Theoretical Mechanics: Week Homework 8

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1 Link

Link to the github repository containing all the materials
Colab

2 Tools used for the tasks

Python (sympy).

3 Task 1

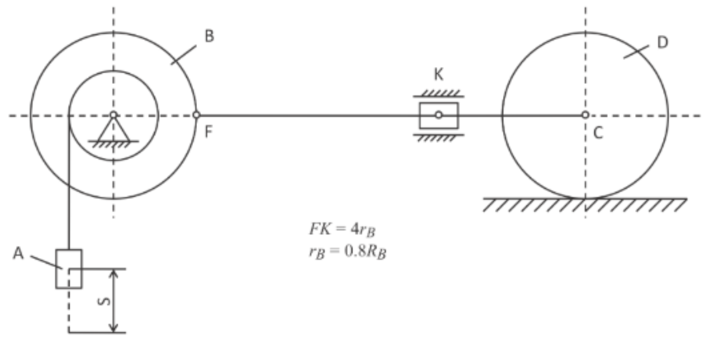
3.1 Task Description

A mechanical system under the gravity force moves from the rest. Define the velocity of object A if it travels distance s from the rest. The masses of the non-deformable ropes are ignored. Neglect the masses of links FK, KC and the piston K.

The task is to:

1. make a plot $v_A(s)$;
2. What will change if we omit the last sentence (Neglect ...). (Explain it and show on equations). Why Yablonskii made these constraints?

Needed variables:
$m_A = 1,\ m_B = 3,\ m_D = 20$ (kg);
$R_B = 20,\ R_D = 20,\ i_{Bx} = 18$ (cm), i_{Bx} - radii of gyration of the body;
$\psi = 0.6$ (cm), where ψ is rolling friction.



3.2 Task Explanation

3.2.1 Neglect masses

Research object: A system of a body A, wheels B, wheel D
Motion: A - plane motion, B - rotational motion, D - rotational and plane motions
Force analysis: $F_{\text{fr}} = \psi N$,

Solution:

Let's solve the task using Euler-Lagrange method.

First, we need to define the kinetic energy of the system.

$$
\begin{aligned}
T &= T_A + T_{B_{\text{rot}}} + T_{D_{\text{rot}}} + T_D \\
T_A &= \frac{m_a \dot{s}^2}{2} \\
T_{B_{\text{rot}}} &= \frac{(\frac{\dot{s}}{r})^2 \cdot J_b}{2} \\
T_{D_{\text{rot}}} &= \frac{m_d V_d^2}{4} \\
T_D &= \frac{m_d V_d^2}{2}
\end{aligned}
$$

Define the potential energy:

$$V = -m_a g s$$

Them, the Lagrangian is the following:

$$L = T - V$$

The generalized force is of the friction force origin.

$$\frac{dx}{ds} = \frac{dK}{ds}$$
$$F_{\text{fr}} = cm_d g$$
$$Q_{\text{fr}} = -F_{\text{fr}} \frac{dx}{ds}$$

Calculate partial derivatives of the Lagrangian with respect to generalized coordinates and velocities.

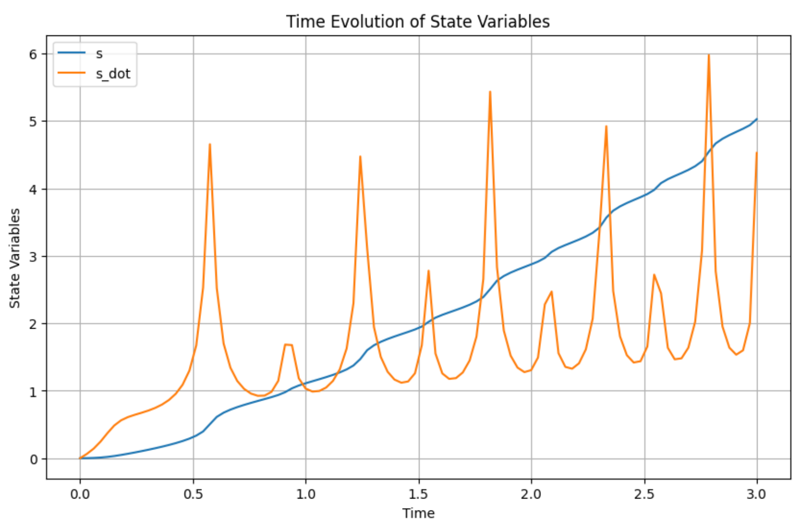
Apply the Euler-Lagrange equations to find the equations of motion. $\frac{\partial L}{\partial \dot{s}}$, $\frac{\partial L}{\partial s}$.

Apply the Euler-Lagrange equations to find the equations of motion.

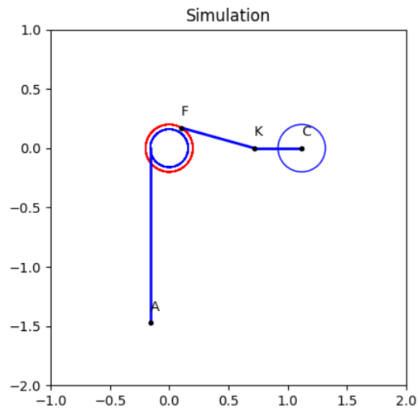
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) - \frac{\partial L}{\partial s} = Q$$

Solving the system computationally will provide us with the results below.

3.3 Plot



3.4 Simulation



4 Task 2

4.1 Task Description

You have a a cart pole. Body 1 is a slider, mass m_1, it moves without friction.

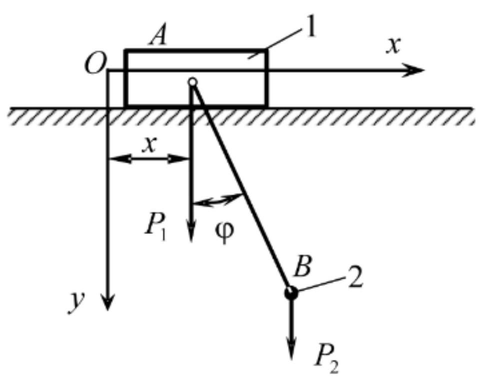
AB is a massless rod with length l. Body 2 with mass m_2 is connected to AB in point B.

It's a 2 DoF system. You should take x and ϕ as a repFresentation of this system. The origin of each coordinate should be the same as on the picture.

Initial conditions:

1. $x = 0,\ \phi = 10°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
2. $x = 0.5,\ \phi = 45°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$
3. $x = 0.5,\ \phi = -135°,\ \dot{x} = 0,\ \dot{\phi} = 0,\ t = 0;$

Parameters: $m_1 = 5\ kg,\ m_2 = 1\ kg,\ l = 1\ m.$



4.2 Task Explanation

Research object: A system of 2 bodies: cart and pole
Motion: A cart - plane motion, pole - rotational motion
Force analysis: $G_1 = m_1 g$, $G_2 = m_2 g, T, N$
Solution:

$$x_2 = x + l\sin(\phi) \qquad \dot{x_2} = \dot{x} + l\dot{\phi}\cos(\phi)$$
$$y_2 = -l\cos(\phi) \qquad \dot{y_2} = l\dot{\phi}\sin(\phi)$$

Let's define the kinetic energy (T):

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2(\dot{x}^2 + 2\dot{x}l\dot{\phi}\cos\phi + l^2\dot{\phi}^2\cos^2\phi + l^2\dot{\phi}^2\sin^2\phi)$$

$$T = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + m_2\dot{x}l\dot{\phi}\cos\phi + \frac{1}{2}m_2 l^2\dot{\phi}^2$$

Define the potential energy (V):

$$V = m_2 g y_2 = -m_2 g l\cos(\phi)$$

Define the Lagrangian (L) as the difference between kinetic and potential energy.

$$L = T - V = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + m_2\dot{x}l\dot{\phi}\cos\phi + \frac{1}{2}m_2 l^2\dot{\phi}^2 + m_2 g l\cos(\phi)$$

Calculate partial derivatives of the Lagrangian with respect to generalized coordinates and velocities.

$$\frac{\partial L}{\partial \dot{x}} = (m_1 + m_2)\dot{x} + m_2 l\dot{\phi}\cos(\phi)$$

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{\phi}} = m_2\dot{x}l\cos(\phi) + m_2 l^2\dot{\phi}$$

$$\frac{\partial L}{\partial \phi} = -m_2\dot{x}l\dot{\phi}\sin(\phi) - m_2 g l\sin(\phi)$$

Apply the Euler-Lagrange equations to find the equations of motion.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = m_2\dot{x}l\cos(\phi) - \dot{\phi}m_2\dot{x}l\sin(\phi) + m_2 l^2\dot{\phi}$$

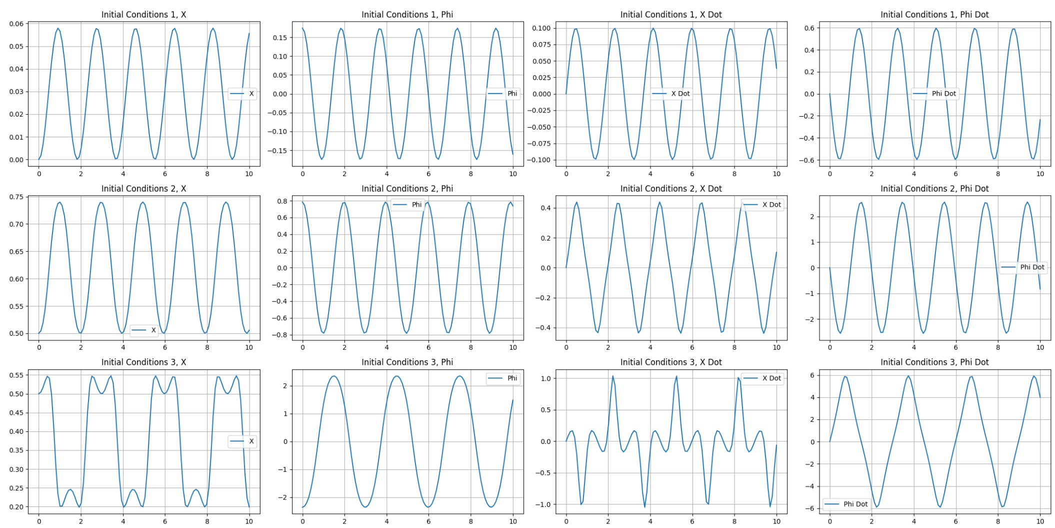
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (m_1 + m_2)\ddot{x} + m_2 l\ddot{\phi}\cos(\phi) - m_2 g l\sin(\phi)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

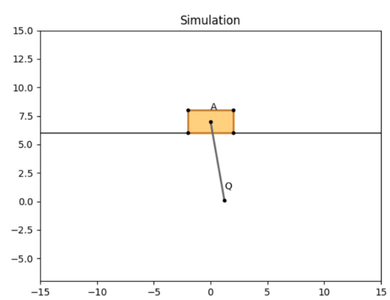
Solving the system computationally will provide us with the results below.

4.3 Plots

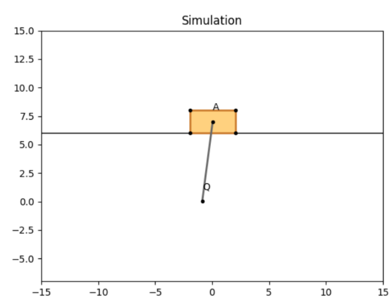


4.4 Simulation

Initial condition 1: $\phi = 10°$, $x = 0$

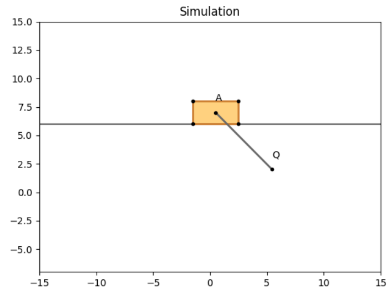


(a)



(b)

Initial condition 2: $\phi = 45°$, $x = 0.5$



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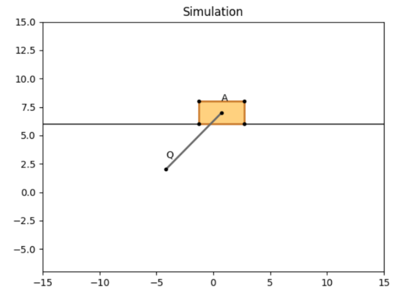
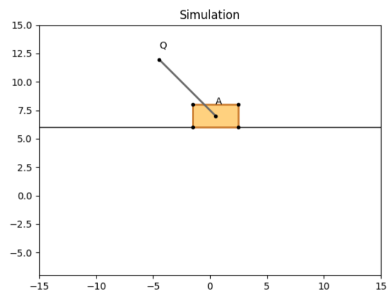
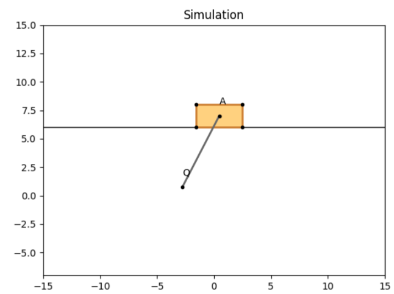


(b)

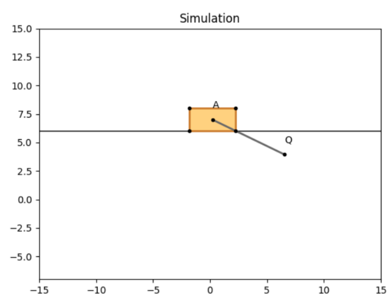
Initial condition 3: $\phi = -135°$, $x = 0.5$



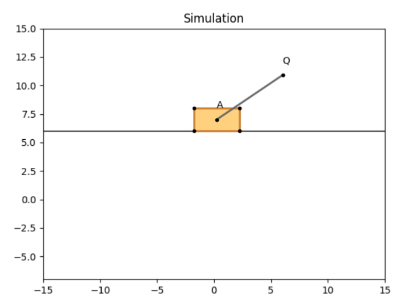
(a)



(b)



(c)



(d)

4.5 Discussion

As we can see, both Newton-Euler and Lagrange-Euler methods provide us with the same solutions.