

Badger Ballistics

University of Wisconsin - Madison

Team Mentor:
Kevin Harnack

Team Faculty Advisor:
Matt Allen -msallen@engr.wisc.edu - 608-890-1619

Student Team Lead:
Brandon Wilson - bwilson8@wisc.edu - 262-672-0756

Student Team Members:
Matt Barrett, Richard Blaschko, Ryan Cape, Mark Diny, Joe Dziekan



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1.0 EXECUTIVE SUMMARY

The University of Wisconsin - Madison Badger Ballistics team is incredibly excited to compete once again in the Midwest High-Power Rocket Competition. With support and guidance from industry professionals and the American Institute of Aeronautics and Astronautics, we are deeply grateful for the opportunity to tackle the challenge of this year's competition. Through a collaborative environment involving university faculty, aerospace professional mentors, and many excited undergraduate engineers we are confident in our approach to this year's task: controlling the roll orientation of a high powered rocket.

The Badger Ballistics rocket is built around a 4" fiberglass body tube. The exterior of the rocket is fashioned with a 5.5:1 Von Karman fiberglass nose cone and four G10 fiberglass stabilizing fins. With the rocket's structural frame and internal components considered, the total dry mass is currently 5.54 kg. An AeroTech J570, provides the necessary 973 N-s of impulse over a 1.9 s burn time to propel the rocket skyward. From these general characteristics, we expect a altitude performance of 3009 ft (917 m), with a maximum velocity and acceleration of 453 ft/s (138 m/s) and 19 Gs respectively.

The competition objectives this year require the rocket to perform a series of roll maneuvers after motor burnout. To master the challenge of roll control, our team chose to use dynamic control surfaces along the bottom of four fins located symmetrically around the rocket's base. The control surfaces are adjusted by high-torque servo motors. The servo motors are operate from the response of a roll control algorithm programmed into the onboard Arduino microcontroller. The roll control algorithm will receives flight data from the onboard inertial-measurement-unit, which calculates an appropriate servo response. As the rocket's roll begins to correctively change, the algorithm continuously receives new data, and hones in on the final desired roll orientation.

The project is currently in its testing phase. We have successfully flown and recovered a test rocket with our data acquisition system, which gave us insight into our mathematical model for altitude prediction. We also successfully flew and recovered the Badger Ballistics rocket with the active roll control system enabled. This flight was nearly a complete success, with some deviations from predictions, which we are currently being reconciled in preparation for the May 2018 competition.



2.0 ROCKET MECHANICS AND ELECTRONICS

2.1 General Design and Dimensions

The primary rocket assembly consists of a nose cone, body tube, four fins: with control surfaces, centering rings, a motor mounting tube, and the avionics bay. The rocket is constructed almost entirely from G10 fiberglass with a density of 0.0650 lb/in³ and a young's modulus of 16.5 GPa measured across the fiber [1]. By using the crossfiber material properties, our team can assume survival in a worst case scenario for material strength. This material was specifically chosen to maximize structural integrity and minimize weight. .

- The body tube constitutes the majority of the rocket's mass with a 4 inch diameter, 52 inches of length, and .065" wall thickness leading to a total volume of 84.259 in³ and ~3 lbs of mass.
- The nose cone is a 5.5:1 Von Karman design constructed primarily from a fiberglass polymer with an added aluminum tip. The Von Karman designs are engineered to minimize drag forces and the weight of the nose cone to optimize flight performance.
- A 60" main parachute attached to a 20 ft kevlar cord, and 24" drogue parachute attached to a 40 ft kevlar cord provide the necessary drag to land the rocket at less than 24 ft/s.
- The rocket fins are waterjet cut from G12 fiberglass.

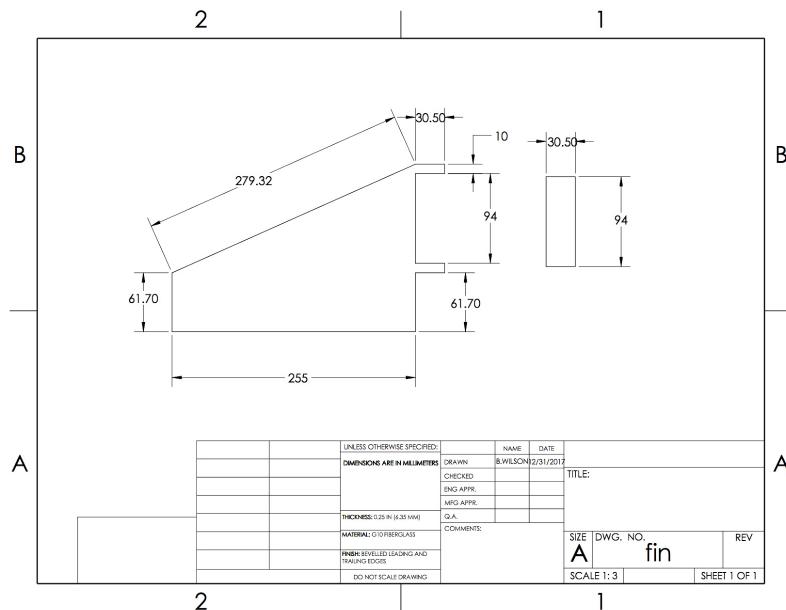


Figure 1. Drawing specifying the dimensions of the rocket fins and control surfaces.



Figure 2. Full rocket assembly.



2.2 Roll Control System Construction

The Badger Ballistics Active Roll Control System consists of four high torque servo motors. Each servo attaches to a control surface by bevel gears and is free to rotate relative to one of the four fins at the rocket's base. Because of the high torque requirement of the servo motors, our team found even the most compact servo motors still required careful planning to fit inside of the rocket's body tube. Figure 3 shows the layout of the servo motor mounting. As shown, our team (quite literally) cut some corners when it came to utilizing the servo mounting holes. This area of the rocket should experience very little stress relative to the servos motion; therefore, they should not be at risk of coming loose.

The servo motors are connected to the Arduino microcontroller via wires running along the exterior of the motor mount tube, inside the body tube. During drogue parachute deployment, the avionics bay detaches from the lower portion of the body tube. This creates an interesting design challenge for separating the wires to the servos from the flight computer. Our team addressed this issue by designing a connection point which runs the wiring parallel to a small length of parachute cord. The cord is slightly shorter than the connection wires, so that when the rocket separates, all of the tensile forces will remain in the parachute cord rather than the wiring.

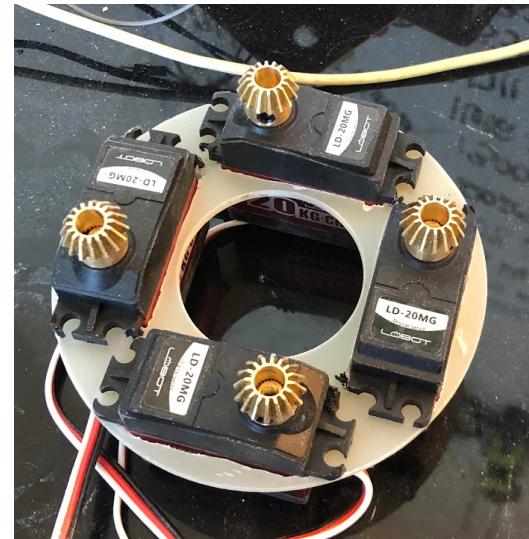


Figure 3. The servo mounting pattern for the roll control assembly.



Figure 4. Servo umbilical from avionics bay detaches from lower section of rocket during drogue parachute deployment



Another interesting challenge was adapting the bevel gears to the control rods. The smallest bore bevel gears we could obtain had a bore size of 0.25", yet the control rods that run through the control surfaces are 3/16". However, due to the thin nature of the control surfaces, our team chose to manufacture a rod transition from the 0.25 in rod to a smaller 3/16 in rod which runs through the rocket fins and control surfaces. To overcome this we shrunk fit a 0.25" outside diameter, 3/16" inside diameter steel tube onto the control rods at the bevel gear attachment point creating the correct mating surface size.

The fins and control surfaces are the most in depth engineered portion of the Badger Ballistics rocket beyond the software and avionics bay. Our team took careful measures to ensure that the fins and control surface assembly would provide minimal aerodynamic drag, absolute structure integrity, and tight design tolerances to ensure smooth dynamic controls. The fins and control surfaces are one of the only pieces of the rocket machined by an outside source. The designs as seen in Figure 1 were cut to spec by Public Missiles. The beveled edges and minimalist design of the fins allow for a smooth fluid transition leading into the control surfaces. A deeper analysis of the assembly's structure integrity can be found in our preliminary design report.

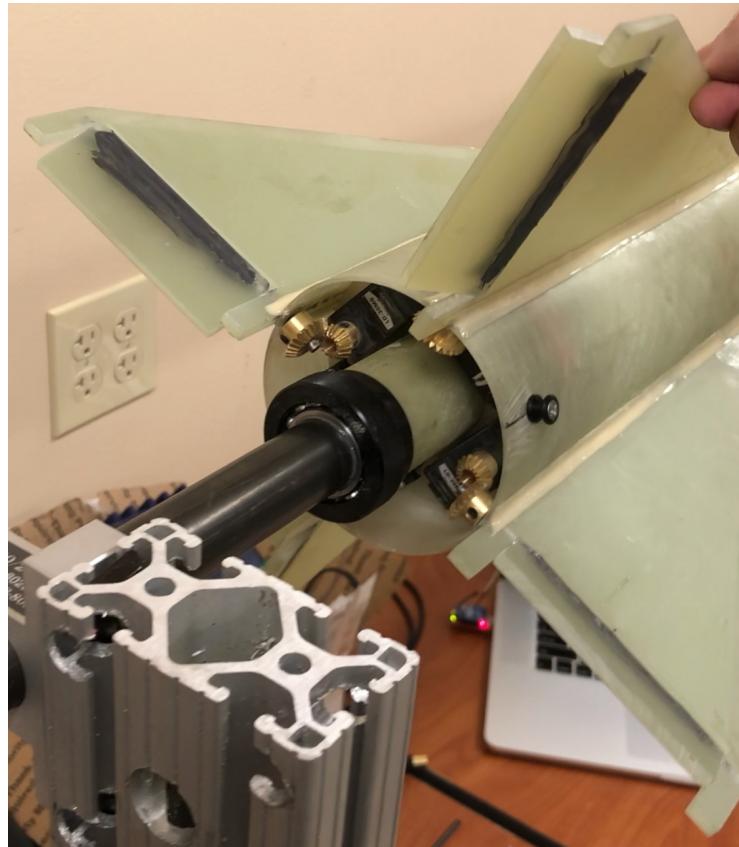


Figure 5. Roll control system at full deflection



2.3 Stability Analysis

The center of gravity location is 139 cm from the nose (measured 137 cm on test launch) and the center of pressure is 156 cm from the tip of the nose cone. This yields a stability caliber of 1.66. OpenRocket plotted the predicted stability caliber vs time (shown in Figure 7), which shows a stability cal of ~2 after motor burnout. The competition requires the stability caliber to be greater than 1, but less than 5 throughout flight. Activation of the roll control will shift the CP back, which will cause the rocket to be overstable. However, this issue should resolve itself as the control surfaces deflect (plus or minus 30 degrees) to either maintain a nulled roll or to roll toward a new orientation. The control surface deflections will push the center of pressure back up the rocket and ultimately result in a stability caliper that shouldn't stray too far from 2.

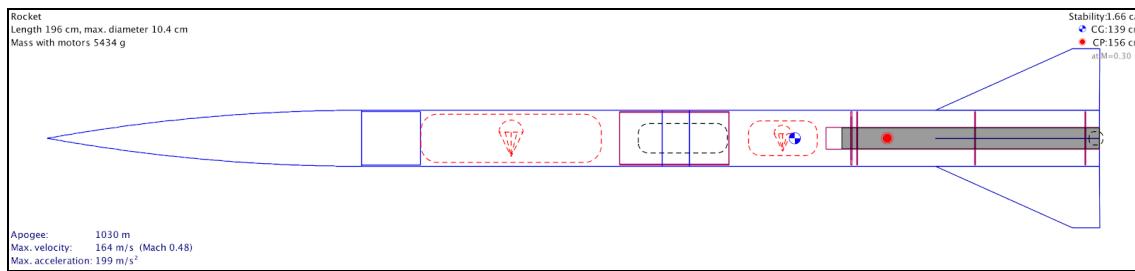


Figure 6. CG and CP locations. Stability 1.66 cal

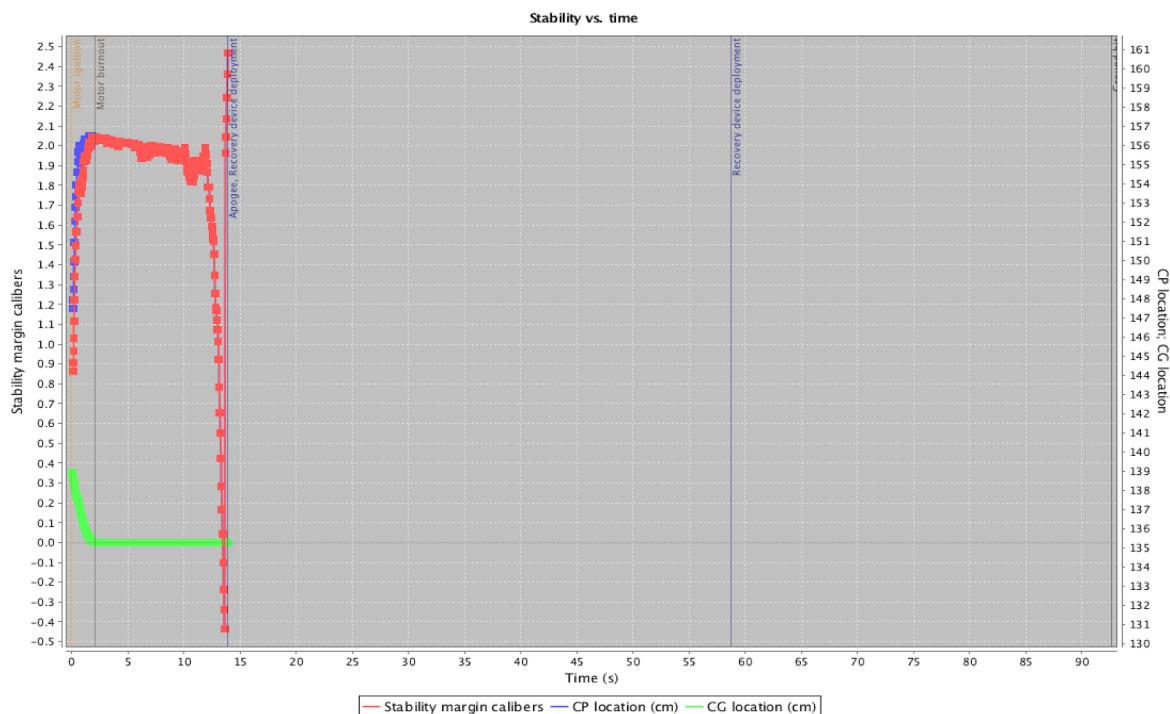


Figure 7. Stability, CP, and CG location vs time. Stability is red, CP is blue, and CG is green.



2.4 Avionics Bay Design

The avionics bay is a key component in the rocket. The competition requirements specify that the avionics bay should be “tough but user friendly”. To achieve this, all of the circuitry and wiring for the flight computer is integrated into a compact circuit board (shown in Figure 8). The components in the bay include:

- Arduino Mega
- Custom made circuit board
- 7.4V battery
- StratoLogger CF altimeter
- BNO055 IMU
- Barometer
- Adafruit Ultimate GPS
- Adafruit SD shield
- LED indicating system
- XBee wireless transmitter

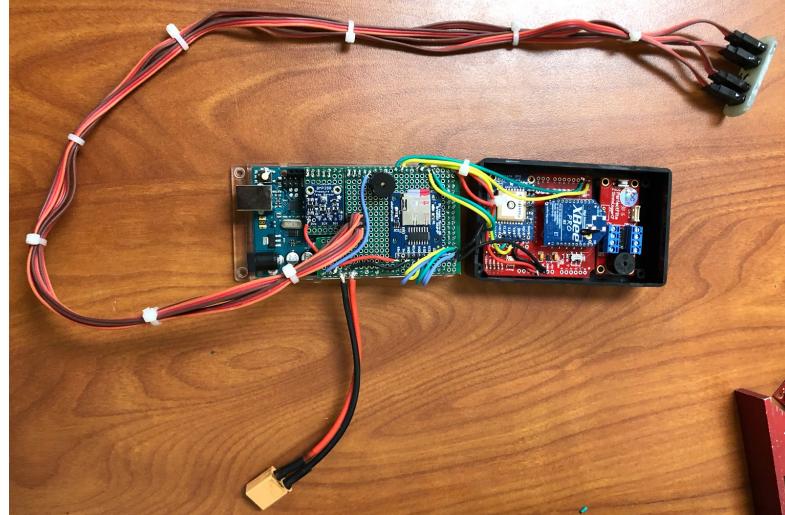


Figure 8. Custom Circuit Board

The avionics bay is constructed of fiberglass, and consists of a “sled” (Figure 9) to which all components will be mounted to. The custom made circuit board will be mounted directly onto the Arduino mega, utilizing all of the pinouts on the arduino. The circuit board consists of a 5V Bus, which will regulate the 7.4V battery input to a 5V output that will be used to power the Arduino, IMU, GPS, SD shield, LED indicating system, FPV, camera, and the XBee wireless transmitter. The 7.4V bus powers the four high torque servos. The GPS and SD shield are soldered directly to the circuit board for compactness and neatness.

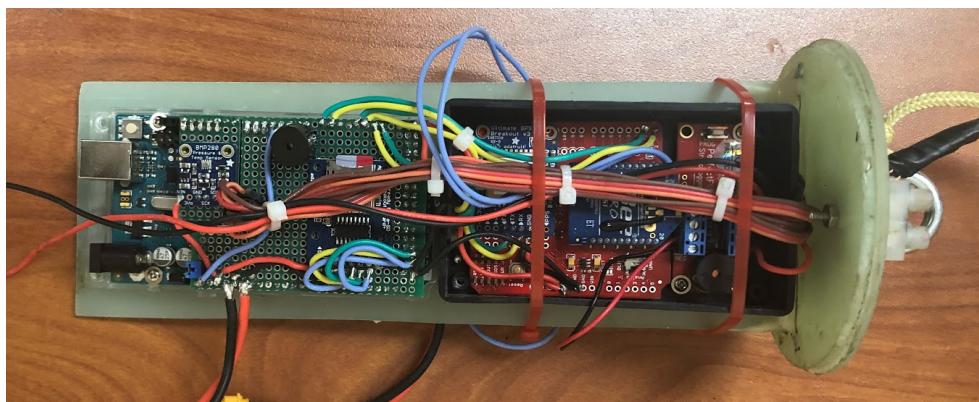


Figure 9. A closeup of the avionics bay mounted on the rocket’s coupling tube.



2.5 Changes Since Preliminary Design

The accelerometer, and gyro were replaced by a Bosch BNO055 absolute orientation sensor. The improved libraries of this sensor reduce the gyroscopic drift, making compass heading outputs more accurate. This sensor also outputs Euler Angles, and Quaternions which allows for visualization of the orientation of the rocket during flight. These additional variables were utilized in the development of a flight data GUI which is discussed further in section 6.3.

3.0 ACTIVE ROLL SYSTEM SUMMARY

3.1 Active Roll System Hardware

The primary hardware components of the roll control system are the 20 kg-cm high-torque servo motors, control surfaces, and bevel gears. The control surfaces are simple fiberglass rectangles that were cut from the main fins. A 3/16 in rod is JB welded in the control surfaces and attaches to the servos via 1.5:1 ratio bevel gears (see Figure 5 from Section 2.2), which reduces the loading on the servos.

3.2 Active Roll System Logic

The Badger Ballistics rocket uses a simple proportional feedback loop to control the roll of the rocket. This can be expressed as: $u(t) = k_p e(t)$, where u is the control signal, k_p is a constant coefficient, and $e(t)$ is the error determined by a setpoint (which will be the angular velocity of the rocket, for nulling roll). This means the deflection of the control surface is directly proportional (and opposite) of the angular velocity to null out any unwanted roll.

Controlling the orientation of the rocket will use the same proportional control, plus an additional integral term to prevent angular drift:

$$u(t) = k_p e(t) + k_I \int e(\alpha) d\alpha .$$

The error for the orientation control will be the difference in angle ($\theta(t) - \theta_{desired}$). Thus, the control sequence will be to first null any roll using the angular velocity as the setpoint for the error, once this is achieved, the IMU will determine the desired angle the rocket needs to roll to, finally the PID setpoint will be that of the difference in rotation angles. Once this is achieved the setpoint will return to the angular velocity to hold the rocket for the desired period of time.



3.3 Active Roll System - Wind Tunnel Testing

As our team began finessing the control algorithm of the rocket, a method of acquiring test data quickly became an issue. The largest wind tunnel available at UW-Madison is roughly four feet in length, which would be far too small to conduct any tests on a six foot rocket. Our team then reached out to Modine Manufacturing in Racine, WI to look into the possibility of utilizing one of their industrial scale wind tunnels. Their primary wind tunnel is capable of producing winds up to 50mph and can easily fit school buses or large semi trucks. Their secondary wind tunnel can reach air speeds of 35 mph and is nearly 150 feet in length. These tunnels are not equipped for aerodynamic testing, however, the potential of blowing strong winds within a large space fit perfectly with what our team needed to conduct roll control testing.

3.3.1 Null Roll Testing

The wind tunnel testing with Modine consisted of six iterations of wind being blown at 35mph. The Badger Ballistics team utilized the first three iterations to test and improve the null roll function of the roll control algorithm. The roll motions of the wind tunnel testing are shown graphically by Figure 11. The sharp spikes in angular velocity are produced as the rocket spins itself up by deflecting the control surfaces to a hard 45 degree angle for ~ 15 seconds. Once a large angular velocity has been established, the onboard microcontroller triggers the null roll algorithm which can be seen during the sharp changes of angular velocity back toward zero. The rocket was



Figure 10. The Badger Ballistics rocket mounted on the test fixture inside of the Modine Manufacturing secondary wind tunnel.

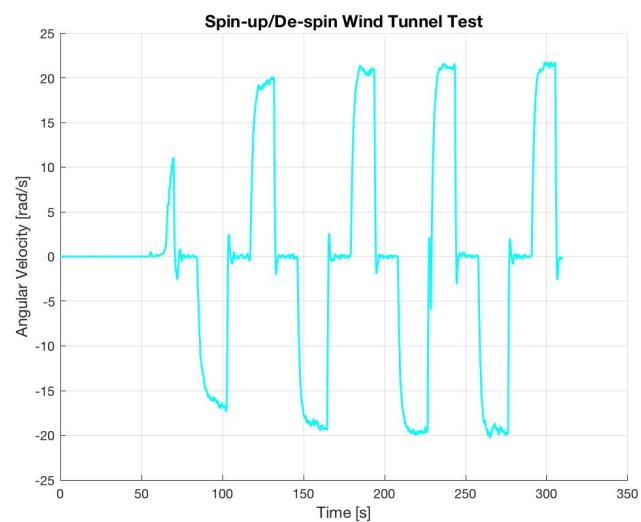


Figure 11. Wind tunnel testing data for null roll: angular velocity vs time.



programmed to repeat this sequence in a loop consisting of 15 second counter-clockwise, 15 second null roll, 15 second clockwise, 15 second null roll. Our team was incredibly impressed and satisfied with the success of the nul roll algorithm. The rocket was able to go from 20.91 rad/s (1198 deg/s) to ~0 rad/s in less than 3 seconds.

3.3.2 Directed Orientation Testing

Iterations 4 through 6 of wind tunnel testing were used to test the roll control algorithm's ability to direct the rocket's heading toward a desired orientation. Our team was successful in honing in on one specific orientation by utilizing the null roll algorithm with a few modifications to direct the rocket toward a specific spot. This data was then used to tweak the variables within our PID controller to avoid over compensations of the roll control system.

The last two trials of wind testing revolved around the ability of our control system to hit multiple orientations during the same run. The results of these experiments were much more volatile. While we were able to hit a few orientations in each run, we would find that the rocket would find itself in a "death spin" when trying to hit certain orientations from certain directions. Our team discovered that when the rocket had to cross the heading mark between 0 and 360, the algorithm would get confused if it was coming from the short side of the roll. This problem is discussed further in section 6.2

4.0 ROCKET OPERATION ASSESSMENT

The test flight of the Badger Ballistics rocket was performed on April 21, 2018 at Richard Bong State Park. It was approximately 50 degrees Fahrenheit, with ~10 mph wind gusts prior to launch. The rocket was flown with an AeroTech J570 motor and was successfully recovered with no damage. Initial assessment of the flight data showed an unexpected angular acceleration during the boost and coast phase that resulted in a 2000 deg/s angular velocity prior to activation of the active roll system. It was determined after the flight that one of the fins were epoxied slightly out of alignment, which likely induced unwanted roll. Attempts are being made to reconcile this to prevent unwanted roll

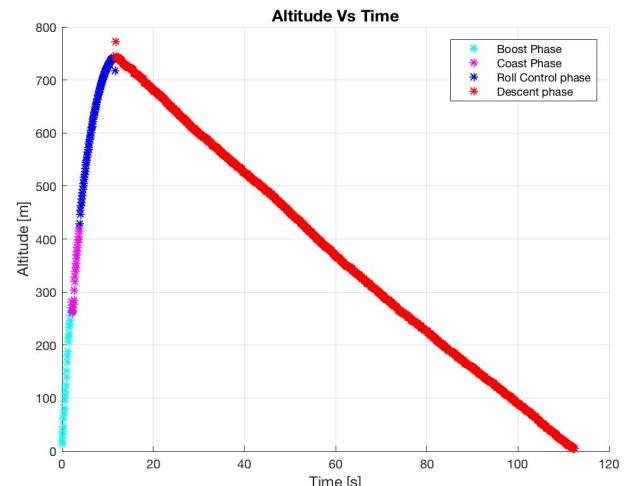


Figure 12. Altitude Vs Time of test flight with color coded data points correlating to phase in flight



prior to activating the roll control system. Figure 12 shows altitude versus time for the duration of the flight with color coded data points for each phase of the flight.

4.1 Launch and Boost Phase Analysis

The boost phase of the rocket performed adequately. The rocket lifted off the launch pad with 13 g's of acceleration and boosted to an altitude of 886 ft during the 2 second rocket engine burn. As can be seen from Figure 13, the angular velocity of the rocket reached -35 rad/s (-2000 deg/s) within the first second of the flight. Shortly after one second into flight the angular velocity unexpectedly becomes positive 35 rad/s. It was discovered after the fact that the maximum output of the on-board gyro is 2000 deg/s, once the absolute value of the maximum output is exceeded, the gyro only outputs the absolute value of the angular velocity. This "tricked" the flight computer into thinking the rocket was rolling counterclockwise at 2000 deg/s, rather than clockwise at 2000 deg/s. Changes have been made in the flight computer to prevent this from happening again.

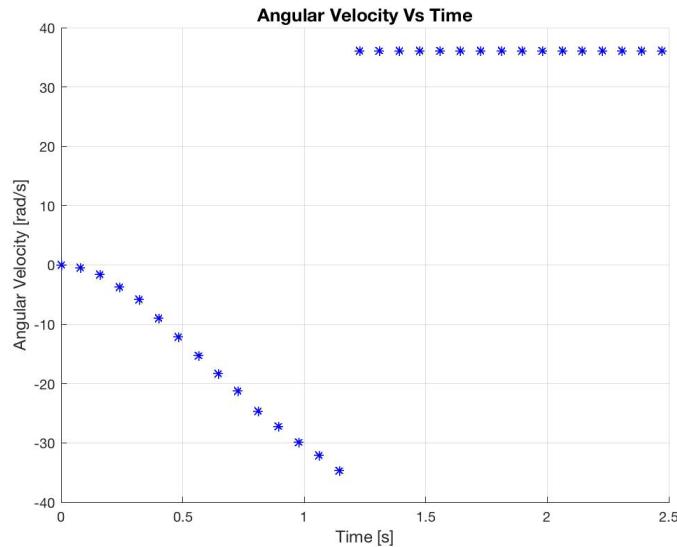


Figure 13. Angular velocity Vs time during boost phase

4.2 Coast Phase Analysis

The rocket coasted for 2 seconds after motor burnout, per competition rules. Four seconds into flight the roll control system was activated, however, due to the erroneous output of the gyro, the roll control surfaces extended in the opposite direction and caused the rocket to spin up even faster. This also induced a large amount of drag force for the entirety of the flight, which caused the rocket to undershoot our desired apogee of 3000 ft by ~500 ft.



4.3 Recovery System and Descent Phase Analysis

The recovery system consisted of a 24 in drogue parachute packed into the lower stage of the rocket with a 2-gram black powder charge, and a main 60 in parachute packed into the upper stage of the rocket with a 2.5 gram black powder ejection charge. 40 ft of kevlar shock cord attached the drogue chute to the lower stage and 30 ft attached the main chute to the upper stage. An onboard GPS relays GPS coordinates to the ground station throughout flight, allowing for easy recovery. A PerfectFlite StratoLogger CF commercial altimeter was used to activate the ejection charges with a 9 Volt battery (separate from the flight computer power). The drogue parachute was set to deploy at apogee, while the main parachute was set to deploy at 700 ft. However, both parachutes unexpectedly deployed at apogee. The reason for this is not entirely known, however we think the two 2-56 shear pins sheared from the upper stage of the rocket when the drogue parachute deployed. To reconcile this, we will be using two 4-40 shear pins on the rocket instead. The rocket safely touched down at 24.25 ft/s, a little faster than predicted, however, mass is being removed from the rocket prior to the competition to reduce the descent velocity, as well as increase our maximum altitude, and a 36 inch parachute will replace the 24 inch drogue.

4.4 Pre & Post-Launch Procedure Assessment

Prior to the test launch, a thorough preflight inspection was conducted to ensure the rocket was safe to fly. The parachutes were inspected and neatly packed into the rocket after ensuring the shock cord was securely attached to the forward and aft sections of the rocket. The black powder ejection charges were then placed into the bay and the rocket was assembled. Arming of the rocket was done by a “wire-wire” switch after the rocket was upright on the launch pad. The StratoLogger CF altimeter then beeped out the altitude of a previous flight, then chirped the “ready” tone. Once the safety officer deemed it safe to launch, the rocket was launched and successfully recovered approximately 300 yards from the launch location. GPS coordinates were successfully transmitted and received at ground station to aid in tracking the rocket. We approached the rocket with caution to verify there were no live black powder charges. After it was deemed safe, the rocket was unarmed, put back together and taken back to the site for the post flight inspection. The post flight inspection showed no signs of damage, or any cause for concern.

5.0 TEST FLIGHT PERFORMANCE

Overall the test flight was a success. Data from the flight was recorded on-board via SD card, and by live telemetry transmitted by the XBee throughout flight. Downward facing video



was not used during the test flight due to budget constraints, but thorough testing of the downward video and LED system will be done once we get a camera. Wind tunnel testing confirmed our null roll function worked, however unexpected spin-up on launch caused a failure to null roll on the test flight. We think we have remedied this and would like to do another test launch to verify this, as well as test the orientation control, however, there are no launch dates between now and the competition.



Figure 14. Left: Brandon and Richard standing with the Badger Ballistics rocket just before the first test launch. Right: The Badger Ballistics rocket after a successful landing.

5.1 Predicted Vs. Actual: Summary

Despite high error rate for many of the flight categories, our team was able to utilize all gathered data in an incredibly fruitful way. Many of the errors which lead to a mismatch between theory and experiment have been identified and are outlined in the following sections.

	Actual	Theoretical	Error %
Maximum Altitude (ft)	2532.82 ft	3013.06 ft	18.96 %
Maximum Velocity (ft/s)	363.26 ft/s	136.111 ft/s	12.15 %
Max Acceleration (g)	14	19	35.71 %



Ground Hit Velocity ft/s	24.2 ft/s	18.4 ft/s	23.97 %
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Table 1. A summary of flight characteristics for the first Badger Ballistics test launch.

The test flight performance data will be contrasted with the theoretical flight predictions which were derived by our team and verified by OpenRocket. The flight prediction software being used by our team has been an ongoing algorithm which is nearly three years in the making. Each new flight and engineering challenge has given more light to the flight prediction code and the mathematical models have continued to grow even since the preliminary design report was written.

Nomenclature:

y = Altitude [m]	κ = Adiabatic Index
F_D = Force of Drag [N]	A = Cross Sectional Area [m]
F_T = Thrust from Rocket Engine [N]	ρ = Density of Air [kg/m ³]
dt = Time Step [s]	M_a = Mach Number
T = Temperature [K]	R_e = Reynolds Number
T_0 = Reference Temperature (293.15 K)	S = Molecular Speed Ratio
P = Pressure [Pa]	R = Gas Constant
P_0 = Reference Pressure (101.325 kPa)	L = Temperature Lapse Rate (0.0065 K/m)
g = Acceleration of gravity on earth's surface	M = Molar Mass of Dry Air (0.0289644 kg/mol)
t = time	γ = Ratio Of Specific Heats [-]
μ = Dynamic Viscosity [Pa-s]	C_D = Coefficient of Drag [-]

One of the requirements of this competition is to reach a minimum altitude of 3000 ft. Since our rocket is propelled by a solid rocket and we are limited to a J size motor, an accurate altitude prediction model based on key vehicle parameters was a necessary step in the design process of the Badger Ballistics rocket. To construct this mathematical model, The equation of motion (EOM) of a rocket during vertical ascent, assuming purely vertical flight can be written as:

$$m(t) \frac{d^2y(t)}{dt^2} + \frac{dm(t)}{dt} \frac{dy(t)}{dt} + F_D - F_T(t) - mg = 0$$

Where $\frac{dm(t)}{dt}$ is the mass flow rate of the rocket engine, and is assumed to be constant-piecewise. The force due to drag is a function of the air density, the coefficient of drag, the cross-sectional area of the vehicle, and the velocity of the vehicle:

$$F_D = \frac{1}{2} \rho(T, P) C_D A \left(\frac{dy(t)}{dt} \right)^2$$

Furthermore, the density of air is a function of pressure and temperature, which are both functions of altitude. Using the Ideal Gas Law, and the fact that Temperature linearly decreases



up to the Tropopause, then becomes constant (model valid up to altitude of ~ 20 km), the equations for temperature, pressure and density can be written as:

$$P(y(t)) = P_0 \left(1 - \frac{L}{y(t)T_0}\right)^{\frac{gM}{RL}}$$

$$T(y(t)) = T_0 - Ly(t) \quad 0 \leq y \leq 10972$$

$$T(y > 10972) = 216.6 \quad y > 10972$$

$$\rho(y(t)) = \frac{PM}{RT} = \frac{P_0 M}{RT(y(t))} \left(1 - \frac{L}{y(t)T_0}\right)^{\frac{gM}{RL}}$$

Modeling the coefficient of drag proved to be more cumbersome. Traditionally, the coefficient of drag is found experimentally through aerodynamic wind tunnel testing. However, this can be extremely costly and time consuming, especially for rockets that go supersonic. To overcome this, the equation of a sphere in continuum and rarefied flows was used [2]. This solution should significantly improve the approximation of the coefficient of drag for regimes of flow where Reynolds number is laminar, transitioning, turbulent, as well as flows in the transonic/supersonic regimes up to Mach 6. While the Badger Ballistics rocket will not reach these velocities, or altitudes, this mathematical model, if thoroughly tested could be extremely useful for future builds.

The equation of the coefficient of drag can be written as:

$$C_D = 24 \left[Re + S \left\{ 4.33 + \left(\frac{3.65 - 1.53 \frac{T_w}{T}}{1 + 0.353 \frac{T_w}{T}} \right) \exp \left(-0.247 \frac{Re}{S} \right) \right\} \right]^{-1}$$

$$+ \exp \left(-\frac{0.5 Ma}{\sqrt{Re}} \right) \left[\frac{4.5 + 0.38(0.03Re + 0.48\sqrt{Re})}{1 + 0.03Re + 0.48\sqrt{Re}} + 0.1Ma^2 + 0.2Ma^8 \right]$$

$$+ \left[1 - \exp \left(-\frac{Ma}{Re} \right) \right] 0.6S \quad 0 \leq Ma \leq 1.75$$

$$C_D = \frac{0.9 + \frac{0.34}{Ma_\infty^2} + 1.86 \left(\frac{Ma_\infty}{Re_\infty} \right)^{1/2} \left[2 + \frac{2}{S_\infty^2} + \frac{1.058}{S_\infty} \left(\frac{T_w}{T} \right)^{1/2} - \frac{1}{S_\infty^4} \right]}{1 + 1.86 \left(\frac{Ma_\infty}{Re_\infty} \right)^{1/2}} \quad 1.75 \leq Ma < 6$$

where ∞ denotes free stream conditions, S is the molecular speed ratio: $S = Ma\sqrt{\gamma/2}$, and T_w/T is the ratio of the object's temperature over the free stream temperature (assumed to be 1). Reynolds Number can be calculated by $Re = \frac{\rho dV}{\mu}$, where d is the diameter of the rocket, V is the velocity and μ is the Dynamic Viscosity, which is a function of temperature, and can be calculated using Sutherland's formula. The Mach number is the ratio the velocity of the rocket and the local speed of sound of the fluid:



$$Ma = \frac{\dot{y}}{c}; \quad c = \sqrt{\kappa RT}$$

The final piece of the equation of motion is the force of thrust, which is a dependent on time. To model this, the thrust vs time data can be obtained from [3] and fit to a high-order polynomial, which can be used in the mathematical model. The final equation can be written in matrix form by separation of variables:

$$\begin{bmatrix} \ddot{y} \\ \ddot{y} \end{bmatrix} = \left[\frac{F_T}{m} - \frac{C_D A}{m} \left(\frac{P_0 M}{RT} \right) \left(1 - \frac{L}{y(1) T_o} \right)^{\frac{gM}{RL}} y(2)^2 - \frac{\dot{m} y(2)}{m} - g \right]$$

To validate this model a high-powered rocket was flown with our data acquisition system on-board and compared to our mathematical model. The rocket flown was a 160 cm long, scratch built rocket, that flew on a CTI J240. Figure 15 shows the thrust data from the J240 fit to a 10th order polynomial to get an equation for thrust vs time.

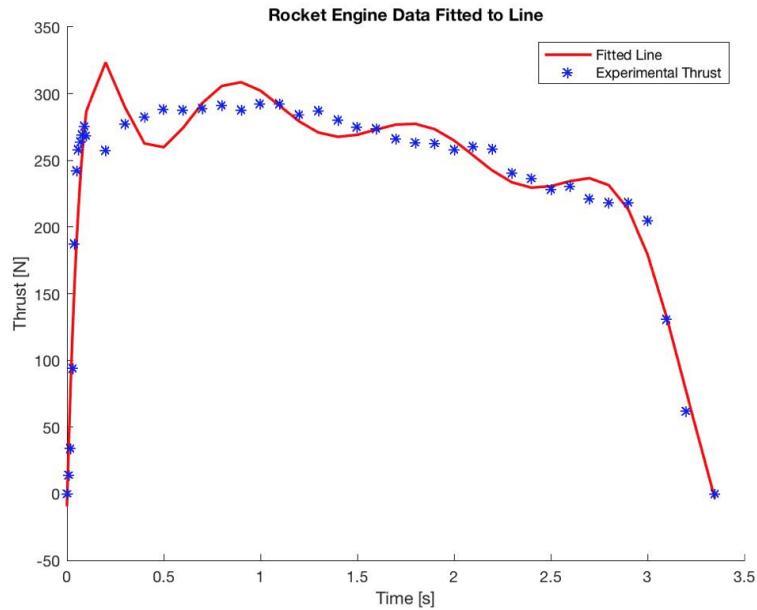


Figure 15. Thrust vs time data from CTI J240 rocket engine fitted to 10th order polynomial

The altitude vs time of the experimental data was plotted against the theoretical model (Figure 16) and resulted in a 96 ft difference in the maximum predicted altitudes (2.03%). While a 96 ft difference seems large, the performance error in commercial rocket engines can be as high as 20%. Additionally, predictions from OpenRocket yielded a 350 ft difference in the maximum predicted altitude (7.35%), which provides evidence this model is more accurate in this particular domain. More testing of this model will be done as we continue to fly rockets to further discover the validity of the model, as well as it's pit-falls.

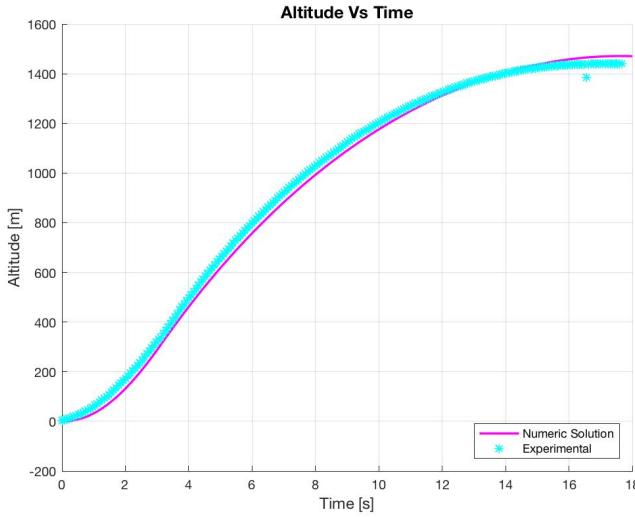


Figure 16. Altitude vs time of test rocket to validate theoretical model for altitude prediction

5.2 Predicted Vs. Actual: Apogee

The altitude prediction model has been described in detail and resulted in a predicted maximum altitude of 3013 ft. However, due to the roll control issue discussed in section 4.1, there was a large amount of additional drag force from the control surfaces at full extension for the entirety of the flight. This caused the rocket to undershoot the predicted altitude by 480 ft (max altitude: 2532.82 ft). The altitude vs time plot (Figure 18) clearly shows a steep change in slope at exactly 4 seconds (when the roll control system activated). This indicates an abrupt change in acceleration due to an increase in drag force.

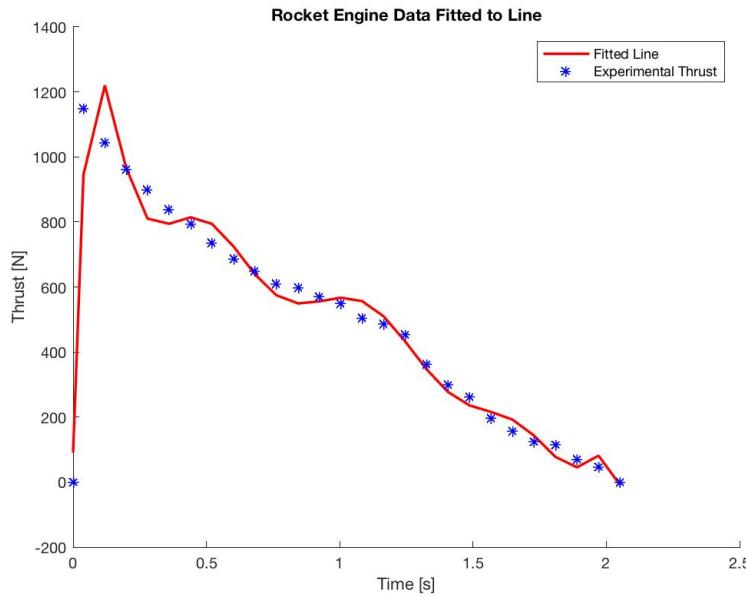


Figure 17. AeroTech J570 thrust vs time data fit to 10th order polynomial

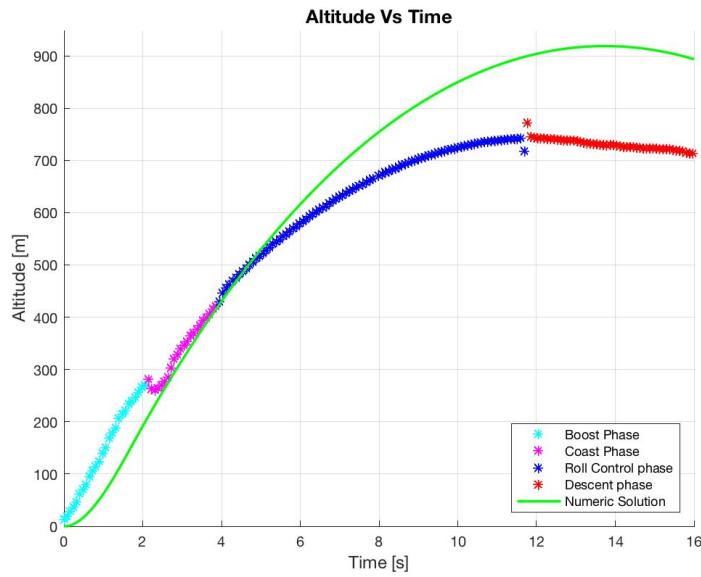


Figure 18. Altitude vs time plot of Badger Ballistics rocket test flight data and theoretical prediction, maximum altitude = 772.20 m = 2532.82 [ft]

5.3 Predicted Vs. Actual: Velocity

The theoretical velocity and acceleration were plotted using the mathematical model. The maximum predicted velocity by this model was 136.1 m/s (Figure 19). The Badger Ballistics rocket didn't experimentally measure velocity, however, the maximum velocity can be estimated by extrapolating the slope of altitude vs time to get the average maximum velocity prior to motor burnout (Figure 20). This resulted in a maximum velocity of 110.75 m/s.

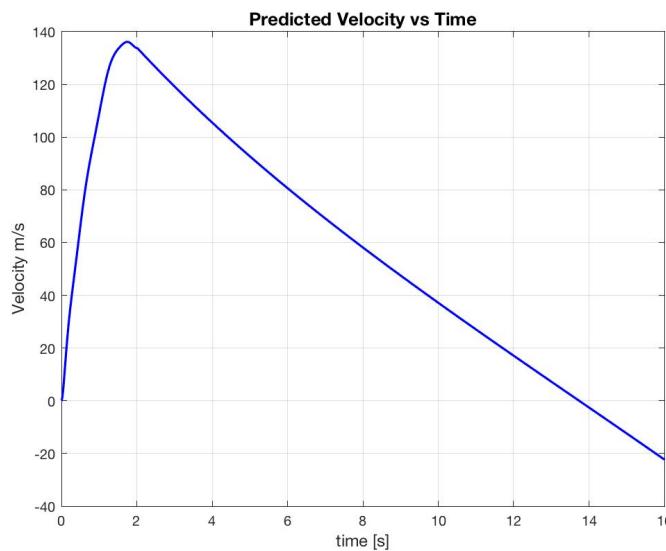


Figure 19. Predicted velocity vs time, max velocity = 136 m/s

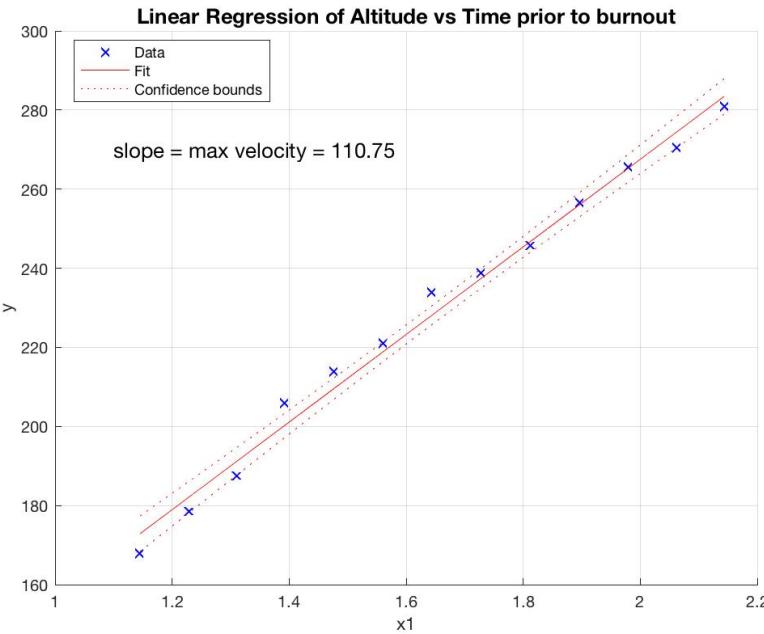


Figure 20. Linear regression of altitude vs time of test flight 0.8 seconds prior to burnout until motor burnout to extrapolate maximum velocity. Max velocity = 110.75 m/s.

5.4 Predicted Vs. Actual: Acceleration

Perhaps one of the worst fit, yet possibly most interesting plots is the absolute value of acceleration vs time plot. As can be seen from Figure 21, the theoretical model predicted a maximum acceleration of 18 g's on launch with a decreasing acceleration similar to the burn profile of the J570 engine, then it asymptotically approaches 1 g. The experimental data, however, shows a large spike at 4 seconds, when the roll control system activated. This spike shows an absolute value of 13.5 g's (should be negative g's but only magnitude obtained), this indicates there was an additional ~ 12 g's of deceleration than predicted, due to the additional drag force from the control surfaces. This additional drag

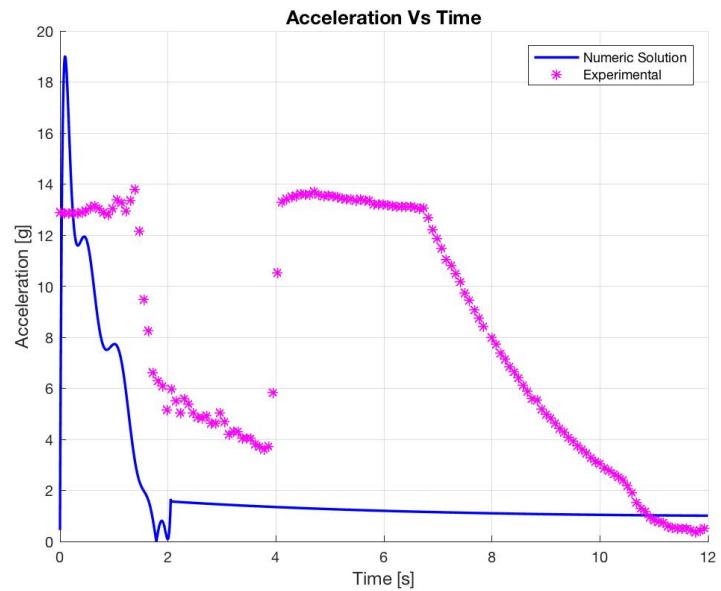


Figure 21. Absolute value of acceleration (magnitude) in g-force



force is likely what prevented the rocket from achieving 3000 ft in altitude. Error checking is now implemented in the flight computer code to prevent the gyro hardware issue for future flights, this will ensure the control surfaces are not at full extension for the entire flight. Additionally, the maximum deflection of the control surfaces was reduced from 45 degree deflections to 30 degree deflections to further reduce unwanted drag.

5.5 Predicted Vs. Actual: Descent Characteristics

OpenRocket was used for descent velocity predictions (Figure 22) and resulted in a ground hit velocity of 5.6 m/s (18.4 ft/s). Extrapolating the slope of the line of the descent from the experimental data showed an average velocity of 7.4 m/s (24.2 ft/s). This is slightly fast, so a 36 inch drogue parachute will replace the 24 inch parachute to ensure the rocket is recovered at a safe speed. (see section 4.3: Recovery System and Descent Phase Analysis for additional info on recovery system).

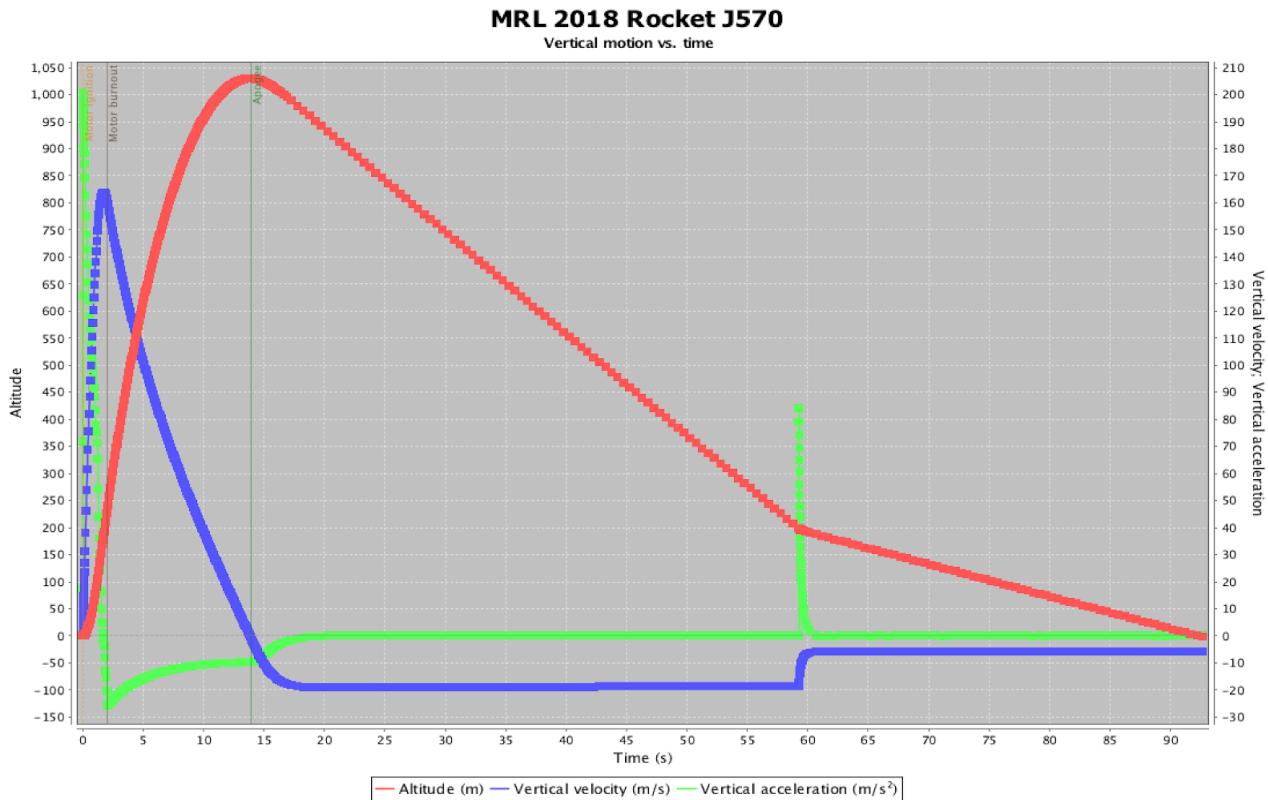


Figure 22. OpenRocket flight predictions



6.0 KEY FINDINGS FINAL IMPROVEMENTS

6.1 Mass Reduction

Since the test flight did not achieve an altitude of 3000 ft (under worst case conditions), efforts are being made to reduce the weight of the rocket. To do this, the shock cord length of the drogue parachute will be shortened from nearly 50 ft to 25 ft, and the main parachute shock cord will be shortened from 21 ft to 15 ft. Furthermore, the U-bolts that are currently used for attachment points will be reduced in size. The steel quick-disconnects that attach the shock cord to the rocket will also be removed and the parachute cord will simply be attached directly to the U-bolts. While this is a very slight reduction in weight, simulations show that a 200 gram downsizing will get us an additional 100 ft of altitude. This combined with reducing the output of the control surfaces to reduce drag, should ensure the rocket gets to at least 3000 ft.

6.2 Roll Control Algorithm

The null roll function proved to work quite effectively during the wind tunnel testing at Modine Manufacturing. However, during this testing, our team found that there were some flaws within the new orientation function that would tell the rocket to roll toward a certain heading by going either clockwise or counterclockwise. The problem arose whenever the rocket's heading would cross the 360 to zero line or vice versa. For a case such as this, the rocket would calculate an erroneous heading "error" and think that it had to severely correct its current position. For example, if the rocket's heading initially was at 350 degree and was desired to go to 20 degrees it should only have to make a 30 degree correction. Yet errors in our code led the control algorithm to believe it had to make a 330 degree correction which caused the roll system to dramatically overshoot the target and sometime would trigger a "death spin". This problem was remedied by incorporating two new conditional statements in the code. One for each approach direction as the heading would cross the zero to 360 line. Further testing is still needed to validate these new methodologies.

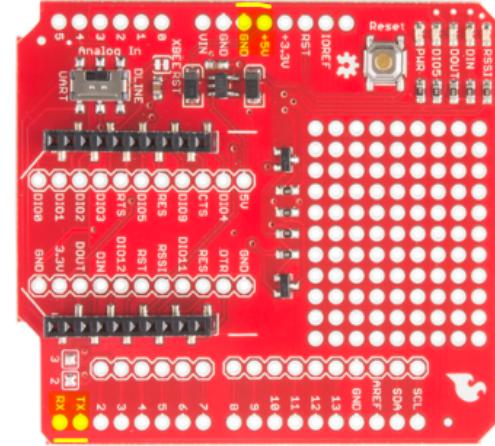
7.0 COMMUNICATIONS SYSTEM SUMMARY

The Badger Ballistics team will achieve active radio communication between the rocket and ground control through the use of an XBee radio system. The system consists of two XBee Pro - Series 1 modules which are capable of transmitting and receiving digital or analog data for an unobstructed distance of one mile. The system will be configured in a way that the two Badger Ballistics' radio modules are only able to transmit and receive to each other. As an added layer of security, any exchange of input commands between the two modules will also require a



two character passcode. A more detailed explanation of the communication system can be found in the Badger Ballistics Preliminary Design Report.

The installation of the XBee radio module into the rocket's avionics bay produced an interesting design challenge. The Sparkfun XBee shield that connects the XBee to the Arduino microcontroller is incredibly large and would cover a number of pins that are needed for other hardware in the electronics circuit. This issue was resolved by installing the XBee onto the Sparkfun shield to regulate voltage, but then installing the shield on its own portion of the PCB in order to free up space on the Arduino itself. The custom circuit board required our team to get creative with wiring the pin ports of the XBee, shield, and PCB to optimize space. After careful research, we were able to reduce the number of pins needed to only four; the "Series In", "Serial Out", "5V" and "GND" pins (highlighted in yellow in Figure).



7.1 Radio Communication Experimental Results

The Badger Ballistics team will be tackling all three portions of the MRL Bonus Communications Challenge. Extensive testing has been conducted to validate and improve the comm system logic. Nearly all of this experimentation has been run in a static, lab room setting. However, we should not expect any system failures during an application launch, aside from transmittance range and accuracy of high-baud data at these distant ranges. During the first test launch on April 28, our team was able to confirm that these issues will not be of concern.

The two way communication portion of the bonus challenge has been successfully implemented via serial transmittance. This logic allows us to quickly send live flight data to ground control. The Badger Ballistics team chose to take this information one step further by processing the data into a live telemetry graphical user interface (Figure 23). This GUI plots acceleration, altitude, compass heading, and angular velocity. This system will also be used as an interface to input/output information to the rocket. The ability to input data to the rocket via the GUI was demonstrated during the Modine wind tunnel testing. Our team was able to successfully input new roll orientations while the rocket was rigged up to the test fixture and the ground control computer was 30 feet away in the lab's control room.

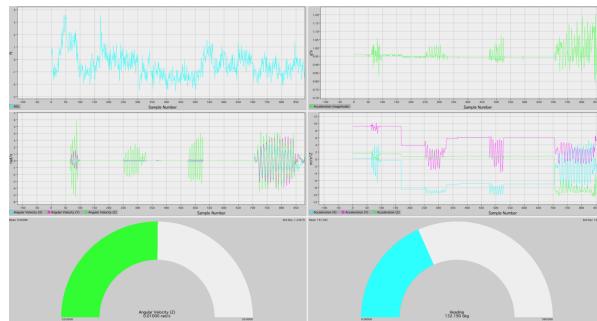


Figure 23. Live telemetry interface with various flight characteristics displayed graphically.

7.2 Communication Capability Improvements

The Badger Ballistics team has yet to implement the final bonus challenge involving onboard computation of a basic mathematical operation involving a limited number of characters. We have begun processing code which will solve the problem with as minimal computational power as possible. This challenge should be well within the scope of our team's Arduino coding capabilities, but due to other systems being more critical, this particular project has been postponed. It may also be necessary to look into utilizing a second set of sending/receiving nodes within the XBee to keep this variable line open for in-flight communication. Due to the high amount of data travelin through the Serial 1 ports, it may be beneficial to give the "Mathematical" function it's own communication line with the rocket.

8.0 BUDGET

Badger Ballistics Preliminary BOM	
<i>University of Wisconsin - Madison</i>	
<i>Midwest Rocket Launch Competition 2017-2018</i>	

The Badger Ballistics team initially projected a budget of \$2000 for the MRL High-Power Rocket competition. Our team did an excellent job in utilizing various University and industrial resources to stay as close to this initial budget as possible. The grand total for this year's MRL competition tallied up to just under \$2400. Most of this excess cost can be accounted for within the lab testing cost category which was something that our team did not initially plan for. Additionally, many of the purchases that were made to build the lab testing fixture will be able to be utilized in future competitions and on other rockets.

On a continued note of lab testing, the Badger Ballistics MRL competition team would like to explicitly recognize the generous donation of test facilities by Modine Manufacturing in Racine, WI. The wind tunnel testing that was conducted within the Modine facilities was

Cost Category	Total Cost
Airframe	\$441.76
Recovery	\$51.10
Electronics	\$297.47
Communications	\$130.75
Lab Testing	\$205.24
Motor	\$431.03
Misc.	\$150.20
Travel / Logistics	\$670.00
Grand Total	\$2,377.55



appraised at nearly \$700 and provided invaluable data which allowed our team to fine tune the variables within our roll control algorithm. Modine was a truly gracious host to the Badger Ballistics team, and in addition to the wind tunnel test facilities, our team had an opportunity to tour the company's entire technical center campus. In the future, our team hopes to potentially utilize some of Modine's other technical capabilities including calorimeter testing, high frequency vibrational testing, and environmental wind tunnel testing.

Overall, our budget was 18.88% above the initial projection. Although this only amounts to ~\$380, this margin of error is definitely cause for concern. In the future, the Badger Ballistics team hopes to be able to secure additional industry sponsorships in order to bring down costs of unique rocket components or to inflate initial budgetary potential.

Communications	Part	Cost	Dealer	Quantity	Total Cost
Airframe					
	Body Tube (4" Fiberglass)	\$105.00	Apogee	1	\$105.00
	Motor Mount Tube (Fiberflass)	\$60.00	Apogee	1	\$60.00
	Fin Material	\$30.00	Public Missiles	4	\$120.00
	1010 Airfoil Rail Buttons	\$7.00	Apogee	2	\$14.00
	Coupler Tube	\$30.00	Apogee	1	\$30.00
	Centering Ring (4" to 38mm) (Fiberflass)	\$14.00	Apogee	2	\$28.00
	Fiberglass Nose Cone (5.5:1 Von Karman)	\$69.00	Madcow	1	\$69.00
	Quick Links	\$7.88	Apogee/Hardware Store	2	\$15.76
Recovery					
	44" LOC Angel Main Parachute	\$0.00	(On Hand)	1	\$0.00
	24" Nylon Drogue Parachute	\$0.00	(On Hand)	1	\$0.00
	Kevlar Shock Cord	\$1.25	Apogee	20	\$25.00
	12" Nomex Chute Protector	\$8.15	Apogee	1	\$8.15
	Ejection Charge Canisters w/ Igniters (10pk)	\$17.95	Wildman	1	\$17.95
Electronics					
	FPV Camera	\$0.00	(On Hand)	2	\$0.00
	Mobius Camera	\$0.00	(Borrowed)	1	\$0.00
	Mobius Camera Shroud	\$0.00	(Borrowed)	1	\$0.00
	Adafruit 10-DOF IMU	\$25.00	Adafruit	1	\$25.00
	Arduino Mega	\$30.00	Arduino	2	\$60.00
	Adafruit SD Shield	\$8.00	Adafruit	2	\$16.00
	8GB Micro SD Card	\$5.00	Amazon	1	\$5.00
	1600 mAh 3S 25'50C Lipo Pack	\$24.00	Amazon	1	\$24.00
	Stratotlogger CF Altimeter (Primary)	\$50.00	-	1	\$50.00
	Terminal Blocks	\$3.41	Apogee	2	\$6.82
	Jolly Logic Altimeter Two Holster	\$10.65	Apogee	1	\$10.65
	Buffer Price for Unspecified Electronics/Replacements	\$100.00	-	-	\$100.00
Communications					
	XBee Pro 60mW - Series 1	\$37.95	Sparkfun	2	\$75.90
	XBee Explorer USB	\$24.95	Sparkfun	1	\$24.95
	XBee Shield	\$14.95	Sparkfun	2	\$29.90
	Soldering Iron	\$0.00	(On Hand)	1	\$0.00
	Soldering Wire	\$0.00	(On Hand)	1	\$0.00
Lab Testing					
	1.5" x 3" 8020 (10 feet)	\$0.00	(On Hand)	1	\$0.00
	8020 Corner Bracket	\$15.00	McMaster Carr	2	\$30.00
	8020 Diagonal Brace	\$20.00	McMaster Carr	2	\$40.00
	8020 Rail-to-Tube Holder	\$35.24	McMaster Carr	1	\$35.24
	Steel Rod	\$50.00	McMaster Carr	1	\$50.00
	Ball Bearings	\$25.00	McMaster Carr	2	\$50.00
	Wind Tunnel Time (Donated)	\$0.00	Modine Manufacturing	-	\$0.00
Motor					
	AeroTech J570	\$80.00	Wildman	5	\$400.00
	AeroPack 38mm Motor Retainer Set	\$31.03	Apogee	1	\$31.03
Misc.					
	Shipping and Tax Estimate For All Orders	\$50.00	-	-	\$50.00
	Black Powder	\$0.00	(On Hand)	-	\$0.00
	2-56 Nylon Shear Pins (20pk)	\$3.10	Apogee	2	\$6.20
	Clear Coat Paint	\$16.00	Hardware Store	1	\$16.00
	Sandpaper (Various Grit)	\$8.00	Hardware Store	1	\$8.00
	Epoxy/West Systems	\$50.00	Wildman	1	\$50.00
	Misc. Hardware (Nuts, Washers, etc.)	\$20.00	Hardware Store	-	\$20.00
Travel / Logistics					
	Registration Fee	\$400.00	-	1	\$400.00
	Hostel Fee (Per Night - double Twin Bed room)	\$90.00	-	3	\$270.00
	Transportation	\$0.00	(Paying Personally)	-	\$0.00



REFERENCES

- [1] Matweb.com. (2018). *G-10 Fiberglass Epoxy Laminate Sheet*. [online] Available at: <http://www.matweb.com/search/datasheet.aspx?matguid=8337b2d050d44da1b8a9a5e61b0d5f85> [Accessed 2 May 2018].
- [2] Charles, B. Henderson, "Drag Coefficients of Spheres in Continuum and Rarefied Flows", *AIAA Journal*, vol. 14, No. 16, pp. 707, June 1976
- [3] J. Coker, "ThrustCurve Hobby Rocket Motor Search", *Thrustcurve.org*, 2008. [Online]. Available: <http://www.thrustcurve.org/motorsearch.jsp?id=294>. [Accessed: 09- Mar- 2018].