Jeremiah Webb

48/48

CS 222 Hw #5 Methods of Proof

1. Prove by direct Proof (formal).

$\neg r$	P_1
$t \to r$	P_2
$p \to q$	P_3
$\neg t \to p$	P_4
$(p \wedge q) \to s$	P_5
$\to s$	conclusion

(I)	$\neg r$	P_1
(II)	$t \to r$	P_2 ✓
(III)	$\neg t$	(I),(II) Modus Tollens
(IV)	$\neg t \to p$	P_4 ✓
(V)	p	(III), (IV) Modus Ponens
(VI)	$p \to q$	P_3 ✓
(VII)	q	(V), (VI) Modus Ponens
(VIII)	$(p \wedge q)$	(V),(VII) rule of conductive ✓ Rule of conjunction, not conductive
(IX)	$(p \wedge q) \to s$	P_5
(X)	$\to s$	(VIII), (IX) Modus Ponens ✓

Scanned with CamScanner

2.

$$\begin{array}{ll} p & P_1 \\ q \vee r & P_2 \\ p \rightarrow \neg r & P_3 \\ q \rightarrow \neg s & P_4 \\ t \rightarrow s & P_5 \\ \hline \neg t & \end{array}$$

(I)	p	P_1
(II)	$p \Rightarrow \neg r$	P_3
(III)	$\neg r$	(I)(II) Modus Ponens
(IV)	$q \vee r$	P_2
(V)	q	(III), (IV) Rule of Disjunctive Syllogism
(VI)	$q \Rightarrow \neg s$	$P4$
(VII)	$\neg s$	Modus Ponens
(VIII)	$t \Rightarrow s$	$P5$
(IX)	$\neg t$	Modus Tollens

Scanned with CamScanner

3. $\mathbb{N} = \{1, 2, 3 \ldots\}$

$x^4 = y^2$

let

Proof: $x^4 = y^2$ Let $x = 2$

$2^4 = 4^2$ $y = 4$

$\therefore$ $16 = 16$

Hence proved

4. n, m only prime integers

$n \cdot m$ only odd integers

Let $n=3$ Prime

$m=7$ Prime

$3 \cdot 7 = 21$ 21 is odd

$\therefore$ Proved

5. $\forall n \in \mathbb{Z}_O, n^2+1 \in \mathbb{Z}E$

Proof: Example

(I) Let $a \in \mathbb{Z}$ definition $n=3$ $n=5$

(II) let $n = 2a+1$ def of odd $3^2+1 = 10$

(III) $n^2+1 = (2a+1)^2+1$ alg $5^2+1 = 26$

(IV) $n^2+1 = 4a^2+4a+2$ Definition of even #

$\boxed{= 2(2a^2+2a+1)}$ Definition of even #

$\therefore$ Steps (I) – (IV) show that

P is true.

Scanned with CamScanner

6. Prove by direct proof

$\forall a, b, c \in \mathbb{N}$, if a divides b, and a divides c, then a divides b+c

$$a|b \wedge a|c \rightarrow a|b+c$$

Proof:

(I) $a|b \wedge a|c$ given

(II) $a|(b+c)$ given

(III) Let $r, s \in \mathbb{Z}$. Define

(IV) $b = ar$ Definition

(V) $c = as$ Definition

(VI) $b+c = ar+as$

$b+c = a(r+s)$ factoring

$\therefore$ $r+s$ is an integer, $a|b+c$ is proven

(I)–(VI) P is true.

Scanned with CamScanner