

# Homework 2: Exponential Families

your name

Due: February 3rd at 11:59 PM

**Problem 1:** Verify that displayed equation 7 in the exponential family notes holds for the binomial distribution, the Poisson distribution, and the normal distribution with both  $\mu$  and  $\sigma^2$  unknown.

**Problem 2:** Show that the second derivative of the map  $h$  (displayed equation 11 in the exponential family notes) is equal to  $-\nabla^2 c(\theta)$  and justify that this matrix is negative definite when the exponential family model is identifiable.

**Problem 3:** The above problem is one of the steps needed to finish the proof of Theorem 2 in the exponential family notes. Finish the proof of Theorem 2.

**Problem 4:** Let  $Y$  be a regular full exponential family with canonical parameter  $\theta$ . Verify that  $Y$  is sub-exponential.

**Problem 5:** In the notes it was claimed that the scalar products of  $\sum_{i=1}^n \{y_i - \nabla c(\theta)\}$  are also sub-exponential (page 15). Show that this is in fact true when the observations  $y_i$  are iid from a regular full exponential family.

**Problem 6:** Derive the MLEs of the canonical parameters of the binomial distribution, the Poisson distribution, and the normal distribution with both  $\mu$  and  $\sigma^2$  unknown.

**Problem 7:** Derive the asymptotic distribution for the MLE of the submodel mean value parameter

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vector  $\hat{\tau}$ .

**Problem 8:** Prove Lemma 1 in the exponential family notes.