Fundamentals of Robotics: Assignment 2

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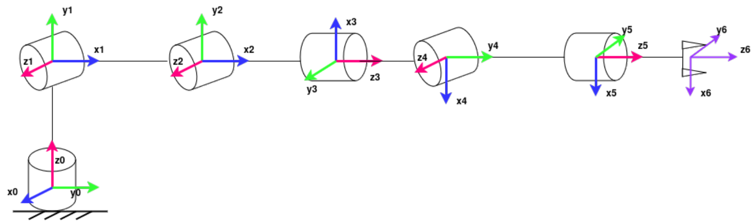
April 20, 2024

1 Link

Colab

2 Configuration of the chosen robot

Manipulator with anntropomorphic elbow and a spherical wrist.



Forward kinematics:

$$^0T_1 = R_z(\theta_1^*)T_z(l_1)R_z(\frac{\pi}{2})R_x(\frac{\pi}{2})$$

$$^1T_2 = R_z(\theta_2^*)T_x(l_2)$$

$$^2T_3 = R_z(\theta_3^*)T_x(l_3)$$

$$^3T_4 = R_z(\theta_4^*)T_z(l_4)R_x(-\frac{\pi}{2})R_z(-\pi)$$

$$^4T_5 = R_z(\theta_5^*)T_y(l_5)R_x(-\frac{\pi}{2})$$

$$^5T_6 = R_z(\theta_6^*)T_z(l_6)$$

$$^0T_6 = {}^0T_1{}^1T_2{}^2T_3{}^3T_4{}^4T_5{}^5T_6$$

Let's define its geometry:

$$l_1 = 1, l_2 = 0.5, l_3 = 0.4, l_4 = 0.4, l_5 = 0.5, l_6 = 0.4$$

And let's define the end-effector position:

$$x_6 = 1.5, y_6 = 0.5, z_6 = 1.5$$

3 Task 1. Derive inverse kinematics for your robot model

Given: End-effector position O_6 and orientation R_6.

Position P_c equals ${}^0T_3$:

$$ {}^0T_3 = {}^0T_1 {}^1T_2 {}^2T_3 $$

$$ {}^0T_3 = R_z(\theta_1^*)T_z(l_1)R_z(\frac{\pi}{2})R_x(\frac{\pi}{2})R_z(\theta_2^*)T_x(l_2)R_z(\theta_3^*)T_x(l_3) $$

$$ \left[\begin{matrix} \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2) & \cos(\theta_1) & \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - \sin(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ -\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\cos(\theta_1)\cos(\theta_2) & \sin(\theta_1) & -\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ -\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3) & 0 & \sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2) \\ 0 & 0 & 0 \end{matrix}\right. $$

We can retrieve the coordinates of the wrist center from the rightmost column of ${}^0T_3$.

$$ \begin{cases} x = -l_2\sin(\theta_1)\cos(\theta_2) + l_3\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - l_3\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ y = l_2\cos(\theta_1)\cos(\theta_2) - l_3\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + l_3\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ z = l_1 + l_2\sin(\theta_2) + l_3\sin(\theta_2)\cos(\theta_2) + l_3\sin(\theta_3)\cos(\theta_2) \end{cases} $$

$$ \begin{cases} x = -\sin(\theta_1)(l_2\cos(\theta_2) - l_3\sin(\theta_2)\sin(\theta_3) + l_3\cos(\theta_2)\cos(\theta_3)) \\ y = \cos(\theta_1)(l_2\cos(\theta_2) - l_3\sin(\theta_2)\sin(\theta_3) + l_3\cos(\theta_2)\cos(\theta_3)) \\ z = l_1 + l_2\sin(\theta_2) + l_3\sin(\theta_2)\cos(\theta_2) + l_3\sin(\theta_3)\cos(\theta_2) \end{cases} $$

$$ -\tan(\theta_1) = -\frac{\sin(\theta_1)}{\cos(\theta_1)} = \frac{x}{y} $$

$$ \theta_1 = -\arctan 2(\frac{x}{y}) $$

Thus,

$$ x_w = $$

$$ y_w = $$

$$ z_w = $$

Since we have already calculated q_1, q_2, q_3, we can calculate q_4, q_5, q_6.

$$ {}^3R_6 = ({}^0R_3^{-1}){}^0R_6 $$

4 Task 2. Solve inverse kinematics for multiple positions

1. Solve the first 3 joints for positioning the wrist
2. Solve the last 3 jooints for orienting the tool

Target reference frame:
Target wrist point: $p_w = t_T - l_6 z_T$
Since $0R_6 = 0R_3 \cdot 3R_6$.

5 Task 3. Track the number of solutions along the way and choose the correct one and closest to the previous (current) configuration.

6 Task 4. Derive the jacobian matrix for your robot model.

7 Task 5. Plan a synchronized trajectory for all 6 joints between two poses. (consider 20Hz controller frequency)

8 Task 5. Use the Jacobian matrix to check for singularities along the planned trajectory