Fuzzy Learning Performance Assessment Based on Decision Making Under Internal Uncertainty

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***Abstract*—The paper delves into decision making with partial knowledge focused on students' behaviour in multiple-choice testing. To address to this problem, we first provided a binary knowledge model and, then, relaxing some assumptions, allowed for the more realistic framework of partial knowledge which, in turn, adds uncertainty to the assessment of students' knowledge due to their incentive to make a guess in multiple-choice testing. The aim of this paper is to propose a fuzzy assessment model formalised according to Reiter's Theory of Diagnosis to reduce this uncertainty and to draw a distinction between the level of students' ability and the degree of guessing. The provided assessment model was tested on modelled answers with respect to the knowledge frameworks and validated in a real-world context. The findings of this research present the fuzzy learning performance assessment model which may enable a teacher to estimate the level of partial knowledge and, thus, to specify a student's score as well as the results of computational experiments that confirm the validity of the theoretical outcomes.**

***Index terms*—Adaptive learning, Bloom's taxonomy, decision making, internal uncertainty, multiple-choice testing, fuzzy logic**

I. Introduction

Many studies have pointed out the importance of assessment for improving learning performance that presents one of the main objectives of educational systems. However, there is a problem of finding effective assessment tools that will enable a teacher to accurately evaluate each student's performance. A promising solution to this problem is e-learning systems [1], [2], [3] that help to overcome the limitations of time and space of traditional teaching and to better suit the needs and preferences of each individual student. Adopting adaptive teaching strategies towards appropriately designed educational content provides personalised assistance to e-learning students according to the computer-based assessment of their learning style, background, motivation and level of knowledge [4], [5], [6], [7], [8].

Computer-based assessment can be categorised into summative and formative [5], [6]. Summative assessment enables teachers to establish whether students have achieved the goals set for them, while formative assessment is defined as judgement combined with prescriptive feedback to assist students in identifying and filling possible gaps between the actual level of knowledge and the required standard traditionally presented as Bloom's taxonomy [9] given as six cognitive levels from the lowest, which suggests recalling and remembering facts, through more complex and abstract mental levels, to the highest order. It pursues the main learning purpose of any teaching methodology - to help students in achieving higher order thinking skills [9], [10], [11] while they acquire knowledge and gain competence. Anderson's revision of the taxonomy [10] considers nineteen types of cognitive processes classified into six major categories: remember, understand, apply, analyse, evaluate, and create. The assessment items, in turn, necessitate applying these cognitive processes to particular types of knowledge.

But, teachers sometimes face the challenge in formulating instructions in accordance with Bloom's cognitive levels. Failure to provide students with a clear learning outcome leads to uncertainty associated with the instructions style. The answer to this problem may lay in applying multiple-choice tests to estimating even higher order thinking skills [12], [13], [14], [15], [16]. Nevertheless, despite a number of advantages over constructed-response tests like wider content sampling and precluding measurement errors introduced by the grader, multiple-choice tests may encourage guessing that results in lowing test reliability in measuring learners knowledge [13], [14]. In psychology, guessing can be formalised as decision making under internal uncertainty [17], [18], thus, students are decision makers with different risk attitudes and level of partial knowledge [18]. The backbone of this treatment is the possibility of applying Item Response Theory and Decision Theory [19] to describe more realistic student's behaviour. Under this setting, a partial knowledge framework implies an ability to rule out some of distractors as incorrect, but incompetence in selecting the correct answer among the remaining ones. Consequently, the internal uncertainty stems from student's incentive to guess and, by that, draws a distinction between the level of ability and the degree of guessing.

As indicated in previous research, there are several sources of uncertainties in computer-based assessment such as linguistic uncertainties [20], [21], intuitionistic uncertainties [22], and uncertainties connected to the degree of confidence in making subjective assessments [23], [24]. However, there seemed little attention to be drawn to the problem of decision-making with the internal uncertainty. The crucial point is to create an assessment model which may enable a teacher to classify

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the answer beyond the mere partition right-wrong in order to ensure better learning performance assessment. As fuzzy logic extends the limiting bivalent sets in a way that allows a smooth transition between sets [33], an extensive body of literature puts forward employing fuzzy logic [20], [21], [25], [26], [27], [28], [29], [30], [31], [32] to cope with uncertainties and to establish transparent human behaviour frameworks. So, it can be highly convenient for describing natural phenomena like the behaviour of students with different level of partial knowledge. The aim of this study is to propose fuzzy learning performance assessment focusing mainly on reducing internal uncertainty stemmed from student's incentive to make a guess in multiple-choice testing.

II. Knowledge models

Before we go any further, we present knowledge models proposed in [18] to describe student's behaviour in multiple-choice testing. For this purpose, we first state a binary knowledge model and then, relaxing some assumptions, allow for a partial knowledge model.

The binary knowledge model implies the assumption that students know the correct answer, but cannot exclude any distractor as incorrect [18]. Under this setting, the k^{th} student knows the correct answer to the i^{th} question if and only if $\theta_k > l_i$, where $\theta_k \in [0, 1]$ is the student's level of knowledge and $l_i \in [0, 1]$ is the difficulty of the i^{th} item. Suppose that one response alternative is correct and $M - 1$ are incorrect. Then, if the student knows the answer, the probability of getting the correct answer is one. Otherwise, the student guesses - the probability is equal to $1/M$. The penalty for each incorrect answer is $p \in [0, 1]$. With respect to this assumption, we can define the student's answers u_{ik} as

$$u_{ik}^B = \big((\theta_k > l_i) + \frac{1}{M}(\theta_k \leq l_i)\big)u_{ik}, \tag{1}$$

$$u_k^B = (1+p)\sum_{i=1}^{N} u_{ik}^B - p\sum_{i=1}^{N}\Big(u_{ik}^B + (u_{ik}^B = 0)\Big),$$

$$\tilde{u}_k^B = \frac{\max\{u_k^B/N, 0\} + pK}{(1+p)K}, \tag{2}$$

where $\tilde{u}_k^B$ is the normalised score based on the binary knowledge model.

According to Decision Theory [18], [19], we can identify the normalised true score for the items the student knows as follows

$$u_{ik}^{BT} = (\theta_k > l_i)u_{ik}, \tag{3}$$

$$u_k^{BT} = (1+p)\sum_{i=1}^{N} u_{ik}^{BT} - p\sum_{k=1}^{N}\Big(u_{ik}^{BT} + (u_{ik}^{BT} = 0)\Big),$$

$$\tilde{u}_k^{BT} = \frac{\max\{u_k^{BT}/N, 0\} + pK}{(1+p)K}. \tag{4}$$

In contrast to the binary knowledge model, the partial knowledge model takes into account that students can rule out some alternatives, but they cannot choose the correct answer [18]. In this case, the knowledge of the answer does not preclude an incorrect answer. For this model, the k^{th} student knows the answer to the i^{th} item if $\theta_k > l_i + v_{ki}$, where v_{ki} is a random variable with the distribution function $F(v_{ki})$. Therefore, the student's answers and the score given can be determined as

$$u_{ik}^P = \big(c_k + (1-c_k)F(\theta_k - l_i)\big)u_{ik}, \tag{5}$$

$$u_k^P = (1+p)\sum_{i=1}^{N} u_{ik}^P - p\sum_{i=1}^{N}\Big(u_{ik}^P + (u_{ik}^P = 0)\Big),$$

$$\tilde{u}_k^P = \frac{\max\{u_k^P/N, 0\} + pK}{(1+p)K}. \tag{6}$$

where c_k is the probability that guess results in a correct answer. Under the assumption that the distribution of v_{ki} is logistic according to Item Response Theory [18], we can write

$$F(\theta_k - l_i) = \frac{1}{1 + \exp\big(-(\theta_k - l_i)/g_i\big)},$$

where g_i is a parameter related to the difference between knowledge and difficulty in defining the probability of knowing the answer to the item.

By anology with (3), (4), we can present the normalised true score as

$$u_{ik}^{PT} = F(\theta_k - l_i)u_{ik}, \tag{7}$$

$$u_k^{PT} = (1+p)\sum_{i=1}^{N} u_{ik}^{PT} - p\sum_{i=1}^{N}\Big(u_{ik}^{PT} + (u_{ik}^{PT} = 0)\Big),$$

$$\tilde{u}_k^{PT} = \frac{\max\{u_k^{PT}/N, 0\} + pK}{(1+p)K}. \tag{8}$$

For the sake of completeness, we should consider the case if students stop answering to the items ordered in increasing difficulty. Assume that they answer n items out of N. Then, we need to refine (2), (4), (6), and (8) replacing N with n_k equal to $n_k = \sum_{i=1}^{N}(u_{ik} + (u_{ik} = 0))$.

As we can see, in comparison with the binary knowledge model based on the level of student's ability θ_k associated with Bloom's taxonomy [9], [10], the partial knowledge model involves an extra parameter c_k which does not relate to any educational standard and, thus, adds uncertainty to the assessment of student's knowledge. In this study, we intend to reduce this uncertainty proposing the fuzzy assessment model which enables us to estimate the level of partial knowledge as

$$\tilde{u}_k^F = \tilde{u}_k^{PT} + w_k, \tag{9}$$

where w_k is an adjustment value based on the degree of guessing. To define (9), we now turn to the fuzzy assessment problem.

III. Fuzzy assessment

Let U is a fuzzy set of student's answers subject to guess results in answering to multiple-choice questions. Then, L is a set of degrees of multiple-choice items difficulty related to Bloom's cognitive levels [9], [10]: *Know*, *Understand*, *Apply*, *Analyse*, *Evaluate*, *Create*. Following this, we define the fuzzy assessment model as

$$L = \{(u, \mu_L(u))|u \in U\}, \tag{10}$$

where $\mu_L(u) : U \to [0,1]$ is the membership function of $L \subset U$.

We introduce the triangular membership function with six center points {0.1,0.26,0.42,0.58,0.74,0.9} as we analyse six cognitive levels and employ the assessment model (10) to assign student's level of partial knowledge to Bloom's tiers. To formalise fuzziness due to the uncertainty of student's knowledge, we intend to apply Reiter's Theory of Diagnosis [34] extended by the authors [35] to the case of fuzzy diagnosis. According to this theory, a fuzzy assessment problem can be clearly defined as a pair (COMPS,TSD), which may be interpreted in terms of the posed problem as follows: COMPS is first-order sentences or the set of assessment parameters related to the partial knowledge framework; TSD is Mamdani's max-min interference mechanism that establishes the fuzzy rule base associated with the relationship between levels of student's ability and guessing probability. The study [34] also pinpoints a specific predicate AB(.) interpreted to mean "abnormal" and the importance of observation OBS for a diagnosis problem. So, taking into consideration all the definitions provided in Section II, we present the fuzzy assessment problem as the set $L \subset$ COMPS such that

$$\text{TSD} \cup \text{OBS}\{\text{AB}(u)|u \in L\} \cup \{\leftarrow \text{AB}(u)|u \in \text{COMPS} - L\}$$

is consistent.

Hereby, we pose this problem in the following way:

- COMPS=$\{U, L, \Theta, G, p\}$, where student's ability level $\theta \in \Theta$ and a parameter $g \in G$;
- TSD based on the rule base presented in Fig.1;
- $U^F = AB(U)$, where the predicate AB(.) maps the score U into the required score U^F applying the COG method;
- OBS = $\{K, N\}$, where K and N are the numbers of students and items, respectively (see Section II).

It is worth noting that, in this study, we address the issues of summative assessment, so considering the formative assessment is beyond the scope of the paper though it raises interesting questions for further research.

IV. Experiments

We conducted a series of computational experiments to test the proposed fuzzy assessment model. For this purpose, we first modelled student's answers to analyse the influence of the knowledge models parameters on assessment results and to make a comparison between the partial knowledge model (6) and the proposed fuzzy assessment model (9). Then, we validated this study in a real-world context.

Taking into account the significant benefits of the R language in solving fuzzy logic problems highlighted in [36], we designed a program to model student's answers under the following settings: the number of students is $K = 10$; the number of items is $N = 6$; the levels of difficulty

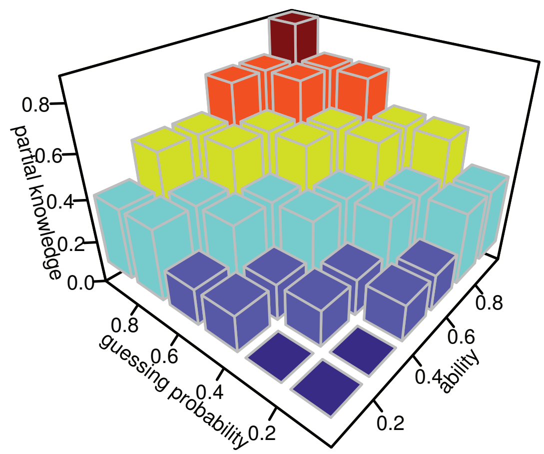


Fig. 1. The rule base for the partial knowledge

associated with the Bloom's cognitive levels set upon a scale $l = \{0.1, 0.26, 0.42, 0.58, 0.74, 0.9\}$; the level of student's ability is given by $\theta = 0.6$; the probability that pure guess results in a correct answer and the discrimination parameter are equal to $c = 0.25$ and $g = 3/2\pi$ [18] for each level, respectively; the penalty for an incorrect answer is $p = 0.1$. It is noteworthy that, according to [18], the optimal penalty is equal to $p > 1/(M-1)$ due to student's incentive to guess under no penalty for wrong answers that may lead to significant assessment errors. On the other hand, a high penalty does not prevent guessing - it introduces a bias in favour of risk takers. As we intended to analyse student's behaviour in guessing, we provided the small value for penalty.

The modelled student's answers are depicted in Fig. 2, where the correct answers are marked by circles.

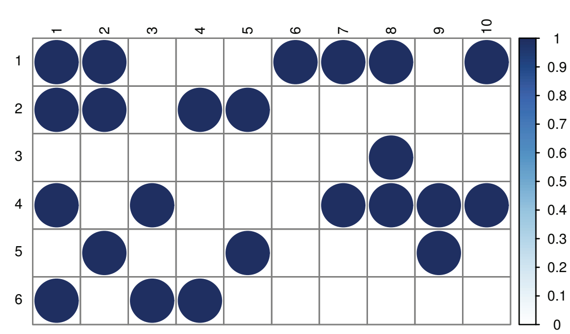


Fig. 2. The modelled answers u_{ik} subject to $N = 6$, $K = 10$: the columns correspond to students; the rows - tests

These answers were transformed according to both the binary knowledge model (1) (see Fig. 3 (a)) and the partial knowledge model (5) (see Fig. 4 (a)). To draw a distinction between the cases when students know a correct answer and do not know, but they guess, we illustrated the answers with no

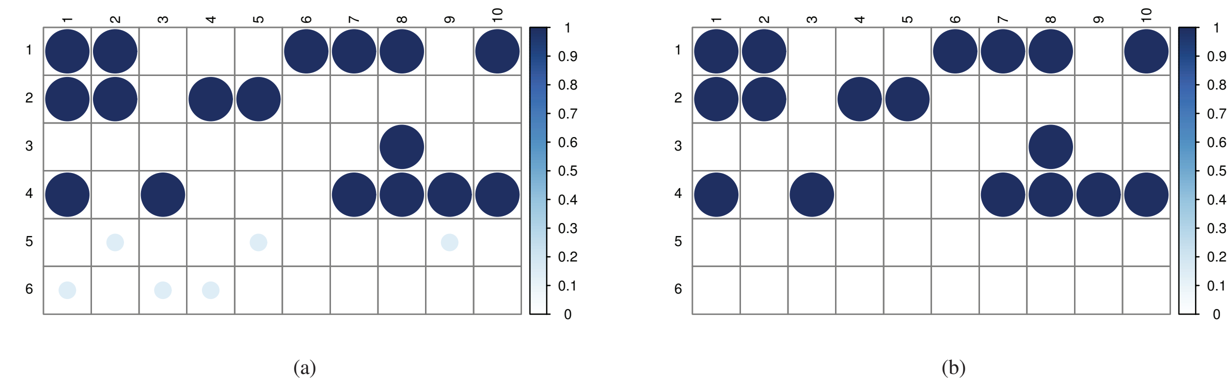


Fig. 3. The answers based on the binary knowledge model: (a) a student either knows or guesses u_{ik}^{B}; (b) a student knows u_{ik}^{BT}

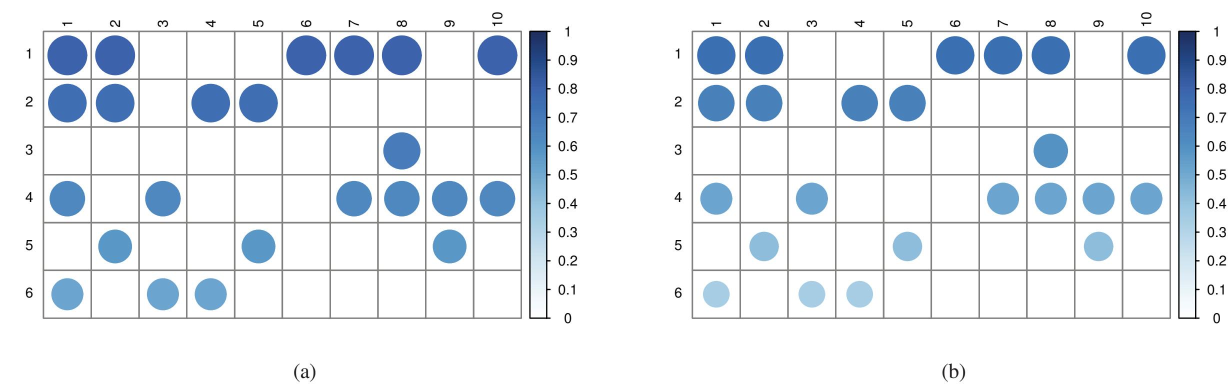


Fig. 4. The answers based on the partial knowledge model: (a) a student either knows or guesses u_{ik}^{P}; (b) a student knows u_{ik}^{PT}

guessing in multiple-choice testing (3), (7) in Fig. 3 (b), Fig. 4 (b), respectively. For the sake of comparison, we also presented the modelled scores based on the definitions presented in Section II with respect to $\theta = 0.6$, c = 0.25 (see Fig. 5 (a)) and $\theta = 0.25$, c = 0.6 (see Fig. 5 (b)).

As we can see from these modelled test results, there is the clear relationship between the level of ability and the degree of guessing: the higher level of ability, the more consistent and reliable the resulting score. This conclusion can be attributed to the fact that the high level of competence leads to reducing pure guess results in guessing. In this case, students prefer rather to guess relying on their partial knowledge. Based on this hypothesis, we presented the rule base (see Fig. 1).

To validate the proposed fuzzy assessment model, we calculated the scores using both (6) and (9) with respect to the item level i which are presented in Table I.

As we can see from the data, the results confirm the validity of the proposed fuzzy assessment model as the resulting scores are directly related to a particular cognitive level and are consistent with different values for the level of ability and the degree of guessing probability.

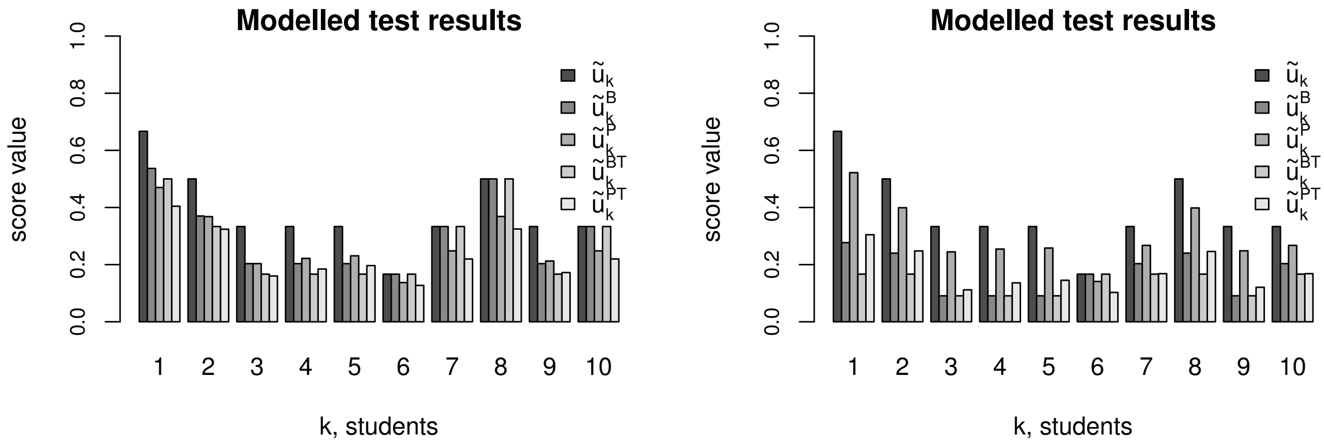
At this point, we tested the validity of the proposed assessment model in a real-world context.

TABLE I
COMPARISON BETWEEN THE ASSESSMENT RESULTS BASED ON THE PARTIAL KNOWLEDGE MODEL AND THE PROPOSED FUZZY MODEL

Item	$\tilde{u}_i^P$	$\tilde{u}_i^F$	$\tilde{u}_i^P$	$\tilde{u}_i^F$	$\tilde{u}_i^P$	$\tilde{u}_i^F$
level	θ=0.8, c=0.4		θ=0.6, c=0.3		θ=0.3, c=0.2	
1	0.5386	0.5147	0.5	0.4753	0.4268	0.4005
2	0.3467	0.338	0.3162	0.3071	0.2606	0.2526
3	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909
4	0.4734	0.4874	0.4131	0.4314	0.3196	0.3479
5	0.2233	0.2238	0.1906	0.1954	0.1439	0.1565
6	0.2096	0.2121	0.1755	0.1849	0.1302	0.1536

We carried out an experiment on a sample of nine Ph.D. students from Samara State Aerospace University. All the participants have expertise in engineering sciences and are under 33 years of age. In addition, they are currently enrolled on a professional development course for academics in English.

The participants were given 30 minutes to complete a set of multiple-choice tests on English prepositions designed according to Bloom's taxonomy to assess: the knowledge of preposition meanings, the sensitivity to the frequencies of linguistic forms, the ability to relate the spatial and temporal



(a) (b)

Fig. 5. The student's resulting scores: (a) $\theta = 0.6$, c = 0.25; (b) $\theta = 0.25$, c = 0.6

meanings of English prepositions, the ability to sense the polysemy of the preposition *over* in spatial meanings and metaphorical extensions.

Each of the tests includes a number of items which, in turn, consist of different numbers of multiple-choice alternatives. Moreover, these tests comprise both single-answer items and multiple-choice items. Several studies have revealed the significance of multiple-choice scoring methods [14], [15], [16] to evaluate the alternatives in valid educational settings, but does not present a clear direction in view of opting for the most optimal solution. Thus far, avoiding the detailed analysis of a number of alternatives for each item, we provided the same value for penalty $p = 0.1$ for each item to experiment with these alternatives. The level of ability was assessed indirectly, so we set the same value $\theta = 0.6$ for each participant to assess guessing in multiple-choice testing. The resulting scores are shown in Fig. 6.

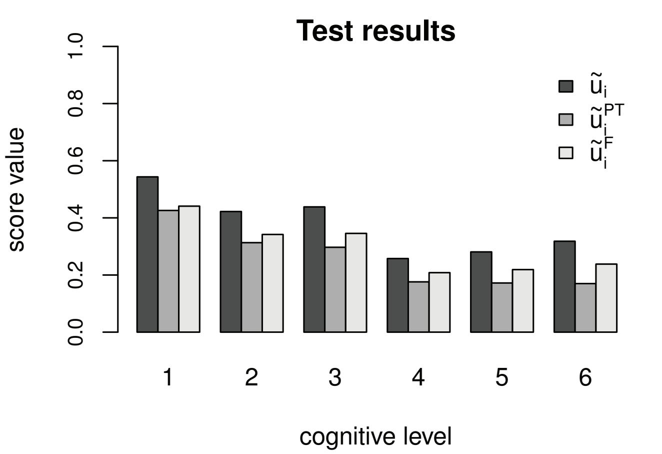


Fig. 6. The result before and after applying the proposed fuzzy assessment model

As can be seen, the resulting scores point to a marked trend in categorising the answers according to the cognitive levels. In addition, there appears to be some evidence linking the level of ability and the level of partial knowledge: in the case $\theta_k < l_i$ the true score and the proposed fuzzy score draw more definite distinction due to the increasing value for guessing probability and, as a consequence, the rising level of partial knowledge.

V. Conclusions and future work

The present research yields the fuzzy learning performance assessment model based on decision making under internal uncertainty stemmed from student's incentive to guess in multiple-choice testing. The proposed assessment tackles the problem of analysing the partial knowledge distinguishing between the level of student's ability associated with Bloom's taxonomy and the degree of guessing probability helping to ensure better learning performance assessment. We applied Reiter's Theory of Diagnosis extended to the case of fuzzy diagnosis to formalise fuzziness and introduce the rule base for partial knowledge.

Furthermore, we tested the validity of the proposed fuzzy assessment model on modelled student's answers with respect to the knowledge framework and a set of parameters that describes the student's behaviour. Going through the results of computational experiments, we conclude that the purpose stated in this study is accomplished as the resulting scores are directly related to a particular cognitive level and are consistent with different values for the level of ability and the degree of guessing probability.

In addition, we validated the proposed model in a real-world context. For this purpose, we carried out an experiment on a sample of nine Ph.D. students from Samara State Aerospace University which were given a set of multiple-choice tests on English prepositions formalised in accordance with Bloom's taxonomy. The results pointed to a marked trend in categorising the answers according to the cognitive levels and demonstrated the relationship between the level of ability and the level of partial knowledge.

The findings of this study raise some issues for further research connected to analysing a number of response alternatives, a value for penalty, and response time as well as considering the dynamics of multiple-choice decision making to extend the proposed results to the case of formative assessment.

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