Strongly correlated systems in atomic and condensed matter physics

Lecture notes

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Chapter 2

Atoms in external field. Magnetic and optical trapping of atoms

2.1 Atomic structure of alkali atoms

Alkali atoms have a single valence electron in the s-orbital. Electron orbital angular momentum $L = 0$ and electron spin $S = 1/2$. Hyperfine coupling mixes electron and nuclear spins

$$\mathcal{H} = A_{\mathrm{HFS}}\,\vec{I}\vec{S} \tag{2.1}$$

At zero field states are characterized by the total angular momentum

$$\vec{F} = \vec{I} + \vec{S} \tag{2.2}$$

Zero field splitting between $F = I + 1/2$ and $F = I - 1/2$ states

$$\Delta E_{hf} = (I + \frac{1}{2})A_{\mathrm{HFS}} \tag{2.3}$$

For ^{23}Na $A_{\mathrm{HFS}} = 1.8$ GHz and for ^{87}Rb $A_{\mathrm{HFS}} = 6.8$ GHz. Both have $I = 3/2$.

Effect of magnetic field comes from electron spin (Zeeman coupling to nuclear spin is negligible)

$$\mathcal{H} = A_{\mathrm{HFS}}\,\vec{I}\vec{S} + g_s\mu_B\,\vec{B}\vec{S} \tag{2.4}$$

with $g_S = 2$ and $\mu_B = 1.4$ MHz/G.

When magnetic field is not too large one can use (assuming field along z)

$$E_{\mathrm{Z}} = g_s\mu_B F_z B + qF_z^2B^2 \tag{2.5}$$

The last term is often referred to as quadratic Zeeman effect $q = h$ 390 Hz/G^2.

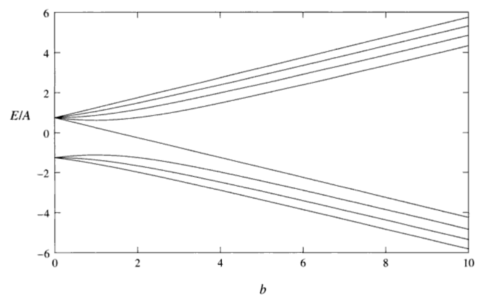

Figure 2.1: Energies of hyperfine levels of an alkali atom with $I = 3/2$ and $A_{\rm HFS} > 0$. The dimensionless magnetic field is $b = 4\mu_B B/A_{\rm HFS}$. Figure taken from [1].

2.2 Trapping of alkali atoms

Magnetic trapping of neutral atoms is due to the Zeman effect. The energy of an atomic state depends on the magnetic field. In an inhomogeneous field an atom experiences a spatially varying potential. For example, when magnetic field is

$$\vec{B} = B'(x, y, -2z) \tag{2.6}$$

the potential is

$$V_{\rm mag}(r) = g_s \mu_B |B(r)| = g_s \mu_B B' (x^2 + y^2 + 4z^2)^{1/2} \tag{2.7}$$

Magnetic trapping is limited by the requirement that the trapped atoms remain in weak field seeking states. For ^{23}Na and ^{87}Rb there are three states $|F = 1, m_F = +1\rangle$, $|F = 2, m_F = +1\rangle$, and $|F = 1, m_F = +2\rangle$.

Optical trapping of alkali atoms uses AC Stark effect. Oscillating electric field (typically at optical frequencies) induces electric dipolar moment

$$\vec{d}(\omega) = \alpha(\omega)\vec{E}(\omega) \tag{2.8}$$

Here $\alpha(\omega)$ is a frequency dependent polarizability. Averaging over fast oscillations of the electric field we find effective potential

$$V_{\rm ACS} = -\langle \vec{d}\vec{E} \rangle = -\frac{\alpha}{2} E^2(r) \tag{2.9}$$

Far-off-resonant optical traps confine atoms regardless of their hyperfine states.

The main lesson of this chapter is that one can use either magnetic coils or optical beams to create confining potential for the atoms. In most cases this potential can be approximated by the quadratic expansion near the minimum. So atoms in a parabolic potential is a good starting point for describing most experiments with ultracold atoms.

2.3 Problems for Chapter 2

Problem 1 Atomic microtraps.

a) A current carrying wire (I_w) and a homogeneous magnetic bias field (B_b) may produce a one-dimensional trapping potential for atoms in the weak-field seeking state. Assuming a simple geometry of a straight wire and the bias field orthogonal to the wire, calculate the form of the confining potential.

b) In the simple design discussed in part a) magnetic field at the center of the trap is zero. This can lead to the so-called Majorana transitions between trapped and untrapped spin states and atom losses. This problem can be circumvented by adding a small B-field component B_{ip} along along the wire direction which, which lifts the energetic degeneracy between the trapped and untrapped states. Show that in this case, the transverse confining potential is parabolic and calculate the confining frequency.

c)* (difficult). One of the problems of the first designs of microtraps was fragmentation of the BECs caused by the random potential. This random potential was traced to fluctuations of the wire width. Explain, why wire width fluctuations lead to random potential fluctuations and explain why experiments observed $d^{-5/2}$ scaling of this random potential with the distance between the trap and the wire.

Bibliography

[1] C.J. Pethick and H. Smith. *Bose-Einstein Condensation in Dilute Gases.* Cambridge University Press, 2002.