Jeremiah webb Final exam CS222

1.

$$B = \begin{cases} b_1 = 7 \\ b_{n+1} = b_n + 2, & n \in \mathbb{N} \end{cases}$$

Plug 1 for n

$b_{n+1} = b_1 + 2$

$7 + 2$

$b_2 = 9$

2.

$b_1 = 7$

$b_{n+1} = b_n + 2$

$b_1 \Rightarrow 7$

$b_{1+1} \Rightarrow b_1 + 2$

$b_1 \Rightarrow 7$ (P_1)

$b_2 \Rightarrow b_1 + 2$ (P_2)

(1) $b_1 \Rightarrow 7$ (P_1)

(2) $b_2 \Rightarrow b_1 + 2$ (P_2) cancel out b_1

(3) $b_2 \Rightarrow 7 + 2$ law of syllogism

(4) $b_2 \Rightarrow 9$

steps 1 - 4 prove $b_2 \Rightarrow 9$

3.

$U = \{g, h, I, J, K, L, m\}$

$A = \{g, h, i, j\}$

$B = \{i, j, k, l\}$

$A \cup B$ $\{g, K, i, j, k, l\}$

7.

20

3 ways to arrange

$20 \cdot 19 \cdot 18$

$= 6,840$

8.

5 movies

$$\frac{5!}{(5-5)!} = 5!$$

$5 \times 4 \times 3 \times 2 \times 1$

$= 120$

Scanned with CamScanner

9.

5 movies

6 ways for last
3x2x1 three
movies

$$\frac{5!}{(5-2)\cdot(5-2)}$$

$$\frac{2}{5} \times \frac{1}{4} \times 3!$$

$\binom{5}{}$ $\quad .01\overline{6}$

10.

$.88 \times .88 \times .88 = \boxed{.681}$

11.

$15+25+22+10+20$

100 lights

one out 100 | 1 out of blue | 1 out of blue | 1 out of 99 lit

$$\frac{1}{100} \cdot \frac{1}{25} + \cdot \frac{1}{24} \cdot \frac{1}{99}$$

.0008

.001

12.

400 attendees

120 attendees ordered pizza and soda

$$\frac{120}{400}$$

13.

p	q	¬p	¬q	$\bar{p} \vee \bar{q}$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

14.

p	q	r	p∨q	$\overline{p \vee q}$	$(\overline{p \vee q}) \wedge R$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	T	F

15.

$\neg(q \to p) \to q$

p	q	$q \to p$	$\neg(q \to p)$	$\neg(q \to p) \to q$
T	T	T	F	T
T	F	T	F	T
F	T	F	T	T
F	F	T	F	T

$\therefore$ Is Tautology

16.

$p: \exists x, y, z \in \mathbb{N} \; xyz = 24$

Let $x = 2$

$y = 2$

$z = 4$

$x \cdot y \cdot z = 24$

$2 \cdot 2 \cdot 4 = 24$

Hence proved

17A.

$$\begin{array}{ll} \neg p & P_1 \\ \neg(q \vee r) \to p & P_2 \\ \neg r & P_3 \\ \hline \therefore q & \end{array}$$

(1) $\neg p$ — P_1

(2) $\neg(q \vee r) \to p$ — P_2

(3) $\neg r$ — P_3

(4) $(q \vee r)$ — (1) (2). Modus Tollens

(5) q — (4) (3) rule of disjunctive syllogism

$\therefore q$

Steps 1 - 5 show that IP is true.

18.

Step 1

let $n=1$

$IP = \frac{3n(n+1)}{2} = 3n$

(I) $\frac{3n(n+1)}{2} = 3n$ Premise

(II) $\frac{3 \cdot 1(1+1)}{2} = 3 \cdot 1$ Substitute

(III) $\frac{6}{2} = 3 = 3$ Math

Conclusion: steps I - III show IP is true for $n=1$.

18. Step 2

Let $n = k$ assume

$$\frac{3k(k+1)}{2} = 3k$$

Step 3

let $n = k+1$

P: $\frac{3n(n+1)}{2} = 3n$ for $n = k+1$ given

$$\frac{3k(k+1)}{2} = 3k$$

(I) $\frac{3n(n+1)}{2} = 3n$ premise

(II) $\frac{3(k+1)\cdot((k+1)+1)}{2} = 3(k+1)$ Substitute

$\frac{3k+1\cdot(k+2)}{} = 3(k+1)$ Math

$$\frac{3k\cdot(k+1)}{2} + 3(k+1) = 3(k+1)$$

$$\frac{(3k+6)\cdot(k+1)}{2} = 3(k+1)$$

$$\frac{3(k+2)\cdot(k+1)}{2} = 3(k+1)$$

Conclusion steps 1-3 show that P is true for $n = k+1$

Step 4

Because the base case is true, and the induction step is true, P is true for all $n \in \mathbb{N}$

19.

Let $A = \{A, b, c\}$

$ARA \{(a,b), (a,c), (b,c)\}$

	a	b	c
a	0	1	1
b	0	0	1
c	0	0	0

20.

	a	b	c
a	0	1	1
b	0	0	1
c	0	0	0

24.

Let $A = \{d, e, f\}$

$B = \{0, 1\}$

$ARB = \{(d,1), (f,0)\}$

Partial function

Every B shows up at least once

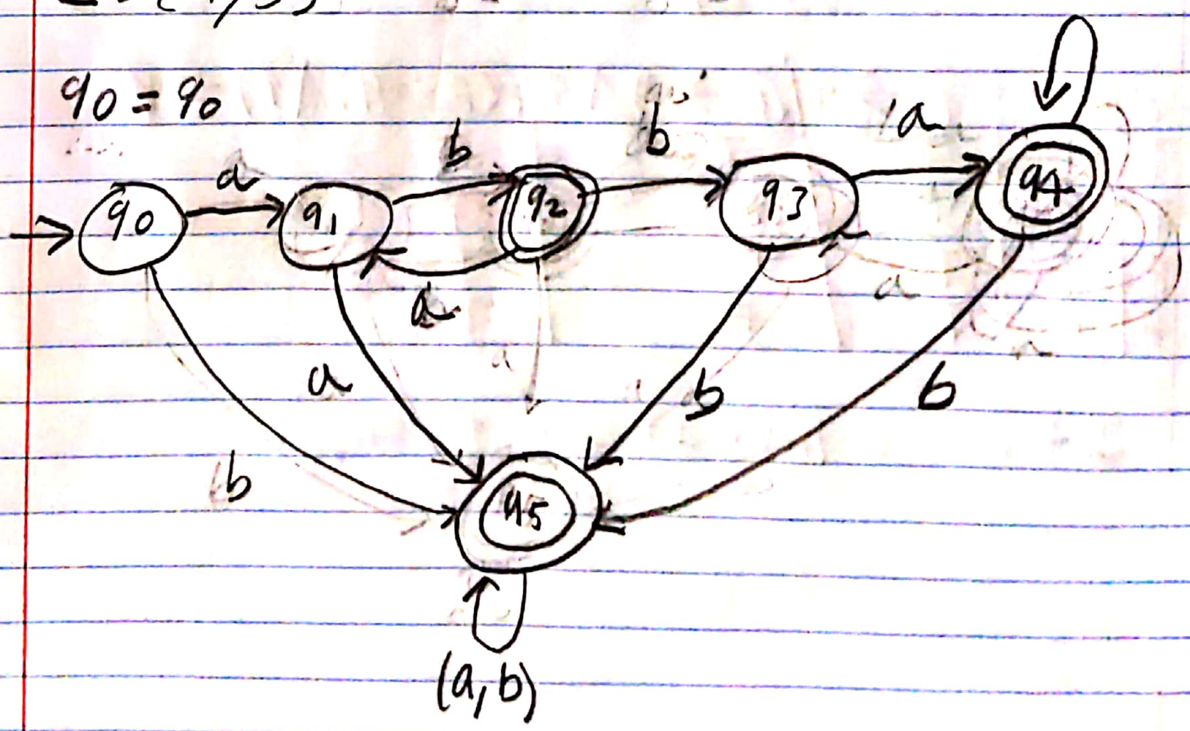
no repeating

25. $M = \{S, \Sigma, q_0, F, \delta\}$

$S = \{q_0, q_1, q_2, q_3, q_4, q_5\} = F = \{q_5\}$

$\Sigma = \{a, b\}$

$q_0 = q_0$



δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_1	q_3
q_3	q_4	q_5
q_4	q_4	q_5
q_5	q_5	q_5

25. contd

26.

I am hungry, then I eat
I eat
____
I am hungry

$(p \to q) \land q \to p$

I am hungry $= p$
I eat $= q$

$p \to q$

$$\begin{array}{l} p \to q \\ q \\ \hline p \end{array}$$

$p \to q$

p	q	$p \to q$	$(p \to q) \land q$	$((p \to q) \land q) \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

It is valid. It is a tautology.