Angular Momentum in Terms of Toroidal and Poloidal Stream Functions

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1 Stream Function Formalism

In anelastic approximations, we always have the condition of divergenceless mass flux,

$$\nabla \cdot (\overline{\rho}\boldsymbol{v}) \equiv 0, \tag{1}$$

where $\overline{\rho}$ is the reference state density (we assume $\overline{\rho} = \overline{\rho}(r)$ is spherically symmetric and time-independent) and $\boldsymbol{v}$ is the fluid velocity. Condition (1) admits a stream function representation for the mass flux,

$$\overline{\rho}\boldsymbol{v} = \nabla \times [\nabla \times (W\hat{\boldsymbol{e}}_r)] + \nabla \times (Z\hat{\boldsymbol{e}}_r), \tag{2}$$

References