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1 Basic

1.1 vimrc

```
se nu rnu ai hls et ru ic is sc
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
se bg=dark
ino {<CR>} {<CR>}<Esc>ko<tab>
```

1.2 readchar [a419b9]

```
inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf +
        fread_unlocked(buf, 1, bufsize, stdin), p = buf;
    return *p++;
}
```

1.3 Black Magic [afb343]

```
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {
    heap h1, h2; // max heap
    h1.push(1), h1.push(3), h2.push(2), h2.push(4);
    h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
    tree<ll, null_type, less<ll>, rb_tree_tag
        , tree_order_statistics_node_update> st;
```

```
tree<ll, ll, less<ll>, rb_tree_tag
    , tree_order_statistics_node_update> mp;
for (int x : {0, 3, 20, 50}) st.insert(x);
assert(st.order_of_key(3) == 1 && st.order_of_key(4) == 2);
assert(*st.find_by_order(2) == 20 && *st.lower_bound(4) == 20);
rope<char> *root[10]; // nsqrt(n)
root[0] = new rope<char>();
root[1] = new rope<char>(*root[0]);
// root[1]->insert(pos, 'a');
// root[1]->at(pos); 0-base
// root[1]->erase(pos, size);
}
// __int128_t, __float128_t
// for (int i = bs._Find_first(); i < bs.size(); i = bs._Find_next(i));
```

1.4 Pragma Optimization [0ca2dd]

```
#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr())|0x8040)
```

1.5 Default Code [fdccdf]

```
#include <bits/stdc++.h>
#define int long long
#define fi first
#define se second
#define all(a) a.begin(),a.end()
#define SZ(a) (int)(a.size())
#define pb push_back
#define eb emplace_back
#define rep(i,a,b) for(int i=a,Z=b;i<Z;++i)
#define rep1(i,a,b) for(int i=a,Z=b;i<=Z;++i)
using namespace std;
using ll = long long;
using pii = pair<int,int>;
using vi = vector<int>;
using vvi = vector<vi>;
using vp = vector<pii>;
using vvp = vector<vp>;
void foo(){
}

signed main(){
    ios::sync_with_stdio(0);cin.tie(0);
    int t = 1;
    // cin >> t;
    while(t--) foo();
}
```

1.6 cmp [57e6a1]

```
struct cmp {
    bool operator()(T a, T b) const {
        return 0;
    }
};
priority_queue<T, vector<T>, cmp> pq;
set<T, cmp> st;
```

1.7 Builtin Functions [4cc12f]

```
int __builtin_popcount(int a); // count number of 1s
int __lg(int a); // floor(log_2(a))
int __gcd(int a,int b); // gcd
iota(a.begin(), a.end(), 1); // fill a, start from 1
bool next_permutation(all(a)); // return 0 if end
```

2 Graph

2.1 BCC Vertex* [6763ed]

```
struct BCC { // 0-base
    int n, dft, nbcc;
    vi low, dfn, bln, stk, is_ap, cir;
    vvi G, bcc, nc;
    void make_bcc(int u) {
        bcc.eb(1, u);
        for (; stk.back() != u; stk.pop_back())
            bln[stk.back()] = nbcc, bcc[nbcc].pb(stk.back());
```

```

    stk.pop_back(), bln[u] = nbcc++;
}
void dfs(int u, int p) {
    int child = 0;
    low[u] = dfn[u] = ++dft, stk.pb(u);
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_ap[u] = 1, bln[u] = nbcc;
                make_bcc(v), bcc.back().pb(u);
            }
        } else if (dfn[v] < dfn[u] && v != p)
            low[u] = min(low[u], dfn[v]);
        if (p == -1 && child < 2) is_ap[u] = 0;
        if (p == -1 && child == 0) make_bcc(u);
    }
BCC(int _n): n(_n), dft(),
    nbcc(), low(n), dfn(n), bln(n), is_ap(n), G(n) {}
void add_edge(int u, int v) {
    G[u].pb(v), G[v].pb(u);
}
void solve() {
    for (int i = 0; i < n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
void block_cut_tree() {
    cir.resize(nbcc);
    for (int i = 0; i < n; ++i)
        if (is_ap[i])
            bln[i] = nbcc++;
    cir.resize(nbcc, 1), nG.resize(nbcc);
    for (int i = 0; i < nbcc && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_ap[j])
                nG[i].pb(bln[j]), nG[bln[j]].pb(i);
} // up to 2 * n - 2 nodes!! bln[i] for id
};

```

2.2 Bridge* [4da29a]

```

struct ECC { // 0-base
    int n, dft, ecnt, necc;
    vector<int> low, dfn, bln, is_bridge, stk;
    vector<vector<pii>> G;
    void dfs(int u, int f) {
        dfn[u] = low[u] = ++dft, stk.pb(u);
        for (auto [v, e] : G[u])
            if (!dfn[v])
                dfs(v, e), low[u] = min(low[u], low[v]);
            else if (e != f)
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            if (f != -1) is_bridge[f] = 1;
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = necc;
            bln[u] = necc++, stk.pop_back();
        }
    }
ECC(int _n): n(_n), dft()
    , ecnt(), necc(), low(n), dfn(n), bln(n), G(n) {}
void add_edge(int u, int v) {
    G[u].pb(pii(v, ecnt)), G[v].pb(pii(u, ecnt++));
}
void solve() {
    is_bridge.resize(ecnt);
    for (int i = 0; i < n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
}; // ecc_id(i): bln[i]

```

2.3 SCC* [7ae459]

```

struct SCC { // 0-base
    int n, dft, nscc;
    vi low, dfn, bln, instack, stk;
    vvi G;
    void dfs(int u) {
        low[u] = dfn[u] = ++dft;
        instack[u] = 1, stk.pb(u);
        for (int v : G[u])
            if (!dfn[v])
                dfs(v), low[u] = min(low[u], low[v]);
            else if (instack[v] && dfn[v] < dfn[u])
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {

```

```

            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = nscc,
                    .back() = nscc, instack[stk.back()] = 0;
            instack[u] = 0, bln[u] = nscc++, stk.pop_back();
        }
    }
SCC(int _n): n(_n), dft(), nscc
    (), low(n), dfn(n), bln(n), instack(n), G(n) {}
void add_edge(int u, int v) {
    G[u].pb(v);
}
void solve() {
    for (int i = 0; i < n; ++i)
        if (!dfn[i]) dfs(i);
}
}; // scc_id(i): bln[i]

```

2.4 2SAT* [f5630a]

```

struct SAT { // 0-base
    int n;
    vector<bool> istrue;
    SCC scc;
    SAT(int _n): n(_n), istrue(n + n), scc(n + n) {}
    int rv(int a) {
        return a >= n ? a - n : a + n;
    }
    void add_clause(int a, int b) {
        scc.add_edge(rv(a), b), scc.add_edge(rv(b), a);
    }
    bool solve() {
        scc.solve();
        for (int i = 0; i < n; ++i) {
            if (scc.bln[i] == scc.bln[i + n]) return false;
            istrue[i] = scc.bln[i] < scc.bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};

```

2.5 MinimumMeanCycle* [3e5d2b]

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)
                for (int j = 0; j < n; ++j)
                    dp[i][j] =
                        min(dp[i - 1][k] + road[k][j], dp[i][j]);
        for (int i = 0; i < n; ++i) {
            if (dp[L][i] >= INF) continue;
            ll ta = 0, tb = 1;
            for (int j = 1; j < n; ++j)
                if (dp[j][i] < INF &&
                    ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                    ta = dp[L][i] - dp[j][i], tb = L - j;
            if (ta == 0) continue;
            if (a == -1 || a * tb > ta * b) a = ta, b = tb;
        }
        if (a != -1) {
            ll g = __gcd(a, b);
            return pll(a / g, b / g);
        }
        return pll(-1LL, -1LL);
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
    }
};

```

2.6 Virtual Tree* [1b641b]

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
}

```

```

if (st[top] != p)
    vG[p].pb(st[top]), --top, st[++top] = p;
st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

2.7 Maximum Clique Dyn* [d50aa9]

```

struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(ALL(r),
                [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt + 1].reset();
            cs[k][p] = 1;
            if (k < lft) r[tp++] = p;
        }
        for (int k = lft; k <= rgt; ++k)
            for (int p = cs[k]._Find_first
                    (); p < N; p = cs[k]._Find_next(p))
                r[tp] = p, c[tp] = k, ++tp;
        dfs(r, c, l + 1, mask);
    }
    void dfs(vector<
        int> &r, vector<int> &c, int l, bitset<N> mask) {
        while (!r.empty()) {
            int p = r.back();
            r.pop_back(), mask[p] = 0;
            if (q + c.back() <= ans) return;
            cur[q++] = p;
            vector<int> nr;
            for (int i : r) if (G[p][i]) nr.pb(i);
            if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
            else if (q > ans) ans = q, copy_n(cur, q, sol);
            c.pop_back(), --q;
        }
    }
    int solve() {
        vector<int> r(n);
        ans = q = 0, iota(ALL(r), 0);
        pre_dfs(r, 0, bitset<N>(string(n, '1')));
        return ans;
    }
}

```

2.8 Dominator Tree* [2da9bb]

```

struct Dominator {
    int n;
    vector<vector<int>> g, r, rdom; int tk;
    vector<int> dfn, rev, fa, sdom, dom, val, rp;
    Dominator
        (int _n) : n(_n), g(n), r(n), rdom(n), tk(0) {
        dfn = rev = fa = sdom = dom =
            val = rp = vector<int>(n, -1);
    }
    void add_edge(int x, int y) { g[x].push_back(y); }
}

```

```

void dfs(int x) {
    rev[dfn[x] = tk] = x;
    fa[tk] = sdom[tk] = val[tk] = tk; tk++;
    for (int u : g[x]) {
        if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
        r[dfn[u]].push_back(dfn[x]);
    }
}
void merge(int x, int y) { fa[x] = y; }
int find(int x, int c = 0) {
    if (fa[x] == x) return c ? -1 : x;
    if (int p = find(fa[x], 1); p != -1) {
        if (sdom[val[x]] > sdom[val[fa[x]]])
            val[x] = val[fa[x]];
        fa[x] = p;
        return c ? p : val[x];
    } else return c ? fa[x] : val[x];
}
vector<int> build(int s) {
    // return the father of each node in dominator tree
    dfs(s); // p[i] = -2 if i is unreachable, p[s] = -1
    for (int i = tk - 1; i >= 0; --i) {
        for (int u : r[i])
            sdom[i] = min(sdom[i], sdom[find(u)]);
        if (i) rdom[sdom[i]].push_back(i);
        for (int u : rdom[i]) {
            int p = find(u);
            dom[u] = (sdom[p] == i ? i : p);
        }
        if (i) merge(i, rp[i]);
    }
    vector<int> p(n, -2); p[s] = -1;
    for (int i = 1; i < tk; ++i)
        if (sdom[i] != dom[i]) dom[i] = dom[dom[i]];
    for (int i = 1; i < tk; ++i)
        p[rev[i]] = rev[dom[i]];
    return p;
}

```

2.9 Vizing's theorem* [2b5b01]

```

namespace vizing { // returns
    edge coloring in adjacent matrix G. 1 - based
const int N = 105;
int C[N][N], G[N][N], X[N], vst[N], n;
void init(int _n) { n = _n;
    for (int i = 0; i <= n; ++i)
        for (int j = 0; j <= n; ++j)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pii> &E) {
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; ++X[u]); };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
    auto flip = [&](int u, int c1, int c2) {
        int p = C[u][c1];
        swap(C[u][c1], C[u][c2]);
        if (p) G[u][p] = G[p][u] = c2;
        if (!C[u][c1]) X[u] = c1;
        if (!C[u][c2]) X[u] = c2;
        return p;
    };
    fill_n(X + 1, n, 1);
    for (int t = 0; t < SZ(E); ++t) {
        int u = E[t]
            .X, v0 = E[t].Y, v = v0, c0 = X[u], c = c0, d;
        vector<pii> L;
        fill_n(vst + 1, n, 0);
        while (!G[u][v0]) {
            L.emplace_back(v, d = X[v]);
            if (!C[v][c]) for (int a = SZ(L) - 1; a >= 0; --a) c = color(u, L[a].X, c);
            else if (!C[u][d]) for (int a = SZ(L) - 1; a >= 0; --a) color(u, L[a].X, L[a].Y);
            else if (vst[d]) break;
            else vst[d] = 1, v = C[u][d];
        }
    }
}

```

```

if (!G[u][v0]) {
    for (; v; v = flip(v, c, d), swap(c, d));
    if (int a; C[u][c0]) {
        for (
            a = SZ(L) - 2; a >= 0 && L[a].Y != c; --a);
        for (; a >= 0; --a) color(u, L[a].X, L[a].Y);
    } else --t;
}
} // namespace vizing

```

2.10 Minimum Clique Cover* [879472]

```

struct Clique_Cover { // 0-base, O(n2^n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n, 1);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0; // probabilistic
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] *= co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

2.11 NumberofMaximalClique* [11fa26]

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsn = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsn++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsn, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

3 Data Structure

3.1 Discrete Trick

```

vector<int> val;
// build
sort(ALL(val));
(ALL(val)), val.resize(unique(ALL(val)) - val.begin());
// index of x
upper_bound(ALL(val), x) - val.begin();
// max idx <= x
upper_bound(ALL(val), x) - val.begin();
// max idx < x
lower_bound(ALL(val), x) - val.begin();

```

3.2 zkwseg [aae74d]

```

template<class T, T (*op)(T, T), T (*e)()>
struct segtree {
    int n, sz;
    vector<T> d;
    explicit segtree(int n): segtree(vector<T>(n, e())) {}
    explicit segtree(const vector<T> &v) {
        n = v.size(), sz = 1;
        while(sz < n) sz <<= 1;
        d.assign(sz*2, e());
        for(int i=0; i<n; ++i) d[sz+i] = v[i];
        for(int i=sz-1; i>=1; --i) d[i] = op(d[i<<1], d[i<<1|1]);
    }
    void set(int p, const T &x) {
        p += sz, d[p] = x;
        while(p>=1) d[p] = op(d[p<<1], d[p<<1|1]);
    }
    T get(int p) const { return d[sz+p]; }
    T prod(int l, int r) const {
        T sl = e(), sr = e();
        l += sz, r += sz+1;
        while(l < r) {
            if(l&1) sl = op(sl, d[l++]);
            if(r&1) sr = op(d[--r], sr);
            l >>= 1, r >>= 1;
        }
        return op(sl, sr);
    }
};

```

3.3 BIT kth* [e39485]

```

int bit[N + 1]; // N = 2 ^ k
int query_kth(int k) {
    int res = 0;
    for (int i = N >> 1; i >= 1; i >>= 1)
        if (bit[res + i] < k)
            k -= bit[res += i];
    return res + 1;
}

```

3.4 Centroid Decomposition* [5a24da]

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[_lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
        for (int i = 1; i <= n; ++i) G[i].clear();
    }
    void add_edge(int a, int b, int w) {
        G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
    }
    void get_cent(
        int u, int f, int &mx, int &c, int num) {
        int mxsz = 0;
        sz[u] = 1;
        for (pll e : G[u])
            if (!done[e.X] && e.X != f) {
                get_cent(e.X, u, mx, c, num);
                sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
            }
        if (mx > max(mxsz, num - sz[u]))
            mx = max(mxsz, num - sz[u]), c = u;
    }
};

```

```

void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pll e : G[u])
        if (!done[e.X] && e.X != f)
            dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}

```

3.5 LiChaoST* [817716]

```

ll INF = 3e18;
struct LiChaoTree { // max
    using L = pair<ll, ll>; // l.first * x + l.second
    int sz;
    vector<L> data;
    vi xs;
    static ll eval
        (L l, ll x) { return l.first * x + l.second; }
    LiChaoTree(vi _xs) : xs(_xs) {
        int n = (int)(xs.size());
        int lg = 1;
        while ((1 << lg) < n) lg++;
        sz = 1 << lg;
        while
            ((int)(xs.size()) < sz) xs.push_back(1e9);
        data = vector<L>(2 * sz, L(0, INF));
    }
    void add(L line, int l, int r) {
        l = lower_bound
            (xs.begin(), xs.end(), l) - xs.begin();
        r = lower_bound
            (xs.begin(), xs.end(), r) - xs.begin();
        add(line, l, r, 0, sz, 1);
    }
    ll query(ll x) {
        int k = (int)(lower_bound
            (xs.begin(), xs.end(), x) - xs.begin());
        assert(0 <=
            k && k < (int)(xs.size()) && xs[k] == x);
        k += sz;
        ll ans = INF;
        while (k >= 1) {
            ans = min(ans, eval(data[k], x));
            k >>= 1;
        }
        return ans;
    }
private:
    void add
        (L line, int ql, int qr, int l, int r, int k) {
            if (qr <= l || r <= ql) {

```

```

                return;
            } else if (ql <= l && r <= qr) {
                int mid = (l + r) / 2;
                ll mx = xs[mid];
                if (eval(line, mx) < eval(data[k], mx)) {
                    swap(line, data[k]);
                }
                if (l + 1 == r) return;
                if (line.first > data[k].first) {
                    add(line, ql, qr, l, mid, 2 * k);
                } else if (line.first < data[k].first) {
                    add(line, ql, qr, mid, r, 2 * k + 1);
                }
            } else {
                int mid = (l + r) / 2;
                add(line, ql, qr, l, mid, 2 * k);
                add(line, ql, qr, mid, r, 2 * k + 1);
            }
        }
    }
}
```

4 Flow/Matching

4.1 Dinic [389161]

```

struct MaxFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int s, t, dis[N], cur[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int tmp = q.front();
            q.pop();
            for (auto &u : G[tmp])
                if (!~dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[tmp] + 1;
                }
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, (int)G[v].size()});
        G[v].pb(edge{u, 0, 0, (int)G[u].size() - 1});
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : G[i]) j.flow = 0;
    }
}

```

4.2 Bipartite Matching* [784535]

```

struct Bipartite_Matching { // 0-base
    int mp[N], mq[N], dis[N + 1], cur[N], l, r;
    vector<int> G[N + 1];
    bool dfs(int u) {
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            int e = G[u][i];
            if (mq[e] == l
                || (dis[mq[e]] == dis[u] + 1 && dfs(mq[e])))
                mp[mq[e]] = u = e, 1;
        }
        return dis[u] = -1, 0;
    }
    bool bfs() {
        queue<int> q;
        fill_n(dis, l + 1, -1);
        for (int i = 0; i < l; ++i)
            if (!~mp[i])
                q.push(i), dis[i] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int e : G[u])
                if (!~dis[mq[e]])
                    q.push(mq[e]), dis[mq[e]] = dis[u] + 1;
        }
        return dis[l] != -1;
    }
    int matching() {
        int res = 0;
        fill_n(mp, l, -1), fill_n(mq, r, l);
        while (bfs()) {
            fill_n(cur, l, 0);
            for (int i = 0; i < l; ++i)
                res += (~mp[i] && dfs(i));
        }
        return res; // (i, mp[i] != -1)
    }
    void add_edge(int s, int t) { G[s].pb(t); }
    void init(int _l, int _r) {
        l = _l, r = _r;
        for (int i = 0; i <= l; ++i)
            G[i].clear();
    }
}

```

4.3 Kuhn Munkres* [4b3863]

```

struct KM { // 0-base, maximum matching
    ll w[N][N], hl[N], hr[N], slk[N];
    int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
    bool vl[N], vr[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            fill_n(w[i], n, -INF);
    }
    void add_edge(int a, int b, ll wei) {
        w[a][b] = wei;
    }
    bool Check(int x) {
        if (vl[x] = 1, ~fl[x])
            return vr[qu[qr++]] = fl[x] = 1;
        while (~x) swap(x, fr[fl[x] = pre[x]]);
        return 0;
    }
    void bfs(int s) {
        fill_n(slk
            , n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
        ql = qr = 0, qu[qr++ ] = s, vr[s] = 1;
        for (ll d;;) {
            while (ql < qr)
                for (int x = 0, y = qu[ql++]; x < n; ++x)
                    if (!vl[x] && slk
                        [x] >= (d = hl[x] + hr[y] - w[x][y])) {
                        if (pre[x] = y, d) slk[x] = d;
                        else if (!Check(x)) return;
                    }
            d = INF;
            for (int x = 0; x < n; ++x)
                if (!vl[x] && d > slk[x]) d = slk[x];
            for (int x = 0; x < n; ++x) {
                if (vl[x]) hl[x] += d;
                else slk[x] -= d;
                if (vr[x]) hr[x] -= d;
            }
            for (int x = 0; x < n; ++x)

```

```

                if (!vl[x] && !slk[x] && !Check(x)) return;
            }
        }
        ll solve() {
            fill_n(fl
                , n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
            for (int i = 0; i < n; ++i)
                hl[i] = *max_element(w[i], w[i] + n);
            for (int i = 0; i < n; ++i) bfs(i);
            ll res = 0;
            for (int i = 0; i < n; ++i) res += w[i][fl[i]];
            return res;
        }
    };

```

4.4 MincostMaxflow* [1c78db]

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    ll dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, ll cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax
                    (e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s
        , int _t, ll &flow, ll &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; BellmanFord(); copy_n(dis, n, pot)) {
            for (int
                i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
            for (int i = t; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                e.flow += up[t], G[e.to][e.rev].flow -= up[t];
            }
        }
        void init(int _n) {
            n = _n, fill_n(pot, n, 0);
            for (int i = 0; i < n; ++i) G[i].clear();
        }
        void add_edge(ll a, ll b, ll cap, ll cost) {
            G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
            G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
        }
    };

```

4.5 Maximum Simple Graph Matching* [0fe1c3]

```

struct Matching { // 0-base
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>> G;
    int Find(int u)
    { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (;;) swap(x, y) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {

```

```

    pre[x] = y, y = match[x];
    if (s[y] == 1) q.push(y), s[y] = 0;
    for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
}
bool Bfs(int r) {
    iota(ALL(fa), 0); fill(ALL(s), -1);
    q = queue<int>(); q.push(r); s[r] = 0;
    for (; !q.empty(); q.pop()) {
        for (int x = q.front(); int u : G[x])
            if (s[u] == -1) {
                if (pre[u] = x, s[u] = 1, match[u] == n) {
                    for (int a = u, b = x, last;
                        b != n; a = last, b = pre[a])
                        last =
                            match[b], match[b] = a, match[a] = b;
                    return true;
                }
                q.push(match[u]); s[match[u]] = 0;
            } else if (!s[u] && Find(u) != Find(x)) {
                int l = LCA(u, x);
                Blossom(x, u, l); Blossom(u, x, l);
            }
    }
    return false;
}
Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
    (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
void add_edge(int u, int v)
{ G[u].pb(v), G[v].pb(u); }
int solve() {
    int ans = 0;
    for (int x = 0; x < n; ++x)
        if (match[x] == n) ans += Bfs(x);
    return ans;
} // match[x] == n means not matched
};

```

4.6 Maximum Weight Matching* [9ffb94]

```

#define REP(i, l, r) for (int i=(l); i<=(r); ++i)
struct WeightGraph { // 1-based
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slk, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slk(nx + 1),
        flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
        match = st = pa = S = vis = slk;
        REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
    }
    int E(edge e)
    { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
    void update_slk(int u, int x, int &s)
    { if (!s || E(g[u][x]) < E(g[s][x])) s = u; }
    void set_slk(int x) {
        slk[x] = 0;
        REP(u, 1, n)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slk(u, x, slk[x]);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (int y : flo[x]) q.push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(ALL(f), xr);
        if (auto pr = it - f.begin(); pr % 2 == 1)
            reverse(1 + ALL(f)), it = f.end() - pr;
        auto res = vector(f.begin(), it);
        return f.erase(f.begin(), it), res;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
        REP(i, 0, SZ(z) - 1) set_match(z[i], z[i ^ 1]);
        set_match(xr, v); f.insert(f.end(), ALL(z));
    }
    void augment(int u, int v) {

```

```

        for (;;) {
            int xnv = st[match[u]]; set_match(u, v);
            if (!xnv) return;
            set_match(v = xnv, u = st[pa[xnv]]);
        }
        int lca(int u, int v) {
            static int t = 0; ++t;
            for (++t; u || v; swap(u, v)) if (u) {
                if (vis[u] == t) return u;
                vis[u] = t, u = st[match[u]];
            }
            return 0;
        }
        void add_blossom(int u, int o, int v) {
            int b = find(n + 1 + ALL(st), 0) - begin(st);
            lab[b] = 0, S[b] = 0, match[b] = match[o];
            vector<int> f = {o};
            for (int t : {u, v}) {
                reverse(1 + ALL(f));
                for (int x = t, y; x != o; x = st[pa[y]])
                    f.pb(x), f.pb(y = st[match[x]]), q.push(y);
            }
            flo[b] = f; set_st(b, b);
            REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
            fill(ALL(flo_from[b]), 0);
            for (int xs : flo[b]) {
                REP(x, 1, nx)
                    if (g[b][x].w == 0 || E(g[xs][x]) < E(g[b][x]))
                        g[b][x] = g[xs][x], g[x][b] = g[x][xs];
                REP(x, 1, n)
                    if (flo_from[xs][x]) flo_from[b][x] = xs;
            }
            set_slk(b);
        }
        void expand_blossom(int b) {
            for (int x : flo[b]) set_st(x, x);
            int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
            for (int x : split_flo(flo[b], xr)) {
                if (xs == -1) { xs = x; continue; }
                pa[xs] = g[x][xs].u, S[xs] = 1, S[x] = 0;
                slk[xs] = 0, set_slk(x), q.push(x), xs = -1;
            }
            for (int x : flo[b])
                if (x == xr) S[x] = 1, pa[x] = pa[b];
                else S[x] = -1, set_slk(x);
            st[b] = 0;
        }
        bool on_found_edge(const edge &e) {
            if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
                int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
                slk[v] = slk[nu] = S[nu] = 0; q.push(nu);
            } else if (S[v] == 0) {
                if (int o = lca(u, v)) add_blossom(u, o, v);
                else return augment(u, v), augment(v, u), true;
            }
            return false;
        }
        bool matching() {
            fill(ALL(S), -1), fill(ALL(slk), 0);
            q = queue<int>();
            REP(x, 1, nx) if (st[x] == x && !match[x])
                pa[x] = S[x] = 0, q.push(x);
            if (q.empty()) return false;
            for (;;) {
                while (SZ(q)) {
                    int u = q.front(); q.pop();
                    if (S[st[u]] == 1) continue;
                    REP(v, 1, n)
                        if (g[u][v].w > 0 && st[u] != st[v]) {
                            if (E(g[u][v]) != 0)
                                update_slk(u, st[v], slk[st[v]]);
                            else if (on_found_edge(g[u][v])) return true;
                        }
                }
                int d = INF;
                REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
                    d = min(d, lab[b] / 2);
                REP(x, 1, nx)
                    if (int
                        s = slk[x]; st[x] == x && s && S[x] <= 0)
                        d = min(d, E(g[s][x]) / (S[x] + 2));
                REP(u, 1, n)
                    if (S[st[u]] == 1) lab[u] += d;
            }
        }
    };

```

```

    else if (S[st[u]] == 0) {
        if (lab[u] <= d) return false;
        lab[u] -= d;
    }
    REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
        lab[b] += d * (2 - 4 * S[b]);
    REP(x, 1, nx)
        if (int s = slk[x]; st[x] == x &&
            s && st[s] != x && E(g[s][x]) == 0)
            if (on_found_edge(g[s][x])) return true;
    REP(b, n + 1, nx)
        if (st[b] == b && S[b] == 1 && lab[b] == 0)
            expand_blossom(b);
    }
    return false;
}
pair<ll, int> solve() {
    fill(ALL(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    fill(ALL(lab), w_max);
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int u, int v, int w)
{ g[u][v].w = g[v][u].w = w; }
};

```

4.7 BoundedFlow* [4a793f]

```

struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!~dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {

```

```

        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(dis, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                add_edge(n + 1, i, cnt[i]), sum += cnt[i];
            else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
        if (sum != maxflow(n + 1, n + 2)) sum = -1;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                G[n + 1].pop_back(), G[i].pop_back();
            else if (cnt[i] < 0)
                G[i].pop_back(), G[n + 2].pop_back();
        return sum != -1;
    }
    int solve(int _s, int _t) {
        add_edge(_t, _s, INF);
        if (!solve()) return -1; // invalid flow
        int x = G[_t].back().flow;
        return G[_t].pop_back(), G[_s].pop_back(), x;
    }
};

```

4.8 Flow Models

- Maximum/Minimum flow with lower bound / Circulation problem
 1. Construct super source S and sink T .
 2. For each edge (x,y,l,u) , connect $x \rightarrow y$ with capacity $u-l$.
 3. For each vertex v , denote by $in(v)$ the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
 4. If $in(v) > 0$, connect $S \rightarrow v$ with capacity $in(v)$, otherwise, connect $v \rightarrow T$ with capacity $-in(v)$.
 - To maximize, connect $t \rightarrow s$ with capacity ∞ (skip this in circulation problem), and let f be the maximum flow from S to T . If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, the maximum flow from s to t is the answer.
 - To minimize, let f be the maximum flow from S to T . Connect $t \rightarrow s$ with capacity ∞ and let the flow from S to T be f' . If $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution. Otherwise, f' is the answer.
 5. The solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow of edge e on the graph.
- Construct minimum vertex cover from maximum matching M on bipartite graph (X,Y)
 1. Redirect every edge: $y \rightarrow x$ if $(x,y) \in M$, $x \rightarrow y$ otherwise.
 2. DFS from unmatched vertices in X .
 3. $x \in X$ is chosen iff x is unvisited.
 4. $y \in Y$ is chosen iff y is visited.
- Minimum cost cyclic flow
 1. Construct super source S and sink T
 2. For each edge (x,y,c) , connect $x \rightarrow y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \rightarrow x$ with $(cost, cap) = (-c, 1)$
 3. For each edge with $c < 0$, sum these cost as K , then increase $d(y)$ by 1, decrease $d(x)$ by 1
 4. For each vertex v with $d(v) > 0$, connect $S \rightarrow v$ with $(cost, cap) = (0, d(v))$
 5. For each vertex v with $d(v) < 0$, connect $v \rightarrow T$ with $(cost, cap) = (0, -d(v))$
 6. Flow from S to T , the answer is the cost of the flow $C+K$
- Maximum density induced subgraph
 1. Binary search on answer, suppose we're checking answer T
 2. Construct a max flow model, let K be the sum of all weights
 3. Connect source $s \rightarrow v, v \in G$ with capacity K
 4. For each edge (u,v,w) in G , connect $u \rightarrow v$ and $v \rightarrow u$ with capacity w
 5. For $v \in G$, connect it with sink $v \rightarrow t$ with capacity $K+2T-(\sum_{e \in E(v)} w(e))-2w(v)$
 6. T is a valid answer if the maximum flow $f < K|V|$
- Minimum weight edge cover
 1. For each $v \in V$ create a copy v' , and connect $u' \rightarrow v'$ with weight $w(u,v)$.
 2. Connect $v \rightarrow v'$ with weight $2\mu(v)$, where $\mu(v)$ is the cost of the cheapest edge incident to v .
 3. Find the minimum weight perfect matching on G' .
- Project selection problem
 1. If $p_v > 0$, create edge (s, v) with capacity p_v ; otherwise, create edge (v, t) with capacity $-p_v$.
 2. Create edge (u, v) with capacity w with w being the cost of choosing u without choosing v .
 3. The mincut is equivalent to the maximum profit of a subset of projects.
 - Dual of minimum cost maximum flow

1. Capacity c_{uv} , Flow f_{uv} , Cost w_{uv} , Required Flow difference for vertex b_u .
2. If all w_{uv} are integers, then optimal solution can happen when all p_u are integers.

$$\begin{aligned} \min \sum_{uv} w_{uv} f_{uv} \\ -f_{uv} \geq -c_{uv} \Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv}) \\ \sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0 \end{aligned}$$

5 String

5.1 KMP [db6286]

```
vector<int> fail_func(const string &s) {
    int n = s.size();
    vector<int> f(n);
    for(int i=1; i<n; ++i) {
        int j = f[i-1];
        while(j > 0 && s[i] != s[j]) j = f[j-1];
        f[i] = j + (s[i] == s[j]);
    }
    return f;
}
int kmp_count(const string &s, const string &p) {
    int n = s.size(), m = p.size();
    vector<int> f = fail_func(p+$");
    int cnt = 0, j = 0;
    for(int i=0; i<n; ++i) {
        while(j > 0 && s[i] != p[j]) j = f[j-1];
        j += (s[i] == p[j]);
        if(j == m) cnt++;
    }
    return cnt;
}
```

5.2 Z-value* [e4e6b3]

```
vi z_value(string s) {
    int n = s.size(), b = 0;
    vi z(n);
    for(int i=1; i<n; ++i) {
        if(z[b]+b < i) z[i] = 0;
        else z[i] = min(z[b]+b-i, z[i-b]);
        while(s[z[i]] == s[z[i]+i]) z[i]++;
        if(z[i]+i > z[b]+b) b = i;
    }
    return z;
}
```

5.3 Manacher* [ae9cc4]

```
int z[N]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {
    string s = "%";
    int l = 0, r = 0;
    for (char c : tmp) s.push_back(c), s.push_back('%');
    for (int i = 0; i < s.size(); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < s.size()
               && s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
}
```

5.4 SAIS* [6f26bc]

```
auto sais(const auto &s) {
    const int n = SZ(s), z = ranges::max(s) + 1;
    if (n == 1) return vector{0};
    vector<int> c(z); for (int x : s) ++c[x];
    partial_sum(ALL(c), begin(c));
    vector<int> sa(n); auto I = views::iota(0, n);
    vector<bool> t(n, true);
    for (int i = n - 2; i >= 0; --i)
        t[i] = (
            s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
    auto is_lms = views::filter([&t](int x) {
        return x && !t[x] && !t[x - 1];
    });
    auto induce = [&] {
        for (auto x = c; int y : sa)
            if (y--) if (!t[y]) sa[x[s[y] - 1]++] = y;
    };
}
```

```
for (auto x = c; int y : sa | views::reverse)
    if (y--) if (t[y]) sa[--x[s[y]]] = y;
};

vector<int> lms, q(n); lms.reserve(n);
for (auto x = c; int i : I | is_lms)
    q[i] = SZ(lms), lms.pb(sa[--x[s[i]]] = i);
induce(); vector<int> ns(SZ(lms));
for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
        int len = min({n - i, n - j, lms[q[i] + 1] - i});
        ns[q[i]] = nz += lexicographical_compare(
            begin(s) + j, begin(s) + j + len,
            begin(s) + i, begin(s) + i + len);
    }
    j = i;
}
fill(ALL(sa), 0); auto nsa = sais(ns);
for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
return induce(), sa;
}

// sa[i]: sa[i]-th suffix
// is the i-th lexicographically smallest suffix.
// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix {
    int n; vector<int> sa, hi, ra;
    Suffix
        (const auto &s, int _n) : n(_n), hi(n), ra(n) {
    vector<int> s(n + 1); // s[n] = 0;
    copy_n(_s, n, begin(s)); // _s shouldn't contain 0
    sa = sais(s); sa.erase(sa.begin());
    for (int i = 0; i < n; ++i) ra[sa[i]] = i;
    for (int i = 0, h = 0; i < n; ++i) {
        if (!ra[i]) { h = 0; continue; }
        for (int j = sa[ra[i] - 1]; max
            (i, j) + h < n && s[i + h] == s[j + h]); ++h;
        hi[ra[i]] = h ? h-- : 0;
    }
}
};
```

5.5 Aho-Corasick Automatan* [794a77]

```
struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], ord[len], top;
    int rnx[len][sigma]; // node actually be reached
    int newnode() {
        fill_n(nx[top], sigma, -1);
        return top++;
    }
    void init() { top = 1, newnode(); }
    int input(string &s) {
        int X = 1;
        for (char c : s) {
            if (!~nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
            X = nx[X][c - 'A'];
        }
        return X; // return the end node of string
    }
    void make_fl() {
        queue<int> q;
        q.push(1), fl[1] = 0;
        for (int t = 0; !q.empty(); ) {
            int R = q.front();
            q.pop(), ord[t++] = R;
            for (int i = 0; i < sigma; ++i)
                if (~nx[R][i]) {
                    int X = rnx[R][i] = nx[R][i], Z = fl[R];
                    for (; Z && !~nx[Z][i]; ) Z = fl[Z];
                    fl[X] = Z ? nx[Z][i] : 1, q.push(X);
                }
            else rnx[R][i] = R > 1 ? rnx[fl[R]][i] : 1;
        }
    }
    void solve() {
        for (int i = top - 2; i > 0; --i)
            cnt[fl[ord[i]]] += cnt[ord[i]];
    }
} ac;
```

5.6 Extended SAM* [64c3b7]

```
struct exSAM {
    int len[N * 2], link[N * 2]; // maxlen, suflink
    int next[N * 2][CNUM], tot; // [0, tot), root = 0
    int lenSorted[N * 2]; // topo. order
    int cnt[N * 2]; // occurrence
    exSAM
```

```

int newnode() {
    fill_n(next[tot], CNUM, 0);
    len[tot] = cnt[tot] = link[tot] = 0;
    return tot++;
}
void init() { tot = 0, newnode(), link[0] = -1; }
int insertSAM(int last, int c) {
    int cur = next[last][c];
    len[cur] = len[last] + 1;
    int p = link[last];
    while (p != -1 && !next[p][c])
        next[p][c] = cur, p = link[p];
    if (p == -1) return link[cur] = 0, cur;
    int q = next[p][c];
    if (len[p] + 1 == len[q]) return link[cur] = q, cur;
    int clone = newnode();
    for (int i = 0; i < CNUM; ++i)
        next[clone][i] = len[next[q][i]] ? next[q][i] : 0;
    len[clone] = len[p] + 1;
    while (p != -1 && next[p][c] == q)
        next[p][c] = clone, p = link[p];
    link[link[cur] = clone] = link[q];
    link[q] = clone;
    return cur;
}
void insert(const string &s) {
    int cur = 0;
    for (auto ch : s) {
        int &nxt = next[cur][int(ch - 'a')];
        if (!nxt) nxt = newnode();
        cnt[cur = nxt] += 1;
    }
}
void build() {
    queue<int> q;
    q.push(0);
    while (!q.empty()) {
        int cur = q.front();
        q.pop();
        for (int i = 0; i < CNUM; ++i)
            if (next[cur][i])
                q.push(insertSAM(cur, i));
    }
    vector<int> lc(tot);
    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(ALL(lc), lc.begin());
    for (int i = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
}
void solve() {
    for (int i = tot - 2; i >= 0; --i)
        cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
}

```

5.7 Main Lorentz [e8313e]

```

vector<pair<int,int>>rep[300005];//[l,r]
void main_lorentz(const string &s, int sft = 0){
    const int n=s.size();
    if(n==1) return;
    const int nu=n/2, nv=n-nu;
    const string u=s.substr(0,nu), v=s.substr(nu), ru(u
        .rbegin(), u.rend()), rv(v.rbegin(), v.rend());
    main_lorentz(u, sft), main_lorentz(v, sft+nu);
    const auto z1=Zalgo(ru), z2=Zalgo(v+'#'+u);
    const auto z3=Zalgo(ru+'#'+rv), z4=Zalgo(v);
    auto get_z = [](){const vector<int> &z, int i){
        return (0<=i && i<(int)z.size()) ? z[i]:0;
    };
    auto add_rep
        = [&](bool left, int c, int l, int k1, int k2){
            const int L=max(1LL, l-k2), R=min(l-left, k1);
            if(L>R) return;
            if(left)rep[l].eb(sft+c-R, sft+c-L);
            else rep[l].eb(sft+c-R-l+1, sft+c-L-l+1);
        };
    for(int cntr=0;cntr<n;cntr++){
        int l, k1, k2;
        if(cntr < nu){
            l=nu-cntr;
            k1=get_z(z1, nu-cntr);
            k2=get_z(z2, nv+1+cntr);
        }
    }
}

```

```

    l=cntr-nu+1;
    k1=get_z(z3, nu+1+nv-1-(cntr-nu));
    k2=get_z(z4, (cntr-nu)+1);
}
if(k1+k2>=l) add_rep(cntr<nu, cntr, l, k1, k2);
}

6 Math
6.1 ax+by=gcd(only exgcd *) [7b833d]

pll exgcd(ll a, ll b) {
    if (b == 0) return pll(1, 0);
    ll p = a / b;
    pll q = exgcd(b, a % b);
    return pll(q.Y, q.X - q.Y * p);
}
/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */

```

6.2 Floor and Ceil [692c04]

```

int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }

```

6.3 Mod Min [9118e1]

```

// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
ll mod_min(ll a, ll m, ll l, ll r) {
    if (a == 0) return l ? -1 : 0;
    if (ll k = (l + a - 1) / a; k * a <= r)
        return k;
    ll b = m / a, c = m % a;
    if (ll y = mod_min(c, a, a - r % a, a - l % a))
        return (l + y * c + a - 1) / a + y * b;
    return -1;
}

```

6.4 Miller Rabin* [da9a82]

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : primes <= 13
// n < 2^64                  7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
bool prime(ll n){ // SCOPE HASH
    vector<ll> tmp = {2,
        325, 9375, 28178, 450775, 9780504, 1795265022};
    for(ll i : tmp)
        if(!Miller_Rabin(i, n)) return false;
    return true;
}

```

6.5 Pollard Rho* [cfe72f]

```

map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2
        == 0) return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
    #define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
    }
}

```

```
    }
    if (d == n) ++p;
    x = f(x, n, p), y = f(f(y, n, p), n, p);
    d = gcd(abs(x - y), n);
}
}
```

6.6 chineseRemainder [a53b6d]

```

ll solve(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    ll p = exgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}

```

6.7 Factorial without prime factor* [c324f3]

```

// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k

```

6.8 QuadraticResidue* [e0bf30]

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >= 1) {
        if (e & 1) {
            tmp = (1LL *
                    g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL
                * f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

6.9 PiCount* [cad6d4]

```

ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<ll> larges(s);

```

```

for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
for (int i = 0; i < s; ++i) {
    roughs[i] = 2 * i + 1;
    larges[i] = (n / (2 * i + 1) + 1) / 2;
}
for (int p = 3; p <= v; ++p) {
    if (smalls[p] > smalls[p - 1]) {
        int q = p * p;
        ++pc;
        if (1LL * q * q > n) break;
        skip[p] = 1;
        for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
        int ns = 0;
        for (int k = 0; k < s; ++k) {
            int i = roughs[k];
            if (skip[i]) continue;
            ll d = 1LL * i * p;
            larges[ns] = larges[k] - (d <= v ? larges
                [smalls[d] - pc] : smalls[n / d]) + pc;
            roughs[ns++] = i;
        }
        s = ns;
        for (int j = v / p; j >= p; --j) {
            int c =
                smalls[j] - pc, e = min(j * p + p, v + 1);
            for (int i = j * p; i < e; ++i) smalls[i] -= c;
        }
    }
}
for (int k = 1; k < s; ++k) {
    const ll m = n / roughs[k];
    ll t = larges[k] - (pc + k - 1);
    for (int l = 1; l < k; ++l) {
        int p = roughs[l];
        if (1LL * p * p > m) break;
        t -= smalls[m / p] - (pc + l - 1);
    }
    larges[0] -= t;
}
return larges[0];

```

6.10 Discrete Log* [da27bf]

```

int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}
int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p;
}

```

6.11 Primes

```
/* 12721 13331 14341 75577 123457 222557  
    556679 999983 1097774749 1076767633 100102021  
    999997771 1001010013 1000512343 987654361 999991231  
    999888733 98789101 987777733 999991921 1010101333  
    1010102101 1000000000039 100000000000037  
    2305843009213693951 4611686018427387847  
    9223372036854775783 18446744073709551557 */
```

6.12 Theorem

$$\begin{aligned}\sum k^1 &= -\frac{1}{12} \\ \sum k^4 &= \frac{1}{30}[n(n+1)(2n+1)(3n^2+3n-1)] \\ \sum k^5 &= \frac{1}{12}[n^2(n+1)^2(2n^2+2n-1)] \\ \sum k^6 &= \frac{1}{42}[n(n+1)(2n+1)(3n^4+6n^3-3n+1)]\end{aligned}$$

- Cramer's rule

$$\begin{aligned}ax+by=e &\quad x = \frac{ed-bf}{ad-bc} \\ cx+dy=f &\Rightarrow y = \frac{af-ec}{ad-bc}\end{aligned}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G , where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G .

- The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
- The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{\text{rank}(D)}{2}$ is the maximum matching on G .

- Cayley's Formula

- Given a degree sequence d_1, d_2, \dots, d_n for each labeled vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
- Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \dots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erdős–Gallai theorem

A sequence of nonnegative integers $d_1 \geq \dots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if

$$d_1 + \dots + d_n \text{ is even and } \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } 1 \leq k \leq n.$$

- Gale–Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \dots \geq a_n$ and b_1, \dots, b_n

$$\text{is bigraphic if and only if } \sum_{i=1}^n a_i = \sum_{i=1}^n b_i \text{ and } \sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$$

- Fulkerson–Chen–Anstee theorem

A sequence $(a_1, b_1), \dots, (a_n, b_n)$ of nonnegative integer pairs

$$\text{with } a_1 \geq \dots \geq a_n \text{ is digraphic if and only if } \sum_{i=1}^n a_i = \sum_{i=1}^n b_i \text{ and } \sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$$

- Pick's theorem

For simple polygon, when points are all integer, we have $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$.

- Möbius inversion formula

- $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right)$
- $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d)$

- Spherical cap

- A portion of a sphere cut off by a plane.
- r : sphere radius, a : radius of the base of the cap, h : height of the cap, θ : $\arcsin(a/r)$.
- Volume = $\pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
- Area = $2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

- Lagrange multiplier

- Optimize $f(x_1, \dots, x_n)$ when k constraints $g_i(x_1, \dots, x_n) = 0$.
- Lagrangian function $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$.
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

- Nearest points of two skew lines

- Line 1: $v_1 = p_1 + t_1 d_1$
- Line 2: $v_2 = p_2 + t_2 d_2$
- $\mathbf{n} = d_1 \times d_2$
- $\mathbf{n}_1 = d_1 \times \mathbf{n}$
- $\mathbf{n}_2 = d_2 \times \mathbf{n}$
- $\mathbf{c}_1 = p_1 + \frac{(p_2-p_1) \cdot \mathbf{n}_1}{d_1 \cdot \mathbf{n}_2} d_1$

$$- c_2 = p_2 + \frac{(p_1-p_2) \cdot \mathbf{n}_1}{d_2 \cdot \mathbf{n}_2} d_2$$

- Derivatives/Integrals

$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$	$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx} \tan x = 1 + \tan^2 x$	$\int \tan ax dx = -\frac{\ln \cos ax }{a}$	$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x)$	$\int xe^{ax} dx = \frac{e^{ax}}{a^2} (ax-1)$	$\int \sqrt{a^2+x^2} = \frac{1}{2} \left(x\sqrt{a^2+x^2} + a^2 \operatorname{asinh}(x/a) \right)$

- Spherical Coordinate

$$(x, y, z) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$$

$$(r, \theta, \phi) = (\sqrt{x^2+y^2+z^2}, \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}), \operatorname{atan2}(y, x))$$

- Rotation Matrix

$$M(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{bmatrix}$$

6.13 Estimation

n	2	3	4	5	6	7	8	9	20	30	40	50	100
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8
n	100	1e3	1e6	1e9	1e12	1e15	1e18						
$d(i)$	12	32	240	1344	6720	26880	103680						
n	1	2	3	4	5	6	7	8	9	10	11	12	13
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7
n	2	3	4	5	6	7	8	9	10	11	12	13	15
B_n	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7	

6.14 General Purpose Numbers

- Bernoulli numbers

$$B_0 = 1, B_1^\pm = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1-x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k : s.t. $\pi(j) > \pi(j+1)$, $k+1$: s.t. $\pi(j) \geq j$, k : s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.15 Tips for Generating Functions

- Ordinary Generating Function $A(x) = \sum_{i \geq 0} a_i x^i$

- $A(rx) \Rightarrow r^n a_n$
- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
- $xA(x)' \Rightarrow na_n$
- $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$

- Exponential Generating Function $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$

- $A(x) + B(x) \Rightarrow a_n + b_n$
- $A^{(k)}(x) \Rightarrow a_{n+k}$
- $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
- $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
- $xA(x) \Rightarrow na_n$

- Special Generating Function

- $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
- $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i}{n-1} x^i$

7 Polynomial

7.1 Fast Fourier Transform [56bdd7]

```
template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans
        (val_t *a, int n, bool inv = false); // see NTT;
    // remember to replace LL with val_t
};
```

7.2 Number Theory Transform* [f68103]

```
//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
    ll w[MAXN];
    ll mpow(ll a, ll n);
    ll minv(ll a) { return mpow(a, P - 2); }
    NTT() {
        ll dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(ll *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^= k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()
        (ll *a, int n, bool inv = false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = a[j + dl] * w[x] % P;
                    if ((a[j] + dl) = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
            if (inv) {
                reverse(a + 1, a + n);
                ll invn = minv(n);
                for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
            }
        }
    }
};
```

7.3 Polynomial Operation [105808]

```
#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
    using vector<ll>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<ll>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev()
        { return reverse(data(), data() + n()), *this; }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if
            (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
    }
```

```
    return *this;
}
Poly& imul(ll k) {
    fi(0, n()) (*this)[i] = (*this)[i] * k % P;
    return *this;
}
Poly Mul(const Poly &rhs) const {
    int m = 1;
    while (m < n() + rhs.n() - 1) m <= 1;
    Poly X(*this, m), Y(rhs, m);
    ntt(X.data(), m), ntt(Y.data(), m);
    fi(0, m) X[i] = X[i] * Y[i] % P;
    ntt(X.data(), m, true);
    return X.isz(n() + rhs.n() - 1);
}
Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
    if (n() == 1) return {ntt.minv((*this)[0])};
    int m = 1;
    while (m < n() * 2) m <= 1;
    Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
    Poly Y(*this, m);
    ntt(Xi.data(), m), ntt(Y.data(), m);
    fi(0, m) {
        Xi[i] *= (2 - Xi[i] * Y[i]) % P;
        if ((Xi[i] %= P) < 0) Xi[i] += P;
    }
    ntt(Xi.data(), m, true);
    return Xi.isz(n());
}
Poly Sqrt()
    const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
    if (n()
        == 1) return {QuadraticResidue((*this)[0], P)};
    Poly
        X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
    return
        X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
}
pair<Poly, Poly> DivMod
    (const Poly &rhs) const { // (rhs.)back() != 0
    if (n() < rhs.n()) return {{0}, *this};
    const int m = n() - rhs.n() + 1;
    Poly X(rhs); X.irev().isz(m);
    Poly Y(*this); Y.irev().isz(m);
    Poly Q = Y.Mul(X.Inv()).isz(m).irev();
    X = rhs.Mul(Q), Y = *this;
    fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
    return {Q, Y.isz(max(1, rhs.n() - 1))};
}
Poly Dx() const {
    Poly ret(n() - 1);
    fi(0,
        ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n())
        ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<ll> _eval(const
    vector<ll> &x, const vector<Poly> &up) const {
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m *
    2) down[i] = down[i / 2].DivMod(up[i]).second;
    down[1] = Poly(up[1])
        .irev().isz(n()).Inv().irev()._tmul(m, *this);
    fi(2, m * 2) down[i]
        = up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
    vector<ll> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<ll> &x) {
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = m - 1; i;
```

```

        > 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
    return up;
}
vector<ll> Eval(const vector<ll> &x) const { // 1e5, 1s
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector<ll> &x, const vector<ll> &y) { // 1e5, 1.4s
    const int m = (int)x.size();
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<ll> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m - 1; i > 0; --i) down[i] = down[i * 2].Mul(up[i * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n())
        if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (ll)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly {1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const ll c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul((k % P).Exp().imul(c).irev().isz(n()).irev());
}
static ll
LinearRecursion(const vector<ll> &a, const vector<ll> &coef, ll n) { // a_n = |sum c_j a_(n-j)
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly {1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;
        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    ll ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
#endif
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

7.4 Newton's Method

Given $F(x)$ where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for β being some constant. Polynomial P such that $F(P) = 0$ can be found iteratively. Denote by Q_k the polynomial such that $F(Q_k) = 0 \pmod{x^{2^k}}$, then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

8 Geometry

8.1 Default Code [2e3f42]

```

using ld = ll;
using pdd = pair<ld, ld>;
using Line = pair<pdd, pdd>;
struct Cir{ pdd O; ld R; };
#define X first
#define Y second
// const ld eps = 1e-7;

```

```

pdd operator+(pdd a, pdd b)
{
    return {a.X + b.X, a.Y + b.Y};
}
pdd operator-(pdd a, pdd b)
{
    return {a.X - b.X, a.Y - b.Y};
}
pdd operator*(ld i, pdd v)
{
    return {i * v.X, i * v.Y};
}
pdd operator*(pdd v, ld i)
{
    return {i * v.X, i * v.Y};
}
pdd operator/(pdd v, ld i)
{
    return {v.X / i, v.Y / i};
}
ld dot(pdd a, pdd b)
{
    return a.X * b.X + a.Y * b.Y;
}
ld cross(pdd a, pdd b)
{
    return a.X * b.Y - a.Y * b.X;
}
ld abs2(pdd v)
{
    return dot(v, v);
}
ld abs(pdd v)
{
    return sqrt(dot(v, v));
}
int sgn(ld v)
{
    return v > 0 ? 1 : (v < 0 ? -1 : 0);
}
// int sgn(
//     ld v){ return v > eps ? 1 : ( v < -eps ? -1 : 0); }
int ori(pdd a, pdd b, pdd c)
{
    return sgn(cross(b - a, c - a));
}
bool collinearity(pdd a, pdd b, pdd c)
{
    return ori(a, b, c) == 0;
}
bool btw(pdd p, pdd a, pdd b)
{
    return collinearity
        (p, a, b) && sgn(dot(a - p, b - p)) <= 0;
}

bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4){
    if(btw(p1, p3, p4) || btw(p2
        , p3, p4) || btw(p3, p1, p2) || btw(p4, p1, p2))
        return true;
    return ori(p1, p2, p3) * ori(p1, p2, p4) < 0 &&
        ori(p3, p4, p1) * ori(p3, p4, p2) < 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4){
    ld a123 = cross(p2 - p1, p3 - p1);
    ld a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123 - p3 * a124) / (a123 - a124);
}
pdd perp(pdd p1)
{
    return pdd(-p1.Y, p1.X);
}
pdd projection(pdd p1, pdd p2, pdd p3)
{
    return p1 +
        p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1);
}
pdd reflection(pdd p1, pdd p2, pdd p3)
{
    return p3 + perp(p2 - p1
        ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2;
}
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0
        , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(
        cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r

```

8.2 PointSegDist* [57b6de]

```

double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(dot(q1 - q0,
        p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

8.3 Heart [468aeb]

```

pair<pdd
    , ld> circenter(pdd a, pdd b, pdd c){ // SCOPE HASH
    pdd m1 = (a+b)/2, m2 = (b+c)/2;
    pdd cent = intersect
        (m1, m1 + perp(b-a), m2, m2 + perp(c-b));
    return {cent, abs(a-cent)};
}
pair<pdd, ld> incenter
    (pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
    ld a =
        abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    pdd cent = (a * p1 + b * p2 + c * p3) / (a + b + c);
    return {cent, abs(a-cent)};
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{
    return (p1 + p2 + p3) / 3;
}
pdd orthcenter(pdd p1, pdd p2, pdd p3)

```

```
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }
```

8.4 point in circle [02a7ca]

```
// return q'
  s relation with circumcircle of tri(p[0],p[1],p[2])
bool inCC(const array<pll, 3> &p, pll q) {
    _int128 det = 0;
    for (int i = 0; i < 3; ++i)
        det += _int128(abs2(p[i]) - abs2(q)) *
            cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}
```

8.5 Convex hull* [9b329e]

```
vector<int> getConvexHull(vector<pdd>& pts){
    vector<int> id(SZ(pts));
    iota(all(id), 0);
    sort(all(id)
        ), [&](int x, int y){ return pts[x] < pts[y]; });
    vector<int> hull;
    for(int tt = 0; tt < 2; tt++){
        int sz = SZ(hull);
        for(int j : id){
            pdd p = pts[j];
            while(SZ(hull) - sz >= 2 &&
                cross(pts
                    [hull.back()] - pts[hull[SZ(hull) - 2]],
                    p - pts[hull[SZ(hull) - 2]]) <= 0)
                hull.pop_back();
            hull.pb(j);
        }
        hull.pop_back();
        reverse(all(id));
    }
    return hull;
}
```

8.6 PointInConvex* [82b81e]

```
bool PointInConvex
    (const vector<pll> &C, pll p, bool strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if (SZ(C) == 0) return false;
    if (SZ(C) < 3) return r && btwangle(p, C[0], C.back());
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori
        (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}
```

8.7 TangentPointToHull* [523bc1]

```
/* The point should be strictly out of hull
   return arbitrary point on the tangent line */
pii get_tangent(vector<pll> &C, pll p) {
    auto gao = [&](int s) {
        return cyc_tsearch(SZ(C), [&](int x, int y)
        { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0
```

8.8 Intersection of line and convex [157258]

```
int TangentDir(vector<pll> &C, pll dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
#define cmpL(i) sign(cross(C[i] - a, b - a))
pii lineHull(pll a, pll b, vector<pll> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if (cmpL(A) < 0 || cmpL(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for (int t = l; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            if (cmpL(m) == cmpL(t) ? l : r) = m;
        }
        return (l + !cmpL(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if (res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if (!
        cmpL(res.X) && !cmpL(res.Y)) // along side i, i+1
        switch ((res.X - res.Y + n + 1) % n) {
            case 0: return pii(res.X, res.X);
            case 2: return pii(res.Y, res.Y);
        }
    /* crossing sides (i, i+1) and (j, j+1)
       crossing corner i is treated as side (i, i+1)
       returned
           in the same order as the line hits the convex */
    return res;
} // convex cut: (r, l)
```

```
(cmpL(m) == cmpL(t) ? l : r) = m;
}
return (l + !cmpL(r)) % n;
};
pii res = pii(gao(B, A), gao(A, B)); // (i, j)
if (res.X == res.Y) // touching the corner i
    return pii(res.X, -1);
if (!
    cmpL(res.X) && !cmpL(res.Y)) // along side i, i+1
    switch ((res.X - res.Y + n + 1) % n) {
        case 0: return pii(res.X, res.X);
        case 2: return pii(res.Y, res.Y);
    }
/* crossing sides (i, i+1) and (j, j+1)
   crossing corner i is treated as side (i, i+1)
   returned
       in the same order as the line hits the convex */
return res;
} // convex cut: (r, l)
```

8.9 minMaxEnclosingRectangle* [180fb8]

```
const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pll> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
    hull(dots);
    double Max = 0, Min = INF, deg;
    int n = SZ(dots);
    dots.pb(dots[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        pll nw = vec(i + 1);
        while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
            u = (u + 1) % n;
        while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
            r = (r + 1) % n;
        if (!i) l = (r + 1) % n;
        while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
            l = (l + 1) % n;
        Min = min(Min, (double)(dot(nw, vec(r)) - dot
            (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
        deg = acos(dot(diff(r
            , l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
        deg = (qi - deg) / 2;
        Max = max(Max, abs(diff
            (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
    }
    return pdd(Min, Max);
}
```

8.10 VectorInPoly* [c6d0fa]

```
// ori(a
    , b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(pll a, pll b, pll c, pll p, int strict) {
    return
        ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector
{cur, p} in counter-clockwise order prv, cur, nxt
bool inside
(pll prv, pll cur, pll nxt, pll p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}
```

8.11 PolyUnion* [434d93]

```
ld rat(pll a, pll b) {
    return sgn(b.X) ? (ld)a.X / b.X : (ld)a.Y / b.Y;
} // all poly. should be ccw
ld polyUnion(vector<vector<pll>> &poly) {
    ld res = 0;
    for (auto &p : poly)
        for (int a = 0; a < SZ(p); ++a) {
            pll A = p[a], B = p[(a + 1) % SZ(p)];
            vector<pair<ld, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < SZ(q); ++b) {
                    pll C = q[b], D = q[(b + 1) % SZ(q)];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        ld sa = cross(D
                            - C, A - C), sb = cross(D - C, B - C);
                        segs.eb(sa / (sa - sb), sgn(sc - sd));
                    }
                }
            }
        }
    }
}
```

```

    }
    if (!sc && !sd &&
        &q < &p && sgn(dot(B - A, D - C)) > 0) {
        segs.eb(rat(C - A, B - A), 1);
        segs.eb(rat(D - A, B - A), -1);
    }
}
sort(all(segs));
for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
ld sum = 0;
int cnt = segs[0].second;
for (int j = 1; j < SZ(segs); ++j) {
    if (!cnt) sum += segs[j].X - segs[j - 1].X;
    cnt += segs[j].Y;
}
res += cross(A, B) * sum;
}
return res / 2;
}

```

8.12 Polar Angle Sort* [b20533]

```

// -1: a // b (if same), 0/1: a < b
int cmp(pll a, pll b, bool same = true) {
#define is_neg(k) (
    sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
int A = is_neg(a), B = is_neg(b);
if (A != B) return A < B;
if (sign(cross(a, b)) == 0)
    return same ? abs2(a) < abs2(b) : -1;
return sign(cross(a, b)) > 0;
}

```

8.13 Half plane intersection* [e13bb6]

```

pll area_pair(Line a, Line b)
{ return pll(cross(a.Y - a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a02X < 0) a12X *= -1, a12Y *= -1;
    return (_int128)
        a02Y * a12X - (_int128) a02X * a12Y > 0; // C^4
}
/* Having solution, check size > 2 */
/* --^- Line.X --^- Line.Y --^- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(all(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    for (auto p : arr) {
        if (cmp(
            dq.back().Y - dq.back().X, p.Y - p.X, 0) == -1)
            continue;
        while (SZ(dq) >= 2 && !isin(p, dq[SZ(dq) - 2], dq.back()))
            dq.pop_back();
        while (SZ(dq) >= 2 && !isin(p, dq[0], dq[1]))
            dq.pop_front();
        dq.pb(p);
    }
    while (SZ(dq) >= 3 && !isin(dq[0], dq[SZ(dq) - 2], dq.back()))
        dq.pop_back();
    while (SZ(dq) >= 3 && !isin(dq.back(), dq[0], dq[1]))
        dq.pop_front();
    return vector<Line>(all(dq));
}

```

8.14 RotatingSweepLine [37eaad]

```

struct Event {
    pll d; int u, v;
    bool operator<(const Event &b) const {
        int ret = cmp(d, b.d, false);
        return ret == -1 ? false : ret; } // no tie-break
};
void rotatingSweepLine(const vector<pll> &p) {
    const int n = SZ(p);
    vector<Event> e; e.reserve(n * (n - 1));
    for (int i = 0; i < n; i++)

```

```

        for (int j = 0; j < n; j++)
            ; j++) // pos[i] < pos[j] when the event occurs
        if (i != j) e.pb(p[j] - p[i], i, j);
    sort(all(e));
    vector<int> ord(n), pos(n);
    iota(all(ord), 0);
    sort(all(ord), [&](int i, int j) { // initial order
        return p[i].Y != p[j].Y ? p[i].Y < p[j].Y : p[i].X < p[j].X; });
    for (int i = 0; i < n; i++) pos[ord[i]] = i;
    // initialize
    for (int i = 0, j = 0; i < SZ(e); i = j) {
        // do something
        vector<pii> tmp;
        for (j < SZ(e) && !(e[i] < e[j]); j++)
            tmp.pb(pii(e[j].u, e[j].v));
        sort(all(tmp), [&](pii x, pii y){
            return pii(pos[x.fi],
                pos[x.se]) < pii(pos[y.fi], pos[y.se]); });
        for (auto [x, y] : tmp) // pos[x] + 1 == pos[y]
            tie(ord[pos[x]], ord[pos[y]], pos[x], pos[y]) =
                make_tuple
                    (ord[pos[y]], ord[pos[x]], pos[y], pos[x]);
    }
}

```

8.15 Minimum Enclosing Circle* [2c7cccd]

```

using ld = long double;
mt19937 rng(950223); // O(n) expected time
pair<ppd, ld> minimumEnclosingCircle(vector<ppd> &pts) {
    shuffle(all(pts), rng);
    pdd c = pts[0];
    ld r = 0;
    for(int i = 1; i < SZ(pts); i++){
        if(abs(pts[i] - c) <= r) continue;
        c = pts[i]; r = 0;
        for(int j = 0; j < i; j++){
            if(abs(pts[j] - c) <= r) continue;
            c = (pts[i] + pts[j]) / 2;
            r = abs(pts[i] - c);
            for(int k = 0; k < j; k++){
                if(abs(pts[k] - c) > r)
                    auto [c,
                        r] = circenter(pts[i], pts[j], pts[k]);
            }
        }
    }
    return {c, r};
}

```

8.16 Intersection of two circles* [f7a2fe]

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.o, o2 = b.o;
    double r1 =
        a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
    if(d < max
        (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
    pdd u = (o1 + o2) * 0.5
        + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) *
        (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v
        = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

8.17 Intersection of polygon and circle* [d4d295]

```

// Divides into multiple triangle, and sum up
const double PI=acos(-1);
double _area(ppd pa, ppd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B
            < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){

```

```

    theta = PI - B - asin(sin(B)/r*a);
    S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
}
else S = .5*sin(C)*a*b;
return S;
}
double area_poly_circle(const
    vector<pdd> poly,const pdd &O,const double r){
double S=0;
for(int i=0;i<SZ(poly);++i)
    S+=_area(poly[i]-0,poly[(i+1)%SZ(poly)
        ]-0,r)*ori(0,poly[i],poly[(i+1)%SZ(poly)]);
return fabs(S);
}

```

8.18 Intersection of line and circle* [76e533]

```

vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
pdd p
    = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
double s = cross
    (b - a, c - a), h2 = r * r - s * s / abs2(b - a);
if (sgn(h2) < 0) return {};
if (sgn(h2) == 0) return {p};
pdd h = (b - a) / abs(b - a) * sqrt(h2);
return {p - h, p + h};
}

```

8.19 Tangent line of two circles [5ac5a5]

```

vector<Line>
    > CCtan(const Cir& c1, const Cir& c2, int sign1){
// sign1 = 1 for outer tang, -1 for inter tang
vector<Line> ret;
double d_sq = abs2(c1.0 - c2.0);
if (sgn(d_sq) == 0) return ret;
double d = sqrt(d_sq);
pdd v = (c2.0 - c1.0) / d;
double c = (c1.R - sign1 * c2.R) / d;
if (c * c > 1) return ret;
double h = sqrt(max(0.0, 1.0 - c * c));
for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
    pdd n = pdd(v.X * c - sign2 * h * v.Y,
                  v.Y * c + sign2 * h * v.X);
    pdd p1 = c1.0 + n * c1.R;
    pdd p2 = c2.0 + n * (c2.R * sign1);
    if (!sgn(p1.X - p2.X) && !sgn(p1.Y - p2.Y))
        p2 = p1 + perp(c2.0 - c1.0);
    ret.pb(Line(p1, p2));
}
return ret;
}

```

8.20 Minkowski Sum* [399d43]

```

void reorder_poly(vector<pdd>& pnts){
    int mn = 0;
    for(int i = 1; i < SZ(pnts); i++)
        if(pnts[i].Y < pnts[mn].Y || (pnts[i].Y ==
            pnts[mn].Y && pnts[i].X < pnts[mn].X)) mn = i;
    rotate(pnts.begin(), pnts.begin() + mn, pnts.end());
}
vector<pdd> minkowski(vector<pdd> P, vector<pdd> Q){
    reorder_poly(P); reorder_poly(Q);
    int psz = SZ(P), qsz = SZ(Q);
    P.pb(P[0]); P.pb(P[1]); Q.pb(Q[0]); Q.pb(Q[1]);
    vector<pdd> ans; int i = 0, j = 0;
    while(i < psz || j < qsz){
        ans.pb(P[i] + Q[j]);
        int t = sgn(cross(P[i + 1]-P[i], Q[j + 1]-Q[j]));
        if(t >= 0) i++; if(t <= 0) j++;
    }
    return ans;
}

```

9 Else

9.1 Cyclic Ternary Search* [9017cc]

```

/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false for all x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(m, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}

```

```

if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
else l = m;
}
return pred(l, r % n) ? l : r % n;
}

```

9.2 Tree Hash* [34aae5]

```

ull seed;
ull shift(ull x) {
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    return x;
}
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u])
        if (i != f)
            sum += shift(dfs(i, u));
    return sum;
}

```

10 Python

10.1 Misc

```

from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction
    ('3.14159').limit_denominator(10).numerator # 22
sys.set_int_max_str_digits(int(1e6))
a, b = map(int, input().split())
a = map(int, input().split()) # array

```