Theoretical Mechanics: Week Homework 8

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1 Link

Link to the github repository containing all the materials

2 Task 1

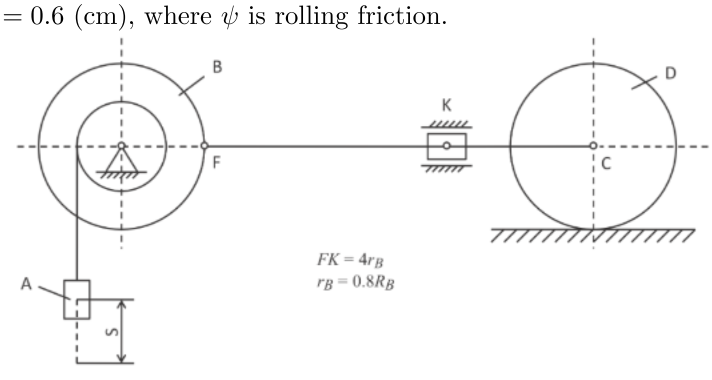
2.1 Task Description

A mechanical system under the gravity force moves from the rest. Define the velocity of object A if it travels distance s from the rest. The masses of the non-deformable ropes are ignored. Neglect the masses of links FK, KC and the piston K.

The task is to:

1. make a plot $v_A(s)$;
2. What will change if we omit the last sentence (Neglect ...). (Explain it and show on equations). Why Yablonskii made these constraints?

Needed variables:
$m_A = 1,\ m_B = 3,\ m_D = 20$ (kg);
$R_B = 20,\ R_D = 20,\ i_{Bx} = 18$ (cm), i_{Bx} - radii of gyration of the body;
$\psi = 0.6$ (cm), where ψ is rolling friction.



2.2 Task Explanation

2.2.1 Neglect masses

Research object: A system of a body A, wheels B, wheel D
Motion: A - plane motion, B - rotational motion, D - rotational and plane motions
Force analysis: $F_{\text{fr}} = \psi N$, **Solution:**

$$T = A, \text{ where } T = \sum T_i, A = \sum A_i$$

$$T = T_A + T_{\text{B\_rot}} + T_{\text{D\_rot}} + T_D$$

$$h = \sin\left(\frac{s \cdot 180}{\pi \cdot r_b}\right) \cdot R_b$$

$$T_A = \frac{m_a \cdot \text{dot\_s}^2}{2}$$

$$y_d = l\sin(\alpha) \qquad \dot{y}_d = l\dot{\alpha}\cos(\alpha)$$
$$x_d = l\cos(\alpha) \qquad \dot{x}_d = -l\dot{\alpha}\sin(\alpha)$$

$$\dot{\alpha} = \omega = \dot{\alpha} = \frac{v_b}{2l\cos\alpha}$$

$$V_d^2 = (l\cos(\alpha)\dot{\alpha})^2 + (-l\sin(\alpha)\dot{\alpha})^2 = l^2(\dot{\alpha})^2 = l\omega^2$$

$$V_d^2 = l^2\omega^2 = l^2 \cdot \frac{v_b^2}{4l^2\cos^2(\alpha)}$$

$$T_{\text{rot}} = \frac{J\omega^2}{2} = \frac{m\rho^2\omega^2}{2} = \frac{m\rho^2}{2}\frac{V_b^2}{4l^2\cos(\alpha)^2}$$

$$T_1 = T + T_{\text{rot}} = \frac{mV_d^2}{2} + \frac{J\omega^2}{2} = \frac{mV_b^2}{2 \cdot 4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{2 \cdot 4l^2\cos^2(\alpha)}$$

$$T_{\text{tot}} = 2T_1 = 2(\frac{mV_b^2}{2 \cdot 4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{2 \cdot 4l^2\cos^2(\alpha)}) = \frac{mV_b^2}{4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{4l^2\cos^2(\alpha)}$$

Work:
2 gravitational forces do job (of a rod 1 and of a rod 2).
Due to the symmetry:

$$A_{\text{tot}} = 2A_G = 2mg\Delta h = 2mg(\frac{h}{2} - l\sin(\alpha))$$

1. When B hits the floor, $\alpha = 0$, so $A_{\text{tot}} = 2mg\frac{h}{2} = mgh$

$$\text{For } \alpha = 0 \text{ we have: } \cos(\alpha) = 1,\ \sin(\alpha) = 0.$$

$$\frac{mV_b^2}{4\cos^2(\alpha)} + \frac{m\rho^2 V_b^2}{4l^2\cos^2(\alpha)} = mgh$$

$$\frac{mV_b^2}{4\cdot 1}(1 + \frac{\rho^2}{l^2}) = mgh$$

$$\frac{V_b^2}{4}(\frac{l^2+\rho^2}{l^2}) = gh$$

$$V_b = \sqrt{\frac{4l^2 gh}{l^2+\rho^2}} = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$$

$$\text{So, } V_b = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$$

2. When B is at the distance $\frac{1}{2}h$ from the floor.

$$\sin(\alpha) = \frac{\frac{h}{2}}{2l} = \frac{h}{4l}$$

$$\text{Thus, } \cos(\alpha)^2 = 1 - \sin(\alpha)^2 = \frac{16l^2 - h^2}{16l^2}$$

$$\frac{mV_b^2}{4\cdot\frac{16l^2-h^2}{16l^2}}(\frac{\rho^2+l^2}{l^2}) = 2mg(\frac{h}{2} - l\sin(\alpha))$$

$$\frac{mV_b^2}{\frac{16l^2-h^2}{4l^2}}(\frac{\rho^2+l^2}{l^2}) = 2mg(\frac{h}{2} - l\frac{h}{4l})$$

$$\frac{4mV_b^2(l^2+\rho^2)}{16l^2-h^2} = \frac{mgh}{2}$$

$$\frac{4V_b^2(l^2+\rho^2)}{16l^2-h^2} = \frac{gh}{2}$$

$$V_b = \sqrt{\frac{gh(16l^2-h^2)}{2\cdot 4(l^2+\rho^2)}} = \frac{1}{2}\sqrt{\frac{gh(16l^2-h^2)}{2(l^2+\rho^2)}}$$

$$\text{So, } V_b = \frac{1}{2}\sqrt{\frac{gh(16l^2-h^2)}{2(l^2+\rho^2)}}$$

Answer:

1.

$$V_b = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$$

2.

$$V_b = \frac{1}{2}\sqrt{\frac{gh(16l^2-h^2)}{2(l^2+\rho^2)}}$$

3 Task 2

3.1 Task Description

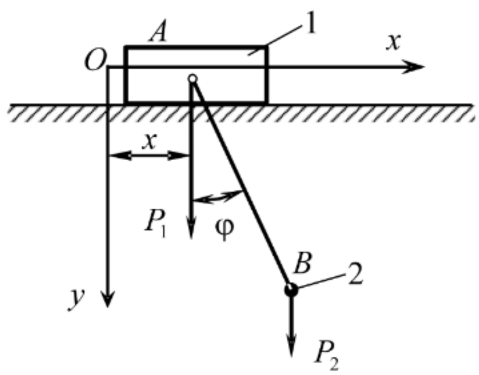
You have a a cart pole. Body 1 is a slider, mass m_1, it moves without friction.
AB is a massless rod with length l. Body 2 with mass m_2 is connected to AB in point B.

It's a 2 DoF system. You should take x and ϕ as a repFresentation of this system. The origin of each coordinate should be the same as on the picture.

Initial conditions:

1. $x = 0,\ \phi = 10°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
2. $x = 0.5,\ \phi = 45°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
3. $x = 0.5,\ \phi = -135°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

Parameters: $m_1 = 5\ kg,\ m_2 = 1\ kg,\ l = 1\ m.$



3.2 Task Explanation

Research object: A system of 2 bodies: cart and pole
Motion: A cart - plane motion, pole - rotational motion
Force analysis: $G_1 = m_1 g$, $G_2 = m_2 g, T, N$
Solution:

$$x_2 = x + l\sin(\phi) \qquad \dot{x_2} = \dot{x} + l\dot{\phi}\cos(\phi)$$

$$y_2 = -l\cos(\phi) \qquad \dot{y_2} = l\dot{\phi}\sin(\phi)$$

Let's define the kinetic energy (T):

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + 2\dot{x}l\dot{\phi}\cos\phi + l^2\dot{\phi}^2\cos^2\phi + l^2\dot{\phi}^2\sin^2\phi)$$

$$T = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + m_2\dot{x}l\dot{\phi}\cos\phi + \frac{1}{2}m_2l^2\dot{\phi}^2$$

Define the potential energy (V):

$$V = m_2gy_2 = -m_2gl\cos(\phi)$$

Define the Lagrangian (L) as the difference between kinetic and potential energy.

$$L = T - V = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + m_2\dot{x}l\dot{\phi}\cos\phi + \frac{1}{2}m_2l^2\dot{\phi}^2 + m_2gl\cos(\phi)$$

Calculate partial derivatives of the Lagrangian with respect to generalized coordinates and velocities.

$$\frac{\partial L}{\partial \dot{x}} = (m_1 + m_2)\dot{x} + m_2l\dot{\phi}\cos(\phi)$$

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{\phi}} = m_2\dot{x}l\cos(\phi) + m_2l^2\dot{\phi}$$

$$\frac{\partial L}{\partial \phi} = -m_2\dot{x}l\dot{\phi}\sin(\phi) - m_2gl\sin(\phi)$$

Apply the Euler-Lagrange equations to find the equations of motion.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = m_2\dot{x}l\cos(\phi) - \dot{\phi}m_2\dot{x}l\sin(\phi) + m_2l^2\dot{\phi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (m_1 + m_2)\ddot{x} + m_2l\ddot{\phi}\cos(\phi) - m_2gl\sin(\phi)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

Solving the system computationally will provide us with the results below.

3.3 Plots

3.4 Simulation

3.5 Discussion

As we can see, both Newton-Euler and Lagrange-Euler methods provide us with the same solutions.