Study Material Template

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3.1 Recall

Let's recall the main formulas in dynamics

Linear Momentum: $p = mv$ (1)

where m is the mass,
v is linear velocity.
Euler's laws

Angular Momentum: $L = I\omega$ (2)

where I is the moment of inertia,
ω is angular velocity.

3.2 Generalized forces. Generalized coordinates

3.3 Intro to dynamics

Dynamics in Robotics stablishes the relation between generalized forces on the robot $\tau(t)$ and robot motion $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$.

We can speak about torques as torques at the joints, external forces or torques applied from the environment.

When we speak about robot motion, we usually speak about joint trajectory or Cartesian trajcetory (of the end-effector).

The dynamic model of a rigid body is described by the following equation:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

So, when we do the dynamics model, we search for:

1. M(q) - inertia of mass matrix
2. C($q, \dot{q}$) - corioulis and centrifugal forces
3. g(q) - gravity forces
4. τ - generalized forces

The task of **Forward dynamics** is obtaining the robot motion $\ddot{q}$ based on applied forces, torques τ.

The task of **Inverse dynamics** is obtaining the forces or torques τ needed to achieve a desired motion $\ddot{q}$.

3.4 Approaches for dynamic modeling

Two core approaches for solving dynamics exist: Newton-Euler (force-torque based) and Euler-Lagrange (energy based).

3.4.1 Euler-Lagrange

Euler-Lagrange approach is based on the system's energy

- Kinetic energy
- Potential energy

Lagrangian

$$L(q, \dot{q}) = T(q, \dot{q}) - U(q), \text{ where } q \text{ are } \textit{generalized} \text{ forces}$$

Euler-Lagrange equation:

Intuition behind the Lagrangian.

3.5 Center of mass

1. The body moves so as if the mass is concentrated in the CoM.
2. All the applied forces are applied to the CoM.

Center of mass for a rigid body:

$$p_c = \frac{1}{m}\int_m \widetilde{p}dm = \frac{1}{m}\int_m \widetilde{p}\rho dV,$$

$\widetilde{p}$ is position of some point of the rigid body.

3.5.1 Tensor of inertia

Let's recall the formula:

$$A \times (B \times C) = (A \cdot C)B - (A \cdot B)$$

It is always symmetric.

3.6 Angular momentum

- angular momentum in frame 0

$$^0h = {}^0I^0w$$

- angular momentum in frame B

$$^Bh = {}^BI^Bw$$

Change from reference frame B to 0

3.7 Kinetics energy of a rigid body

3.8 Potential energy of a rigid body

3.9 Generalized forces. Generalized coordinates