Outline of Chapter 5: Polynomial

Polynomial Algebra $\sim F[x]$ is a PID (Principal Ideal) Domain)

- 9 Division Theorem, Divisibility
- The (Not A!) Greatest Common Divisor (GCD) and Relatively Prime
- ~> Chinese Reminder Theorem (CRT) ~> Lagrange Interpolation Formula
- ~~~ (Chapter 7) Jordan-Chevalley Decomposition
- In PID, Irreducible ⇔ Prime
- (PID \Rightarrow UFD) Unique Factorization
- Repeated Factor $\rightsquigarrow (f(x), f'(x)) = 1$?

(illusion) Lecture 1

Review of §5.1 Basic Concepts

County the
$$\sum_{i+j=2p} (-i)^i \mid j \mid \lambda^{2p} = \sum_{i=0}^{2p} (\gamma)^i = 1$$
.

$$f(x) = \left\{ \sum_{k=0}^{2n} (-1)^k x^k \right\} \left\{ \sum_{k=0}^{2n} x^k \right\}$$

那么其中 x^{2p} 的系数均为 1, x^{2p-1} 的系数均为 0. $(1 \le p \le 2n^2, p \in \mathbb{N}^*)$

- $F[x] \text{ is an (integral) domain:} \qquad b_{x} \qquad a_{x} \qquad b_{x} \qquad a_{x} \qquad b_{x} \qquad a_{x} \qquad b_{x} \qquad a_{x} \qquad b_{x} \qquad$
 - $f(x),g(x),h(x)\in F[x],$ if $f(x)h(x)=g(x)h(x),\ h(x)\neq 0,$ then f(x)=g(x)h(x)

$$g(x)$$
. (14)

$$\left(\int_{-\infty}^{\infty} g(x) - g(x)\right) = 0$$
 $\int_{-\infty}^{\infty} f(x) = g(x)$

明
$$f(x), g(x), h(x) \in \mathbf{R}[x]$$
 and we have $xf^2(x) + xg^2(x) = h^2(x)$, then $f(x) = g(x) = h(x) = 0$.

Notes:

• 回顾 $f(x), g(x) \in \mathbf{R}[x], f^2(x) + q^2(x) = 0 \rightarrow f(x) = g(x) = 0$.

• 上述两个结论在 $\mathbf{C}[x]$ 上述成立吗?

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Determine all polynomials
$$f(x) \in F[x]$$
 such that $f[f(x)] = f^n(x)$, where $n \in \mathbb{N}^*$ is a given positive integer.

Hint: $f(x) \neq 0 \leadsto \deg f[f(x)] = (\deg f(x))^2$.

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Review of §5.2 Division Theorem

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Thm 3

Suppose $f(x), g(x) \in F[x]$ and $g(x) \neq 0$, then there exist unique polynomials $q(x), r(x) \in F[x]$ such that

$$f(x) = q(x)g(x) + r(x), \qquad (1)$$

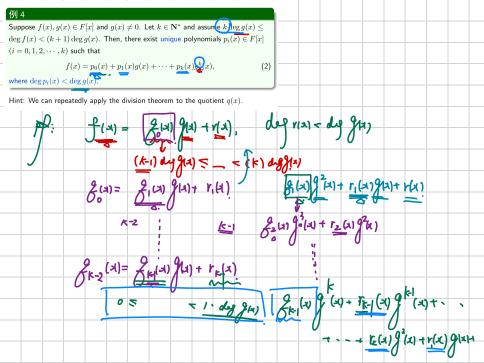
where $\deg r(x) < \deg g(x)$.

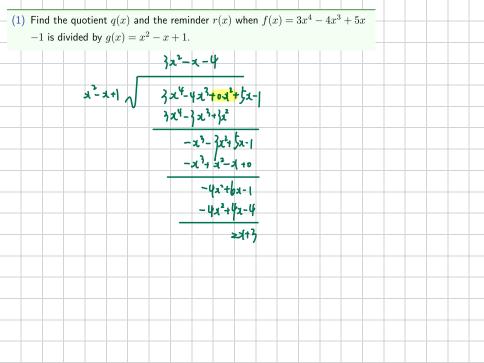
Notes:

- 若 r(x) = 0, 那么 $\deg 0 = -\infty < \deg g(x)$ 也成立;
- (带余除法与数域扩大无关) 若 $F \subseteq K$, 在 K[x] 中存在 $q(x), \tilde{r}(x) \in K[x]$
- (带余除法与数域扩大无关) 若 $F \subseteq K$,在 K[x] 中存在 $\tilde{q}(x), \tilde{r}(x) \in K[x]$ 满足 \mathfrak{P}_F

$$f(x) = \tilde{q}(x)g(x) + \tilde{r}(x) \leadsto \tilde{q}(x) = q(x), \ \tilde{r}(x) = r(x).$$

Thm 3 Suppose $f(x), g(x) \in F[x]$ and $g(x) \neq 0$, then there exist unique polynomials $q(x), r(x) \in F[x]$ such that f(x) = q(x)q(x) + r(x),(1)where $\deg r(x) < \deg g(x)$. **(**k) 6



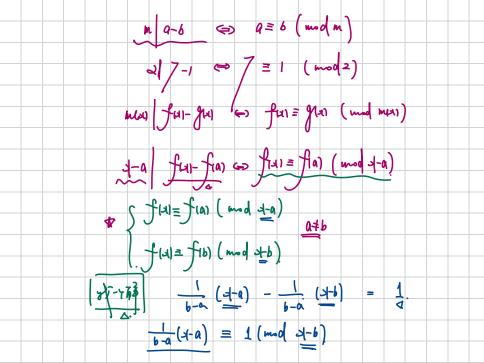


(2) Prove that when
$$f(x) \in F[x]$$
 is divided by $(x-a)(x-b)$, the reminder $r(x)$ is
$$\frac{f(a)-f(b)}{a-b}x+\frac{af(b)-bf(a)}{a-b}.$$

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$$\int (x) = \int (a) \left(a - b \right) = 1 \quad (mod + a).$$

$$\int (x) = \int (a) \left(a - b \right) = 1 \quad (x - a) - (b)$$

$$\int (x) = \int (a) \left(a - b \right) = 1 \quad (x - b) = 1 \quad ($$

Review of §5.2 Divisibility

Thm 6

Given conditions in Division Theorem, if we have r(x) = 0, i.e., f(x) = q(x)g(x), we say that g(x) divides f(x) (or g(x) is a divisor of f(x)), and we denote this as $g(x) \mid f(x)$. Otherwise, we write $g(x) \nmid f(x)$.

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● 整除有自反性和传递性,以及相伴性(associate):

$$f(x) = \tilde{q}(x)g(x) \leadsto \tilde{q}(x) = q(x).$$

(1)
$$f(x), g(x) \in F[x], f(x^2) \mid g(x^2) \Rightarrow f(x) \mid g(x);$$
(2) $f(x), g(x) \in F[x], f^2(x) \mid g^2(x) \Rightarrow f(x) \mid g(x);$
(3) Given $a \neq 0, d, n \in \mathbb{N}^*, x^m - a^m \mid x^n - a^n \Leftrightarrow m \mid n.$

(1) $f(x), g(x) \in F[x], f^2(x) \mid g^2(x) \Rightarrow f(x) \mid g(x);$
(3) $f(x) = f(x), f(x) = f(x), f(x) = f(x)$

(1) $f(x), g(x) \in F[x], f(x) \mid g(x) \Rightarrow f(x) \mid g(x);$
(3) $f(x) = f(x), f(x) = f(x)$

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(29)

$$\Rightarrow x^{2} | x \Rightarrow y \Rightarrow x^{2} | x \Rightarrow y \Rightarrow x^{2} \Rightarrow x^$$

1)
$$[m \mid n] \Rightarrow n = km$$

$$x = (x^{m} - a)^{m} = (x^{m})^{k} - (a^{m})^{k}$$

$$x = (x^{m} - a)^{m} = (x^{m})^{k} - (a^{m})^{k}$$

$$x = (x^{m} - a)^{m} = (x^{m})^{k} + (x^{m})^{k} + (x^{m})^{k} + (x^{m})^{k}$$

$$x = (x^{m} - a)^{m} = x^{m} - a^{m} + x^{m} +$$

Examples

Slogan:
$$g(x) \mid f_k(x) \leadsto g(x) \mid \sum_k h_k(x) f_k(x)$$
, For all $h_k(x) \in F[x]$.

例 8

- (1) $x^2 + x + 1 \mid x^{3n} + x^{3m+1} + x^{3p+2}$, For all $n, m, p \in \mathbb{N}^*$;
- (2) If $m, n, p \in \mathbb{N}^*$ have the same parity, prove that $x^2 x + 1 \mid x^{3n} x^{3m+1}$ $+x^{3p+2}$. Check the converse of this proposition is also true.

Notes:

$$\int [w_i] = 0 \Rightarrow |-w_i| \int [x_i] \frac{w_i + w_2}{2} (+w_1, +w_2) = 1$$

- (§5.5 Polynomial Functions) Alternative: $x^2+x+1=(x-\omega_1)(x-\omega_2)=0,$ $\omega_i^3=1, \omega_i\neq 1 \Longrightarrow \omega_i^{3n}+\omega_i^{3m+1}+\omega_i^{3p+2}=1+\omega_i^1+\omega_i^2=0.$ When we have $x^2+x+1\left|\sum_i x^{a_i}\left(a_i\in\mathbb{N}^*\right)\right.$?

(1)
$$x^2 + x + 1$$
 | $x^{3n} + x^{3m+1} + x^{3p+2}$, For all $n, m, p \in \mathbb{N}^*$; | $x^3 + x + 1$ | $x^{3n} + x^{3m+1} + x^{3p+2}$? | $x^3 + x + 1$ | $x^{3n} + x^{3m+1} + x^{3p+2}$? | $x^{3n} + x^{3m+1} + x^{3m+1}$

$$\Rightarrow |\lambda|^{2} + |\lambda| + |\lambda|^{2}$$

$$def(|\lambda|^{2} + |\lambda|^{2}) = |\partial f(|\lambda|^{2} + |\lambda|^{2}) = 2.$$

$$\Rightarrow |\lambda|^{2} + |\lambda|^{2}$$

(2) If
$$m, n, p \in \mathbb{N}^*$$
 have the same parity, prove that $x^2 - x + 1 \mid x^{3n} - x^{3m+1} + x^{3p+2}$. Check the converse of this proposition is also true.

+ス3(トメセン)

$$|A| = |A| + |A|$$