Study Material Template

Your Name

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3.1 INtuition behind poles

In control theory, the system state-space equation

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

has the transfer function

$$G(s) = C(sI - A)^{-1}B + D.$$

Since $(sI - A)^{-1} = \mathrm{adj}(sI - A)\det(sI - A)$, where $\mathrm{adj}(sI - A)$ is the adjugate of $sI - A$, the poles of $G(s)$ are the numbers that satisfy $\det(sI - A) = 0$. This is exactly the characteristic equation of matrix A, whose solutions are the eigenvalues of A.

3.2 LQR

In pole-placement method, we want to place the poles in the specific spots (or, we choose specific eigenvalues). But it is not intuitive where to place them, especially for complex systems, systems with numerous actuators. So, the new method is proposed. The key concept of the method lies in optimization of choosing K.

In LQR we find an optimal K by choosing parameters that are important to us, specifically how well the system performs and how much effort it takes to reach this performance.

If $Q >> R$, then we are turning the problem of Let J be an additive cost function:

$$J(x_0, p(x,t)) = \int_0^{\infty} g(x,u)$$

Q - how bad if x is not where it is supposed to be. Q - nonnegative, positive semidefinite.

if the system is a positions, velocity, and $Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$ we penalize for

Suppose there is the best control law:

$$u = -kx$$

that minimizes the quadratic cost function.

$$J = \int x^T Q x + u^T R u$$

Hamiltonian-Jacobi-Bellman (HJB)

$$\min_u [g(x,u) + \frac{dJ}{dxf(x,u)}] = 0$$

Cost on effectiveness and energy to reach this effectiveness.

3.3 Subtopic 2.1

3.4 Subtopic 2.2

4 Controllability. Observability

Controllability

A system is controllable on $t_0 \le t \le t_f$ if it is possible to find a control input $u(t)$ that would drive the system to a desired state $x(t_f)$ from any initial state x_0.

Observability

A system is observable on $t_0 \le t \le t_f$ if it is possible to exactly estimate the state of the system $x(t_f)$, given any initial estimation error.

Observability (Alternative)

A system is observable on $t_0 \le t \le t_f$ if any initial state x_0 is uniquely determined by the output $y(t)$ on that interval.