Preliminary Design Report

Prepared by:
The University of Iowa AIAA
Midwest High Power Rocketry Competition Team

Submitted on: March 20, 2015

Table of Contents:

Executive Summary

This report contains all information about the preliminary design of the University of Iowa’s rocket. Included are descriptions of components, including the fins, airframe, propulsion system, coupling interface, and payload system. Within each description are details and equations utilized as well as the reasoning behind design decisions. Diagrams of components and the overall design are given. Along with the design descriptions, analysis of the rocket design is included with appropriate figures and plots of resulting simulation data. This analysis was used to provide the predicted performance of the rocket as designed. Safety analysis is also included, and all relevant safety procedures are discussed. Finally, the included appendices give breakdowns of equations used as well as external sources utilized during the design of the rocket.

Design Features of Rocket

Rocket Fin Specifications

The team initially searched online for equations to consider when choosing fin shape, location, and size. *Source 2* in *Appendix B* was the first document considered, which outlines the basic fluid mechanics behind rocket drag. After reviewing the document, it was determined that smaller fins are better in general for reducing drag, and that 3 fins work better than 4 when aerodynamic efficiency is concerned. Members then observed common rocket fin designs and concluded that the majority of high-power rockets had trapezoidal fins. According to *Source 3* in *Appendix B*, the shape is easier to manufacture than elliptical fins, easier to calculate the forces generated on them, and easier to taper for aerodynamic efficiency. The decision was made to create 4 trapezoidal fins on both the booster and dart so as to make fin alignment easier.

An Excel spreadsheet was created to begin optimization of the trapezoidal fin design based on the centers of gravity for the booster, dart, and booster/dart combination. The ideal rocket would have a center of pressure just below the center of gravity (CG) in all three cases. This is what the team was optimizing for during calculation iterations. The calculations used were acquired from *Source 4* in *Appendix B* and the equations are listed in *Appendix D*. A 1/16 in (0.1588 cm) thick woven carbon fiber reinforced polymer plate was purchased as the material from which the fins would be cut. A water-jet CNC cutter was used to cut out the appropriate geometry for each fin. Models of the rocket fin designs can be found in *Appendix C* as *Figures 2* and *3*.

Rocket Body Specifications

The body of the booster and dart were both originally designed to be composed of composite materials created from a wet-layup procedure by AIAA team members. Complications arose during the materials testing portion of construction and a backup design had to be utilized. Compression tests on Blue Tube 2.0, a type of vulcanized craft paper for large model rockets, resulted in minimal deformation of the material. The relatively low cost and easy access to the material through Apogee Rocket Components supported the use of Blue Tube 2.0 as the main material for the construction of both the dart and booster bodies.

The current design of the dart is 116.27 cm in length and 5.74 cm in maximum diameter. This includes the nosecone (24.13 cm), dual-deployment system bay (15.24 cm), electronics rail (15.4 cm), aluminum tailcone/coupling section (28.7 cm), and parachute bays (42.9 cm total). The current design of the booster is approximately 100 cm in length and 10.196 cm in maximum diameter. This includes the nosecone (20 cm), motor (14.3 cm), parachute bay (approximately 20 cm), and empty space. These dimensions can be seen in *Figure 11* of *Appendix C*. The ratio of dart to booster diameter was based on literature found when researching drag separation. While the weight of the rocket is not entirely known at this time, high-end estimates based on the components indicate that both the booster and dart will be no more than 2.268 kg each.

Nosecone Specifications

The nosecone of the dart and booster were both bought through Apogee Rocket Components and are constructed of polypropylene plastic. Both nosecones had a standard ogive profile and were 24.13 cm in length. They each include a small tab on the inside for the connection of a parachute and shock cord. Due to packing constraints and construction techniques, these tabs were removed. For the dart, a bolt has been run through the nosecone for the attachment of the streamers and shock cord. The tip of the booster nosecone has been removed so as to provide a flat surface for the interface between the booster and dart.

Propulsion Design Specifications

The motor to be used for both the test flight and competition flight is the Cesaroni I445 motor. This motor will be used for all flights of the rocket in order to ensure consistency. This will also allow for accurate comparisons between the performances of the test flight and the competition flight. The motor will be placed in a Cesaroni 54 mm 1-Grain Case with a rear closure. A Pro-54 delay adjustment tool will be used to ensure that the motor ejection does not occur before separation of the dart and booster. Specific data regarding the motor is given in *Table 1* in *Appendix A.* This table includes information regarding the motor and propellant weight, the average thrust, maximum thrust, the burn time, and impulse. Specifications for the motor were taken from *Source 1* in *Appendix B.*

Recovery System Design Specifications

As final weights were not available, design specifications were calculated using estimated recovered weights of 5 lb (2.268 kg) for the dart and 5 lb (2.268 kg) for the booster. This recovered booster weight does not include the weight of the motor, as that was assumed to be nominal after burnout. A descent rate of 15 ft/s (4.572 m/s) was also assumed for each component.

Streamers were selected for use with a dual deployment system instead of the typical drogue chute in order to save money and decrease drift distance. An estimated decent rate of 50 ft/s (15.24 m/s) was used to calculate the size requirement of the streamers. After calculations were completed it was decided that two streamers would be used to increase drag. The selected streamers were 6 inch by 60 inch (15.24 cm by 152.4 cm) Nylon streamers. This size was selected because the surface area will provide enough drag to slow the components during decent while still being small enough to easily pack into the dart.

The base guideline for parachute calculations was that 3.5 square feet (3250 square cm) of parachute would be required for every pound of recovered weight. Using this value, it was determined that a 50 inch (127 cm) parachute would be acceptable for both the booster and the dart. However, 50 inches is not a standard size for parachutes, so the decision was made to increase the size to the next-largest standard parachute sized, which was 52 inch (132.1 cm). Since the dart and booster were estimated to have the same recovered weight, 52 inch (132.1 cm) Angel parachutes were purchased for both components.

Planned Construction Solutions & Techniques

The majority of construction followed standard rocketry construction procedures, including sizing of airframe tubes, attachment of fins, coupling section sizing, bulkhead placement, and general component construction. All machining operations were completed using university-supplied hand and power tools, as well as CNC tools such as an end-mill, lathe, and water-jet. However, the custom-designed parts required additional and innovative manufacturing techniques.

The section that required the most non-traditional rocket construction solutions was the dart/booster interface. As no standard drag-separation hardware was deemed acceptable, a custom dart tailcone will be turned from solid aluminum stock using a lathe. To decrease weight, the aluminum coupler will be bored out to remove excess material. As the shaft on the end of the tailcone needed to connect to the booster, the booster nosecone was heavily modified. This involved trimming off the tip, attaching a centering ring to the back, and then inserting and securing a section of Blue Tube.

To hold all of the electronics, a custom rail was designed and 3D printed using a composite-based additive manufacturing process with carbon fibers and Nylon as the materials. In order to connect the electronics rail, tabs were designed into the 3D printed part. These tabs locked the rail into slots cut in the back portion of the dart. While this increased the construction complexity, it allowed for faster and easier access to the electronics. The electronics rail was also designed to attach to the aluminum coupling section via protrusions and screws at the back of the rail.

As the fins were to be made from a woven carbon fiber composite sheet, typical cutting operations would not have been sufficient to produce quality cuts. Therefore, with the help of trained operators, a water-jet was used to cut the composite sheet into the desired shapes.

Design Features of Payload System

In order to meet competition requirements, it was decided that team members would create a custom avionics system in order to minimize the weight and size of the components. Separate devices were chosen to accomplish the tasks of collecting video and flight data. The video collection device consists of one Mini DVR 808 #16 V3 –Lens D Car Key Chan Micro Camera HD 720P Pocket Camcorder which was modified with a 24 position 0.5mm pitch 6in Flat Flex Cable in order to extend the camera from the rest of the avionics system. The data collection system consists of an Arduino Micro, an ITG3200/ADXL345 Accelerometer/Gyroscope combo, and an Adafruit MicroSD card breakout board. A 3D printed carbon fiber Nylon composite will be used to seat the avionics system within the dart.

To implement the video collection device, the enclosure was removed from the key chain micro camera and the camera was detached from the circuit board. Solder paste was reflowed onto one end of the flat flex cable lead with the use of a hot-air rework station. The camera leads were then placed over the cable leads and the solder was reflowed to join the leads together. This process is less likely to produce shorts between leads or damage cables than the use of a soldering iron. The open end of the flex cable was then attached to the circuit board via a solderless clamp to complete the device.

The data collection system uses the Arduino as the main computer to collect data from the combo board, convert it into useful information, and then save it to the MicroSD card. The Arduino requires a 7-12V power source, so a 9V battery was chosen to power it. Since the combo board and MicroSD card breakout board are designed to interface with the Arduino, no additional circuitry is required (resistors, capacitors, etc.). The code will need to take byte data from the combo board and convert it to usable accelerometer and gyroscope data. This data will need to be run through a filter, which corrects for drift in the gyroscope. The data will then be organized and stored on the SD card using standard read/write functions.

A 3D printed enclosure is being created to encapsulate all of the electronics. This includes the data collection system, video collection device, radio tracker, and the Jolly Logic AltimeterTwo. The enclosure, or electronics rail, is roughly 6 inches (15.24 cm) long and attaches to the aluminum coupling section at the base of the dart by means of two M4 bolts. The customized electronics rail (e-rail) has a diameter of 53 mm and uses 3 54 mm OD O-rings along its length in order to create an interference fit between the e-rail and the dart body. The e-rail is attached to the dart via two tabs that allow for quick removal of the enclosure to access flight data. The e-rail will be printed in two halves that fit together and are held in place by O-rings. The enclosure will be printed so that the directionality of the carbon fibers supports the length of the e-rail and any parts on the e-rail supporting acting forces will not have delamination issues. Due to the carbon fibers in the thermoplastic, the material strength will far exceed any forces acting on the e-rail.

Diagram of Rocket

The center of pressure (CP), which is the balance point of all aerodynamic forces on an object, was calculated both by hand and by computer simulation. The simulation program used to calculate the center of pressure was Open Rocket. Both of these calculations used the Barrowman method, which assumes incompressible flow and no viscous forces. The calculations were done by assigning each component a weighted value, the stability derivative, summing the values multiplied by the distance from a reference line to the center of pressure for the specific component, and then dividing by the total stability derivative. In this case, the reference line was the nose of the rocket. The diagrams from Open Rocket of the booster, dart and booster/dart combination can be found in *Figures 2*, *3* and *4* respectively.

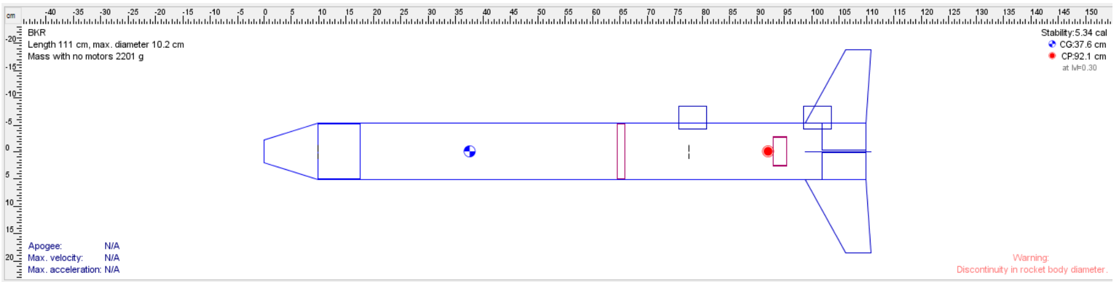

Figure 2: CG (blue) and CP (red) of rocket booster

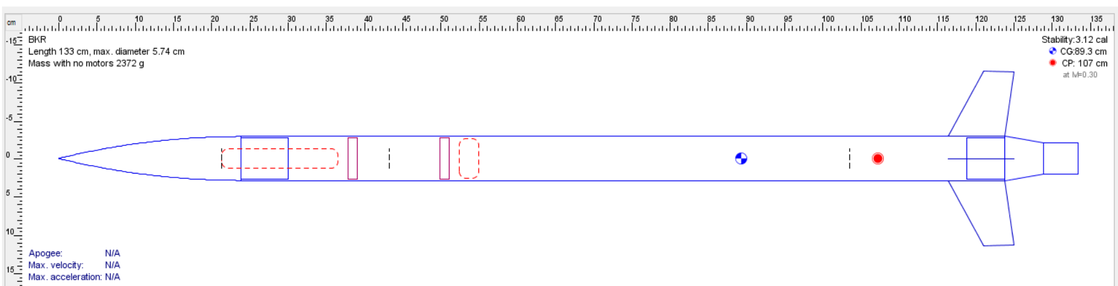

Figure 3: CG (blue) and CP (red) of dart

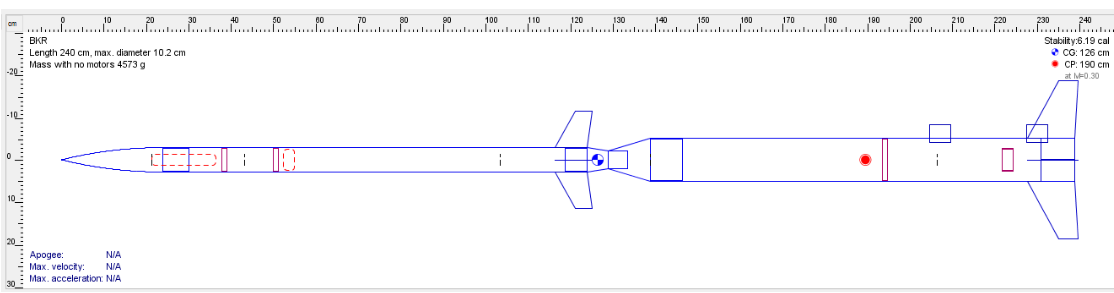

Figure 4: CG (blue) and CP (red) of combined booster/dart

The center of pressure equation can be found in *Appendix D*, *Equation 1*. The stability derivative ($C_{N\alpha}$) value for the nose cone was calculated from *Equations 2* and *3*; the $C_{N\alpha}$ represents the slope of the restoring force as a function of angle of attack for a given component. Values for the fins were calculated from *Equations 4*, *5* and *6*; and the $C_{N\alpha}$ values for the tail cone/booster tip combination were calculated from *Equations 7* and *8*. Using the Barrowman method, $C_{N\alpha}$ of the airframe tube is 0 because the effect of the airframe tube is negligible. An explanation of dimension variables can be found in *Appendix A*, *Table 5*. The equations and data were written out in Excel before being entered into Open Rocket to retrieve the final data values.

Analysis of Anticipated Performance

The flight analysis of the rocket was calculated using Open Rocket software. A dummy motor was added to the dart to allow the two stages to separate during simulation. The dummy motor was downloaded from *Source 5* in *Appendix B.* The software allowed input of the dimensions and weights of the rocket components along with the impulse of the competition motor, and returned a graph with specific flight data. A total of seven variables were calculated to produce the resulting graphs including time (s), altitude (m), total velocity (m/s), CP location (cm), CG location (cm) and drag force (N). These graphs can be found in *Appendix C* as *Figures 7, 8, 9* and *10.*

The separation occurs around 9 seconds, as seen by the change in CP and CG. The dart then continues to a maximum altitude of 600 m and a separation distance of 20 m. This is low and indicates a flaw in either the simulation accuracy or the actual design of the rocket, most likely due to too large of a rocket mass. Modification of the simulation after rocket construction as well as the test launch will determine the accuracy of the initial simulation.

The estimated flight time is 155 seconds, with a horizontal distance less than 800m for average wind speeds of 5 and 10m/s. Wind speeds will most likely not exceed this magnitude. Even with an increase in altitude for the actual launch the horizontal distance should not create a problem for recovery. The calculated velocity at impact for the dart and booster are 10 m/s and 12 m/s, respectively. This will be slow enough to prevent damage to the fins and electronic components.

The CP is located 60cm below the CG for the combined rocket and 18cm below the CG for the dart by itself, which makes the rocket stable for both stages of flight. These distances are sufficient to account for the predicted changes in CP and CG values due to burnout and separation.

Innovation

Initially, the rocket design was based on the idea of creating the dart and booster airframes using hand-made composite materials. The wet layup process, however, proved to be more difficult and inaccurate than anticipated. The result was that the carbon, glass, and Kevlar fabric sleeves that could not be removed from the PVC forming mandrel. It was decided that a composite rocket was still a good idea for strength and weight concerns, but hand-construction of the components would not be used. Instead, Blue Tube 2.0 tubes would be used for the dart and booster airframes.

The team was able to create many unique features for the rocket. Each major rocket component was scratch-built, and the entire rocket design was largely based around the coupling system. Team members used Loki-style rocket designs to draw ideas from due to their simple drag-separation systems. The final design consists of an aluminum-on-Blue Tube 2.0 coupler. The aluminum piece will be made on a lathe before being attached to the bottom of the dart. The end of the dart will then be inserted into a length of Blue Tube 2.0 inside the booster nosecone. This will hold the two components together until separation occurs during flight. A model of the aluminum coupler can be found in *Appendix C, Figure 1*. This design allows for a simple yet effective drag-separation system.

The electronics rail was designed to be a compact custom enclosure based on the components required. It was determined that using a standard electronics platter to hold all electronic components would require a large volume and weight. Due to this issue, the electronics team chose to design and 3D print a custom enclosure out of plastic. This allows the electronics to be efficiently packed and contained within the dart. A more detailed analysis of the electronics rail is described in the *Design Features of Payload System* section.

Team members designed the fins for both the booster and dart. The designs were based on calculations described in the *Rocket Fin Specifications* section. This was done in an attempt to maximize stability while decreasing possible drag. The booster and dart fins are the same geometry on different scales. The connection of the booster and dart fins will be different, as the booster fins will be slid into the booster body and anchored down on both the inside and outside in order to increase stability. The dart fins will simply be attached to the outside of the dart airframe due to the limited space available within the dart.

Safety

Designed for Safe Flight and Recovery

The rocket's design was based largely around the drag separation of the dart and booster. The motor contains an ejection charge, which will be used to deploy the parachute after drag-separation occurs. This ejection charge could damage the dart if it were to go off before separation. To prevent this, a delay tool was added to the design. This allows the ejection charge deployment to be delayed until after the dart and booster have separated.

The dart contains a dual deployment system to keep it from traveling extensive distances. The dual deployment system typically utilizes a drogue chute and a regular parachute, but streamers were selected instead of a drogue chute for this rocket. The streamers are more cost-effective while still providing additional drag for the dart during decent. To ensure a safe landing for the dart, the selected parachute is slightly larger than necessary for the dart's weight.

Materials-Handling Procedures

For the low-powered model rocket flights, a 24-pack of C6-5 motors was purchased. The motors were kept in a metal drawer when not in use. This is where leftover motors will remain until used or discarded.

The construction of the rocket will require the use of various epoxies. When these epoxies are used, gloves and respirators will be worn as safety precautions. When not in use, the epoxy will be stored in a flammable cabinet as a cautionary measure. Safety goggles will be worn at all times when constructing all aspects of the rocket.

An enamel-based spray paint will be used to paint the rocket after construction. Since aerosol cans are highly flammable, these will also be stored in the flammable cabinet when not in use. The painting process will occur outdoors away from any possible bystanders or in a location with excellent ventilation.

Black powder, used for the dual deployment system, will be stored in a flammable cabinet along with the other materials. Specific measurements of black powder will eventually be made based on cavity volumes and the pressures needed to break the shear pins. These measurements will be decided once the rocket is more fully constructed.

Planned Assembly Procedures

For both the test flight and the competition flight, three teams will be created to better streamline the assembly of the rocket and ensure that everything is prepared for the flight. The electronics team will construct and prepare the electronics rail. This team will be made up of the members who designed the electronics rail and wired all of the electronics. During assembly, these members will ensure that all of the electronics are working and that the camera is on and ready to record the flight. The second team will prepare the parachute and streamers for the dart and ensure that the dual deployment system is ready. The black powder charges will be placed in the dual deployment system at this point. Then, the competition altimeter will be activated and secured in the electronics rail. The electronics rail will then be inserted into the dart and secured. As the dart is being prepared, the third team will prepare the booster. This team will be responsible for packing the booster parachute, placing the motor in the casing, and ensuring that the motor is centered within the booster.

Once both the booster and dart are prepared, the rocket will be placed on the launch stand and the two components will be put together. The data collection system and dual deployment system will be activated via pull-pin switches at this time. A toggle switch is usually used to control the dual deployment system, but it was decided that a pull-pin would provide a better visual than a button to indicate that the systems were active. This ensures safe handling of the black powder charges and proper activation of the internal measurement units.

Planned Pre- and Post-Launch Procedures

Teams will assemble the rocket before launch, as described in the *Planned Assembly Procedures* section. The project supervisor will have a checklist and will coordinate with each team during the assembly process. This will ensure that everything is properly assembled and that nothing has been forgotten. The pre-launch procedure will also be coordinated with the safety officer in order to comply with safety guidelines.

Post-launch, the radio tracker will be used to find the location of the dart. Once it is determined that all charges have been deployed and the sections have separated appropriately, the team will recover the dart and booster. Then, the teams will remove the altimeters from each component to retrieve altitude data. The electronics rail will then be removed from the dart so that the flight data and video can be accessed.

Budget

The estimated costs associated with this competition can be found in *Appendix A* under *Table 2*, *3* and *4*. The Iowa Space Grant Consortium was kind enough to offer sponsorship of the team. They will be covering the cost of competition registration as well as the cost of hiring a judge. $950 was granted towards construction costs. The team has also reached out to departments within the College of Engineering at the University of Iowa and to date has received $1,200 from the college. The team is awaiting responses from potential corporate sponsors and to date has been granted $1,000 from companies outside of the university towards this project.

Appendix A: Tables

Table 1: Motor Specifics

Diameter	54.0 mm
Length	14.3 cm
Total Weight	575 g
Propellant Weight	213 g
Average Thrust	442.7 N
Maximum Thrust	526.2 N
Total Impulse	474.9 N-s
Burn Time	1.1 s
Specific Impulse	228 s

Table 2: High-Powered Rocket Competition materials list and budget

Project	***Material***	***Projected Cost***
High-Powered Rocket		
	2 x PNC Nose Cone	$41.48
	Aero Pack 54P TC Retainer	$43.87
	2 x 52" Angel Parachute	$168.00
	2 x Nylon 6" x 60" Streamers	$15.98
	4 sq ft economy Dragon plate	$123.00
	Motor Mount	$11.99
	Aero Pack Retainer P	$28.89
	Thrust Plate	$30.14
	G5000 RocketPoxy	$38.25
	Blue Ebay	$39.95
	LOC Stiffy Body Coupler	$5.78
	PNC-56A Transition Piece	$13.96
	Large Airfoiled Rail Buttons	$10.00
	Ball bearing swivel	$5.00
	U-Bolt assembly	$5.49
	4 x Quick Links	$13.00
	Madcow Parachute Protector	$8.51
	Dual Deployment System	$33.95
	Materials Testing	$500.00
		$1,137.24

Table 3: Specific expenses for high-powered rocket material testing

Project	***Material***	***Cost***
High-Powered Rocket		
	1 x (11021) 3.9in (98mm) LOC Body Tube	$11.50
	1 x (10505) 98mm Blue Tube	$38.95
	Aervoe 20 Oz. Mold Releas & Protector (Case of 12) 3470	$64.90
	TotalBoat 5:1 Epoxy Kit (Gallon, Slow Hardener)	$109.99
	1 x 26.45 4.0 inch carbon fiber Light sleeves (cost for 5ft)	
	1 x 21.45 4.0 inch Kevlar sleeves (cost for 5ft)	
	1 x 16.95 4.0 inch lght Fiberglass sleeves (cost for 5ft)	
	1 x 71.82 5.0 inch diameter Treated Shrink Tubing (cost for 18ft)	$ 136.67
	4 x 2X4-8' Stud	$11.52
	2"X5' Solid PVC pipe	$4.64
	3"X5' Cell core PVC pipe	$6.40
	2 x 1"X18" black nipple (solid weight)	$9.72
	Venom latex 100CT S-M gloves	$9.97
	3M Latex respirator w/ valve	$6.37
	3M paint/sand valved respirator	$6.97
	1 PK Jubilee paper towel	$1.18
	Heavy duty shears	$4.99
	2 x LOC instant mix 5min 14M	$7.54
	1 QT mix and measure cup	$1.18
	7X2 Construction Screw	$3.79
	3M Sandpaper fine	$2.17
	3 x 3" Galvanized strap-bulk	$3.33
	1X8-3' #2 Quality board	$2.45
	Shipping and tax	$71.89
		$516.12

Table 4: High-Powered Rocket Competition Travel Expenses

Project	***Expense***	***Cost***
Rocketry Competition Traveling Expenses		
	3 nights in hotel	$1,680
	3 cars, 2 trips	$262.08
		$1,942.08

Table 5: Center of Pressure Dimensional Variables and Functions

n	Number of fins
l_s	Fin span length
d_n	Diameter at nose base
l_m	Fin mid chord length
l_r	Fin root chord length
l_t	Fin tip chord length
l_n/X_b	Nose length
d_u	Upstream diameter
d_d	Downstream diameter
d_b	Average diameter (booster or dart)
d_f	Diameter at fin root
X_c	Distance to tailcone
X_f	Distance to root of fin
$C_{N\alpha}$ (R)	Total $C_{N\alpha}$ for rocket
$C_{N\alpha}$ (n)	$C_{N\alpha}$ for nose
$C_{N\alpha}$ (c)	$C_{N\alpha}$ for diameter change
$C_{N\alpha}$ (f)	$C_{N\alpha}$ for fin set
K_{fb}	Interference effect coefficient
X_{cp}	Distance to total center of pressure
X_{cp} (n)	Distance to C_p for nosecone
X_{cp} (c)	Distance to C_p for diameter change
X_{cp} (f)	Distance to C_p for fin set

Appendix B: Sources

Source 1: Motor Specifics, http://www.thrustcurve.org/motorsearch.jsp?id=704

Source 2: General Fin Design, http://www.oldrocketplans.com/pubs/Estes/estTR-11/TR-11.pdf

Source 3: Trapezoidal Fin Desing, http://www.nakka-rocketry.net/fins.html

Source 4: Fin Calculations, http://cambridgerocket.sourceforge.net/AerodynamicCoefficients.pdf

Source 5: Dummy Motor Source, https://sites.google.com/site/theskydartteam/openrocket

Appendix C: Figures

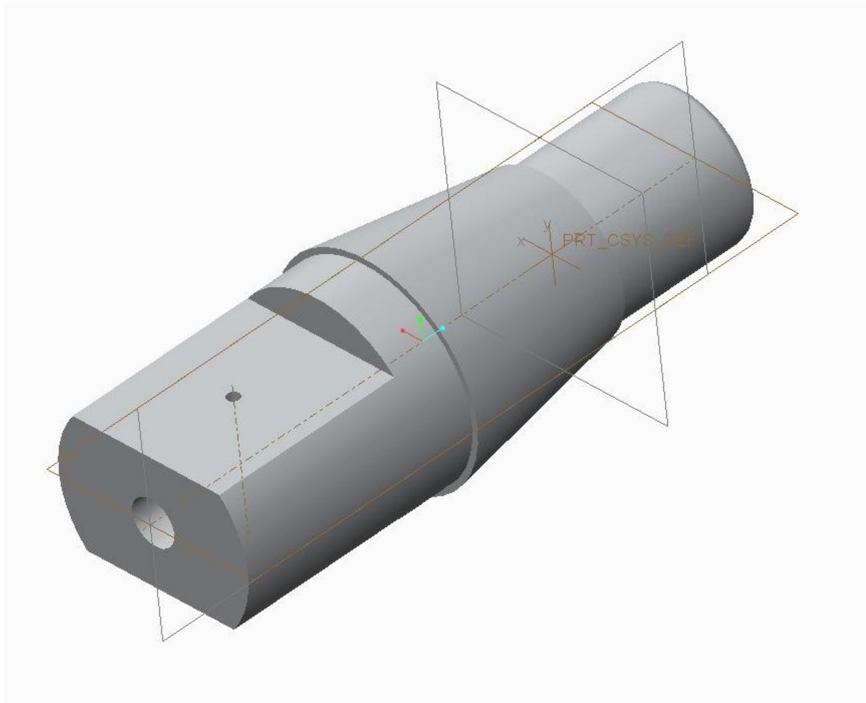

Figure 1: Aluminum tail cone coupler for dart

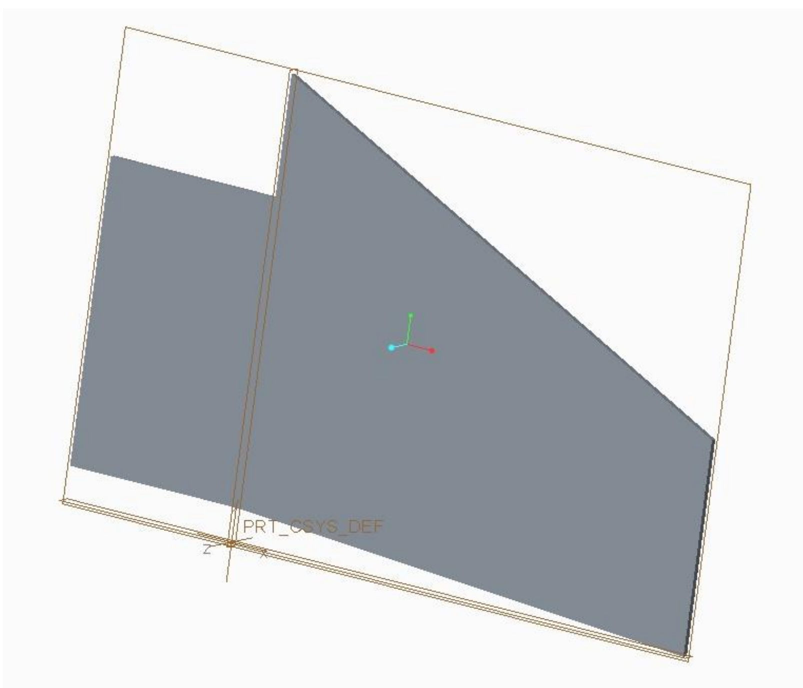

Figure 5: Booster fin design

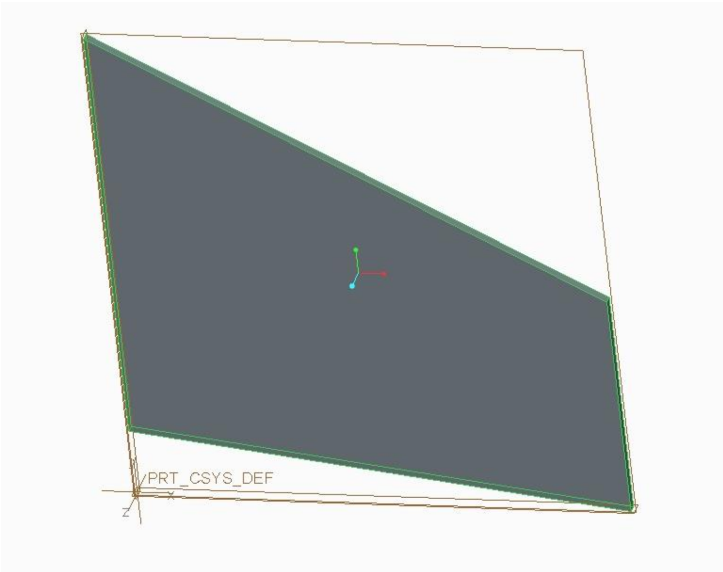

Figure 6: Dart fin design

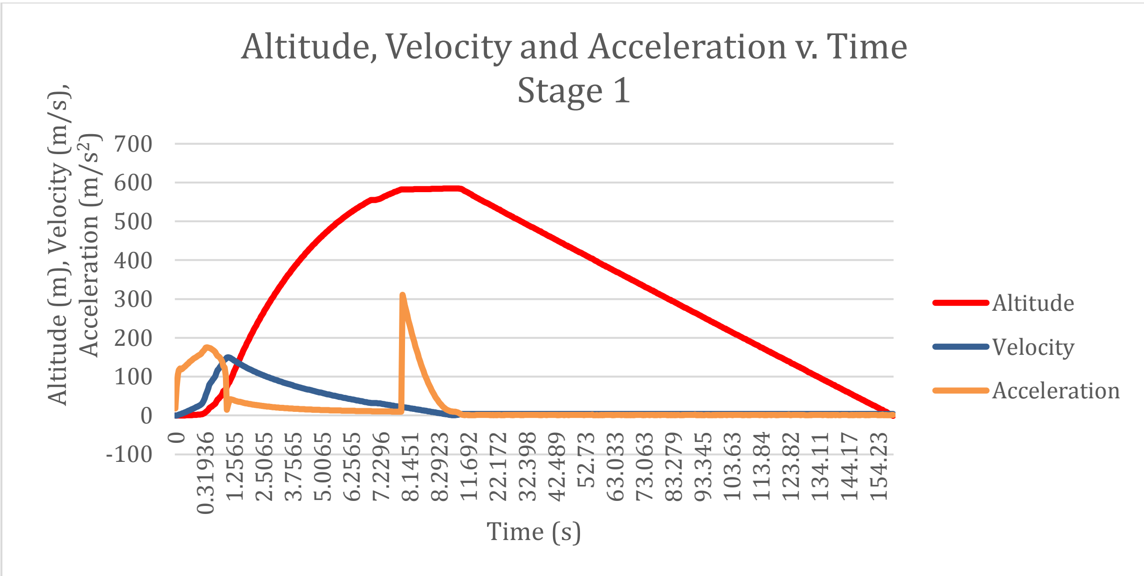

Figure 7: Stage 1 Rocksim flight performance analysis

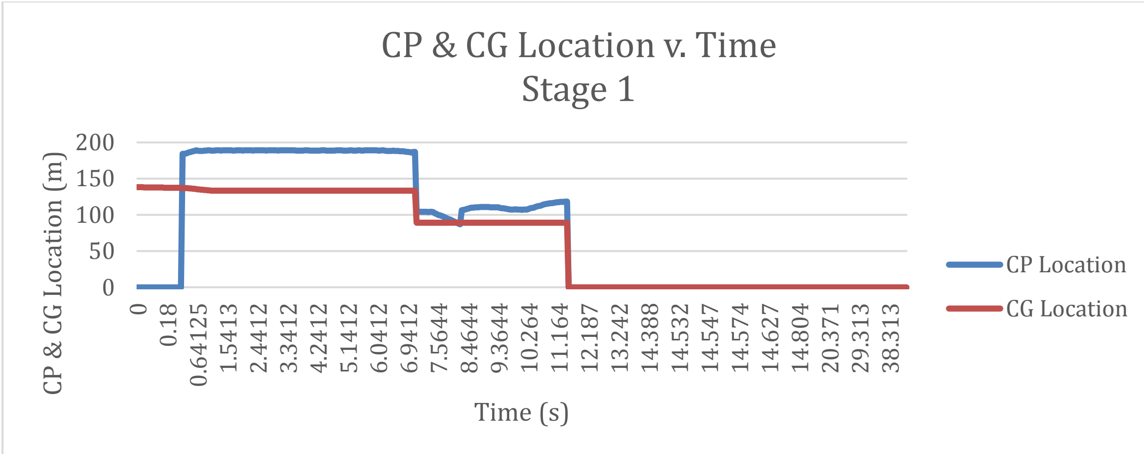

Figure 8: Stage 1 Rocksim CP & CG analysis

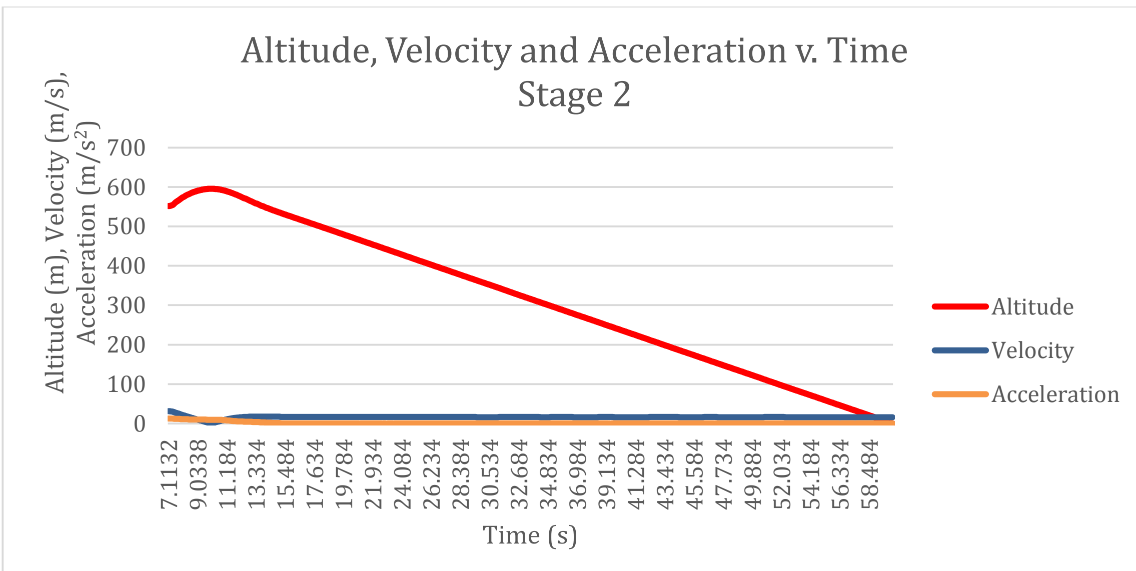

Figure 9: Stage 2 Rocksim flight performance analysis

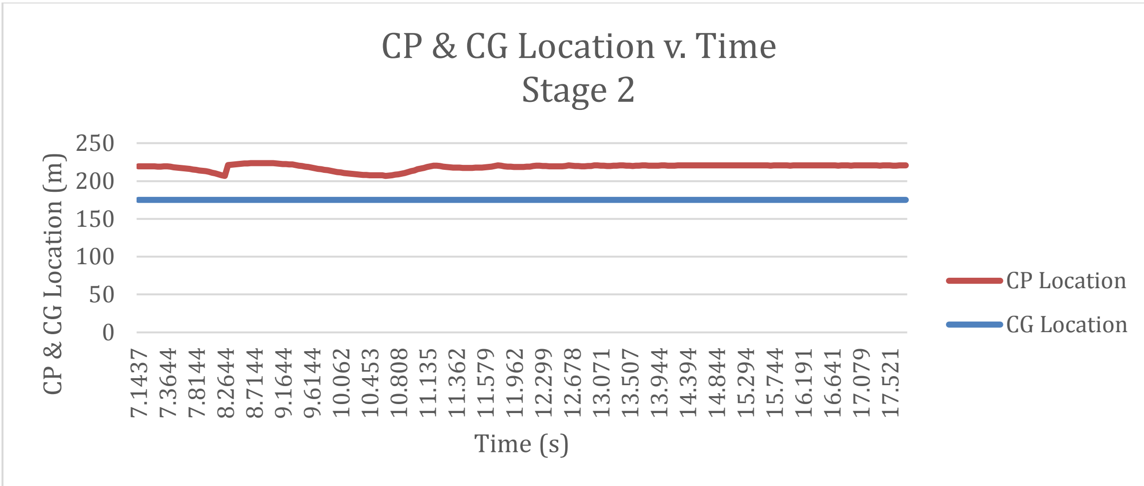

Figure 10: Stage 2 Rocksim CP & CG analysis

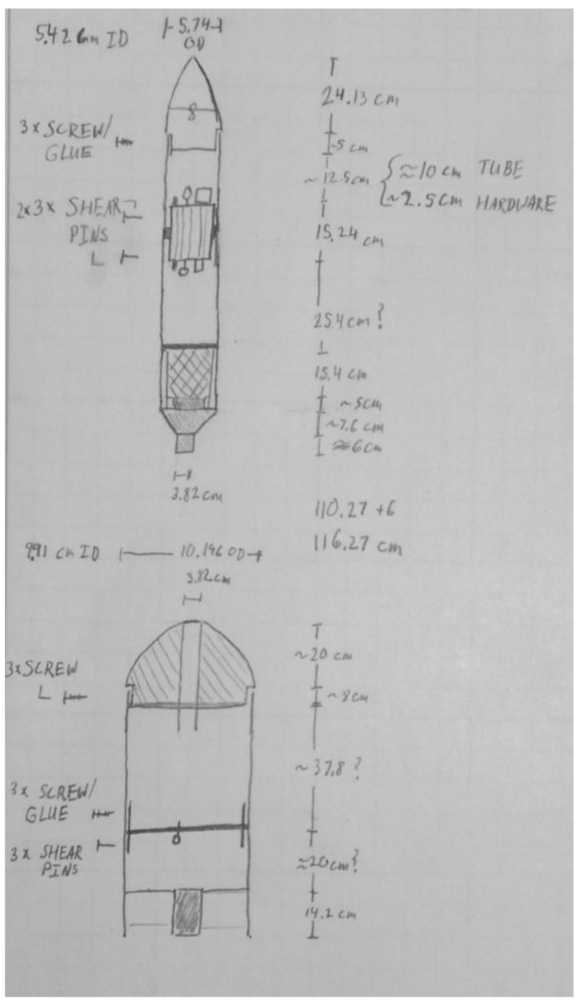

Figure 11: Dimensions for current rocket design (Note: Fins not shown)

Appendix D: Equations

Note: All parameter definitions can be found in *Table 5* of *Appendix A.*

Equation 1:

$$C_{N\alpha(R)} = \sum_{P \in R} C_{N\alpha(P)}$$

Equation 2:

$$C_{N\alpha(n)} = 2$$

Equation 3:

$$C_{N\alpha(c)} = 2\left[\left(\frac{d_d}{d_n}\right)^2 - \left(\frac{d_u}{d_n}\right)^2\right]$$

Equation 4:

$$C_{N\alpha(f)} = K_{fb}\frac{4n\left(\frac{l_s}{d_n}\right)^2}{1+\sqrt{1+\left(\frac{2l_m}{l_r+l_t}\right)^2}}$$

Equation 5:

$$K_{fb} = 1 + \frac{\frac{d_f}{2}}{(l_s + \frac{d_f}{2})}$$

Equation 6:

$$X_{cp(R)} = \frac{\sum_{P \in R} C_{N\alpha(P)} X_{cp(P)}}{C_{N\alpha(R)}}$$

Equation 7:

$$\text{Ogive: } X_{cp(n)} = 0.466 l_n$$

Equation 8:

$$X_{cp(c)} = X_c + \frac{l_c}{3}\left[1 + \frac{1 - \frac{d_u}{d_d}}{1 - \left(\frac{d_u}{d_d}\right)^2}\right]$$

Equation 9:

$$X_{cp} = X_f + \frac{l_m(l_r + 2l_t)}{3(l_r + l_t)} + \frac{1}{6}\left[l_r + l_t - \frac{l_r l_t}{l_r + l_t}\right]$$