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Pole position problem for Meixner filters

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ABSTRACT

This paper is motivated by previous research that demonstrates the importance of the explicit solution to the pole position problem. The main purpose of this paper is to solve the two-parameter pole position problem for the Meixner filters with an extra parameter. To attain the objective, we extended the theoretical results provided for the discrete Laguerre filters and proposed the approach to optimizing the extra parameter using the connection coefficients method. The present research yields a series of computational experiments to test this approach, to verify the theoretical results, and to point up the positive outcomes of using the Meixner filters.

1. Introduction

Seydnejad and Ebrahimi [1] stress the importance of an explicit solution to the pole position problem in broadband beamforming. This study indicates that finding the optimal pole of the discrete Laguerre filters enables to obtain a better beamformer response and, by that, to ensure the output without any distortion. According to an extensive body of knowledge on using the Meixner-like functions in signal processing applications [2–6], the Meixner filters can be an effective alternative to the discrete Laguerre filters in attempt to improve the results to a greater degree. This fact is attributed to an extra parameter that permits providing better series expansions [2]. As a consequence, a number of studies have set out the solutions to the extra parameter optimization problem [7,8]. But, there seemed little attention to be drawn to the explicit solution to this problem. Thus, the present study raises the issue of solving the two-parameter pole position problem drawing primarily on the work [9] that provides the theoretical outcomes in solving the pole position problem for the discrete Laguerre filters that needs to be extended. In addition, this research casts doubt on Brinker's statement [2] that “Meixner filters are not realizable since Meixner functions do not have a rational z-transform” putting forward rational Meixner filters as rational Laplace transforms of the generalized Laguerre functions [9] converted into z-transforms.

2. Material and methods

2.1. Problem statement

For each fixed pole $\gamma \in \Gamma$, where $\Gamma = \{\gamma \in \mathbb{R} : \gamma > 0\}$, and $\alpha \in \{\alpha \in \mathbb{R} : \alpha > -1\}$, $k \in \mathbb{N}_0$ in the Hilbert space $L_2(\mathbb{R}^+)$, the generalized Laguerre functions $L_k(\tau, \gamma, \alpha)$ given by $\langle \exp(-\gamma\tau/2) L_k(\gamma\tau, \alpha) \rangle_{k=0}^{\infty}$ are orthogonal with respect to the nonnegative weight function $\omega(\tau, \alpha) = \tau^{\alpha}$ over the interval $\tau \in \mathbb{R}^+$ and the norm $\|L_k(\gamma, \alpha)\|^2 = \Gamma(k+\alpha+1)/(k!\gamma^{\alpha+1})$. Therefore, the Laplace transform $\Lambda_k(s, \gamma, \alpha)$ of $L_k(\tau, \gamma, \alpha)$ can be defined as [9]

$$\Lambda_k(s, \gamma, \alpha) = \left(\frac{\gamma}{s+\gamma/2}\right)^{\alpha+1} \left(\frac{s-\gamma/2}{s+\gamma/2}\right)^{k}. \tag{1}$$

The filters $\Lambda_k(s, \gamma, \alpha)$ can be mapped onto a rational z-transform $G_k(z, \xi, \alpha)$ with real pole $|\xi| < 1$ using the

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following modified bilinear transformation:

$$\Lambda_k(s,\gamma,\alpha)\mapsto\frac{z}{z+1}\Lambda_k\left(a\frac{z-1}{z+1},\gamma,\alpha\right)=\frac{(-1)^k 2a}{a+\gamma/2}G_k\left(z,\frac{a-\gamma/2}{a+\gamma/2},\alpha\right);$$

$$G_k(z,\xi,\alpha)\mapsto\frac{2a}{a+s}G_k\left(\frac{a+s}{a-s},\xi,\alpha\right)=\frac{(-1)^k}{1+\xi}\Lambda_k\left(s,2a\frac{1-\xi}{1+\xi},\alpha\right). \quad (2)$$

Applying (2) to (1), we can introduce the Meixner filters $G_k(z,\xi,\alpha)$ as

$$G_k(z,\xi,\alpha)=\frac{(1-\xi^2)z}{z-\xi}\left(\frac{(1-\xi)(z+1)}{z-\xi}\right)^{\alpha}\left(\frac{1-\xi z}{z-\xi}\right)^{k}. \quad (3)$$

It is noteworthy that, for each given parameter $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\wedge\alpha\neq 0\}$, (1) and, consequently, (3) can be appertained to the class of almost orthogonal systems [10] that are suitable for the analysis of imperfect systems consisting of parameters, values of which are not ideally precise. We denote the system of almost orthogonal Meixner filters $\langle G_k^{\epsilon}(z,\xi,\alpha)\rangle_{k=0}^{\infty}$ as $\langle G_k(z,\xi,\alpha)\rangle_{k=0}^{\infty}$, where $\epsilon\in\{\epsilon>0\wedge\epsilon\lll 1\}$ and $\lim_{\epsilon\to 0}G_k^{\epsilon}(z,\xi,\alpha)=\tilde{G}_k(z,\xi,\alpha)$, where $\tilde{G}_k(z,\xi,\alpha)$ is the class of orthogonal Meixner filters.

If we assume $G_k(z,\xi,\alpha)$ completeness on a circle of radius higher than 1 and lower than $1/|\xi|$, we can express $H(z)$, which is guaranteed to be analytic in the exterior of the closed unit circle, as

$$\Delta_m(\xi,\alpha)=\left\|H-\sum_{k=0}^{m}\beta_k(\xi,\alpha)G_k(\xi,\alpha)\right\|^2, \quad (4)$$

where the coefficients $\beta_k(\xi,\alpha)$ are given by

$$\beta_k(\xi,\alpha)=\langle H,G_k(\xi,\alpha)\rangle/(1-\xi^2). \quad (5)$$

With reference to (4), we can state the two-parameter pole position problem as $(\xi,\alpha)=\arg\min_{\substack{|\xi|<1\\ \alpha>-1}}\Delta_m(\xi,\alpha)$ for each fixed $m\in\mathbb{N}_0$.

To cope with expansions in nonorthogonal filters [10,11], we specify the posed problem considering $(\xi,\alpha,\beta_k)=\arg\min_{\substack{|\xi|<1\\ \alpha>-1}}\Delta_m(\xi,\alpha,\beta_k)$ for each fixed $m\in\mathbb{N}_0$, where β_k is a set of the coefficients (5) for given ξ and α. Then, to exclude ϵ, we need to require

$$\frac{\partial\Delta_m(\xi,\alpha,\beta_k)}{\partial\beta_k}=0;\quad\frac{\partial^2\Delta_m(\xi,\alpha,\beta_k)}{\partial\beta_k^2}>0. \quad (6)$$

In Section 2.3, we intend to study this issue in more detail. Initially, we concentrate on the extension of the theorem provided in [9] for the discrete Laguerre filters to the case of the Meixner filters.

2.2. Extension of the theorem

Let the pole position problem $\xi=\arg\min_{|\xi|<1}\Delta_m(\xi,\alpha,\beta_k)$ require the following necessary and sufficient conditions for optimality:

$$\frac{\partial\Delta_m(\xi,\alpha,\beta_k)}{\partial\xi}=0;\quad\frac{\partial^2\Delta_m(\xi,\alpha,\beta_k)}{\partial\xi^2}>0. \quad (7)$$

Then, to prove the uniqueness of the condition that leads to the optimal solution and, by that, to extend the theorem postulated and proven in [9], we first simplify the necessary condition (7) by analogy with the proof of Lemma 1 [9] that results in $\beta_{m+1}(\xi,\alpha)\beta_m(\xi,\alpha)\lambda_{m+1,m}(\xi,\alpha)=0$, where $\lambda_{k,n}(\xi,\alpha)=\langle\partial G_k(z,\xi,\alpha)/\partial\xi,G_n(z,\xi,\alpha)\rangle$ [12,13]. Owing to the fact that the connection coefficient $\lambda_{m+1,m}(\xi,\alpha)\neq 0$, we can conclude that at least one of two stated coefficients is equal to zero. Considering the sufficient condition for optimality (7) enables to specifying this result. We now turn to further extension of the results provided in [9] pointing to the fact that $\beta_{m+1}(\xi,\alpha)=0$ is the unique condition for the Meixner filters $G_k(z,\xi,\alpha)$ to solve the posed pole position problem.

As indicated in [9], the body of the evidence on condition uniqueness is derived from Lemma 2 that requires $\beta_k(\xi,\alpha+1)=\beta_k(\xi,\alpha)-\beta_{k+1}(\xi,\alpha)$. Making a point about the fact that studying the Meixner filters was beyond the scope of that paper, this recurrence relation was employed to offer the same lemmas and theorem to examine the case of the discrete Laguerre filters. In present study, we extend this result to the case of the Meixner filters. For this purpose, we introduce the following lemmas:

Lemma 1. *If $|\xi|<1$, $k\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, then the system $\langle G_k(z,\xi,\alpha)\rangle_{k=0}^{\infty}$ satisfies the following recurrence relation*:

$$G_k(z,\xi,\alpha+1)=G_k(z,\xi,\alpha)+G_{k+1}(z,\xi,\alpha).$$

Proof. The correctness of Lemma 2 explicitly follows from (3). □

Corollary 1. *Suppose $|\xi|<1$, $k\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$. Then the coefficients $\beta_k(\xi,\alpha)$ for the system $\langle G_k(z,\xi,\alpha)\rangle_{k=0}^{\infty}$ satisfy*

$$\beta_k(\xi,\alpha+1)=\beta_k(\xi,\alpha)+\beta_{k+1}(\xi,\alpha). \quad (8)$$

Proof. Applying Lemma 1 to the coefficients definition (5) results in (8). □

Lemma 2. *If $|\xi|<1$, $m\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, $\beta_m(\xi,\alpha)\beta_{m+2}(\xi,\alpha)<0$ subject to $\beta_{m+1}(\xi,\alpha)=0$.*

Proof. For given $|\xi|<1$, $m\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$, it is evident that $\beta_m(\xi,\alpha)=\beta_m(\xi,\alpha+1)$ and $\beta_{m+2}(\xi,\alpha)=\beta_{m+1}(\xi,\alpha+1)$ if $\beta_{m+1}(\xi,\alpha)=0$. Hence, we can replace $\beta_m(\xi,\alpha)\beta_{m+2}(\xi,\alpha)<0$ with $\beta_m(\xi,\alpha+1)\beta_{m+1}(\xi,\alpha+1)<0$. Further, applying Corollary 1 gives $\beta_m(\xi,\alpha+1)\beta_m(\xi,\alpha+2)-\left(\beta_m(\xi,\alpha+1)\right)^2<0$. To prove the correctness of the inequality, we require $|\beta_m(\xi,\alpha+1)|>|\beta_m(\xi,\alpha+2)|$ that can be written as $|\beta_m(\xi,\alpha+1)|>|\beta_m(\xi,\alpha+1)+\beta_{m+1}(\xi,\alpha+1)|$. In accordance with the triangle inequality $|\beta_m(\xi,\alpha+1)+\beta_{m+1}(\xi,\alpha+1)|\leq|\beta_m(\xi,\alpha+1)|+|\beta_{m+1}(\xi,\alpha+1)|$, $\beta_m(\xi,\alpha)\beta_{m+2}(\xi,\alpha)<0$ as claimed. □

At this point we extend the main theorem postulated in [9] to the case of the Meixner filters by introducing some $\theta\in\Theta$ composed of $\gamma\in\Gamma$ and $|\xi|<1$.

Theorem 3. *Suppose that $\theta\in\Theta$, $m\in\mathbb{N}$, and $\alpha\in\{\alpha\in\mathbb{R}:\alpha>-1\}$. Then, account for the fact that $\beta_k(\theta,\alpha+1)=\beta_k(\theta,\alpha)\mp\beta_{k+1}(\theta,\alpha)$, the condition $\beta_{m+1}(\theta,\alpha)=0$ leads to the solution to the problem $\theta=\arg\min_{\theta\in\Theta}\Delta_m(\theta,\alpha)$, while the equation $\beta_m(\theta,\alpha)=0$ achieves the solution to the problem $\theta=\arg\max_{\theta\in\Theta}\Delta_m(\theta,\alpha)$.*

Proof. A proof is identical to that given for the theorem in [9].□

2.3. Solution to the two-parameter pole position problem

In this section, we supplement the conditions (6) and (7) with the following:

$$\frac{\partial \Delta_m(\xi,\alpha,\beta_k)}{\partial \alpha}=0;\quad \frac{\partial^2 \Delta_m(\xi,\alpha,\beta_k)}{\partial \alpha^2}>0, \tag{9}$$

which correspond to $\alpha = \arg\min_{\alpha>-1}\Delta_m(\xi,\alpha,\beta_k)$, to turn to the problem $(\xi,\alpha,\beta_k)=\arg\min_{\substack{|\xi|<1\\ \alpha>-1}}\Delta_m(\xi,\alpha,\beta_k)$ posed in Section 2.1.

For the sake of compactness, we present the approach to solving this problem considering the first derivatives in the conditions (6), (7), and (9). But, it can be simply extended to the appropriate second derivatives.

Let us solve the following equations:

$$\frac{\partial\left\|f-\sum_{k=0}^{m}\beta_k\psi_k(\omega)\right\|^2}{\partial\beta_k}=0;\quad \frac{\partial\left\|f-\sum_{k=0}^{m}\beta_k\psi_k(\omega)\right\|^2}{\partial\omega}=0, \tag{10}$$

where the model of f expressed as a sum of the almost orthogonal system $\langle\psi_k(\omega)\rangle_{k=0}^{\infty}$ includes a set of optimization parameters $\omega\in\Omega$. Then, working out the inner product of the sum in (10) results in $\langle\psi_n(\omega),f\rangle=\sum_{k=0}^{m}\langle\psi_n(\omega),\psi_k(\omega)\rangle\beta_k$, where β_k is the coefficients (5) defined for given ω. Computing the connection coefficients $\nu_{k,n}(\omega)=\langle\psi_k(\omega),\psi_n(\omega)\rangle$ leads to $\tilde{\beta}_k=\sum_{n=0}^{m}\nu_{k,n}^{-1}(\omega)\beta_n$, where $\tilde{\beta}_k$ is the coefficients (5) which correspond to the orthogonal system $\langle\tilde{\psi}_k(\omega)\rangle_{k=0}^{\infty}$. Following this result, we can define $\sum\langle\partial\psi_n(\omega)/\partial\omega,\psi_k(\omega)\rangle$ $\nu_{k,l}^{-1}(\omega)\beta_l$ as the inner product $\langle P(\omega)\partial\psi_k^{k,l}(\omega)\ /\partial\omega,f\rangle$, where $P(\omega)$ is the projection operator into the finite-dimensional subspace spanned by $\psi_k(\omega)$, while $\langle Q(\omega)\partial\psi_k(\omega)/\partial\omega,f\rangle=0$, where $Q(\omega)$ is the projection operator into the orthogonal complement so that $P(\omega)+Q(\omega)=I$ [11]. Then, the solution to the posed problem (10) can be obtained by varying the finite-dimensional subspace spanned by $\psi_k(\omega)$ in the Hilbert space so that the component of the vector $\partial\psi_k(\omega)/\partial\omega$ orthogonal to the span of $\psi_k(\omega)$ is orthogonal to f.

Below is a step-by-step procedure to solve the two-parameter pole position problem applying the described approach:

1. Determine the coefficients $\tilde{\beta}_k$ based on β_k;
2. Define a set of optimization parameters $\omega=(\xi,\alpha)$;
3. Compute the sets of connection coefficients that correspond to the inner products $\langle\partial G_k(\xi,\alpha)/\partial\xi, G_k(\xi,\alpha)\rangle$ and $\langle\partial G_k(\xi,\alpha)/\partial\alpha, G_k(\xi,\alpha)\rangle$;
4. Define a set of equations for (α,ξ) calculating the complements of $\partial G_k(\xi,\alpha)/\partial\xi$ and $\partial G_k(\xi,\alpha)/\partial\alpha$ orthogonal to $G_k(\xi,\alpha)$.

3. Computational experiments

To test the proposed approach and to bear out the theoretical outcomes, we now provide a couple of examples chosen from the literature. The first example was previously considered in [9] to illustrate the pole position problem solving results for the discrete Laguerre filters. So, for the sake of comparison, we provided the same example to prove the validity of the proposed extension to the Meixner filters. The choice of the second example is motivated by the literature review. As noted in [2–6], using the Meixner-like functions is a better option to represent functions with a slow initial start in comparison with the Laguerre functions. In attempt to highlight the cases where the Meixner filters can be a constructive alternative to the discrete Laguerre filters, we conducted a series of computational experiments to compare the performance of these two families and to probe into the possibility of using the rational Meixner filters to solve two-parameter pole position problem taking the example provided in [2].

Example 1. First we consider the transfer function $H(z)$ of a supersonic jet engine inlet described in [14], which is given by $H(z)=z(2.034z^6\ -4.9825z^5+6.57z^4-5.8189z^3+3.636z^2-1.4105z+0.2997)/(z^7-2.46z^6+3.433z^5-3.333z^4+2.546z^3-1.584z^2+0.7478z-0.252)$. Solving the pole position problem gives the normalized truncation error curves $\delta_m(\xi,\alpha)=\Delta_m(\xi,\alpha)/\|H\|^2$ for fixed $\alpha=0.15$ and $m=[0,5]$ depicted in Fig. 1(a). The markers used to illustrate the solutions to $\beta_{m+1}(\xi,\alpha)=0$. In addition, we conducted a series

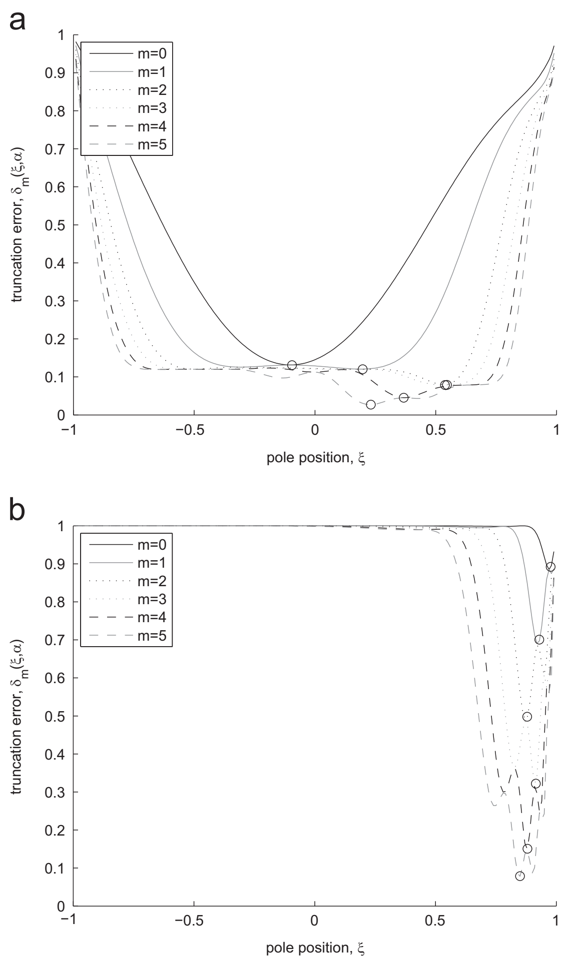

Fig. 1. (a) The normalized truncation error curves $\delta_m(\xi,\alpha)$ correspond to $m=[0,5]$ and $\alpha=0.15$ (Example 1). (b) The normalized truncation error curves $\delta_m(\xi,\alpha)$ correspond to $m=[0,5]$ and $\alpha=0.2$ (Example 2).

of the computational experiments in the case of $\alpha=\{-0.05, 0, 0.15\}$ (see Table 1) to estimate the influence of the parameter α on the following results: the numerical values of the solutions ξ, the normalized truncation error values $\delta_m(\xi,\alpha)$, and the Meixner coefficients $\beta_m(\xi,\alpha)$, $\beta_{m+2}(\xi,\alpha)$ to support Lemma 2 for the optimal pole positions.

Example 2. The other example is the z-transform $H(z)$ of the function having a slow start presented in [2] $H(z)=(z/(z-(0.9+0.1j)))^4+(z/(z-(0.9-0.1j)))^4$. In this case, Fig. 1(b) demonstrates the error curves $\delta_m(\xi,\alpha)$ and marks the solutions for $\alpha=0.2$ and $m=[0,5]$. By analogy, Table 2 shows the validity of the theoretical results for $\alpha=\{-0.1, 0, 0.2\}$.

As can be seen from the data in Fig. 1 and Tables 1 and 2, the results supporting Lemma 2 and Theorem 3 are consistent across all the computational experiments. However, we need to study the issue of using the proposed Meixner filters for each of the examples from the viewpoint of implementation.

The study [14] provides a solution to a model reduction problem making a point about a relatively high order of a Laguerre model to present the function from Example 1. In comparison with the discrete Laguerre filters, the rational Meixner filters include additional all-pass filters, which in case of noninteger α should be considered in terms of fractional order systems. For this reason, using the Meixner filters may not be suitable. In attempt to reduce the complexity of the Meixner model by optimizing an extra parameter, we employed the approach proposed in Section 2.3. The two-dimensional truncation error surface is plotted in Fig. 2(a) for fixed $m=5$. The results are presented in Table 3, where the highlighted values correspond to the solutions to the two-parameter pole position problem. As we can see, the Meixner model in case of

Table 1
The numerical values of the computational experiments results for $m=[0,5]$ and $\alpha=\{-0.05, 0, 0.15\}$ (Example 1).

m	ξ	$\delta_m(\xi,-0.05)$	$\beta_m(\xi,-0.05)$	$\beta_{m+2}(\xi,-0.05)$
0	0.05559	0.154114	2.04136	−0.3921
1	0.34404	0.135662	−0.76907	0.40725
2	0.61506	0.070919	1.26555	−0.14208
3	0.52393	0.067878	0.45445	−0.12571
4	0.42947	0.065522	0.31422	−0.25751
5	0.24111	0.02341	0.68916	−0.07024
m	ξ	$\delta_m(\xi,0)$	$\beta_m(\xi,0)$	$\beta_{m+2}(\xi,0)$
0	0.0163	0.144027	2.044403	−0.3362
1	0.31272	0.126422	−0.7951	0.29546
2	0.59511	0.065033	1.28178	−0.06933
3	0.54317	0.064591	0.27292	−0.13786
4	0.41511	0.058449	0.38066	−0.24986
5	0.23812	0.023641	0.66331	−0.06816
m	ξ	$\delta_m(\xi,0.15)$	$\beta_m(\xi,0.15)$	$\beta_{m+2}(\xi,0.15)$
0	−0.09505	0.130929	2.03612	−0.30639
1	0.19776	0.120073	−0.49302	0.10444
2	0.5385	0.079	0.98101	−0.00502
3	0.5467	0.078784	0.25825	−0.12917
4	0.36745	0.045114	0.43795	−0.34008
5	0.23136	0.027222	0.66968	−0.07525

Table 2
The numerical values of the computational experiments results for $m=[0,5]$ and $\alpha=\{-0.1, 0, 0.2\}$ (Example 2).

m	ξ	$\delta_m(\xi,-0.1)$	$\beta_m(\xi,-0.1)$	$\beta_{m+2}(\xi,-0.1)$
0	0.98163	0.925025	−3354.25	2435.27
1	0.93477	0.755271	−3266.51	3431.97
2	0.88481	0.544809	−3150.14	2866.34
3	0.83713	0.395557	−2655.37	2004.39
4	0.88618	0.186432	3001.42	−1838.69
5	0.853434	0.092719	2317.38	−1171.3
m	ξ	$\delta_m(\xi,0)$	$\beta_m(\xi,0)$	$\beta_{m+2}(\xi,0)$
0	0.97878	0.910969	−3275.66	2372.03
1	0.93262	0.730402	−3131.2	3335.49
2	0.88302	0.522051	−2959.77	2845.29
3	0.91943	0.365431	3268.31	−2545.7
4	0.88439	0.170942	2861.92	−1763.46
5	0.85283	0.086198	2165.88	−1141.8
m	ξ	$\delta_m(\xi,0.2)$	$\beta_m(\xi,0.2)$	$\beta_{m+2}(\xi,0.2)$
0	0.97317	0.885844	−3090.62	2241.75
1	0.92778	0.693985	−2914.57	3085.99
2	0.87924	0.496094	−2614.25	2721.86
3	0.91508	0.320957	3143.74	−2260.16
4	0.88074	0.151216	2594.43	−1598.88
5	0.8499	0.07881	1878.62	−1070.58

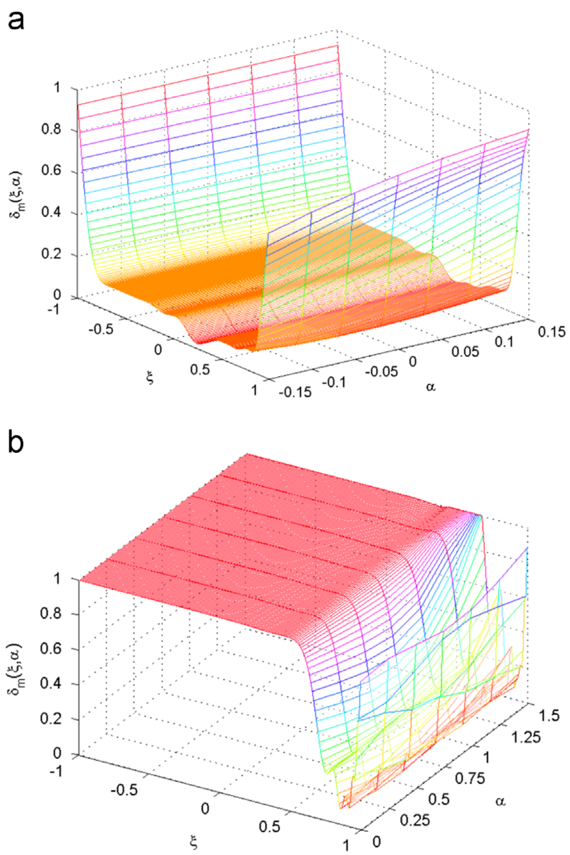

Fig. 2. (a) The normalized truncation error curves $\delta_m(\xi,\alpha)$ correspond to $m=5$ and $\alpha=[-0.15, 0.15]$ (Example 1). (b) The normalized truncation error curves $\delta_m(\xi,\alpha)$ correspond to $m=5$ and $\alpha=[0, 1.25]$ (Example 2).

Table 3
The numerical values of the computational experiments results for fixed $m=5$.

Example 1			Example 2		
α	ξ	$\delta_5(\xi,\alpha)$	α	ξ	$\delta_5(\xi,\alpha)$
−0.15	0.24998	0.024196	0	0.85283	0.086198
−0.1	0.24579	0.023609	0.25	0.84918	0.077589
−0.05	**0.24183**	**0.023406**	0.5	0.89844	0.065408
0	0.23812	0.023641	0.75	0.89377	0.054573
0.05	0.23537	0.024348	1	0.88914	0.04664
0.1	0.23289	0.02554	1.25	0.88461	0.040736
0.15	0.23071	0.027214	**1.5**	**0.88019**	**0.036267**

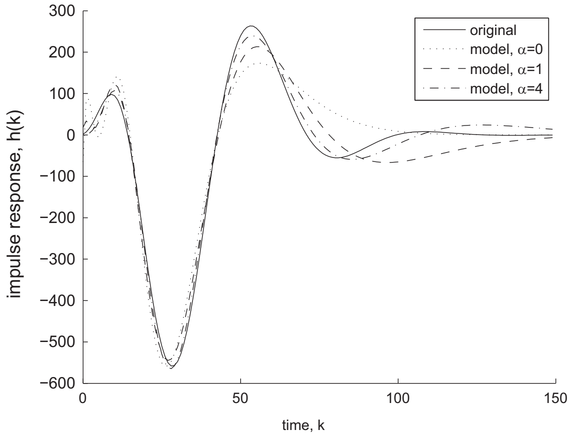

Fig. 3. The impulse response function and its approximations correspond to $m=5$ and $\alpha=\{0,1,4\}$ (Example 2).

$\alpha=-0.05$ provides better result, but this contribution is rather not significant.

If we look at Example 2, we see that this system is not within the model of the Meixner filters. It is due to the fact that the poles are complex-valued even if they are chosen close to real axis. Nevertheless, this example enables us to reveal the benefits of using Meixner filters. The results of applying the proposed approach are shown in Fig. 2(b) and Table 3, according to which we can conclude that increasing the value of α leads to better approximation results. Moreover, this parameter can be used to emphasize the low-frequency components at the expense of high-frequency components [2]. We provided the approximation results for given $\alpha=\{0,1,4\}$ and optimal poles $\xi=\{0.85283, 0.88914, 0.88057\}$ in Fig. 3. These results are similar to that presented in [2], but the structure of the proposed rational filters seems to be less complex due to using explicit formulas in lieu of the cascaded filter structure described in [2,4].

4. Conclusion

In this study, we posed the two-parameter pole position problem for the Meixner filters. To solve this problem, we first extended the theoretical findings in solving the pole position problem for the discrete Laguerre filters to the case of the proposed Meixner filters. Then, we proposed an approach to solving $(\xi,\alpha)=\arg\min_{|\xi|<1,\ \alpha>-1}\Delta_m(\xi,\alpha)$ based on the connection coefficients method. To test this approach, to verify the theoretical results, and to point up the positive outcomes of using the Meixner filters in signal processing applications, we carried out a series of computational experiments, the results of which confirm the validity of the theoretical outcomes and raise some questions for further research: considering the possibility for extending the results to the Kautz filters and Malmquist–Takenaka systems; drawing the similarity between the connection coefficients for ξ and α to reduce computational costs in solving the two-parameter pole position problem.

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