Theoretical Mechanics: Week Homework 5

Ekaterina Mozhegova

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1 Tools used for solving the tasks

Python (sympy, matplotlib).

2 Task 1

2.1 Link for the Code

Colab

2.2 Task Description

The legend shall speak that this situation was in WW2. There are two actors in this story: a sniper and an officer. Both knew about each other's existence. There was a river between them. The officer was always sitting in a trench, but the sniper knew his location and already calculated the distance to the target (L, m). After a while cargo ship appeared, which blocked the direct vision of the trench. The officer decided to stand up to stretch his legs. The sniper assumed that it might happen and made a shot, hitting the officer. Let's check this story.

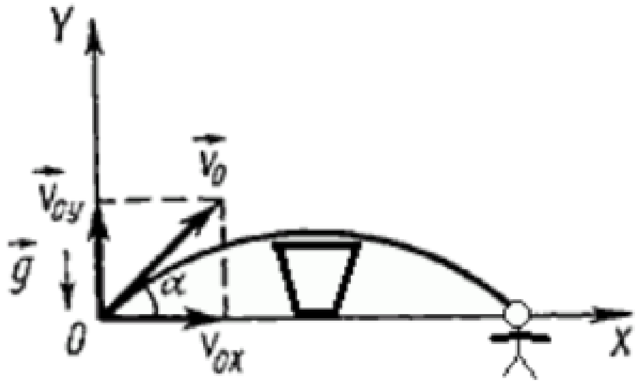
Needed variables:

- $m = 13.6\,\mathrm{g}$
- $L = 1500$
- $M_1 = 1.3 * 10^{-5}$
- $v_0 = 870\,\mathrm{m/s}$

Find:

- α
- $h_{\max}$

- $F_C(v^2) = -kv\vec{v}$



Task 1 (Yablonskii (eng) S3)

2.3 Task explanation

Neglect the air drag

Research Object: Bullet (a particle)
Motion: Projectile motion
Conditions:

Initial:

- $x_0 = 0$
- $y_0 = 0$
- $\dot{x}_0 = v_0 \cos(\alpha)$
- $\dot{y}_0 = v_0 \sin(\alpha)$

Final:

- $x_f = L$
- $y_f = 0$
- $\dot{x}_0 = ?$
- $\dot{y}_0 = ?$

Force Analysis

$$\begin{cases} ma_x &= 0 \\ ma_y &= -mg \end{cases}$$

$$\begin{cases} \int ma_x &= \int 0 \\ \int ma_y &= \int -mg \end{cases}$$

$$\begin{cases} mv_x &= C \\ mv_y &= -mgt + C_2 \end{cases}$$

Projections

$$X: \quad v_0 \cos(\alpha) t = L \tag{1}$$

$$Y: \quad v_0 \sin(\alpha) t - \frac{gt^2}{2} = 0 \tag{2}$$

Solution:
From (1), we get that $t = \dfrac{L}{v_0 \cos(\alpha)}$.
Substituting t into (2), we get:

$$v_0 \sin(\alpha) \frac{L}{v_0 \cos(\alpha)} - g \frac{L^2}{2v_0^2 \cos^2(\alpha)} = 0$$

Simplifying, we have:

$$\frac{2v_0^2 \sin(\alpha)\cos(\alpha) L - gL^2}{2v_0^2 \cos^2(\alpha)} = 0$$

Further simplification gives:

$$L\left(\frac{v_0^2 \sin(2\alpha) - gL}{2v_0^2 \cos^2(\alpha)}\right) = 0$$

Finally:

$$\sin(2\alpha) = \frac{gL}{v_0^2}$$

Two roots exist: $0.556°$**,** $89.4436°$**.**

Since there is no air drag, the time to rise t_{rise} equals the time to fall t_{fall}, which is half of the total time t_{total}:

$$t_{\text{rise}} = t_{\text{fall}} = \frac{t_{\text{total}}}{2}$$

The maximum height $h_{\max}$ can be calculated as:

$$h_{\max} = v_0 \sin(\alpha)\left(\frac{t_{\text{total}}}{2}\right) - \frac{g\left(\frac{t_{\text{total}}}{2}\right)^2}{2}$$

We get a solution for each α.
Heights: 3.64184 m, 38613.7051 m

Air drag

Research Object: Bullet (a particle)
Motion: Projectile motion
Conditions:

Initial:

- $x_0 = 0$
- $y_0 = 0$
- $\dot{x}_0 = v_0 \cos(\alpha)$
- $\dot{y}_0 = v_0 \sin(\alpha)$

Final:

- $x_f = L$
- $y_f = 0$
- $\dot{x}_0 =?$
- $\dot{y}_0 =?$

Force Analysis:

$$m\vec{a} = \sum \vec{F} = m\vec{g} + \vec{F}_c$$

Projections of Forces:
Y:

$$ma_y = -mg - kv_y\sqrt{v_x^2 + v_y^2}$$

X:

$$ma_x = -kv_x\sqrt{v_x^2 + v_y^2}$$

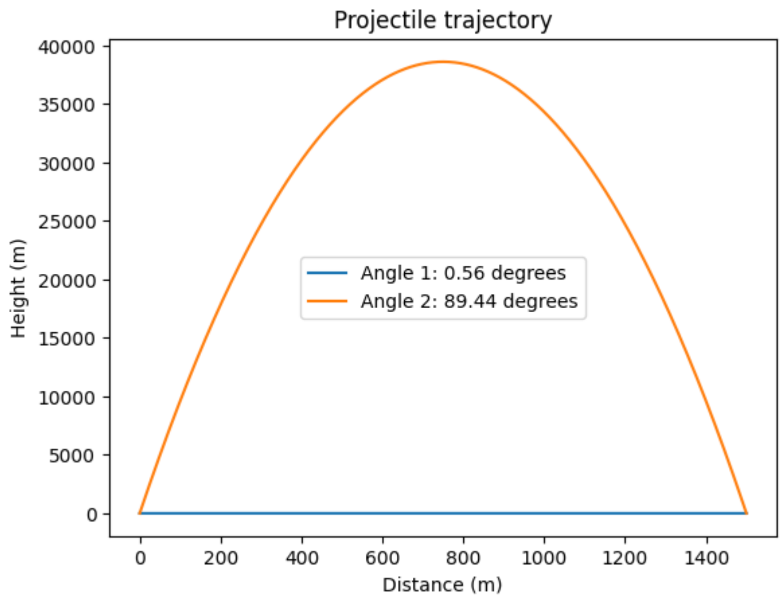
Where:

$$a_y = -g - \frac{k}{m}v_y\sqrt{v_x^2 + v_y^2}$$

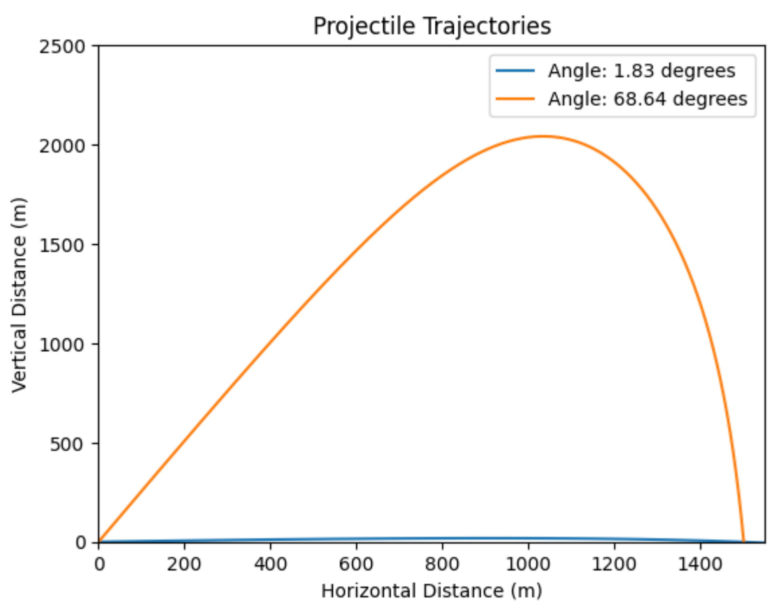
$$a_x = -\frac{k}{m}v_x\sqrt{v_x^2 + v_y^2}$$

After integrating, we get the possible **angles:** $1.83°$**,** $68.64°$.
And **heights:** $18.6337\,m, 2041.3772\,m$.

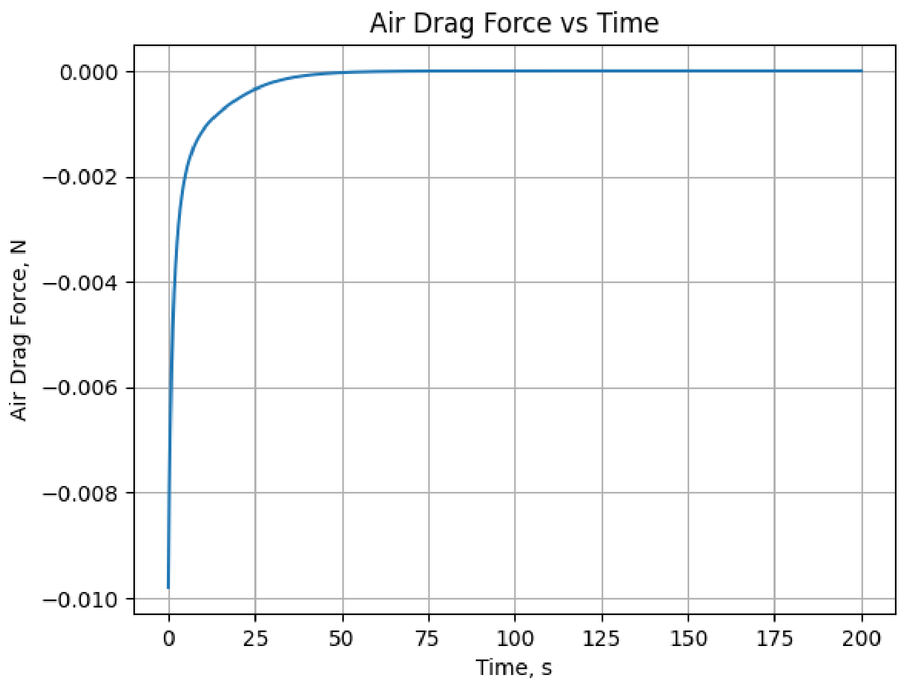
2.4 Plots



Trajectory of a projectile without air drag



Trajectory of a projectile with air drag



Air resistance

3 Task 2

3.1 Link to the simulation

Colab

3.2 Task Description

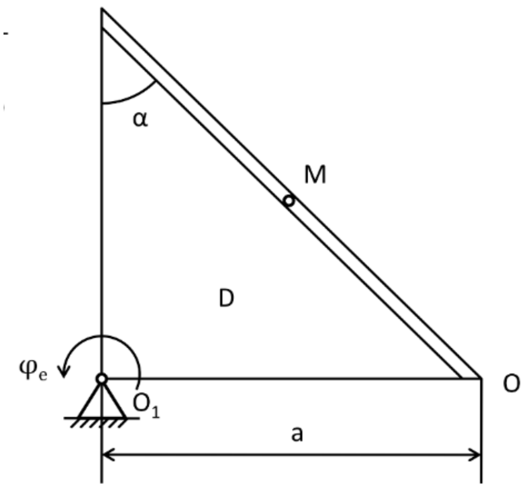
A particle M (mass m) is moving inside of the cylindrical channel of the moving object D. The object D has a radius r. No friction between M and D.

Determine the equation of the relative motion of this particle $x = f(t)$. Also you need to find the pressure force the particle acting on the channel wall.

At the end, you should provide:

1. simulate this mechanism (obtain all positions);
2. show all acceleration components, inertial forces, gravity force and N;
3. plot of the particle $x(t)$, till the time, while point won't leave the channel;
4. plot $N(t)$, till the time, while point won't leave the channel.

Needed variables:
$m = 0.02,\ \omega = \pi,\ a = 60,\ \alpha = 45°;$
Initial conditions: $t_0 = 0,\ x_0 = 0,\ \dot{x}_0 = 0.4.$



Task 2 (Yablonskii (eng) S6)

3.3 Task Explanation

3.3.1 Research Object

The system consists of a point M (a particle) and a tube inside a rotating triangle.

3.3.2 Motion

M moves rectilinearly, while OA undergoes rotational motion.

3.3.3 Kinematic Analysis

It is necessary to determine the directions of all accelerations (inertial force components).

3.3.4 Force Analysis

The forces acting are: $m\vec{g}$, $\vec{N}$, and inertial forces components.

The tangential component of inertial force:

$$\vec{\Phi}^{tr} = -ma_n^{tr} = -m\omega^2 R,$$

where $R = O1M(\phi)$.

The Coriolis force:

$$\vec{\Phi}_{\text{cor}} = -m\mathbf{a}^{\text{cor}} = -2m \cdot \omega \times \mathbf{v}^{\text{rel}}$$

3.3.5 Solution

Since the triangle rotates with a constant angular speed, $\epsilon = 0$, and so $a_\tau = 0$.

Projections:

$$X : -mg\sin(\beta) - m\omega^2 R\sin(\gamma) = ma_x \quad (1)$$
$$Y : N + 2mg\omega N^{\mathrm{rel}} - mg\cos(\beta) + m\omega^2 R\cos(\gamma) = 0 \quad (3)$$

By the law of cosines:

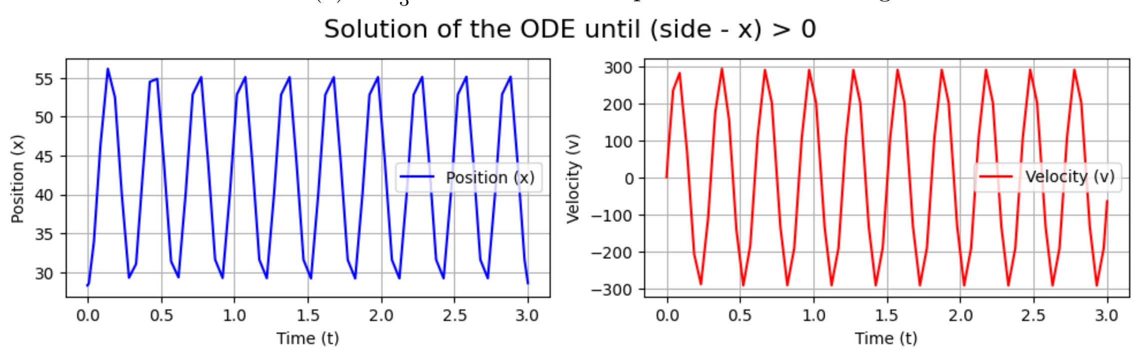
$$(a\sqrt{2})^2 = (a\sqrt{2} - x)^2 + R^2 - 2(a\sqrt{2} - x)R\cos(\beta)$$

$$\cos\beta = \frac{(a\sqrt{2})^2 - (a\sqrt{2} - x)^2 - R^2}{2(a\sqrt{2} - x)}$$

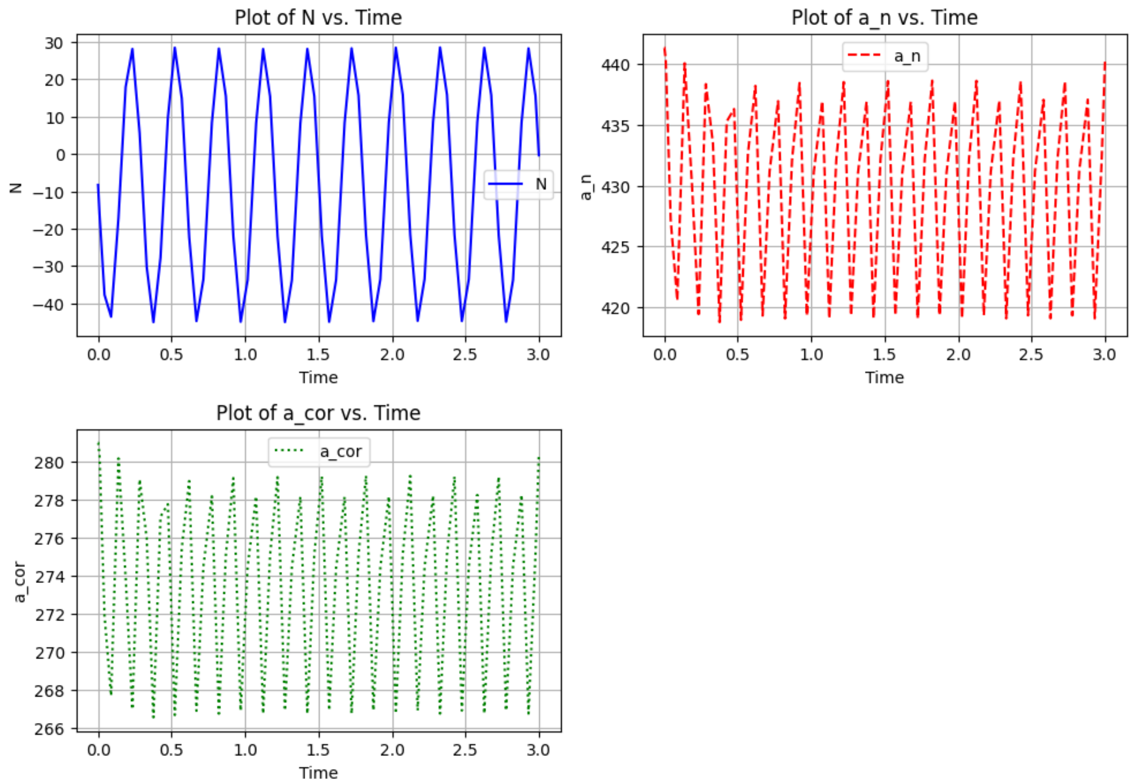
We need to integrate Equation (1) while the point is inside the tube. Then, substitute the values into the second equation to get N.

3.4 Plots

From the initial position $= 0$, the point will fly away out of the tube. Let's choose initial condition: $X(0) = \frac{a\sqrt{2}}{3}$. In this case the plots are the following.

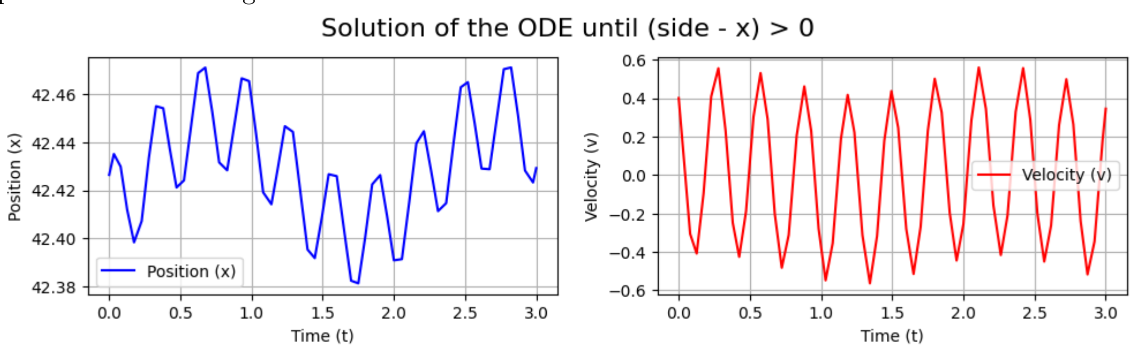


Position of the particle $x(t)$ and velocity $v(t)$

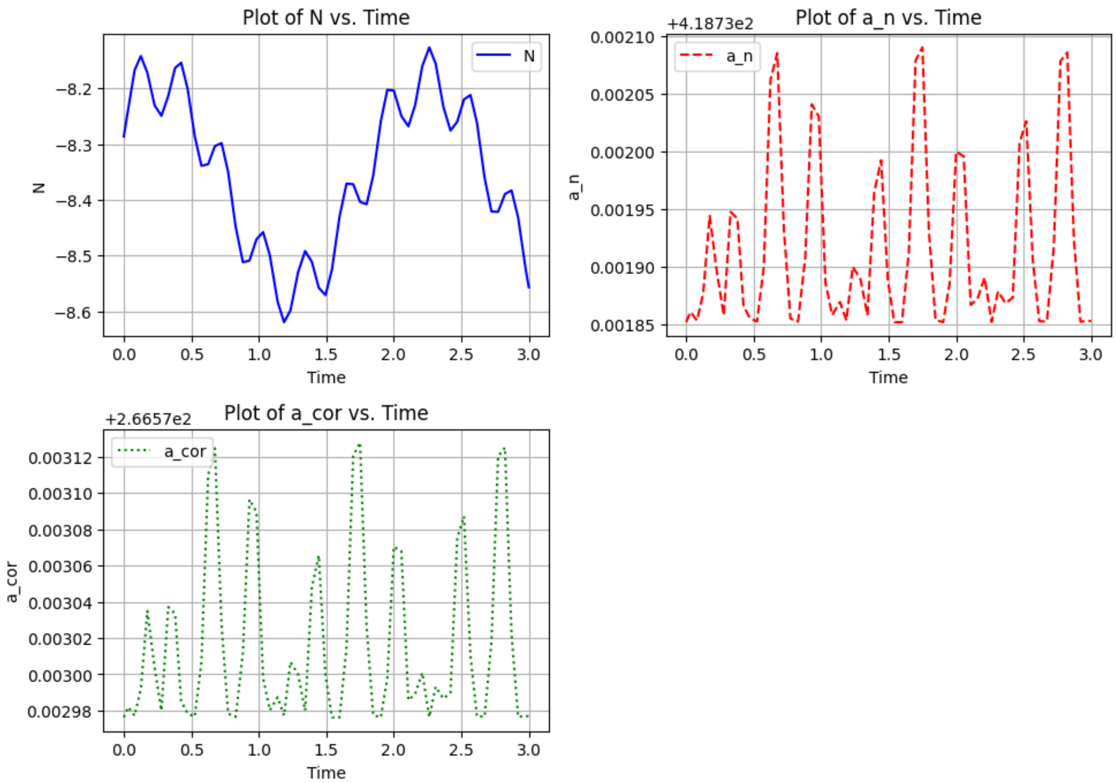


$N(t)$ and acceleration components, a_n, a_{cor}

For the initial position $X(0) = \frac{a\sqrt{2}}{2}$ (the midpoint of AO). In this case the plots are the following.



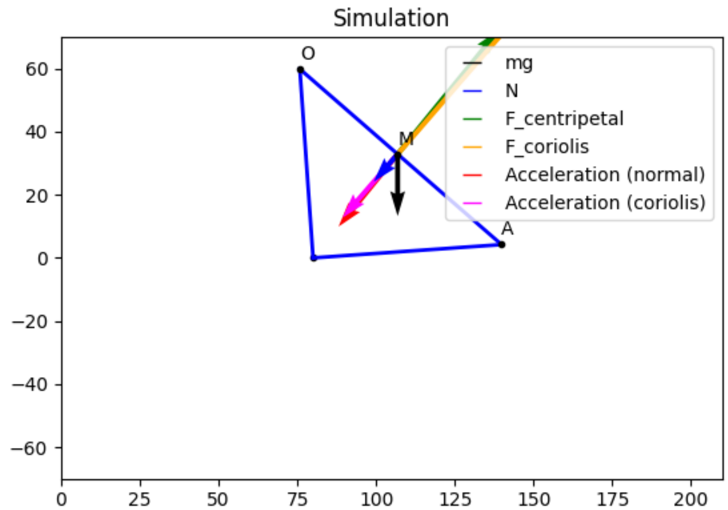
Position of the particle $x(t)$ and velocity $v(t)$



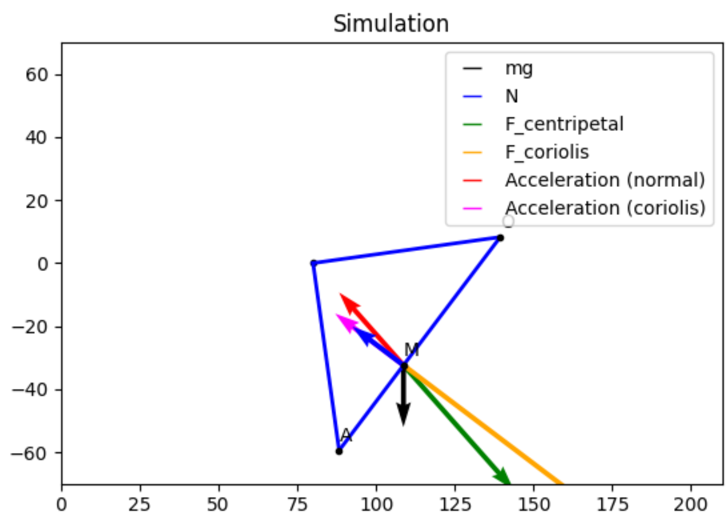
$N(t)$ and acceleration components, a_n, a_{cor}

3.5 Simulation

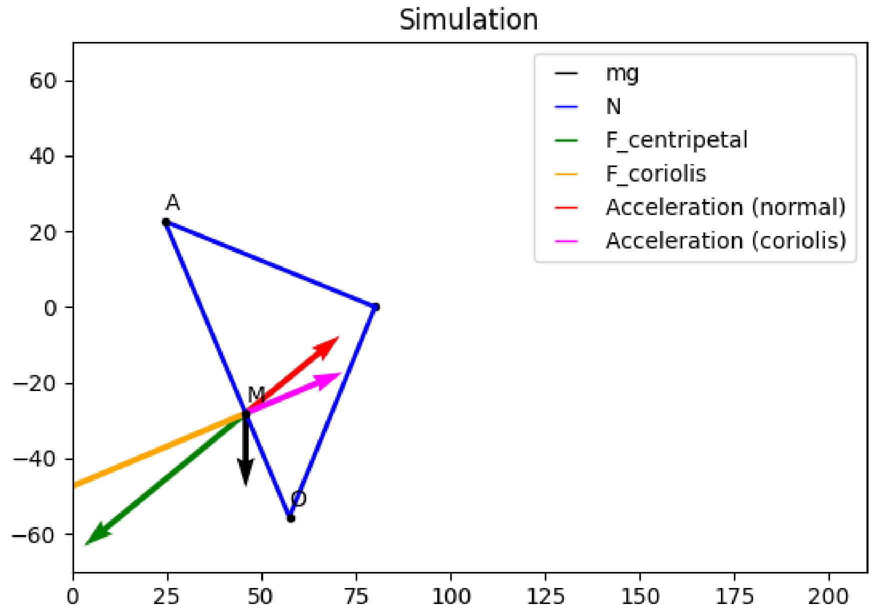
The initial position $X(0) = \frac{a\sqrt{2}}{3}$.



$N(t)$ and acceleration components, a_n, a_{cor}



$N(t)$ and acceleration components, a_n, a_{cor}



$N(t)$ and acceleration components, a_n, a_{cor}