

Assignment problems: (i) T.C

$$\textcircled{1} \quad T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{2}\right) + n & n>1 \end{cases}$$

Substitution Method.

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$= 2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$\Rightarrow 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

$$\Rightarrow 2^2 \left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2^2} \right) + 2n$$

$$\Rightarrow 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2 n = k$$

\downarrow k times

$$\Rightarrow 2^k T\left(\frac{n}{2^k}\right) + kn$$

$$\Rightarrow 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot n$$

$$\Rightarrow n \log_2 2 T\left(\frac{n}{n \log_2 2}\right) + \log_2 n \cdot n$$

$$\Rightarrow n \cdot T(1) + \log_2 n \cdot n$$

$$\Rightarrow \underline{n + n \log_2 n}$$

$$T.C. \Rightarrow O(n \log_2 n)$$

problem

$$T(n) = \begin{cases} 1 & n=1 \\ 8T\left(\frac{n}{2}\right) + n^2 & n>1 \end{cases}$$

$$T(n) = 8T\left(\frac{n}{2}\right) + n^2$$

$$\Rightarrow 8\left(8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right) + n^2$$

$$\Rightarrow 8^2 T\left(\frac{n}{2^2}\right) + 3n^2 \Rightarrow \frac{8^2 \times n^2}{4} + n^2$$

$$\Rightarrow 8^2 \left(8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right) + 3n^2$$

$$\Rightarrow 8^3 T\left(\frac{n}{2^3}\right) + 7n^2$$

$$\frac{n}{2^k} = 1$$

k times

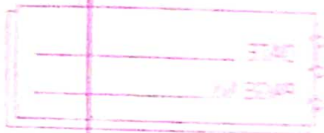
$$k = \log_2 n$$

$$\Rightarrow \boxed{7n^2}$$

$$\Rightarrow 8^k T\left(\frac{n}{2^k}\right) + k n (2^k - 1) n^2$$

$$\Rightarrow 8^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + (2^{\log_2 n} - 1) n^2$$

$$\Rightarrow n^{\log_2 8} T\left(\frac{n}{n^{\log_2 2}}\right) + (2^{\log_2 n} - 1) n^2$$



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$$\Rightarrow n^3 T\left(\frac{n}{n}\right) + n^3 - n^2$$

$$\Rightarrow n^3 + n^3 - n^2$$

T.C. $\Rightarrow O(n^3)$