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Unique condition for generalized Laguerre functions to solve pole position problem

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A B S T R A C T

Previous research indicates that solution to pole optimization problem for the generalized Laguerre functions can be found by vanishing at least one of two clearly stated Laguerre coefficients. The aim of this paper is to prove uniqueness of a certain coefficient leading to the optimal solution. To achieve this purpose, we employed connection coefficients method to work out specific recurrence relations suitable for the continuous generalized Laguerre functions in the case of the optimal pole position. The proposed results were extended to the discrete Laguerre functions using modified bilinear transform and introducing the rational z-transform of the Meixner-like functions. The findings of this research present a postulated and proved theorem and conducted computational experiments to support the theoretical results.

1. Introduction

Clowes's work [1] laid the foundation for the pole optimization problem and stated that the solution to this problem attains by equating at least one of a pair of certain Laguerre coefficients to zero. In fact, this analytical solution has a major drawback concerned with the necessity for finding the roots of high-order polynomials, which can entail an enormous computational cost. This point, as a consequence, led to a substantial number of studies [2–10] into implicit, but improved methods to avoid the mentioned disadvantage. However, Clowes's explicit solution is still of considerable interest in signal processing applications. A vivid example is applying the explicit solution to design broadband beamformer in [11]. This work indicates that finding the optimal pole of the Laguerre filters improves the response of the time-domain filters and, by that, helps to obtain a better beamformer response for the desired signal that should appear at the output of the beamformer without any distortion. Putting an emphasis on the fact that, according to the research results of Masnadi-Shirazi and Aleshams [12] in the field of the discrete Laguerre functions, there is no need to vanish both stated coefficients because one of them can be entirely neglected, the authors [11] employed the improved explicit solution that enables to reduce the computational complexity to one half. A number of attempts to advance Clowes's result were also made for the continuous Laguerre functions. In particular, these works [13–15] carry out an exploration of the sign of two certain coefficients product to exclude the insufficient Laguerre coefficient. In the following papers [4,5,16], the authors point up uniqueness of the satisfied Laguerre coefficient to solve the pole optimization problem as a special case, but there seemed to provide little convincing explanation. So, the main purpose of the present research is to extend previous research results and to prove uniqueness of the certain coefficient that leads to the optimal solution for the generalized Laguerre functions.

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2. Material and methods

2.1. Problem statement

Let $L_k^\alpha(\tau,\gamma)$ denote the continuous generalized Laguerre functions, which are given by $\langle \exp(-\gamma\tau/2)L_k^\alpha(\gamma\tau)\rangle_{k=0}^{\infty}$, obtained from the Laguerre polynomials [17] for each fixed pole $\gamma \in \Gamma$, where $\Gamma=\{\gamma\in\mathbb{R}:\gamma>0\}$, $k\in\mathbb{N}_0$, and each given parameter $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$ in the Hilbert space $L_2(\mathbb{R}^+)$. This orthogonal system with nonnegative weight function $\omega^\alpha(\tau)=\tau^\alpha$ satisfies the following inner product over the interval $\tau\in\mathbb{R}^+$:

$$\langle L_k^\alpha(\tau,\gamma), L_n^\alpha(\tau,\gamma)\rangle_{\omega^\alpha(\tau)} = \|L_k^\alpha(\gamma)\|^2\delta_{k,n}, \tag{1}$$

where $\delta_{k,n}$ is Kronecker delta function and the norm $\|L_k^\alpha(\gamma)\|^2=\Gamma(k+\alpha+1)/k!\gamma^{\alpha+1}$. Further, for some $\gamma\in\Gamma$, fixed $m\in\mathbb{N}_0$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, a function $f(\tau)$ with regard to $\int_0^\infty (f(\tau))^2\,d\tau<\infty$ can be expressed as a truncated Laguerre series with the following truncation error:

$$\Delta_m^\alpha(\gamma)=\sum_{k=m+1}^{\infty}\left(\beta_k^\alpha(\gamma)\right)^2\left\|L_k^\alpha(\gamma)\right\|^2, \tag{2}$$

where the Laguerre coefficients $\beta_k^\alpha(\gamma)$ are given by

$$\beta_k^\alpha(\gamma)=\frac{\langle f(\tau), L_k^\alpha(\tau,\gamma)\rangle_{\omega^\alpha(\tau)}}{\|L_k^\alpha(\gamma)\|^2}. \tag{3}$$

The problem of finding of the $\gamma=\arg\min_{\gamma\in\Gamma}\Delta_m^\alpha(\gamma)$ for each fixed $m\in\mathbb{N}_0$ and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$ is the pole position problem.

2.2. Problem solution

The posed problem requires the following necessary and sufficient conditions for optimality, respectively:

$$\frac{d\Delta_m^\alpha(\gamma)}{d\gamma}=0;\quad \frac{d^2\Delta_m^\alpha(\gamma)}{d\gamma^2}>0. \tag{4}$$

Setting the derivative of the truncation error (2) with respect to γ equal to zero and using (3), we can write that

$$\sum_{k=m+1}^{\infty}\frac{d}{d\gamma}\left(\frac{1}{\|L_k^\alpha(\gamma)\|^2}\right)\|L_k^\alpha(\gamma)\|^4\left(\beta_k^\alpha(\gamma)\right)^2 + 2\sum_{k=m+1}^{\infty}\beta_k^\alpha(\gamma)\left\langle f(\tau),\frac{\partial L_k^\alpha(\tau,\gamma)}{\partial\gamma}\right\rangle_{\omega^\alpha(\tau)}=0. \tag{5}$$

To simplify this equation, we now formulate the following lemma.

Lemma 1. *For some $\gamma\in\Gamma$, $k\in\mathbb{N}$, $n\in\mathbb{N}_0$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, the connection coefficients $\lambda_{k,n}^\alpha(\gamma)$ between the generalized Laguerre functions $L_k^\alpha(\tau,\gamma)$ and its first-order derivatives with respect to γ are equal to*

$$\lambda_{k,n}^\alpha(\gamma)=\begin{cases} -\dfrac{d}{d\gamma}\left(\dfrac{1}{\|L_k^\alpha(\gamma)\|^2}\right)\dfrac{\|L_k^\alpha(\gamma)\|^2}{2} & \text{if } n=k;\\ -\lambda_{n,k}^\alpha(\gamma) & \text{if } n\in\{k-1,k+1\};\\ 0 & \text{otherwise}.\end{cases} \tag{6}$$

Proof. According to the connection coefficients method [18,19], we can write that $\partial L_k^\alpha(\tau,\gamma)/\partial\gamma=\sum_{n=0}^\infty \lambda_{k,n}(\gamma)L_n^\alpha(\tau,\gamma)$, where the connection coefficients can be exactly determined from $\lambda_{k,n}^\alpha(\gamma)=\frac{\left\langle \frac{\partial L_k^\alpha(\tau,\gamma)}{\partial\gamma}, L_n^\alpha(\tau,\gamma)\right\rangle_{\omega^\alpha(\tau)}}{\|L_n^\alpha(\gamma)\|^2}$. Finding the derivative of both sides of (1) with respect to γ and, after that, simplifying this expression leads to (6). □

Applying Lemma 1 to (5) gives

$$\sum_{k=m+1}^{\infty}\left(\beta_k^\alpha(\gamma)\beta_{k-1}^\alpha(\gamma)\lambda_{k,k-1}^\alpha(\gamma)+\beta_k^\alpha(\gamma)\beta_{k+1}^\alpha(\gamma)\lambda_{k,k+1}^\alpha(\gamma)\right)=0. \tag{7}$$

Taking into account that $\lambda_{k,n}^\alpha(\gamma)=-\lambda_{n,k}^\alpha(\gamma)$ if $n\in\{k-1,k+1\}$ (6), we can simplify (7) by collecting like terms

$$\beta_{m+1}^\alpha(\gamma)\beta_m^\alpha(\gamma)\lambda_{m+1,m}^\alpha(\gamma)=0. \tag{8}$$

Lemma 1 implies that $\lambda_{m+1,m}^\alpha(\gamma)\neq 0$. Therefore, at least one of two clearly stated Laguerre coefficients is equal to zero. Considering the sufficient condition for optimality (4), we prove that $\beta_{m+1}^\alpha(\gamma)=0$ is the unique condition that leads to the solution to the posed problem. Moreover, we provide the evidence that vanishing the inappropriate Laguerre coefficient $\beta_m^\alpha(\gamma)$ solves the other problem $\gamma=\arg\max_{\gamma\in\Gamma}\Delta_m^\alpha(\gamma)$. Before we go any further, it is required to formulate and to prove a set of the supporting lemmas.

Lemma 2. *If $\gamma\in\Gamma$, $k\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, then the coefficients $\beta_k^\alpha(\gamma)$ for the system $\langle L_k^\alpha(\tau,\gamma)\rangle_{k=0}^\infty$ satisfy the following recurrence relation*:

$$\beta_k^{\alpha+1}(\gamma)=\beta_k^\alpha(\gamma)-\beta_{k+1}^\alpha(\gamma). \tag{9}$$

Proof. Let $\tau L_k^{\alpha+1}(\tau,\gamma)=\sum_{n=0}^\infty \eta_{k,n}^\alpha(\gamma)L_n^\alpha(\tau,\gamma)$, where the connection coefficients $\eta_{k,n}^\alpha(\gamma)$ are given by

$$\eta_{k,n}^\alpha(\gamma)=\frac{1}{\|L_n^\alpha(\gamma)\|^2}\left\langle \tau L_k^{\alpha+1}(\tau,\gamma), L_n^\alpha(\tau,\gamma)\right\rangle_{\omega^\alpha(\tau)}. \tag{10}$$

Applying the well-known recurrence relation [17] to the system $\langle L_k^\alpha(\tau,\gamma)\rangle_{k=0}^\infty$, we can write

$$L_k^\alpha(\tau,\gamma)=L_k^{\alpha+1}(\tau,\gamma)-L_{k-1}^{\alpha+1}(\tau,\gamma). \tag{11}$$

After that, substituting (11) into (10) to obtain $\eta_{k,n}^\alpha(\gamma)$, and then, using (3) readily gives (9). □

Corollary 1. *For each $\gamma\in\Gamma$, $m\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, if $\beta_{m+1}^\alpha(\gamma)=0$, then $\beta_m^\alpha(\gamma)=\beta_m^{\alpha+1}(\gamma)$ and $\beta_{m+2}^\alpha(\gamma)=-\beta_{m+1}^{\alpha+1}(\gamma)$.*

Proof. The correctness of Corollary 1 explicitly follows from Lemma 2 subject to $\beta_{m+1}^\alpha(\gamma)=0$. □

Lemma 3. *If $\gamma\in\Gamma$, $m\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, then*

$$|\beta_m^{\alpha+1}(\gamma)|>|\beta_m^{\alpha+2}(\gamma)|\quad \textit{subject to } \beta_{m+1}^\alpha(\gamma)=0. \tag{12}$$

Proof. Using Lemma 2, the inequality (12) can be written as $|\beta_m^{\alpha+1}(\gamma)|>|\beta_m^{\alpha+1}(\gamma)-\beta_{m+1}^{\alpha+1}(\gamma)|$. Taking into account the triangle inequality $|\beta_m^{\alpha+1}(\gamma)-\beta_{m+1}^{\alpha+1}(\gamma)|\geq|\beta_m^{\alpha+1}(\gamma)|-|\beta_{m+1}^{\alpha+1}(\gamma)|$, we can conclude that (12) is true. □

Lemma 4. *If $\gamma\in\Gamma$, $m\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, then*

$$\beta_m^\alpha(\gamma)\beta_{m+2}^\alpha(\gamma)<0\quad \textit{subject to } \beta_{m+1}^\alpha(\gamma)=0. \tag{13}$$

Proof. Using Corollary 1, we can present (13) in the form $\beta_m^{\alpha+1}(\gamma)\beta_{m+1}^{\alpha+1}(\gamma)>0$. Applying Lemma 2, we can write that $\left(\beta_m^{\alpha+1}(\gamma)\right)^2-\beta_m^{\alpha+1}(\gamma)\beta_m^{\alpha+2}(\gamma)>0$. According to Lemma 3, that is true. □

At this point we postulate the main theorem.

Theorem 5. *For each $\gamma \in \Gamma$, $m \in \mathbb{N}$, and $\alpha \in \{\alpha \in \mathbb{R}: \alpha > -1\}$, the condition $\beta^{\alpha}_{m+1}(\gamma) = 0$ leads to the solution to the problem $\gamma = \arg\min_{\gamma \in \Gamma} \Delta^{\alpha}_{m}(\gamma)$, as well as, the equation $\beta^{\alpha}_{m}(\gamma) = 0$ achieves the solution to the problem $\gamma = \arg\max_{\gamma \in \Gamma} \Delta^{\alpha}_{m}(\gamma)$.*

Proof. Assume that the theorem is true. Using the left-hand side of (8), the sufficient condition (4) for each of the coefficients can be written as

1. If $\beta^{\alpha}_{m+1}(\gamma) = 0$, then
$$\frac{d^2 \Delta^{\alpha}_{m}(\gamma)}{d\gamma^2} = \frac{d\beta^{\alpha}_{m+1}(\gamma)}{d\gamma} \beta^{\alpha}_{m}(\gamma) \lambda^{\alpha}_{m+1,m}(\gamma) > 0;$$
2. If $\beta^{\alpha}_{m}(\gamma) = 0$, then
$$\frac{d^2 \Delta^{\alpha}_{m}(\gamma)}{d\gamma^2} = \beta^{\alpha}_{m+1}(\gamma) \frac{d\beta^{\alpha}_{m}(\gamma)}{d\gamma} \lambda^{\alpha}_{m+1,m}(\gamma) < 0.$$

Let $\beta^{\alpha}_{m+1}(\gamma) = 0$. Then, using Lemma 1 to find the derivative
$$\frac{d\beta^{\alpha}_{m+1}(\gamma)}{d\gamma} = \frac{\lambda^{\alpha}_{m+1,m}(\gamma)}{\|L^{\alpha}_{m+1}(\gamma)\|^2} \beta^{\alpha}_{m}(\gamma) - \frac{\lambda^{\alpha}_{m+2,m+1}(\gamma)}{\|L^{\alpha}_{m+1}(\gamma)\|^2} \beta^{\alpha}_{m+2}(\gamma),$$
we can write
$$\frac{(\lambda^{\alpha}_{m+1,m}(\gamma))^2}{\|L^{\alpha}_{m+1}(\gamma)\|^2} (\beta^{\alpha}_{m}(\gamma))^2 - \frac{\lambda^{\alpha}_{m+1,m}(\gamma) \lambda^{\alpha}_{m+2,m+1}(\gamma)}{\|L^{\alpha}_{m+1}(\gamma)\|^2} \beta^{\alpha}_{m}(\gamma) \beta^{\alpha}_{m+2}(\gamma) > 0. \tag{14}$$

Account for the fact that, according to Lemma 1, $\lambda^{\alpha}_{m+1,m}(\gamma) \lambda^{\alpha}_{m+2,m+1}(\gamma) > 0$ and applying Lemma 4, the inequality (14) is true.

Let $\beta^{\alpha}_{m}(\gamma) = 0$. By analogy,
$$\frac{\lambda^{\alpha}_{m,m-1}(\gamma) \lambda^{\alpha}_{m+1,m}(\gamma)}{\|L^{\alpha}_{m}(\gamma)\|^2} \beta^{\alpha}_{m-1}(\gamma) \beta^{\alpha}_{m+1}(\gamma) - \frac{(\lambda^{\alpha}_{m+1,m}(\gamma))^2}{\|L^{\alpha}_{m}(\gamma)\|^2} (\beta^{\alpha}_{m+1}(\gamma))^2 < 0 \tag{15}$$
implies that the inequality (15) is true. Consequently, the theorem is proved. □

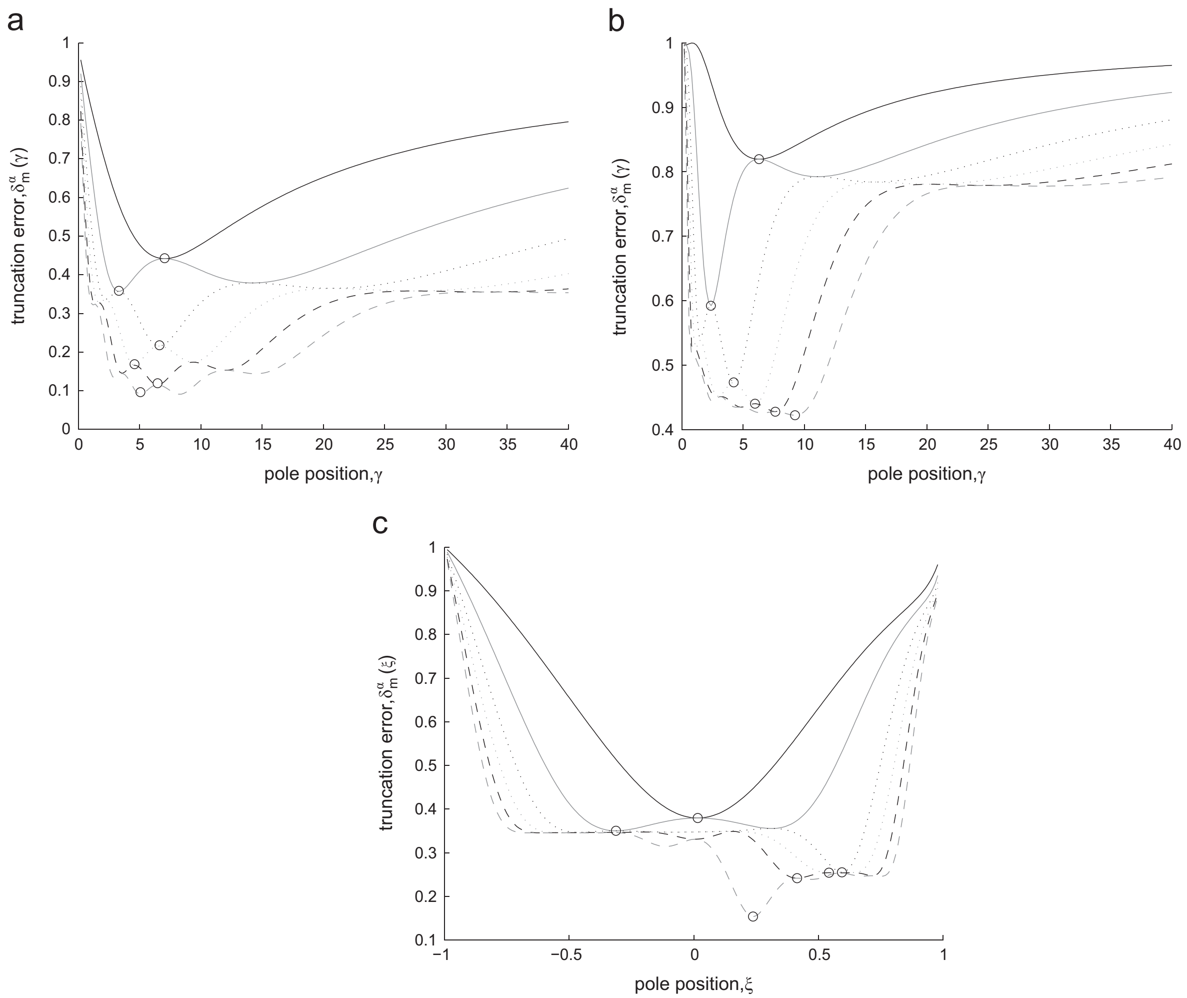

Fig. 1. (a) The normalized truncation error curves $\delta^{\alpha}_{m}(\gamma)$ correspond to $m = 0..5$ and $\alpha = -0.25$ (Example 1). (b) The normalized truncation error curves $\delta^{\alpha}_{m}(\gamma)$ correspond to $m = 0..5$ and $\alpha = 0.25$ (Example 2). (c) The normalized truncation error curves $\delta_{m}(\xi)$ correspond to $m = 0..5$ (Example 3).

2.3. Extension of the proposed solution

We now extend the proposed results to the case of the discrete Laguerre functions and filters using the modified bilinear transformation [4,5]. The Laplace transform $\Lambda_k^\alpha(s,\gamma)$ of $L_k^\alpha(\tau,\gamma)$ is given by

$$\Lambda_k^\alpha(s,\gamma)=\left(\frac{\gamma}{s+\gamma/2}\right)^{\alpha+1}\left(\frac{s-\gamma/2}{s+\gamma/2}\right)^k. \tag{16}$$

It is noteworthy that (16) differs from the Laplace transform thoroughly discussed in [4,5,20]. Applying the modified bilinear transform to $\Lambda_k^\alpha(s,\gamma)$, we can introduce a rational z-transform with real pole $|\xi|<1$ as

$$G_k^\alpha(z,\xi)=\frac{(1-\xi^2)z}{z-\xi}\left(\frac{(1-\xi)(z+1)}{z-\xi}\right)^\alpha\left(\frac{1-\xi z}{z-\xi}\right)^k. \tag{17}$$

Considering the Meixner-like functions [20] that correspond to (17) is beyond the scope of this paper, so, we focus mainly on the case of $\alpha=0$ that reduces to z-transform of the Laguerre sequences $g_k(n,\xi)$ [3–5] with the norm $\|g_k(\xi)\|^2=1-\xi^2$. For these discrete functions, it is easy to confirm the validity of Lemma 1 [3,5]. However, the body of our evidence is derived from Lemma 2 that requires $\beta_k^{\alpha+1}(\xi)=\beta_k^\alpha(\xi)-\beta_{k+1}^\alpha(\xi)$, where $\beta_k^\alpha(\xi)=\langle H(z), G_k^\alpha(z,\xi)\rangle/\|g_k^\alpha(\xi)\|^2$. In fact, this recurrence relation is true for another rational z-transform similar to (17), which can be defined by replacing $((1-\xi)(z+1))^\alpha$ on $((1+\xi)(z-1))^\alpha$. Drawing attention to the fact that we analyze the case of $\alpha=0$, we employ this rational z-transform to offer the same lemmas and theorem to extend the results to the case of the discrete Laguerre functions and filters.

3. Computational experiments

We conducted a series of computational experiments presented in the form of three examples to support the theoretical results: the first two examples illustrate the problem solving results in the case of the continuous generalized Laguerre functions; the third example is provided to show the validity of the proposed results in the case of the discrete Laguerre functions considering the transfer function of a supersonic jet engine inlet described in [7].

Example 1. Let $f(\tau)=\exp(-\tau)(\cos(3\tau)+\sin(3\tau)/3)$. Solving the pole position problem, we obtained normalized truncation error curves $\delta_m^\alpha(\gamma)=\Delta_m^\alpha(\gamma)/\|f\|^2$ for fixed $\alpha=-0.25$ and m=0..5 depicted in Fig. 1(a). Using the circle markers, we provided an illustration of the solutions to $\beta_{m+1}^\alpha(\gamma)=0$. The numerical values of the solutions γ for some given γ_0 and the Laguerre coefficients $\beta_m^\alpha(\gamma)$, $\beta_{m+2}^\alpha(\gamma)$ for the optimal pole positions are presented in Table 1.

Example 2. Let $f(\tau)=\sin(\tau)\cos(1.75\tau)/\tau$. The normalized truncation error curves $\delta_m^\alpha(\gamma)$ for fixed $\alpha=0.25$ and m=0..5 with the markers and the numerical values are shown in Fig. 1(b) and Table 1, respectively.

Example 3. Let $H(z)=z(2.034z^6-4.9825z^5+6.57z^4-5.8189z^3+3.636z^2-1.4105z+0.2997)/(z^7-2.46z^6+3.433z^5-3.333z^4+2.546z^3-1.584z^2+0.7478z-0.252)$.

Table 1
The numerical values of the computational experiments results.

Example 1			
m	$\gamma(\gamma_0=6.5)$	$\beta_m^\alpha(\gamma)$	$\beta_{m+2}^\alpha(\gamma)$
0	7.04	1.268	−0.669
1	3.31	0.565	−0.334
2	6.619	−0.656	0.334
3	4.586	−0.404	0.213
4	6.473	0.331	−0.178
5	5.068	0.22	−0.12
Example 2			
m	$\gamma(\gamma_0=5)$	$\beta_m^\alpha(\gamma)$	$\beta_{m+2}^\alpha(\gamma)$
0	6.307	1.623	−1.24
1	2.374	1.005	−0.442
2	4.244	−1.109	0.304
3	5.983	0.916	−0.218
4	7.638	−0.729	0.173
5	9.24	0.585	−0.148
Example 3			
m	$\xi(\xi_0=0.48)$	$\beta_m^0(\xi)$	$\beta_{m+2}^0(\xi)$
0	0.016	2.043	−0.336
1	−0.312	0.789	−0.105
2	0.595	0.828	−0.045
3	0.543	0.192	−0.097
4	0.415	0.315	−0.207
5	0.238	0.626	−0.064

In this case, Fig. 1(c) shows the normalized truncation error curves $\delta_m(\xi)=\Delta_m^0(\xi)/\|H\|^2$ marking the solutions to $\beta_{m+1}^0(\xi)=0$. The results of the computational experiments are presented in Table 1 by analogy.

Before we reach any conclusion, it is important to state that, from the data in Fig. 1 and Table 1, where the optimal pole positions are highlighted, Lemmas 3, 4, and, therefore, Theorem 5 are valid. As we have seen, the results are consistent for both the continuous and discrete Laguerre functions across the three separate tests.

4. Conclusions

To bring this paper to close, we summarize the main points here: the present research reveals that for the continuous generalized Laguerre functions only one of two possible conditions $\beta_m^\alpha(\gamma)=0$ and $\beta_{m+1}^\alpha(\gamma)=0$ leads to the solution to the posed pole position problem $\gamma=\arg\min_{\gamma\in\Gamma}\Delta_m^\alpha(\gamma)$; the other condition solves the other problem $\gamma=\arg\max_{\gamma\in\Gamma}\Delta_m^\alpha(\gamma)$. To confirm the findings, we provided the evidence pointing to the fact that the sufficient condition in the case of the continuous generalized Laguerre functions is $\beta_{m+1}^\alpha(\gamma)=0$. The evidence is based on the postulated and proved theorem and a set of supporting lemmas, which rely on the connection coefficients method. After that, we extended the proposed results to the discrete Laguerre functions using the modified bilinear transformation and introducing the rational z-transforms of the Meixner-like functions. To support the theoretical results, the present research yields a series of the computational experiments, going through the results of which, we conclude that the purpose, stated in this paper, is accomplished.

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