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1.

$P: \nexists n \in \mathbb{N} \ |F| = n$

Suppose F is Finite

(1) $\neg P: \exists n \in \mathbb{N} \ |F| = n$ Premise $\forall i \in (1, 2 \ldots n-2)$

(2) $F = \{F_1, F_2, \ldots F_n\}, f_i + f_{i+1} = f_{i+2}$ Definitions

(3) $f_{n-1}, f_n \in F$ Definition

(4) $f_{n-1} + f_n \in F$

(5) $f_{n-1} + f_n \neq f_i, \forall i \in \{1, 2, \ldots, n\}$ and $f_{n-1} + f_n \in F$

This is contradictory, then supposition is false

Therefore f is infinite.

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2. Square of an integer must be even so integer must be even.

$P: x \in \mathbb{N}, x^2 \in \mathbb{Z}E, x \in \mathbb{Z}E$

Proof

$\neg P: x \in \mathbb{N}, x^2 \in \mathbb{Z}o, x \in \mathbb{Z}o$ Premise

(I) let $a \in \mathbb{N}$ Definition

(II) let $x = 2a+1$ Definition of odd

(III) $x^2 = (2a+1)^2$ Algebra

$x^2 = 4a^2 + 4a + 1$ Definition of odd

$\boxed{\begin{array}{l}\text{Thus if } x \text{ is odd so is } x^2 \\ \therefore \text{ Thus if } x \text{ is even } x^2 \text{ is even } \neg P \Rightarrow Fo \text{ P is true}\end{array}}$

Prove by Induction

3. $\mathbb{P}: \forall n \in N, \exists m \in N, n^3+2n=3m$

Step 1:

Let $m=1$

$P: \sum_{i=1}^{m} i^3+2i = 3m$

(I) $\sum_{i=1}^{m} i^3+2i$ Premise

(II) $\sum_{i=1}^{1} i^3+2i$ Sub $1=m$

(III) 3 definition of sub Σ

(IV) $3 \cdot 1$ Math

(V) $3m$ Substitute $m=1$

Conclusion

Steps (I)-(V) show that $\mathbb{P}$ is true for $m=1$

Step 2: Induction Hypothesis

Let $m=k$, assume $\sum_{i=1}^{k} (i^3+2i) = 3k$

Step 3: Induction step

$P: \sum_{i=1}^{m} (i^3+2i) = 3m$ for $m=k+1$ $\sum_{i=1}^{k} (i^3+2i) = 3$

(I) $\sum_{i=1}^{m} (i^3+2i)$ Premise

(II) $\sum_{i=1}^{k+1} (i^3+2i)$ Substitution $m=k+1$

$k^3+3k+1 + 2k+2$

(III) $\sum_{i=1}^{k} (i^3+2i) + ((k+1)^3 + (2\cdot(k+1)))$ def of Σ

(IV) $3k+3k + k^3+1 + 2k+2$ Induction Hypothesis

$k^3 + 6k + 2k + 3$

$k^3 + 8k + 3$

$\frac{b \pm \sqrt{4ac - b^2}}{2a}$

not possible $\left(\frac{64 \pm \sqrt{12-64}}{2}\right)$

$m = k+1$ False

Step 4

Although base case is true, the induction did Not pass because $m \neq k+1$, thus P is false for all $n \in \mathbb{N}$ and $m \in \mathbb{N}$

Prove by induction

4. $\mathbb{P}$: $\forall n \in \mathbb{N}, a \neq 1, a^0 + a^1 + \ldots a^n = \frac{a^{n+1} - 1}{a-1}$

Let $a = 2$

Let $n = 1$

$\mathbb{P}$: $\forall n \in \mathbb{N}, a \neq 1, \sum_{i=1}^{n} a^i = \frac{a^{i+1} - 1}{a-1}$

(I) $\sum_{i=1}^{n} (2^i)$ Premise

(II) $\sum_{i=1}^{1} (2^i)$ Sub $n = 1$

(III) $2^1 = 2$ def of substitution

(IV) $\frac{2^{1+1} - 1}{2-1}$ Substitue $1 = n$

(V) $\frac{2^2 - 1}{2-1} = \frac{4-1}{1} = 3$ Math

$2 \neq 3$

Conclusion: Steps (I)-(V) proves that $\mathbb{P}$ is not true for $n = 1$.

Thus induction fails