Score: /48

CS 222

Name: _____

Homework 5: Methods of Proof (Direct Formal, Existence, Direct General)

Due date: Friday 3/12/21 Submit the assignment via Canvas Assignments. Upload homework as one pdf document. A scanner app like Cam Scanner will make this possible. Any HW submitted after the due date will have a penalty.

1. Prove by direct proof (formal). (10 points)

$$\begin{array}{ll} & \neg r \\ & t \rightarrow r \\ & p \rightarrow q \\ & \neg t \rightarrow p \\ & (p \wedge q) \rightarrow s \\ \hline \rightarrow & s \end{array}$$

2. Prove by direct proof (formal). (10 points)

$$\begin{array}{ll} & p \\ & q \vee r \\ & p \rightarrow \neg r \\ & q \rightarrow \neg s \\ & t \rightarrow s \\ \hline \Rightarrow & \neg t \end{array}$$

3. Prove by existence proof. (4 points)

$$\mathbb{P}\text{: } \exists\, x, y \in \mathbb{Z} \text{ such that } x^4 = y^2$$

4. Prove by existence proof. (4 points)

$$\mathbb{P}\text{: } \exists\, \mathrm{n}, \mathrm{m} \in \mathbb{Z}_{\mathrm{p}}, \mathrm{nm} \in \mathbb{Z}_{O}.$$

5. Prove by direct proof (general). (10 points)

$$\mathbb{P}\text{: } \forall n \in \mathbb{Z}_{O}, n^2 + 1 \in \mathbb{Z}_{E}$$

6. Prove by direct proof (general). (10 points)

$$\mathbb{P}: \ \forall a, b, c \in \mathbb{N}, \text{if } a \text{ divides } b \text{ and } a \text{ divides } c, \text{then } a \text{ divides } b + c$$