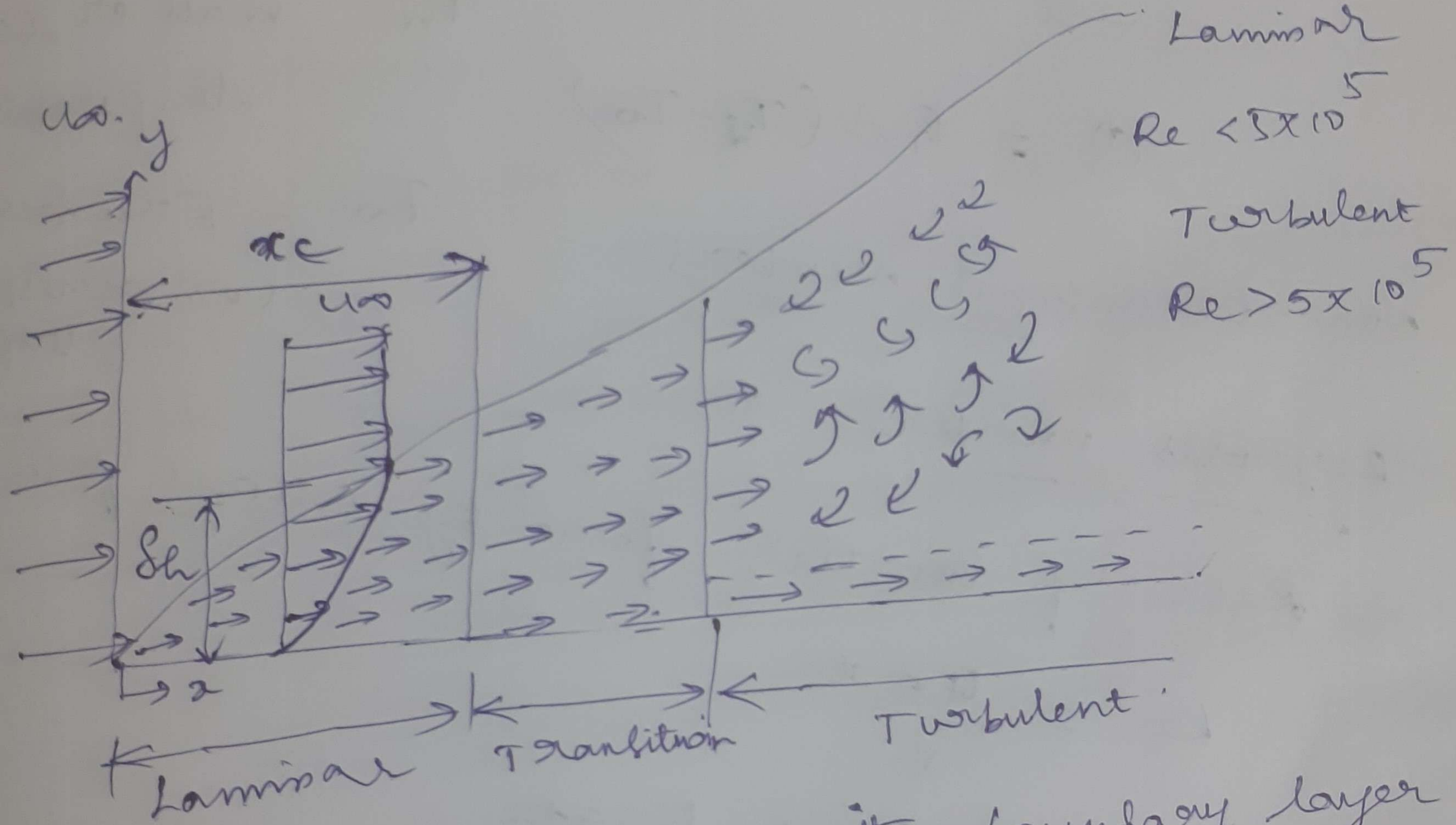
II - Convection

Forced convection

Boundary layer concept



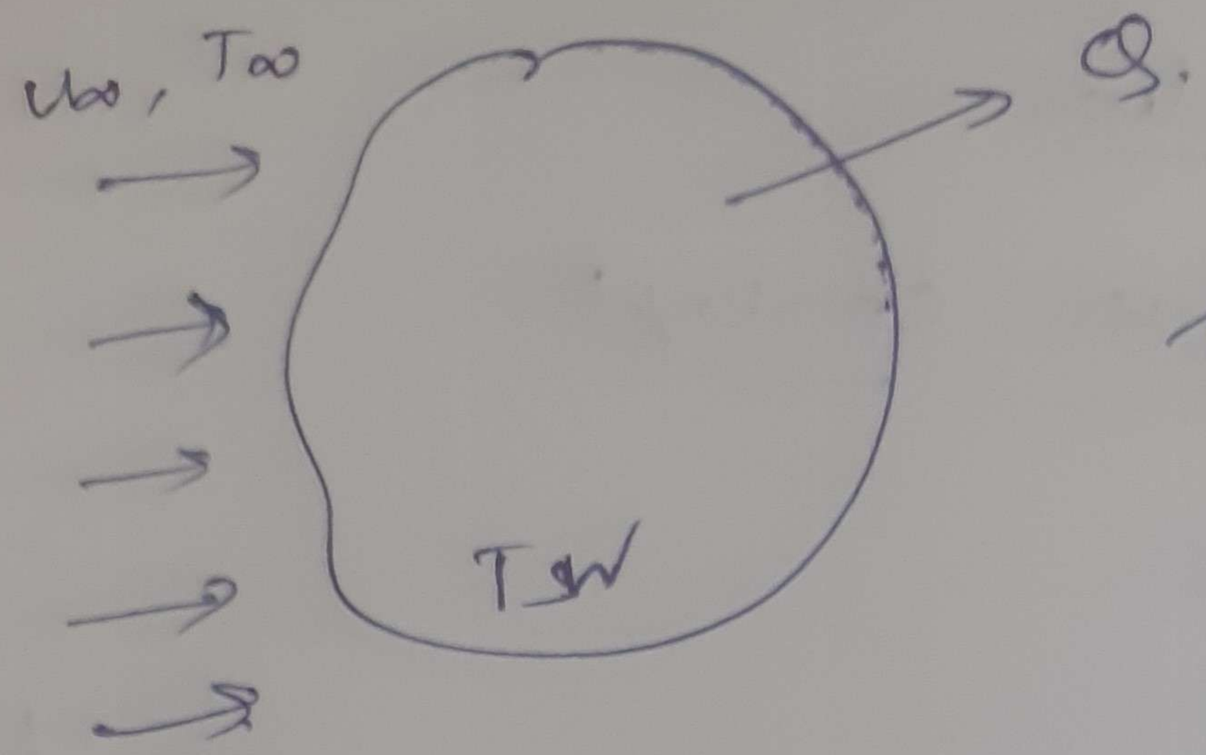
Development of velocity boundary layer on flat plate

x_c - critical distance

u_∞ - Free stream velocity

δ_h - Boundary layer thickness (velocity) (Hydrodynamic)

Boundary layer thickness from which fluid velocity equals 99% of free stream velocity ($u = 0.99 u_\infty$)



h - convective heat transfer coefficient W/m²K

A - surface area

T_W - wall or hot temperature

T_∞ - Free stream (or) atmospheric temperature

$$Q = hA(T_W - T_\infty)$$

non-dimensional numbers

Reynolds number

Ratio of inertial force by viscous force

$$Re = \frac{u_\infty x}{\nu}$$

x - characteristic length

$x \rightarrow D$ for cylinder (or) sphere

$\rightarrow L$ for flat plate

ν - kinematic viscosity m²/s

u_∞ - Free stream velocity m/s

Prandtl number

$$Pr = \frac{\mu C_p}{k} = \frac{\nu}{\alpha}$$

Ratio of molecular diffusivity of momentum and molecular diffusivity of mass.

Nusselt number

$$Nu = \frac{h x}{k}$$

Ratio of temperature gradient by conduction and convection at surface.

$x \rightarrow D$ for cylinder (or) sphere

$\rightarrow L$ for flat plate

k - Thermal conductivity of fluid w/m k.

Stanton number

$$St = \frac{Nu}{Re\,Pr}$$

Ratio of wall heat transfer rate to heat transfer by conduction.

1) Air at 25°C and at atmospheric pressure is flowing over a flat plate at 3 m/s. The plate is 1 m wide and temperature is 75°C. Calculate the following at location of 1 m from the leading edge.

(i) Hydrodynamic boundary layer thickness

(ii) Thermal boundary layer thickness

iii) Local heat transfer co-efficient

iv) Local friction co-efficient

v) Average heat transfer co-efficient

<u>Given data</u>

$T_\infty = 25°C$ $\qquad x = 1 m.$

$u_\infty = 3$ m/s.

$T_w = 75°C$

property values of air at

$T_f = \frac{T_w + T_\infty}{2} = \frac{75+25}{2} = 50°C$

From D.B. P. 34.

$Pr = 0.698$

$K = 28.26 \times 10^{-3} = 0.02826$ W/mK

$\nu = 17.95 \times 10^{-6}$ m²/s

$$Re_x = \frac{u_\infty x}{\nu} = \frac{3 \times 1}{17.95 \times 10^{-6}}$$

$$= 1.7 \times 10^5 < 5 \times 10^5$$

Flow is laminar

For external flow, From D.B. P. 113

∴ Flat plate

Hydrodynamic boundary layer thickness $\delta_{hx} = 5x\, Re_x^{-0.5}$

$$= 5 \times 1 \times 1.7 \times 10^{5}$$

$$\delta_{hx} = 0.012 \text{ m}$$

Thermal boundary thickness

$$\delta_{Tx} = \delta_{hx}\, Pr^{-0.333}$$

$$\delta_{Tx} = 0.012 \times (0.698)^{-0.333}$$

$$\delta_{Tx} = 0.0136 \text{ m}$$

For const. wall temperature

$$Nu_x = 0.332\, Re_x^{0.5}\, Pr^{0.333}$$

$$= 0.332 \times (1.7\times10^5)^{0.5} \times (0.698)^{0.333}$$

$$Nu_x = 121.42$$

$$Nu_x = \frac{h_x\, x}{K}$$

$$121.42 = \frac{h_x \times 1}{0.02826}$$

local heat transfer co-efficient $h_x = 3.431\ W/m^2K$

For Flat plate

$$h = 2h_x$$

h - Average heat transfer co-efficient

$$h = 2\times3.431$$

$$= 6.863\ W/m^2K$$

Local friction co-efficient

$$C_{fx} = 0.664\, Re^{-0.5}$$

$$= 0.664\,(1.7\times10^5)^{-0.5}$$

$$C_{fx} = 1.61\times10^{-3}$$

2) Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Assuming unit depth in 'z'-direction. Calculate the heat transfer at the first 0.4 m of length of the plate. The plate is heated over its entire length to a temperature of 60°C

Given data

$T_\infty = 27°C$ $\quad x = 0.4\,m$

$u_\infty = 2\,m/s$

$T_w = 60°C$

From D.B. P. 34.

property values of air at

$$T_f = \frac{T_w + T_\infty}{2} = \frac{60+27}{2} =$$

$$T_f = 43.5°C \simeq 45°C$$

$Pr = 0.699$

$\nu = 16.96 \times 10^{-6}\,m^2/s$

$K = 26.56 \times 10^{-3}\,W/mK$

$$Re = \frac{u_\infty x}{\nu} = \frac{2 \times 0.4}{16.96 \times 10^{-6}}$$

$$Re = 47169 < 5 \times 10^5$$

Flow is Laminar. From D.B. P. 113

At constant wall temperature

$$Nu_x = 0.332 \, Pr^{0.333} \, Re^{0.5}$$

$$= 0.332 \, (0.699)^{0.333} \, (47169)^{0.5}$$

$$Nu_x = 63.99$$

$$Nu_x = \frac{h_x x}{k}$$

$$64 = \frac{h_x \times 0.4}{26.56 \times 10^{-3}}$$

$$h_x = 4.249$$

For flat plate

$$h = 2 h_x = 2 \times 4.249$$

$$= 8.5 \, W/m^2 K$$

$$Q = hA(T_w - T_\infty)$$

$$= 8.5 \times (0.4 \times 1)(60 - 27)$$

$$Q = 112.2 \, W$$

3) Air at a standard conditions of 100 kPa 30°C flows over a flat plate at 20 m/s. The plate is 0.6 m square section and it is maintained at 90°C. Calculate the heat transfer from the plate at a distance of 0.3 m and 0.6 m from leading edge

Given data

$T_\infty = 30°C$ $\quad T_w = 90°C$

$u_\infty = 20$ m/s.

$x = 0.6$ m $\quad$ From D.B. P 34.

property values of air

at $T_f = \frac{T_w + T_\infty}{2} = \frac{90+30}{2} = 60°C$

$Pr = 0.696$

$\nu = 18.97 \times 10^{-6}$ m²/s

$K = 0.02896$ W/mK

At $x_1 = 0.3$

$Re_x = \frac{u_\infty x}{\nu} = \frac{20 \times 0.3}{18.97 \times 10^{-6}}$

$= 3.16 \times 10^5 < 5 \times 10^5 \rightarrow$ Laminar

At $x = 0.6$

$$Re_x = \frac{U_\infty x}{\nu} = \frac{20 \times 0.6}{18.97 \times 10^{-6}}$$

$$= 6.32 \times 10^5 > 5 \times 10^5 \rightarrow \text{Turbulent}$$

$\therefore$ Flow is laminar turned Turbulent

From D.B. P. 115

For combined Laminar - Turbulent

$$Nu_L = Pr^{0.333}(0.037\, Re_L^{0.8} - 871)$$

$$Nu_L = (0.696)^{0.333}(0.037 \times (6.32 \times 10^5)^{0.8} - 871)$$

$$Nu_L = 662$$

$$Nu = \frac{h_x x}{k}$$

$$662 = \frac{h_x \times 0.6}{0.02896}$$

$$h_x = 32 \text{ W/m}^2\text{K}$$

For flat plate

$$h = 2h_x = 64 \text{ W/m}^2\text{K}$$

$$Q = hA(T_w - T_\infty)$$

$$= 64 \times (0.6 \times 0.6)(90 - 30)$$

$$Q = 13.8 \text{ W}$$

4) An air stream at 0°C is flowing along heated plate at a speed of 75 m/s. The plate is 45 cm long and 60 cm wide. Assuming transition of boundary layer takes place at Reynolds no. 5×10^5. Calculate the average value of friction co-efficient and heat transfer co-efficient from the full length of the plate. Also calculate the rate energy dissipation from the plate.

<u>Given data</u>

$T_\infty = 0°C$

$T_w = 90°C$

$L = 45$ cm
$= 45 \times 10^{-2}$ m.

$W = 60$ cm
$= 60 \times 10^{-2}$ m

$u_\infty = 75$ m/s.

From D.B. P. 34
property values of air at

$T_f = \frac{0+90}{2} = 45°C$

$\nu = 17.95 \times 10^{-6}$ m²/s.

$Pr = 0.698$

$K = 0.0282$ W/mK

$$Re_x = \frac{U_\infty x}{\nu} = \frac{75 \times 0.45}{17.9 \times 10^{-6}}$$

$$= 1.88 \times 10^6 > 5 \times 10^5$$

Transition of boundary layer at $Re = 5 \times 10^5$ (Given)

$\therefore$ For combined Laminar-turbulent

From D.B. P.115

$$C_{fL} = 0.074\, Re_L^{-0.2} - 1742\, Re^{-1}$$

$$= 0.074 (1.88 \times 10^6)^{-0.2} - 1742 (1.88 \times 10^6)^{-1}$$

$$= 3.188 \times 10^{-3}$$

$$Nu_L = Pr^{0.333} (0.037\, Re_L^{0.8} - 871)$$

$$= (0.698)^{0.333} (0.037 (1.88 \times 10^6)^{0.8} - 871)$$

$$= 2658$$

$$Nu_L = \frac{h_x \cdot x}{k}$$

$$2659 = \frac{h_x \times 0.45}{0.02826}$$

$h_x = 167 \ W/m^2K$

$h = 2h_x$

$h = 2 \times 167 = 334 \ W/m^2K$

$Q = hA(T_w - T_\infty)$

$= 334 \times 0.45 \times 0.6 (90 - 0)$

$Q = 8116.2 \ W$

Relation between fluid friction and heat transfer (Reynolds Colburn analogy)

Stanton no. $St_x = \dfrac{Nu_x}{Re_x \, Pr}$

From D.B. p.113, For Laminar flow,

$Nu_x = 0.332 \, Re_x^{0.5} \, Pr^{0.333}$

$$St_x = \frac{0.332 \, Re_x^{1/2} \, Pr^{1/3}}{Re \cdot Pr}$$

$= 0.332 \, Re_x^{1/2 - 1} \, Pr^{1/3 - 1}$

$St_x = 0.332 \, Re^{-1/2} \, Pr^{-2/3}$

$$\frac{St}{Pr^{-2/3}} = 0.332\, Re^{-1/2} \quad -(1)$$

$$St_x\, Pr^{2/3} = 0.332\, Re^{-1/2}$$

From D. B. P. 113 - For Laminar flow,

we get,

$$C_{fx} = 0.664\, Re_x^{-0.5}$$

$$C_{fx} = 2 \times 0.332\, Re^{-1/2}$$

$$\frac{C_{fx}}{2} = 0.332\, Re^{-1/2} \quad -(2)$$

Equating (1) and (2)

$$\frac{St}{Pr^{-2/3}} = \frac{C_{fx}}{2} \quad -(3)$$

and

$$C_{fx} = \frac{\tau_w}{\frac{\rho u_\infty^2}{2}} \quad -(4)$$

Equation (3) and (4) are called Reynolds colburn analogy. It expresses the relation between fluid friction

1) Air at 27°C and 1 atm pressure flows on a flat plate at 2 m/s. Plate is heated to 60°C over its entire length. Calculate the drag force on the first 0.4m length of the plate using the analogy between fluid friction and heat transfer.

Given data

$T_\infty = 27°C$ $\quad u_\infty = 2\ m/s$

$T_w = 60°C$

$$\text{Shear stress} = \frac{\text{Shear force (Drag Force)}}{\text{Area}}$$

$$\text{Shear force (Drag force)} = \text{Shear stress} \times \text{Area}$$
$$= \tau_w \times \text{Area}$$

property values of air at

$$T_f = \frac{60+27}{2} = 43.5°C \simeq 45°C$$

From D.B. P. 34.

$\nu = 17.95 \times 10^{-6}\ m^2/s$ $\quad \rho = 1.093\ kg/m^3$

$P_r = 0.698$

$k = 0.0286\ W/mK$

$$Re = \frac{u_\infty x}{\nu} = \frac{2 \times 0.4}{17.95 \times 10^{-6}}$$

$$Re = 0.0445 \times 10^{6}$$

$$= 0.445 \times 10^{5} < 5 \times 10^{5}$$

Flow is laminar

From D.B. P. 113

$$C_f = 0.664\, Re_x^{-0.5}$$

$$= 0.664 \times (0.445 \times 10^{5})^{-0.5}$$

$$C_f = 3.15 \times 10^{-3}$$

From D.B. P. 114

$$\tau_w = \frac{C_f \rho u_\infty^2}{2}$$

$$\tau_w = \frac{3.15 \times 10^{-3} \times 1.093 \times 2^2}{2}$$

Shear stress $\tau_w = 0.0068\ N/m^2$

Drag Force $= 0.0068 \times 0.4 \times 1$

$$= 0.00272\ N$$

Internal flow

mass flow rate $\dot{m} = \rho A_c u_\infty$

$Re = \frac{u_\infty D}{\nu}$

$Nu = \frac{hD}{k}$

property values to be evaluated at mean temp $T_m = \frac{T_i + T_o}{2}$

$Re < 2300$ – Laminar.

> 2300 – Turbulent.

1) Liquid Sodium is to be heated from 120°C to 180°C at a rate of 2.3 kg/s in a 2.5 cm diameter tube with a constant wall temperature. Calculate the heat transfer rate

Given data

$T_i = 120°C$

$T_o = 180°C$

$\dot{m} = 2.3$ kg/s.

$D = 2.5$ cm $= 2.5 \times 10^{-2}$ m.

property values of Liquid Sodium

$$T_m = \frac{T_i + T_o}{2}$$

$$T_m = \frac{120+180}{2} = 150°C$$

From D.B. P. 33.

$$\rho = 916 \ kg/m^3$$

$$k = 84.9 \ W/mK.$$

$$\nu = 0.594 \times 10^{-6} \ m^2/s.$$

$$Pr = 0.0087$$

$$\dot{m} = \rho A_c u_\infty$$

$$\dot{m} = \rho \times \frac{\pi}{4} \times D^2 \times u_\infty$$

$$2.3 = 916 \times \frac{\pi}{4} \times u_\infty$$

$$u_\infty = 5.11 \ m/s.$$

$$Re = \frac{u_\infty \times D}{\nu} = \frac{5.11 \times 0.025}{0.594 \times 10^{-6}}$$

$$Re = 0.2156 \times 10^6 > 2300$$

Flow is turbulent

For liquid metals and const. wall temp.

From D.B. P. 127

$$Nu = 5 + 0.025 [Re_D Pr]^{0.8}$$

$$= 5 + 0.0025\,(0.2152 \times 10^{6} \times 0.0087)$$

$$Nu = 15.37$$

$$Nu = \frac{hD}{k}$$

$$15.37 = \frac{h \times 2.5 \times 10^{-2}}{84.9}$$

$$h = 52200.5 \ W/m^2K$$

$$Q = hA\,(T_0 - T_i) \qquad [l = 1m]$$

$$= 52200 \times \pi \times 2.5 \times 10^{-2} \times 1$$

$$(180 - 120)$$

$$Q = 245.9 \times 10^{3} \ W$$

2) A fine wire having diameter 0.025 mm is placed in a 100 kPa air stream at 25°C having flow velocity 50 m/s. An electric current is passed through the wire raising the surface temperature to 50°C. Calculate the heat loss per unit length of wire.

Given data

$T_\infty = 25°C$

$T_w = 50°C$

Geometry : cylinder

$D = 0.025 \times 10^{-3}$ m.

$u_\infty = 50$ m/s.

property values of air at

$$T_f = \frac{T_w + T_\infty}{2} = \frac{50+25}{2} = 37.5°C \simeq 40°C$$

From D.B. p. 34.

$\nu = 16.96 \times 10^{-6}$ m²/s.

$Pr = 0.669$

$K = 0.02756$ W/mK

$$Re = \frac{u_\infty D}{\nu} = \frac{5 \times 0.025 \times 10^{-3}}{16.96 \times 10^{-6}}$$

$Re = 73.7$

From D.B. P. 116

Flow over cylinder

$$Nu = C \cdot Re_D^{m} \cdot Pr^{0.333}$$

For $Re = 73.7$

$C = 0.683$

$m = 0.466$

$$= 0.683 \times (73.7)^{0.466} (0.689)^{0.333}$$

$Nu = 4.4$

$$Nu = \frac{hD}{K}$$

$$h = \frac{4.4 \times 0.02756}{0.025 \times 10^{-3}}$$

$h = 4884 \ W/m^2K$

$$Q = hA(T_w - T_\infty)$$

$$= 4884 \times \pi \times 0.025 \times 10^{-3} \times 1 \ (50-25)$$

$Q = 9.59 \ W$

Air at 100 kpa and 35°c flow across a 50 mm diameter cylinder a velocity of 50 m/s. The surface is at 150°c. Calculate the heat loss per unit length of the cylinder.

Given data

Geometry : cylinder

$D = 50 \times 10^{-3} m$

$T_\infty = 35°c$

$u_\infty = 50 m/s$

$T_w = 150°c$

property values of air at

$$T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°c \simeq 95°c$$

From D.B. p. 34.

$\nu = 23.13 \times 10^{-6} m^2/s$

$Pr = 0.688$

$k = 0.0321 W/mK$

$$Re = \frac{u_\infty D}{\nu} = \frac{50 \times 50 \times 10^{-3}}{23.13 \times 10^{-6}}$$

$Re = 0.108 \times 10^6$

From D.B. P.116

Flow over cylinder

$$Nu = C\,Re^{m}\,Pr^{0.333}$$

$$C = 0.0266 \qquad m = 0.805$$

$$Nu = 0.0266 \times (0.1080 \times 10^{6})^{0.805} \times (0.688)^{0.333}$$

$$= 264.4$$

$$Nu = \frac{hD}{K}$$

$$264.4 = \frac{h \times 50 \times 10^{-3}}{0.0321}$$

$$h = 170.1 \ W/m^2K$$

$$Q = hA(T_w - T_\infty) \qquad [l = 1m]$$

$$= 170.1 \times \pi \times 50 \times 10^{-3} \times 1 \times (150 - 35)$$

$$Q = 3.07 \ W$$

Free convection

Grashof no. $Gr = \frac{\text{Buoancy force}}{\text{viscous force}}$

$$Gr = \frac{g \beta x^3 \Delta T}{\nu^2}$$

g - acceleration due to gravity m/s^2

β - volume expansion co-efficient, K^{-1}

$\beta = \frac{1}{T_f} \rightarrow K$

x - characteristic length

x - L for plane surface

x - D for cylinder

ΔT - $(T_w - T_\infty)$

ν - Kinematic viscosity m^2/s.

$Gr < 10^9$ Laminar

$> 10^9$ Turbulent

1) A horizontal pipe 0.3 m diameter is maintained at temperature of 250°C in a room where the ambient condition is at 15°C. Calculate the heat loss per meter length of the pipe.

Given data

$D = 0.3\ m$

$T_w = 250°C$

$T_\infty = 15°C$

property values of air at

$T_f = \frac{T_w + T_\infty}{2} = \frac{250+15}{2} = 132.5°C \simeq 135°C$

From D.B. P. 34.

$\alpha = 26.25 \times 10^{-6}\ m^2/s$

$Pr = 0.685$

$k = 0.0341\ W/mK$

$\beta = \frac{1}{T_f} = \frac{1}{(132.5+273)} = 2.466 \times 10^{-3}\ K^{-1}$

For D.B. P.135

$Gr_x = \frac{g\beta x^3 \Delta T}{\nu^2}$

$$Gr = \frac{9.81 \times 2.466 \times 10^{-3} \times (0.3)^3 \times (250-15)}{(26.625 \times 10^{-6})^2}$$

$$Gr = 2.165 \times 10^{8}$$

$$Gr\,Pr = 2.165 \times 10^{8} \times 0.689$$

$$= 0.149 \times 10^{9} = 1.49 \times 10^{8} < 10^{9}$$

$\therefore$ Flow is Laminar.

From. D.B. p.136

$$Nu = 0.59 (Gr\,Pr)^{0.25}$$

(For higher values of $Gr\,Pr$)

$$= 0.59 (1.49 \times 10^{8})^{0.25}$$

$$Nu = 65.18$$

$$Nu = \frac{hD}{k}$$

$$65.18 = \frac{h \times 0.3}{0.0341}$$

$$h = 7.4 \ W/m^2 K$$

$$Q = hA(T_w - T_\infty)$$

$$= 7.4 \times \pi \times 0.3 \times 1 \times (250-15)$$

$$Q = 1.638 \times 10^{3} \ W$$

2) A large vertical plate 4m high is maintained at 60°C and exposed to atmospheric air at 10°C. calculate the heat transfer from the plate if it is 10 m wide.

Given data

$T_w = 60°C$ Height = 4 m

width = 10m

$T_\infty = 10°C$

property values of air at

$$T_f = \frac{60+10}{2} = 35°C$$

From D.B. p. 34.

$\nu = 16.48 \times 10^{-6}\ m^2/s$

$Pr = 0.695$

$K = 0.0271\ W/mK$

$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)} = 3.246 \times 10^{-3}\ K$$

$$Gr_x = \frac{g \beta x^3 \Delta T}{\nu^2}$$

$$= \frac{9.81 \times 3.246 \times 10^{-3} \times 10^{3} \times 50}{16.48 \times 10^{-6}}$$

$$= 5.864 \times 10^{12}$$

$$Gr\,Pr = 5.864 \times 10^{12} \times 0.695$$

$$= 4.078 \times 10^{12} > 10^{9}$$

Flow is turbulent

From D.B. p. 136

$$Nu = 0.1\,(Gr\,Pr)^{0.333}$$

$$= 0.1\,(4.078 \times 10^{12})^{0.333}$$

$$= 1597.7$$

$$Nu = \frac{h_x x}{k}$$

$$1597.7 = \frac{h_x \times 10}{0.02715}$$

$$h_x = 4.33 \ W/m^2K$$

$$h = 2h_x = 8.67 \ W/m^2K$$

$$Q = hA(T_w - T_\infty)$$

$$= 8.67 \times 4 \times 10\,(60 - 10)$$

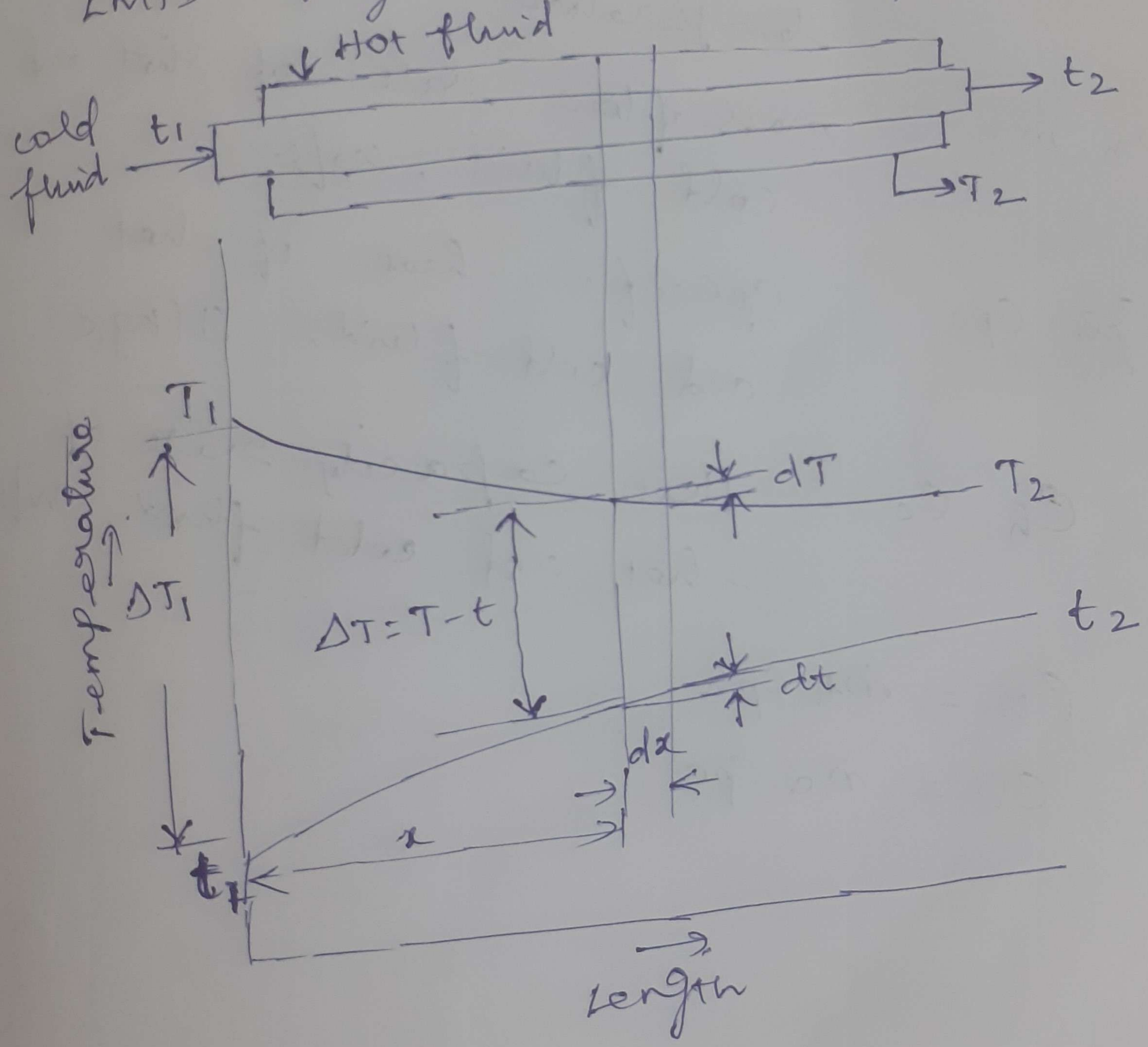
$$Q = 17.35 \times 10^{3} \ W$$

Module-III - Heat Exchangers

Heat Exchanger Analysis

Expression for LMTD - parallel flow

LMTD - Logarithmic Mean Temp. Difference



T_1, T_2 - Hot fluid inlet and outlet temperature °C

t_1, t_2 - cold fluid inlet and outlet temperature

$\dot{m}_h, \dot{m}_c$ - mass flow rate of hot and cold fluid, kg/s.

C_{ph}, C_{pc} - Specific heat of hot and cold fluid, J/kgK

C_h, C_c - Heat capacity rate of hot and cold fluid, W/K

$$C_h = \dot{m}_h C_{ph}$$

$$C_c = \dot{m}_c C_{pc}$$

The heat transfer between the cold and hot fluids for a differential element of length dx is $[Q = UA\Delta T]$

$$dQ = U\,dA\,\Delta T$$

Energy balance Equation,

Heat lost by the hot fluid = Heat gained by the cold fluid

$$Q = -\dot{m}_h C_{ph}(T_2 - T_1) = \dot{m}_c C_{pc}(t_2 - t_1)$$

$$Q = -C_h(T_2 - T_1) = C_c(t_2 - t_1)$$

$$dQ = -C_h\,dT = C_c\,dt$$

$$dT = -\frac{dQ}{C_h} \quad -(1)$$

$$dt = \frac{dQ}{C_c} \quad -(2)$$

$(1) - (2)$

$$dT - dt = -\frac{dQ}{C_h} - \frac{dQ}{C_c}$$

$$d(T-t) = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$d(T-t) = -U\,dA\,\Delta T\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d(T-t)}{\Delta T} = -U\,dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

For whole heat exchanger

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(T-t)}{(T-t)} = \int_0^A -U\,dA\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

$$\left[\ln(T-t)\right]_{\Delta T_1}^{\Delta T_2} = -U\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\int_0^A dA$$

$$\left[\ln \Delta T\right]_{\Delta T_1}^{\Delta T_2} = -U\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\left[A\right]_0^A$$

$$\left[\ln \Delta T_2 - \ln \Delta T_1\right] = -U\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\cdot\left[A-0\right]$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right] \quad -(3)$$

we know that

$$Q = -C_h(T_2-T_1)$$

$$Q = C_h(T_1-T_2)$$

and $Q = C_c(t_2-t_1)$

$$C_h = \frac{Q}{(T_1-T_2)} \quad -(4)$$

$$C_c = \frac{Q}{(t_2-t_1)} \quad -(5)$$

Substituting (4) & (5) in (3)

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_1 - T_2}{Q} + \frac{t_2 - t_1}{Q}\right)$$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{UA}{Q}\left[T_1 - T_2 + t_2 - t_1\right]$$

$$\ln\left[\frac{(T_1 - t_1)}{(T_2 - t_2)}\right] = \frac{UA}{Q}\left[(T_1 - t_1) - (T_2 - t_2)\right]$$

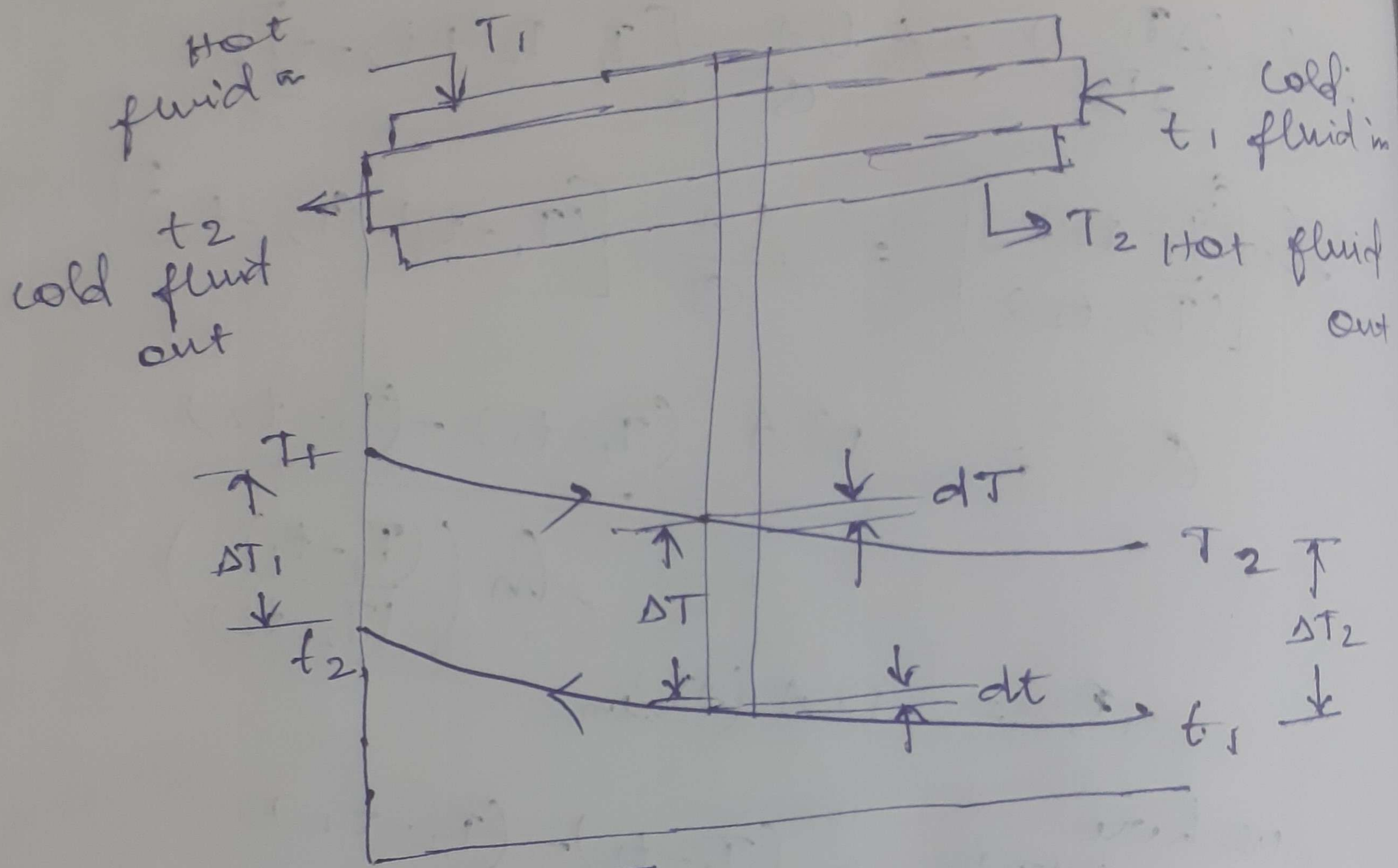
$$Q = UA\,\frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left(\frac{(T_1 - t_1)}{(T_2 - t_2)}\right)}$$

$$Q = UA\,(\Delta T)_{lm}$$

where

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left(\frac{(T_1 - t_1)}{(T_2 - t_2)}\right)}$$

Expression for LMTD - counter flow



$$dQ = U\,dA\,\Delta T$$

$$Q = -C_h(T_2 - T_1) = -C_c(t_2 - t_1)$$

$$dQ = -C_h\,dT = -C_c\,dt$$

$$dT = -\frac{dQ}{C_h} \qquad dt = -\frac{dQ}{C_c}$$

$$dT - dt = -\frac{dQ}{C_h} - \left(\frac{-dQ}{C_c}\right)$$

$$d(T-t) = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$d(T-t) = -U\,dA\,\Delta T\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\frac{d(T-t)}{\Delta T} = -U\,dA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\frac{d(T-t)}{(T-t)} = -U\,dA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\int_{\Delta T_1}^{\Delta T_2}\frac{d(T-t)}{(T-t)} = \int_0^A -U\,dA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\left[\ln(T-t)\right]_{\Delta T_1}^{\Delta T_2} = -U\left(\frac{1}{C_h}-\frac{1}{C_c}\right)\left[A\right]_0^A$$

$$\left[\ln \Delta T\right]_{\Delta T_1}^{\Delta T_2} = -UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$\left(\ln \Delta T_2 - \ln \Delta T_1\right) = -UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$+\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = +UA\left(\frac{1}{C_h}-\frac{1}{C_c}\right)$$

$$Q = C_h(T_1-T_2)$$

$$Q = C_c(t_2-t_1)$$

$$C_h = \frac{Q}{T_1-T_2} \qquad C_c = \frac{Q}{t_2-t_1}$$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = UA\left(\frac{T_1 - T_2}{Q} - \frac{t_2 - t_1}{Q}\right)$$

$$\ln\left(\frac{\Delta T_1}{\Delta T_2}\right) = \frac{UA}{Q}\left[(T_1 - t_2) - (T_2 - t_1)\right]$$

$$Q = UA\,\frac{\left[(T_1 - t_2) - (T_2 - t_1)\right]}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$Q = UA\,\frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{(T_1 - t_2)}{T_2 - t_1}\right)}$$

$$Q = UA(\Delta T)_{lm}.$$

where $(\Delta T)_{lm} = \dfrac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$

In a double pipe counter flow heat exchanger 10,000 kg/hr of an oil having a specific heat of 2095 J/kgK is cooled from 80°C to 50°C by 8000 kg/hr of water entering at 25°C. Determine the heat exchanger area for an overall heat transfer co-efficient of 300 W/m²K. Take Cp for water as 4180 J/kg K.

Given data

Hot fluid : oil

cold fluid : water

$\dot{m}_h = 10000$ kg/hr = 2.78 kg/s.

$C_{ph} = 2095$ J/kgK.

$T_1 = 80°C$

$T_2 = 50°C$

$\dot{m}_c = 8000$ kg/hr = 2.2 kg/s.

$t_1 = 25°C$

$U = 300$ W/m²K

$C_{pc} = 4180$ J/kgK

using energy balance equation

Heat lost by the hot fluid = Heat gained by the cold fluid

$$\dot{m}_h C_{ph} (T_1 - T_2) = \dot{m}_c C_{pc} (t_2 - t_1)$$

$$2.78 \times 2095 (80-50) = 2.2 \times 4180 (t_2 - 25)$$

$$\boxed{t_2 = 44^\circ C}$$

From D.B. P. 152

For counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$= \frac{(80-44) - (50-25)}{\ln\left(\frac{80-44}{50-25}\right)}$$

$$(\Delta T)_{lm} = 30.16^\circ C$$

$$Q = \dot{m}_h C_{ph} (T_1 - T_2)$$

$$= 2.78 \times 2095 (80-50)$$

$$= 174.72 \times 10^3 \, W$$

From D.B. P. 152

$Q = UA(\Delta T)_{lm}$

$174.72 \times 10^3 = 300 \times A \times 30.16$

$A = 19.3\ m^2$

2) Hot oil with a capacity rate 2500 W/K flows through a double pipe heat exchanger. It enters at 360°C and leaves at 300°C. Cold fluid enters at 30°C and leaves at 200°C. If the overall heat transfer co-efficient is 800 W/m²K. Determine the heat exchanger area required for (i) parallel flow (ii) counter flow.

Hot fluid: oil

$C_h = 2500\ W/K$

$T_1 = 360°C$

$T_2 = 300°C$

$t_1 = 30°C$

$t_2 = 200°C$

$U = 800\ W/m^2K$

Heat capacity rate $\left\{ \begin{array}{l} C_h = \dot{m}_h C_{ph} \\ C_c = \dot{m}_c C_{pc} \end{array} \right.$

parallel flow from D.B. P. 152

$$Q = U A (\Delta T)_{lm}$$

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left(\frac{T_1 - t_1}{T_2 - t_2}\right)}$$

$$= \frac{(360 - 30) - (300 - 200)}{\ln\left(\frac{360 - 30}{300 - 200}\right)}$$

$$(\Delta T)_{lm} = 192.64\ ^\circ C$$

$$Q = m C_h (T_1 - T_2)$$

$$= 2500 (360 - 300)$$

$$Q = 150 \times 10^3\ W$$

$$Q = U A (\Delta T)_{lm}$$

$$150 \times 10^3 = 800 \times A \times 192.64$$

$$A = 0.973\ m^2$$

counter flow

$$Q = U.A\,(\Delta T)_{lm}.$$

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}.$$

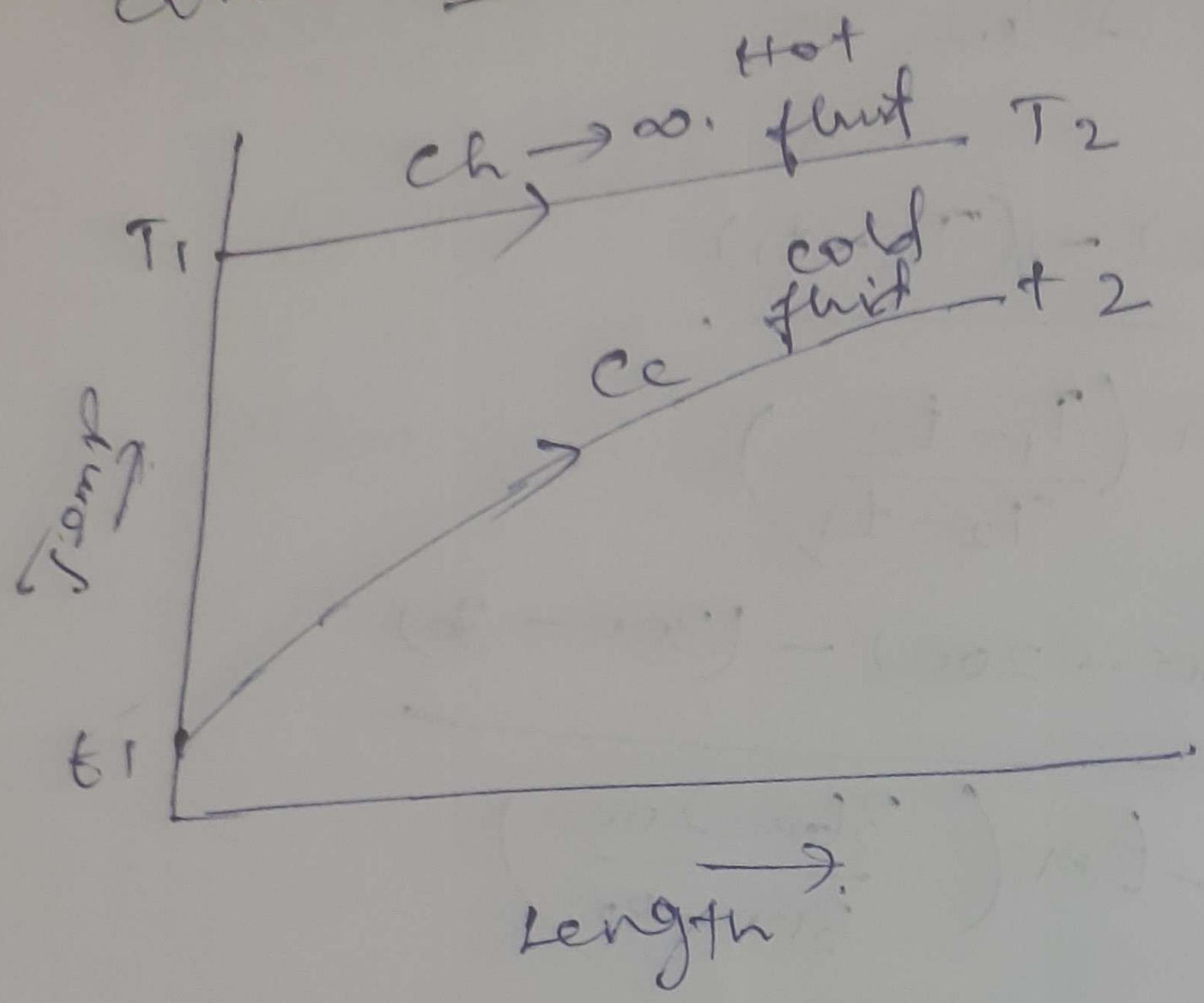
$$= \frac{(360 - 200) - (300 - 30)}{\ln\left(\frac{360 - 200}{300 - 30}\right)}$$

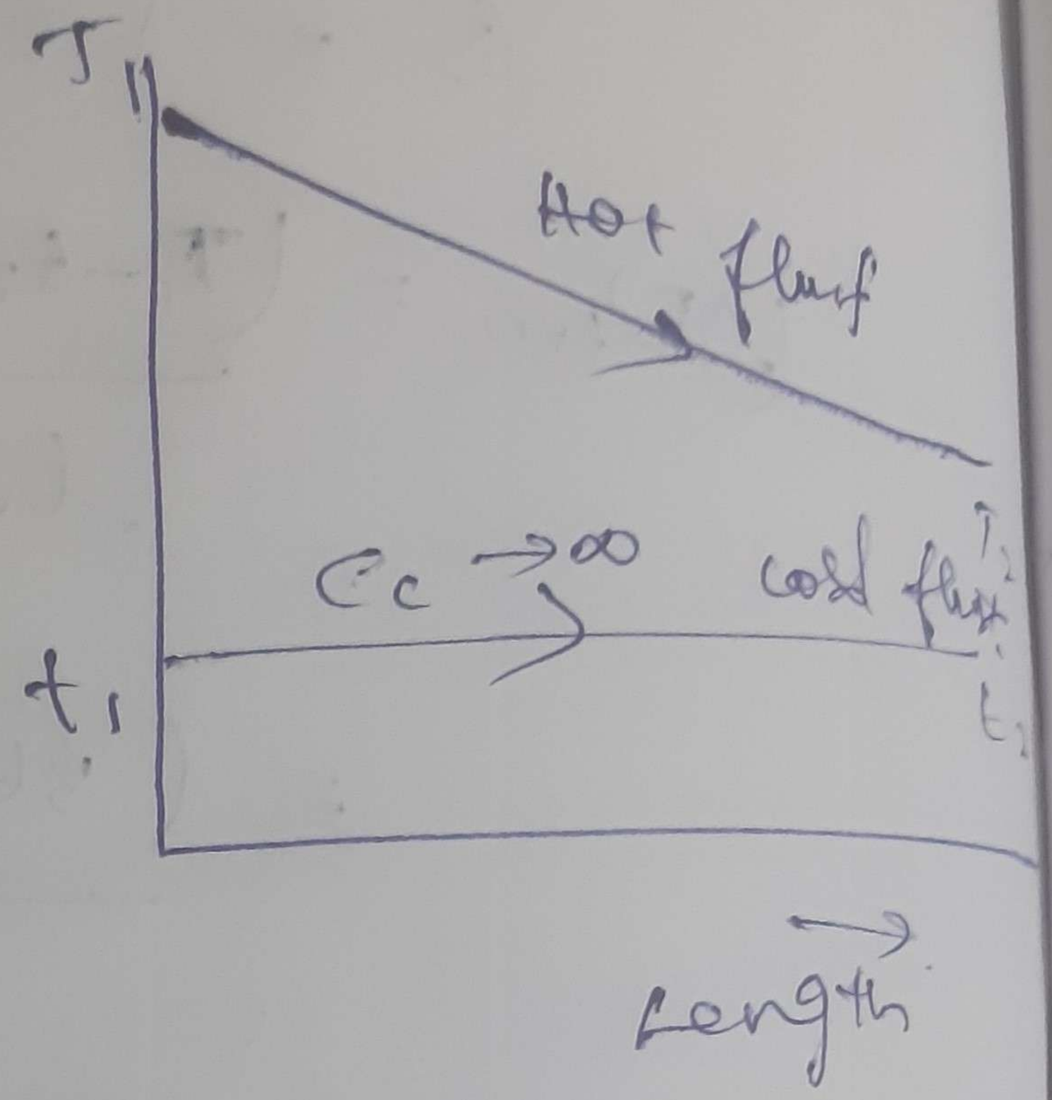
$$(\Delta T)_{lm} = 210.2\,^\circ C$$

$$150 \times 10^3 = 800 \times A \times 210.2$$

$$A = 0.891\ m^2$$

condenser and Evaporators





1) Saturated steam at 120°C is condensing on the outer tube surface of a single pass heat exchanger, the heat transfer co-efficient is 1800 W/m²K. Determine the surface area of a heat exchanger capable of heating 1000 kg/hr of water from 20°C to 90°C. Also calculate the rate of condensation of steam. Assume enthalpy of evaporation as 2200 kJ/kg

Given data

Hot fluid : steam

Cold fluid : water

$T_1 = 120^\circ C = T_2$

$U = 1800 \ W/m^2K$.

$\dot{m}_c = 1000 \ kg/hr = 0.277 \ kg/s$.

$t_1 = 20^\circ C$

$t_2 = 90^\circ C$

$h_{fg} = 2200 \ kJ/kg$

from D.B. p. 152

$Q = UA(\Delta T)_{lm}$.

For counterflow,

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$= \frac{(120 - 90) - (120 - 20)}{\ln\left(\frac{120 - 90}{120 - 20}\right)}$$

$(\Delta T)_{lm} = 58.1^\circ C$

$Q = \dot{m}_c C_{pc} (t_2 - t_1) = \dot{m}_s h_{fg}$

$Q = 0.277 \times 4180 (90 - 20)$

$Q = 81.42 \times 10^3 \, W.$

C_p for water $= 4180 \, J/kg$

$Q = UA(\Delta T)_{lm}.$

$81.42 \times 10^3 = 1800 \times A \times 58.1$

$A = 0.778 \, m^2$

$Q = \dot{m}_s h_{fg}$

$81.42 \times 10^3 = \dot{m}_s \times 2200 \times 10^3$

$\dot{m}_s = 0.037 \, kg/s.$

Multipass and Cross flow heat exchanger

$Q = F\,U\,A\,(\Delta T)_{lm}$.

F - correction factor

$(\Delta T)_{lm} \Rightarrow$ counter flow

Flow area for n tubes

$A_c = \frac{\pi}{4} d^2 \times n$

Surface area

$A = \pi d l \times n_p$

1) In a food processing plant water is to be cooled from 18°C to 6.5°C by using brine solution entering at an inlet temp. of -1.1°C and leaving at 2.9°C. What area is required when using a shell and tube heat exchanger with water making one shell pass and brine making two tube passes. Assume an overall heat transfer co-efficient of 850 W/m²K and a design heat load of 6000 W.

Given data

Hot fluid : water

Cold fluid : Brine

$T_1 = 18°C$ $\quad t_1 = -1.1°C$

$T_2 = 6.5°C$ $\quad t_2 = 2.9°C$

$U = 850\ W/m^2K$ $\quad Q = 6000\ W$

(1) To find Area (A)

from D.B. p. 152

$Q = UA(\Delta T)_{lm} F$

To find F from Graph.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{2.9 + 1.1}{18 + 1.1} = 0.209$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{18 - 6.5}{2.9 + 1.1} = 2.87$$

from D.B. p. 159

$F = 0.97$

For counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$= \frac{(18-2.9)-(6.5+1.1)}{\ln\left(\frac{18-2.9}{6.5+1.1}\right)}$$

$$(\Delta T)_{lm} = 10.9\ ^{\circ}C$$

$$Q = FUA(\Delta T)_{lm}$$

$$6000 = 0.97 \times 850 \times A \times 10.9$$

$$A = 0.661\ m^2$$

9) Water at the rate of 4 kg/s is heated from 38°C to 55°C in a shell and tube type heat exchanger. The water is to flow inside tubes of 2 cm diameter with an average velocity of 35 cm/s. Hot water available at 95°C and at the rate of 2 kg/s is used as the heating medium on the shell side. If the length of the tubes must not be more than 2 m. Calculate no. of tube passes, the no. of tubes per pass and length of the tubes for one pass shell assuming $U = 1500\ W/m^2K$.

Given data

Hot fluid : Hot water

cold fluid : cold water

$\dot{m}_c = 4\ kg/s$

$t_1 = 38°C$

$t_2 = 55°C$

$d = 2 \times 10^{-2} m$

$V = 35\ cm/s$

$= 35 \times 10^{-2}\ m/s$

$T_1 = 95°C$

$\dot{m}_h = 2\ kg/s$

$U = 1500\ W/m^2K$

Assuming one tube passes $n_p = 1$

$$Q = UA(\Delta T)_{lm}$$

$$Q = \dot{m}_h C_{ph}(T_1 - T_2)$$

C_p for water = 4180 J/kgK

$$Q = \dot{m}_c C_{pc}(t_2 - t_1)$$

$$\therefore Q = 4 \times 4180 (55 - 38)$$

$$Q = 284.65 \times 10^3 \, W$$

For counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left(\frac{T_1 - t_2}{T_2 - t_1}\right)}$$

$$Q = \dot{m}_h C_{ph}(T_1 - T_2)$$

$$284.65 \times 10^3 = 2 \times 4180 (95 - T_2)$$

$$T_2 = 61^\circ C$$

$$(\Delta T)_{lm} = \frac{(95 - 25) - (61 - 38)}{\ln\left(\frac{95 - 55}{61 - 38}\right)}$$

$$(\Delta T)_{lm} = 30.7^\circ C$$

$$A = \frac{Q}{U\,(\Delta T)_{lm}} = \frac{284.65 \times 10^3}{1500 \times 30.72}$$

Surface area $A = 6.17\ m^2$

Flow area

$$A_c = \frac{\pi}{4} d^2 \times n$$

$$\dot{m}_c = \rho A_c V$$

$$4 = 1000 \times A_c \times 35 \times 10^{-2}$$

$$A_c = 0.1142\ m^2$$

$$0.1142 = \frac{\pi}{4} \times (2 \times 10^{-2})^2 \times n.$$

$$n = 36.38 \simeq 36$$

Surface area $A = \pi d l n \times n_p.$

$$6.17 = \pi \times 2 \times 10^{-2} \times l \times 36 \times 1.$$

$$l = 2.731\ m.$$

Here, $l > 2\ m.$

But, l should be less than $2m$.

∴ considering two tube passes,

$$Q = UA'(\Delta T)_{lm} F$$

To find F

From D.B P. 159.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{55-38}{95-38} = 0.3$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{95-61}{55-38} = 2$$

$F = 0.9$

$$284.6 \times 10^3 = 1500 \times A' \times 30.7 \times 0.9$$

$$A' = 6.8\ m^2$$

$$A' = \pi d l n_p$$

$$6.8 = \pi \times 2 \times 10^{-2} \times l \times 36 \times 2$$

$$\boxed{l = 1.57\ m}$$

$n = 36$

$n_p = 2$

$l = 1.57\ m$

Effectiveness - NTU method

$$\epsilon = \frac{Q}{Q_{max}}$$

$$Q = \dot{m}_h C_{ph} (T_1 - T_2) = \dot{m}_c C_p (t_2 - t_1)$$

$$Q = C_h (T_1 - T_2) = C_c (t_2 - t_1)$$

$$Q_{max} = C_{min} (T_1 - t_1)$$

$C_{min} \Longrightarrow$ minimum value of C_h and C_c

$$NTU = \frac{UA}{C_{min}}$$

NTU - No. of Transfer units

1) water enters a cross flow heat exchanger (both fluids unmixed) at 5°c and flow at the rate 4600 kg/hr to cool 4000 kg/hr of air that initially at 40°c. Assume over all heat transfer co-efficient is 150 W/m²K for a heat exchanger surface area of 25 m². Calculate the exit temperature of air and water.

Given data

Hot fluid : Air

Cold fluid : water

$t_1 = 5°c$

$\dot{m}_c = 4600 \text{ kg/hr} = 1.27 \text{ kg/s}$

$\dot{m}_h = 4000 \text{ kg/hr} = 1.11 \text{ kg/s}$

$T_1 = 40°c$

$U = 150 \text{ W/m}^2\text{K}$

$A = 25 \text{ m}^2$

To find T_2 and t_2

C_p for water = 4180 J/kg/K

C_p for air = 1005 J/kg/K.

$$\epsilon = \frac{Q}{Q_{max}}.$$

$\dot{m}_h C_{ph} = 1.27 \times 4180 = C_h$

$= 5308.6$ W/K

$\dot{m}_c C_{pc} = 1.11 \times 1005 = C_c$

$= 1115.5$ W/K

$C_c < C_h$

$\therefore C_{min} = C_c = 1115.5$ W/K.

$Q_{max} = C_{min}(T_1 - t_1)$

$= 1115.5(40-5)$

$Q_{max} = 39.04 \times 10^3$ W

To find ϵ

From D.B.P. 166

Cross-flow - Both fluids unmixed

$$\frac{C_{min}}{C_{max}} = \frac{1115.5}{5308.6} = 0.21$$

$$NTU = \frac{UA}{C_{min}}$$

$$NTU = \frac{150 \times 25}{1115.5} = 3.36$$

From Graph

$$\epsilon = 0.85$$

$$Q = \epsilon Q_{max}$$

$$= 0.85 \times 39.04 \times 10^3$$

$$Q = 33186.12 \text{ W}$$

$$Q = C_h (T_1 - T_2)$$

$$33186.12 = 5308.6 (40 - T_2)$$

$$\boxed{T_2 = 10.25^\circ C}$$

$$Q = C_c (t_2 - t_1)$$

$$33186.12 = 1115.5 (t_2 - 5)$$

$$\boxed{t_2 = 11.25^\circ C}$$

1) Water enters a counter flow double pipe heat exchanger at 15°C, flowing at the rate 1300 kg/hr. It is heated by oil (C_p = 2000 J/kgK) flowing at the rate of 550 kg/hr from the inlet temperature of 94°C. Heat exchanger area of $1 m^2$ and overall heat transfer co-efficient of 1075 W/m^2K determine the heat transfer and outlet temperature of water and oil

Given data

Hot fluid : oil

cold fluid : water

$T_1 = 94°C$

$A = 1 m^2$

$U = 1075$ W/m^2K

$t_1 = 15°C$

$\dot{m}_c = 1300$ kg/hr

$= 0.361$ kg/s

$C_{ph} = 2000$ J/kgK

$\dot{m}_h = 550$ kg/hr

$= 0.152$ kg/s

To find T_2 and t_2

C_p for water $= 4180$ J/kg/K

$$\epsilon = \frac{Q}{Q_{max}}$$

$\dot{m}_h C_{ph} = 0.152 \times 2000 = C_h$

$= 305.4$ W/K

$\dot{m}_c C_{pc} = 0.361 \times 4180 = C_c$

$= 1508.6$ W/K.

$C_h < C_c$

$\therefore C_{min} = C_h = 305.4$ W/K

$Q_{max} = C_{min}(T_1 - t_1)$

$= 305.4(94-15)$

$Q_{max} = 24.12 \times 10^3$ W

To find ϵ :

From D. B. P. 164.

$$\frac{C_{min}}{C_{max}} = \frac{305.4}{1508.6} = 0.202$$

$$NTU = \frac{UA}{C_{min}} = \frac{1075 \times 1}{305.4} = 3.51$$

From Graph $\epsilon = 0.95$

$$\epsilon = \frac{Q}{Q_{max}}$$

$$Q = 0.95 \times 24.12 \times 10^3$$

$$Q = 22.92 \times 10^3 \, W$$

$$Q = C_h (T_1 - T_2)$$

$$22.92 \times 10^3 = 305.4 (94 - T_2)$$

$$\boxed{T_2 = 18.9^\circ C}$$

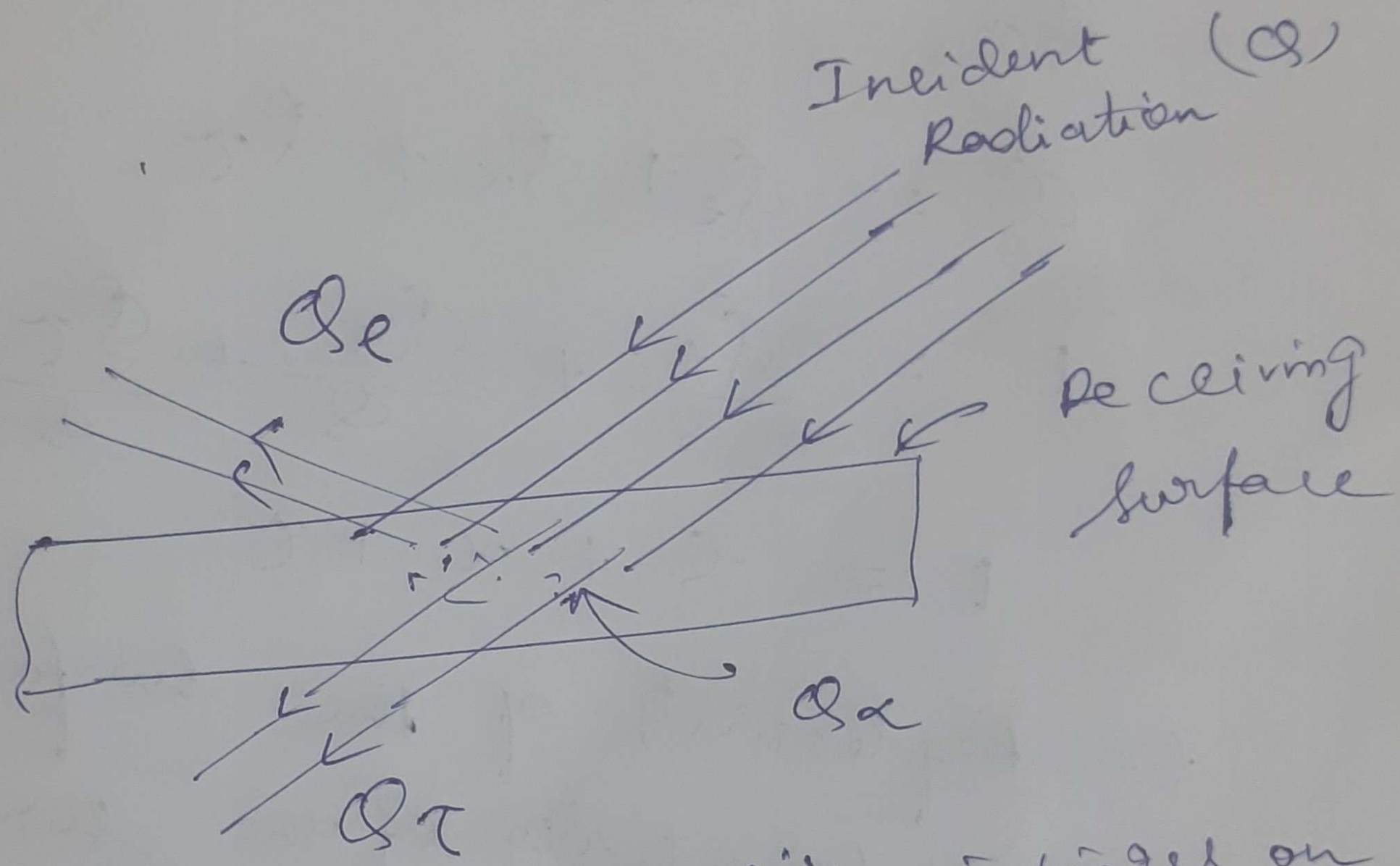
$$Q = C_c (t_2 - t_1)$$

$$22.92 \times 10^3 = 1508.6 (t_2 - 15)$$

$$\boxed{t_2 = 30.19^\circ C}$$

Radiation

Absorption, Reflection and Transmission



When incident radiation, impinges on a surface, the following may happen,

(i) a part is reflected back

(ii) a part is Transmitted

(iii) The reminder is absorbed

$Q \rightarrow$ Heat received by the surface (Incident radiation)

$Q_\rho \rightarrow$ Heat reflected by the surface

Q_α - Heat absorbed by the surface

Q_τ - Heat transmitted by the surface

$$Q = Q_\rho + Q_\alpha + Q_\tau$$

$$1 = \frac{Q_\rho}{Q} + \frac{Q_\alpha}{Q} + \frac{Q_\tau}{Q}$$

$$1 = \rho + \alpha + \tau$$

ρ - Reflectivity of the surface

α - Absorptivity of the surface

τ - Transmissivity of the surface

$\rho = 0 \rightarrow$ non reflecting surface

$\rho = 1 \rightarrow$ Perfect reflector

$\tau = 0 \rightarrow$ Opaque surface

$\tau = 1 \rightarrow$ Transparent surface

$\alpha = 0 \rightarrow$ Non absorbing surface

$\alpha = 1 \rightarrow$ Perfect absorbing surface (Black body)

Black body

A black body is an ideal body that absorbs all incident radiant energy and reflects or transmits none. For black body

$\alpha = 1 \quad \rho = 0 \quad \tau = 0.$

No actual body is perfectly black.

The concept of black body is an idealisation with which radiation characteristics of real bodies can be compared.

④ Laws of Radiation

Stefan Boltzmann's law

$E_b \propto T^4$

$E_b = \sigma T^4$

E_b - Emissive power of a black surface

$E_b = \frac{Q}{A}$ W/m^2

Kirchoff's law

At thermal Equilibrium

$\epsilon = \alpha$

Wien's law

wave length corresponding to maximum energy emission is inversely proportional to the absolute temperature of the body

$\lambda_m \propto \frac{1}{T}$

$\lambda_m = c \frac{1}{T}$

$\lambda_m T = c$

$c = 2.898 \times 10^{-3}$ mmK

λ - wave length in μm

(5)

Fraction of emissive power of black body $= \dfrac{\text{Black body radiant energy upto the wave length } \lambda}{\text{Total emissive power of black (At same temp)}}$

$$= \frac{E_b(0-\lambda)}{E_b} = \frac{E_b(0-\lambda)}{\sigma T^4}$$

Fraction of emissive power in the wave length band of λ_1 and λ_2

$$E_b(\lambda_1-\lambda_2) = \frac{E_b(0-\lambda_1) - E_b(0-\lambda_2)}{\sigma T^4}$$

Emissive power in the wave length band λ_1 and λ_2

$$E_b(\lambda_1-\lambda_2) = \frac{E_b(0-\lambda_1) - E_b(0-\lambda_2)}{\sigma T^4} \times \sigma T^4$$

1) A black body is kept at a temperature of 727°c. Calculate the thermal radiation emitted by the surface in the wave length band 1 μm and 5 μm.

Given data

$\lambda_1 = 1\ \mu m$ $\qquad T = 727°C = 1000\ K$

$\lambda_2 = 5\ \mu m$

$\lambda_1 T = 1 \times 1000 = 1000\ \mu m K$

$\lambda_2 T = 5 \times 1000 = 5000\ \mu m K$

$$E_b(\lambda_1 - \lambda_2) = \frac{E_{b(0-\lambda_1)} - E_{b(0-\lambda_2)}}{\sigma T^4} \times \sigma T^4$$

From D.B. p. 83.

for $\lambda_1 T = 1000\ \mu m K$

$$\frac{E_{b0-\lambda_1}}{\sigma T^4} = 0.000321$$

and $\lambda_2 T = 5000\ K$

$$\frac{E_{b0-\lambda_2}}{\sigma T^4} = 0.633747$$