Fuzzy Learning Performance Assessment Based on Decision Making Under Internal Uncertainty

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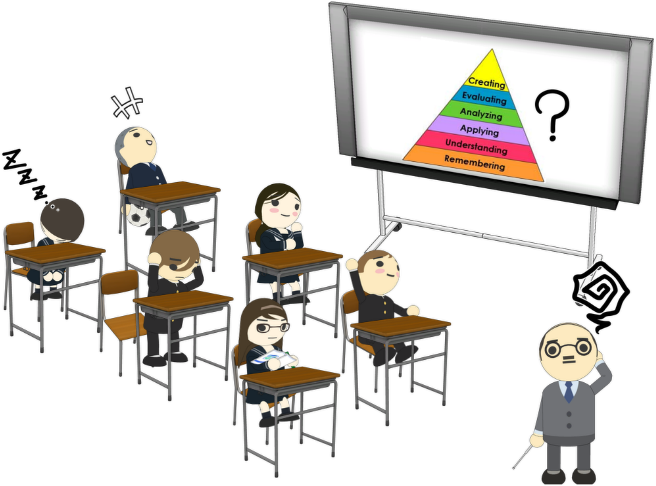
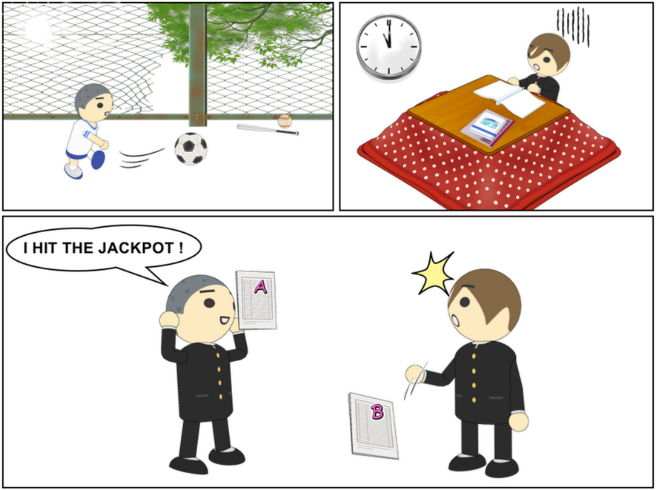
Outline

Bloom's taxonomy



Problem statement



Binary knowledge model

The score definitions based on a binary knowledge model

$$u_{ik}^{B} = \left((\theta_k > l_i) + \frac{1}{M}(\theta_k \leq l_i)\right)u_{ik}, \tag{1}$$

$$u_{ik}^{BT} = (\theta_k > l_i)u_{ik}, \tag{2}$$

$$u_{k}^{B/BT} = (1+p)\sum_{i=1}^{N} u_{ik}^{B/BT} - p\sum_{i=1}^{N}\left(u_{ik}^{B/BT} + (u_{ik}^{B/BT} = 0)\right),$$

$$\tilde{u}_{k}^{B/BT} = \frac{\max\{u_{k}^{B/BT}/N, 0\} + pK}{(1+p)K}, \tag{3}$$

u_{ik} is the k^{th} student's answer to the i^{th} item; $\theta_k \in [0,1]$ is the k^{th} student's level of ability; $l_i \in [0,1]$ is the difficulty of the i^{th} item; K, N, M are the numbers of students, items, and alternatives, respectively; $p \in [0,1]$ is a penalty for each incorrect answer.

Partial knowledge model

The score definitions based on a partial knowledge model

$$\begin{aligned} u_{ik}^{P} &= \big(c_k + (1 - c_k)F(\theta_k - l_i)\big)u_{ik}, & (4) \\ u_{ik}^{PT} &= F(\theta_k - l_i)u_{ik}, & (5) \\ u_k^{P/PT} &= (1+p)\sum_{i=1}^{N} u_{ik}^{P/PT} - p\sum_{i=1}^{N}\Big(u_{ik}^{P/PT} + (u_{ik}^{P/PT} = 0)\Big), \\ \tilde{u}_k^{P/PT} &= \frac{\max\{u_k^{P/PT}/N, 0\} + pK}{(1+p)K}, & (6) \end{aligned}$$

c_k is the probability that the k^{th} student's guess results in a correct answer; $F(\theta_k - l_i) = \dfrac{1}{1 + \exp\big(-(\theta_k - l_i)/g_i\big)}$, g_i is a parameter related to the difference between knowledge and difficulty in defining the probability of knowing the answer; $n_k = \sum_{i=1}^{N}(u_{ik} + (u_{ik} = 0))$

Definitions

$$\tilde{u}_k^F = \tilde{u}_k^{PT} + w_k,$$

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Fuzzy assessment model (FAM)

Let U is a fuzzy set of answers subject to guess results in answering. Then, L is a set of degrees of items difficulty related to Bloom's cognitive levels. Then, FAM can be given as $L = \{(u, \mu_L(u))|u \in U\}$, where $\mu_L(u) : U \to [0, 1]$ is the membership function of $L \subset U$.

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Fuzzy assessment problem (FAP)

According to Reiter's Theory of Diagnosis, FAP can be defined as a pair $(\mathbf{COMPS}, \mathbf{TSD})$ if the set $L \subset \mathbf{COMPS}$ such that

$$\mathbf{TSD} \cup \mathbf{OBS}\{\mathbf{AB}(u) | u \in L\} \cup \{\leftarrow \mathbf{AB}(u) | u \in \mathbf{COMPS} - L\}$$

is consistent.

Fuzzy assessment

Step-by-step procedure

Step 1: Define **COMPS**$=\{U, L, \Theta, G, p\}$, where $\theta \in \Theta$, $g \in G$, and $\mu_L(u)$ is the triangular membership function with six center points $\{0.1, 0.26, 0.42, 0.58, 0.74, 0.9\}$;

Step 2: Establish **TSD** as Mamdani's max-min interference mechanism based on the rule base for partial knowledge presented in Fig.1:

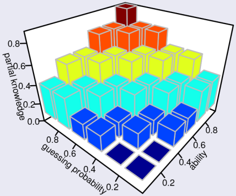


Fig. 1. The rule base

Step 3: Compute $U^F = \mathbf{AB}(U)$, where the predicate $\mathbf{AB}(.)$ maps the score U into the required score U^F applying the COG method;

Step 4: Provide **OBS** $= \{K, N\}$, where K and N are the numbers of students and items, respectively.

Parameters

Simulation parameters

- number of students is $K = 10$
- level of ability is $\theta = 0.6$
- number of items is $N = 6$
- degree of guessing is $c = 0.25$
- discrimination parameter is $g = 3/2\pi$
- value for penalty $p = 0.1$

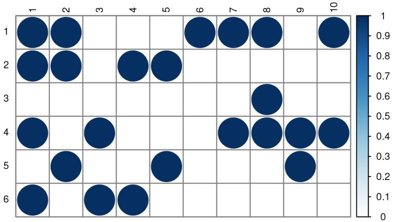


Fig. 2. The modelled answers u_{ik} subject to $N = 6$, $K = 10$: the columns correspond to students; the rows - tests

Knowledge models

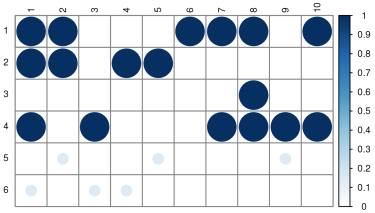


Fig. 3(a) Student either knows or guesses u_{ik}^{B} (1)

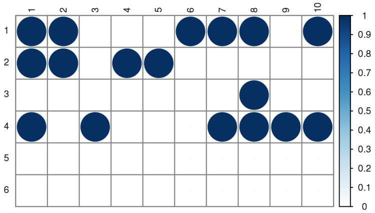


Fig. 3(b) Student knows u_{ik}^{BT} (2)

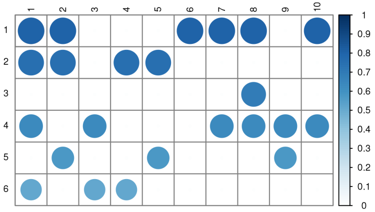


Fig. 4(a) Student either knows or guesses u_{ik}^{P} (4)

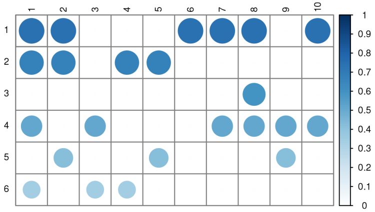


Fig. 4(b) Student knows u_{ik}^{PT} (5)

Test results

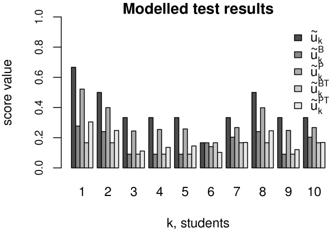
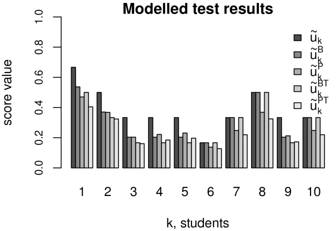


Fig. 5(a) $\theta = 0.6$, $c = 0.25$ (3), (6)



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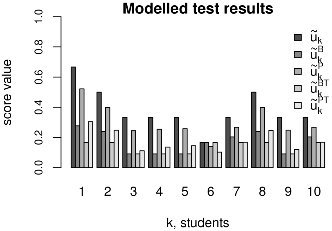

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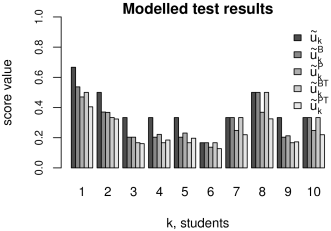
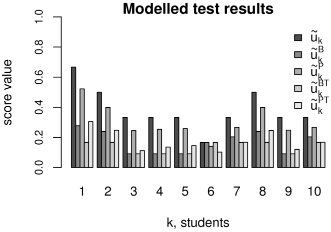


Fig. 5(b) $\theta = 0.25$, c $= 0.6$ (3), (6)

Higher level of ability,

more consistent and reliable score.

Test results



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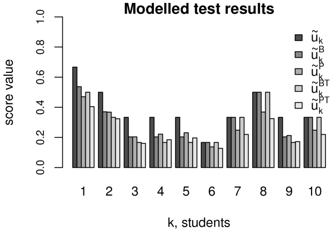

Fig. 5(b) $\theta = 0.25$, c $= 0.6$ (3), (6)

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Comparison between the assessment results

Item level	$\tilde{u}_i^P$	$\tilde{u}_i^F$	$\tilde{u}_i^P$	$\tilde{u}_i^F$	$\tilde{u}_i^P$	$\tilde{u}_i^F$
	θ=0.8, c=0.4		θ=0.6, c=0.3		θ=0.3, c=0.2	
1	0.5386	0.5147	0.5	0.4753	0.4268	0.4005
2	0.3467	0.338	0.3162	0.3071	0.2606	0.2526
3	0.0909	0.0909	0.0909	0.0909	0.0909	0.0909
4	0.4734	0.4874	0.4131	0.4314	0.3196	0.3479
5	0.2233	0.2238	0.1906	0.1954	0.1439	0.1565
6	0.2096	0.2121	0.1755	0.1849	0.1302	0.1536

Model validation

Piloting

- 9 PhD students
- 30 minutes for multiple-choice tests
- on English prepositions
- levelled according to Bloom's taxonomy
- FAM parameters are $\theta = 0.6$ and $p = 0.1$

Results

1. Marked trend in categorising the answers according to the cognitive levels
2. More definite distinction between true score and fuzzy score in the case $\theta_k < l_i$

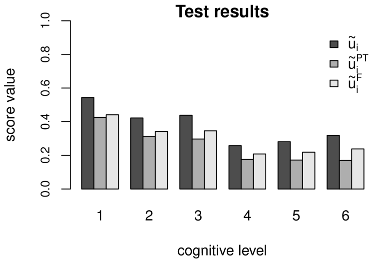

Fig. 6. The result before and after applying the proposed FAM

Conclusions

1. Fuzzy assessment based on decision making with partial knowledge was proposed and formalised according to Reiter's Theory of Diagnosis
2. Provided assessment model was tested on modelled answers with respect to the knowledge frameworks and validated in a real-world context
3. Results showed the required relationship between the level of ability and the level of partial knowledge
4. The purpose stated in this study is accomplished

Future work

- To analyse a number of responses alternatives, a value for penalty, and response time
- To study the dynamics of multiple-choice decision making to extend the proposed results to the case of formative assessment

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