Lecture: engineering interesting dispersion relations for ultracold atoms

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Various condensed-matter phenomena rely on "interesting" single-particle structure: e.g., the quantum Hall effect, topological edge states, charge-density-wave and spin-density-wave instabilities due to nesting, etc. To realize these phenomena using cold atoms, one must first be able to engineer the appropriate single-particle Hamiltonians. This lecture introduces some tools that can be used to this end. I will primarily talk about recent experimental advances in creating magnetic fields and spin-orbit coupling for neutral matter [1], but the ideas are more general and can also be used to explore questions that have nothing to do with gauge fields or topology.

LIGHT-MATTER INTERACTIONS

We are looking for ways to manipulate the motion of single atoms using the light-matter interaction. The basic situation is always as follows (Fig. 1a): you are given an atom with some set of "low-lying" energy levels (henceforth "internal ground states") and some set of highly excited energy levels. Lasers can be used to drive transitions between the internal ground states and the highly excited levels. In general one wants to drive these transitions off-resonantly (for reasons that will be clear in a minute) so that the excited levels are used only as virtual intermediate states. These levels can then be "adiabatically eliminated," leaving us with a ground-states-only problem.

Simplest case: dipole trap/optical lattice

Consider an atom with one ground state $|g\rangle$ and one excited state $|e\rangle$, separated by a frequency ω_A. Thus $H_0 = \omega_A |e\rangle\langle e|$. Let us drive the transition with a monochromatic laser that has matrix elements $V = \Omega(|g\rangle\langle e| + h.c.)\cos(\omega_L t)$. Let $\Delta \equiv \omega_L - \omega_A$ and assume $|\Delta| \ll \omega_L, \omega_A$. There is a standard way to deal with this problem by "transforming into the rotating frame," but I prefer a slightly different perspective (sometimes called the "dressed state" picture) [2].

Let us use the fact that many-photon coherent states are sharply peaked at some average number, and move to a number basis, "requantizing" the interaction term. Thus, $V \simeq f(x)(a^\dagger + a)(|g\rangle\langle e| + h.c.)$ and the laser is a quantum field with the Hamiltonian $\omega_L a^\dagger a$. When $\omega_L \simeq \omega_A$ (rotating-wave limit), the eigenstates of the light-matter system split up into near-degenerate (w.r.t ω_L) 2×2 manifolds with photon and atom states $|N, g\rangle, |N-1, e\rangle$. These are connected by matrix elements $\sim f(x)\sqrt{N} \equiv \Omega(x)$ and have an internal splitting Δ. When $\Delta > \Omega$ the excited state only affects the initial state perturbatively. Thus, the total energy of the system when the atom is at a position x is $\Omega(x)^2/\Delta$, which is called the "light shift." As long as the atoms are moving sufficiently slowly compared with Δ, the light shift can be treated as a quasistatic potential.

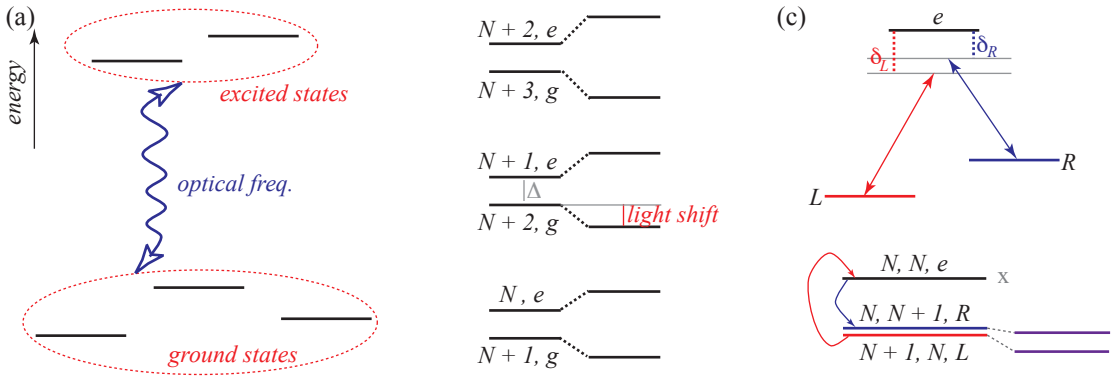

FIG. 1. (a) General level structure. (b) Dressed-state picture for a two-level atom. (c) Three-level atom setup and dressed-state picture, before and after adiabatic elimination of the excited state.

Three-level atom, two lasers

Now let us take an atom with three states: two low-lying states L, R and an excited state e. We now drive the system with two lasers arranged as in Fig. 1c. The global state is specified by N_L, N_R and the atomic state (N_L and N_R are the photon numbers in the relevant electromagnetic modes); the global spectrum splits into 3×3 submanifolds, spanned by $|N_L, N_R, L\rangle, |N_L - 1, N_R, e\rangle, |N_L - 1, N_R + 1, R\rangle$. (Other states are decoupled either by having large denominators or by having zero matrix element—once the atom is in R it can only scatter back into L.) Perturbatively eliminating the excited state (we assume its detuning is large), we get the reduced 2×2 Hamiltonian

$$H_{eff} = H_0 + \frac{g_L(x)g_R(x)}{\Delta}|N_L, N_R, L\rangle\langle N_L - 1, N_R + 1, R| + h.c. \tag{1}$$

$$+\frac{g_L(x)^2}{\Delta}|N_L, N_R, L\rangle\langle N_L, N_R, L| + \frac{g_R(x)^2}{\Delta}|N_L - 1, N_R + 1, R\rangle\langle N_L - 1, N_R + 1, R| \tag{2}$$

where $H_0 = E_0(|N_L, N_R, L\rangle\langle N_L, N_R, L| - |N_L - 1, N_R + 1, R\rangle\langle N_L - 1, N_R + 1, R|)$ is the (bare) detuning between the two atom + light levels. Evidently this way of labeling the states is highly redundant, so from here on we shall be more economical and label these states $|\mathcal{L}\rangle, |\mathcal{R}\rangle$.

In some form or other, the idea of driving two-photon transitions (as in line 1 of the above equation) through virtual intermediate states is central to all the dispersion-engineering protocols that we shall discuss.

More levels

This analysis generalizes directly to the case of more levels either downstairs or upstairs. Having more ground-state levels just gives us more possibilities for two-photon transitions, whereas having more excited levels gives us more intermediate states to sum over. In the general case, where the ground states are labeled i and the excited levels α, one ends up with a general Hamiltonian of the form

$$H_{eff} = H_0(i) + \frac{1}{\Delta_\alpha}\sum_{ij,\alpha} g_i^\alpha g_j^\alpha |i\rangle\langle j| \tag{3}$$

where $H_0(i)$ consists of terms in the original Hamiltonian that acted only on the reduced manifold. Note that in practice the "slow" states in this procedure are hyperfine sublevels of the atomic ground state. The reason is that these are much longer-lived than optically excited states, which tend to have lifetimes $\sim 1\mu s$ (though there are exceptions). Note also that the sum over α is not to be taken cavalierly; destructive interference is often important.

GAUGE FIELDS FOR NEUTRAL ATOMS: NIST APPROACH

So far the most important application of these ideas has been to create gauge fields and spin-orbit coupling for neutral atoms. The motivation is of course that neutral atoms do not experience a Lorentz force, which is inconvenient if you want to use them to simulate the quantum Hall effect etc. Our objective is to engineer an effective Lorentz force, i.e., to massage the atomic dispersion until it looks like $(\mathbf{p} - e\mathbf{A})^2$, where $\mathbf{A}$ is a vector potential (in some gauge) whose curl is the desired magnetic field. For example in 2D you can get a constant magnetic field by picking $A_x \sim By$, $A_y = 0$ (Landau gauge). NB there is no underlying physical basis for "gauge invariance," so with ultracold atomic experiments one is free to measure gauge-dependent quantities if one likes.

We now turn to the Spielman group's pioneering experiments [3, 4] realizing a magnetic field in 2D. One can split the problem up into two parts: (1) the problem of creating a vector potential, i.e., a 1D dispersion of the form $(p-\alpha)^2$ and (2) the problem of making α vary spatially.

Vector potentials through Raman transitions

One uses Raman transitions between two hyperfine ground states of Rb. The lasers generating the Raman transition are at an angle to one another (Fig. 2a) so that the Raman transition imparts a momentum kick along x. In our terminology, $g_L \sim e^{i\mathbf{k_1}\cdot\mathbf{x}}, g_R \sim e^{-i\mathbf{k_2}\cdot\mathbf{x}}$, so the Raman coupling takes the form

$$\Omega \exp(i\mathbf{K}\cdot\mathbf{x})|\mathcal{L}\rangle\langle\mathcal{R}| + h.c. \tag{4}$$

In other words, whenever an atom undergoes a Raman transition, it gets a fixed-magnitude momentum kick. We choose axes such that $\mathcal{K} = k_L\hat{\mathbf{x}}$. The total Hamiltonian in the low-energy sector is then

$$H_{eff} = \frac{p_x^2}{2M}\mathbb{I} + \beta(|\mathcal{L}\rangle\langle\mathcal{L}| - |\mathcal{R}\rangle\langle\mathcal{R}|) + \Omega\sum_{p_x}|p_x,\mathcal{L}\rangle\langle p_x + k_L,\mathcal{R}| + h.c. \tag{5}$$

(Some uniform Stark-shift terms have been omitted.) Evidently each $p,\mathcal{L}$ only hybridizes with $p+k_L,\mathcal{R}$, so you can solve for the dispersion as a function of Ω,β. For small Ω, hybridization is only effective for $p \simeq k_L/2$, and one has a double-well dispersion (we will come back to this), but for large Ω, the dispersion is a flat-bottomed parabolic shape and its minimum is sensitive to β (Fig 2h).

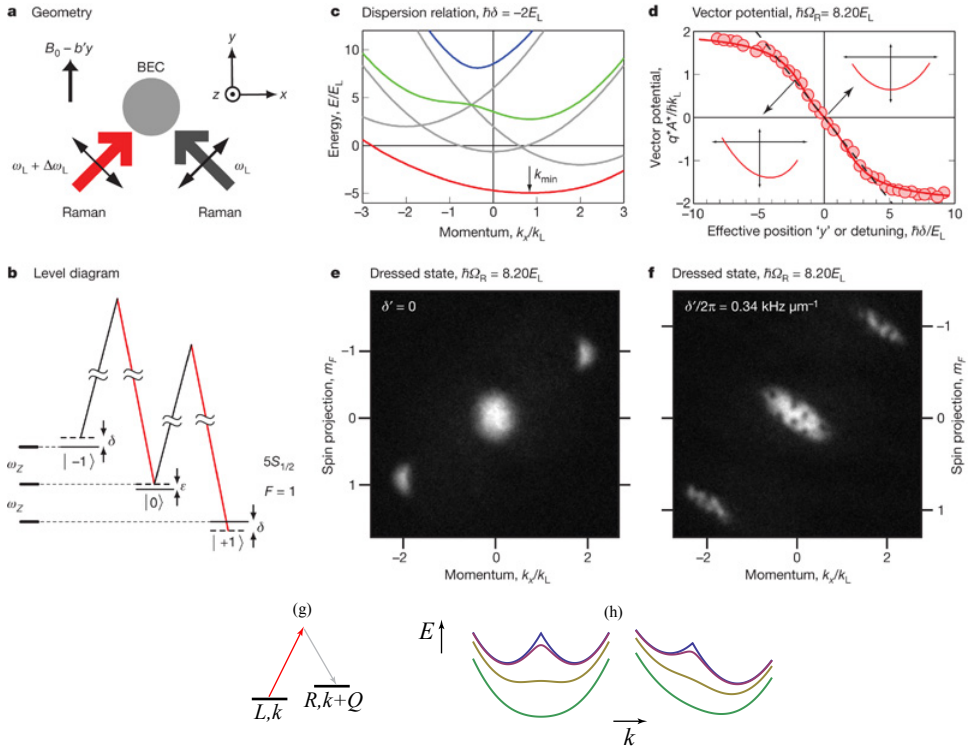

FIG. 2. Parts (a)-(f) are reproduced from Ref. [3]. (a) shows the orientations of the lasers. (b) shows the level structure used in the experiment. For technical reasons there are three rather than two levels downstairs but this does not change anything important. (c) shows the resulting dispersion—note that as the intensity of the Raman beams increases the dispersion minimum moves inwards. (d) Vector potential gradient (leading to magnetic field). (e) shows a BEC loaded into the NIST setup with a constant vector potential, and (f) shows what happens when the vector potential has a gradient. The vortices are a telltale sign that the BEC is in an effective field. (g) shows the Raman-coupled levels in the simplified model discussed in the text, and (h) shows the evolution of the dressed dispersion, both when one is on two-photon resonance (left) and when one is detuned from two-photon resonance (right).

From vector potentials to magnetic fields

This step is fairly simple. The two internal ground states are typically split by a Zeeman field. By introducing a field gradient, therefore, one can tune β and thereby realize a magnetic field. A particularly clear demonstration: a Bose-Einstein condensate loaded into the Spielman setup naturally develops an Abrikosov vortex lattice (Fig. 2f).

Limitations

The main drawback of this setup is that it is prone to heating. We did not discuss this but it is well known that if you drive a transition at a detuning Δ, the heating rate is $\sim 1/\Delta^2$. Normally this is parametrically smaller at large detuning than the couplings $\sim 1/\Delta$. However, because in this case the Raman transitions going through the two intermediate virtual states interfere destructively (see Ref. [1] sec. 4.4.1), the total Raman coupling at detunings larger than the fine-structure splitting (a few nm) actually falls off as $1/\Delta^2$. Thus, one cannot parametrically separate the heating rate and the coupling, and as a consequence the lifetime of NIST-type experiments (in units of the Raman coupling) is inherently limited.

FREEBIES: "ABELIAN" SPIN-ORBIT COUPLING

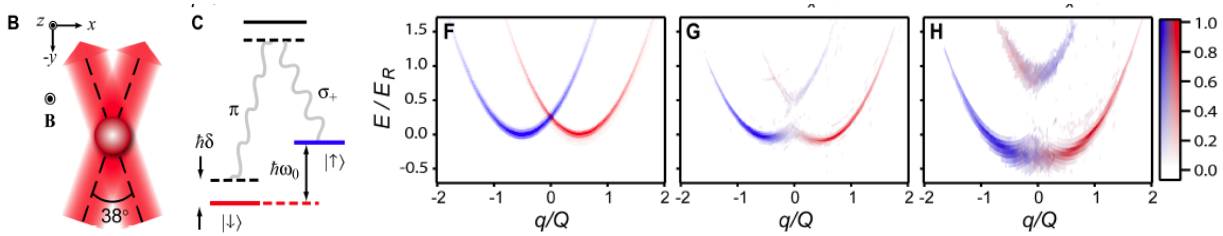

FIG. 3. B,C show experimental setup used in Ref. [5] to realize SOC with fermionic lithium. F-H show spin-dependent dispersion relations as measured through spin-injection spectroscopy (see Ref. [5] for details). Blue regions are spin-up, red regions are spin-down.

A feature of the NIST gauge-field setup is that, almost as a byproduct, it realizes a dispersion with two "spin"-split parabolas. This is a form of spin-orbit coupling ($H \sim p^2 + p_x\sigma_x$) analogous to that in 1D Rashba-coupled wires, which are candidate systems for realizing Majorana fermions. Although the first (NIST) experiments on these systems involved bosons, a Chinese group from Shanxi and Tsinghua Univs. and the Zwierlein group at MIT have demonstrated a similar Hamiltonian for fermions (Li-6). The MIT group in particular were able to load fermions adiabatically into the lowest SOC band and to map out the bands spectroscopically.

There have been a lot of theory papers on nonabelian (e.g., pure Rashba) SOC but as far as I know none of these have proved experimentally realizable. To get a flavor of these let us look very briefly at a proposal by Campbell et al. [6], illustrated in the following figure.

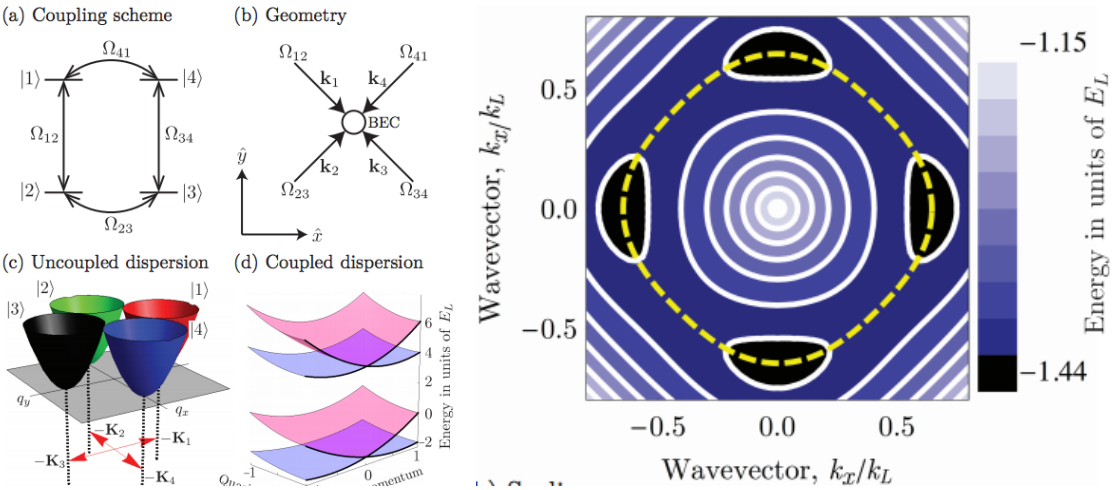

FIG. 4. Scheme for pure Rashba SOC. Numerical dispersion calculations (right panel) show that the dispersion minimum is vaguely circular with some fourfold anisotropy.

Realizing a pure Rashba SOC is interesting for a number of reasons. For pure Rashba coupling $H = p^2/M - (\mathbf{p} \times \sigma) \cdot \mathbf{z}$, the ground state would have a degenerate minimum in momentum space at some momentum p_0. It is currently somewhat unclear what the ground state of interacting bosons with this dispersion relation should be. Unlike the usual situation (in which bosons condense into the lowest single-particle state), here the lowest single-particle state is highly degenerate. Interactions play a crucial role in breaking this degeneracy; however, whether they give rise to some Bose-condensed ground state or to an exotic, non-condensed, possibly fractionalized phase is not yet conclusively established.

GAUGE FIELDS IN LATTICES

One can approach a band-structure problem perturbatively from one of two limits: the free-electron/momentum-space limit, in which one starts with plane waves and then adds backscattering, or the tight-binding/position-space limit, in which one begins with atomic orbitals and then adds hybridization. Similarly there are two conceptual approaches to realizing gauge fields: one of them (the NIST approach) begins with free particles; the other begins with single-site localized particles, and reintroduces hopping in a controlled way. A "magnetic field" on a lattice is specified through its Aharonov-Bohm phases, i.e., by specifying the flux through every plaquette. We now discuss how this flux can be engineered.

Tool: photon-assisted tunneling

The way this is done is photon-assisted tunneling. Let us first discuss this for a simple two-site problem. Take the two sites to have some hopping J and some static offset $\Delta \gg J$. Then the wavefunctions take the form $|1\rangle - (J/\Delta)|2\rangle$ and $|2\rangle + (J/\Delta)|1\rangle$. Imagine driving site 1 with a two-photon transition of frequency $\sim \Delta$, thus driving resonant transitions between the two sites. [Why a two-photon transition? (1) The detuning between neighboring site energies is in the kHz-MHz range so a single optical photon is far too energetic. (2) Microwaves/rf pulses might have the right frequency but their wavelength is far too long to be useful.] One sees that the matrix element for this transition is given by $\Omega(x)J/\Delta$ where Ω is the matrix element (Rabi frequency) of the two-photon transition. You can think of the eigenstates as (no photons, 1) $\pm$ (1 photon, 2). A key observation is that with this setup it is easy to make the hopping amplitudes complex: you just use a traveling wave to stimulate the transitions.

Staggered-flux lattice

Although the most dramatic success of the real-space approach has been realizing the Hofstadter Hamiltonian, a much more transparent version of the scheme is discussed in Ref. [7]. In that work, a *staggered* flux lattice was realized; the complexities of the new scheme arise from having to rectify this staggered flux.

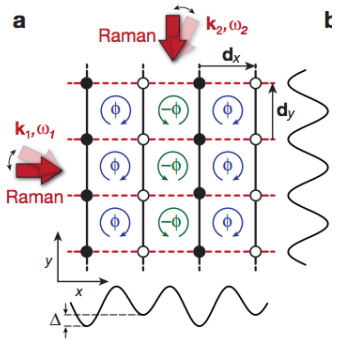

FIG. 5. Setup for realizing staggered magnetic flux lattices [7]. Hopping is free along y but is inhibited by a superlattice along x; hopping is restored via a two-photon process that compensates the detuning. As discussed in the text, the effective magnetic flux forms stripes parallel to the y axis.

For the staggered flux lattice one starts with a lattice where hopping is free along one direction (say y) but suppressed by a superlattice with offset Δ in the other direction. (Tunneling by two sites is of course still allowed but has an amplitude J^2/Δ which will turn out to be small.) Now we turn on two traveling-wave laser beams propagating at an angle θ with respect to one another. Let us simplify and take one to be along the y axis and the other along the x axis. Then, at some given point along the lattice, the amplitude $\Omega(x)$ for the two-photon process is $\sim e^{ik_1 x - ik_2 y}$ (say for the downhill direction; the uphill transition naturally has the conjugate phase). If we fix x, then the phase of the hopping term oscillates as a function of y. Note that photon-assisted tunneling is irrelevant along vertical bonds. Thus, the total flux through a unit plaquette is $e^{2ik_2 y}$, which is nonzero and can be as large as π.

If you prefer, you can also think about these systems as being subject to a periodic potential *moving* through the lattice (i.e., a "shaken optical lattice"). This point of view is taken in Ref. [7].

The main drawback of this setup as described is that the flux is spatially modulated along x. (However it is uniform along y, so one can create two-leg ladders with constant flux.) This spatially modulated flux can be rectified as follows.

Uniform-flux lattice

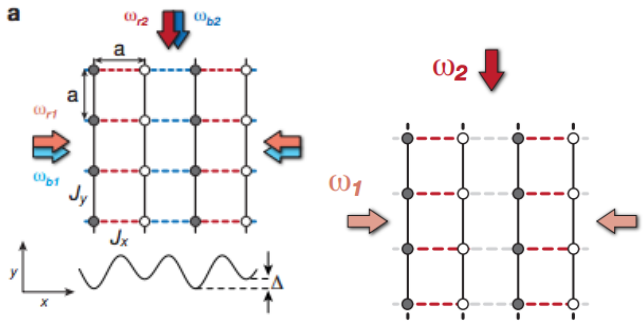

FIG. 6. Technique for rectifying the flux lattice [9]. A set of red lasers induces laser-assisted tunneling that is large on half the bonds, whereas a set of blue lasers induces laser-assisted tunneling that is large on the other half. As the right panel suggests, when the laser-assisted tunneling is absent one finds that there is no tunneling across the gray bonds.

There is a simple way to do this which is to use a uniform gradient along x instead of a superlattice [8]. However, this has various scalability problems. Instead, the Bloch group have developed an ingenious way of rectifying the staggered-flux lattice [9]. One uses the same superlattice but instead of using a two-photon transition with traveling waves, one uses a standing wave along x. It is helpful to exaggerate a little and take this standing wave's periodicity such that it has nodes at *every other bond* along x. Now at an antinodal plaquettes the same staggered-flux argument analysis goes through and gives us a ladder of uniform flux, but at the nodal plaquettes there is no flux at all. This is "half-wave" rectification; it gives us a uniform flux at half the sites and no flux at the other half. Now one can do the same thing with a different standing-wave/traveling-wave pair that only affects the other half, such that the detunings are opposite or the traveling wave is traveling in the opposite direction. This gives us a completely rectified flux pattern.

Spin-dependent flux lattice

Finally, one can take this uniform flux lattice idea and put the internal-state structure back in. This leads to a spin-dependent uniform flux lattice where opposite spins experience opposite fields, and thus (in the topological regime) have counterpropagating edge modes. This was done by the Ketterle group [10]. It is a sort of "quantum spin-Hall effect", but a somewhat special limit of this in which the topological index is $Z \times Z$ rather than Z_2. The most surprising prediction of Kane and Mele [11] is that edge states should persist in this model even if one adds some specific spin-flip terms (which correspond in the condensed matter context to Rashba couplings). It would be exciting if the Ketterle setup could be adapted to test this prediction, but to my knowledge this hasn't yet been done.

COMPARISONS/CHALLENGES

The momentum-space and position-space approaches described above are complementary. The momentum-space approach allows us to realize quantum Hall systems that are relatively close to condensed matter. These are essentially "continuum" quantum Hall states, in which the underlying lattice does not play an important role (in other words, the flux per unit plaquette is tiny). In this regime, the quantum Hall state is almost rotation-invariant—this near-rotation-invariance allows one to realize interesting phenomena such as quantum Hall nematics, magnetorotons [12], etc. On the other hand, the flux-lattice approach has the virtue of pushing condensed-matter ideas beyond the accessible parameter regime in condensed-matter systems, and thus perhaps has more "new physics." Moreover, although the (fractional/integer) QHE and the Chern insulator are essentially similar states, there are some differences: for example, a Chern insulator such as the Hofstadter model can have bands with Chern numbers larger than one, whereas this is not true with the QHE. There are some interesting ideas [13] about the unusual excitations ("genons") of fractional Chern insulators derived from partially filled bands with higher Chern numbers.

A major challenge for all of these proposals is heating. Let us briefly comment on this. There are actually two distinct issues here. The first is that there are vacuum electromagnetic modes into which the excited levels can radiate. These radiative decoherence processes typically go as $1/\Delta^2$ where Δ is the detuning from the dressing levels. If one took the unrealistic limit of a strictly monochromatic electromagnetic field, these processes would not matter. However, there are also processes involving the absorption of one photon by a large number of atomic excitations (the "many-body" continuum). In generic interacting systems driven at finite frequencies, it is believed that these processes always eventually heat the system to infinite temperature, but of course this might take a very long time.

[1] N. Goldman, G. Juzeliunas, P. Ohberg, I. B. Spielman, arXiv:1308.6533

[2] The issue here is that because the system is subject to a time-dependent drive, the notion of "energies" is not well-defined; yet we want to use the language of energy shifts to talk about the physics. There are three conceptually different ways of making sense of this. The first is the semiclassical perspective (used in Metcalf and Van der Straten and other laser-cooling books) in which the atom is regarded as a driven classical dipole; in this perspective the fundamental quantities are *forces* rather than energies but the dynamics can be thought of in terms of effective "energies." The second is the Floquet approach [see, e.g., Bukov et al., arXiv:1407.4803], in which one uses the time-periodic equivalent of Bloch's theorem to define "quasienergies" for periodically driven systems. These quasienergies are similar to regular energies for many purposes. The third approach is to use a full quantum mechanical description of the energy levels of the atoms and the light field.

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