Doubling Ratio

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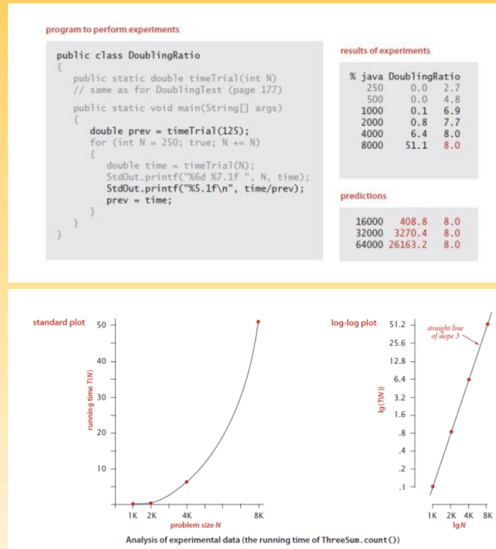
The following experiment can be used to predict whether an algorithm that is believed to have polynomial grown of some amount, N^b, the doubling ratio experiment can help approximate b.

$$ratio = \frac{time_{cur}}{time_{prev}}$$



1

Doubling Ratio

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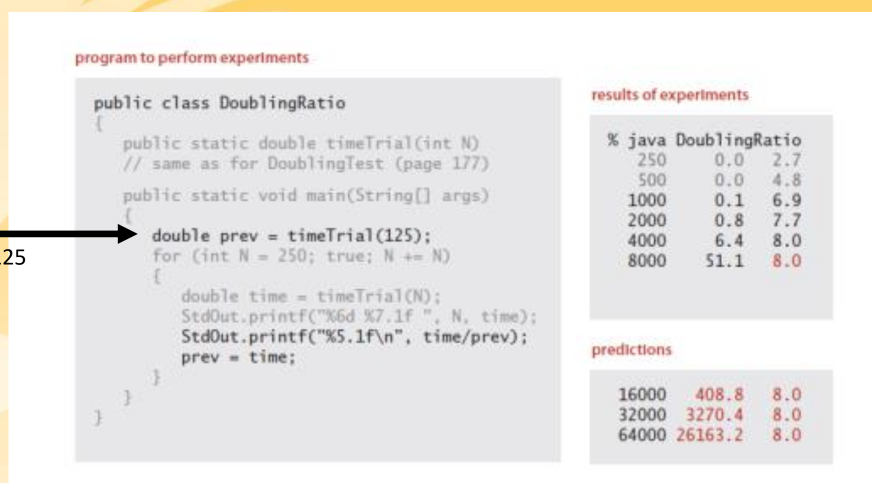
Runs algorithm with initial N=125
Returns runtime


program to perform experiments

```
public class DoublingRatio
{
   public static double timeTrial(int N)
   // same as for DoublingTest (page 177)
   public static void main(String[] args)
   {
      double prev = timeTrial(125);
      for (int N = 250; true; N += N)
      {
         double time = timeTrial(N);
         StdOut.printf("%6d %7.1f ", N, time);
         StdOut.printf("%5.1f\n", time/prev);
         prev = time;
      }
   }
}
```

results of experiments

```
% java DoublingRatio
  250    0.0   2.7
  500    0.0   4.8
 1000    0.1   6.9
 2000    0.8   7.7
 4000    6.4   8.0
 8000   51.1   8.0
```

predictions

```
16000   408.8   8.0
32000  3270.4   8.0
64000 26163.2   8.0
```


2

Doubling Ratio

Loop from N=250 and doubles itself with each iteration (N+=N)

For each iteration… algorithm is re-ran and the ratio between the new and old runtimes is computed.

program to perform experiments

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public class DoublingRatio
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predictions

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Fast at first, but eventually, it will slow down and then converge.

Doubling Ratio

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$$ratio = \frac{time_{cur}}{time_{prev}}$$

- The ratio will converge to a ratio of 2^b if the algorithm's true order is N^b, where b is some unknown polynomial.
- e.g. Assume some unknown algorithm is $\sim N^2$, but we don't know that because we don't understand the code, but we can run an experiment.
  - $N = 16, T_1 = 268$
  - $N = 32, T_2 = 1027$
  - Knowing the doubling ratio and having doubled our inputs, we observe a convergence of ratio at
    - $Ratio = \frac{T2}{T1} = \frac{1027}{268} = 3.83$
    - $b = log_2(3.83) = 1.93734$ (round up to 2)
  - Therefore, order is $\sim N^2$

program to perform experiments

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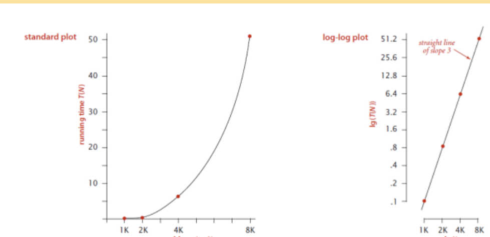

Analysis of experimental data (the running time of ThreeSum.count())

Doubling Ratio

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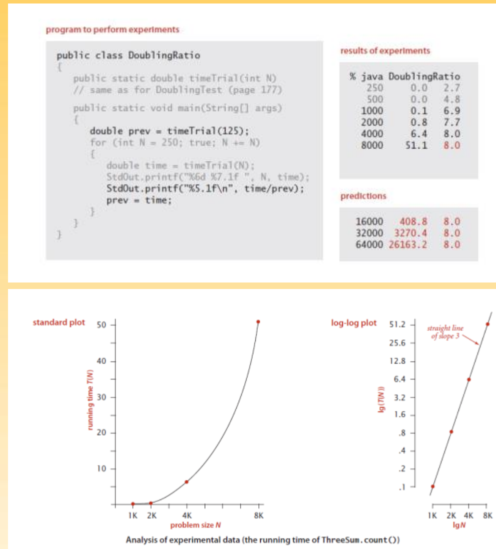
$$ratio = \frac{time_{cur}}{time_{prev}}$$

- The ratio will converge to a ratio of 2^b if the algorithm's true order is N^b, where b is some unknown polynomial.
- By forcing the size to be double, we can observe how the result scales.



Doubling Ratio

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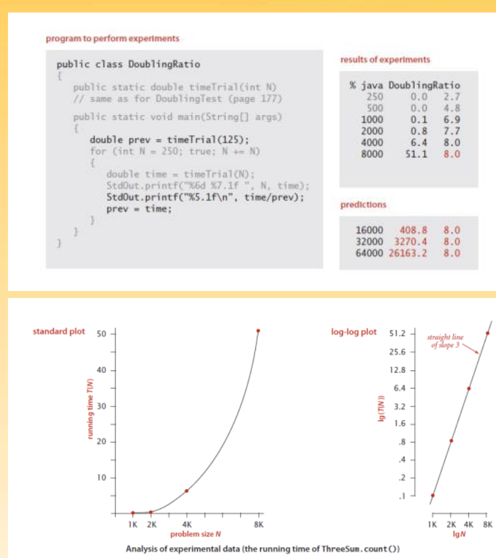
- The ratio will converge to a ratio of 2^b if the algorithm's true order is N^b, where b is some unknown polynomial.
- By forcing the size to be double, we can observe how the result scales.
- Once we have our convergence, we can solve for b.
  - e.g. 2^b = 8 after convergence in the ThreeSum
  - Which means that $b = 3$ AND the algorithm's order of growth is N^3.



Doubling Ratio

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Proposition C. (Doubling ratio) If $T(N) \sim a N^b \lg N$ then $T(2N)/T(N) \sim 2^b$.

Proof: Immediate from the following calculation:

$$\begin{aligned} T(2N)/T(N) &= a(2N)^b \lg(2N) / a N^b \lg N \\ &= 2^b (1 + \lg 2 / \lg N) \\ &\sim 2^b \end{aligned}$$

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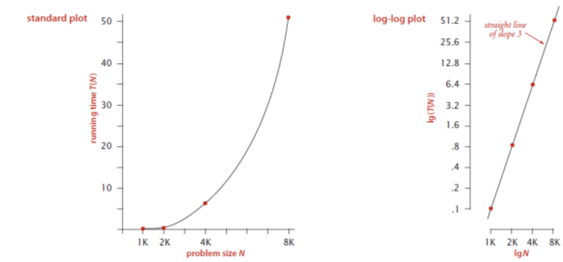

Analysis of experimental data (the running time of ThreeSum.count())

With our model developed, we can compare predict runtime given larger values of N where the experiment would no longer be feasible.