

## Homework 1: Set Theory

**Due date: Friday 2/5** Submit the assignment via Canvas Assignments. Upload homework as one pdf document. A scanner app like Cam Scanner will make this possible.

Print and write work on this worksheet. Write **clearly** and show **all work** for full credit.

True or False. (1 point each)

1. False  $45 \in \{1, 3, 5, 7, 9, \dots\}$
2. False  $8 \notin \{x | x \in \mathbb{N} \wedge x \geq 8\}$
3. True  $\{b, c, d\} \subseteq \{b, c, d\}$
4. False  $\{\} \subset \{\}$
5. True for any set  $S$ ,  $\emptyset \subseteq S$
6. True  $T \subseteq S$  and  $S \subseteq T$  if and only if  $S = T$
7. False  $|\{1, 2, 3\}| = |\{1, 2, \{3, 4\}\}|$

0 1 2 3    1, 2 1, 3 2, 3 1 2 3

0 1 2 3    4

8. Edit the false statements in #1-7 to make them true. Answers may vary. (2 points)

1.  $45 \in \{5, 10, 15, \dots\}$
2.  $8 \in \{x \mid x \in \mathbb{N} \wedge x \leq 8\}$
3.  $\{1\} \subseteq \{2, 3, 4\}$
4.  $\{1\} \subseteq \{2, 3\}$
7.  $\{1, 2, 3\} \subseteq \{1, 2, \{3, 4\}\}$

9. Let  $A = \{x \mid x, k \in \mathbb{N}, x = k^3, k < 4\}$ . List all the subsets of A. (2 points)

$$A = \{x \mid x, k \in \mathbb{N}, x = k^3, k < 4\} = \{1, 8, 27\}$$

$k = 1, 2, 3$   
 $x = 1, 8, 27$

Subsets:

$$\emptyset, \{1\}, \{8\}, \{27\}, \{1, 8\}, \{1, 27\}, \{8, 27\}, \{1, 8, 27\}$$

10. Consider some set,  $T$ , created by the formula  $3^n$ , where  $n$  is a natural number and  $n \geq 1$ .

Provide three representations:

a. Enumeration (2 points)

$$T = \{3, 9, 27, \dots\}$$

$$T = \{x \mid x, n \in \mathbb{N}, x = 3^n, n \geq 1\}$$

$$n = 1, 2, 3, 4, \dots$$

$$x = 3, 9, 27, \dots$$

b. Formal Rule (2 points)

$$T = \{x \mid x, n \in \mathbb{N}, x = 3^n, n \geq 1\}$$

c. Recursive Formula (2 points)

$$T = \left\{ \begin{array}{l} t_1 = 3 \\ t_2 = 9 \\ t_n = 3^n \end{array} \right\}, n \in \mathbb{N}, n \geq 1$$

11. Let  $U = \{0, 1, 2, \dots, 10\}$ ,  $A = \{0, 2, 4, 6, 8, 10\}$  and  $B = \{0, 1, 2, 3, 4, 5, 6\}$ . Find:

d.  $\overline{A \cup B}$  (2 points)

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 8, 10\}$$

$$\overline{A \cup B} = \{7, 9\}$$

e.  $\bar{A} \cap \bar{B}$  (2 points)

$$\bar{A} = \{1, 3, 5, 7, 9\}$$

$$\bar{B} = \{7, 8, 9, 10\}$$

$$\bar{A} \cap \bar{B} = \{7, 9\}$$

12. Use a membership table to establish  $\overline{A \cup B} = \bar{A} \cap \bar{B}$  (4 points)

A	B	$A \cup B$	$\overline{A \cup B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cap \bar{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

1:  $X \in$  given set

0:  $X \notin$  given set