Theoretical mechanics. Homework 1

Ekaterina Mozhegova

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1 Task 1

1.1 Tools

Python (Matplotlib, scipy, sympy)

1.2 Link to the simulation

https://colab.research.google.com/drive/18xTHbIfrOZOSFk18J3NwOZv4AvnBLmiQ?usp=sharing

1.3 Task description

You should find:

1. Simulate the move of $\vec{O}$ for $t = [0..10]$.

$$\vec{O} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3\cos(2t)\cos(t) + 0.82 \\ 3\cos(2t)\sin(t) + 0.82 \end{bmatrix}$$

2. Find and draw plots v, a, a_n, a_τ, κ (Osculating circle) with respect to t;

3. Find $y(x)$, $\vec{v}$, $\vec{a}$, $\vec{a}_n$, $\vec{a}_\tau$ and show it on the simulation.

1.4 Task explanation

1) Projections of velocity on x and y are found as $\dot{x}$ and $\dot{y}$ respectively, then the total velocity is found by the following formula: $v = \sqrt{v_x^2 + v_y^2}$.

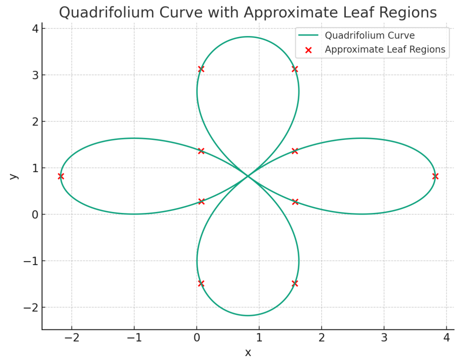
2) Projections of the acceleration are $\ddot{x}$ and $\ddot{y}$.

Tangential acceleration $a_\tau = \dfrac{\vec{a} \cdot \vec{v}}{|\vec{v}|}$. Since $a = \sqrt{a_\tau^2 + a_n^2}$, the normal acceleration a_n can be found as $a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{a_x^2 + a_y^2 - a_\tau^2}$.

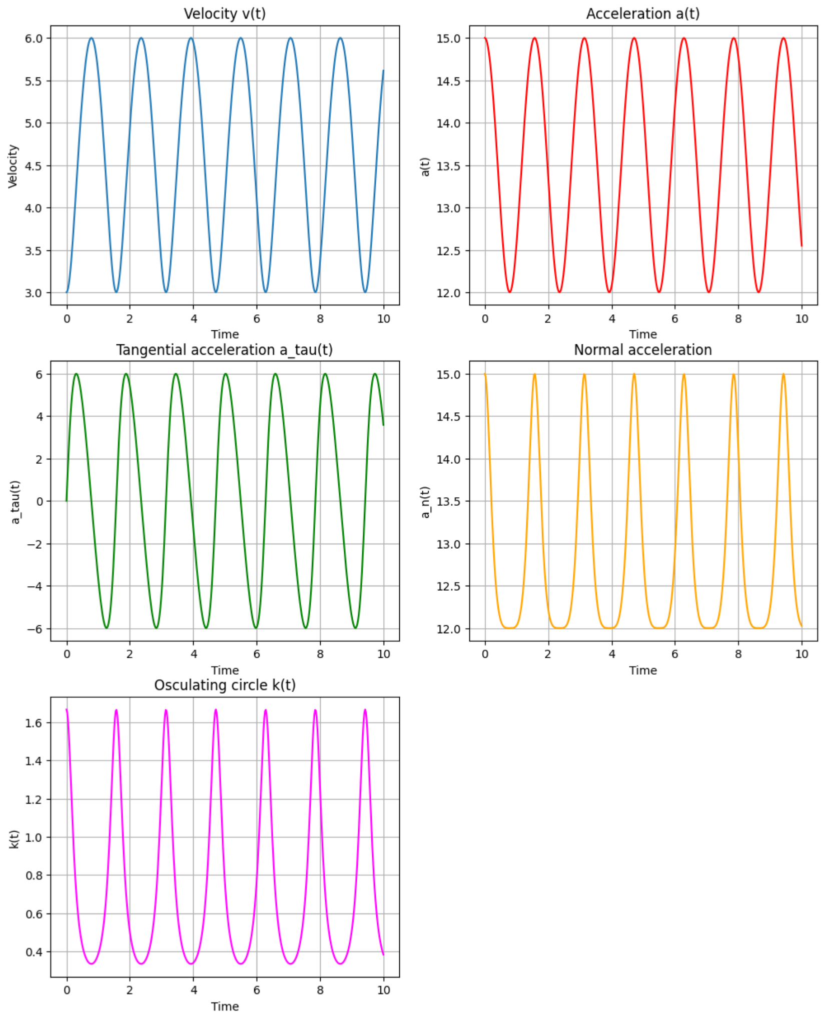
3) The curvature (K) of a curve can be found as a cross product of the velocity and acceleration vectors over the cube of the magnitude of the velocity vector:

$$K = \frac{\|\mathbf{r}'(t)\|^3}{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}$$
$$K = \frac{|v_x a_y - v_y a_x|}{v^3}$$

4) To find y(x), we can apply a method of interpolation for the generated points x and y. However, the function represented by the provided parametric equation does not constitute a function in the algebraic sense, as it lacks a strict one-to-one correspondence between x and y values. For greater precision, the curve should be treated as a piecewise function. To improve the result we can consider the following marked approximate regions where the leaves might be located.



1.5 Plots



1.6 Screenshots from simulation

Several screenshots, in some interesting positions. Example: parabola — mid-way of left branch, root, somewhere in right branch.

2 Task 2

2.1 Tools used for the task

GeoGebra

2.2 Link to the simulation

https://www.geogebra.org/calculator/jveychw3

2.3 Task description

You should solve the task, till the M point travels s:

1. Simulate this mechanism (obtain all positions of bodies 1, 2, 3)
2. Velocity for M(draw plots for magnitudes and show vectors on simulation);
3. Accelerations (tangent, normal, overall) for M(draw plots for magnitudes and show vectors on simulation);
4. Draw plots of angular velocities for 2, 3 bodies.

If $R_2 = 40, r_2 = 30, R_3 = 15, x = x(t) = 3 + 80t^2$, and $s_M = 0.5$.

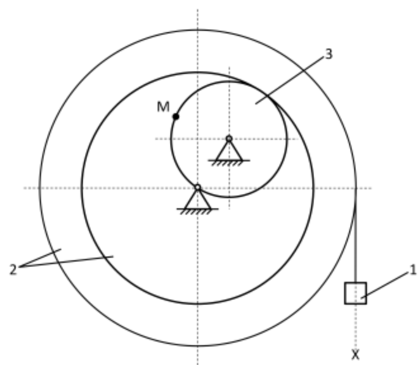


Figure 1: Task 2

2.4 Task explanation

To implement a simulation, it is necessary to know the time interval. Find t by the following way:

1) $x(t) = 3 + 80t^2$

$v(x) = \dot{x} = 160t$

$\omega = \dfrac{160t}{40} = 4t$

$\omega_2 = 4t$

Linear velocities of the ineer part of the big wheel is equal to the linear velocity of the small wheel.

$\omega_2 r_2 = \omega_3 R_3$

$v_3 = \omega_3 R_3 = \omega_2 r_2$

$s = \int v_3(t)dt = 5$

$s = \int \omega_3 R_3 dt = \int \omega_2 r_2 dt = \int 4tr_2 dt = 2t^2 r_2 \Rightarrow t = \sqrt{\dfrac{s}{4r_2}} \approx 0.28868$

Once you found time interval t, the simulated objects can be represented as functions of t, which are moving in a way corresponding the time change.

2.5 Plots

5. Plots. Put needed plots. Don't forget to make an appropriate title, legend, and axes description.

2.6 Screenshots from simulation

Several screenshots, in some interesting positions. Example: parabola — mid-way of left branch, root, somewhere in right branch.

3 Task 3

3.1 Link to the simulation

https://www.geogebra.org/calculator/erfh2pn8

3.2 Task description

You should find:

1. Simulate this mechanism (obtain all positions.) (xi (t), yi (t), where i is A, B, C point)
2. Velocities for B, C (draw plots for magnitudes and show vectors on simulation);
3. Accelerations for B and C (draw plots for magnitudes and show vectors on simulation);
4. Draw a plot of angular velocity of body BA.

$y_A(t) = 22.5 + \sin(5\pi t)$, where $t \in [0, 10]$ sec; AB = 45, AC = 30.

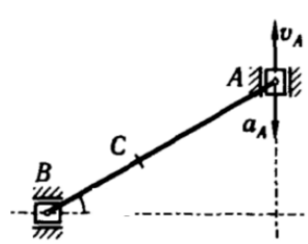


Figure 2: Task 3

3.3 Task explanation

1) We can find the law of motion for the body B from the right triangle by the Pythagorean's theorem. $x_b(t) = \sqrt{AB^2 - (y_a(t))^2}$.

Velocity $u_b = \dot{x}_b(t)$.

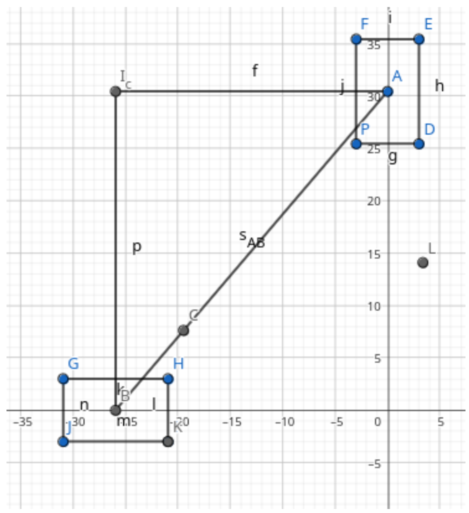


Figure 3: Detect Ic

2) To find the velocity at point C, first we need to find the instantaneous center of zero velocity I_c.

I_c lies at the point of intersection of lines perpendicular to the vectors of A's and B's velocities.

Then, $v_c = \omega_{\mathrm{AB}} \cdot CI_c$

3) Let's find ω_{AB}.

$\omega_{\mathrm{AB}} = \dfrac{v_a}{AB}$, where $v_a = \dot{y}_a(t)$.

3.4 Plots

5. Plots. Put needed plots. Don't forget to make an appropriate title, legend, and axes description.

3.5 Screenshots from simulation

Several screenshots, in some interesting psositions. Example: parabola — midway of left branch, root, somewhere in right branch.