

Different forms of the fluid equations

1. FULLY COMPRESSIBLE

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u} \quad \text{where} \quad \frac{D}{Dt} := \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \quad (1)$$

$$\rho \frac{D\mathbf{u}}{Dt} = -2\rho \boldsymbol{\Omega}_0 \times \mathbf{u} - \nabla P + \rho \mathbf{g} + \nabla \cdot \mathbf{D} + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B} \quad (2a)$$

$$\text{where} \quad D_{ij} = \rho \nu \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} (\nabla \cdot \mathbf{u}) \delta_{ij} \right] \quad (2b)$$

$$\rho T \frac{DS}{Dt} = \nabla \cdot (\rho \kappa C_p \nabla T) + Q + D_{ij} \frac{\partial u_i}{\partial x_j} + \frac{1}{\mu} |\nabla \times \mathbf{B}|^2 \quad (3)$$

$$S = S(P, T) \quad (4)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}) \quad (5)$$

$$\nabla \cdot \mathbf{B} \equiv 0 \quad (6)$$

2. ANELASTIC

$$\nabla \cdot (\bar{\rho} \mathbf{u}) \equiv 0 \quad (7)$$

$$\nabla \cdot \mathbf{B} \equiv 0 \quad (8)$$

$$\bar{\rho} \frac{D\mathbf{u}}{Dt} = -2\bar{\rho}\Omega_0 \hat{e}_z \times \mathbf{u} - \nabla P' + \rho' g \hat{e}_r + \nabla \cdot \mathbf{D} + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (9a)$$

$$\text{where } D_{ij} = \bar{\rho} \bar{v} \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} (\nabla \cdot \mathbf{u}) \delta_{ij} \right], \quad (9b)$$

$$\bar{\rho} \bar{T} \left(\frac{DS'}{Dt} + u_r \frac{d\bar{S}}{dr} \right) = -\nabla \cdot (\bar{\rho} \bar{T} \bar{\kappa} \nabla S') + Q_{\text{rad}} + D_{ij} \frac{\partial u_i}{\partial x_j} + \frac{1}{\mu} |\nabla \times \mathbf{B}|^2, \quad (10)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\bar{\eta} \nabla \times \mathbf{B}) \quad (11)$$

$$\frac{\rho'}{\bar{\rho}} = \frac{P'}{\bar{P}} - \frac{T'}{\bar{T}} = \frac{P'}{\gamma \bar{P}} - \frac{S'}{C_p} \quad (12)$$

$$\frac{d\bar{P}}{dr} = -\bar{\rho} g, \quad \text{where } g = \frac{GM_\odot}{r^2} \quad (13)$$

$$\frac{1}{\bar{T}} \frac{d\bar{T}}{dr} - \left(\frac{\gamma - 1}{\gamma} \right) \frac{1}{\bar{P}} \frac{d\bar{P}}{dr} = \frac{1}{C_p} \frac{d\bar{S}}{dr} = \frac{\bar{N}^2}{g} \quad (14)$$

3. LABELED INDUCTION EQUATION

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u} \times \mathbf{B})}_{\text{stretching}} - \underbrace{\nabla \times (\eta \nabla \times \mathbf{B})}_{\text{diffusion}} \quad (15)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{u}_i \times \mathbf{B})}_{\text{stretching}} - \underbrace{\nabla \times (\eta \nabla \times \mathbf{B})}_{\text{diffusion}} - \underbrace{\frac{ck_B}{en_e} \nabla n_e \times \nabla T_e}_{\text{battery}} \quad (16)$$