

Homework 2: problems about exponential families

your name

Due: February 14th at 11:59 PM

This homework set will cover problems concerning exponential family theory. All derivations must be typed. Screenshots of work done with pen and paper will not be accepted.

Problem 1 [10 points]: Verify that displayed equation 7 in the exponential family notes holds for the binomial distribution, the Poisson distribution, and the normal distribution with both μ and σ^2 unknown.

Problem 2 [20 points]: This problem concerns the proof of Theorem 3 in the exponential family notes. Do the following:

- **part a** [10 points]: Show that the second derivative of the map h is equal to $-\nabla^2 c(\theta)$ and justify that this matrix is negative definite when the exponential family model is identifiable.
- **part b** [10 points]: Finish the proof of Theorem 3.

Note that part a will be referenced later in this course. Hence, it is treated as its own sub-problem.

Problem 3 [20 points]: Let $y \in \mathbb{R}$ be a regular full exponential family with canonical parameter $\theta \in \mathbb{R}$. Do the following:

- **part a** [15 points]: Verify that y is sub-exponential.
- **part b** [5 points]: In the notes it was claimed that the scalar products of $\sum_{i=1}^n \{y_i - \nabla c(\theta)\}$ are sub-exponential (see the “Finite sample concentration of MLE” section in the exponential family notes). Verify that this is true when y_i are iid realizations from a regular full exponential family.

Problem 4 [10 points]: Derive the MLEs of the canonical parameters of the binomial distribution, and the normal distribution with both μ and σ^2 unknown.

Problem 5 [10 points]: Derive the asymptotic distribution for $\hat{\tau}$, the MLE of the submodel mean value parameter vector. Hint: use the [Delta method](#).

Problem 6 [20 points]: Do the following:

- **part a** [10 points]: Prove Lemma 1 in the exponential family notes.

- **part b** [10 points]: Provide a brief explanation of why this Lemma is important without using any mathematical symbols.

The above six problems are worth 90 points in total. 10 points will be allocated for presentation and correct submission of the homework.