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CS 222 HW #5 Methods of Proof

1. Prove by direct Proof (formal).

$\neg r$	P_1
$t \to r$	P_2
$p \to q$	P_3
$\neg t \to p$	P_4
$(p \wedge q) \to s$	P_5
$\to s$	conclusion

(I)	$\neg r$	P_1
(II)	$t \to r$	P_2
(III)	$\neg\neg t$	(I),(II) Modus Tollens
(IV)	$\neg t \to p$	P_4
(V)	p	(III), (IV) Modus Ponens
(VI)	$p \to q$	P_3
(VII)	q	(V), (VI) Modus Penons
(VIII)	$(p \wedge q)$	(V), (VII) rule of conductive
(IX	$(p \wedge q) \to s$	$P5$
(X)	$\to s$	(VIII), (IX) Modus Ponens

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2.

p	P_1
$q \vee r$	P_2
$p \rightarrow \neg r$	P_3
$q \rightarrow \neg s$	P_4
$t \rightarrow s$	P_5
$\neg t$	

(I)	p	P_1
(II)	$p \Rightarrow \neg r$	P_3
(III)	$\neg r$	(I)(II) Modus Ponens
(IV)	$q \vee r$	P_2
(V)	q	(III), (IV) Rule of Disjunctive Syllogism
(VI)	$q \rightarrow \neg s$	$P4$
(VII)	$\neg s$	Modus Ponens
(VIII)	$t \rightarrow s$	$P5$
(IX)	$\neg t$	Modus Tollens

3. $\mathbb{N} = \{1, 2, 3 \ldots\}$

$x^4 = y^2$

let

Proof:

Let $x = 2$, $y = 4$

$x^4 = y^2$

$2^4 = 4^2$

$16 = 16$

Hence proved

4. n, m only prime integers

$n \cdot m$ only odd integers

Let $n = 3$ Prime

$m = 7$ Prime

$3 \cdot 7 = 21$ 21 is odd

$\therefore$ Proved

5. $\forall n \in \mathbb{Z}_O, n^2 + 1 \in \mathbb{Z}_E$

Proof:

Example

(I) Let $a \in \mathbb{Z}$ definition

$n = 3$ $n = 5$

(II) let $n = 2a + 1$ def of odd

$3^2 + 1 = 10$ ✓

(III) $n^2 + 1 = (2a+1)^2 + 1$ alg

$5^2 + 1 = 26$ ✓

(IV) $n^2 + 1 = 4a^2 + 4a + 2$ Definition of even #

$\boxed{= 2(2a^2 + 2a + 1)}$ Definition of even #

$\therefore$ Steps (I) – (IV) show that P is true.

6. Prove by direct proof

$\forall a, b, c \in \mathbb{N}$, if a divides b, and a divides c, then a divides b+c

$$a|b \wedge a|c \rightarrow a|b+c$$

Proof:

(I) $a|b \wedge a|c$ given

(II) $a|(b+c)$ given

(III) Let $r, s \in \mathbb{Z}$. Define

(IV) $b = ar$ Definition

(V) $c = as$ Definition

(VI) $b+c = ar+as$

$b+c = a(r+s)$ factoring

$\therefore$ $r+s$ is an integer, $a|b+c$ is proven

(I)–(VI) P is true!