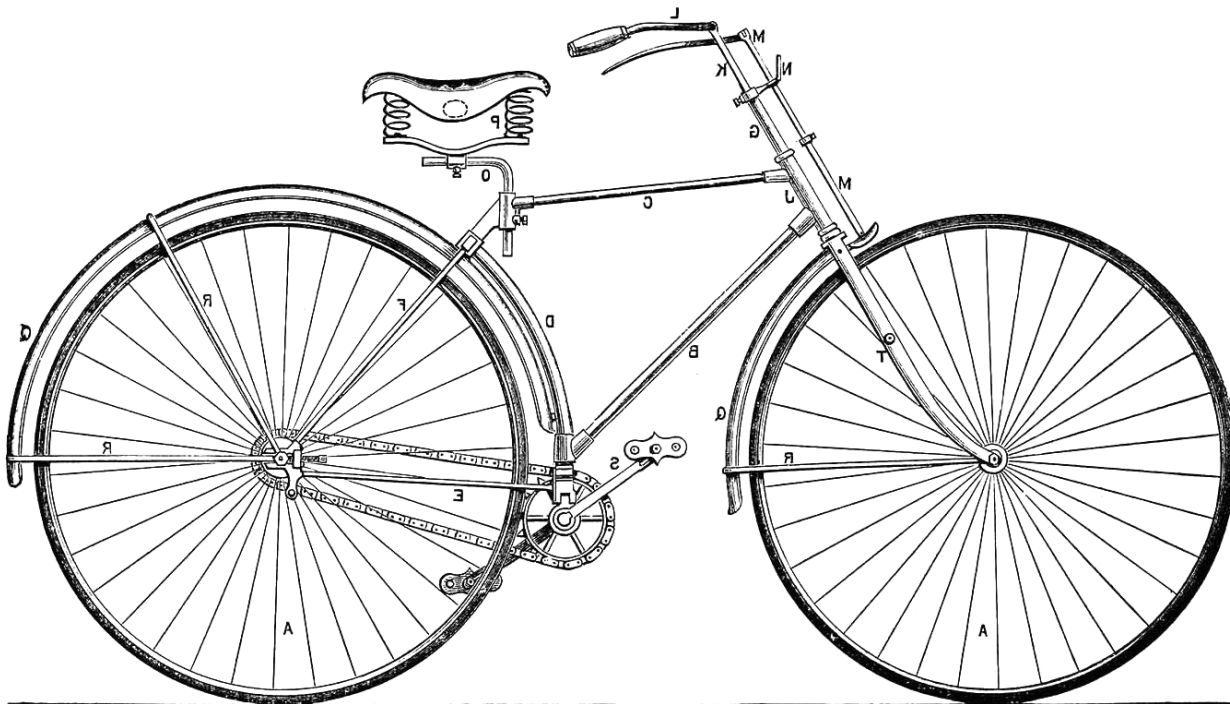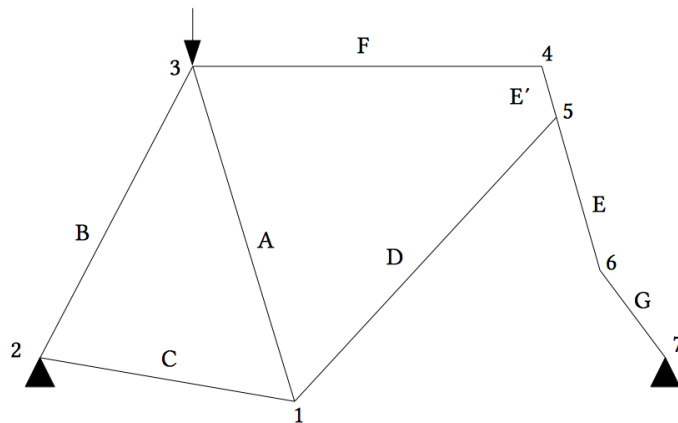

Diamond Frame

By the 1890s, however, pretty much every manufacturer had settled on the now-standard diamond frame for bicycles. Model the frame with an fixed rear wheel point (keypoint 2) and an x-fixed front wheel point (keypoint 7); this allows the frame to relax but not sink into the ground. The load of the rider is again concentrated at the seat (keypoint 3). Define the keypoints listed in the table for the frame (check your units!).

Although pipe elements are available in ANSYS, you will use a hollow circular beam (since our interest is in the forces and loading rather than the internal/external pressure differential). Define the element type as BEAM188. Change the sub-type to ◯ and define the inner and outer radii to be 0.014 (m) and 0.015 (m), respectively, yielding a ⌀ 3 cm frame tube.

	x/cm	y/cm
1	0.0	0.0
2	-40.8	7.0
3	-16.3	53.6
4	39.6	53.6
5	41.9	46.6
6	48.9	21.0
7	59.4	7.0

Diamond frames saw the introduction of dozens of new materials, from stainless steel pipes to carbon-fiber composites. To give the other frames a fair shake, rather than a high-tech material we'll handicap slightly by using a structural steel. Create a `Structural→Linear→Elastic→Isotropic` material with E = 200 GPa, v = 0.26, and ρ = 7830 kg·m⁻³.

Mesh with one division per line. Note that E and E′ are two separate elements in this case.

Apply constraints and loads: keypoint 2 should be constrained for all degrees of freedom; keypoint 3 should carry the load of a stationary human adult; keypoint 7 should be constrained in all degrees of freedom *except* `UX`—this allows the frame to relax slightly under loading.

Answer the following questions (and include the same plots as in the two previous examples):

- What is the maximum displacement? Report your nodal displacement (vector sum).

- How close is the material to failure?

- Plot the von Mises stress by element. Where is it at a maximum, and what is that value? Report the tabulated stress.

Run the simulation once more on the basis of a jolt, 4410 N, and report the same again. Is there a reason that bicycle frames have looked essentially the same for more than a century? Did this set of models have sufficient accuracy to illuminate that result? Engineers found that the diamond frame distributes stresses better than other models and requires less material.

☞Aside: Frame Failure Before Computation

When it has been too expensive or computationally difficult to model a bicycle frame, engineers have hung heavy weights from the bicycle frame while the entire assembly was uniformly heated in a furnace. The frame would then fail at the point of greatest stress.

Ferreira2014: M. B. Ferreira *et al.* (2014) Numerical and experimental evaluation of the use of a g lass fiber laminated composite materials as r einforcement in timber beams. *International Journal of Composite Materials 2014, 4* :2, pp. 73–82. DOI:10.5923/j.cmaterials.20140402.06

Lake2013: J. Lake. (2013) Biomechanics and Newton's laws: Force–time curves and human movement.