Supplementary Materials for "A Two-Population Algorithm for Large-Scale Multiobjective Optimization Based on Fitness-Aware Operator and Adaptive Environmental Selection"

Bingdong Li, Yan Zhang, Peng Yang, *Member, IEEE,* Xin Yao, *Fellow, IEEE,* and Aimin Zhou*, *Senior Member, IEEE*

Due to space limitation, the hypervolume (HV) results are shown in the following Tables I,II,III,IV. Specifically, Table I and Table II show the HV values of DGEA, LMOCSO, LMOEA-DS, Two_Arch2, TriMOEA-TA&R, CCGDE3 and LSTPA on 2-objective and 3-objective LSMOP1−LSMOP9, where the best result on each test instance is shown in gray background. Table III and Table IV demonstrate the HV values of the compared algorithms on 2-objective and 3-objective DTLZ7, MaF11 and LSMOP9 corresponding to the median run.

TABLE I: HV values of DGEA, LMOCSO, LMOEA-DS, Two_Arch2, TriMOEA-TA&R, CCGDE3 and LSTPA on 2-objective LSMOP1−LSMOP9, where the best result on each test instance is shown in gray background. '\' indicates that the indicator values cannot be calculated in reasonable time, due to the algorithm type setting of PlatEMO, hereinafter. A larger HV value indicates better performance.

No.	Dim.	DGEA	LMOCSO	LMOEA-DS	Two_Arch2	TriMOEA-TA&R	CCGDE3	LSTPA
LSMOP1	100	5.79e-1(2.86e-4)−	5.80e-1(3.37e-4)−	4.74e-1(1.94e-2)−	3.49e-1(3.41e-2)−	1.14e-1(4.45e-2)−	1.99e-4(8.93e-4)−	5.82e-1(3.75e-5)
	200	5.76e-1(1.29e-3)−	5.79e-1(7.93e-4)−	3.88e-1(7.53e-3)−	3.14e-1(5.60e-2)−	1.15e-1(3.91e-2)−	0.00e+0(0.00e+0)−	5.82e-1(4.63e-5)
	500	5.66e-1(2.26e-3)−	5.75e-1(6.34e-3)−	3.52e-1(2.73e-3)−	2.77e-1(5.88e-2)−	1.35e-1(3.67e-2)−	0.00e+0(0.00e+0)−	5.82e-1(4.83e-5)
	1000	5.03e-1(1.10e-2)−	5.74e-1(4.81e-3)−	3.37e-1(2.70e-3)−	2.40e-1(5.49e-2)−	1.09e-1(3.03e-2)−	0.00e+0(0.00e+0)−	5.82e-1(7.38e-5)
	2000	3.43e-1(2.96e-3)−	4.90e-1(8.36e-2)−	3.10e-1(1.34e-3)−	2.21e-1(1.77e-2)−	1.44e-1(2.91e-2)−	0.00e+0(0.00e+0)−	5.81e-1(1.52e-4)
LSMOP2	100	5.50e-1(3.21e-2)−	5.29e-1(1.83e-2)−	5.44e-1(2.79e-3)−	5.65e-1(1.04e-3)−	2.91e-1(5.90e-3)−	3.43e-1(1.13e-2)−	5.77e-1(1.01e-3)
	200	5.64e-1(1.63e-3)−	5.53e-1(7.88e-3)−	5.55e-1(1.09e-3)−	5.63e-1(9.85e-4)−	3.99e-1(2.89e-3)−	4.07e-1(4.12e-3)−	5.77e-1(3.20e-4)
	500	5.72e-1(3.94e-4)−	5.50e-1(4.40e-3)−	5.68e-1(4.55e-4)−	5.63e-1(9.38e-4)−	4.93e-1(7.94e-4)−	4.91e-1(9.08e-4)−	5.75e-1(8.59e-3)
	1000	5.76e-1(2.10e-4)+	5.62e-1(4.39e-4)−	5.73e-1(3.18e-4)+	5.65e-1(5.49e-4)−	5.34e-1(7.13e-5)−	5.32e-1(6.74e-4)−	5.68e-1(4.99e-3)
	2000	5.78e-1(2.65e-4)+	5.69e-1(3.92e-4)−	5.77e-1(4.19e-4)+	5.68e-1(1.57e-4)−	5.57e-1(9.96e-5)−	5.55e-1(5.40e-4)−	5.72e-1(2.10e-4)
LSMOP3	100	5.10e-2(6.65e-2)≈	5.39e-2(4.24e-2)≈	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	5.58e-2(3.96e-2)
	200	4.60e-2(4.53e-2)≈	2.27e-2(3.73e-2)≈	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	2.82e-2(4.00e-2)
	500	4.73e-2(4.57e-2)≈	2.73e-3(9.06e-3)−	0.00e+0(0.00e+0)−	2.05e-2(2.19e-2)≈	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	3.42e-2(3.94e-2)
	1000	6.25e-2(4.09e-2)+	1.12e-4(5.04e-4)−	0.00e+0(0.00e+0)−	5.04e-2(1.61e-2)≈	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	4.32e-2(3.33e-2)
	2000	4.15e-2(4.59e-2)+	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	5.45e-2(1.96e-2)+	1.53e-6(6.88e-4)−	0.00e+0(0.00e+0)≈	1.53e-4(6.88e-4)
LSMOP4	100	5.64e-1(3.06e-3)−	5.54e-1(1.41e-3)−	5.52e-1(1.59e-3)−	5.31e-1(5.02e-3)−	2.31e-1(1.27e-2)−	2.54e-1(1.70e-2)−	5.79e-1(1.20e-3)
	200	5.69e-1(1.79e-3)−	5.65e-1(5.05e-4)−	5.60e-1(1.08e-3)−	5.57e-1(1.83e-3)−	3.21e-1(7.32e-3)−	3.49e-1(8.26e-3)−	5.81e-1(5.15e-4)
	500	5.74e-1(6.82e-4)−	5.74e-1(5.78e-4)−	5.50e-1(2.29e-3)−	5.72e-1(3.98e-4)−	4.28e-1(1.66e-3)−	4.45e-1(4.23e-3)−	5.82e-1(1.26e-4)
	1000	5.75e-1(1.67e-3)−	5.75e-1(5.63e-4)−	5.58e-1(7.39e-4)−	5.77e-1(2.25e-4)−	4.91e-1(7.73e-4)−	4.96e-1(3.34e-3)−	5.82e-1(1.81e-4)
	2000	5.69e-1(1.07e-3)−	5.68e-1(5.12e-4)−	5.68e-1(2.55e-4)−	5.77e-1(2.22e-4)−	5.32e-1(3.61e-4)−	5.31e-1(1.53e-3)−	5.80e-1(6.28e-4)
LSMOP5	100	3.42e-1(1.37e-3)−	3.45e-1(3.41e-4)−	1.83e-1(3.64e-2)−	1.76e-1(9.35e-3)−	9.50e-2(1.85e-2)−	0.00e+0(0.00e+0)−	3.46e-1(2.12e-4)
	200	3.38e-1(1.30e-3)−	3.19e-1(7.81e-2)−	1.88e-1(2.98e-2)−	1.80e-1(1.26e-2)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	3.46e-1(3.52e-4)
	500	3.23e-1(2.83e-3)−	1.75e-1(1.19e-1)−	1.98e-1(1.65e-2)−	1.80e-1(1.13e-2)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	3.46e-1(4.15e-4)
	1000	3.11e-1(3.00e-3)−	9.49e-2(1.86e-2)−	1.87e-1(9.87e-3)−	1.84e-1(9.66e-3)−	1.84e-3(9.66e-3)−	0.00e+0(0.00e+0)−	3.46e-1(5.29e-4)
	2000	1.70e-1(7.65e-4)−	5.30e-2(3.65e-2)−	1.41e-1(1.81e-2)−	1.76e-1(3.09e-3)−	5.30e-5(3.65e-2)−	0.00e+0(0.00e+0)−	3.43e-1(1.25e-3)
LSMOP6	100	1.30e-1(6.83e-2)+	2.94e-2(2.46e-2)+	5.82e-2(6.84e-3)+	0.00e+0(0.00e+0)−	9.05e-2(1.17e-4)+	0.00e+0(0.00e+0)−	8.71e-3(2.06e-2)
	200	1.80e-1(6.79e-2)+	3.37e-2(1.13e-2)+	8.30e-2(9.31e-3)+	0.00e+0(0.00e+0)≈	9.04e-2(4.36e-5)+	0.00e+0(0.00e+0)≈	3.20e-3(9.35e-3)
	500	1.68e-1(5.89e-2)+	4.48e-2(6.00e-3)+	1.24e-1(8.73e-3)+	0.00e+0(0.00e+0)−	8.89e-2(2.96e-4)+	0.00e+0(0.00e+0)−	1.44e-2(2.41e-5)
	1000	1.48e-1(6.94e-2)+	5.79e-2(2.19e-3)+	1.32e-1(5.06e-3)+	3.07e-2(9.96e-4)−	8.75e-2(3.25e-4)+	2.70e-2(4.74e-4)−	5.29e-2(4.24e-14)
	2000	1.04e-1(4.05e-2)+	5.90e-2(2.91e-6)−	1.19e-1(4.98e-3)+	6.36e-2(7.53e-3)−	7.95e-2(3.57e-4)+	5.90e-2(1.06e-4)−	7.19e-2(1.07e-5)
LSMOP7	100	2.28e-2(7.08e-2)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	2.36e-3(6.23e-3)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
	200	9.67e-3(3.16e-2)≈	5.53e-5(2.47e-4)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	4.85e-4(2.17e-3)
	500	0.00e+0(0.00e+0)≈	1.08e-4(4.85e-4)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	8.73e-3(2.69e-2)≈	0.00e+0(0.00e+0)≈	2.75e-2(8.50e-2)
	1000	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	4.51e-3(2.02e-2)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	4.31e-3(1.93e-2)
	2000	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)≈	2.99e-2(4.19e-2)+	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
LSMOP8	100	3.11e-1(1.82e-2)−	3.08e-1(7.51e-3)−	2.91e-1(7.26e-3)−	1.43e-1(1.04e-2)−	9.09e-2(1.80e-7)−	0.00e+0(0.00e+0)−	3.34e-1(3.99e-3)
	200	3.18e-1(8.20e-3)−	3.02e-1(5.37e-3)−	2.95e-1(4.65e-3)−	1.55e-1(4.30e-3)−	9.09e-2(1.05e-8)−	0.00e+0(0.00e+0)−	3.38e-1(3.06e-3)
	500	3.27e-1(2.05e-3)−	3.23e-1(1.27e-3)−	2.74e-1(5.06e-3)−	1.73e-1(1.75e-4)−	9.09e-2(8.28e-9)−	0.00e+0(0.00e+0)−	3.43e-1(2.38e-3)
	1000	3.31e-1(6.54e-4)−	3.34e-1(7.55e-4)−	2.26e-1(5.19e-3)−	1.78e-1(5.21e-3)−	9.09e-2(7.47e-9)−	0.00e+0(0.00e+0)−	3.46e-1(5.45e-4)
	2000	3.04e-1(2.42e-3)−	3.37e-1(1.90e-4)−	1.34e-1(3.08e-3)−	1.91e-1(4.14e-3)−	9.09e-2(1.22e-6)−	0.00e+0(0.00e+0)−	3.40e-1(1.80e-3)
LSMOP9	100	2.33e-1(1.44e-2)+	2.36e-1(2.13e-3)+	9.09e-2(4.27e-17)−	9.09e-2(3.45e-16)−	7.59e-2(2.41e-2)−	0.00e+0(0.00e+0)−	2.25e-1(4.34e-2)
	200	2.24e-1(1.81e-2)−	2.32e-1(7.97e-3)−	9.33e-2(1.09e-2)−	9.09e-2(9.79e-16)−	8.55e-2(2.56e-2)−	0.00e+0(0.00e+0)−	2.41e-1(5.57e-4)
	500	1.82e-1(1.53e-2)−	2.33e-1(4.70e-3)−	1.51e-1(2.00e-2)−	9.09e-2(4.22e-6)−	8.59e-2(7.19e-4)−	0.00e+0(0.00e+0)−	2.41e-1(2.90e-4)
	1000	2.03e-1(7.06e-3)−	2.35e-1(2.50e-3)−	1.77e-1(1.48e-2)−	9.11e-2(1.64e-5)−	8.32e-2(2.06e-4)−	0.00e+0(0.00e+0)−	2.41e-1(4.74e-4)
	2000	2.04e-1(3.89e-2)−	2.25e-1(5.54e-4)−	1.85e-1(1.16e-2)−	9.13e-2(1.26e-5)−	8.30e-2(3.99e-4)−	0.00e+0(0.00e+0)−	2.31e-1(9.98e-4)
+/−/≈		10/27/8	5/32/8	7/32/6	1/36/8	6/35/4	0/38/7	

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that of LSTPA, respectively.

TABLE II: HV values of DGEA, LMOCSO, LMOEA-DS, Two_Arch2, TriMOEA-TA&R, CCGDE3 and LSTPA on 3-objective LSMOP1−LSMOP9, where the best result on each test instance is shown in gray background. A larger HV value indicates better performance.

No.	Dim.	DGEA	LMOCSO	LMOEA-DS	Two_Arch2	TriMOEA-TA&R	CCGDE3	LSTPA
LSMOP1	100	6.12e-1(4.31e-2)−	8.31e-1(3.90e-3)−	6.31e-1(4.19e-2)−	6.20e-1(2.99e-2)−	2.17e-1(4.93e-2)−	0.00e+0(0.00e+0)−	8.39e-1(1.82e-3)
	200	4.95e-1(6.02e-2)−	8.28e-1(5.31e-3)−	5.42e-1(5.38e-2)−	5.76e-1(4.67e-2)−	1.79e-1(4.19e-2)−	0.00e+0(0.00e+0)−	8.40e-1(2.76e-4)
	500	4.53e-1(7.63e-2)−	8.23e-1(5.33e-3)−	4.47e-1(8.07e-3)−	5.74e-1(4.42e-2)−	1.92e-1(5.60e-2)−	0.00e+0(0.00e+0)−	8.40e-1(1.28e-4)
	1000	4.37e-1(1.47e-2)−	8.16e-1(9.46e-3)−	3.84e-1(8.67e-3)−	5.40e-1(5.67e-2)−	2.29e-1(9.74e-3)−	0.00e+0(0.00e+0)−	8.40e-1(1.16e-4)
	2000	3.74e-1(8.43e-2)−	6.16e-1(1.44e-2)−	3.56e-1(2.69e-3)−	4.71e-1(2.88e-2)−	2.22e-1(1.17e-2)−	0.00e+0(0.00e+0)−	8.31e-1(8.76e-3)
LSMOP2	100	7.87e-1(4.02e-3)−	8.01e-1(2.34e-3)−	7.56e-1(5.52e-3)−	8.16e-1(2.72e-3)≈	8.16e-3(2.00e-3)	5.87e-1(5.74e-3)−	8.16e-1(2.00e-3)
	200	8.09e-1(3.98e-3)−	8.09e-1(2.19e-3)−	7.83e-1(4.46e-3)−	8.06e-1(3.15e-3)−	8.06e-3(3.15e-3)−	7.01e-1(6.63e-3)−	8.20e-1(1.66e-3)
	500	8.27e-1(1.14e-3)−	8.24e-1(1.01e-3)−	7.98e-1(6.01e-3)−	8.16e-1(1.01e-3)−	7.96e-1(5.14e-4)−	7.76e-1(5.13e-3)−	8.30e-1(8.25e-4)
	1000	8.33e-1(5.74e-4)−	8.30e-1(7.10e-4)−	8.01e-1(5.10e-3)−	8.22e-1(1.28e-3)−	8.34e-3(5.98e-4)	7.98e-1(5.43e-3)−	8.34e-1(5.98e-4)
	2000	8.37e-1(4.74e-4)+	8.33e-1(3.95e-4)−	8.04e-1(5.16e-3)−	8.29e-1(8.04e-4)−	8.35e-3(3.31e-4)	8.13e-1(4.23e-3)−	8.35e-1(3.31e-4)
LSMOP3	100	1.99e-1(3.33e-2)+	9.84e-2(1.24e-2)−	9.27e-2(1.85e-3)−	4.31e-1(3.66e-2)+	1.52e-1(4.20e-2)≈	0.00e+0(0.00e+0)−	1.46e-1(5.40e-2)
	200	1.86e-1(2.24e-2)+	9.21e-2(1.92e-3)≈	9.10e-2(2.48e-4)≈	2.03e-1(1.12e-1)+	1.82e-1(3.90e-2)+	0.00e+0(0.00e+0)−	1.15e-1(3.11e-2)
	500	1.75e-1(2.44e-2)+	9.12e-2(7.74e-4)≈	9.11e-2(2.37e-4)≈	5.05e-2(8.49e-2)−	1.89e-1(3.23e-2)+	0.00e+0(0.00e+0)−	9.48e-2(3.49e-2)
	1000	1.60e-1(2.78e-2)+	9.09e-2(4.00e-4)+	9.12e-2(2.49e-4)≈	2.06e-3(6.35e-3)−	2.17e-1(1.35e-2)+	0.00e+0(0.00e+0)−	9.01e-2(2.28e-2)
	2000	9.23e-2(5.95e-3)+	2.72e-2(4.27e-2)−	9.12e-2(2.90e-4)+	0.00e+0(0.00e+0)−	2.29e-1(1.79e-2)+	0.00e+0(0.00e+0)−	5.91e-2(4.45e-2)
LSMOP4	100	6.69e-1(2.88e-2)−	7.89e-1(1.79e-2)−	5.71e-1(2.97e-2)−	7.83e-1(1.09e-2)−	2.62e-1(1.57e-2)−	2.00e-1(1.03e-2)−	8.23e-1(1.01e-2)
	200	7.19e-1(1.13e-2)−	8.08e-1(8.09e-3)−	6.72e-1(8.92e-3)−	8.10e-1(6.42e-3)−	4.18e-1(1.09e-2)−	3.77e-1(1.72e-2)−	8.27e-1(3.88e-3)
	500	7.81e-1(1.69e-3)−	8.19e-1(2.81e-3)−	7.45e-1(4.74e-3)−	8.18e-1(2.63e-3)−	6.31e-1(3.24e-3)−	6.07e-1(8.19e-3)−	8.33e-1(1.60e-3)
	1000	8.10e-1(1.43e-3)−	8.24e-1(1.58e-3)−	7.78e-1(4.44e-3)−	8.18e-1(1.90e-3)−	8.37e-3(5.04e-4)	7.09e-1(8.47e-3)−	8.37e-1(5.04e-4)
	2000	8.25e-1(6.80e-4)−	8.19e-1(7.88e-4)−	7.93e-1(5.30e-3)−	8.06e-1(1.38e-3)−	8.06e-3(1.38e-3)−	7.70e-1(4.91e-3)−	8.29e-1(9.95e-4)
LSMOP5	100	3.69e-1(6.57e-2)−	1.97e-1(1.82e-1)−	3.98e-1(4.35e-3)−	4.04e-1(1.02e-2)−	1.32e-1(9.63e-2)−	0.00e+0(0.00e+0)−	5.46e-1(1.73e-2)
	200	3.69e-1(6.55e-2)−	1.10e-1(8.62e-2)−	3.96e-1(5.32e-3)−	3.88e-1(2.07e-2)−	5.20e-3(1.07e-1)	0.00e+0(0.00e+0)−	5.20e-1(1.07e-1)
	500	3.83e-1(1.78e-4)−	9.08e-2(1.18e-4)−	3.90e-1(5.52e-3)−	3.66e-1(3.79e-2)−	1.43e-1(4.93e-2)−	0.00e+0(0.00e+0)−	5.54e-1(9.36e-4)
	1000	3.54e-1(9.02e-2)−	8.99e-2(4.02e-3)−	3.83e-1(8.24e-3)−	2.79e-1(2.98e-2)−	1.89e-1(7.71e-2)−	0.00e+0(0.00e+0)−	5.50e-1(2.61e-2)
	2000	3.81e-1(1.35e-3)−	9.07e-2(4.29e-4)−	3.43e-1(4.28e-3)−	2.59e-1(2.66e-2)−	9.09e-2(3.79e-8)−	0.00e+0(0.00e+0)−	5.46e-1(1.04e-2)
LSMOP6	100	3.53e-2(1.99e-2)+	1.99e-2(2.94e-2)≈	5.93e-3(4.88e-3)≈	2.05e-2(2.82e-2)≈	1.36e-2(2.63e-2)≈	0.00e+0(0.00e+0)−	2.07e-2(2.24e-2)
	200	4.67e-2(1.22e-2)+	7.13e-3(2.64e-2)≈	2.22e-3(4.52e-3)+	7.25e-3(1.58e-2)≈	7.13e-3(2.15e-2)≈	0.00e+0(0.00e+0)≈	2.84e-5(1.27e-4)
	500	5.87e-2(2.56e-2)+	2.36e-2(5.06e-2)≈	2.64e-3(4.57e-3)−	0.00e+0(0.00e+0)≈	4.74e-3(2.12e-2)≈	0.00e+0(0.00e+0)≈	6.50e-3(2.44e-2)
	1000	6.62e-2(2.34e-2)+	6.58e-2(7.32e-2)≈	5.16e-3(5.48e-3)≈	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	0.00e+0(0.00e+0)−	4.31e-2(7.93e-2)
	2000	6.92e-2(2.69e-2)+	0.00e+0(0.00e+0)≈	1.19e-2(4.51e-3)+	0.00e+0(0.00e+0)≈	1.74e-2(2.91e-2)+	0.00e+0(0.00e+0)≈	0.00e+0(0.00e+0)
LSMOP7	100	1.37e-2(3.33e-2)−	9.09e-2(1.48e-6)+	0.00e+0(0.00e+0)−	9.45e-3(1.87e-2)−	9.08e-2(1.87e-5)+	0.00e+0(0.00e+0)−	8.98e-2(2.34e-3)
	200	1.34e-2(3.29e-2)−	9.05e-2(1.34e-3)+	0.00e+0(0.00e+0)−	7.52e-2(3.41e-2)−	9.08e-2(5.63e-6)+	0.00e+0(0.00e+0)−	9.03e-2(1.15e-3)
	500	3.57e-2(3.99e-2)−	8.47e-2(1.10e-2)≈	0.00e+0(0.00e+0)−	1.90e-1(4.91e-2)+	9.08e-2(1.65e-6)+	0.00e+0(0.00e+0)−	9.08e-2(2.24e-5)
	1000	5.72e-2(3.05e-2)−	8.72e-2(5.55e-3)≈	1.51e-2(5.77e-3)−	1.99e-1(6.60e-2)+	9.09e-2(1.99e-7)−	0.00e+0(0.00e+0)−	9.72e-2(2.89e-2)
	2000	7.91e-2(1.49e-2)−	8.51e-2(2.74e-3)−	4.90e-2(5.77e-3)−	8.20e-2(2.87e-2)≈	9.08e-2(1.02e-6)+	0.00e+0(0.00e+0)−	9.05e-2(4.57e-4)
LSMOP8	100	4.26e-1(4.26e-2)−	7.58e-2(1.30e-2)−	3.89e-1(1.48e-2)−	3.69e-1(1.16e-2)−	9.37e-2(3.08e-2)−	0.00e+0(0.00e+0)−	5.42e-1(1.69e-2)
	200	4.62e-1(3.51e-2)−	7.96e-2(7.07e-3)−	3.87e-1(1.52e-2)−	3.68e-1(9.80e-3)−	8.80e-2(6.86e-4)−	2.69e-3(2.53e-3)−	5.55e-1(2.76e-3)
	500	4.93e-1(1.51e-2)−	8.88e-2(1.91e-3)−	4.12e-1(8.69e-3)−	3.88e-1(9.57e-3)−	9.07e-2(1.23e-4)−	5.26e-2(1.51e-3)−	5.57e-1(1.35e-3)
	1000	4.75e-1(9.09e-2)−	8.97e-2(7.03e-4)−	4.24e-1(8.94e-3)−	3.93e-1(3.68e-3)−	9.07e-2(5.13e-4)−	6.88e-2(1.33e-3)−	5.58e-1(5.60e-4)
	2000	4.41e-1(1.20e-1)−	9.01e-2(5.44e-4)−	4.17e-1(1.00e-2)−	3.93e-1(2.46e-3)−	9.08e-2(2.64e-4)−	7.72e-2(8.81e-4)−	5.56e-1(9.14e-4)
LSMOP9	100	7.05e-2(5.37e-3)−	1.18e-1(8.50e-2)−	1.91e-1(1.03e-4)−	1.49e-1(9.94e-3)−	8.28e-2(1.23e-2)−	0.00e+0(0.00e+0)−	2.50e-1(1.29e-2)
	200	9.12e-2(4.84e-2)−	1.14e-1(7.90e-2)−	1.91e-1(3.37e-4)−	1.54e-1(1.63e-2)−	7.99e-2(1.54e-2)−	0.00e+0(0.00e+0)−	2.51e-1(1.26e-2)
	500	5.41e-2(6.22e-3)−	1.21e-1(8.62e-2)−	1.90e-1(3.70e-4)−	1.47e-1(2.98e-6)−	8.02e-2(7.13e-3)−	0.00e+0(0.00e+0)−	2.51e-1(1.27e-2)
	1000	6.12e-2(2.97e-2)−	6.59e-2(3.46e-2)−	1.90e-1(3.89e-4)−	9.11e-2(1.64e-5)−	7.98e-2(4.22e-3)−	0.00e+0(0.00e+0)−	2.40e-1(2.47e-2)
	2000	5.96e-2(3.11e-2)−	5.38e-3(1.31e-2)−	1.90e-1(3.20e-4)−	1.49e-1(6.82e-3)−	8.23e-2(2.48e-3)−	0.00e+0(0.00e+0)−	2.37e-1(2.51e-2)
+/−/≈		11/34/0	3/33/9	3/37/5	4/35/6	9/27/4	0/42/3	

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that of LSTPA, respectively.

TABLE III: HV values of the compared algorithms on 2-objective DTLZ7, MaF11 and LSMOP9 corresponding to the median run, where the best results on each test instance is shown in gray background. A larger HV value indicates better performance.

No.	Dim.	DGEA	LMOCSO	LMOEA-DS	Two_Arch2	TriMOEA-TA&R	CCGDE3	LSTPA
DTLZ7	100	2.42e-1(1.55e-5)−	2.38e-1(1.48e-2)−	2.41e-1(2.21e-4)−	1.94e-1(9.97e-2)−	2.42e-1(2.34e-5)+	0.00e+0(0.00e+0)−	2.42e-1(1.05e-6)
	200	2.42e-1(4.85e-6)≈	2.42e-1(6.30e-5)−	2.41e-1(2.92e-4)−	2.42e-1(3.09e-3)−	2.42e-1(2.28e-5)+	0.00e+0(0.00e+0)−	2.42e-1(1.65e-5)
	500	2.42e-1(3.82e-6)≈	2.42e-1(4.49e-5)−	2.41e-1(2.79e-4)−	1.82e-1(1.08e-1)−	2.42e-1(2.65e-5)+	0.00e+0(0.00e+0)−	2.42e-1(5.31e-5)
	1000	2.42e-1(1.39e-6)+	2.42e-1(6.87e-5)−	2.41e-1(2.05e-4)−	1.82e-1(1.08e-1)−	2.42e-1(1.81e-5)+	0.00e+0(0.00e+0)−	2.42e-1(7.06e-5)
	2000	2.42e-1(6.04e-6)+	2.42e-1(5.03e-5)−	2.41e-1(2.25e-4)−	1.82e-1(1.08e-1)−	2.42e-1(2.13e-5)+	0.00e+0(0.00e+0)−	2.42e-1(8.28e-5)
MaF11	100	5.79e-1(9.56e-3)−	6.01e-1(3.70e-3)−	6.20e-1(2.89e-3)+	6.11e-1(5.76e-3)−	6.27e-1(6.44e-4)+	4.57e-1(1.07e-2)−	6.16e-1(3.36e-3)
	200	5.68e-1(7.27e-3)−	5.95e-1(3.94e-3)−	6.18e-1(3.29e-3)+	5.84e-1(7.66e-3)−	6.22e-1(8.25e-4)+	4.43e-1(9.82e-3)−	6.04e-1(5.09e-3)
	500	5.66e-1(6.44e-3)−	5.87e-1(2.78e-3)≈	6.15e-1(3.77e-3)+	5.48e-1(6.90e-3)−	5.89e-1(7.91e-3)≈	4.27e-1(1.11e-2)−	5.87e-1(3.38e-3)
	1000	5.65e-1(4.60e-3)−	5.86e-1(3.00e-3)≈	6.17e-1(3.40e-3)+	5.27e-1(5.98e-3)−	5.41e-1(6.36e-3)−	4.29e-1(1.19e-2)−	5.85e-1(3.32e-3)
	2000	5.62e-1(3.78e-3)−	5.90e-1(3.70e-3)+	6.19e-1(3.09e-3)+	5.09e-1(5.21e-3)−	5.19e-1(4.98e-3)−	4.24e-1(1.02e-2)−	5.78e-1(1.64e-3)
LSMOP9	100	2.33e-1(1.44e-2)+	2.36e-1(2.13e-3)+	9.09e-2(4.27e-17)−	9.09e-2(3.45e-16)−	7.59e-2(2.41e-2)−	0.00e+0(0.00e+0)−	2.25e-1(4.34e-2)
	200	2.24e-1(1.81e-2)−	2.32e-1(7.97e-3)−	9.33e-2(1.09e-2)−	9.09e-2(9.79e-16)−	8.55e-2(2.56e-2)−	0.00e+0(0.00e+0)−	2.41e-1(5.57e-4)
	500	1.82e-1(1.53e-2)−	2.33e-1(4.70e-3)−	1.51e-1(2.00e-2)−	9.09e-2(4.22e-6)−	8.59e-2(7.19e-4)−	0.00e+0(0.00e+0)−	2.41e-1(2.90e-4)
	1000	2.03e-1(7.06e-3)−	2.35e-1(2.50e-3)−	1.77e-1(1.48e-2)−	9.11e-2(1.64e-5)−	8.32e-2(2.06e-4)−	0.00e+0(0.00e+0)−	2.41e-1(4.74e-4)
	2000	2.04e-1(3.89e-2)−	2.25e-1(5.54e-4)−	1.85e-1(1.16e-2)−	9.13e-2(1.26e-5)−	8.30e-2(3.99e-4)−	0.00e+0(0.00e+0)−	2.31e-1(9.98e-4)
+/−/≈		3/10/2	2/11/2	5/10/0	0/15/0	7/7/1	0/15/0	

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that of LSTPA, hereinafter

TABLE IV: HV values of the compared algorithms on 3-objective DTLZ7, MaF11 and LSMOP9 corresponding to the median run, where the best result on each test instance is shown in gray background. A larger HV value indicates better performance.

No.	Dim.	DGEA	LMOCSO	LMOEA-DS	Two_Arch2	TriMOEA-TA&R	CCGDE3	LSTPA
DTLZ7	100	2.54e-1(3.06e-4)−	2.54e-1(1.46e-4)−	2.68e-1(1.82e-3)+	2.79e-1(6.30e-4)+	2.69e-1(4.70e-4)+	0.00e+0(0.00e+0)−	2.55e-1(1.33e-4)
	200	2.54e-1(3.10e-4)−	2.54e-1(2.74e-4)−	2.67e-1(1.47e-3)+	2.77e-1(8.00e-4)+	2.69e-1(6.38e-4)+	0.00e+0(0.00e+0)−	2.55e-1(4.05e-5)
	500	2.54e-1(2.49e-4)−	2.54e-1(2.88e-4)−	2.66e-1(2.02e-3)+	2.79e-1(6.27e-4)+	2.69e-1(4.32e-4)+	0.00e+0(0.00e+0)−	2.55e-1(6.06e-5)
	1000	2.54e-1(2.66e-4)−	2.54e-1(2.55e-4)−	2.65e-1(1.59e-3)+	0.00e+0(0.00e+0)−	2.69e-1(7.50e-4)+	0.00e+0(0.00e+0)−	2.55e-1(2.60e-5)
	2000	2.54e-1(3.86e-4)−	2.54e-1(1.49e-4)−	2.64e-1(2.42e-3)+	2.79e-1(5.61e-4)+	2.69e-1(5.99e-4)+	0.00e+0(0.00e+0)−	2.55e-1(5.35e-5)
MaF11	100	8.24e-1(1.13e-2)−	8.90e-1(5.90e-3)−	8.75e-1(3.38e-3)−	9.13e-1(4.99e-3)+	9.23e-1(1.00e-3)+	6.82e-1(2.11e-2)−	8.95e-1(5.52e-3)
	200	8.12e-1(8.68e-3)−	8.84e-1(3.71e-3)≈	8.65e-1(8.09e-3)−	8.81e-1(7.16e-3)≈	9.15e-1(2.19e-3)+	6.73e-1(1.41e-2)−	8.86e-1(5.57e-3)
	500	7.89e-1(1.04e-2)−	8.75e-1(5.29e-3)−	8.59e-1(7.66e-3)−	8.53e-1(5.91e-3)−	8.52e-1(6.33e-3)−	6.51e-1(1.25e-2)−	8.78e-1(4.36e-3)
	1000	7.71e-1(8.72e-3)−	8.72e-1(3.75e-3)−	8.53e-1(8.44e-3)−	8.25e-1(4.20e-3)−	8.11e-1(4.55e-3)−	6.37e-1(1.02e-2)−	8.77e-1(5.71e-3)
	2000	7.61e-1(1.04e-2)−	8.68e-1(4.69e-3)−	8.55e-1(5.26e-3)−	8.09e-1(4.90e-3)−	7.84e-1(6.11e-3)−	6.31e-1(8.50e-3)−	8.83e-1(8.86e-3)
LSMOP9	100	7.05e-2(5.37e-3)−	1.18e-1(8.50e-2)−	1.91e-1(1.03e-4)−	1.49e-1(9.94e-3)−	8.28e-2(1.23e-2)−	0.00e+0(0.00e+0)−	2.50e-1(1.29e-2)
	200	9.12e-2(4.84e-2)−	1.14e-1(7.90e-2)−	1.91e-1(3.37e-4)−	1.54e-1(1.63e-2)−	7.99e-2(1.54e-2)−	0.00e+0(0.00e+0)−	2.51e-1(1.26e-2)
	500	5.41e-2(6.22e-3)−	1.21e-1(8.62e-2)−	1.90e-1(3.70e-4)−	1.47e-1(2.98e-6)−	8.02e-2(7.13e-3)−	0.00e+0(0.00e+0)−	2.51e-1(1.27e-2)
	1000	6.12e-2(2.97e-2)−	6.59e-2(3.46e-2)−	1.90e-1(3.89e-4)−	1.52e-1(1.37e-2)−	7.98e-2(4.22e-3)−	0.00e+0(0.00e+0)−	2.40e-1(2.47e-2)
	2000	5.96e-2(3.11e-2)−	5.38e-3(1.31e-2)−	1.90e-1(3.20e-4)−	1.49e-1(6.82e-3)−	8.23e-2(2.48e-3)−	0.00e+0(0.00e+0)−	2.37e-1(2.51e-2)
+/−/≈		0/15/0	0/14/1	5/10/0	5/9/1	7/8/0	0/15/0	