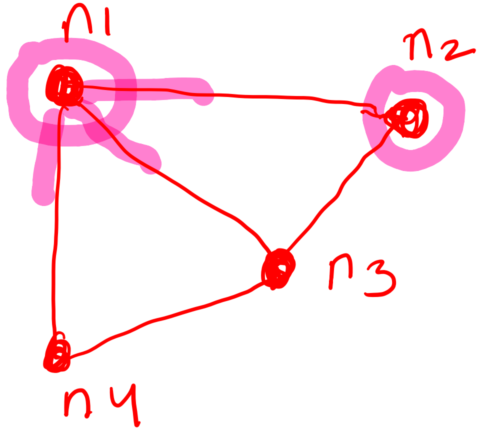
Module 10

Graphs

Graph: A graph, G is non-empty finite set of nodes N and edges E.

Nodes (or vertices): represent things and are identified $N = \{n_1, n_2, n_3, \ldots, n_K\}$ $|N| = K$

Edges: are of the form $\{n_i, n_j\}$, and they represent relationship between nodes.

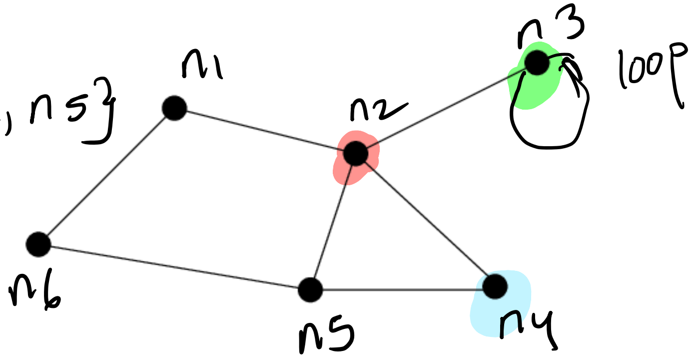


Degree: # of edges that lead to a node.

degree $(n_1) = 3$
degree $(n_2) = 2$

Ex 1: Given the following graph, identify:

a. **Nodes:** $N = \{n_1, n_2, n_3, n_4, n_5, n_6\}$



b. **Edges:** $E = \{\{n_1, n_2\}, \{n_1, n_6\}, \{n_2, n_3\}, \{n_2, n_4\}, \{n_2, n_5\}, \{n_3, n_3\}, \{n_4, n_5\}, \{n_5, n_6\}\}$

Adjacency: Two nodes are adjacent, if they are connected by a edge.

Ex 2: Represent adjacency of the graph above using:

a. **Adjacency Lists**

$n_1 : n_2, n_6$

$n_2 : n_1, n_3, n_5, n_4$

$n_3 : n_2, n_3$

$n_4 : n_2, n_5$

$n_5 : n_2, n_4, n_6$

$n_6 : n_1, n_5$

b. **Adjacency Matrix**

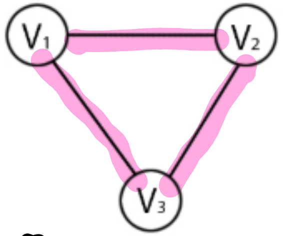
1 : adjacent

0 : not adjacent

$$
\begin{array}{c|cccccc}
 & n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \\
\hline
n_1 & 0 & 1 & 0 & 0 & 0 & 1 \\
n_2 & 1 & 0 & 1 & 1 & 1 & 0 \\
n_3 & 0 & 1 & 1 & 0 & 0 & 0 \\
n_4 & 0 & 1 & 0 & 0 & 1 & 0 \\
n_5 & 0 & 1 & 0 & 1 & 0 & 1 \\
n_6 & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

Directed Graph (digraph): is a graph which is made up of nodes connected with edges where direction is associated.

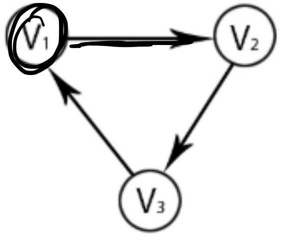
Undirected Graph



$V = \{V_1, V_2, V_3\}$

$E = \{\{V_1, V_2\}, \{V_2, V_3\}, \{V_1, V_3\}\}$

Directed Graph



$V = \{V_1, V_2, V_3\}$

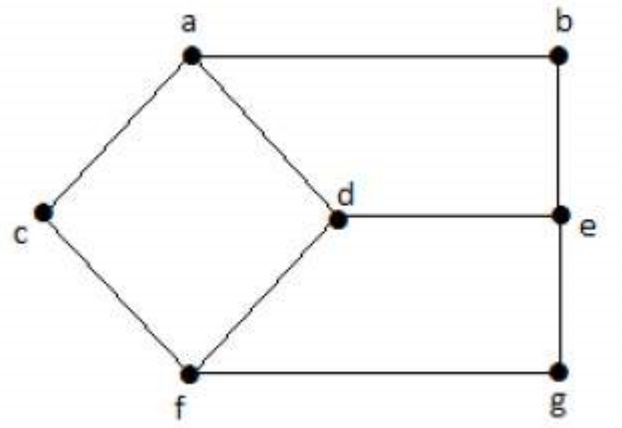
$E = \{(V_1, V_2), (V_2, V_3), (V_3, V_1)\}$

Walk : $V_1 \to V_3$: $V_1 V_3$, $V_1 V_2 V_3$.

Path: is a walk with no repeated vertices or edges. (Open).

Cycle: a closed path.

<u>Ex 3</u>: Given the following graph, identify:



a. **Paths from $a \to g$:**

adeg, acfg, abeg, acfdeg, ...

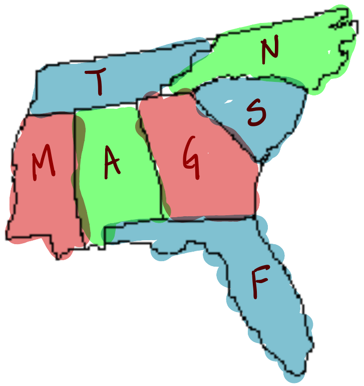
b. **Cycles from $\boxed{d \to d}$:**

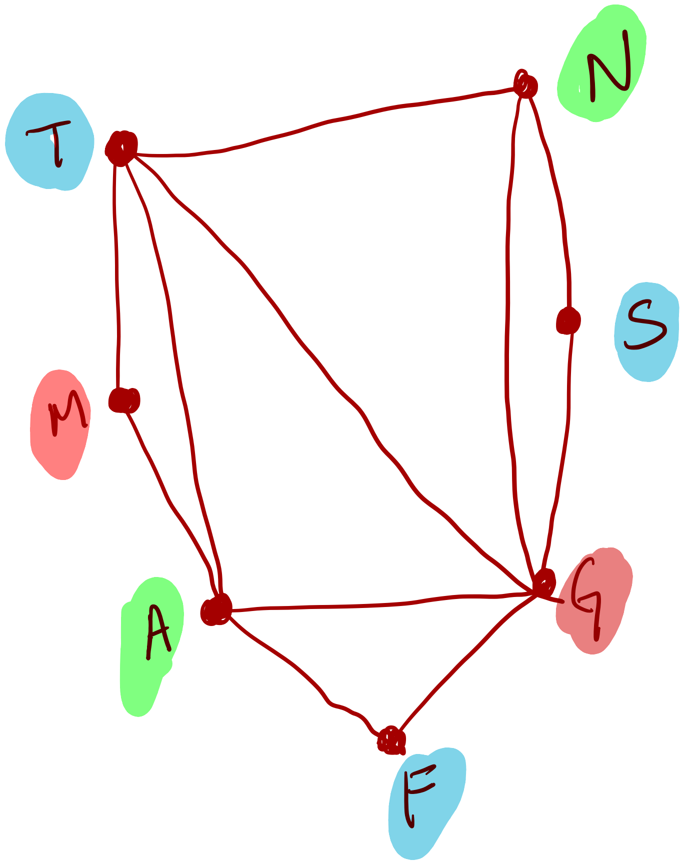
dacfd, debad, degfd, dabegfd

...

Graph Coloring

Q: Color the map with minimum number of colors such that states that share the boundary do not have same color?

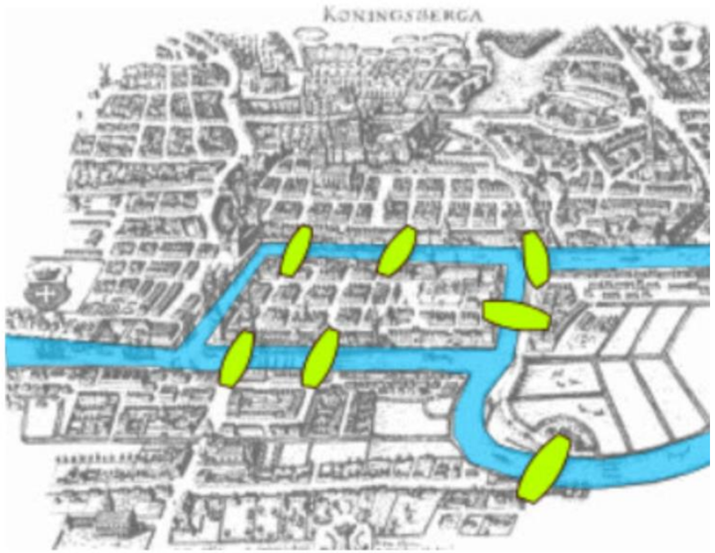


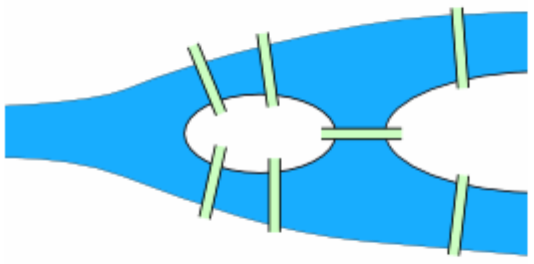


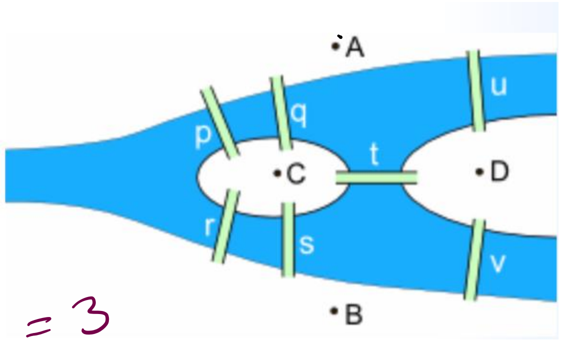
3 colors

The Seven Bridges of Konigsberg

Q: can you take a walk through the town, visiting each part of the town and crossing each bridge only once?





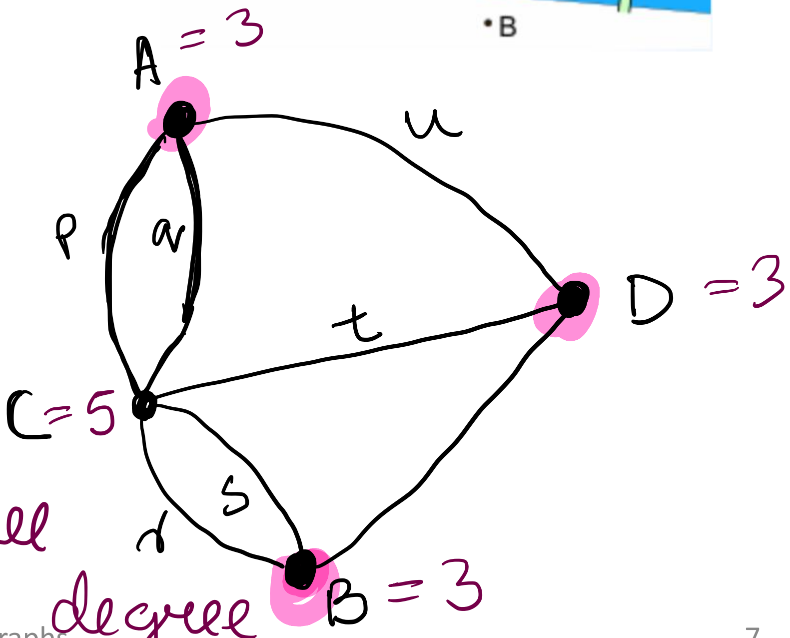


NO !

Clue : Euler path

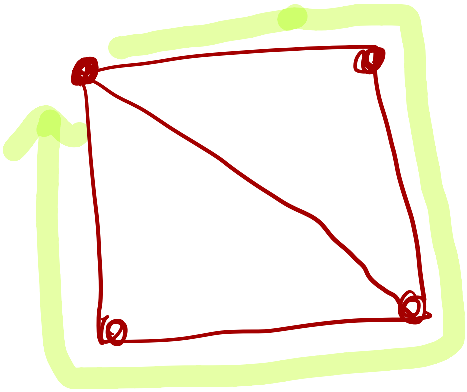
① Count the vertices with odd degree (0 or two)

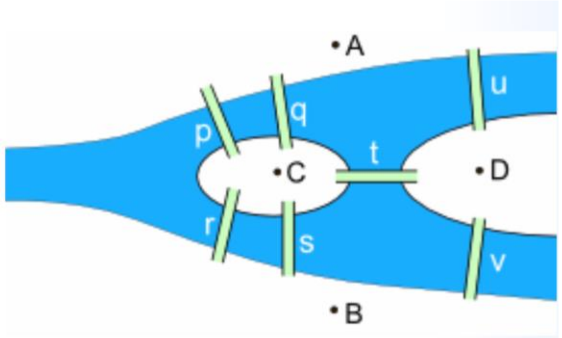
② start with vertice that have odd degree and end up in the other node odd degree.



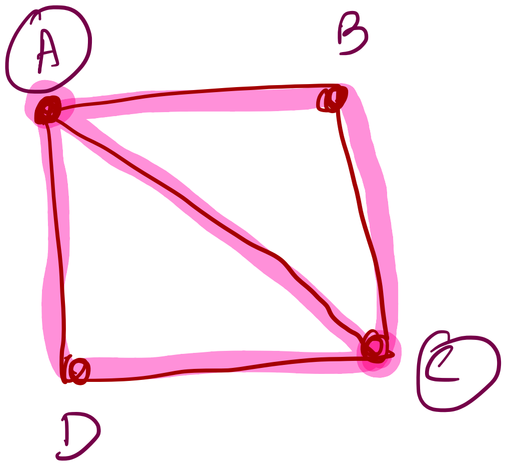
The Seven Bridges of Konigsberg:

Simple path: A walk around the graph such that you visit every vertex/node Once.



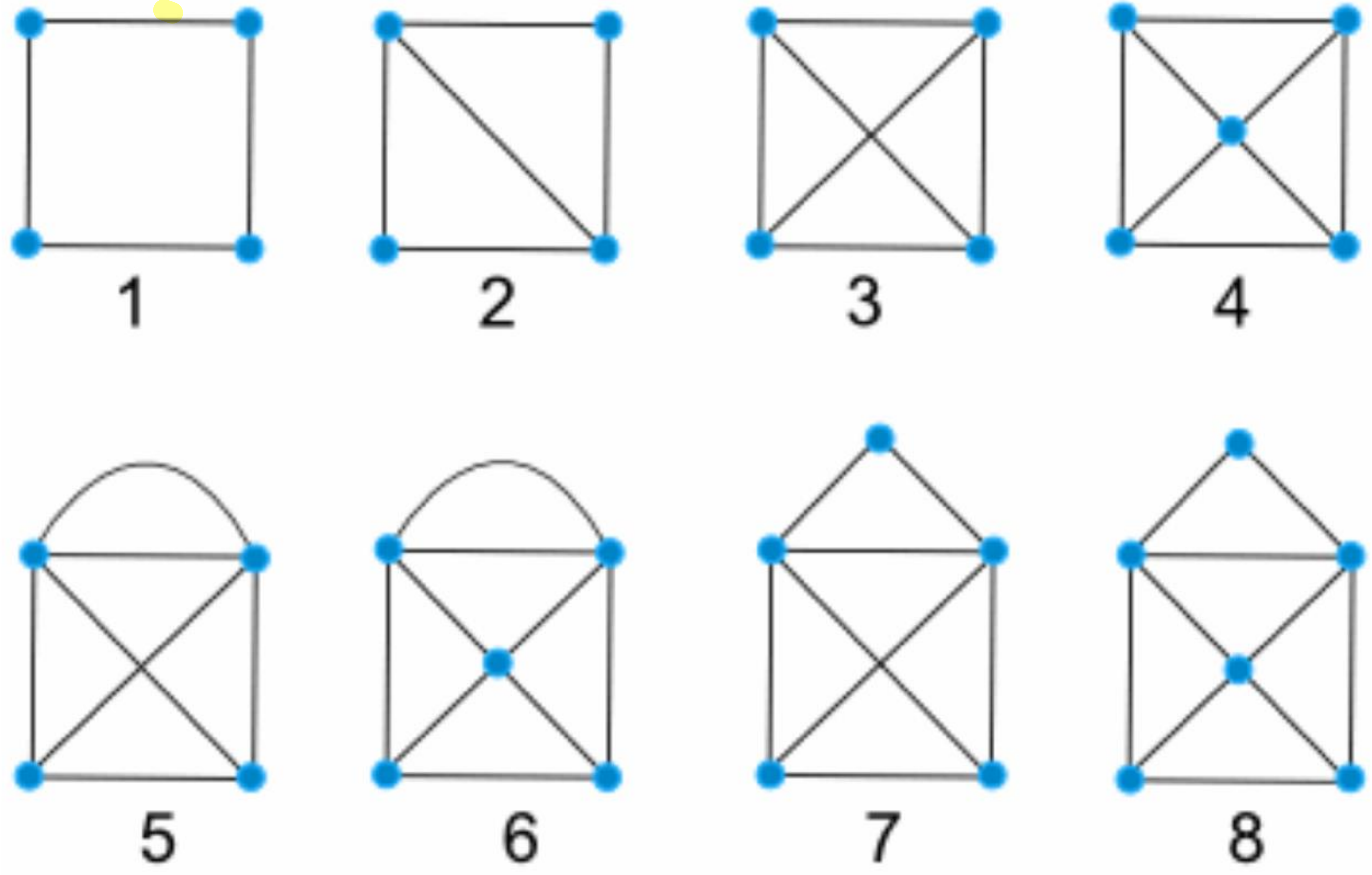


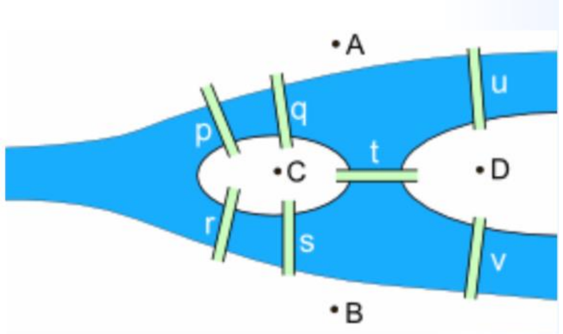
Euler path: A walk around a graph that visits every edge once is called a euler path.

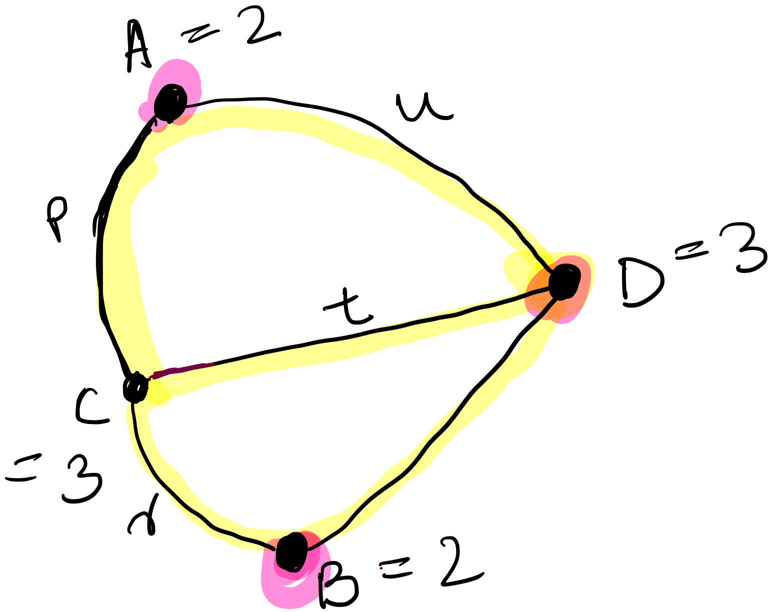


The Seven Bridges of Konigsberg

TRY To Find Euler path for graph for 1 - 10







Traveling Salesman Visualization

Optional
Click to watch
the video.

Max Flow

Ford-Fulkerson Algorithm

Optional
Click the link to
watch the video