Fundamentals of Robotics: Assignment 2

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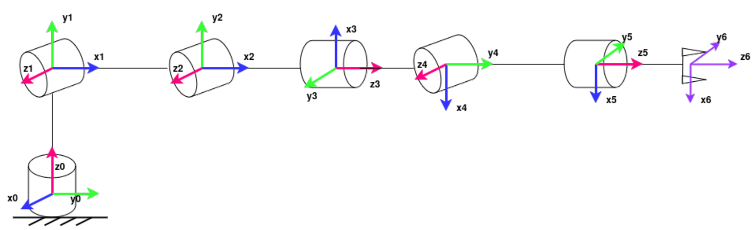
1 Link

Colab

2 Configuration of the chosen robot

Manipulator with antropomorphic elbow and a spherical wrist.



Forward kinematics:

$$^0T_1 = R_z(\theta_1^*)T_z(l_1)R_z(\frac{\pi}{2})R_x(\frac{\pi}{2})$$

$$^1T_2 = R_z(\theta_2^*)T_x(l_2)$$

$$^2T_3 = R_z(\theta_3^*)T_x(l_3)$$

$$^3T_4 = R_z(\theta_4^*)T_z(l_4)R_x(-\frac{\pi}{2})R_z(-\pi)$$

$$^4T_5 = R_z(\theta_5^*)T_y(l_5)R_x(-\frac{\pi}{2})$$

$$^5T_6 = R_z(\theta_6^*)T_z(l_6)$$

$$^0T_6 = {}^0T_1{}^1T_2{}^2T_3{}^3T_4{}^4T_5{}^5T_6$$

3 Task 1. Derive inverse kinematics for your robot model

3.1 q_1

Given: End-effector position and orientation ${}^0T_6$.

First, let's define the configuration of the wrist center by solving forward kinematics. The end-effector position corresponds to the position of the wrist center; the 6th joint defines the end-effector orientation.

$$\begin{aligned}
T_{01} &= R_z(\theta_1) \cdot T_z(l_1) \cdot R_z(\frac{\pi}{2}) \cdot R_x(\frac{\pi}{2}) \\
T_{12} &= R_z(\theta_2) \cdot T_x(l_2) \\
T_{23} &= R_z(\theta_3) \cdot T_x(l_3 + l_4) \cdot R_y(\frac{\pi}{2}) \cdot R_z(\frac{\pi}{2}) \\
T_{\text{wrist_center}} &= T_0 \cdot T_{01} \cdot T_{12} \cdot T_{23}
\end{aligned}$$

Let's solve the forward kinematics problem for the wrist center symbolically. Notice the translation column of the transformation matrix. From it we can retrieve the value of $q_1 = \theta_1$.

$$\begin{matrix} x \\ y \\ z \end{matrix} = \begin{matrix} 0.8\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - 0.8\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) - 1.0\sin(\theta_1)\cos(\theta_2) \\ -0.8\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + 0.8\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 1.0\cos(\theta_1)\cos(\theta_2) \\ 0.8\sin(\theta_2)\cos(\theta_3) + 1.0\sin(\theta_2) + 0.8\sin(\theta_3)\cos(\theta_2) + 0.3 \end{matrix}$$

We can see that:

$$\frac{x}{y} = -\frac{\sin(\theta_1)}{\cos(\theta_1)}$$

$$\tan(\theta_1) = -\frac{x}{y}$$

$$\theta_1 = -\arctan(\frac{x}{y})$$

3.2 q_3

We can find the value of $q_3 = \theta_3$ by the cosine theorem.

$$q3 = \arccos\left(\frac{(z - l1)^2 + (x^2 + y^2) - l2^2 - (l3 + l4)^2}{2 \cdot l2 \cdot (l3 + l4)}\right)$$

3.3 q_2

$$r = \sqrt{x^2 + y^2}$$

$$s = z - l1$$

$$q_2 = \arcsin\left(\frac{(l2 + (l3 + l4)\cos(q_3)) \cdot s - (l3 + l4)\sin(q_3) \cdot r}{r^2 + s^2}\right)$$

3.4 q_4

We know that

$$^0R_6 = {}^0R_3 \cdot {}^3R_6$$
$$^3R_6 = ({}^0R_3^T){}^0R_6$$

We have already calculated the angles q_1, q_2, and q_3, so we can calculate $^0R_3^T$. Additionally, we know $^0R_6$ since the end-effector position is provided to us. Thus, with $^3R_6$, we can determine all remaining angles.

Let's solve $^3R_6$ symbolically,

$$\begin{matrix} \sin(\theta_4)\sin(\theta_6) - \cos(\theta_4)\cos(\theta_5)\cos(\theta_6) & \sin(\theta_4)\cos(\theta_6) + \sin(\theta_6)\cos(\theta_4)\cos(\theta_5) & \sin(\theta_5)\cos(\theta_4) \\ -\sin(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_6)\cos(\theta_4) & \sin(\theta_4)\sin(\theta_6)\cos(\theta_5) - \cos(\theta_4)\cos(\theta_6) & \sin(\theta_4)\sin(\theta_5) \\ \sin(\theta_5)\cos(\theta_6) & -\sin(\theta_5)\sin(\theta_6) & \cos(\theta_5) \end{matrix}$$

In order to find q_4 we can solve $\frac{sin(\theta_4)\sin(\theta_5)}{\sin(\theta_5)\cos(\theta_4)}$.

If $\sin(\theta_5) \neq 0$, then $q_4 = \arctan(\frac{\sin(\theta_4)}{\cos(\theta_4)})$.

3.5 q_5

$$(\sin(\theta_5)\cos(\theta_4))^2 + (\sin(\theta_4)\sin(\theta_5))^2 = \sin(\theta_5)^2$$
$$\theta_5 = \arctan 2(\sqrt{(\sin(\theta_5)\cos(\theta_4))^2 + (\sin(\theta_4)\sin(\theta_5))^2}, \cos(\theta_5))$$

3.6 q_6

Analogically to q_4, we can find the value of q_6 from $^3R_6$:

$$\frac{-\sin(\theta_5)\sin(\theta_6)}{\sin(\theta_5)\cos(\theta_6)}$$

If $\sin(\theta_5) \neq 0$, then $q_6 = \arctan(\frac{-\sin(\theta_6)}{\cos(\theta_6)})$.

4 Task 4. Derive the jacobian matrix for your robot model.

$$J = [J_1 \quad J_2 \quad \dots \quad J_6]$$

In screw theory, the Jacobian is defined as:

$$J = \begin{bmatrix} U_{i-1} \times (O_n - O_{i-1}) \\ U_{i-1} \end{bmatrix}$$

U_{i-1} is the column vector corresponding to the axis of rotation matrix (in our case, z). O_{i-1} is the translation vector of transformation matrix for T_{i-1}.

<u>Singularity</u>
To check the singularity of the Jacobian, we need to check whether its determinant is not equal to 0.

5 Task 5. Plan a synchronized trajectory for all 6 joints between two poses. (consider 20Hz controller frequency)