DRAFT VERSION JULY 5, 2024
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Different forms of the fluid equations

1. FULLY COMPRESSIBLE

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\boldsymbol{u} \quad \text{where} \quad \frac{D}{Dt} \coloneqq \frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla \tag{1}$$

$$\rho\frac{D\boldsymbol{u}}{Dt} = -2\rho\boldsymbol{\Omega}_0\times\boldsymbol{u} - \nabla P + \rho\boldsymbol{g} + \nabla\cdot\boldsymbol{D} + \frac{1}{\mu}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} \tag{2a}$$

$$\text{where} \quad D_{ij} = \rho\nu\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}(\nabla\cdot\boldsymbol{u})\delta_{ij}\right] \tag{2b}$$

$$\rho T\frac{DS}{Dt} = \nabla\cdot(\rho\kappa C_{\mathrm{p}}\nabla T) + Q + D_{ij}\frac{\partial u_i}{\partial x_j} + \frac{1}{\mu}|\nabla\times\boldsymbol{B}|^2 \tag{3}$$

$$S = S(P, T) \tag{4}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) - \nabla\times(\eta\nabla\times\boldsymbol{B}) \tag{5}$$

$$\nabla\cdot\boldsymbol{B} \equiv 0 \tag{6}$$

2. ANELASTIC

$$\nabla \cdot (\overline{\rho}\boldsymbol{u}) \equiv 0 \tag{7}$$

$$\nabla \cdot \boldsymbol{B} \equiv 0 \tag{8}$$

$$\overline{\rho}\frac{D\boldsymbol{u}}{Dt} = -2\overline{\rho}\Omega_0\hat{\boldsymbol{e}}_z \times \boldsymbol{u} - \nabla P' + \rho' g\hat{\boldsymbol{e}}_r + \nabla \cdot \boldsymbol{D} + \frac{1}{\mu}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}, \tag{9a}$$

$$\text{where} \quad D_{ij} = \overline{\rho}\,\overline{\nu}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}(\nabla \cdot \boldsymbol{u})\delta_{ij}\right], \tag{9b}$$

$$\overline{\rho}\overline{T}\frac{DS'}{Dt} = \nabla \cdot (\overline{\rho}\overline{T}\kappa\nabla S') + Q_{\mathrm{rad}} + D_{ij}\frac{\partial u_i}{\partial x_j} + \frac{1}{\mu}|\nabla \times \boldsymbol{B}|^2, \tag{10}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) - \nabla \times (\overline{\eta}\nabla \times \boldsymbol{B}) \tag{11}$$

$$\frac{\rho'}{\overline{\rho}} = \frac{P'}{\overline{P}} - \frac{T'}{\overline{T}} = \frac{P'}{\gamma\overline{P}} - \frac{S'}{C_{\mathrm{p}}} \tag{12}$$

$$\frac{d\overline{P}}{dr} = -\overline{\rho}g, \quad \text{where} \quad g = \frac{GM_\odot}{r^2} \tag{13}$$

$$\frac{1}{\overline{T}}\frac{d\overline{T}}{dr} - \left(\frac{\gamma - 1}{\gamma}\right)\frac{1}{\overline{P}}\frac{d\overline{P}}{dr} = \frac{1}{C_{\mathrm{p}}}\frac{d\overline{S}}{dr} = \frac{\overline{N^2}}{g} \tag{14}$$