CS 222 Test 2 Jeremiah Webb 4/4/21

1.

r= He is guilty

q= He goes to prison

$\neg r \to \neg q$

$\neg q \to \neg r$

$\neg r \land q \not\equiv$

$r \land \neg q \not\equiv$

$r \lor q \not\equiv$

2. Negation

If he is not innocent and he does not go free

3.A. P: You do not study

q: You struggle

B.

$$\frac{\begin{array}{c} p \to q \\ \neg q \end{array}}{\neg p}$$

$((p \to q) \land \neg q) \to \neg p$

c. $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

p	q	$p \rightarrow q$	$\neg q$	$((p \rightarrow q) \wedge \neg q$	$[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$
F	F	T	F	F	T
F	T	F	T	F	T
T	F	T	F	F	T
T	T	T	T	T	T

4D. It is a Tautology.

5.

Existence a, b, c all real #s

let $a = 5$ $\quad a^2 - b^2 = c^2$

$b = 4$ $\quad 5^2 - 4^2 = c^2$

$c = 4$ $\quad 25 - 16 = c^2$

$19 = c^2$

$3 = c$

$5^2 - 4^2 = 3^2$

$25 - 16 = 9$

$9 = 9$

Proven ✓

Scanned with CamScanner

5A. Formal

P: $\neg(p \vee \neg q)$ P_1

$q \to r$ P_2

$\therefore r$

(1) $\neg(p \vee \neg q)$ P_1

(2) $\neg p \wedge q$ (1) Demorgans Rule

(3) q (2) Rule of conductive simplification

(4) $q \to r$ P_2

(5) r (3),(4), Modus Ponens

Steps (1)-(5) show that $\mathbb{P}$ is true.

5B. Prove by direct proof general

$P: \forall n \in \mathbb{Z}_O, n(n+3) \in \mathbb{Z}_E$

n^2+3n

Proof:

(1) let $a \in \mathbb{Z}$ Definition

(2) $n = 2a+1$ Definition of odd

(3) $n^2+3n = (2a+1)^2+3\cdot(2a+1)$ Algebra

(4) $n^2+3n = 4a^2+4a+1+6a+3$ Algebra

(5) n^2+3n $4a^2+10a+4$ Algebra

(6) $n^2+3n = 4(a^2+2.5a+1)$ Definition of even

Steps (1)-(6) show that $\mathbb{P}$ is true.

6D. Prove by Induction

TP: $\forall n \in \mathbb{N}$, $(-1)^{2n+1} = -1$

Step 1: Base case

let $n = 1$

TP: $(-1)^{2n+1} = -1$ for $n = 1$

(1) $(-1)^{2n+1}$ Premise

(2) $(-1)^{2\cdot 1+1} = (-1)^3$ Substitute $n = 1$

(3) $(-1)\cdot(-1)\cdot(-1) = -1$ Math

conclusion: steps (1)-(3) show TP is true for $n = 1$.

Step 2: Induction Hypothesis

let $n = k$, assume $(-1)^{2k+1} = -1$

Step 3: Induction Step

let $n = k+1$

TP: $(-1)^{2n+1} = -1$ for $n = k+1$ given $(-1)^{2k+1} = -1$

(1) $(-1)^{2n+1}$ Premise

(2) $(-1)^{2\cdot(k+1)+1}$ Substitute $k+1$

(3) $(-1)^{2k+1+2} = (-1)^{2k+1}\cdot(-1)^2$ algebra

(4) (-1) Math

conclusion steps (1)-(4) show that TP is true for $n = k+1$.

Step 4: Induction Proof conclusions
Because both the base case
and Induction step are
true P is true for
$\forall n \in \mathbb{N}$

7.
Initial: If he is guilty, then he
goes to prison

Negation: He is guilty and
he does not go to prison.