

9 每日练习 9 (Due: 2025/1/22 22:00)

1. 已知 $\alpha, \beta \in (0, \frac{\pi}{2})$, $\sin 2\alpha = m \sin 2\beta$, $\tan(\alpha + \beta) = n \tan(\alpha - \beta)$, 则

A. $m = \frac{1-n}{1+n}$

B. $m = \frac{1+n}{1-n}$

C. $n = \frac{m-1}{m+1}$

D. $n = \frac{m+1}{m-1}$

2. (多选) 设 A, B 是一个随机试验中的两个事件, 若 $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(AB) = \frac{1}{4}$, 则

A. $P(\bar{B}) = \frac{2}{3}$

B. $P(B|A) = \frac{1}{2}$

C. $P(\bar{A}B) = \frac{1}{12}$

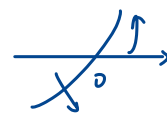
D. $P(A \cup B) = \frac{5}{6}$

3. 已知 F_1, F_2 分别为双曲线 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ($a > 0, b > 0$) 的左、右焦点, 过 F_2 的直线 l 与圆 $O: x^2 + y^2 = a^2$ 相切于点 M , 若 $|MF_1| = 3|OM|$, 则双曲线的渐近线方程为 _____.

4. 已知函数 $f(x) = (a+2)e^x + ae^{-x} - 2x$ ($a \in \mathbb{R}$).

(1) 若 $a = 0$, 求 $f(x)$ 的极值;

(2) 讨论 $f(x)$ 的单调性.

(1) \bullet $a=0, f(x) = 2e^x - 2x \quad f'(x) = 2e^x - 2$ 

$f_{\min} = f(0) = 2$


(2) \bullet $f'(x) = (a+2)e^x - ae^{-x} - 2 = \frac{(a+2)(e^x)^2 - 2e^x - a}{e^x}$ $\begin{matrix} a+2 & & +a \\ & \times & \\ 1 & & -1 \end{matrix}$


$\left[(a+2)e^x + a \right] (e^x - 1) = (a+2) \left[e^x + \frac{a}{a+2} \right] (e^x - 1)$

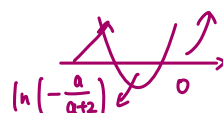
$= \frac{\left[(a+2)e^x + a \right] (e^x - 1)}{e^x}$

$\underbrace{-\frac{a}{a+2} \leq 0}_{\Delta}$

即 $\frac{a}{a+2} \geq 0 \Rightarrow a \leq -2$ 或 $a > 0$

(1) $a = -2$  或 $a > 0$

(2) $a < -2$  11

(3) $-2 < a < 0$, $-\frac{a}{a+2} < 1$ 

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B. $m = \frac{1+n}{1-n}$

C. $n = \frac{m-1}{m+1}$

D. $n = \frac{m+1}{m-1}$ ✓

D

$$\frac{\sin 2\alpha}{\sin 2\beta} = \frac{\sin \alpha \cos \alpha}{\sin \beta \cos \beta} = m$$

$$\frac{\sin(\alpha+\beta) \cos(\alpha-\beta)}{\cos(\alpha+\beta) \sin(\alpha-\beta)} = n = \frac{\sin 2\alpha + \sin 2\beta}{\sin 2\alpha - \sin 2\beta} = \frac{m+1}{m-1}$$

和差化积

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

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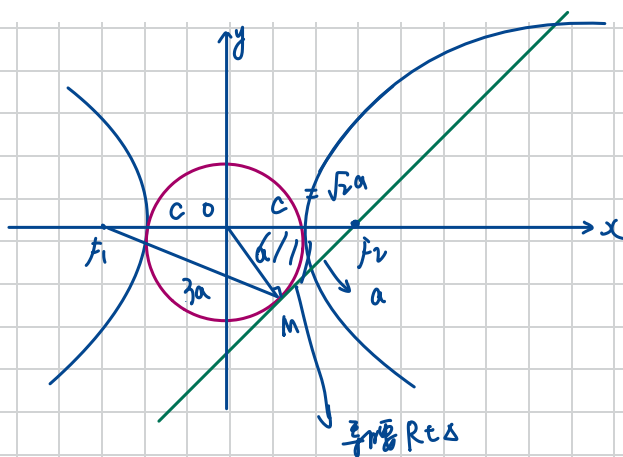
D. $P(A \cup B) = \frac{5}{6}$ X

$\frac{\frac{1}{4}}{\frac{1}{2}}$

$P(B) - P(AB)$

$\frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

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$\frac{c}{a} = \sqrt{2}$

$(\frac{b}{a})^2 = \frac{c^2 - a^2}{a^2} = 2 - 1 = 1 = 1^2$