Module 12

Functions

Function: A relation is a function where "each member" of set A participates at most once in the relation.

iff " If and only if " ↔

$$f : A \rightarrow B$$

$A R_B$ is a function iff $\forall a \in A$, $(a, b_i) \in {_A}R_B$, $\nexists (a, b_j) \in {_A}R_B$
$$i \neq j$$

* No two ordered pairs may have the same element a.
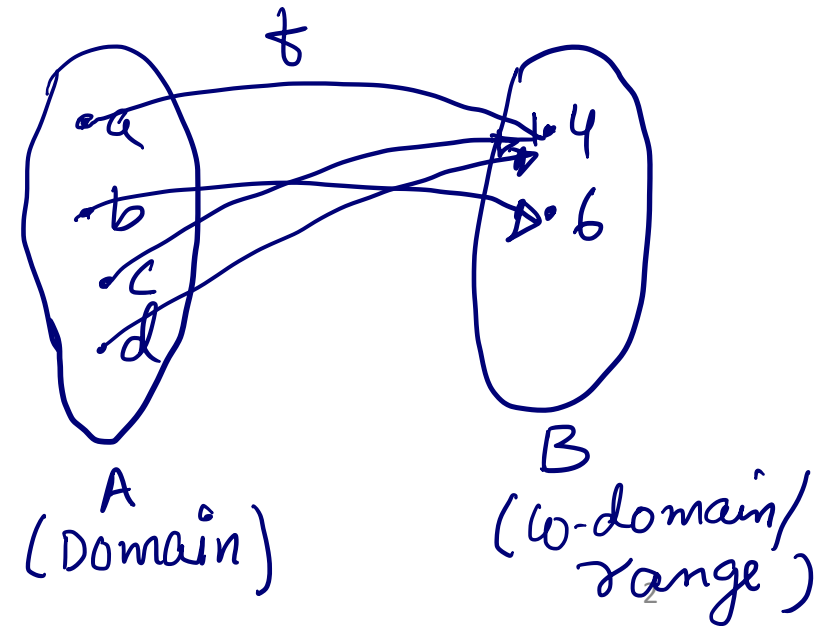
* it may have ordered pairs with same element b.

Representation of a function:

① $f(x) = x^2$ $\mathbb{Z} \rightarrow \mathbb{W}$

② $A = \{a, b, c, d\}$ $B = \{4, 6\}$
$$f = \{(a, 4), (b, 6), (c, 4), (d, 4)\}$$

f

A
(Domain)

B
(co-domain/
range)

Characteristic of functions

1. Total vs Partial Function

A function is total iff every member of A participates in the domain of the function.

A function is _partial_ otherwise.

$A = \{a, b, c, d\}$ $B = \{4, 6\}$

Total: $f = \{(a, 4), (b, 6), (c, 6), (d, 4)\}$

Partial: $f = \{(a, 4), (b, 6)\}$

2. One-to-One Function (Injective)

$$f : A \longrightarrow B$$
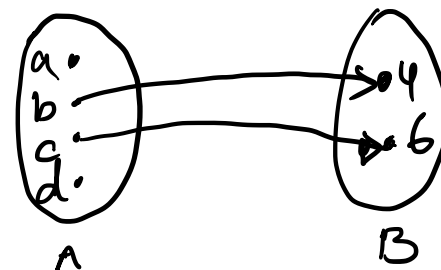$$\text{(domain)} \quad \text{(range)}$$
$$X \qquad Y$$

A function f is one-to-one iff every member of set B appears in the range at most once.

$$A = \{a, b, c, d\} \qquad B = \{4, 6\}$$

Ex: One-to-One function : $f = \{(b, 4), (c, 6)\}$

not one-to-one function: $f = \{(a, 4), (b, 4), (c, 6)\}$

** one-to-one function defines set of reversible.
   "Inverse exist" $f^{-1} : B \longrightarrow A$

3. Onto Function (Surjective)

A function ONTO iff every member of set B appears in the range at least once.

$$A = \{a, b, c, d\} \qquad B = \{4, 6\}$$

Onto: $f : \{(a, 4), (b, 6), (c, 4)\}$

not onto: $f : \{(a, 4), (b, 4), (c, 4), (d, 4)\}$
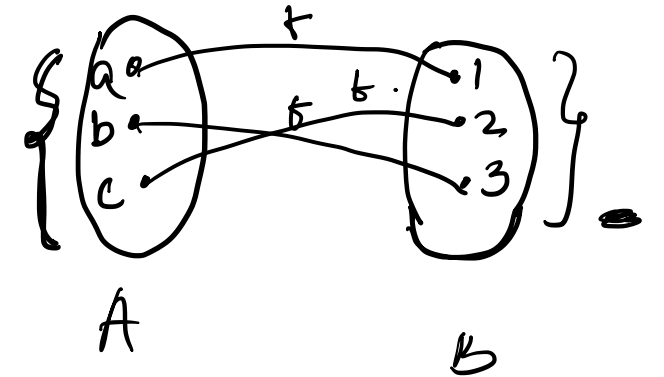
(because 6 is not included)

4. One-to-One and Onto Function (Bijective)

A function is one-to-one and onto iff every member of set B appears EXACTLY ONCE.

$$A = \{a, b, c, d\} \qquad B = \{4, 6\}.$$

$$f : \{(a, 4), (b, 6)\}$$

if $f : A \to B$ is total and one-to-one then $|A| \leq |B|$.

if $f : A \to B$ is onto, then $|A| \geq |B|$

if $f : A \to B$ is total and onto and one-to-one, then $|A| = |B|$.

Ex1 : A={2,4,6} B={x,y,z}

a)   f={(2,x), (4,y), (4,z)} Is this a function?   NOT A function

b)   f={(2,x),(4,x),(6,x)} function, or no?   YES IT is a function .

Ex2 : A={2,4,6} B={x,y,z}

f={(2,x),(4,x),(6,x)} Is function f total or partial or one to one or onto?

Total ✓
NOT a one-to-one function, x appears more than once .
Not a onto function, y and z dont appear .

Ex3 : A={2,4,6}   B={x,y,z}

f={(2,y), (4,z)} Is this a function? What about one-to-one or onto characteristics for this function f?

Yes it is a partial function.

One-to-one → Yes!

Onto : NO (element x is not used).

Ex4 : A={2,4,6} B={x,y,z}

Write a function example $_AR_A$ that is partial, one-to-one function?

$$_AR_A = f = \{ (2,x), (4,y) \}$$

Ex5 : A={2,4,6} B={x,y,z}

Write a function example $_AR_A$ that is partial, one-to-one function!

(repeated
Same as EX4)

Ex6 : A={2,4,6} B={x,y,z}

Write a function example $_BR_B$ that is NOT onto!

$Ex: BR_A = f = \{(2,X), (4,X), (6,Y)\}$

(it is not onto
because element z is
not included)