Module 7

Methods of Proof: Direct General

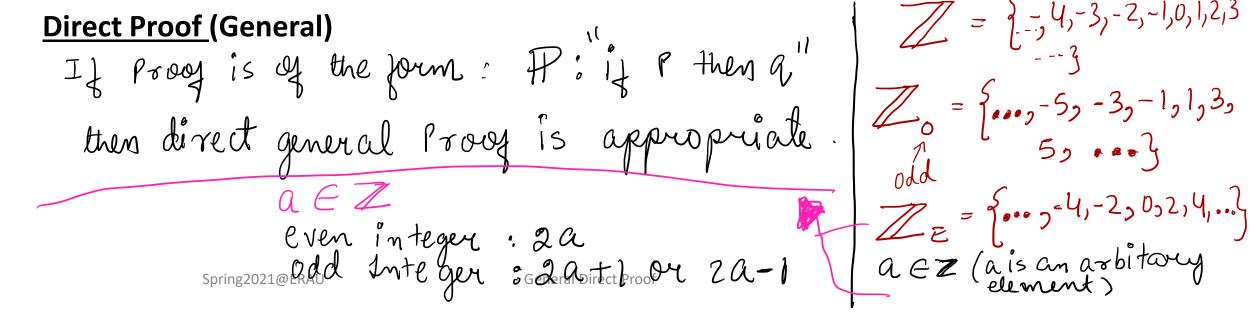
Rule of Universal Specification

If an open statement is true for all possible replacements in the designated universe, a finite universe of \mathcal{H} \mathcal{H}

If $\forall x p(x)$ is true, then we know that p(a) is true, for each a in the universe for x.

Rule of Universal Generalization

Universal generalization is the rule of inference that states that $\forall x P(x)$ is true, given the premise that P(c) is true for all elements c in the domain. Universal generalization is used when we show that $\forall x P(x)$ is true by taking an arbitrary element c from the domain and showing that P(c) is true. The element c that we select must be an arbitrary, and not a specific, element of the domain.



Ex 1: Prove the Theorem: Every odd integer is the difference of two perfect squares.

If there exists an odd integer, then its equal to difference of two perfect squares.

 $P: \forall n \in \mathbb{Z}, \tilde{n} \in \mathbb{Z}_o$ then $\exists s, t \in \mathbb{Z}, \text{ such that}$

 $\forall n \in \mathbb{Z}, n \in \mathbb{Z}_0 \longrightarrow \exists s, t \in \mathbb{Z}, such that n = s^2 t^2$

 $9 = 25 - 16 = 5^{2} + 4^{2}$ $(-5) = 4 - 9 = 2^2 - 5^2$ $\bigcap = (a+1)^2 - a^2$

20013 defination (i) Let $a \in \mathbb{Z}$ (ii) Let n = 2a+1 defination of odd

(III) $n = a^2 + 2a + 1 - a^2$ algebra (factoring)

 $(11) \cap = (a+1)^2 - a^2$

 $(V) n = 5^2 a^2$

S=a+1

100 Steps i-V Show that It is valid

-1+9 = 8 $\underline{\mathsf{Ex}\; 2}\colon \mathbb{P}\colon \forall \mathsf{m}, \mathsf{n} \in \underline{\mathbb{Z}_o}(\widehat{\mathsf{m}}) + (\widehat{\mathsf{n}}) \in \mathbb{Z}_E$ 5+3=8 Proof 8 17+1=18 defination (i) Let $a,b \in \mathbb{Z}$ defination of odd (ii) Let m = 2a+1dyn of odd. (iii) Let n = 2b+1 algebra (iV) m+n = 2a+1+2b+1 (V) m+n = 2a+2b+2algebra (y^i) m+n = 2(a+b+1)defination of even oo steps (i)-(vi) show that It is true.

Proof:

(i)
$$a \mid b$$
 given | Premise

(ii) Let $C \in \mathbb{Z}$ define

(iii) $b = Ca$ defination of $a \mid b$

(iv) $b+1 = Ca+1$ algebra

(v) $b+1 = a(c+1/a)$ algebra | factoring

(vi) $C+1/a \notin \mathbb{Z}$, $a \nmid b$ defination of $a \nmid b$

° o Steps (i) - (vi) proove P is valid.

Spring 2021@ERAU General Direct Proof

 $\underline{\mathsf{Ex}\; \mathsf{4}}\colon \mathbb{P}\colon \forall \mathsf{m}\in \mathbb{Z},\; \mathsf{4}|\mathsf{m})\to \mathsf{2}|\mathsf{m}$ Proof, Premise / given (i) 4/m define (ii) Let CEZ defination of a 1 b (II) m = 4C jactoring (1) m = 2(2C)defination of alb (V) 2 $C \in \mathbb{Z}$, So $2 \mid m$ Show Pistone.

(For Fun) (i) Let a = x what is a? what is x? explanations? (ii) ata = x+a 2a = x+a (iV) 2a-2x = x+a-2x 2 (a-x) = a-x dividing by zero ??