Homework 2: Exponential Families

your name

Due: February 3rd at 11:59 PM

Problem 1: Verify that displayed equation 7 in the exponential family notes holds for the binomial distribution, the Poisson distribution, and the normal distribution with both μ and σ^2 unknown.

Problem 2: Show that the second derivative of the map h (displayed equation 11 in the exponential family notes) is equal to $-\nabla^2 c(\theta)$ and justify that this matrix is negative definite when the exponential family model is identifiable.

Problem 3: The above problem is one of the steps needed to finish the proof of Theorem 2 in the exponential family notes. Finish the proof of Theorem 2.

Problem 4: Let Y be a regular full exponential family with canonical parameter vector θ. Verify that Y is sub-exponential.

Problem 5: In the notes it was claimed that the negative and/or a scalar products of $\sum_{i=1}^n \{y_i - \nabla c(\theta)\}$ are also sub-exponential (page 15). Show that this is in fact true when the observations y_i are iid from a regular full exponential family.

Problem 6: Derive the MLEs of the canonical parameters of the binomial distribution, the Poisson distribution, and the normal distribution with both μ and σ^2 unknown.

Problem 7: Derive the asymptotic distribution for the MLE of the submodel mean value parameter vector $\hat{\tau}$.

Problem 8: Prove Lemma 1 in the exponential family notes.