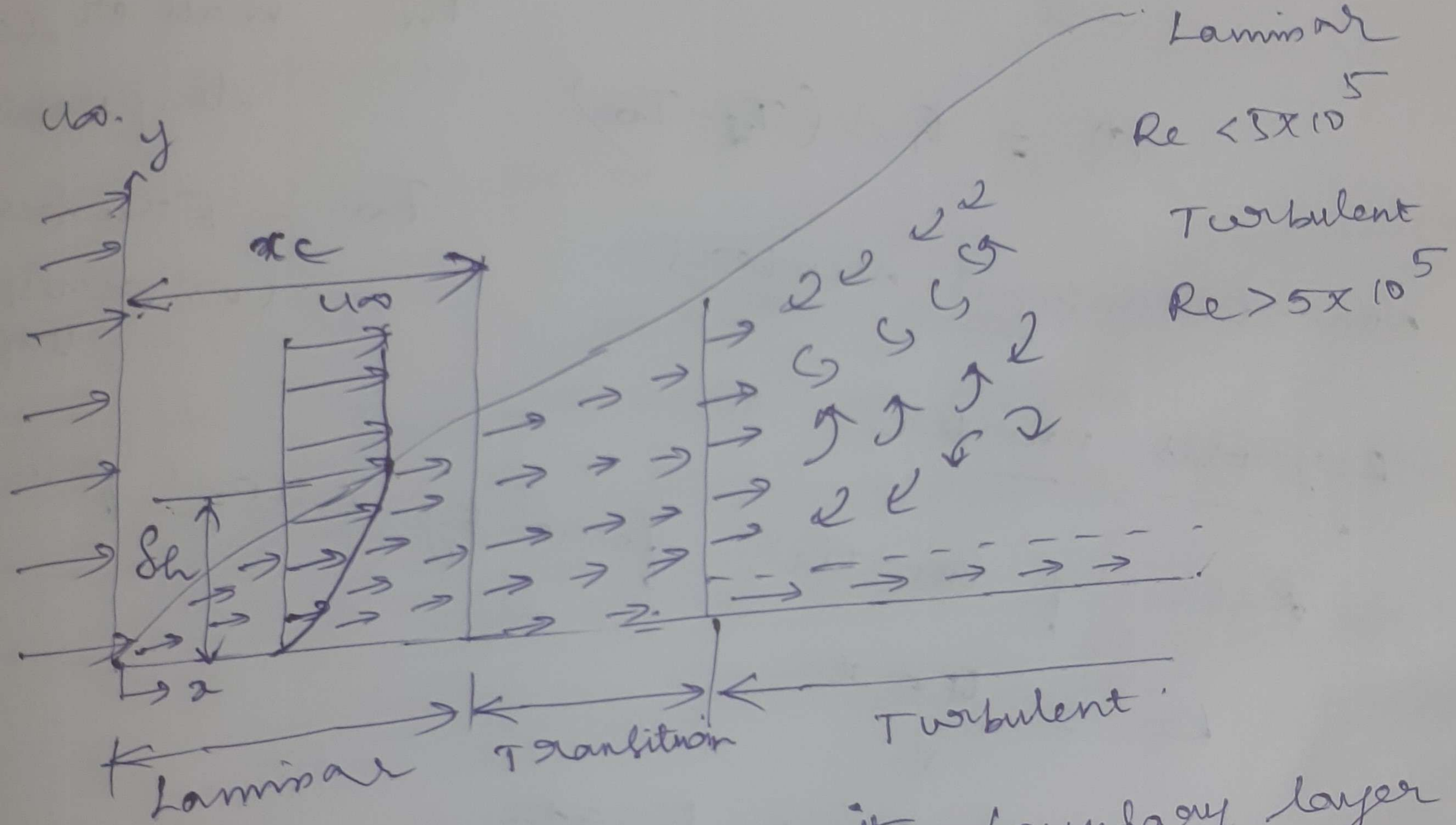
II - Convection

Forced convection

Boundary layer concept



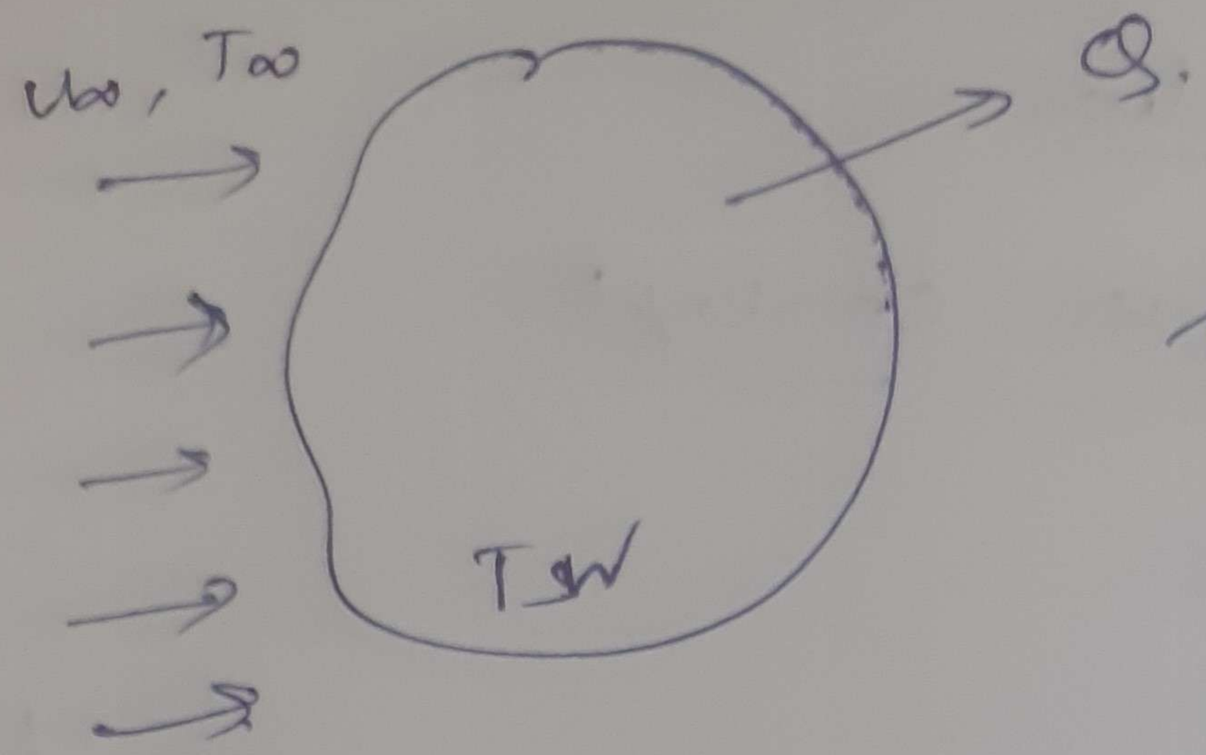
Development of velocity boundary layer on flat plate

x_c - critical distance

u_∞ - Free stream velocity

δ_h - Boundary layer thickness (velocity) (Hydrodynamic)

Boundary layer thickness from which fluid velocity equals 99% of free stream velocity ($u = 0.99 u_\infty$)



h - convective heat transfer coefficient W/m²K

A - surface area

T_W - wall or hot temperature

T_∞ - Free stream (or) atmospheric temperature

$$Q = hA(T_W - T_\infty)$$

non-dimensional numbers

Reynolds number

Ratio of inertial force by viscous force

$$Re = \frac{u_\infty x}{\nu}$$

x - characteristic length

$x \rightarrow D$ for cylinder (or) sphere

$\rightarrow L$ for flat plate

ν - kinematic viscosity m²/s

u_∞ - Free stream velocity m/s

Prandtl number

$$Pr = \frac{\mu C_p}{k} = \frac{\nu}{\alpha}$$

Ratio of molecular diffusivity of momentum and molecular diffusivity of mass.

Nusselt number

$$Nu = \frac{h x}{k}$$

Ratio of temperature gradient by conduction and convection at surface.

$x \rightarrow D$ for cylinder (or) sphere

$\rightarrow L$ for flat plate

k - Thermal conductivity of fluid w/m k.

Stanton number

$$St = \frac{Nu}{Re\,Pr}$$

Ratio of wall heat transfer rate to heat transfer by conduction.

1) Air at 25°C and at atmospheric pressure is flowing over a flat plate at 3 m/s. The plate is 1 m wide and temperature is 75°C. Calculate the following at location of 1 m from the leading edge.

(i) Hydrodynamic boundary layer thickness

(ii) Thermal boundary layer thickness

iii) Local heat transfer co-efficient

iv) Local friction co-efficient

v) Average heat transfer co-efficient

<u>Given data</u>

$T_\infty = 25°C$ $\qquad x = 1 m.$

$u_\infty = 3$ m/s.

$T_w = 75°C$

property values of air at

$T_f = \frac{T_w + T_\infty}{2} = \frac{75+25}{2} = 50°C$

From D.B. P. 34.

$Pr = 0.698$

$K = 28.26 \times 10^{-3} = 0.02826$ W/mK

$\nu = 17.95 \times 10^{-6}$ m²/s

$$Re_x = \frac{u_\infty x}{\nu} = \frac{3 \times 1}{17.95 \times 10^{-6}}$$

$$= 1.7 \times 10^5 < 5 \times 10^5$$

Flow is laminar

For external flow, From D.B. P. 113

∴ Flat plate

Hydrodynamic boundary layer thickness $\delta_{hx} = 5x\, Re_x^{-0.5}$

$$= 5 \times 1 \times 1.7 \times 10^{5}$$

$$\delta_{hx} = 0.012 \text{ m}$$

Thermal boundary thickness

$$\delta_{Tx} = \delta_{hx}\, Pr^{-0.333}$$

$$\delta_{Tx} = 0.012 \times (0.698)^{-0.333}$$

$$\delta_{Tx} = 0.0136 \text{ m}$$

For const. wall temperature

$$Nu_x = 0.332\, Re_x^{0.5}\, Pr^{0.333}$$

$$= 0.332 \times (1.7\times10^5)^{0.5} \times (0.698)^{0.333}$$

$$Nu_x = 121.42$$

$$Nu_x = \frac{h_x\, x}{K}$$

$$121.42 = \frac{h_x \times 1}{0.02826}$$

local heat transfer co-efficient $h_x = 3.431\ W/m^2K$

For Flat plate

$$h = 2h_x$$

h - Average heat transfer co-efficient

$$h = 2\times3.431$$

$$= 6.863\ W/m^2K$$

Local friction co-efficient

$$C_{fx} = 0.664\, Re^{-0.5}$$

$$= 0.664\,(1.7\times10^5)^{-0.5}$$

$$C_{fx} = 1.61\times10^{-3}$$

2) Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Assuming unit depth in 'z'-direction. Calculate the heat transfer at the first 0.4 m of length of the plate. The plate is heated over its entire length to a temperature of 60°C

Given data

$T_\infty = 27°C$ $\quad x = 0.4\,m$

$u_\infty = 2\,m/s$

$T_w = 60°C$

From D.B. P. 34.

property values of air at

$$T_f = \frac{T_w + T_\infty}{2} = \frac{60+27}{2} =$$

$$T_f = 43.5°C \simeq 45°C$$

$Pr = 0.699$

$\nu = 16.96 \times 10^{-6}\,m^2/s$

$K = 26.56 \times 10^{-3}\,W/mK$

$$Re = \frac{u_\infty x}{\nu} = \frac{2 \times 0.4}{16.96 \times 10^{-6}}$$

$$Re = 47169 < 5 \times 10^5$$

Flow is Laminar. From D.B. P. 113

At constant wall temperature

$$Nu_x = 0.332 \, Pr^{0.333} \, Re^{0.5}$$

$$= 0.332 \, (0.699)^{0.333} \, (47169)^{0.5}$$

$$Nu_x = 63.99$$

$$Nu_x = \frac{h_x x}{k}$$

$$64 = \frac{h_x \times 0.4}{26.56 \times 10^{-3}}$$

$$h_x = 4.249$$

For flat plate

$$h = 2 h_x = 2 \times 4.249$$

$$= 8.5 \, W/m^2 K$$

$$Q = hA(T_w - T_\infty)$$

$$= 8.5 \times (0.4 \times 1)(60 - 27)$$

$$Q = 112.2 \, W$$

3) Air at a standard conditions of 100 kPa 30°C flows over a flat plate at 20 m/s. The plate is 0.6 m square section and it is maintained at 90°C. Calculate the heat transfer from the plate at a distance of 0.3 m and 0.6 m from leading edge

Given data

$T_\infty = 30°C$ $\quad T_w = 90°C$

$u_\infty = 20$ m/s.

$x = 0.6$ m $\quad$ From D.B. P 34.

property values of air

at $T_f = \frac{T_w + T_\infty}{2} = \frac{90+30}{2} = 60°C$

$Pr = 0.696$

$\nu = 18.97 \times 10^{-6}$ m²/s

$K = 0.02896$ W/mK

At $x_1 = 0.3$

$Re_x = \frac{u_\infty x}{\nu} = \frac{20 \times 0.3}{18.97 \times 10^{-6}}$

$= 3.16 \times 10^5 < 5 \times 10^5 \rightarrow$ Laminar

At $x = 0.6$

$$Re_x = \frac{U_\infty x}{\nu} = \frac{20 \times 0.6}{18.97 \times 10^{-6}}$$

$$= 6.32 \times 10^5 > 5 \times 10^5 \rightarrow \text{Turbulent}$$

$\therefore$ Flow is laminar turned Turbulent

From D.B. P. 115

For combined Laminar - Turbulent

$$Nu_L = Pr^{0.333}(0.037\, Re_L^{0.8} - 871)$$

$$Nu_L = (0.696)^{0.333}(0.037 \times (6.32 \times 10^5)^{0.8} - 871)$$

$$Nu_L = 662$$

$$Nu = \frac{h_x x}{k}$$

$$662 = \frac{h_x \times 0.6}{0.02896}$$

$$h_x = 32 \text{ W/m}^2\text{K}$$

For flat plate

$$h = 2h_x = 64 \text{ W/m}^2\text{K}$$

$$Q = hA(T_w - T_\infty)$$

$$= 64 \times (0.6 \times 0.6)(90 - 30)$$

$$Q = 13.8 \text{ W}$$

4) An air stream at 0°C is flowing along heated plate at a speed of 75 m/s. The plate is 45 cm long and 60 cm wide. Assuming transition of boundary layer takes place at Reynolds no. 5×10^5. Calculate the average value of friction co-efficient and heat transfer co-efficient from the full length of the plate. Also calculate the rate energy dissipation from the plate.

<u>Given data</u>

$T_\infty = 0°C$

$T_w = 90°C$

$L = 45$ cm
$= 45 \times 10^{-2}$ m.

$W = 60$ cm
$= 60 \times 10^{-2}$ m

$u_\infty = 75$ m/s.

From D.B. P. 34
property values of air at

$T_f = \frac{0+90}{2} = 45°C$

$\nu = 17.95 \times 10^{-6}$ m²/s.

$Pr = 0.698$

$K = 0.0282$ W/mK

$$Re_x = \frac{U_\infty x}{\nu} = \frac{75 \times 0.45}{17.9 \times 10^{-6}}$$

$$= 1.88 \times 10^6 > 5 \times 10^5$$

Transition of boundary layer at $Re = 5 \times 10^5$ (Given)

$\therefore$ For combined Laminar-turbulent

From D.B. P.115

$$C_{fL} = 0.074\, Re_L^{-0.2} - 1742\, Re^{-1}$$

$$= 0.074 (1.88 \times 10^6)^{-0.2} - 1742 (1.88 \times 10^6)^{-1}$$

$$= 3.188 \times 10^{-3}$$

$$Nu_L = Pr^{0.333} (0.037\, Re_L^{0.8} - 871)$$

$$= (0.698)^{0.333} (0.037 (1.88 \times 10^6)^{0.8} - 871)$$

$$= 2658$$

$$Nu_L = \frac{h_x \cdot x}{k}$$

$$2659 = \frac{h_x \times 0.45}{0.02826}$$

$h_x = 167 \ W/m^2K$

$h = 2h_x$

$h = 2 \times 167 = 334 \ W/m^2K$

$Q = hA(T_w - T_\infty)$

$= 334 \times 0.45 \times 0.6 (90 - 0)$

$Q = 8116.2 \ W$

Relation between fluid friction and heat transfer (Reynolds Colburn analogy)

Stanton no. $St_x = \dfrac{Nu_x}{Re_x \, Pr}$

From D.B. p.113, For Laminar flow,

$Nu_x = 0.332 \, Re_x^{0.5} \, Pr^{0.333}$

$$St_x = \frac{0.332 \, Re_x^{1/2} \, Pr^{1/3}}{Re \cdot Pr}$$

$= 0.332 \, Re_x^{1/2 - 1} \, Pr^{1/3 - 1}$

$St_x = 0.332 \, Re^{-1/2} \, Pr^{-2/3}$

$$\frac{St}{Pr^{-2/3}} = 0.332\, Re^{-1/2} \quad -(1)$$

$$St_x\, Pr^{2/3} = 0.332\, Re^{-1/2}$$

From D. B. P. 113 - For Laminar flow,

we get,

$$C_{fx} = 0.664\, Re_x^{-0.5}$$

$$C_{fx} = 2 \times 0.332\, Re^{-1/2}$$

$$\frac{C_{fx}}{2} = 0.332\, Re^{-1/2} \quad -(2)$$

Equating (1) and (2)

$$\frac{St}{Pr^{-2/3}} = \frac{C_{fx}}{2} \quad -(3)$$

and

$$C_{fx} = \frac{\tau_w}{\frac{\rho u_\infty^2}{2}} \quad -(4)$$

Equation (3) and (4) are called Reynolds colburn analogy. It expresses the relation between fluid friction

1) Air at 27°C and 1 atm pressure flows on a flat plate at 2 m/s. Plate is heated to 60°C over its entire length. Calculate the drag force on the first 0.4m length of the plate using the analogy between fluid friction and heat transfer.

Given data

$T_\infty = 27°C$ $\quad u_\infty = 2\ m/s$

$T_w = 60°C$

$$\text{Shear stress} = \frac{\text{Shear force (Drag Force)}}{\text{Area}}$$

$$\text{Shear force (Drag force)} = \text{Shear stress} \times \text{Area}$$
$$= \tau_w \times \text{Area}$$

property values of air at

$$T_f = \frac{60+27}{2} = 43.5°C \simeq 45°C$$

From D.B. P. 34.

$\nu = 17.95 \times 10^{-6}\ m^2/s$ $\quad \rho = 1.093\ kg/m^3$

$P_r = 0.698$

$k = 0.0286\ W/mK$

$$Re = \frac{u_\infty x}{\nu} = \frac{2 \times 0.4}{17.95 \times 10^{-6}}$$

$$Re = 0.0445 \times 10^{6}$$

$$= 0.445 \times 10^{5} < 5 \times 10^{5}$$

Flow is laminar

From D.B. P. 113

$$C_f = 0.664\, Re_x^{-0.5}$$

$$= 0.664 \times (0.445 \times 10^{5})^{-0.5}$$

$$C_f = 3.15 \times 10^{-3}$$

From D.B. P. 114

$$\tau_w = \frac{C_f \rho u_\infty^2}{2}$$

$$\tau_w = \frac{3.15 \times 10^{-3} \times 1.093 \times 2^2}{2}$$

Shear stress $\tau_w = 0.0068\ N/m^2$

Drag Force $= 0.0068 \times 0.4 \times 1$

$$= 0.00272\ N$$

Internal flow

mass flow rate $\dot{m} = \rho A_c u_\infty$

$Re = \frac{u_\infty D}{\nu}$

$Nu = \frac{hD}{k}$

property values to be evaluated at mean temp $T_m = \frac{T_i + T_o}{2}$

$Re < 2300$ – Laminar.

> 2300 – Turbulent.

1) Liquid Sodium is to be heated from 120°C to 180°C at a rate of 2.3 kg/s in a 2.5 cm diameter tube with a constant wall temperature. Calculate the heat transfer rate

Given data

$T_i = 120°C$

$T_o = 180°C$

$\dot{m} = 2.3$ kg/s.

$D = 2.5$ cm $= 2.5 \times 10^{-2}$ m.

property values of Liquid Sodium

$$T_m = \frac{T_i + T_o}{2}$$

$$T_m = \frac{120+180}{2} = 150°C$$

From D.B. P. 33.

$$\rho = 916 \ kg/m^3$$

$$k = 84.9 \ W/mK.$$

$$\nu = 0.594 \times 10^{-6} \ m^2/s.$$

$$Pr = 0.0087$$

$$\dot{m} = \rho A_c u_\infty$$

$$\dot{m} = \rho \times \frac{\pi}{4} \times D^2 \times u_\infty$$

$$2.3 = 916 \times \frac{\pi}{4} \times u_\infty$$

$$u_\infty = 5.11 \ m/s.$$

$$Re = \frac{u_\infty \times D}{\nu} = \frac{5.11 \times 0.025}{0.594 \times 10^{-6}}$$

$$Re = 0.2156 \times 10^6 > 2300$$

Flow is turbulent

For liquid metals and const. wall temp.

From D.B. P. 127

$$Nu = 5 + 0.025 [Re_D Pr]^{0.8}$$

$$= 5 + 0.0025\,(0.2152 \times 10^{6} \times 0.0087)$$

$$Nu = 15.37$$

$$Nu = \frac{hD}{k}$$

$$15.37 = \frac{h \times 2.5 \times 10^{-2}}{84.9}$$

$$h = 52200.5 \ W/m^2K$$

$$Q = hA\,(T_0 - T_i) \qquad [l = 1m]$$

$$= 52200 \times \pi \times 2.5 \times 10^{-2} \times 1$$

$$(180 - 120)$$

$$Q = 245.9 \times 10^{3} \ W$$

2) A fine wire having diameter 0.025 mm is placed in a 100 kPa air stream at 25°C having flow velocity 50 m/s. An electric current is passed through the wire raising the surface temperature to 50°C. Calculate the heat loss per unit length of wire.

Given data

$T_\infty = 25°C$

$T_w = 50°C$

Geometry : cylinder

$D = 0.025 \times 10^{-3}$ m.

$u_\infty = 50$ m/s.

property values of air at

$$T_f = \frac{T_w + T_\infty}{2} = \frac{50+25}{2} = 37.5°C \simeq 40°C$$

From D.B. p. 34.

$\nu = 16.96 \times 10^{-6}$ m²/s.

$Pr = 0.669$

$K = 0.02756$ W/mK

$$Re = \frac{u_\infty D}{\nu} = \frac{5 \times 0.025 \times 10^{-3}}{16.96 \times 10^{-6}}$$

$Re = 73.7$

From D.B. P. 116

Flow over cylinder

$$Nu = C \cdot Re_D^{m} \cdot Pr^{0.333}$$

For $Re = 73.7$

$C = 0.683$

$m = 0.466$

$$= 0.683 \times (73.7)^{0.466} (0.689)^{0.333}$$

$Nu = 4.4$

$$Nu = \frac{hD}{K}$$

$$h = \frac{4.4 \times 0.02756}{0.025 \times 10^{-3}}$$

$h = 4884 \ W/m^2K$

$$Q = hA(T_w - T_\infty)$$

$$= 4884 \times \pi \times 0.025 \times 10^{-3} \times 1 \ (50-25)$$

$Q = 9.59 \ W$

Air at 100 kpa and 35°c flow across a 50 mm diameter cylinder a velocity of 50 m/s. The surface is at 150°c. Calculate the heat loss per unit length of the cylinder.

Given data

Geometry : cylinder

$D = 50 \times 10^{-3} m$

$T_\infty = 35°c$

$u_\infty = 50 m/s$

$T_w = 150°c$

property values of air at

$$T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°c \simeq 95°c$$

From D.B. p. 34.

$\nu = 23.13 \times 10^{-6} m^2/s$

$Pr = 0.688$

$k = 0.0321 W/mK$

$$Re = \frac{u_\infty D}{\nu} = \frac{50 \times 50 \times 10^{-3}}{23.13 \times 10^{-6}}$$

$Re = 0.108 \times 10^6$

From D.B. P.116

Flow over cylinder

$$Nu = C\,Re^{m}\,Pr^{0.333}$$

$$C = 0.0266 \qquad m = 0.805$$

$$Nu = 0.0266 \times (0.1080 \times 10^{6})^{0.805} \times (0.688)^{0.333}$$

$$= 264.4$$

$$Nu = \frac{hD}{K}$$

$$264.4 = \frac{h \times 50 \times 10^{-3}}{0.0321}$$

$$h = 170.1 \ W/m^2K$$

$$Q = hA(T_w - T_\infty) \qquad [l = 1m]$$

$$= 170.1 \times \pi \times 50 \times 10^{-3} \times 1 \times (150 - 35)$$

$$Q = 3.07 \ W$$

Free convection

Grashof no. $Gr = \frac{\text{Buoancy force}}{\text{viscous force}}$

$$Gr = \frac{g \beta x^3 \Delta T}{\nu^2}$$

g - acceleration due to gravity m/s^2

β - volume expansion co-efficient, K^{-1}

$\beta = \frac{1}{T_f} \rightarrow K$

x - characteristic length

x - L for plane surface

x - D for cylinder

ΔT - $(T_w - T_\infty)$

ν - Kinematic viscosity m^2/s.

$Gr < 10^9$ Laminar

$> 10^9$ Turbulent

1) A horizontal pipe 0.3 m diameter is maintained at temperature of 250°C in a room where the ambient condition is at 15°C. Calculate the heat loss per meter length of the pipe.

Given data

$D = 0.3\ m$

$T_w = 250°C$

$T_\infty = 15°C$

property values of air at

$T_f = \frac{T_w + T_\infty}{2} = \frac{250+15}{2} = 132.5°C \simeq 135°C$

From D.B. P. 34.

$\alpha = 26.25 \times 10^{-6}\ m^2/s$

$Pr = 0.685$

$k = 0.0341\ W/mK$

$\beta = \frac{1}{T_f} = \frac{1}{(132.5+273)} = 2.466 \times 10^{-3}\ K^{-1}$

For D.B. P.135

$Gr_x = \frac{g\beta x^3 \Delta T}{\nu^2}$

$$Gr = \frac{9.81 \times 2.466 \times 10^{-3} \times (0.3)^3 \times (250-15)}{(26.625 \times 10^{-6})^2}$$

$$Gr = 2.165 \times 10^{8}$$

$$Gr\,Pr = 2.165 \times 10^{8} \times 0.689$$

$$= 0.149 \times 10^{9} = 1.49 \times 10^{8} < 10^{9}$$

$\therefore$ Flow is Laminar.

From. D.B. p.136

$$Nu = 0.59 (Gr\,Pr)^{0.25}$$

(For higher values of $Gr\,Pr$)

$$= 0.59 (1.49 \times 10^{8})^{0.25}$$

$$Nu = 65.18$$

$$Nu = \frac{hD}{k}$$

$$65.18 = \frac{h \times 0.3}{0.0341}$$

$$h = 7.4 \ W/m^2 K$$

$$Q = hA(T_w - T_\infty)$$

$$= 7.4 \times \pi \times 0.3 \times 1 \times (250-15)$$

$$Q = 1.638 \times 10^{3} \ W$$

2) A large vertical plate 4m high is maintained at 60°C and exposed to atmospheric air at 10°C. calculate the heat transfer from the plate if it is 10 m wide.

Given data

$T_w = 60°C$ Height = 4 m

width = 10m

$T_\infty = 10°C$

property values of air at

$$T_f = \frac{60+10}{2} = 35°C$$

From D.B. p. 34.

$\nu = 16.48 \times 10^{-6}\ m^2/s$

$Pr = 0.695$

$K = 0.0271\ W/mK$

$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)} = 3.246 \times 10^{-3}\ K$$

$$Gr_x = \frac{g \beta x^3 \Delta T}{\nu^2}$$

$$= \frac{9.81 \times 3.246 \times 10^{-3} \times 10^{3} \times 50}{16.48 \times 10^{-6}}$$

$$= 5.864 \times 10^{12}$$

$$Gr\,Pr = 5.864 \times 10^{12} \times 0.695$$

$$= 4.078 \times 10^{12} > 10^{9}$$

Flow is turbulent

From D.B. p. 136

$$Nu = 0.1\,(Gr\,Pr)^{0.333}$$

$$= 0.1\,(4.078 \times 10^{12})^{0.333}$$

$$= 1597.7$$

$$Nu = \frac{h_x x}{k}$$

$$1597.7 = \frac{h_x \times 10}{0.02715}$$

$$h_x = 4.33 \ W/m^2K$$

$$h = 2h_x = 8.67 \ W/m^2K$$

$$Q = hA(T_w - T_\infty)$$

$$= 8.67 \times 4 \times 10\,(60 - 10)$$

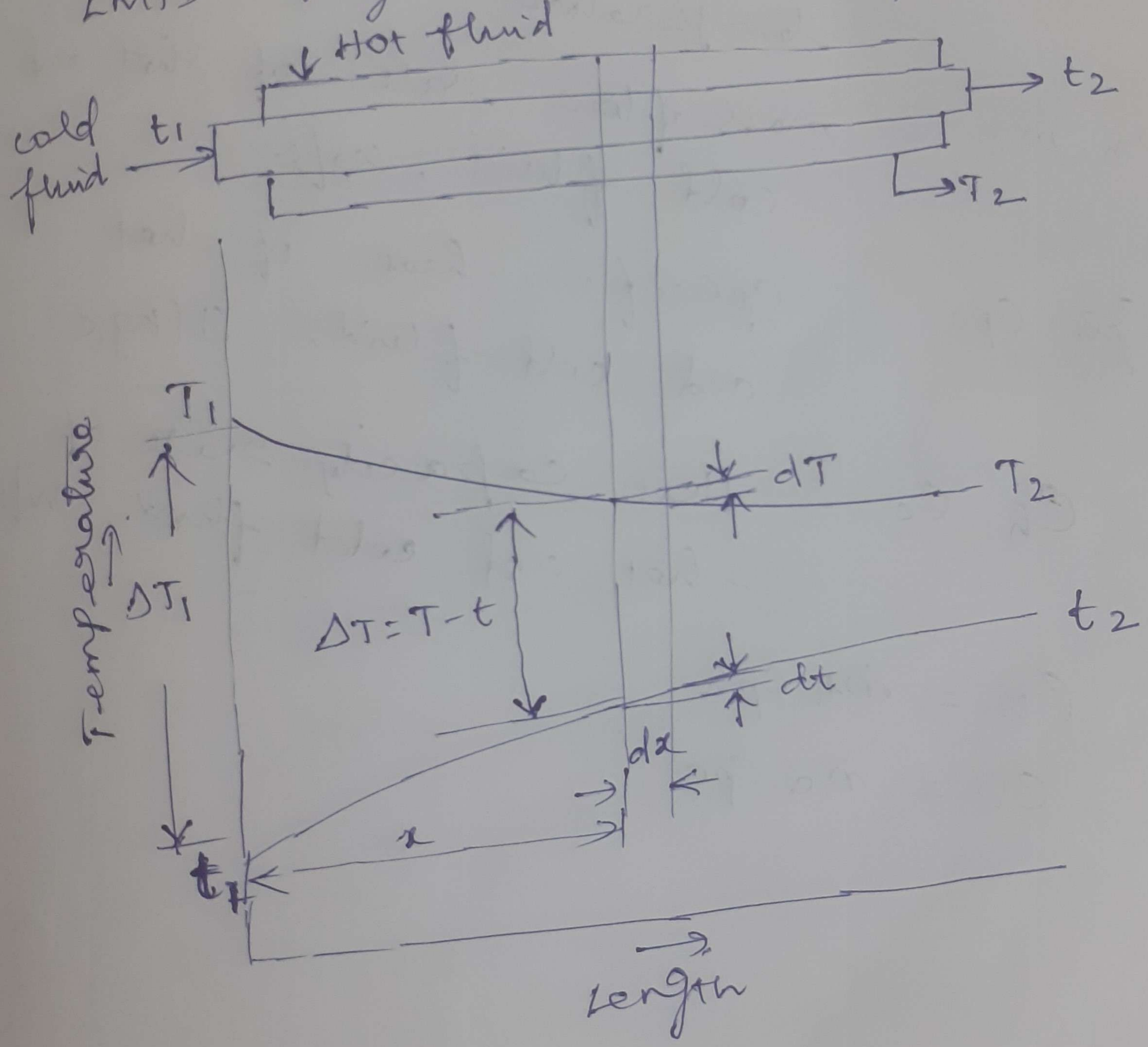
$$Q = 17.35 \times 10^{3} \ W$$

Module-III - Heat Exchangers

Heat Exchanger Analysis

Expression for LMTD - parallel flow

LMTD - Logarithmic Mean Temp. Difference



T_1, T_2 - Hot fluid inlet and outlet temperature °C

t_1, t_2 - cold fluid inlet and outlet temperature

$\dot{m}_h, \dot{m}_c$ - mass flow rate of hot and cold fluid, kg/s.

C_{ph}, C_{pc} - Specific heat of hot and cold fluid, J/kgK

C_h, C_c - Heat capacity rate of hot and cold fluid, W/K

$$C_h = \dot{m}_h C_{ph}$$

$$C_c = \dot{m}_c C_{pc}$$

The heat transfer between the cold and hot fluids for a differential element of length dx is $[Q = UA\Delta T]$

$$dQ = U\,dA\,\Delta T$$

Energy balance Equation,

Heat lost by the hot fluid = Heat gained by the cold fluid

$$Q = -\dot{m}_h C_{ph}(T_2 - T_1) = \dot{m}_c C_{pc}(t_2 - t_1)$$

$$Q = -C_h(T_2 - T_1) = C_c(t_2 - t_1)$$

$$dQ = -C_h\,dT = C_c\,dt$$

$$dT = -\frac{dQ}{C_h} \quad -(1)$$

$$dt = \frac{dQ}{C_c} \quad -(2)$$

$(1) - (2)$

$$dT - dt = -\frac{dQ}{C_h} - \frac{dQ}{C_c}$$

$$d(T-t) = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d(T-t) = -U\,dA\,\Delta T\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d(T-t)}{\Delta T} = -U\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

For whole heat exchanger

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(T-t)}{(T-t)} = \int_0^A -U\,dA\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

$$\left[\ln(T-t)\right]_{\Delta T_1}^{\Delta T_2} = -U\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\int_0^A dA$$

$$\left[\ln \Delta T\right]_{\Delta T_1}^{\Delta T_2} = -U\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\left[A\right]_0^A$$

$$\left[\ln \Delta T_2 - \ln \Delta T_1\right] = -U\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\cdot\left[A-0\right]$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right] \quad -(3)$$

we know that

$$Q = -C_h(T_2-T_1)$$

$$Q = C_h(T_1-T_2)$$

and $Q = C_c(t_2-t_1)$

$$C_h = \frac{Q}{(T_1-T_2)} \quad -(4)$$

$$C_c = \frac{Q}{(t_2-t_1)} \quad -(5)$$

Substituting (4) & (5) in (3)

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_1 - T_2}{Q} + \frac{t_2 - t_1}{Q}\right)$$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{UA}{Q}\left[T_1 - T_2 + t_2 - t_1\right]$$

$$\ln\left[\frac{(T_1 - t_1)}{(T_2 - t_2)}\right] = \frac{UA}{Q}\left[(T_1 - t_1) - (T_2 - t_2)\right]$$

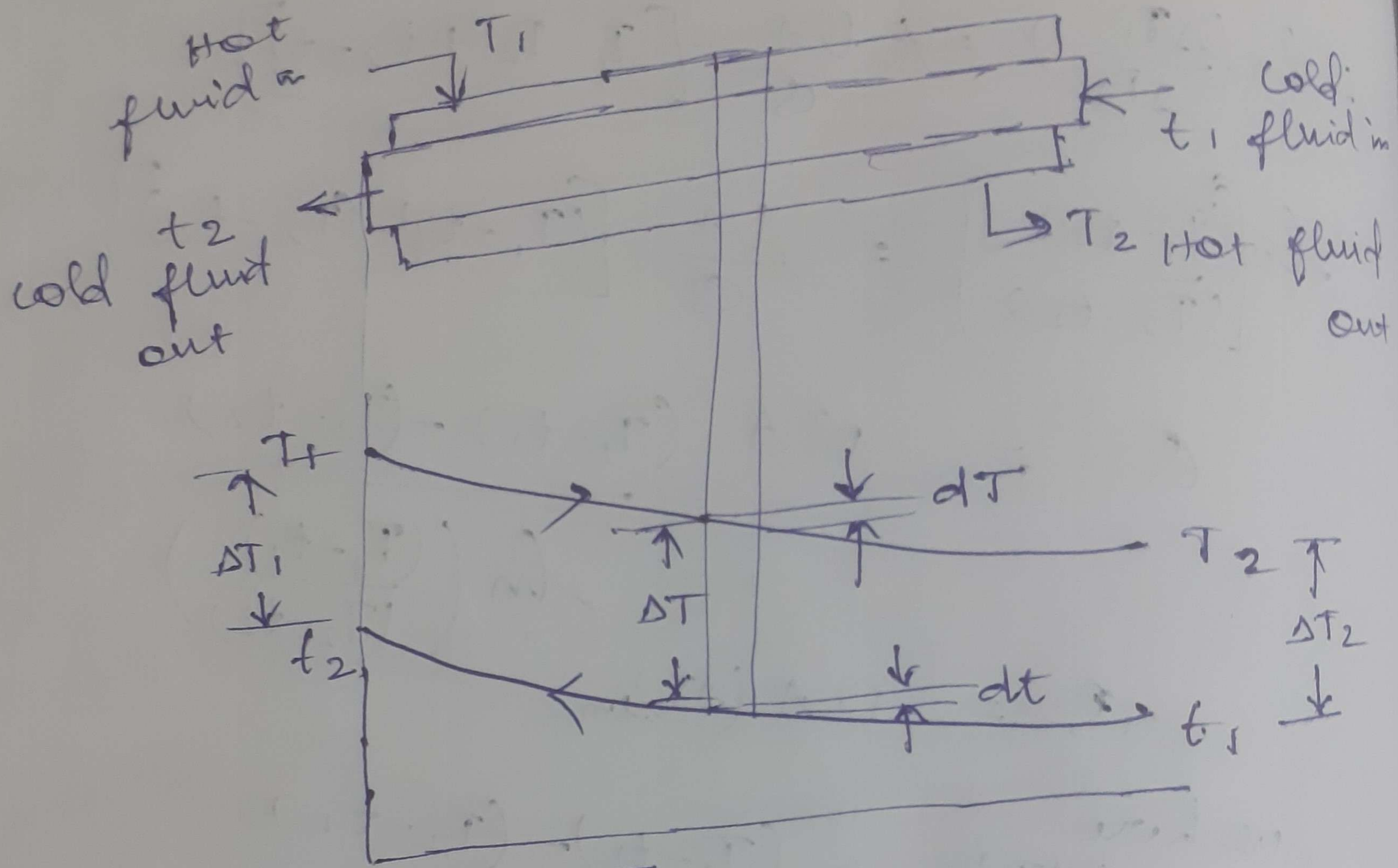
$$Q = UA\,\frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left(\frac{(T_1 - t_1)}{(T_2 - t_2)}\right)}$$

$$Q = UA\,(\Delta T)_{lm}$$

where

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left(\frac{(T_1 - t_1)}{(T_2 - t_2)}\right)}$$

Expression for LMTD - counter flow



$$dQ = U\,dA\,\Delta T$$

$$Q = -C_h(T_2 - T_1) = -C_c(t_2 - t_1)$$

$$dQ = -C_h\,dT = -C_c\,dt$$

$$dT = -\frac{dQ}{C_h} \qquad dt = -\frac{dQ}{C_c}$$

$$dT - dt = -\frac{dQ}{C_h} - \left(\frac{-dQ}{C_c}\right)$$

$$d(T-t) = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$d(T-t) = -U\,dA\,\Delta T\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\frac{d(T-t)}{\Delta T} = -U\,dA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\frac{d(T-t)}{(T-t)} = -U\,dA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\int_{\Delta T_1}^{\Delta T_2}\frac{d(T-t)}{(T-t)} = \int_0^A -U\,dA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\left[\ln(T-t)\right]_{\Delta T_1}^{\Delta T_2} = -U\left(\frac{1}{C_h}-\frac{1}{C_c}\right)\left[A\right]_0^A$$

$$\left[\ln \Delta T\right]_{\Delta T_1}^{\Delta T_2} = -UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\left(\ln \Delta T_2 - \ln \Delta T_1\right) = -UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$+\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = +UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$Q = C_h(T_1-T_2)$$

$$Q = C_c(t_2-t_1)$$

$$C_h = \frac{Q}{T_1-T_2} \qquad C_c = \frac{Q}{t_2-t_1}$$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = UA\left(\frac{T_1 - T_2}{Q} - \frac{t_2 - t_1}{Q}\right)$$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{UA}{Q}\left[(T_1 - t_2) - (T_2 - t_1)\right]$$

$$Q = UA\,\frac{\left[(T_1 - t_2) - (T_2 - t_1)\right]}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$Q = UA\,\frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{(T_1 - t_2)}{T_2 - t_1}\right)}$$

$$Q = UA(\Delta T)_{lm}.$$

where $(\Delta T)_{lm} = \dfrac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$

In a double pipe counter flow heat exchanger 10,000 kg/hr of an oil having a specific heat of 2095 J/kgK is cooled from 80°C to 50°C by 8000 kg/hr of water entering at 25°C. Determine the heat exchanger area for an overall heat transfer co-efficient of 300 W/m²K. Take Cp for water as 4180 J/kg K.

Given data

Hot fluid : oil

cold fluid : water

$\dot{m}_h = 10000$ kg/hr = 2.78 kg/s.

$C_{ph} = 2095$ J/kgK.

$T_1 = 80°C$

$T_2 = 50°C$

$\dot{m}_c = 8000$ kg/hr = 2.2 kg/s.

$t_1 = 25°C$

$U = 300$ W/m²K

$C_{pc} = 4180$ J/kgK

using energy balance equation

Heat lost by the hot fluid = Heat gained by the cold fluid

$$\dot{m}_h C_{ph} (T_1 - T_2) = \dot{m}_c C_{pc} (t_2 - t_1)$$

$$2.78 \times 2095 (80-50) = 2.2 \times 4180 (t_2 - 25)$$

$$\boxed{t_2 = 44^\circ C}$$

From D.B. P. 152

For counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$= \frac{(80-44) - (50-25)}{\ln\left(\frac{80-44}{50-25}\right)}$$

$$(\Delta T)_{lm} = 30.16^\circ C$$

$$Q = \dot{m}_h C_{ph} (T_1 - T_2)$$

$$= 2.78 \times 2095 (80-50)$$

$$= 174.72 \times 10^3 \, W$$

From D.B. P. 152

$Q = UA(\Delta T)_{lm}$

$174.72 \times 10^3 = 300 \times A \times 30.16$

$A = 19.3\ m^2$

2) Hot oil with a capacity rate 2500 W/K flows through a double pipe heat exchanger. It enters at 360°C and leaves at 300°C. Cold fluid enters at 30°C and leaves at 200°C. If the overall heat transfer co-efficient is 800 W/m²K. Determine the heat exchanger area required for (i) parallel flow (ii) counter flow.

Hot fluid: oil

$C_h = 2500\ W/K$

$T_1 = 360°C$

$T_2 = 300°C$

$t_1 = 30°C$

$t_2 = 200°C$

$U = 800\ W/m^2K$

Heat capacity rate $\left\{ \begin{array}{l} C_h = \dot{m}_h C_{ph} \\ C_c = \dot{m}_c C_{pc} \end{array} \right.$

parallel flow from D.B. P. 152

$$Q = U A (\Delta T)_{lm}$$

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left(\frac{T_1 - t_1}{T_2 - t_2}\right)}$$

$$= \frac{(360 - 30) - (300 - 200)}{\ln\left(\frac{360 - 30}{300 - 200}\right)}$$

$$(\Delta T)_{lm} = 192.64\ ^\circ C$$

$$Q = m C_h (T_1 - T_2)$$

$$= 2500 (360 - 300)$$

$$Q = 150 \times 10^3\ W$$

$$Q = U A (\Delta T)_{lm}$$

$$150 \times 10^3 = 800 \times A \times 192.64$$

$$A = 0.973\ m^2$$

counter flow

$$Q = U.A\,(\Delta T)_{lm}.$$

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}.$$

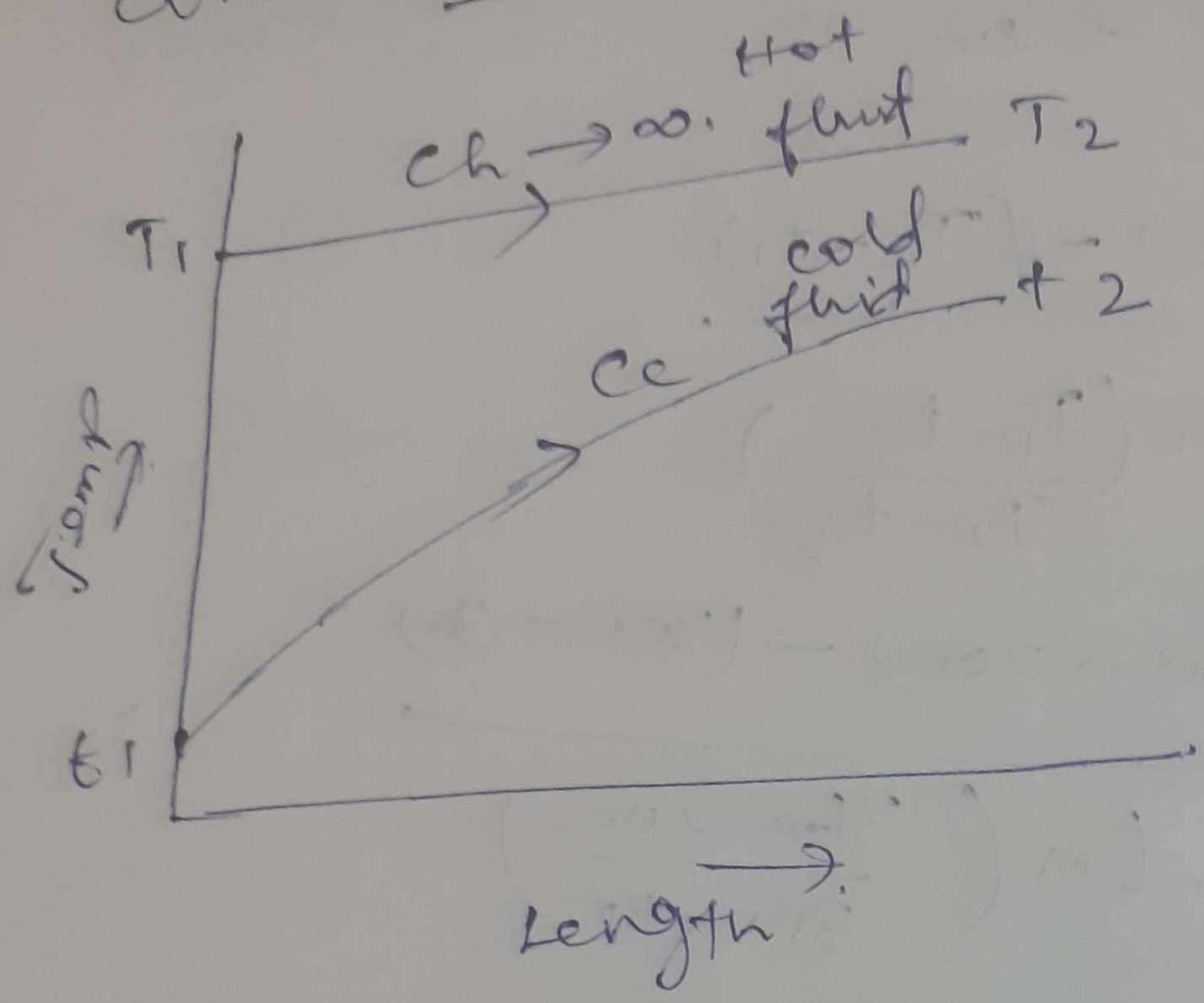
$$= \frac{(360 - 200) - (300 - 30)}{\ln\left(\frac{360 - 200}{300 - 30}\right)}$$

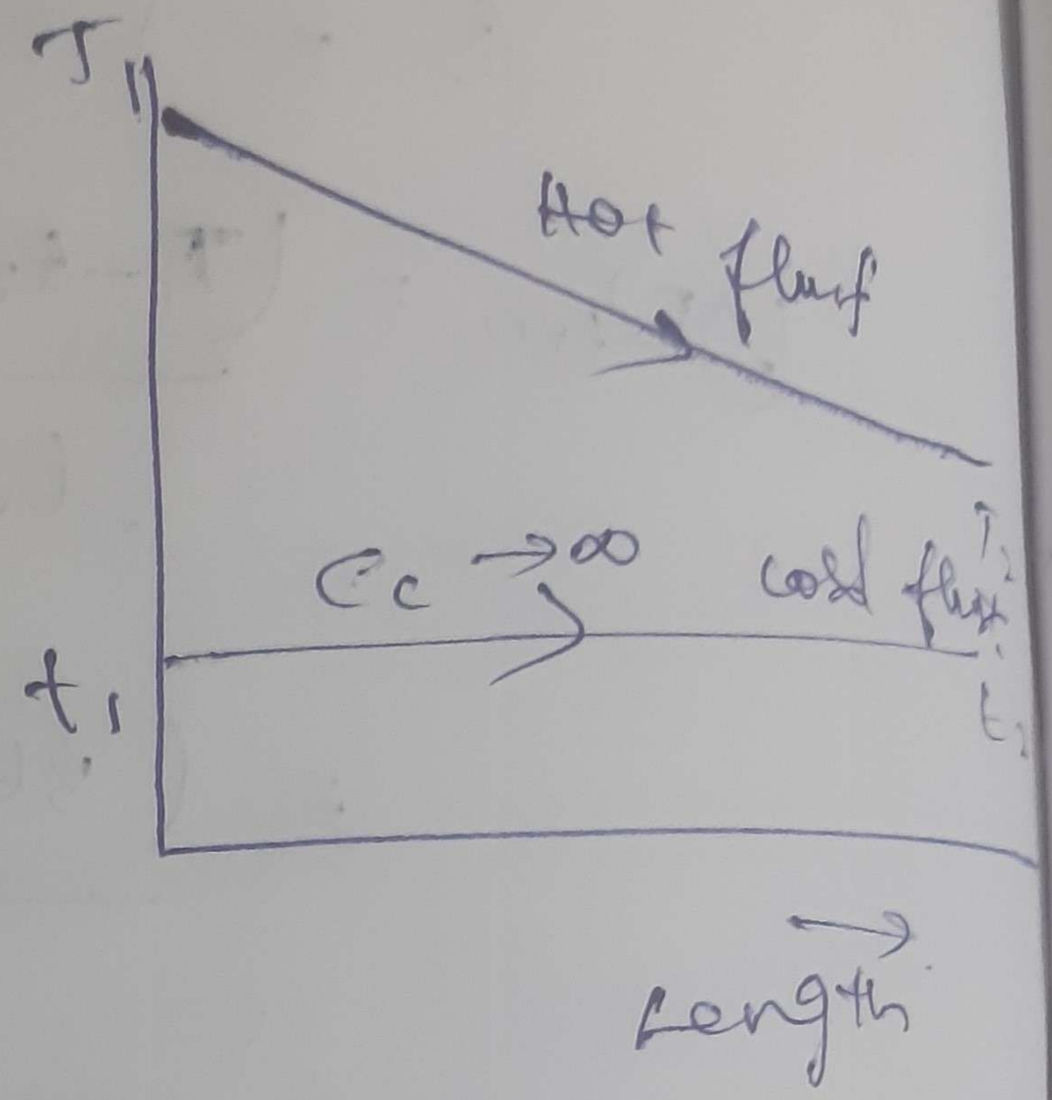
$$(\Delta T)_{lm} = 210.2\,^\circ C$$

$$150 \times 10^3 = 800 \times A \times 210.2$$

$$A = 0.891\ m^2$$

condenser and Evaporators





1) Saturated steam at 120°C is condensing on the outer tube surface of a single pass heat exchanger, the heat transfer co-efficient is 1800 W/m²K. Determine the surface area of a heat exchanger capable of heating 1000 kg/hr of water from 20°C to 90°C. Also calculate the rate of condensation of steam. Assume enthalpy of evaporation as 2200 kJ/kg

Given data

Hot fluid : steam

Cold fluid : water

$T_1 = 120^\circ C = T_2$

$U = 1800 \ W/m^2K$.

$\dot{m}_c = 1000 \ kg/hr = 0.277 \ kg/s$.

$t_1 = 20^\circ C$

$t_2 = 90^\circ C$

$h_{fg} = 2200 \ kJ/kg$

from D.B. p. 152

$Q = UA(\Delta T)_{lm}$.

For counterflow,

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$= \frac{(120 - 90) - (120 - 20)}{\ln\left(\frac{120 - 90}{120 - 20}\right)}$$

$(\Delta T)_{lm} = 58.1^\circ C$

$Q = \dot{m}_c C_{pc} (t_2 - t_1) = \dot{m}_s h_{fg}$

$Q = 0.277 \times 4180 (90 - 20)$

$Q = 81.42 \times 10^3 \, W.$

C_p for water $= 4180 \, J/kg$

$Q = UA(\Delta T)_{lm}.$

$81.42 \times 10^3 = 1800 \times A \times 58.1$

$A = 0.778 \, m^2$

$Q = \dot{m}_s h_{fg}$

$81.42 \times 10^3 = \dot{m}_s \times 2200 \times 10^3$

$\dot{m}_s = 0.037 \, kg/s.$

Multipass and Cross flow heat exchanger

$Q = F\,U\,A\,(\Delta T)_{lm}$.

F - correction factor

$(\Delta T)_{lm} \Rightarrow$ counter flow

Flow area for n tubes

$A_c = \frac{\pi}{4} d^2 \times n$

Surface area

$A = \pi d l \times n_p$

1) In a food processing plant water is to be cooled from 18°C to 6.5°C by using brine solution entering at an inlet temp. of -1.1°C and leaving at 2.9°C. What area is required when using a shell and tube heat exchanger with water making one shell pass and brine making two tube passes. Assume an overall heat transfer co-efficient of 850 W/m²K and a design heat load of 6000 W.

Given data

Hot fluid : water

Cold fluid : Brine

$T_1 = 18°C$ $\quad t_1 = -1.1°C$

$T_2 = 6.5°C$ $\quad t_2 = 2.9°C$

$U = 850\ W/m^2K$ $\quad Q = 6000\ W$

(1) To find Area (A)

from D.B. p. 152

$Q = UA(\Delta T)_{lm} F$

To find F from Graph.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{2.9 + 1.1}{18 + 1.1} = 0.209$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{18 - 6.5}{2.9 + 1.1} = 2.87$$

from D.B. p. 159

$F = 0.97$

For counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$= \frac{(18-2.9)-(6.5+1.1)}{\ln\left(\frac{18-2.9}{6.5+1.1}\right)}$$

$$(\Delta T)_{lm} = 10.9\ ^{\circ}C$$

$$Q = FUA(\Delta T)_{lm}$$

$$6000 = 0.97 \times 850 \times A \times 10.9$$

$$A = 0.661\ m^2$$

9) Water at the rate of 4 kg/s is heated from 38°C to 55°C in a shell and tube type heat exchanger. The water is to flow inside tubes of 2 cm diameter with an average velocity of 35 cm/s. Hot water available at 95°C and at the rate of 2 kg/s is used as the heating medium on the shell side. If the length of the tubes must not be more than 2 m. Calculate no. of tube passes, the no. of tubes per pass and length of the tubes for one pass shell assuming $U = 1500\ W/m^2K$.

Given data

Hot fluid : Hot water

cold fluid : cold water

$\dot{m}_c = 4\ kg/s$

$t_1 = 38°C$

$t_2 = 55°C$

$d = 2 \times 10^{-2} m$

$V = 35\ cm/s$

$= 35 \times 10^{-2}\ m/s$

$T_1 = 95°C$

$\dot{m}_h = 2\ kg/s$

$U = 1500\ W/m^2K$

Assuming one tube passes $n_p = 1$

$$Q = UA(\Delta T)_{lm}$$

$$Q = \dot{m}_h C_{ph}(T_1 - T_2)$$

C_p for water = 4180 J/kgK

$$Q = \dot{m}_c C_{pc}(t_2 - t_1)$$

$$\therefore Q = 4 \times 4180 (55 - 38)$$

$$Q = 284.65 \times 10^3 \, W$$

For counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$Q = \dot{m}_h C_{ph}(T_1 - T_2)$$

$$284.65 \times 10^3 = 2 \times 4180 (95 - T_2)$$

$$T_2 = 61^\circ C$$

$$(\Delta T)_{lm} = \frac{(95 - 25) - (61 - 38)}{\ln\left(\frac{95 - 55}{61 - 38}\right)}$$

$$(\Delta T)_{lm} = 30.7^\circ C$$

$$A = \frac{Q}{U\,(\Delta T)_{lm}} = \frac{284.65 \times 10^3}{1500 \times 30.72}$$

Surface area $A = 6.17\ m^2$

Flow area

$$A_c = \frac{\pi}{4} d^2 \times n$$

$$\dot{m}_c = \rho A_c V$$

$$4 = 1000 \times A_c \times 35 \times 10^{-2}$$

$$A_c = 0.1142\ m^2$$

$$0.1142 = \frac{\pi}{4} \times (2 \times 10^{-2})^2 \times n.$$

$$n = 36.38 \simeq 36$$

Surface area $A = \pi d l n \times n_p.$

$$6.17 = \pi \times 2 \times 10^{-2} \times l \times 36 \times 1.$$

$$l = 2.731\ m.$$

Here, $l > 2\ m.$

But, l should be less than $2m$.

∴ considering two tube passes,

$$Q = UA'(\Delta T)_{lm} F$$

To find F

From D.B P. 159.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{55-38}{95-38} = 0.3$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{95-61}{55-38} = 2$$

$F = 0.9$

$$284.6 \times 10^3 = 1500 \times A' \times 30.7 \times 0.9$$

$$A' = 6.8\ m^2$$

$$A' = \pi d l n_p$$

$$6.8 = \pi \times 2 \times 10^{-2} \times l \times 36 \times 2$$

$$\boxed{l = 1.57\ m}$$

$n = 36$

$n_p = 2$

$l = 1.57\ m$

Effectiveness - NTU method

$$\epsilon = \frac{Q}{Q_{max}}$$

$$Q = \dot{m}_h C_{ph} (T_1 - T_2) = \dot{m}_c C_p (t_2 - t_1)$$

$$Q = C_h (T_1 - T_2) = C_c (t_2 - t_1)$$

$$Q_{max} = C_{min} (T_1 - t_1)$$

$C_{min} \Longrightarrow$ minimum value of C_h and C_c

$$NTU = \frac{UA}{C_{min}}$$

NTU - No. of Transfer units

1) water enters a cross flow heat exchanger (both fluids unmixed) at 5°c and flow at the rate 4600 kg/hr to cool 4000 kg/hr of air that initially at 40°c. Assume over all heat transfer co-efficient is 150 W/m²K for a heat exchanger surface area of 25 m². Calculate the exit temperature of air and water.

Given data

Hot fluid : Air

Cold fluid : water

$t_1 = 5°c$

$\dot{m}_c = 4600 \text{ kg/hr} = 1.27 \text{ kg/s}$

$\dot{m}_h = 4000 \text{ kg/hr} = 1.11 \text{ kg/s}$

$T_1 = 40°c$

$U = 150 \text{ W/m}^2\text{K}$

$A = 25 \text{ m}^2$

To find T_2 and t_2

C_p for water = 4180 J/kg/K

C_p for air = 1005 J/kg/K.

$$\epsilon = \frac{Q}{Q_{max}}.$$

$\dot{m}_h C_{ph} = 1.27 \times 4180 = C_h$

$= 5308.6$ W/K

$\dot{m}_c C_{pc} = 1.11 \times 1005 = C_c$

$= 1115.5$ W/K

$C_c < C_h$

$\therefore C_{min} = C_c = 1115.5$ W/K.

$Q_{max} = C_{min}(T_1 - t_1)$

$= 1115.5(40-5)$

$Q_{max} = 39.04 \times 10^3$ W

To find ϵ

From D.B.P. 166

Cross-flow - Both fluids unmixed

$$\frac{C_{min}}{C_{max}} = \frac{1115.5}{5308.6} = 0.21$$

$$NTU = \frac{UA}{C_{min}}$$

$$NTU = \frac{150 \times 25}{1115.5} = 3.36$$

From Graph

$$\epsilon = 0.85$$

$$Q = \epsilon Q_{max}$$

$$= 0.85 \times 39.04 \times 10^3$$

$$Q = 33186.12 \text{ W}$$

$$Q = C_h (T_1 - T_2)$$

$$33186.12 = 5308.6 (40 - T_2)$$

$$\boxed{T_2 = 10.25^\circ C}$$

$$Q = C_c (t_2 - t_1)$$

$$33186.12 = 1115.5 (t_2 - 5)$$

$$\boxed{t_2 = 11.25^\circ C}$$

1) Water enters a counter flow double pipe heat exchanger at 15°C, flowing at the rate 1300 kg/hr. It is heated by oil (C_p = 2000 J/kgK) flowing at the rate of 550 kg/hr from the inlet temperature of 94°C. Heat exchanger area of $1 m^2$ and overall heat transfer co-efficient of 1075 W/m^2K determine the heat transfer and outlet temperature of water and oil

Given data

Hot fluid : oil

cold fluid : water

$T_1 = 94°C$

$A = 1 m^2$

$U = 1075$ W/m^2K

$t_1 = 15°C$

$\dot{m}_c = 1300$ kg/hr

$= 0.361$ kg/s

$C_{ph} = 2000$ J/kgK

$\dot{m}_h = 550$ kg/hr

$= 0.152$ kg/s

To find T_2 and t_2

C_p for water $= 4180$ J/kg/K

$$\epsilon = \frac{Q}{Q_{max}}$$

$\dot{m}_h C_{ph} = 0.152 \times 2000 = C_h$

$= 305.4$ W/K

$\dot{m}_c C_{pc} = 0.361 \times 4180 = C_c$

$= 1508.6$ W/K.

$C_h < C_c$

$\therefore C_{min} = C_h = 305.4$ W/K

$Q_{max} = C_{min}(T_1 - t_1)$

$= 305.4(94-15)$

$Q_{max} = 24.12 \times 10^3$ W

To find ϵ :

From D. B. P. 164.

$$\frac{C_{min}}{C_{max}} = \frac{305.4}{1508.6} = 0.202$$

$$NTU = \frac{UA}{C_{min}} = \frac{1075 \times 1}{305.4} = 3.51$$

From Graph $\epsilon = 0.95$

$$\epsilon = \frac{Q}{Q_{max}}$$

$$Q = 0.95 \times 24.12 \times 10^3$$

$$Q = 22.92 \times 10^3 \, W$$

$$Q = C_h (T_1 - T_2)$$

$$22.92 \times 10^3 = 305.4 (94 - T_2)$$

$$\boxed{T_2 = 18.9^\circ C}$$

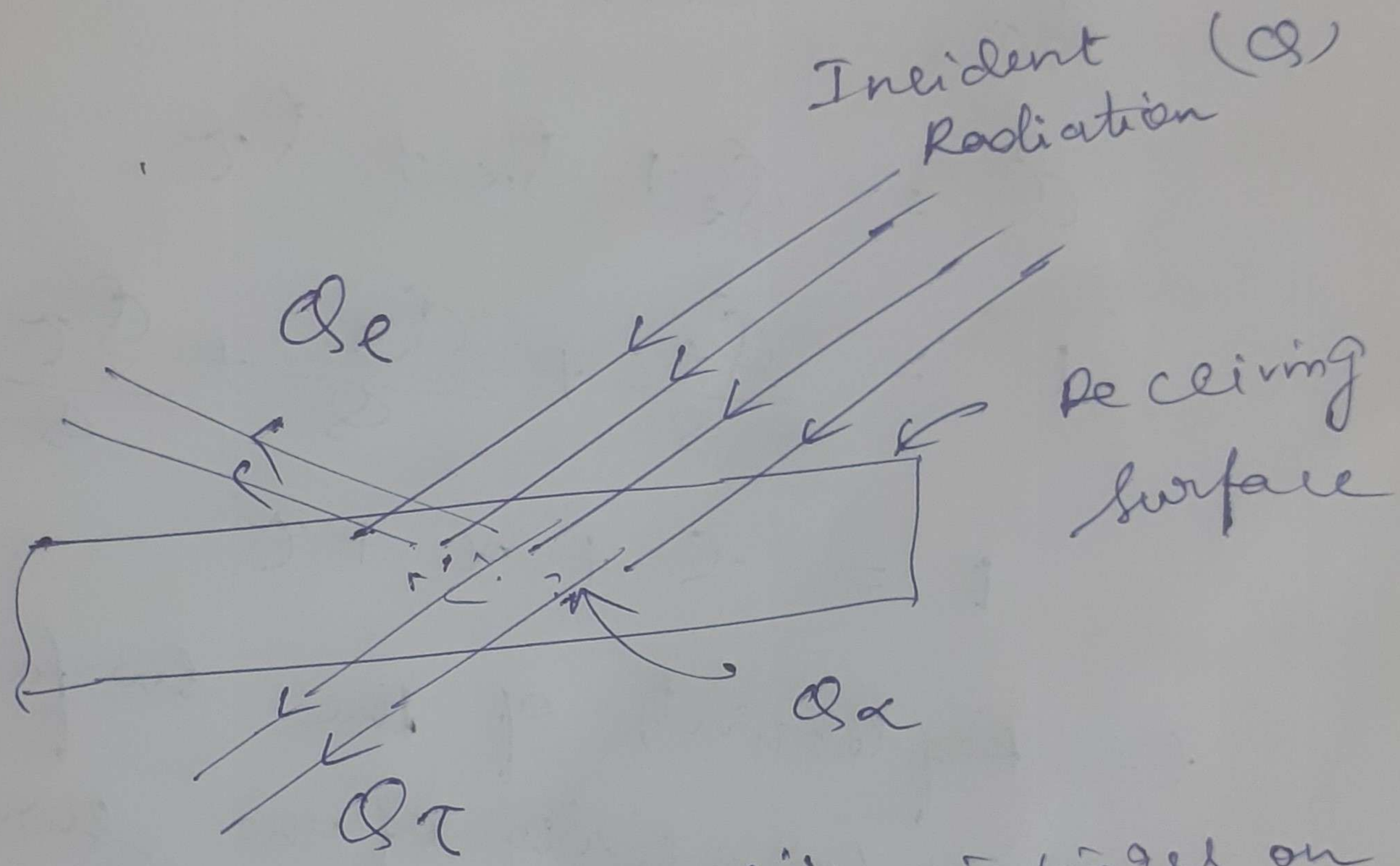
$$Q = C_c (t_2 - t_1)$$

$$22.92 \times 10^3 = 1508.6 (t_2 - 15)$$

$$\boxed{t_2 = 30.19^\circ C}$$

Radiation

Absorption, Reflection and Transmission



When incident radiation, impinges on a surface, the following may happen,

(i) a part is reflected back

(ii) a part is Transmitted

(iii) The reminder is absorbed

$Q \rightarrow$ Heat received by the surface (Incident radiation)

$Q_\rho \rightarrow$ Heat reflected by the surface

Q_α - Heat absorbed by the surface

Q_τ - Heat transmitted by the surface

$$Q = Q_\rho + Q_\alpha + Q_\tau$$

$$1 = \frac{Q_\rho}{Q} + \frac{Q_\alpha}{Q} + \frac{Q_\tau}{Q}$$

$$1 = \rho + \alpha + \tau$$

ρ - Reflectivity of the surface

α - Absorptivity of the surface

τ - Transmissivity of the surface

$\rho = 0 \rightarrow$ non reflecting surface

$\rho = 1 \rightarrow$ Perfect reflector

$\tau = 0 \rightarrow$ Opaque surface

$\tau = 1 \rightarrow$ Transparent surface

$\alpha = 0 \rightarrow$ Non absorbing surface

$\alpha = 1 \rightarrow$ Perfect absorbing surface (Black body)

Black body

A black body is an ideal body that absorbs all incident radiant energy and reflects or transmits none. For black body

$\alpha = 1 \quad \rho = 0 \quad \tau = 0.$

No actual body is perfectly black.

The concept of black body is an idealisation with which radiation characteristics of real bodies can be compared.

④ Laws of Radiation

Stefan Boltzmann's law

$E_b \propto T^4$

$E_b = \sigma T^4$

E_b - Emissive power of a black surface

$E_b = \frac{Q}{A}$ W/m^2

Kirchoff's law

At thermal Equilibrium

$\epsilon = \alpha$

Wien's law

wave length corresponding to maximum energy emission is inversely proportional to the absolute temperature of the body

$\lambda_m \propto \frac{1}{T}$

$\lambda_m = c \frac{1}{T}$

$\lambda_m T = c$

$c = 2.898 \times 10^{-3}$ mmK

λ - wave length in μm

(5)

Fraction of emissive power of black body $= \dfrac{\text{Black body radiant energy upto the wave length } \lambda}{\text{Total emissive power of black (At same temp)}}$

$$= \frac{E_b(0-\lambda)}{E_b} = \frac{E_b(0-\lambda)}{\sigma T^4}$$

Fraction of emissive power in the wave length band of λ_1 and λ_2

$$E_b(\lambda_1-\lambda_2) = \frac{E_b(0-\lambda_1) - E_b(0-\lambda_2)}{\sigma T^4}$$

Emissive power in the wave length band λ_1 and λ_2

$$E_b(\lambda_1-\lambda_2) = \frac{E_b(0-\lambda_1) - E_b(0-\lambda_2)}{\sigma T^4} \times \sigma T^4$$

1) A black body is kept at a temperature of 727°c. Calculate the thermal radiation emitted by the surface in the wave length band 1 μm and 5 μm.

Given data

$\lambda_1 = 1\ \mu m$ $\qquad T = 727°C = 1000\ K$

$\lambda_2 = 5\ \mu m$

$\lambda_1 T = 1 \times 1000 = 1000\ \mu m K$

$\lambda_2 T = 5 \times 1000 = 5000\ \mu m K$

$$E_b(\lambda_1 - \lambda_2) = \frac{E_{b(0-\lambda_1)} - E_{b(0-\lambda_2)}}{\sigma T^4} \times \sigma T^4$$

From D.B. p. 83.

for $\lambda_1 T = 1000\ \mu m K$

$$\frac{E_{b0-\lambda_1}}{\sigma T^4} = 0.000321$$

and $\lambda_2 T = 5000\ K$

$$\frac{E_{b0-\lambda_2}}{\sigma T^4} = 0.633747$$

7

$= (0.000321 - 0.63374) \times 5.67 \times 10^{-8} \times (1000)^4$

$E_{b(T_1-T_2)} = -35915.25 \ W/m^2$

2) A black body is at a temperature of 927°C. Determine the fraction of the radiation emitted by the surface in the wave length band 2 μm and 6 μm.

Given data

$T = 927°C = 1200 K$

$T_1 = 2 \mu m$ $\qquad T_1 T = 2 \times 1200$

$= 2400 \ \mu mK$

$T_2 = 6 \mu m$ $\qquad T_2 T = 6 \times 1200$

$= 7200 \ \mu mK$

$$\frac{E_{b(0-T_1)} - E_{b(0-T_2)}}{\sigma T^4}$$

From D.B. p. 82.

For $T_1 T = 2400 \ \mu mK$

$$\frac{E_{b(0-T_1)}}{\sigma T^4} = 0.14025$$

For $T_2 T = 7200$ mmK

$$\frac{E_b(0-T_2)}{\sigma T^4} = 0.8192$$

$$E_b(T_1-T_2) = 0.14025 - 0.8192$$

$$= 0.678$$

Radiative heat exchange between two surfaces

Emissivity

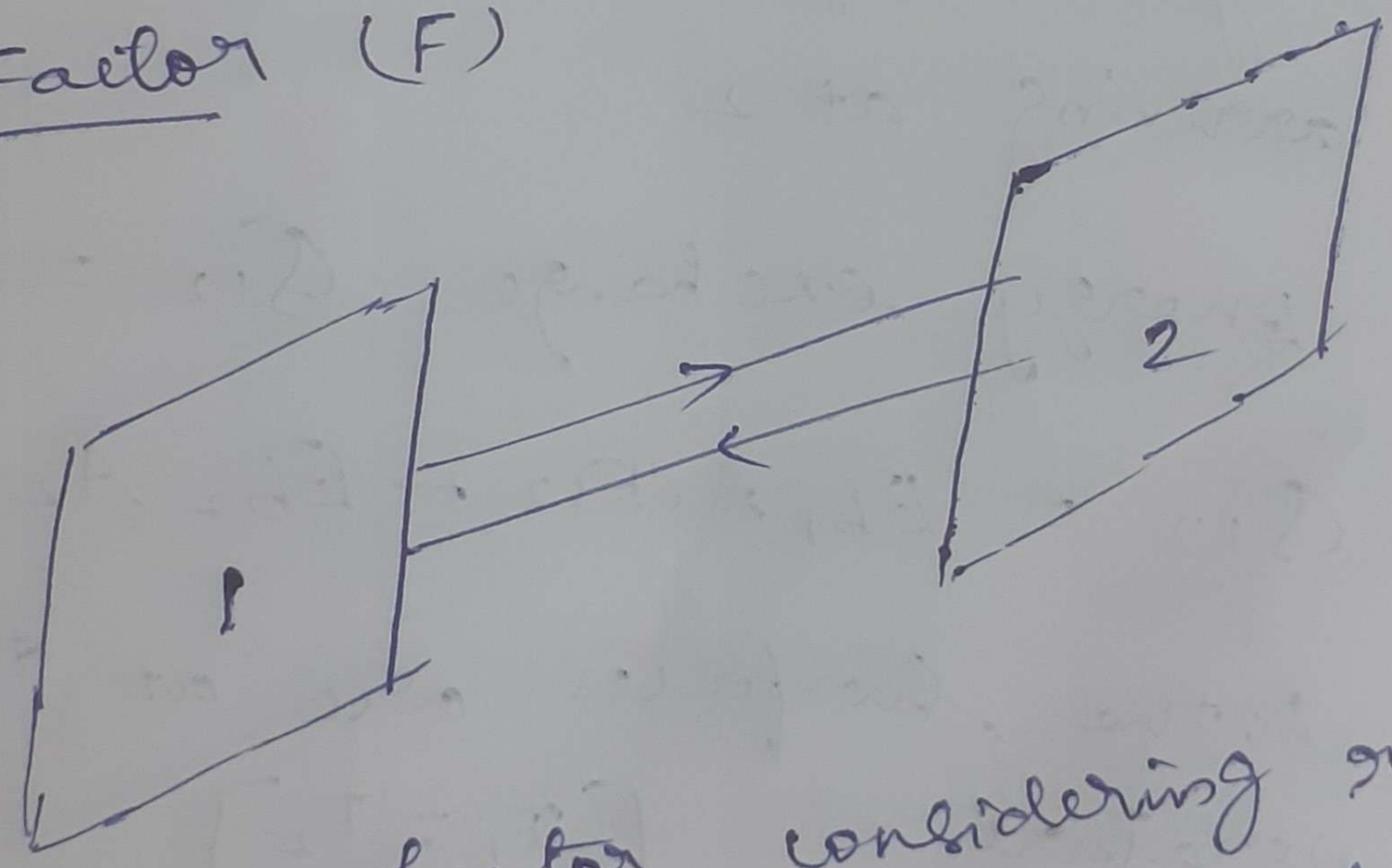
Ratio of emissive power of a grey body and black body

$$\epsilon = \frac{E}{E_b}. \qquad E_b = \sigma T^4$$

$$E = \epsilon E_b$$

$$E = \epsilon \sigma T^4 \ W/m^2.$$

Shape Factor (F)



F_{12} - Shape factor considering radiation heat transfer from surface 1 to 2

F_{21} - Shape Factor considering radiation from 2 to 1

(10) Shape factor is the fraction of radiating energy that is diffused from one surface element and strikes the other surface directly without interruption

Considering surfaces 1 and 2

Energy leaving the surface 1 and arriving 2 $Q_1 = E_{b1} A_1 F_{12}$

Energy leaving 1 and arriving at 2 $Q_2 = E_{b2} A_2 F_{21}$

Net Energy exchange $Q_{12} = Q_1 - Q_2$

$$Q_{12} = E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21}$$

If bothe surfaces are at the same temperature $[T_1 = T_2]$

$Q_{12} = 0$ and $E_{b1} = E_{b2}$

$$\therefore E_{b1} A_1 F_{12} = E_{b2} A_2 F_{21}$$

$$\boxed{A_1 F_{12} = A_2 F_{21}}$$

Known as Reciprocal theorem

we have

$$Q_{12} = E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21}$$

since $A_1 F_{12} = A_2 F_{21}$

$$Q_{12} = A_1 F_{12} (E_{b1} - E_{b2})$$

(or)

$$Q_{12} = A_2 F_{21} (E_{b1} - E_{b2})$$

$E_b = \sigma T^4$

For a black body

$$Q_{12} = A_1 F_{12} (\sigma T_1^4 - \sigma T_2^4)$$

$$Q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

we know that

$$\epsilon = \frac{E}{E_b} = \frac{\frac{Q}{A}}{E_b}.$$

For Grey body,

$$\frac{Q}{A} = \epsilon E_b.$$

$$Q = A \epsilon E_b = A \epsilon \sigma T^4$$

For surfaces 1 and 2

$$\therefore Q_{12} = \epsilon A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

For Grey body

where ϵ - Equivalent emissivity for two surfaces

(i) For Finite parallel plate (or) Disc

$$Q = \epsilon A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

where $\epsilon = \dfrac{1}{\left(\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1\right)}$

(ii) For Infinite parallel plate (or) disc

For infinite parallel plate, all the radiation emitted by one surface will be received by the other surface.

Hence shape Factor $F_{12} = 1$

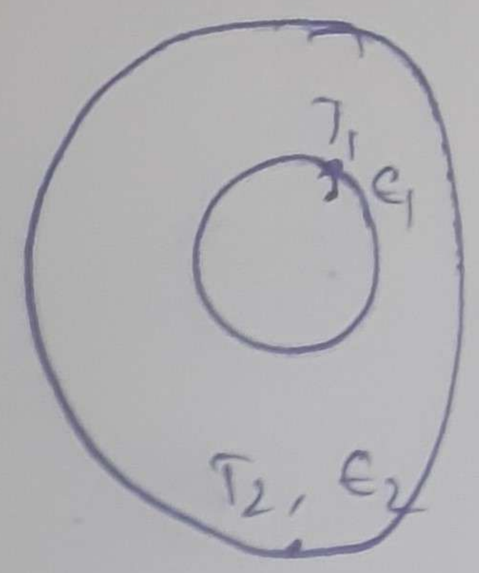
and considering unit area

$$Q = \epsilon \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\left(\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1\right)}$$

iii) For concentric cylinder or spheres

$$\boxed{F_{12} = 1}$$



$$Q = A_1 \epsilon \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

1) The emissivities of two large parallel plates maintained at a temperature of 800°C and 300°C are 0.3 and 0.5 respectively. Find the net radiant heat exchange per unit area of these plates.

Given data

$T_1 = 800°C = 1073 K$

$T_2 = 300°C = 573 K$

$\epsilon_1 = 0.3$

$\epsilon_2 = 0.5$

For infinite plate

$$Q = \epsilon \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1} = \frac{1}{\frac{1}{0.3} + \frac{1}{0.5} - 1}$$

$$\epsilon = 0.23$$

$$\boxed{\sigma = 5.67 \times 10^{-8} \, W/m^2 K}$$

$$Q = 0.23 \times 5.67 \times 10^{-8} \left((1073)^4 - (573)^4\right)$$

$$Q = 15.8 \, KW$$

2) Two concentric parallel discs face each other. The diameter of discs are 1 m and the distance between them is also 1 m. The surface emissivities are 0.2 and 0.6 and their temperatures are 300°C and 80°C

Given data

Dia = 1 m

Distance = 1 m

$T_1 = 300°C = 573 K$

$T_2 = 80°C = 353 K$

$\epsilon_1 = 0.2$

$\epsilon_2 = 0.6$

For finite parallel surfaces

$$Q = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}$$

$$\epsilon = \frac{1}{\left(\frac{1}{0.2} + \frac{1}{0.6} - 1\right)} = 0.176$$

From D.B. P. 91

$$\text{Ratio} = \frac{\text{Dia}}{\text{Distance}} = \frac{1}{1} = 1$$

curve 15 → Direct radiation between the planes.

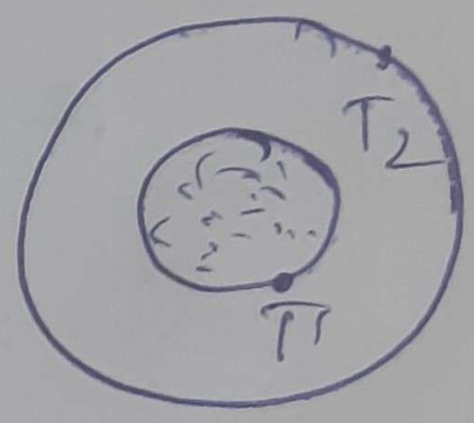
$$F_{12} = 0.18$$

$$Q = 0.176 \times \frac{\pi}{4} \times (1)^2 \times 0.18 \times 5.67 \times 10^{-8} \left((573)^4 - (353)^4\right)$$

$$Q = 130.52 \text{ W}$$

3) The surface of a double walled spherical vessel used for storing liquid oxygen are covered with a layer of silver having a emissivity of 0.03. The temperature of the outer surface of the inner wall is -153°C and the temperature of inner surface of the outer wall is 27°C. Spheres are 21 cm and 30 cm in diameter with a space between them is evacuated. Calculate the radiating heat transfer through the wall into the vessel.

<u>Given data</u>



$\epsilon_1 \sim \epsilon_2 = 0.03$ (same material)

$T_1 = -153°C = 120\ K$

$T_2 = 27°C = 300\ K$.

$D_1 = 21\ cm = 21 \times 10^{-2}\ m$

$D_2 = 30\ cm = 30 \times 10^{-2}\ m$.

$r_1 = 10.5 \times 10^{-2}\ m$

$r_2 = 15 \times 10^{-5}\ m$.

For concentric disk (or) sphere

$$Q = A_1 \epsilon \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\epsilon_1 + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

For sphere

$$A_1 = 4\pi r_1^2 = 4\pi \times (10.5 \times 10^{-2})^2 = 0.138$$

$$A_2 = 4\pi r^2 = 4\pi \times (15 \times 10^{-2})^2 = 0.282 \, m^2$$

$$\epsilon = \frac{1}{\frac{1}{0.03} + \frac{0.138}{0.282}\left(\frac{1}{0.03} - 1\right)}$$

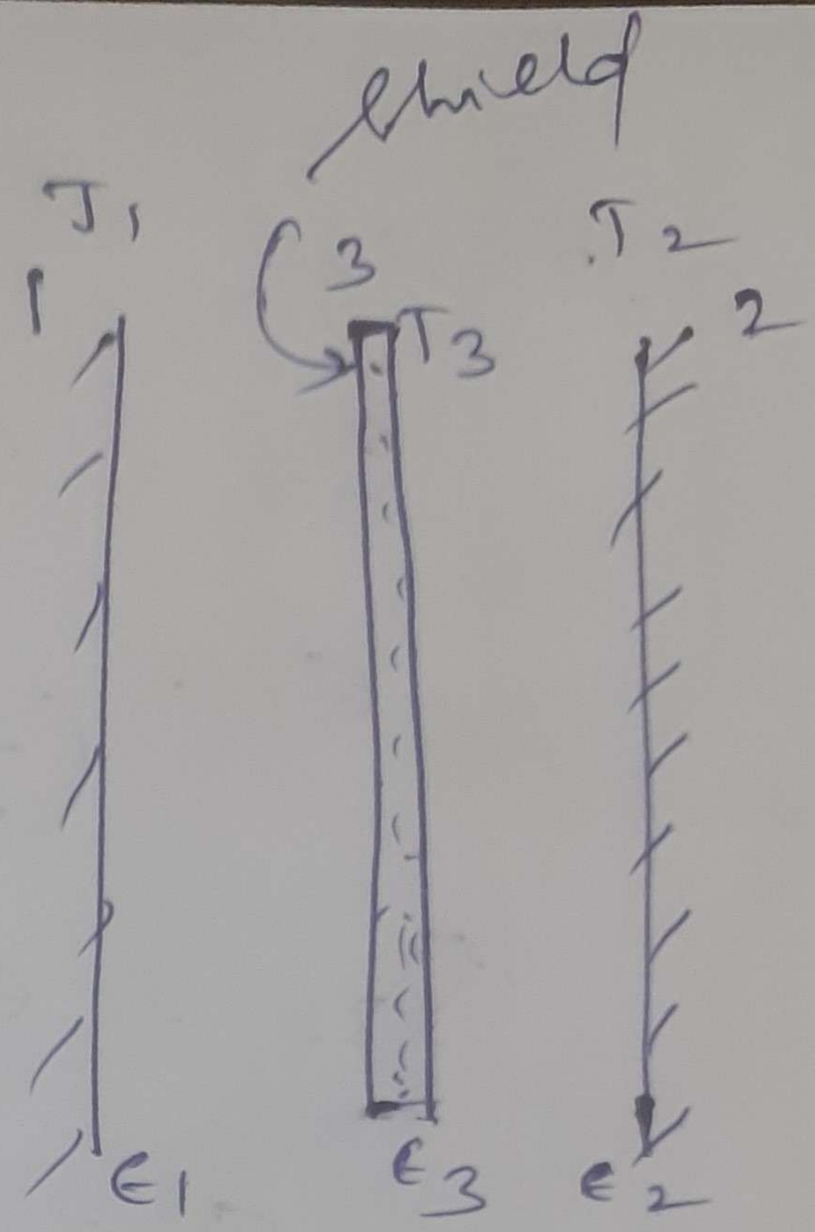
$$\epsilon = 0.0203$$

$$Q = 0.138 \times 0.0203 \times 5.67 \times 10^{-8} \left((120)^4 - (300)^4\right)$$

$$Q = -1.26 \, W$$

Radiation Shield

Radiation shields are used to reduce the heat transfer by radiation between the surfaces by effectively increasing the surface emittance without actually removing heat from the source



without Shield

Assuming infinite parallel planes

$$Q_{12} = \epsilon \sigma (T_1^4 - T_2^4)$$

$$e = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}$$

with Shield

$$Q_{12} = \epsilon \sigma (T_1^4 - T_2^4)$$

$$e = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right) + \left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right)}$$

Also, $Q_{13} = Q_{3-2}$

$$Q_{13} = \epsilon \sigma (T_1^4 - T_3^4)$$

$$\epsilon = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right)}$$

$$Q_{32} = \epsilon \sigma (T_3^4 - T_2^4)$$

$$\epsilon = \frac{1}{\left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right)}$$

1) Two large parallel plates with emissivities of 0.6 and 0.8 are at 1000 K and 500 K respectively. Determine the percentage reduction in heat transfer by placing a shield of emissivity of 0.1 between the plates. Also find the shield temperature

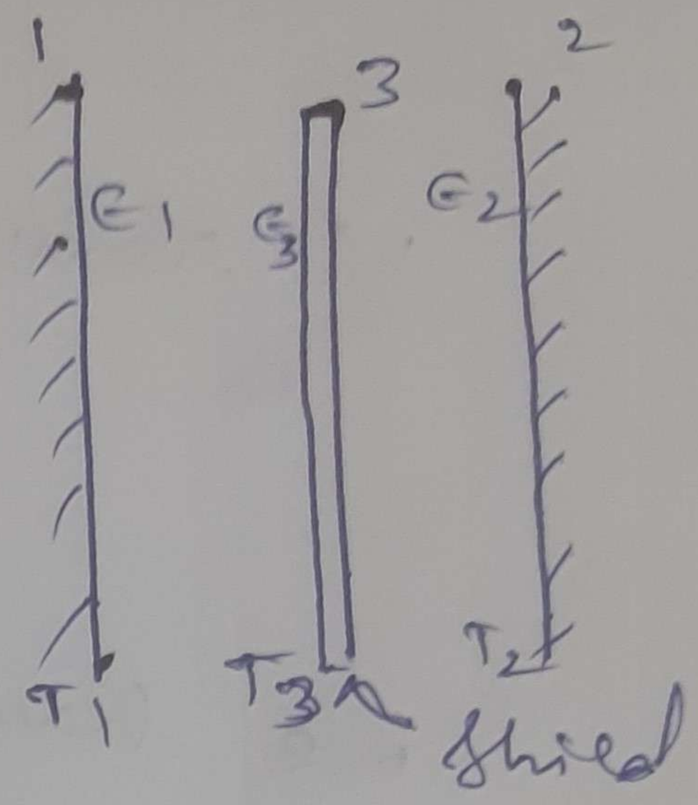
Given data

$\epsilon_1 = 0.6$ $\quad T_1 = 1000\ K$

$\epsilon_2 = 0.8$ $\quad T_2 = 500\ K$

$\epsilon_3 = 0.1$

(i) without shield



$$Q_{1-2} = \epsilon \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$\epsilon = \frac{1}{\frac{1}{0.6} + \frac{1}{0.8} - 1}$$

$$\epsilon = 0.521$$

$$Q_{12} = 0.521 \times 5.67 \times 10^{-8} ((1000)^4 - (500)^4)$$

$$Q_{12} = 27.73 \times 10^3 \, W$$

(ii) without shield

$$Q_{12} = \epsilon \sigma (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right) + \left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right)}$$

$$\epsilon = \frac{1}{\left(\frac{1}{0.6} + \frac{1}{0.1} - 1\right) + \left(\frac{1}{0.1} + \frac{1}{0.8} - 1\right)}$$

$$\epsilon = 0.047$$

$$Q_{12}' = 0.047 \times 5.67 \times 10^{-8} ((1000)^4 - (500)^4)$$

$$Q_{12}' = 2.54 \times 10^3 \, W$$

$$\% \text{ reduction in } Q = \frac{27.73\times10^3 - 2.54\times10^3}{27.73\times10^3}\times100$$

$$= 90.8\%$$

we have,

$$Q_{13} = Q_{32}$$

$$\epsilon_{13}\,\phi\,(T_1^4 - T_3^4) = \epsilon_{32}\,\phi\,(T_3^4 - T_2^4)$$

$$\epsilon_{13} = \frac{1}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_3}-1} = \frac{1}{\frac{1}{0.6}+\frac{1}{0.1}-1} = 0.093$$

$$\epsilon_{32} = \frac{1}{\left(\frac{1}{\epsilon_3}+\frac{1}{\epsilon_2}-1\right)} = \frac{1}{\left(\frac{1}{0.1}+\frac{1}{0.8}-1\right)} = 0.097$$

$$0.093\left((1000)^4 - T_3^4\right) = 0.097\left(T_3^4 - (500)^4\right)$$

$$\frac{(1000)^4 - T_3^4}{T_3^4 - (500)^4} = 1.041$$

$$1000^4 - T_3^4 = \left(1.041\times T_3^4\right) - \left(1.041\times(500)^4\right)$$

$$1000^4 + 1.041\times(500)^4 = 1.041\,T_3^4 + T_3^4$$

$$1.065\times10^{12} = T_3^4\,(1.041+1)$$

$$T_3 = \left(\frac{1.065 \times 10^{12}}{2.041} \right)^{1/4}$$

$$\boxed{T_3 = 849.9\ K}$$

Radiation from Gases

The radiation from gases differs from that of solids in the following way,

The radiation from solid is at all wavelength where as gases radiate over a specific wave length range (or) band within the thermal spectrum

CO_2 and H_2O (water vapour) are the most important absorbing gases in combustion products.

Mean Beam length (L_m)

It is the distance between the medium and the surface

$$L_m = 3.6 \frac{V}{A}$$

V - volume of medium (gas)

A - Surface area (Area of enclosure)

Emissivity of gaseous mixture (ϵ_g)

For a total pressure of $P = 1$ atm.

$$\epsilon_g = \epsilon_{H_2O} + \epsilon_{CO_2} - \Delta\epsilon$$

ϵ_{H_2O} - emissivity of H_2O

ϵ_{CO_2} - emissivity of CO_2

$\Delta\epsilon$ - overall correction factor

For a total pressure other than 1 atm.

$$\epsilon_g = \epsilon_{H_2O} C_{H_2O} + \epsilon_{CO_2} C_{CO_2} - \Delta\epsilon$$

C_{H_2O} - correction factor for H_2O

C_{CO_2} - correction factor for CO_2

Radiant heat exchange between gas and enclosure (T_w)

$$\frac{Q}{A} = \sigma(\epsilon_g T_g^4 - \alpha_g T_w^4)$$

1) A heating furnace of total surface area of $30\ m^2$ and volume $10\ m^3$ is maintained at a temperature of $1000^\circ C$ over its entire volume. The total pressure of the combustion gases is 2 atm. The partial pressure of water vapour is 0.1 atm and that of CO_2 is 0.3 atm. Calculate the emissivities of the gaseous mixture

Given data

$A = 30\ m^2$

$T = 1000^\circ C = 1273\ K$

$V = 10\ m^3$

$P_T = 2$ atm

$p_{H_2O} = 0.1$ atm

$p_{CO_2} = 0.3$ atm.

To find ϵ_g -

For total pressure other than 1 atm.

$$\epsilon_g = \epsilon_{CO_2} C_{CO_2} + \epsilon_{H_2O} C_{H_2O} - \Delta\epsilon$$

from D.B.P. 106 (Graph)

$p_{CO_2} \cdot L = 0.3 \times 1.2$

$= 0.36$ atm.m.

$T = 1273\ K$

$L = 3.6 \times \frac{V}{A}$

$= 3.6 \times \frac{10}{30}$

$L = 1.2\ m.$

$$\boxed{\epsilon_{CO_2} = 0.16}$$

From D. B. p. 107 (Graph)

$P_T = 2$ atm.

$p_{CO_2} \cdot L = 0.3 \times 1.2 = 0.36$ atm.m.

$$\boxed{C_{CO_2} = 1.11}$$

From D.B. P. 108

$p_{H_2O} \cdot L = 0.1 \times 1.2 = 0.12$ atm.m.

$T = 1273$ K

$$\boxed{\epsilon_{H_2O} = 0.12}$$

From D. B. P. 109

$$\frac{p_{H_2O} + P_T}{2} = \frac{0.12 + 2}{2} = 1.06 \text{ atm.m}$$

$p_{H_2O} \cdot L = 0.12$ atm.m.

$$\boxed{C_{H_2O} = 1.43}$$

From. D. B. P. 110

$$\frac{p_{H_2O}}{p_{CO_2}+p_{H_2O}} = \frac{0.1}{0.1+0.3} = 0.25$$

At $T = 1273$ K.

$$\boxed{\Delta\epsilon = 0.035}$$

$$\epsilon_g = \epsilon_{CO_2} C_{CO_2} + \epsilon_{H_2O} C_{H_2O} - \Delta\epsilon$$

$$= (0.16 \times 1.11) + (0.12 \times 1.43) - 0.035$$

$$\boxed{\epsilon_g = 0.313}$$

2) A furnace in the form a cube has gas in it at 1500K. The analysis of gas is 16% CO_2 and 10% H_2O and the rest are non radiating gases. Determine the emissivity of the gaseous mixture. The total pressure is 1 atm. Assume Beam length is 1.34 m.

Given data

$T = 1500$ K.

$$p_{CO_2} = \frac{16}{100} \times 1 = 0.16 \text{ atm.}$$

$p_{H_2O} = \frac{10}{100} \times 1 = 0.1$ atm.

$L = 1.34$ atm

For 1 atm.

$\epsilon_g = \epsilon_{CO_2} + \epsilon_{H_2O} - \Delta\epsilon$

From D.B. p. 106

$p_{CO_2} \cdot L_m = 0.16 \times 1.34 = 0.214$ atm.m

$T = 1500$ K.

$\epsilon_{CO_2} = 0.12$

From D.B. p. 108

$p_{H_2O} \cdot L = 0.1 \times 1.34 = 0.134$ atm.m.

$T = 1500$ K.

$\epsilon_{H_2O} = 0.105$

From D.B. p. 110

$$\frac{p_{H_2O}}{p_{CO_2} + p_{H_2O}} = \frac{0.1}{0.16 + 0.1}$$

$= 0.38$

$T = 1500$ K

$\Delta\epsilon = 0.04$.

$\epsilon_g = 0.12 + 0.105 - 0.04$

$\epsilon_g = 0.185$

3) A cubical furnace 0.5 m side has interior wall that are essentially black. Inside the furnace the gas 20% CO_2 and 80% N_2 at total pressure of 1 atm. The gas temperature is 1500 K and the walls are to be maintained at 300°C. Calculate the radiation heat exchange between the wall and gas.

Given data

cube side $a = 0.5$ m.

$$p_{CO_2} = \frac{20}{100} \times 1 = 0.2$$

$$T_g = 1500 \text{ K}$$

$$T_w = 300°C = 573 \text{ K}.$$

$$L_m = 3.6 \times \frac{V}{A}$$

For cube

$$\text{volume} = a^3 = 0.5^3 = 0.125 \text{ m}^3$$

$$\text{Area} = 6a^2 = 6 \times (0.5)^2 = 1.5 \text{ m}^2$$

$$L_m = 3.6 \times \frac{V}{A} = 3.6 \times \frac{0.125}{1.5}$$

$$L_m = 0.3 \text{ m}$$

For 1 atm pressure

$$\epsilon_g = \epsilon_{CO_2} + \epsilon_{H_2O} - \Delta\epsilon$$

From D.B. P.106

$$p_{CO_2} \cdot L_m = 0.2 \times 0.3 = 0.06$$

$$T = 1500\,K$$

$$\epsilon_{CO_2} = 0.075$$

Since no H_2O is present in the gas ϵ_{H_2O} is negligible

$$\therefore \epsilon_g = 0.075$$

$$\frac{Q}{A} = \sigma\left(\epsilon_g T_g^4 - \alpha_g T_w^4\right)$$

$$\alpha_g = \epsilon_{CO_2}\left(\frac{T_g}{T_w}\right)^{0.65} + \epsilon_{H_2O}\left(\frac{T_g}{T_w}\right)^{0.45} - \Delta\epsilon$$

$$\alpha_g = 0.075\left(\frac{1500}{573}\right)^{0.65}$$

$$\alpha_g = 0.14$$

$$\frac{Q}{A} = 5.67\times10^{-8}\left(0.075\times(1500)^4 - 0.14\times(573)^4\right)$$

$$\frac{Q}{A} = 20.67\times10^{3}\ W/m^2$$

$$Q = 20.67\times10^{3}\times1.5$$

$$Q = 31.008\times10^{3}\ W$$

Mass Transfer

Fick's Law of Diffusion

Mass (molar) flux of a substance per unit area is proportional to the concentration gradient

$$N_{Ax} \propto \frac{dC_A}{dx}$$

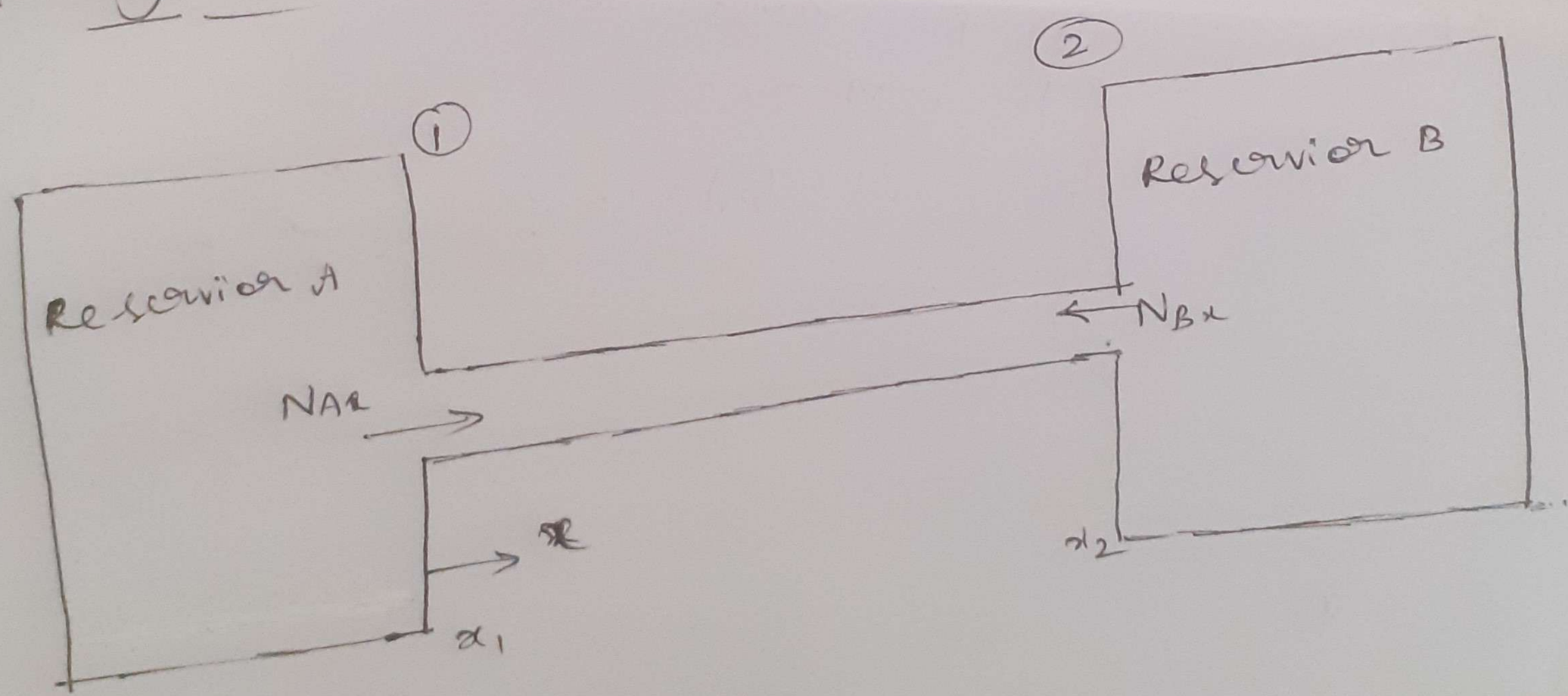
$$N_{Ax} = -D_{AB} \frac{dC_A}{dx}$$

N_{Ax} - molar flux ... kmol/sec.m^2

D_{AB} - Diffusion co-efficient ... m^2/sec

$\frac{dC_A}{dx}$ - concentration gradient ... kmol/m^3.m.

Steady State Equimolar counter Diffusion



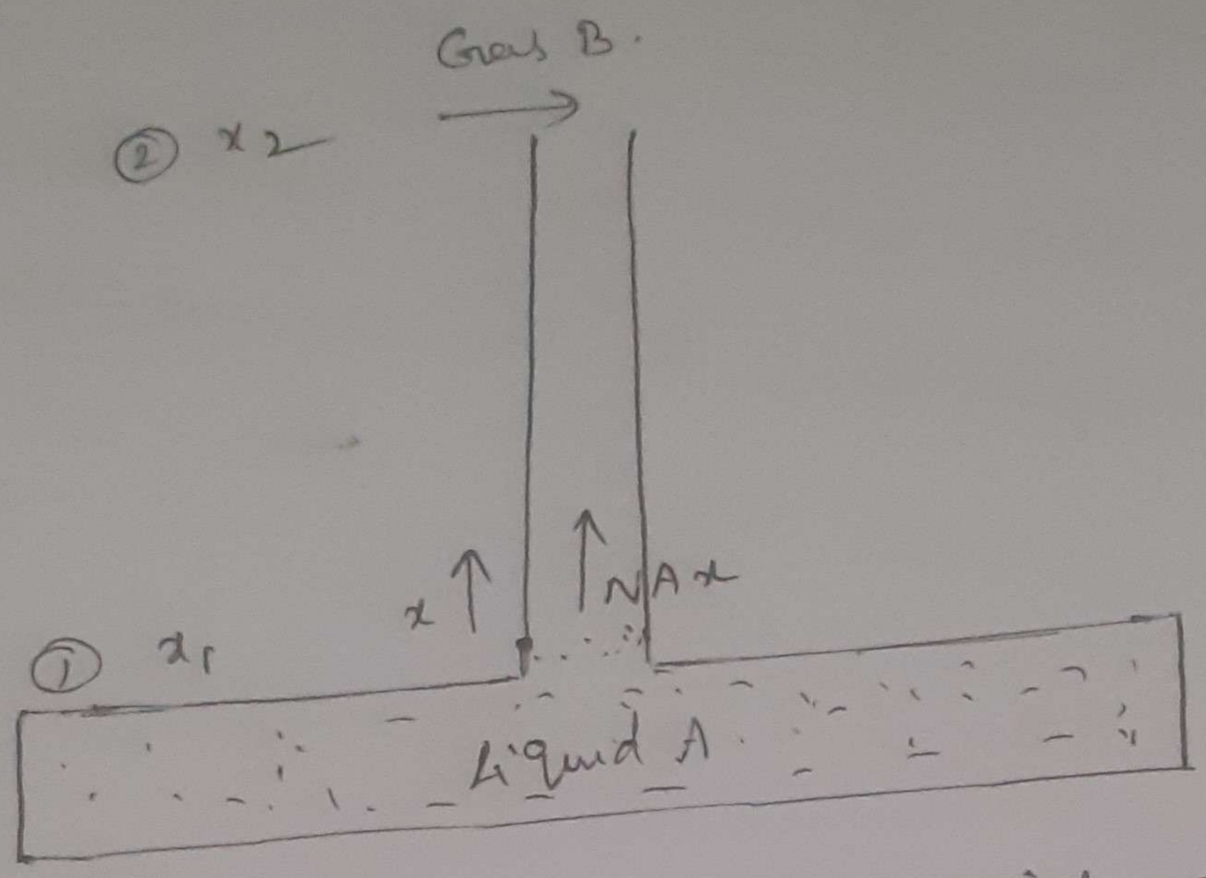
$$N_{Ax} = -N_{Bx}$$

$$N_{Ax} = \frac{D_{AB}}{R_u T (x_2 - x_1)} (P_{A1} - P_{A2})$$

Mass transfer rate $\dot{m}_A$ = Molar transfer rate × molecular weight

$$\dot{m}_A = N_A \times M_A$$

Steady State Diffusion Through Stagnant Gas Film



Evaporation of a liquid into a gas.

$$N_{Ax} = \frac{D_{AB}}{R_u T (x_2 - x_1)} P \ln\left(\frac{(P - p_{A2})}{(P - p_{A1})}\right)$$

$$N_{Bx} = 0.$$

). Ammonia diffuses through an air column 4 cm long. Determine the mass of ammonia diffuses per unit area per hour, If the partial pressures at the column extremeties are 0.6 bar and 0.1 bar. The total pressure is 1 bar. Assume Temperature of the system is 30°C and take $D_{AB} = 0.28 \times 10^{-4}\ m^2/s$. Assume Equimolar counter diffusion.

Given data

species A - Ammonia $x_2 - x_1 = 4\ cm = 4 \times 10^{-2}\ m$. $R_u = 8314\ J/kgmol\,K$.

species B - Air $P_{A1} = 0.6\ bar = 0.6 \times 10^5\ N/m^2$

$P_{A2} = 0.1\ bar = 0.1 \times 10^5\ N/m^2$

$T = 30°C = 303\ K$.

$D_{AB} = 0.28 \times 10^{-4}\ m^2/s$.

$$N_A = \frac{D_{AB}}{R_u T (x_2 - x_1)} (P_{A1} - P_{A2})$$

$$= \frac{0.28 \times 10^{-4}}{8314 \times 303 \times (4 \times 10^{-2})} (0.6 - 0.1) \times 10^5$$

Molar flux $N_A = 1.389 \times 10^{-5}\ kgmol/sec.m^2 = N_B$.

mass Transfer rate $\dot{m}_A = N_A \times M_A \times A$

$\dot{m}_A = 1.389 \times 10^{-5} \times 17 \times 1$

$\dot{m}_A = 2.36 \times 10^{-4}\ kg/s$.

$\dot{m}_A = 0.8496\ kg/hr$.

Two reservoirs A and B contains CO_2 and N_2 at 1 bar and 290 K. The partial pressure of CO_2 at the two ends are 0.8 bar and 0.2 bar respectively. The reservoirs are connected by a duct of cross sectional area of $0.1\ m^2$, 0.5 m long. Calculate the diffusion rate of CO_2. Assume diffusion co-efficient $D = 0.17 \times 10^{-4}\ m^2/s$.

Given data

$T = 290\ K$.

$P_{A1} = 0.8\ bar = 0.8 \times 10^5\ N/m^2$

$P_{A2} = 0.2\ bar = 0.2 \times 10^5\ N/m^2$

$A = 0.1\ m^2$

$(x_2 - x_1) = 0.5\ m$

$D_{AB} = 0.17 \times 10^{-4}\ m^2/s$.

This is the case of equimolar counter diffusion,

Molar flux $N_A = \dfrac{D_{AB}}{R_u T (x_2 - x_1)} (P_{A1} - P_{A2})$ $\qquad R_u = 8314\ J/kmol\,K$

$= \dfrac{0.17 \times 10^{-4}}{8314 \times 290 \times 0.5} (0.8 \times 10^5 - 0.2 \times 10^5)$

$N_A = 8.46 \times 10^{-7}\ kmol/s$.

$\dot{m}_A = N_A \times M_A \times A$

For CO_2, Molecular wt $M_A = 44$

$= 8.46 \times 10^{-7} \times 44 \times 0.1$

Diffusion rate $\dot{m}_A = 3.72 \times 10^{-6}\ kg/s$.

ny hydrogen and oxygen are equimolar counter diffusion the total pressure is 1 atm and the temperature is 20°C, the mass diffusivity for the mixture is 0.41 m²/s. The partial pressures at the two planes perpendicular to the direction of diffusion and 0.95 cm apart are 120mm and 40mm of Mercury. calculate the rate of diffusion of mixture in Kmol/m²/hr

Given data

$T = 20°C = 293 K$.

$(x_2 - x_1) = 0.95 \text{ cm} = 0.95 \times 10^{-2} \text{ m}$

$D_{AB} = 0.41 \text{ m}^2/s$

$P_{A1} = 120 \text{ mm of Hg}$

$P_{A2} = 40 \text{ mm of Hg}$

$1 \text{ atm} = 760 \text{ mm of Hg}$

$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$

$$P_{A1} = \frac{120}{760} = 15.99 \times 10^3 \text{ N/m}^2$$

$$P_{A2} = \frac{40}{760} = 5.33 \times 10^3 \text{ N/m}^2$$

$$N_A = \frac{D_{AB}}{R_u T (x_2 - x_1)} (P_{A1} - P_{A2})$$

$R_u = 8314 \text{ J/kmol K}$

$$= \frac{0.41}{8314 \times 293 \times (0.95 \times 10^{-2})} (15.99 \times 10^3 - 5.33 \times 10^3)$$

$= 0.188 \text{ kmol/m}^2.s$

$= 680.7 \text{ kmol/m}^2.hr$

Estimate the diffusion rate of water at 27°C from the bottom of a test tube 20 mm in diameter and 4 cm long into dry air at 27°C, Take the diffusion co-efficient of water in air as $0.26 \times 10^{-4}\ m^2/s$

Given data

$T = 27°C = 300\ K.$ Species A - Water

$D = 20\ mm = 20 \times 10^{-3}\ m.$ Species B - Air

$x_2 - x_1 = 4\ cm = 4 \times 10^{-2}\ m$

$D_{AB} = 0.26 \times 10^{-4}\ m^2/s.$

This is the case of steady state diffusion thru stagnant gas film,

$$N_{Ax} = \frac{D_{AB}}{R_u T (x_2 - x_1)} P \ln\left(\frac{P - p_{A2}}{P - p_{A1}}\right)$$

The partial pressure from steam table, at 27°C

$p_s = 0.03531\ bar = p_{A1}$

Since the air is dry at $x = x_2$

$p_{A2} = 0.$

$$N_{Ax} = \frac{0.26 \times 10^{-4}}{8.314 \times 300 \times (4 \times 10^{-2})} \times 1 \times 10^5 \ln\left(\frac{(1 \times 10^5 - 0)}{(1 \times 10^5 - 0.353 \times 10^5)}\right)$$

molar Transfer rate $N_{Ax} = 1.135 \times 10^{-5}\ kgmol/sec.m^2.$

Mass Transfer rate $\dot{m}_A = N_{Ax} \times M_A \times A$

$= 1.135 \times 10^{-5} \times 18 \times \frac{\pi}{4} \times (20 \times 10^{-3})^2$

$\dot{m}_A = 6.418 \times 10^{-8}\ kg/sec$

A well is 40 m deep and 9 m diameter and 1 atm. pressure is at 25°C. The air at the top having a relative humidity 50%. Determine the rate of diffusion of water through the well.

Assume $D_{AB} = 2.58 \times 10^{-5}\ m^2/s$.

Given data

Species A - Water $\quad x_2 - x_1 = 40\,m$

Species B - Air $\quad D = 9\,m$

$P = 1\,atm = 1.0132 \times 10^5\ N/m^2$

$RH = 50\%$

$D_{AB} = 2.58 \times 10^{-5}\ m^2/s$

$T = 25°C = 298\,K$

This is the case of steady state diffusion thru stagnant gas film.

$$N_{Ax} = \frac{D_{AB}}{R_u T (x_2 - x_1)} P \ln\left(\frac{P - p_{A2}}{P - p_{A1}}\right)$$

From steam table, at 25°C $\quad [1\,atm = 1.0132 \times 10^5\ N/m^2]$

$p_s = 0.03169\ bar = p_{A1}$

At the top $(x = x_2)$ $\quad RH = 50\%$

$\therefore p_{A2} = 0.5 \times 0.03169$

$= 0.0158\ bar$

$$N_{Ax} = \frac{2.58 \times 10^{-5}}{8314 \times 298 \times (40)} \times 1.0132 \times 10^5 \ln\left(\frac{(1 - 0.0158) \times 1.013 \times 10^5}{(1 - 0.03169) \times 1.013 \times 10^5}\right)$$

Molar transfer rate $N_A = 4.293 \times 10^{-10}\ kgmol/sec.\,m^2$

Mass transfer rate $\dot{m}_A = N_A \cdot M_A \times A$

$= 4.293 \times 10^{-10} \times 18 \times \frac{\pi}{4} \times 9^2$

$= 4.84 \times 10^{-7}\ kg/sec$

An open pan 20 cm in diameter and 20 mm in deep filled with water to a level of 10 mm and is exposed to dry air at 25°C. Assuming diffusion co-efficient of 0.25×10^{-4} m²/s, calculate the diffusion rate of water.

Given data

$D = 20 \times 10^{-2}$ m

$(x_2 - x_1) = 20 \times 10^{-3}$ m.

$T = 25°C = 298$ K.

$D_{AB} = 0.25 \times 10^{-4}$ m²/s.

This is the case of steady state diffusion through stagnant gas film

$$N_{Ax} = \frac{D_{AB}}{R_u T (x_2 - x_1)} P \ln\left(\frac{P - P_{A2}}{P - P_{A1}}\right)$$

From steam table

At 25°C

$P_s = 3.169 \times 10^3$ N/m² = P_{A1}

Since the air is dry $P_{A2} = 0$.

$$N_{Ax} = \frac{0.25 \times 10^{-4}}{8314 \times 298 \times 10 \times 10^{-3}} 1 \times 10^5 \ln\left(\frac{1 \times 10^5 - 0}{1 \times 10^5 - 3.169 \times 10^3}\right)$$

Molar transfer rate $N_{Ax} = 3.24 \times 10^{-6}$ kmol/s.m² [∵ for water $M_A = 18$]

Diffusion rate $\dot{m}_A = N_{Ax} \times M_A \times A$

$= 3.24 \times 10^{-6} \times 18 \times \frac{\pi}{4} \times (10 \times 10^{-3})^2$

$\dot{m}_A = 1.86 \times 10^{-6}$ kg/s.