

Homework 1: Set Theory

Due date: Friday 2/5 Submit the assignment via Canvas Assignments. Upload homework as one pdf document. A scanner app like Cam Scanner will make this possible.

Print and write work on this worksheet. Write **clearly** and show **all work** for full credit.

True or False. (1 point each)

1. False $45 \in \{1, 3, 5, 7, 9, \dots\}$ **⊖ It is a set of all odd numbers**
2. False $8 \notin \{x | x \in \mathbb{N} \wedge x \geq 8\}$
3. True $\{b, c, d\} \subseteq \{b, c, d\}$
4. False $\{\} \subset \{\}$
5. True for any set S , $\emptyset \subseteq S$
6. True $T \subseteq S$ and $S \subseteq T$ if and only if $S = T$

7. False **⊖** $|\{1, 2, 3\}| = |\{1, 2, \{3, 4\}\}|$ **⤴ This is one element**
- $\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 1, 2 & 1, 3 & 2, 3, 1, 2, 3 \\ 0 & 1 & 2 & 3 & 4 & \dots & \end{array}$

8. Edit the false statements in #1-7 to make them true. Answers may vary. (2 points)

1. $45 \in \{5, 10, 15, \dots\}$

2. $8 \in \{x \mid x \in \mathbb{N} \wedge x \leq 8\}$ ✓

3. $\{1\} \subseteq \{2\}$

4. $\{1\} \subseteq \{2\}$ ✓

7. $\{1, 2, 3\} \subseteq \{1, 2, \{3, 4\}\}$

9. Let $A = \{x \mid x, k \in \mathbb{N}, x = k^3, k < 4\}$. List all the subsets of A. (2 points)

A = {1, 8, 27}

$k = 1, 2, 3$
 $x = 1, 8, 27$

$A = \{x \mid x, k \in \mathbb{N}, x = k^3, k < 4\} = \{1, 8, 27\}$ ✓

Subsets:

$\emptyset, \{1\}, \{8\}, \{27\}, \{1, 8\}, \{1, 27\}, \{8, 27\}, \{1, 8, 27\}$

10. Consider some set, T , created by the formula 3^n , where n is a natural number and $n \geq 1$.

Provide three representations:

a. Enumeration (2 points)

$T = \{x \mid x, n \in \mathbb{N}, x = 3^n, n \geq 1\}$

$T = \{3, 9, 27, \dots\}$ ✓

$n = 1, 2, 3, 4, \dots$
 $x = 3, 9, 27, \dots$

b. Formal Rule (2 points)

$T = \{x \mid x, n \in \mathbb{N}, x = 3^n, n \geq 1\}$ ✓

c. Recursive Formula (2 points)

$T = \left\{ \begin{array}{l} t_1 = 3 \\ t_2 = 9 \\ t_n = 3^n \end{array} \right\}, n \in \mathbb{N}, n \geq 1$

not needed

$t_n = 3t_{n-1}$

Must refer to previous elements to be recursive

11. Let $U = \{0, 1, 2, \dots, 10\}$, $A = \{0, 2, 4, 6, 8, 10\}$ and $B = \{0, 1, 2, 3, 4, 5, 6\}$. Find:

d. $\overline{A \cup B}$ (2 points)

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 8, 10\}$$

$$\overline{A \cup B} = \{7, 9\}$$

e. $\bar{A} \cap \bar{B}$ (2 points)

$$\bar{A} = \{1, 3, 5, 7, 9\}$$

$$\bar{B} = \{7, 8, 9, 10\}$$

$$\bar{A} \cap \bar{B} = \{7, 9\}$$

12. Use a membership table to establish $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (4 points)

A	B	$A \cup B$	$\overline{A \cup B}$	\bar{A}	\bar{B}	$\bar{A} \cap \bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

1: $X \in$ given set

0: $X \notin$ given set