Theoretical Mechanics: Big Homework 2

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1 Link

Link to the GitHub repository containing all the materials Colab

2 Task Description

1. Obtain the required measurements of the stand (like needed masses, lengths and so on).
2. Gather the positions and velocities of the stand. You should run the same experiment 3 times each.

   Initial conditions:

   - $x = 0$, $\phi = 15°$, $\dot{x} = 0$, $\dot{\phi} = 0$, $t = 0$;
   - $x = 0.25$ m, $\phi = 45°$, $\dot{x} = 0$, $\dot{\phi} = 0$, $t = 0$;
   - $x = 0.25$ m, $\phi = -135°$, $\dot{x} = 0$, $\dot{\phi} = 0$, $t = 0$;
3. Substitute real data to your math model from HW 7, 8 (you can choose any method you like) and compare the results (propose and justify the metric).
4. Explain what affects the difference between the math model and real data. Is the difference significant?
5. If so, change the model (add new forces, change the object representation), gather new needed data and compare it again.
6. Make a conclusion.

2.1 What report should contain

- The list of used tools and applications (I gathered a trajectory dataset using x tool), etc.
- The list of data you gathered from the stand and how did you do it.

- Show how you conducted experiments. Is there any difference when you did the same experiment? Show it using plots and/or other metrics like *std*, *mse* and so on.
- Show the way how you chose the metrics for trajectory comparison, how you justify the answer.
- If the error is too large, explain how you wanted to change your model and why you chose such a path.
- Summarize your experience.

3 Tools

Python was used for data handling and solving dynamics. Arduino was used for obtaining the data from the encoders of the stand.

4 Methodology

4.1 Data retrieving

To measure the state (position and velocity) of our system, we used the data from the encoders of the stand. Firstly, we changed the encoders tape to a new one since the suggested one provided us with noisy, inaccurate results. In GitHub, you can find the Arduino script for reading the data from the encoders.

For both encoders, we measured the maximum range of values. Knowing the length of the rails (428 mm) and the maximum possible rotation angle of the rod (360°), we can convert to the desired unit of measurement.

We conducted 3 experiments for each set of initial conditions and obtained datasets of 200 values for each experiment.

1			2.6			4.12			4.25
2	Test1	Position	Angle	Test2	Position	Angle	Test3	Position	Angle
3	20:54:53.164	249.99	-131.60	20:58:03.512	249.35	-130.91	20:59:40.725	248.49	-130.69
4	20:54:53.211	250.13	-129.52	20:58:03.559	249.35	-130.13	20:59:40.771	248.92	-128.78
5	20:54:53.258	250.13	-124.54	20:58:03.605	249.92	-127.40	20:59:40.817	249.31	-123.97
6	20:54:53.305	250.10	-116.39	20:58:03.652	250.02	-121.72	20:59:40.911	249.20	-116.13
7	20:54:53.352	249.99	-104.82	20:58:03.698	249.99	-112.84	20:59:40.956	249.02	-104.91
8	20:54:53.399	249.31	-89.57	20:58:03.744	249.49	-100.49	20:59:41.002	247.45	-90.04
9	20:54:53.446	245.16	-70.59	20:58:03.836	247.20	-84.54	20:59:41.048	243.70	-71.45
10	20:54:53.492	235.55	-47.84	20:58:03.882	241.95	-64.78	20:59:41.093	235.66	-49.09
11	20:54:53.539	220.75	-21.45	20:58:03.929	231.33	-41.17	20:59:41.139	221.97	-23.05
12	20:54:53.586	208.60	5.81	20:58:03.975	216.39	-14.26	20:59:41.185	208.93	4.85
13	20:54:53.680	211.39	30.90	20:58:04.021	206.42	13.13	20:59:41.231	209.21	30.55
14	20:54:53.715	228.05	52.21	20:58:04.068	212.36	37.48	20:59:41.278	224.19	52.69
15	20:54:53.762	253.38	69.72	20:58:04.114	231.44	58.19	20:59:41.371	249.24	70.80
16	20:54:53.809	277.90	83.41	20:58:04.160	257.92	74.92	20:59:41.417	274.79	85.10
17	20:54:53.856	294.20	93.16	20:58:04.208	281.47	87.88	20:59:41.464	292.05	95.46
18	20:54:53.903	299.99	98.80	20:58:04.254	295.66	96.80	20:59:41.510	299.02	101.70
19	20:54:53.950	297.20	100.27	20:58:04.346	299.13	101.57	20:59:41.556	296.52	103.74
20	20:54:53.997	291.98	98.19	20:58:04.393	295.41	102.44	20:59:41.603	290.69	102.31

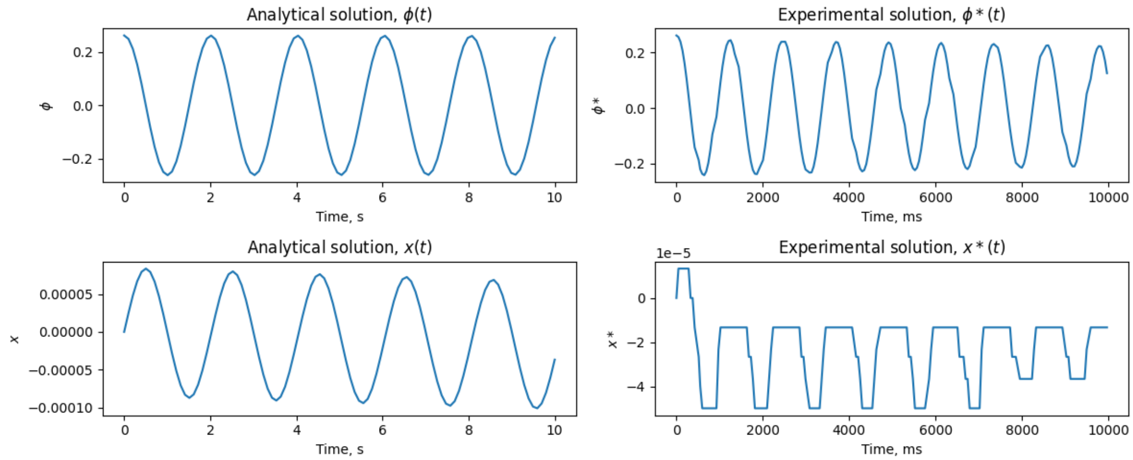
Figure 1: Example: extract of the obtained data table

4.2 Data visualization

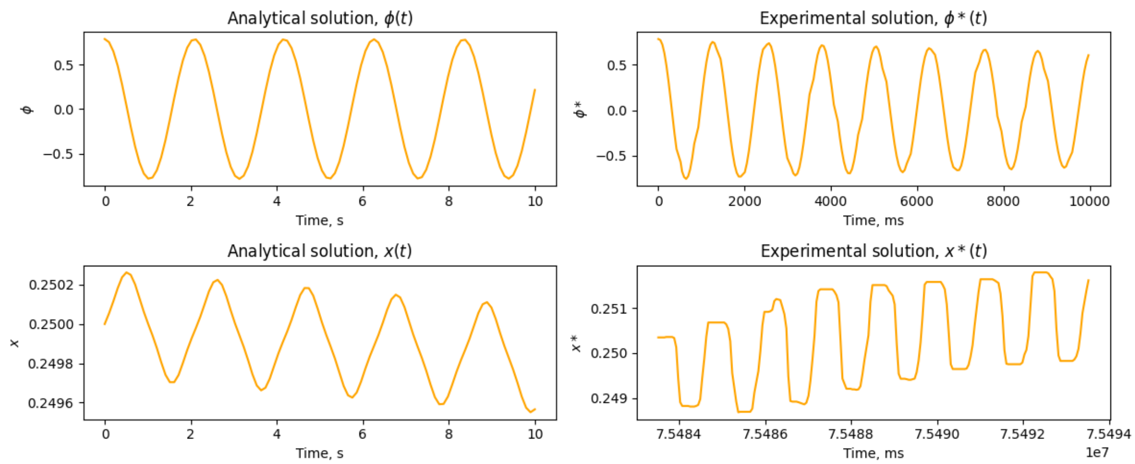
For each set of initial conditions, we calculated the mean value of the state (angle and position) from three experiments.

The plotted values are as follows.

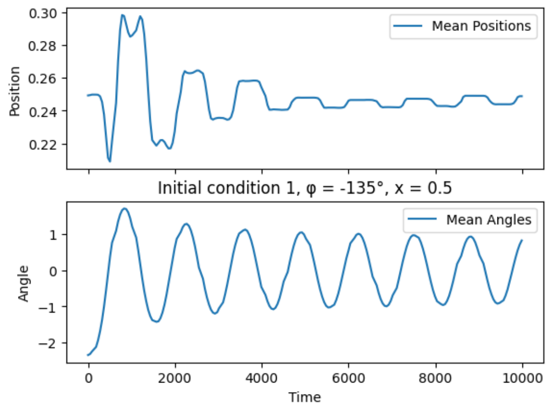
Initial condition 1: $\phi = 10°$, $x = 0$



Initial condition 2: $\phi = 45°$, $x = 0.5$



Initial condition 3: $\phi = -135°$, $x = 0.5$



5 Task Explanation. Analytical solution

Research object: A system of 2 rods: rod 1 and rod 2
Motion: Rod 1 - rotational and plane motions, rod 2 - rotational and plane motions
Force analysis: $G_1 = m_1 g$, $G_2 = m_2 g$, F_{fr}, F_{drag}
Solution:

Let's solve the system using Newton-Euler method.

To accurately model the real-world system, it is necessary to incorporate air drag and friction into our dynamics equations.

$$\text{eq1:} \quad -T\sin(\phi) + m_1\ddot{x} + km_1 g \tanh(1000\dot{x}) = 0$$

$$\text{eq2:} \quad N - m_1 g - T\cos(\phi) = 0$$

$$\text{eq3:} \quad T\sin(\phi) + m_2\left(\ddot{\phi}\cdot l\cos(\phi) - \dot{\phi}^2\cdot l\sin(\phi) + \ddot{x}\right) + k_{\text{air}}\sin(\phi)\tanh(1000\dot{x}) = 0$$

$$\text{eq4:} \quad T\cos(\phi) - m_2 g - m_2\left(\ddot{\phi}\cdot l\sin(\phi) + \dot{\phi}^2\cdot l\cos(\phi)\right) + k_{\text{air}}\cos(\phi)\tanh(1000\dot{x}) = 0$$

F_{fr} changes the sign, and to handle it, we introduce $\tanh(1000\dot{x})$.

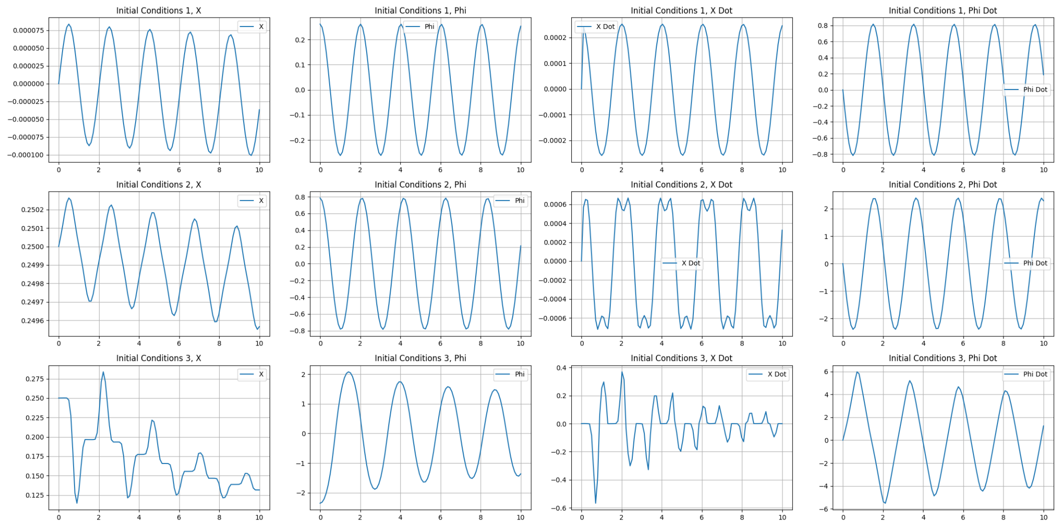
Substitute constants (e.g., masses) with the values we measured.

1. Mass of the rod's head - 154g.
2. Mass of the cart - 266g.
3. The mass of the rod, 11g, we can assume negligibly small.

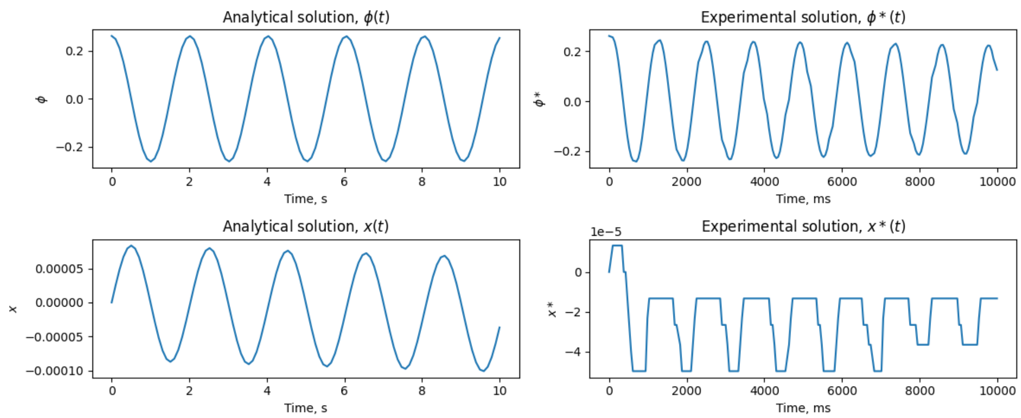
Take the average value of k_{air} in indoor conditions similar to ours, $k_{\text{air}} = 0.5$ and $k = 0.2$.

Solve computationally with sympy.

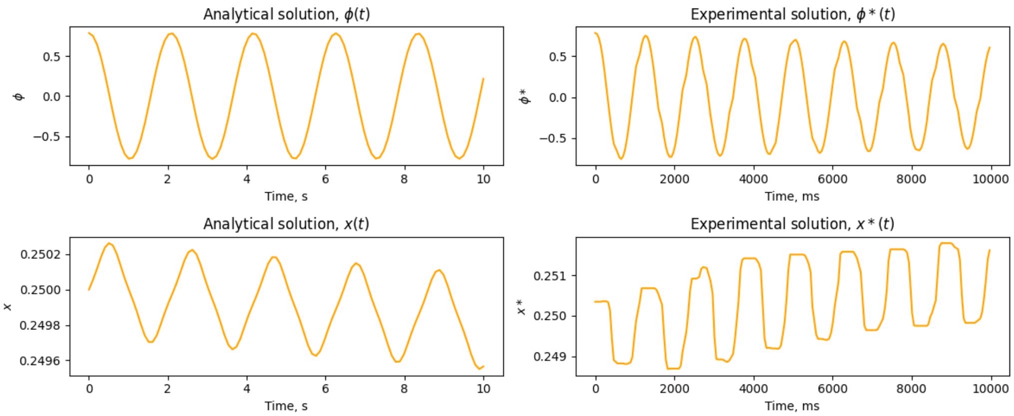
6 Plots



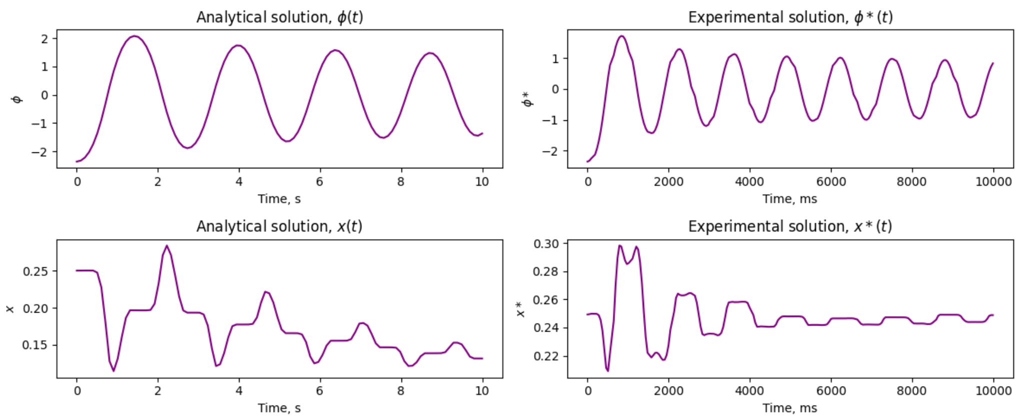
Initial condition 1: $\phi = 10°$, $x = 0$



Initial condition 1: $\phi = 45°$, $x = 0.25$



Initial condition 1: $\phi = -135°$, $x = 0.25$



7 Discussion

From the plots, we observe discrepancies in the positions of x, while ϕ is predicted quite accurately. Although we made assumptions and theoretical suggestions concerning the coefficients of friction and air resistance, their precise values are unknown to us. For more accurate results, determining these values is necessary.

Additionally, in our experiment, we neglected the mass of the rod due to its relatively small value. However, for future, more accurate research, considering this mass might be reasonable.

Also, the issue with x might arise from high friction in the bearings, causing the cart to deviate slightly from its initial position. Determining this friction coefficient in practice is a non-trivial task, which could be a subject for further research.

8 Team management:

Mikhail and I changed the tape in the cart-pole system to overcome the excessively high error. We connected an Arduino to it to obtain the required measurements of the stand. Timur, Leonid, Andrey, Elina, Reza, and I measured the values, while Mikhail and Yura solved the dynamics, data parsing - me.