Practice Problems

① $f(n) = n - 10$ $\quad g(n) = n + 10$

$f(n) = \Theta(g(n))$.

$f(n) \leq c_1 \cdot g(n)$ $\qquad$ $f(n) \geq c_2 \cdot g(n)$

$n - 10 \leq c_1 \cdot n + 10$ $\qquad$ $n - 10 \geq c_2 \cdot n + 10$

$c = 1$ $\qquad$ $c = \frac{1}{2}$

$f(n) = O(g(n))$ $\qquad$ $f(n) = \Omega(g(n))$

We can say that $f(n) = \Theta(g(n))$ as it holds true for both Ω as well as O

(2) $f(n) = n \quad g(n) = n$

$f(n) = \Theta(g(n))$

$f(n) \leq c_1 \cdot g(n)$ $\quad$ $f(n) \geq c_2 \cdot g(n)$

$n \leq c_1 \cdot n$ $\quad$ $n \geq c_2 n$.

where $c = 1$ $\quad$ where $c = 1$

$\therefore$ True $\quad$ $\therefore$ True.

$\Rightarrow f(n) \geq \Theta(g(n))$ True

(3) $64^{\log_2 n} \cdot 32^{\log_2 n} = \Theta(n^5)$

$n^{\log_2 64} \cdot n^{\log_2 32}$

$\log_2 64 \Rightarrow \log_2 2^6 \Rightarrow 6$

$\log_2 32 \Rightarrow \log_2 2^5 \Rightarrow 5$

$\Rightarrow n^6 \cdot n^5$

$\Rightarrow n^{11}$

$f(n) \leq c \cdot g(n)$

$n^{11} \leq c \cdot (n^5) \longrightarrow$ false

↳ if $c = n^6$ we have c is dependent so we can't take

④ $\frac{4^n}{2^n} = O(2^n)$

$\frac{2^n * 2^n}{2^n} = O(2^n)$

$\boxed{2^n \leq c \cdot 2^n}$ where $c = 1$

⇓

True.

⑤ $128^{\log_2 n} \cdot n^2 = O(n^9)$

$n^{\log_2 128} \Rightarrow n^7 \cdot n^2 \leq c \cdot n^9$

$\{\log_2 128 \Rightarrow \log_2 2^7 \Rightarrow 7\}$

$\Rightarrow n^7 \cdot n^2 = O(n^9)$

$\Rightarrow \boxed{n^9 \leq c \cdot n^9}$ where $c = 1$ → True