Module 5

Direct Formal Proof

Ex 1: $\mathbb{P}$: p P_1

$p \to q$ P_2

$(q \wedge p) \to s$ P_3

$\therefore$ (s) ← conclusion.

(i) p P_1

(ii) $p \to q$ P_2

(iii) q (i), (ii), Modus Ponens

(iv) $p \wedge q$ (i), (iii), Rule of Conjunction

(v) $(q \wedge p) \to s$ P_3

(vi) (s) (iv), (v), Modus Ponens.

Steps (i)-(vi) show that $\mathbb{P}$ is true.

Rules of Inference

Rule of Inference	Name of Rule
p, $p \to q$, $\therefore q$	Rule of Detachment (Modus Ponens)
$p \to q$, $q \to r$, $\therefore p \to r$	Law of the Syllogism
$p \to q$, $\neg q$, $\therefore \neg p$	Modus Tollens
p, q, $\therefore p \wedge q$	Rule of Conjunction
$p \vee q$, $\neg p$, $\therefore q$	Rule of Disjunctive Syllogism
$\neg p \to F_o$, $\therefore p$	Rule of Contradiction
$p \wedge q$, $\therefore p$	Rule of Conjunctive Simplification
p, $\therefore p \vee q$	Rule of Disjunctive Amplification
$p \wedge q$, $p \to (q \to r)$, $\therefore r$	Rule of Conditional Proof
$p \to r$, $q \to r$, $\therefore (p \vee q) \to r$	Rule for Proof by Cases
$p \to q$, $r \to s$, $p \vee r$, $\therefore q \vee s$	Rule of the Constructive Dilemma
$p \to q$, $r \to s$, $\neg q \vee \neg s$, $\therefore \neg p \vee \neg r$	Rule of the Destructive Dilemma

Ex 2: $\mathbb{P}$: p P_1

$p \to (q \vee r)$ P_2

$q \to s$ P_3

$\neg s$ P_4

$r \to t$ P_5

$\therefore$ t ← conclusion.

(i) p P_1

(ii) $p \to (q \vee r)$ P_2

(iii) $(q \vee r)$ (i), (ii) Modus Ponens.

(iv) $q \to s$ P_3

(v) $\neg s$ P_4

(vi) $\neg q$ (iv), (v), Modus tollens

(vii) r (iii), (vi), Rule of Disjunctive Syllogism.

(viii) $r \to t$ P_5

(ix) t (vii), (viii), Modus ponens

∴ (i)-(ix) show $\mathbb{P}$ is true or valid. ∎

Rules of Inference

Rule of Inference	Name of Rule
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$p \to q$ $q \to r$ $\therefore p \to r$	Law of the Syllogism
$p \to q$ $\neg q$ $\therefore \neg p$	Modus Tollens
p q $\therefore p \wedge q$	Rule of Conjunction
$p \vee q$ $\neg p$ $\therefore q$	Rule of Disjunctive Syllogism
$\neg p \to F_o$ $\therefore p$	Rule of Contradiction
$p \wedge q$ $\therefore p$	Rule of Conjunctive Simplification
p $\therefore p \vee q$	Rule of Disjunctive Amplification
$p \wedge q$ $p \to (q \to r)$ $\therefore r$	Rule of Conditional Proof
$p \to r$ $q \to r$ $\therefore (p \vee q) \to r$	Rule for Proof by Cases
$p \to q$ $r \to s$ $p \vee r$ $\therefore q \vee s$	Rule of the Constructive Dilemma
$p \to q$ $r \to s$ $\neg q \vee \neg s$ $\therefore \neg p \vee \neg r$	Rule of the Destructive Dilemma

Ex 3: $\mathbb{P}$: p P_1

$p \to (q \wedge r)$ P_2

$r \to t$ P_3

$\therefore$ t

(i) p P_1

(ii) $p \to (q \wedge r)$ P_2

(iii) $(q \wedge r)$ (i), (ii) Modus Ponens.

(iv) r (iii), Rule of conjunctive simplification.

(v) $r \to t$ P_3

(vi) t (iv), (v), rule of Modus Ponens

$\therefore$ steps (i)-(vi) show $\mathbb{P}$ is valid/true.

Rules of Inference

Rule of Inference	Name of Rule
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$p \to q$ $q \to r$ $\therefore\ p \to r$	Law of the Syllogism
$p \to q$ $\neg q$ $\therefore\ \neg p$	Modus Tollens
p q $\therefore\ p \wedge q$	Rule of Conjunction
$p \vee q$ $\neg p$ $\therefore\ q$	Rule of Disjunctive Syllogism
$\neg p \to F_o$ $\therefore\ p$	Rule of Contradiction
$p \wedge q$ $\therefore\ p$	Rule of Conjunctive Simplification
p $\therefore\ p \vee q$	Rule of Disjunctive Amplification
$p \wedge q$ $p \to (q \to r)$ $\therefore\ r$	Rule of Conditional Proof
$p \to r$ $q \to r$ $\therefore\ (p \vee q) \to r$	Rule for Proof by Cases
$p \to q$ $r \to s$ $p \vee r$ $\therefore\ q \vee s$	Rule of the Constructive Dilemma
$p \to q$ $r \to s$ $\neg q \vee \neg s$ $\therefore\ \neg p \vee \neg r$	Rule of the Destructive Dilemma

Ex 4: $\mathbb{P}$: $(f \vee s) \rightarrow m$ P_1

$m \rightarrow c$ P_2

$\neg c$ P_3

$\therefore \neg s$ ← conclusion

(i) $m \rightarrow c$ P_2

(ii) $\neg c$ P_3

(iii) $\neg m$ (i), (ii), Modus Tollens

(iv) $(f \vee s) \rightarrow m$ P_1

(v) $\neg (f \vee s)$ (iii), (iv), Modus Tollens.

(vi) $\neg f \wedge \neg s$ (v), Demorgans Rule.

(vii) $\neg s$ (vi), Rule of Conjuctive simplification.

∴ Steps (i) - (vii) show $\mathbb{P}$ is true

Rules of Inference

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$p \rightarrow q$, $q \rightarrow r$, $\therefore p \rightarrow r$	Law of the Syllogism
$p \rightarrow q$, $\neg q$, $\therefore \neg p$	Modus Tollens
p, q, $\therefore p \wedge q$	Rule of Conjunction
$p \vee q$, $\neg p$, $\therefore q$	Rule of Disjunctive Syllogism
$\neg p \rightarrow F_o$, $\therefore p$	Rule of Contradiction
$p \wedge q$, $\therefore p$	Rule of Conjunctive Simplification
p, $\therefore p \vee q$	Rule of Disjunctive Amplification
$p \wedge q$, $p \rightarrow (q \rightarrow r)$, $\therefore r$	Rule of Conditional Proof
$p \rightarrow r$, $q \rightarrow r$, $\therefore (p \vee q) \rightarrow r$	Rule for Proof by Cases
$p \rightarrow q$, $r \rightarrow s$, $p \vee r$, $\therefore q \vee s$	Rule of the Constructive Dilemma
$p \rightarrow q$, $r \rightarrow s$, $\neg q \vee \neg s$, $\therefore \neg p \vee \neg r$	Rule of the Destructive Dilemma

Ex 5: $\mathbb{P}$:

$\neg q$ P1

$p \to r$ P2

$p \vee q$ P3

$\therefore$ r ← conclusion

(i) $\neg q$ P1

(ii) $q \vee p$ P3

(iii) p (i),(ii) Rule of Disjunctive Syllogism.

(iv) $p \to r$ P2

(v) r (iii), (iv) Modus Ponens.

steps (i) - (v) show $\mathbb{P}$ is valid

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$p \to r$ $q \to r$ $\therefore (p \vee q) \to r$	Rule for Proof by Cases
$p \to q$ $r \to s$ $p \vee r$ $\therefore q \vee s$	Rule of the Constructive Dilemma
$p \to q$ $r \to s$ $\neg q \vee \neg s$ $\therefore \neg p \vee \neg r$	Rule of the Destructive Dilemma

Ex 6: $\mathbb{P}$:

r — P_1

$p \to \neg q$ — P_2

$r \to q$ — P_3

$\therefore \neg p$

conclusion.

(i) r — P_1

(ii) $r \to q$ — P_3

(iii) q — (i), (ii) Modus Ponens

(iv) $p \to \neg q$ — P_2

(v) $\neg p$ — (iii), (iv) Modus Tollens

∴ (i)-(v) show that $\mathbb{P}$ is valid

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$p \to q$ $r \to s$ $\neg q \vee \neg s$ $\therefore \neg p \vee \neg r$	Rule of the Destructive Dilemma