~Stability

Using Barrowman's equations, the center of pressure was approximated both with and without the braking mechanism. Moreover, this allowed us to not only determine the ideal theoretical location of the braking mechanism but also the safe range of values for the ideal location of the center of mass relative to the nose cone and center of pressure.

Methodology

One of the most accurate methods of finding center of pressure is through the use of Barrowman's Equations. At an angle of attack of 0 degrees, the aerodynamic normal force or the aerodynamic "reaction" force caused on the body tube is 0 as long as the surface is smooth and symmetric (no change of length from its starting point until ending point). Barrowman's Formulas do not take lift generated from the body tube due to the change in angle of attack (caused by wind or overstability) into consideration, mathematical formulas were later derived from the former equations to take lift into consideration when calculating the center of pressure of a rocket body. However, this normal aerodynamic force was found to be negligible for angles of attack less than or equal to 10 degrees and/or at subsonic speeds.

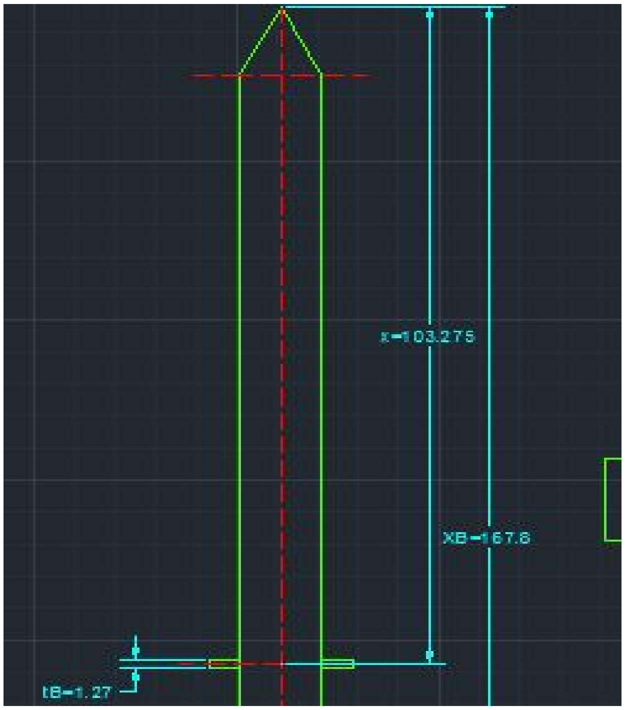
Assumptions

1. *The angle of attack of the rocket is less than ten degrees.*
2. *The effects of compressibility can be neglected ($Ma < 0.4$).*
3. *Viscous forces are negligible.*
4. *Lift forces on the rocket body tube can be neglected ($(N_F)_L = 0$).*
5. *The airflow over the rocket is smooth and does not change rapidly.*
6. *The nose of the rocket comes smoothly to a point .*

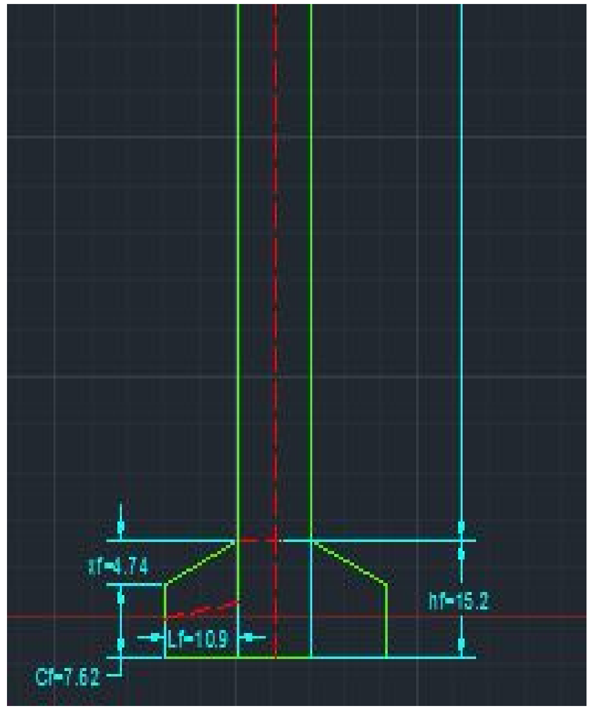
Approach

To Find center of pressure for rocket, we need the C_p location ($\bar{x}_N$, $\bar{x}_F$) of nose and fins. We must also find the aerodynamic normal forces on the fins and nose tip. We will then use these variables to find the location of center of pressure when the braking mechanism is retracted. We will then compare this model approximation to the reference value obtained through open rocket. Upon confirming our results, we will then derive a formula for the normal aerodynamic forces on the extracted braking mechanism from the same equation used for calculating normal aerodynamic forces applied on the N amount of fins. Finally, we will factor the braking mechanism into our calculations for center of pressure and measure the resultant change in center of pressure with respect to the placement of the mechanism itself.

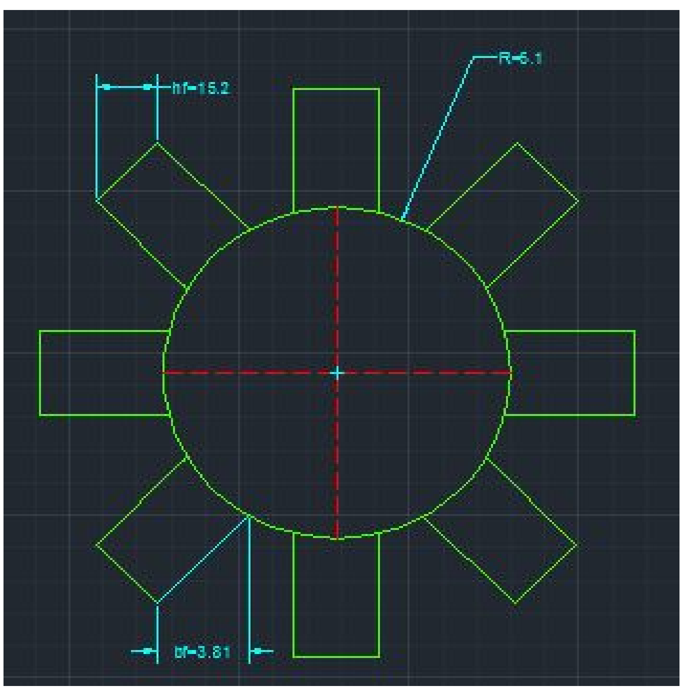
Schematic design (in scale)



Side View: Top Section.



Side View: Bottom Section.



Top View

Calculating C_p Without Braking Mechanism

OpenRocket Simulations (reference)

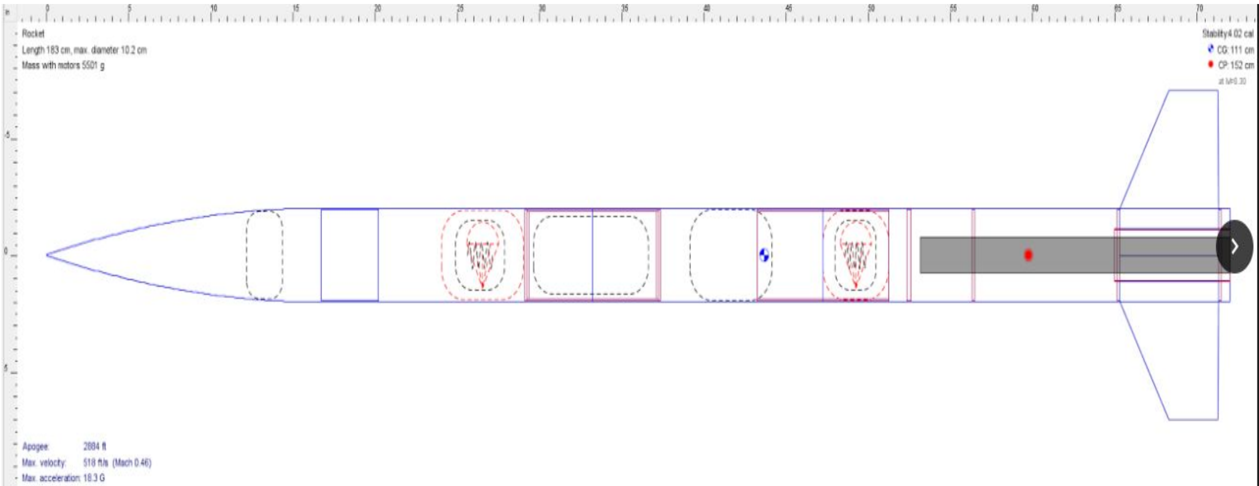


Figure 1. Openrocket simulation on of Norman on J class motor. 4.02 Cal Stability Margin

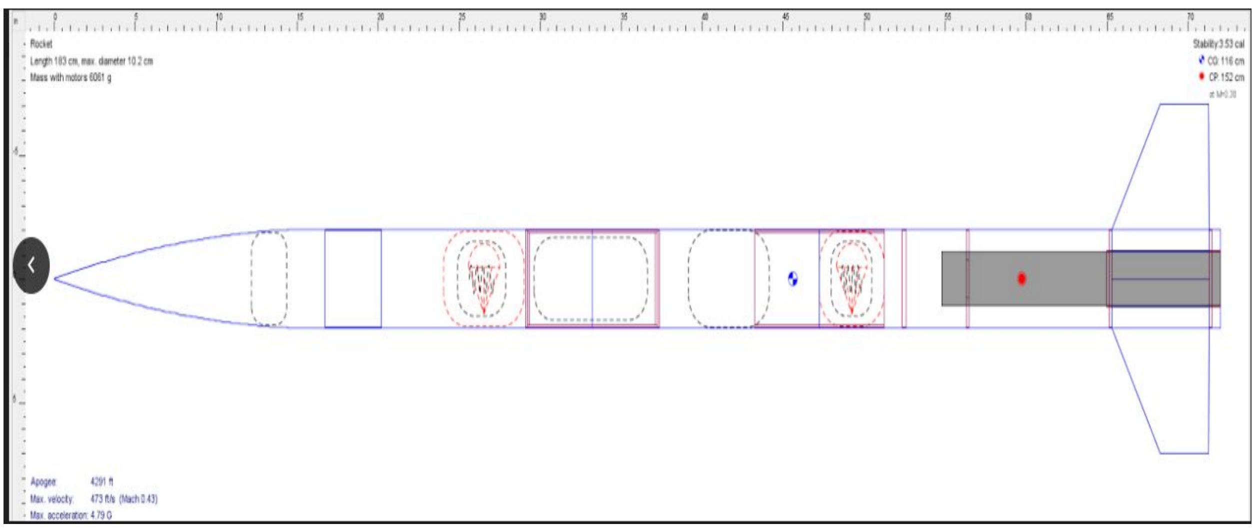


Figure 2. Openrocket simulation of Norman on K class motor: 3.53 Cal Stability Margin

Note: A Downward shift in center of gravity of Gravity, whist switching from J to K, cases the decrement of the stability margin. **Center of pressure is independent from mass.**

Equations Used

	Centroid (cm)	**Normal Aerodynamic Force**
Fins (x4)	$\bar{x}_F = \left[\frac{xf(hf+2cf)}{3(hf+cf)} \right] + x_b + \frac{1}{6} \left[(h_f+c_f) - \frac{hf\,cf}{hf+cf} \right]$	$(C_N)_F = \left[\frac{R}{wf+R} + 1 \right] \left[\frac{4N\left(\frac{wf}{2R}\right)^2}{1+\sqrt{1+\left(\frac{2lf}{hf+cf}\right)}} \right]$ (Where N is the number of fins)
Nose Cone	$\bar{x}_N = \frac{2}{3} l_N$	$(C_N)_N = 2$

Main plug-in equation for Center of pressure:

$$\overline{X} = \left[(C_N)_N \bar{x}_N + (C_N)_F \bar{x}_F \right] \div \left[(C_N)_N + (C_N)_F \right]$$

Results

	Reference (cm)	**Barrowman's Model (cm)**
J Class motor	**152**	**151**
K Class motor	**152**	**151**

- Percent error for J-class & K-class motor was found to be -1.37 percent. (based on data at the time).
- Reference in this case was open an openrocket simulation.

Calculating C_p With Braking Mechanism:

Since the braking mechanism is a non-conventional component on a rocket. We must divide it into three components. The top plate that is perpendicular to the wind, two identical sides parallel to the wind, and the front size of the plate that is parallel to the wind as well. (w/resp to chord line).

Air Flow

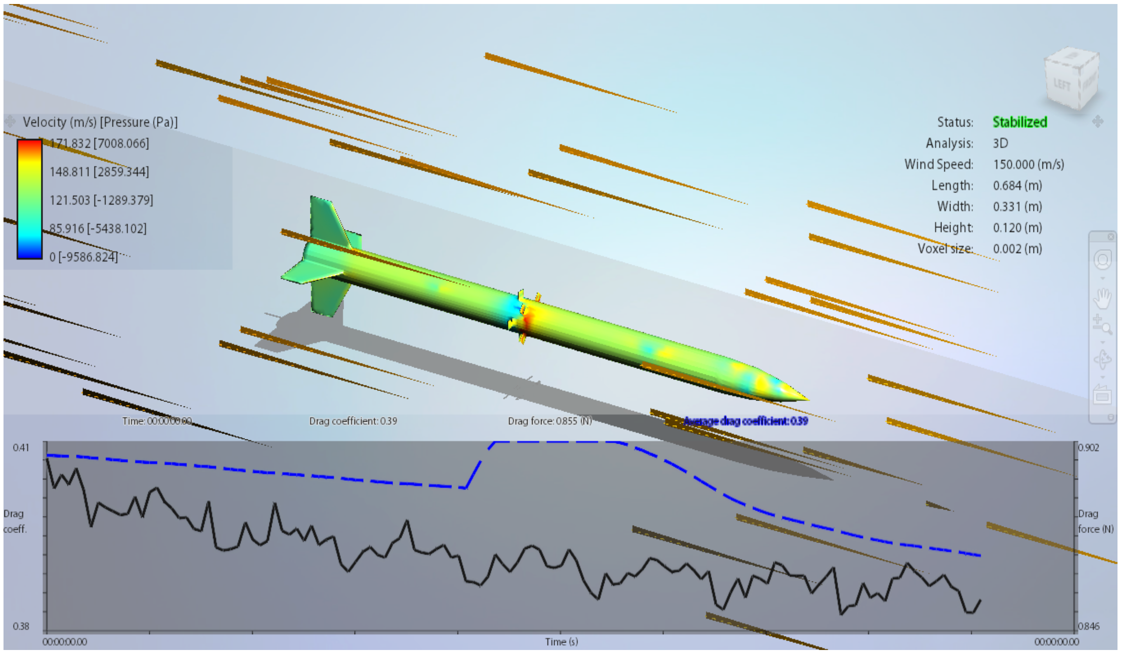


Figure C1. Flow Design wind tunnel Simulation showing pressure at all points & Active drag plot of a scaled down model rocket. (scaled down cm to in)

As we can see, air flows symmetrically about the chord line in all cases except that of the front and top plate of our braking mechanism. However, that can be modeled with the identical front plate of the B/M on the opposing side of the rocket. Furthermore, just like conventional components, the aerodynamic coefficient of side plate and the front plate can be found using the barrowman equations since their chord line is parallel to the wind and air flows symmetrically through the two sides and the two front sides on opposing sides of the rocket. However, since the top plate is both perpendicular to the wind and air does not flow symmetrically about the plate's chord line, we must model it by performing an analysis of the axial and lift forces versus the normal and drag forces in order to prove that the aerodynamic Normal coefficient of the top plate is simply its drag coefficient. We can then factor these three components (w/resp to their centroids) into the main plug in equation. The braking mechanism in this case was placed at the ideal location found using the equations derived below and a scaled down version of Norman was found to be **stabilized** on Autodesk flow design.

Analysis of B/M

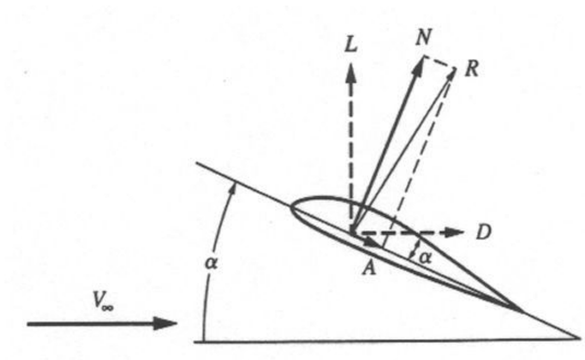
Since barrowman's equations for finding normal coefficients are only applicable for aerodynamic components on the rocket such as the fins. Furthermore, all these components have negligible thicknesses at the leading edge. Moreover, the 3rd dimension/thickness can be ignored. Hence, barrowman's equation for finding normal force coefficient on the fin is two Dimensional. Therefore, the fins are modeled as two symmetrical/identical plates separated by a chord line that is parallel to the direction of the wind. One fin plate on the visible front side of the 2-D schematic (refer to schematic diagram) and the other on the opposite (hidden) side. Similarly, the B/M has two side plates, a front plate, a back plate inside the rocket tube (not needed) and the top plate

Side Plate & front plates

The Normal coefficient for these parallel components (w/ respect to chord line) is simply derived from the equation used earlier for the fins of the rocket. Where Cf is equal to zero. The fins are modeled as two parallel plates separated by a parallel chord like (to its neutral axis) and connected together by a rounded leading edge and a trailing edge that plays a negligible role in determining the center of pressure. Similarly, the bottom plate of the Braking mechanism can be ignored. Upon breaking down this rectangular cube, The top plate can be modeled separately by looking into the the aerodynamic forces on the top section of the plate. Moreover, the front and side plates can be modeled as two parallel with a leading edge separated by a chord line, just like the fins.

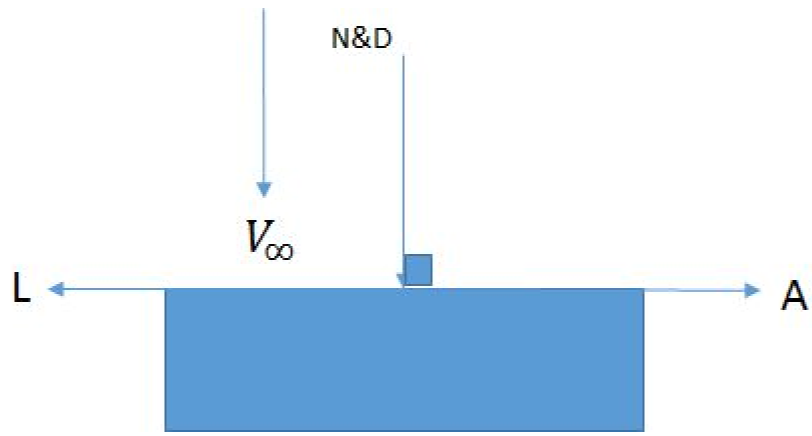
Top Plate

Model used



This figure shows the Difference between lift (L) and drag (D) versus normal force (N) and axial force (A) on an airplane wing at a certain and of attack, alpha. However, since the plane wings are aerodynamic and designed to minimize drag and maximize lift at an angle of attack of 0 with respect to the wind. This model is only valid for angle of attack of 90 degrees, At this angle of attack the wing is no more aerodynamic than the top plate of our B/M with respect to the wind. In Fact, at angles of attack close to 90 degrees, the plane wing is simply a Trapezoidal plate perpendicular to the wind. Moreover, this is what causes an aircraft to stall.

Proof and derivation



- Alpha is equal to 90 degrees, alpha is the angle the wind attacks the geometric chord line of this rectangular plate,
- Alpha is the angle of attack

Equations

1. L=Ncos(α)-Asin(α)
2. A=-Lsin(α)+Dcos(α)
3. N=Lcos(α)+Dsin(α)
4. A=-Lsin(α)+Dcos(α)

- The lift force is defined to be perpendicular to the velocity vector while drag is defined to be parallel to it.
- No matter what the angle of attack (α) is, lift and drag always maintain the same orientation to velocity.
- Normal and Axial Forces are NOT measure with respect to velocity, but with respect to the geometry of the airfoil itself .
- Key geometrical parameter: Chord line is defined from leading to trailing edge of the airfoil.

Analysis based off model

3. N=Lcos(90)+D sin(90)

Therefore, N=D

1. L=N cos(90)-A sin(90)

Similarly, L=-A

D=1/2pV^2S Cd (Where S is the area of the top plate)

Similarly,

N=1/2pV^2SC n

Therefore, C $n = Cd$

Conclusion: At 90 degree angle of attack on B/M, Drag force equals Normal force, lift and axial forces cancel out, in fact, any change in the angle of attack will result in stabilization of rocker as a whole due to the resulting lift and axial forces.

Analyzing model used

Alpha [deg]	Lift (CL) [-]	Drag (CD) [-]	Normal (CN) [-]	Axial (CA) [-]
0	0.0000	0.0000	0.0000	0.0000
10	0.8628	0.0209	0.8533	-0.1293
15	1.0314	0.0577	1.0112	-0.2112
17	0.5802	0.2164	0.6181	0.0373
23	0.5752	0.3378	0.6615	0.0862
33	0.8826	0.6971	1.1199	0.1039
45	0.9620	1.0828	1.4459	0.0854
55	0.8579	1.4210	1.6561	0.1123
70	0.5603	1.6585	1.7501	0.0407
80	0.3273	1.8010	1.8305	-0.0096
90	0.0744	1.8377	1.8377	-0.0744
100	-0.1835	1.7581	1.7632	-0.1246
110	-0.4265	1.6362	1.6834	-0.1589
120	-0.6298	1.5037	1.6171	-0.2065
130	-0.8132	1.2601	1.4880	-0.1870
140	-0.8975	0.9425	1.2934	-0.1451
150	-0.7041	0.6038	0.9117	-0.1708
160	-0.5802	0.3126	0.6521	-0.0953
170	-0.8132	0.1325	0.8239	0.0107
180	0.0000	0.0000	0.0000	0.0000

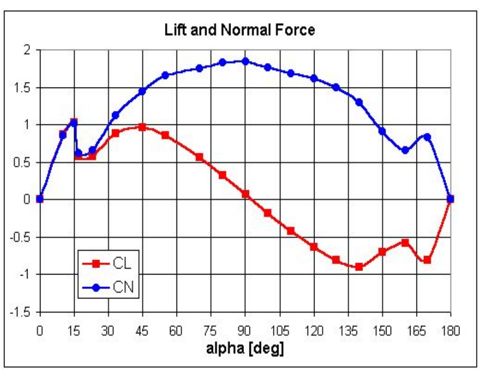


Figure D1 Comparison of Lift and Normal for coef.

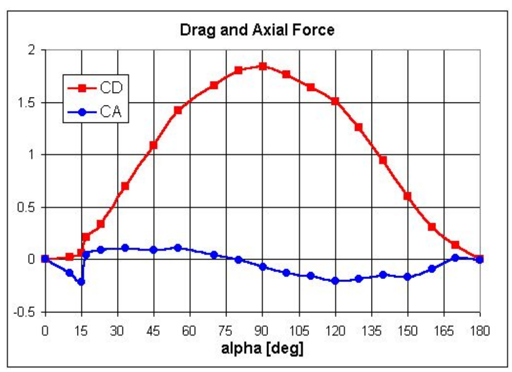


Figure D2 Comparison of Drag and Axial Coefficients

Normal force was assumed to be 5000 N and Axial force was assumed to be 1200 N on the wing in this case. Upon running a matlab simulation to calculate the aerodynamic coefficient at various angles of attacks, i was able to determine that in fact, my observations were correct. As shown in the table above, at an angle of attack of 90. The normal coefficient is equal to the drag coefficient and is at its maximum value. The lift generated (absolute value) is at minimum. Moreover, the wing is no longer aerodynamic. The axial force and lift cancel out. Hence, No lift is generated. Finally, this is an ideal model for determining the contribution of the top plate of our braking mechanism which is close to 90 degrees at all times. This model is only an ideal one for our purpose at angles of attack (alpha) very close to 90 degrees which is the orientation of the top plate with respect to the wind at an angle of attack of 0.

Equations used

	Normal Aerodynamic Force	**Centroid (cm)**
Breaking mechanism(TOP PLATE)	$(C_N)_{TB/M} = (C_D)_{TB/M}$	$\overline{X}_{\mathbf{TB/M}} = x$ *Where x is the position of the upper tip of the B/M from nose.
Breaking mechanism (symmetrical side plates)	$(C_N)_{SB/M} = \left[\frac{R}{Bf+R} + 1\right]\left[\frac{4N(\frac{Bf}{2R})^2}{1+\sqrt{1+(\frac{Bf}{Th})^2}}\right]$	$\overline{X}_{\mathbf{SB/M}} = x + (T_h/2)$
Breaking mechanism (front plate).	$(C_N)_{FB/M} = \left[\frac{R}{wf+R} + 1\right]\left[\frac{2N(\frac{Zf}{2R})^2}{1+\sqrt{1+(\frac{Zf}{Th})^2}}\right]$	$\overline{X}_{\mathbf{FB/M}} = x + (T_h/2)$
Fins (x4)	$(C_N)_F = \left[\frac{R}{wf+R} +1\right]\left[\frac{4N(\frac{wf}{2R})^2}{1+\sqrt{1+(\frac{2lf}{hf+cf})^2}}\right]$	$\bar{x}_F = \left[\frac{xf(hf+2cf)}{3(hf+cf)}\right] + x_b + \frac{1}{6}\left[(h_f+c_f) - \frac{hfcf}{hf+cf}\right]$
Nose Cone	$(C_N)_N = 2$ approx. 2 for most nose cone shapes.	$\bar{x}_N = \frac{2}{3} l_N$

Main plug-in equation for Center of pressure:

$$\overline{X} = \left[(C_N)_N\bar{x}_N + (C_N)_F\bar{x}_F + (C_N)_{TB/M}\bar{x}_{\mathbf{TB/M}} + (C_N)_{SB/M}\overline{X}_{\mathbf{SB/M}} + (C_N)_{FB/M}\overline{X}_{\mathbf{FB/M}}\right] \div \left[(C_N)_N + (C_N)_F + (C_N)_{TB/M} + (C_N)_{FB/M} + (C_N)_{SB/M}\right]$$

Results

The ideal location for the placement of the braking mechanism was determined to be approximately 103.275 cm from the nose cone in order to keep the center of pressure from shifting.

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