Theoretical Mechanics: Week Homework 7

Ekaterina Mozhegova

March 13, 2024

1 Link

Link to the github repository containing all the materials Colab

2 Task 1

3 Tools used for the task

Python (sympy, scipy). Simulink

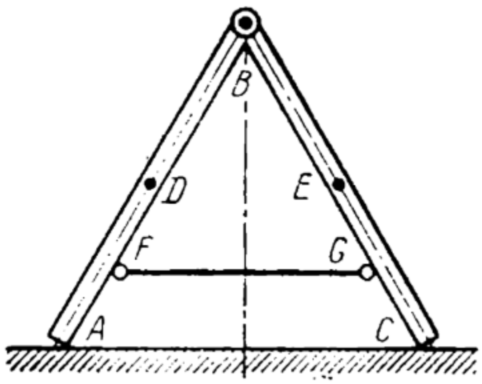
3.1 Task Description

A step ladder ABC, hinged at B, rests on a smooth horizontal floor, as shown on the figure. $AB = BC = 2l$.

The centres of gravity are at the midpoints D and E of the rods. The radius of gyration of each part of the ladder about the axis passing through the center of gravity is p.

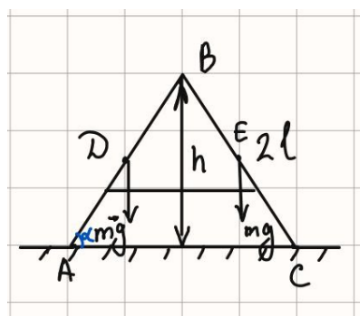
The distance between B and the floor is h. At the certain moment the ladder collapses due to the rupture of a ling FG between the two halves of the ladder. Neglecting the effect of friction in the hinge, determine:

1. the velocity v_1 of the point B at the moment, when it hits the floor;
2. the velocity v_2 of point B at the moment, when it is at a distance $\frac{1}{2}h$ from the floor.



3.2 Task Explanation

Research object: A system of 2 rods: rod 1 and rod 2
Motion: rod 1 - rotational and plane motions, rod 2 - rotational and plane motions
Force analysis: $G_1 = m_1 g$, $G_2 = m_2 g$



Solution:

$$T = A, \text{ where } T = \sum T_i, A = \sum A_i$$

$$T = T_1 + T_2 = 2T_1 (\text{due to the system's symmetry})$$

$$T_1 = \frac{mv_d^2}{2} + \frac{J\omega^2}{2} = \frac{mv_d^2}{2} + \frac{m\rho^2\omega^2}{2}$$

(plane motion and rotational motion)

$$v_d^2 = \dot{y}_d^2 + \dot{x}_d^2$$

$$y_d = l\sin(\alpha) \qquad \dot{y}_d = l\dot{\alpha}\cos(\alpha)$$

$$x_d = l\cos(\alpha) \qquad \dot{x}_d = -l\dot{\alpha}\sin(\alpha)$$

$$\dot{\alpha} = \omega = \dot{\alpha} = \frac{v_b}{2l\cos\alpha}$$

$$V_d^2 = (l\cos(\alpha)\dot{\alpha})^2 + (-l\sin(\alpha)\dot{\alpha})^2 = l^2(\dot{\alpha})^2 = l\omega^2$$

$$V_d^2 = l^2\omega^2 = l^2 \cdot \frac{v_b^2}{4l^2\cos^2(\alpha)}$$

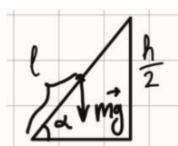
$$T_{\text{rot}} = \frac{J\omega^2}{2} = \frac{m\rho^2\omega^2}{2} = \frac{m\rho^2}{2}\frac{V_b^2}{4l^2\cos(\alpha)^2}$$

$$T_1 = T + T_{\text{rot}} = \frac{mV_d^2}{2} + \frac{J\omega^2}{2} = \frac{mV_b^2}{2\cdot 4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{2\cdot 4l^2\cos^2(\alpha)}$$

$$T_{\text{tot}} = 2T_1 = 2(\frac{mV_b^2}{2\cdot 4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{2\cdot 4l^2\cos^2(\alpha)}) = \frac{mV_b^2}{4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{4l^2\cos^2(\alpha)}$$

Work:
2 gravitational forces do the work (G_1 of a rod 1 and G_2 of a rod 2).
Due to the symmetry:

$$A_{\text{tot}} = 2A_G = 2mg\Delta h = 2mg(\frac{h}{2} - l\sin(\alpha))$$



1. When B hits the floor, $\alpha = 0$, so $A_{\text{tot}} = 2mg\frac{h}{2} = mgh$

For $\alpha = 0$ we have: $\cos(\alpha) = 1$, $\sin(\alpha) = 0$.

$$\frac{mV_b^2}{4\cos^2(\alpha)} + \frac{m\rho^2V_b^2}{4l^2\cos^2(\alpha)} = mgh$$

$$\frac{mV_b^2}{4\cdot 1}(1 + \frac{\rho^2}{l^2}) = mgh$$

$$\frac{V_b^2}{4}(\frac{l^2+\rho^2}{l^2}) = gh$$

$$V_b = \sqrt{\frac{4l^2gh}{l^2+\rho^2}} = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$$

$$\text{So, } V_b = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$$

2. When B is at the distance $\frac{1}{2}h$ from the floor.

$$\sin(\alpha) = \frac{\frac{h}{2}}{2l} = \frac{h}{4l}$$

$$\text{Thus, } \cos(\alpha)^2 = 1 - \sin(\alpha)^2 = \frac{16l^2 - h^2}{16l^2}$$

$$\frac{mV_b^2}{4 \cdot \frac{16l^2 - h^2}{16l^2}}(\frac{\rho^2 + l^2}{l^2}) = 2mg(\frac{h}{2} - l\sin(\alpha))$$

$$\frac{mV_b^2}{\frac{16l^2 - h^2}{4l^2}}(\frac{\rho^2 + l^2}{l^2}) = 2mg(\frac{h}{2} - l\frac{h}{4l})$$

$$\frac{4mV_b^2(l^2 + \rho^2)}{16l^2 - h^2} = \frac{mgh}{2}$$

$$\frac{4V_b^2(l^2 + \rho^2)}{16l^2 - h^2} = \frac{gh}{2}$$

$$V_b = \sqrt{\frac{gh(16l^2 - h^2)}{2 \cdot 4(l^2 + \rho^2)}} = \frac{1}{2}\sqrt{\frac{gh(16l^2 - h^2)}{2(l^2 + \rho^2)}}$$

$$\text{So, } V_b = \frac{1}{2}\sqrt{\frac{gh(16l^2 - h^2)}{2(l^2 + \rho^2)}}$$

Answer:

1.
$$V_b = 2l\sqrt{\frac{gh}{l^2 + \rho^2}}$$

2.
$$V_b = \frac{1}{2}\sqrt{\frac{gh(16l^2 - h^2)}{2(l^2 + \rho^2)}}$$

4 Task 2

4.1 Task Description

You should solve this problem using:

1. **Newton-Euler** method;
2. Model-oriented design applications (*SimInTech*, or MATLAB Simulink).

Tasks

1. To derive a differential equation of the motion, using **Newton-Euler** approach.

2. To create plots $x(t),\ \phi(t), \dot{x}(t), \dot{\phi}(t)$.

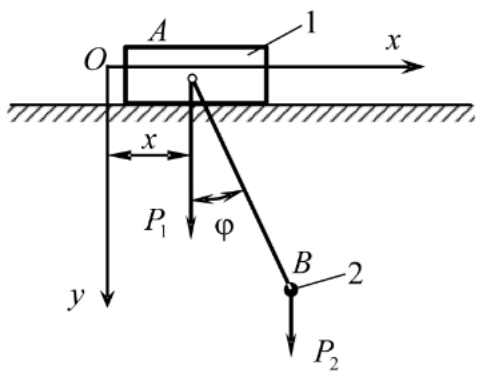
3. To make a simulation of this system. Show velocities and accelerations for 1, 2 bodies (coding approach).

Artifacts

1. Report in *.pdf* or in *.md*.

2. For **Newton-Euler** method — code, GIFs, plots.

3. For **SimInTech** — *.prt*, for **Simulink** — *.slx* file which contains a description of the system, GIFs, plots.



4.2 Task Explanation

Research object: A system of 2 bodies: cart and pole
Motion: A cart - plane motion, pole - rotational motion
Force analysis: $G_1 = m_1 g$, $G_2 = m_2 g, T, N, m_2\ddot{x}$ - inertial force
Solution:
(1) Cart:

$$\begin{cases} \text{X:} \quad -T_x - m_1\ddot{x} = 0 \\ \text{Y:} \quad N = G_1 + T_y \end{cases}$$

(2) Pole:

$$\begin{cases} \text{X:} \quad T_x = m_2 a_{2x} + m_2\ddot{x} \\ \text{Y:} \quad T_y - G_2 = m_2 a_{2y} \end{cases}$$

$$J\epsilon = J\ddot{\phi} = -G_2 l \cos(\frac{\pi}{2} - \phi) - m_2\ddot{x} l \cos(\phi)$$

Acceleration of the pole $\vec{a}_2$ is the sum of tangential and centripetal acceleration:

$$\vec{a}_2 = \vec{\dot{\phi}}^2 \cdot l + \vec{\ddot{\phi}} \cdot l$$

Projections of $veca_2$ are the following:

$$a_{2x} = -\dot{\phi}^2 l \sin(\phi) + \ddot{\phi} l \cos(\phi)$$

$$a_{2y} = \dot{\phi}^2 l \cos(\phi) + \ddot{\phi} l \sin(\phi)$$

So,

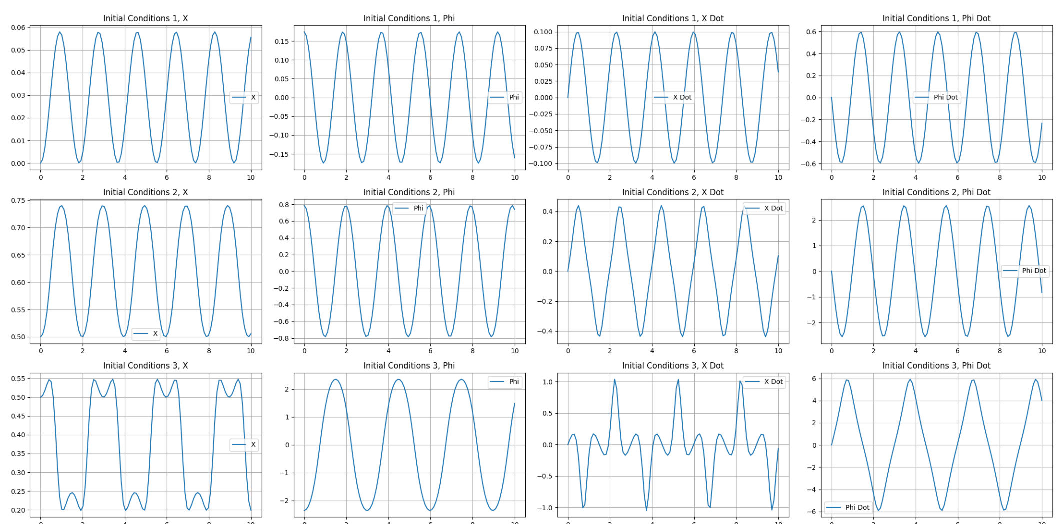
$$\begin{cases} -T_x - m_1\ddot{x} = 0 \\ N = G_1 + T_y \\ T_x = m_2(-\dot{\phi}^2 l \sin(\phi) + \ddot{\phi} l \cos(\phi)) + m_2\ddot{x} \\ T_y - G_2 = m_2(\dot{\phi}^2 l \cos(\phi) + \ddot{\phi} l \sin(\phi)) \\ J\ddot{\phi} = -G_2 l \cos(\frac{\pi}{2} - \phi) - m_2\ddot{x} l \cos(\phi) \end{cases}$$

Or:

$$\begin{cases} -T_x - m_1\ddot{x} = 0 \\ N = m_1 g + T_y \\ T_x = m_2(-\dot{\phi}^2 l \sin(\phi) + \ddot{\phi} l \cos(\phi)) + m_2\ddot{x} \\ T_y - m_2 g = m_2(\dot{\phi}^2 l \cos(\phi) + \ddot{\phi} l \sin(\phi)) \\ J\ddot{\phi} = -m_2 g l \cos(\frac{\pi}{2} - \phi) - m_2\ddot{x} l \cos(\phi) \end{cases}$$

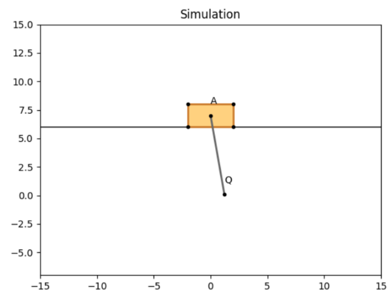
Solving the system computationally will provide us with the results below.

4.3 Plots

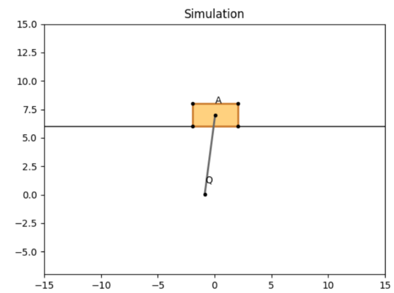


4.4 Simulation

Initial condition 1: $\phi = 10°$, $x = 0$

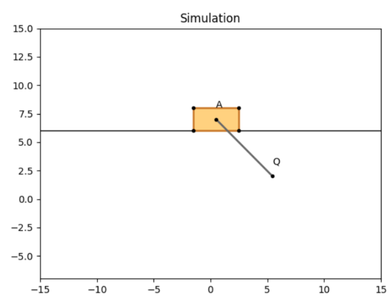


(a)

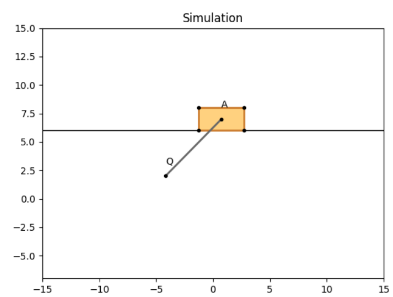


(b)

Initial condition 2: $\phi = 45°$, $x = 0.5$

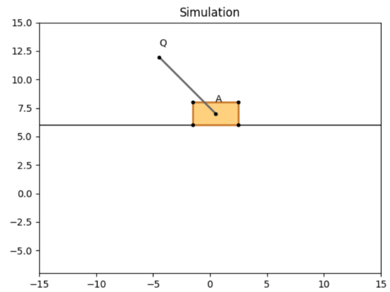


(a)

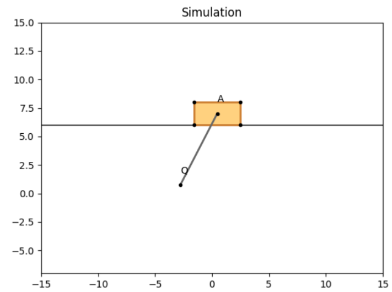


(b)

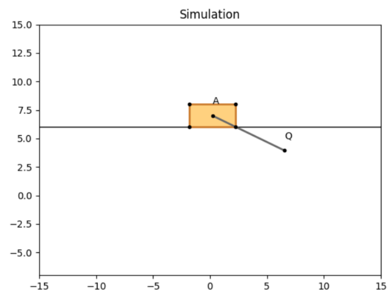
Initial condition 3: $\phi = -135°$, $x = 0.5$



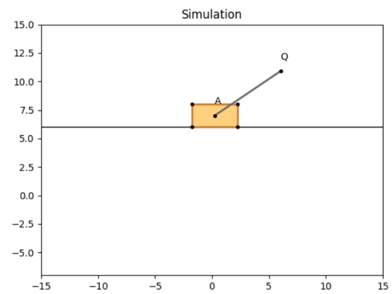
(a)



(b)



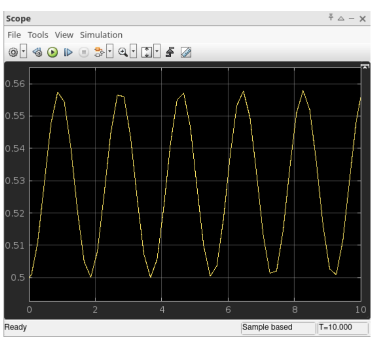
(c)



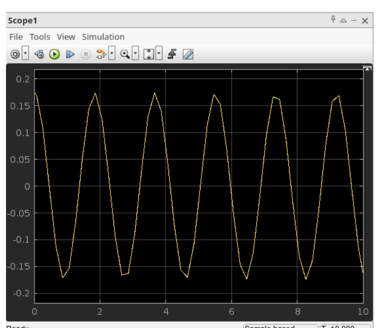
(d)

5 Simulink

Initial condition 1: $\phi = 10°$, $x = 0$

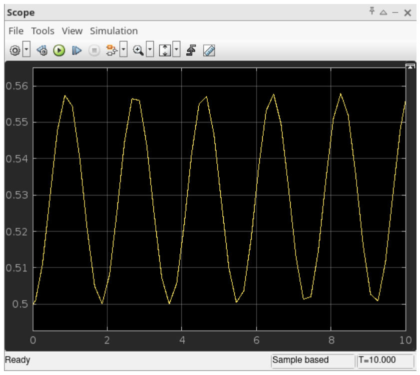


(a) x

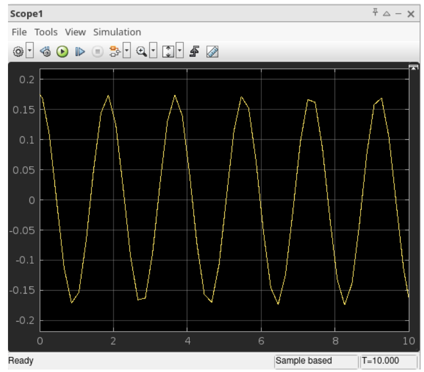


(b) ϕ

Initial condition 2: $\phi = 45°$, $x = 0.5$

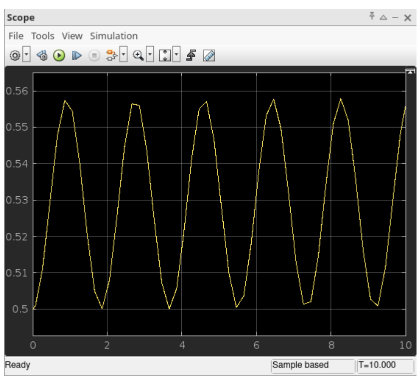


(a) x

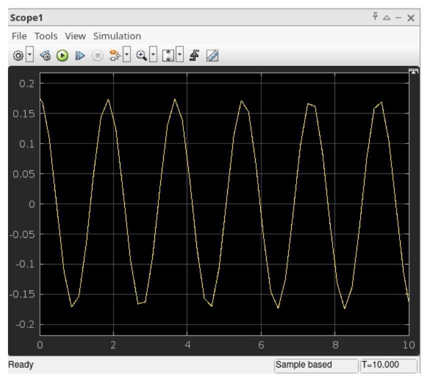


(b) ϕ

Initial condition 3: $\phi = -135°$, $x = 0.5$



(a) x



(b) ϕ

The plots approximately coincide with those obtained using the coding method.