Theoretical Mechanics: Week Homework 3

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1 Tools used for solving the tasks

Python (sympy, matplotlib).

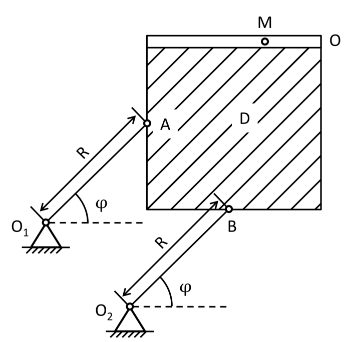
2 Task 1

2.1 Link to the Code

Colab

2.2 Task Description

You should find an absolute velocity and coriolis acceleration, and absolute acceleration of particle M at the time $t = t_1$.
Needed variables:
$OM = s_r(t) = f_3(t) = 2t^3 + 3t$;
$\phi(t) = f_2(t) = \frac{1}{24}\pi t^2$;
$t_1 = 2,\ R = 15$.



Task 1 (Yablonskii (eng) K-5)

2.3 Task explanation

Velocity

1) Relative
$v_M^{rel} = \dot{s}_r = 6t^2 + 3$
$v_M^{rel}(t_1) = 6(2^2) + 3 = 27$
2) Transport
$\omega = \dot{\phi(t)} = \frac{\pi}{12}t$
$\epsilon = \dot{\omega} = \frac{\pi}{12}$
$\omega(t_1) = \frac{\pi}{6}$
$v_M^{tr}(t_1 = 2) = \omega(t_1 = 2)R = \frac{\pi}{6} \cdot 15 = \frac{5\pi}{2}$
$\phi(t_1 = 2) = \frac{1}{24}\pi(2)^2 = \frac{\pi}{6}$
The velocity components and total velocity are given by:

$$\begin{aligned}
v_M^y &= v_M^{tr} \sin\left(\frac{\pi}{2} - \phi\right), \\
v_M^y(t = t_1) &= v_M^{tr} \sin\left(\frac{\pi}{2} - \phi\right) = 5\frac{\pi}{2} \sin\left(\frac{\pi}{2} - \frac{\pi}{6}\right), \\
v_M^x &= v_M^{tr} \cos\left(\frac{\pi}{2} - \phi\right) + v_M^{rel}, \\
v_M^x(t = t_1) &= v_M^{tr} \cos\left(\frac{\pi}{2} - \phi\right) + v_M^{rel} = 5\frac{\pi}{2} \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right) + 27, \\
V_M^{tot} &= \sqrt{{v_M^y}^2 + {v_M^x}^2} = 31.6661.
\end{aligned}$$

Acceleration

No coriolis acceleration since the square body is in a translatory motion, it does not rotate.
$a_M^{rel} = \dot{v_M^{rel}} = 12t$
$a_M^{rel}(t_1) = \dot{v_M^{rel}}(t_1) = 24$

Given constants:

$$\begin{aligned}
a_M^{rel} &= 24, \\
\epsilon &= \frac{\pi}{12}, \\
\omega &= \frac{\pi}{6}.
\end{aligned}$$

The acceleration components and total acceleration are:

$$\begin{aligned}
a_\tau^{tr} &= \epsilon R, \\
a_n^{tr} &= \omega^2 R, \\
a_M^y &= a_t \sin\left(\frac{\pi}{2} - \phi\right) - a_{ntr} \sin(\phi), \\
a_M^x &= a_M^{rel} + a_\tau^{tr} \cos\left(\frac{\pi}{2} - \phi\right) + a_n^{tr} \cos(\phi), \\
a_M^{tot} &= \sqrt{{a_M^y}^2 + {a_M^x}^2} = 29.5555.
\end{aligned}$$

Answer: $v_M^{tot} = 31.6661, a_M^{tot} = 29.5555, a_M^{cor} = 0.$

3 Task 2

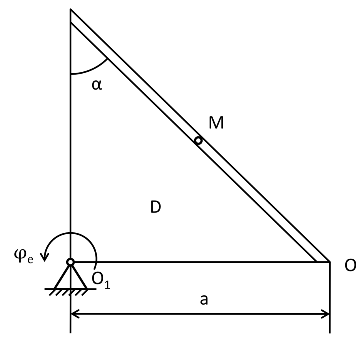
3.1 Link to the code

Colab

3.2 Task Description

You should find:

1. simulate this mechanism (obtain all positions);
2. Find absolute, transport and relative velocities and accelerations for M;
3. Find t, when M reaches O point;
4. draw plots $v_{rel},\ v_{tr},\ a_{tr},\ a_{rel},\ a$ respect to time.



Task 2 (Yablonskii (eng) S6)

Needed variables:
$\phi_e = f_1(t) = 0.2t^3 + t$;
$OM = s_r = f_2(t) = 5\sqrt{2}(t^2 + t)$;
$a = 60,\ \alpha = 45$.

3.3 Task explanation

First, let's calculate the time when M leaves the channel (i.e., when the point M travels the distance OA).

$$5\sqrt{2}(t^2 + t) = OA = 60\sqrt{2} \Rightarrow t = 3$$

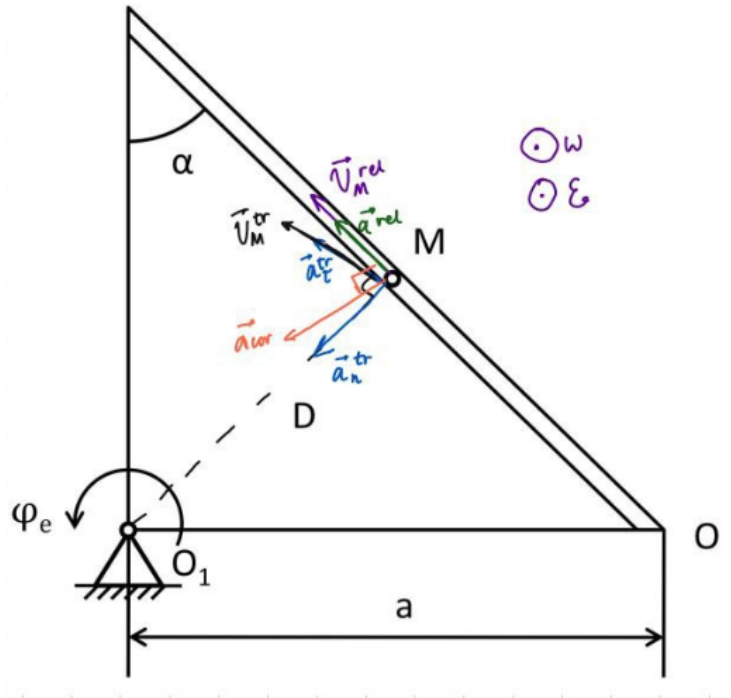
1) Relative
$v_M^{rel} = \dot{s}_r = 10\sqrt{2}t + 5\sqrt{2}$.
$a_M^{rel} = \ddot{s}_r = 10\sqrt{2}$

2) Transport
$\omega_e = \dot{\phi}_e = 0.6t^2 + 1$
$v_M^{tr} = \omega_e \cdot MO_1$
$a_\tau^{tr} = \dot{\omega_e} \cdot MO_1 = \epsilon_e \cdot MO_1$
$a_n^{tr} = \omega_e^2 \cdot MO_1$

3) The Coriolis acceleration, a_M^{cor}, is given by

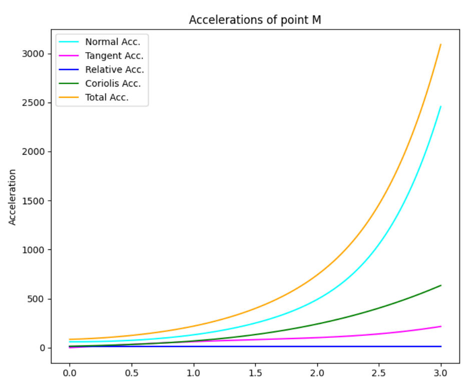
$$a_M^{\mathrm{cor}} = 2\mathbf{w}_{\mathrm{tr}} \times \mathbf{v}_M^{\mathrm{rel}}$$

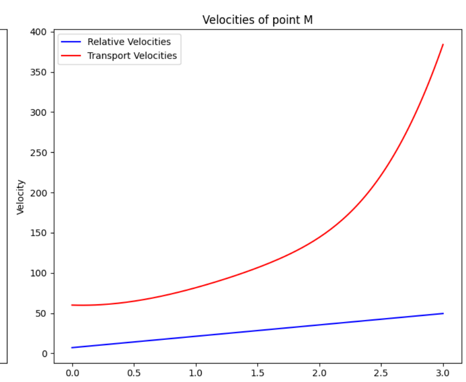
The Coriolis acceleration is directed to the center of rotation - O_1.
The directions of all velocities and accelerations is shown in the picture.



Task 2 (Yablonskii (eng) S6)

3.4 Plots





3.5 Simulation