# Module 6

Methods of Proof: Existence

**Existence Proof** If $\mathbb{P}$ is of the form $\mathbb{P}: \exists x, y, z \ldots$ then existance proof appropriate.

**Ex 1:** $\mathbb{P}$: $\exists n \in \mathbb{N}$ such that $n = s^2 + t^2$, where $s, t \in \mathbb{N}, s \neq t$.

$\mathbb{N} = \{1, 2, 3, \ldots\}$.

Proof: Let $s = 1$, $t = 2$ and $n = 5$

$n = s^2 + t^2$

$5 = 2^2 + 1^2$

$5 = 4 + 1$

$5 = 5$.

Hence prooved

**Ex 2:** $\mathbb{P}$: $\exists a, b, c \in \mathbb{Z}$ such that $3a + 6b - 2c = 20$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

Proof: Let $a = 2$, $b = 2$, $c = -1$

$3a + 6b - 2c = 20$

$3(2) + 6(2) - 2(-1) = 20$

$6 + 12 + 2 = 20$

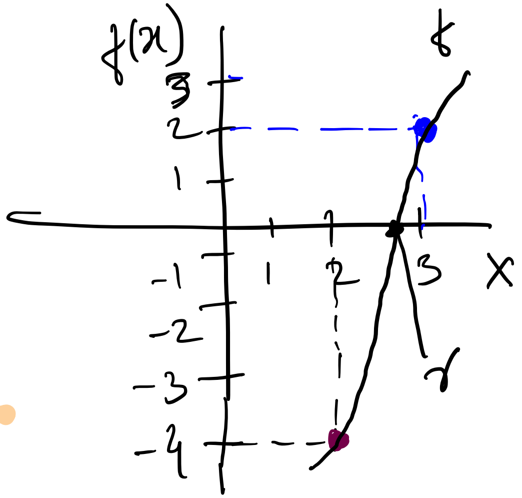
$20 = 20$.

Hence prooved

**<u>Ex 3</u>:** $\mathbb{P}$: **Let** $\mathbf{f(x) = x^3 - 3x^2 + 2x - 4}$**, then** $\exists r \in \mathbb{R}$**, such that** $f(r) = 0$**, and** $2 < r < 3$

Proof: Let $x = 2$ $\quad f(2) = 2^3 - 3(2)^2 + 2(2) - 4$
$= 8 - 12 + 4 - 4 = -4$

Let $x = 3$ $\quad f(3) = 3^3 - 3(3)^2 + 2(3) - 4$
$= 27 - 27 + 6 - 4 = 2$




$f(x)$ is a polynomial, so it is smooth and continuous.

$\therefore \exists r$ such that $2 < r < 3$ and $f(r) = 0$

Hence prooved

Intermediate value theorem (IVT).

**Ex 4: Disprove:** $\forall x, y \in \mathbb{N}, \sqrt{x^2+y^2} = x+y$

Proof: $x = 3$ and $y = 4$

$$\sqrt{3^2+4^2} \stackrel{?}{=} 3+4$$

$$\sqrt{9+16} \stackrel{?}{=} 7$$

$$\sqrt{25} \stackrel{?}{=} 7$$

$$5 \neq 7.$$

$\boxed{\exists x, y \in \mathbb{N}, \sqrt{x^2+y^2} \neq x+y}$ Hence, proved