MA345 Differential Equations & Matrix Method

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COAS.301.12

MODULE I - 1ST ORDER ODE

Week 1: 1st order ODE: Assignment: Homework 1

1.1 Background

- 1.2 Solutions and Initial Value Problems
- 2.2 Separable Equations

Week 2: 1st order ODE: Assignment: Homework 2

- 2.3 Linear Equations
- 2.4 Exact Equations

Week 3: 1st order ODE: Substitutions / summary Assignment: Homework 3

- 2.5 Special Integrating Factors
- 2.6 Substitutions and Transformations

MODULE II - 2ND ORDER LINEAR ODE

Week 5: 2nd order linear ODE: Characteristic equation Assignment: Homework 4

- 4.2 Homogeneous Linear Equations: The General Solution
- 4.3 Characteristic Equations with Complex Roots
- 6.2 Higher Order Homogeneous Linear Equations with Constant Coefficients

Week 6: 2nd order linear ODE: Undetermined Coefficients Assignment: Homework 5

- 4.4 Nonhomogeneous Equations: The Method of Undetermined Coefficients
- 4.5 The Superposition Principle and Undetermined Coefficients Revisited
- 6.3 Undetermined Coefficients and the Annihilator Method

Week 7: 2nd order linear ODE: Variation of Parameters Assignment: Homework 6

- 4.6 Variation of Parameters
- 6.4 Method of Variation of Parameters

QUIZ 2

Solve $(e^{2y} - y\cos xy)\,dx + (2xe^{2y} - x\cos xy + 2y)\,dy = 0.$

$M = \frac{\partial f}{\partial x}$ $\qquad$ $N = \frac{\partial f}{\partial y}$ $\qquad$ $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$f(x,y) = ?$

f ⟶ $\int M\,dx = xe^{2y} - y\,\frac{\sin xy}{y} + g(y) + 0$ $\qquad$ $\int \cos 3x\,dx = \frac{\sin 3x}{3}$

compare!

$\int N\,dy = 2x\,\frac{e^{2y}}{2} - x\,\frac{\sin xy}{x} + y^2 + h(x)$ $\qquad$ $\int e^{3x}\,dx = \frac{e^{3x}}{3}$

$f(x,y) = xe^{2y} - \sin xy + y^2$

$$xe^{2y} - \sin xy + y^2 = C$$

Solve $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y(1 - x^2)}, \quad y(0) = 2.$

M N

$(\cos x \sin x - xy^2)\,dx + y(1-x^2)\,dy = 0$

$\dfrac{\partial M}{\partial y} = -2xy = \dfrac{\partial N}{\partial x}$

f :

$\int M\,dx = \int \cos x \sin x\,dx - \int xy^2\,dx = -\dfrac{\cos^2 x}{2} - \dfrac{x^2y^2}{2} + g(y)$

$u = \cos x$

$du = -\sin x\,dx$

$\int N\,dy = \int (y - yx^2)\,dy = \dfrac{y^2}{2} - \dfrac{y^2x^2}{2} + h(x)$

compare!

$\dfrac{y^2}{2} - \dfrac{y^2x^2}{2} - \dfrac{\cos^2 x}{2} = C$

$y^2(1-x^2) - \cos^2 x = k$ general solution

IC: $y(0) = 2$

$4 - 1 = k \quad k = 3$

Particular Solution

$y^2(1-x^2) - \cos^2 x = 3$

$$\overset{M}{2xy}\,dx + \overset{N}{(x^2-1)}\,dy = 0$$

Exact:

$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x}$

$$f = \begin{cases} \int M\,dx = \int 2xy\,dx = x^2y + g(y) \\ \int N\,dy = \int (x^2-1)\,dy = x^2y - y + h(x) \end{cases}$$

+C

Compare

$$x^2y - y = C \qquad y = \frac{C}{x^2-1}$$

Linear:

Standard form:

$$\frac{dy}{dx} + \underbrace{\frac{2x}{x^2-1}}_{P(x)}\,y = 0$$

$$g(x) = e^{\int \frac{2x}{x^2-1}dx} = e^{\ln|x^2-1|} = x^2-1$$

$u = x^2-1$

$du = 2x\,dx$

$$\left(y(x^2-1)\right)' = 0 \Rightarrow \int dx \Rightarrow y(x^2-1) = C \qquad y = \frac{C}{x^2-1}$$

Separation of variables

$$dy(x^2-1) = -2xy\,dx$$

$$\int \frac{dy}{y} = \int -\frac{2x}{x^2-1}\,dx$$

$u = x^2-1$

$du = 2x\,dx$

$$\ln|y| = -\ln|x^2-1| + C_1$$

$$e^{\ln|y|} = e^{-\ln|x^2-1|+C_1} = A\left(e^{\ln|x^2-1|}\right)^{-1}$$

$$y = A(x^2-1)^{-1} = \frac{A}{x^2-1}$$