Newtonian Mechanics

Mechanics is the study of the motion of objects and the effect of forces acting on those objects. It is the foundation of several branches of physics and engineering. **Newtonian,** or **classical, mechanics** deals with the motion of **ordinary** objects—that is, objects that are large compared to an atom and slow moving compared with the speed of light. A model for Newtonian mechanics can be based on **Newton's laws of motion:**†

1. When a body is subject to no resultant external force, it moves with a constant velocity.
2. When a body is subject to one or more external forces, the time rate of change of the body's momentum is equal to the vector sum of the external forces acting on it.
3. When one body interacts with a second body, the force of the first body on the second is equal in magnitude, but opposite in direction, to the force of the second body on the first.

Experimental results for more than two centuries verify that these laws are extremely useful for studying the motion of ordinary objects in an **inertial reference frame**—that is, a reference frame in which an undisturbed body moves with a constant velocity. It is Newton's second law, which applies only to inertial reference frames, that enables us to formulate the equations of motion for a moving body. We can express Newton's second law by

$$\frac{dp}{dt} = F(t, x, v) ,$$

where $F(t, x, v)$ is the resultant force on the body at time t, location x, and velocity v, and $p(t)$ is the momentum of the body at time t. The momentum is the product of the mass of the body and its velocity—that is,

$$p(t) = mv(t)$$

—so we can express Newton's second law as

$$m\frac{dv}{dt} = ma = F(t, x, v) , \tag{1}$$

where $a = dv/dt$ is the acceleration of the body at time t.

Typically one substitutes $v = dx/dt$ for the velocity in (1) and obtains a second-order lifferential equation in the dependent variable x. However, in the present section, we will focus on ituations where the force F does not depend on x. This enables us to regard (1) as a *first*-order equation

$$m\frac{dv}{dt} = F(t, v) \tag{2}$$

n $v(t)$.

To apply Newton's laws of motion to a problem in mechanics, the following general procedure may be useful.

Procedure for Newtonian Models

(a) Determine *all* relevant forces acting on the object being studied. It is helpful to draw a simple diagram of the object that depicts these forces.

(b) Choose an appropriate axis or coordinate system in which to represent the motion of the object and the forces acting on it. Keep in mind that this coordinate system must be an inertial reference frame.

(c) Apply Newton's second law as expressed in equation (2) to determine the equations of motion for the object.

$$V = \frac{gm}{k} + \left(V_0 - \frac{gm}{k}\right) e^{-\frac{k}{m}t} \qquad V = \frac{dx}{dt}$$

$$x = \int V\,dt$$

general solution

$$\boxed{x(t) = \int V\,dt = \frac{gm}{k}t - \frac{m}{k}\left(V_0 - \frac{gm}{k}\right) e^{-\frac{k}{m}t} + c_1}$$

IC $t = 0$ $\quad x(0) = \underline{\underline{0}}$

$$0 = -\frac{m}{k}\left(V_0 - \frac{gm}{k}\right) + c_1$$

$$c_1 = \frac{m}{k}\left(V_0 - \frac{gm}{k}\right)$$

$$\boxed{x(t) = \frac{mg}{k}t - \frac{m}{k}\left(V_0 - \frac{gm}{k}\right) e^{-\frac{k}{m}t} + \frac{m}{k}\left(V_0 - \frac{gm}{k}\right)}$$

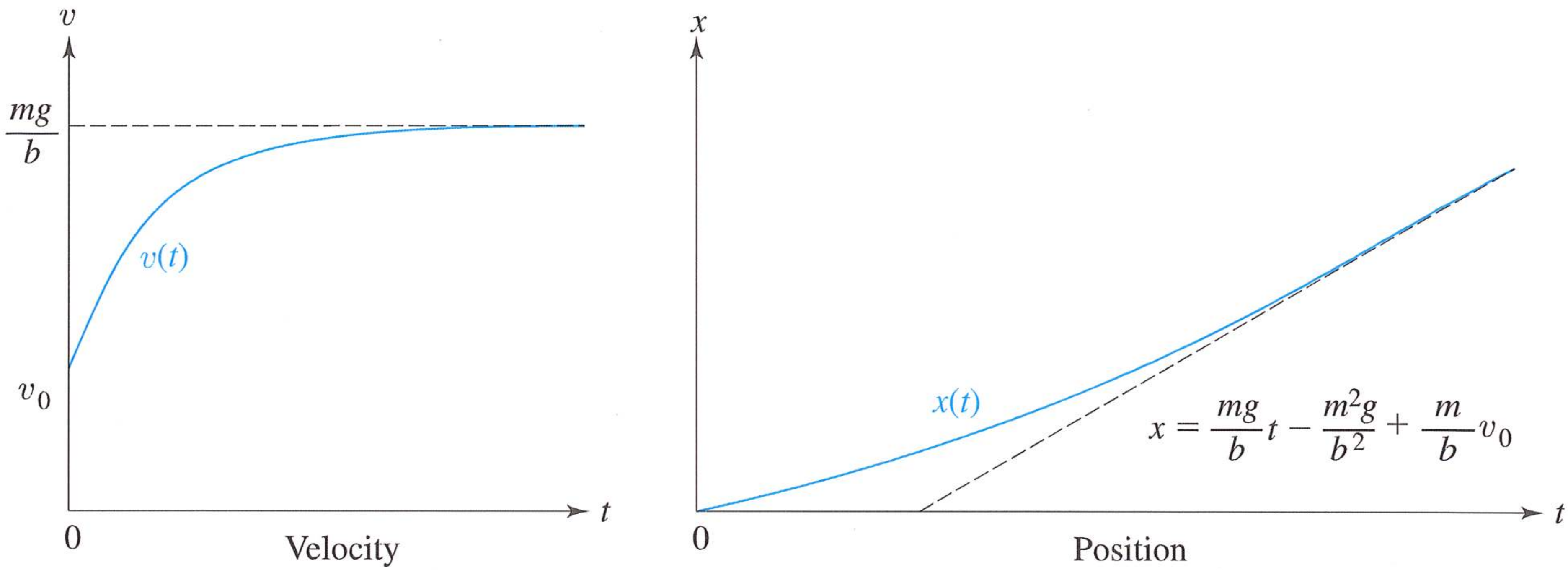

Figure 3.8 Graphs of the position and velocity of a falling object when $v_0 < mg/b$

$$y' - \frac{y}{x} = 3xe^{x} \qquad y(1) = 3e-3$$

$$P(x) = -\frac{1}{x} \qquad g(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln|x|} = x^{-1} = \frac{1}{x}$$

$$(g(x)\cdot y)' = Q(x)\cdot g(x)$$

$$\int \left(\frac{1}{x}y\right)' dx = \int 3\cancel{x}e^{x}\cdot\frac{1}{\cancel{x}}\,dx$$

$$\frac{1}{x}y = 3e^{x} + C$$

$$\boxed{y = 3xe^{x} + Cx}$$

general solution

$$3e-3 = 3e + C$$

$$C = -3$$

$$\boxed{y = 3xe^{x} - 3x}$$