The effect of anomalous electron viscosity on edge transport

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Abstract

The introduction of an anomalous electron viscosity on the order of the anomalous electron thermal diffusivity has been shown to limit the size of edge-localized modes during nonlinear drift-MHD simulations [1]. Kinematic electron viscosity μ_e is equivalent to hyper-resistivity $\eta_H=\mu_e d_e^2=2.8\text{x}10^{-6}\mu_e$ cm^2 ($n_e/10^{19}$m^{-3}), where d_e is the electron skin depth. If set by anomalous processes [2], μ_e may increase with temperature as in gyro-Bohm scaling and could achieve values on the order of ~1 m^2/s at the plasma edge. Thus, hyper-resistivity will dominate resistivity, η_R=0.021 m^2/s $T_{keV}^{-3/2}$, at small enough distances and high enough temperatures. For the parameters above, hyper-resistivity dominates for spatial scales below $(\mu_e/\eta_R)^{1/2}d_e$~1 cm. By regulating the amount of reconnection, μ_e has a strong impact on transport due to magnetic perturbations and in the stability of the hyper-resistive ballooning mode [3]. Thus, it may play an important role in setting the structure of the H-mode pedestal [1].

Hyper-resistivity can increase the rate of reconnection γ by increasing the width of the reconnection zone δ relative to its length L [4]. If plasma flows out of the reconnecting layer at the Alfvén speed V_A, conservation of mass limits the reconnection rate by the aspect ratio of the layer $\gamma L/V_A\sim\delta/L$. Hyper-resistivity increases both scales to $S_H^{-1/4}$ where the hyper-Lundquist number is $S_H\sim L^3V_A/\eta_H$ instead of $S_R^{-1/2}$ where the resistive Lundquist number is $S_R\sim LV_A/\eta_R$. Hyper-resistivity also increases the ability of external perturbations to cause reconnection and transport. Shielding currents eliminate reconnection in a plasma that is rotating faster than a critical frequency [5]. The critical frequency is larger in the hyper-resistive case, $\sim S_H^{-1/5}$ rather than $\sim S_R^{-1/3}$, when inertia dominates viscosity.

[1] X. Q. Xu, B. Dudson, P. B. Snyder, et al., Phys. Rev. Lett. **105**, 175005 (2010).

[2] H. Biglari and P. H. Diamond, Phys. Fluids B **5**, 3838 (1993).

[3] M. Yagi, K. Itoh, S.-I. Itoh, et al, Phys. Fluids B **5**, 3702 (1993).

[4] P. K. Kaw, E. J. Valleo and P. H. Rutherford, Phys. Rev. Lett. **43**, 1398 (1979).

[5] R. Fitzpatrick, Phys. Plasmas **5**, 3325 (1998).

Outline

- **Motivation: Recent BOUT++ results show a marked decrease in ELM size with increasing electron viscosity**
  - What is the physical explanation for the observations?
  - Electron viscosity is equivalent to hyper-resistivity
- **Linear & non-linear theory of hyper-resistive reconnection**
  - Understand role of hyper-resistivity/electron viscosity in reconnection
  - Is there a fast mechanism for ELM reconnection?
- **Implications for anomalous transport**
- **Conclusions for ELM simulations**

Motivation: electron viscosity can potentially have a large impact on resistive transport and reconnection phenomena

- **The equivalence between electron viscosity and hyper-resistivity**
  - Neglecting electron inertia and terms of order m_e/m_i

$$en(\mathbf{E}+\mathbf{V_e}\times\mathbf{B}) = en\eta_R\mathbf{J} + m_e n\mu_e\nabla^2\mathbf{V}_e$$

$$\partial_t\mathbf{A} = -\nabla\phi + \mathbf{V_e}\times\mathbf{B} - \eta_R\mathbf{J} + \frac{\mu_e m_e}{e^2 n}\nabla^2\mathbf{J}$$

- **Thus, we find that there is a hyper-diffusion of magnetic flux**
  - Since Ampere's Law

$$\nabla^2\mathbf{A} = -\frac{4\pi}{c}\mathbf{J} \quad \Rightarrow \quad D_H = \frac{\eta_H}{4\pi c^2} = \mu_e d_e^2 \approx 2.8\times10^{-6}\mu_e \times\left(\frac{n}{10^{13}cm}\right)$$

$$d_e = \frac{c}{\omega_{pe}}$$

- **Hyper-resistivity always dominates resistivity at small enough spatial scales**

$$\eta_R < \eta_H/\delta^2$$

$$\delta^2 < \eta_H/\eta_R \quad \Rightarrow \quad \left(\delta/d_e\right)^2 < \mu_e/D_R$$

$$D_R = \frac{\eta_R}{4\pi c^2}$$

What is the source of anomalous electron viscosity? What is the possible scaling with plasma parameters?

- Biglari & Hahm proved that it is difficult to generate significant anomalous viscosity with **electrostatic perturbations***
  - Nonetheless, ion-scale modes cause significant electron transport at the gyro-Bohm level

$$\mu_e \sim D_{gB} = C_s \rho_s^2 / a \sim T^{3/2} \sim 0.1 - 10 m^2/s$$

- The edge plasma may be subject to **magneto-static perturbations** that cause tearing of the flux surfaces & reconnection of the field lines
  - Quasilinear estimate

$$\mu_e \sim 2D_M = 2\sqrt{\frac{2}{\pi}} V_T \int d\ell \frac{BB}{B_0^2} \sim T^{1/2} \sim 1 - 100 m^2/s$$

- Anomalous viscosity increases with temperature and hyper-resistivity decreases as 1/density, so that $\eta_H \sim T^{1/2}/n$ or $T^{3/2}/n$
  - Thus, it dominates resistivity $\eta_R \sim n^0/T^{3/2}$ at low enough collisionality
  - If μ_e =1 m2/s and Te~1 keV, η_H dominates η_R for spatial scales below

$$\delta < (\mu_e/\eta_R)^{1/2} d_e \sim 1 \text{ cm}$$

*H. Biglari and P. H. Diamond, Phys. Fluids B **5**, 3838 (1993).*

Drift MHD ELM-model implemented within BOUT++ code allows one to explore non-ideal effects on ELM stability*

- **Drift-MHD ELM model**

$$\frac{\partial \varpi}{\partial t} + \mathbf{v}_E \cdot \nabla \varpi = B_0 \nabla_\| J_\| + 2\vec{b}_0 \times \vec{\kappa}_0 \cdot \nabla P, \quad (1)$$

$$\frac{\partial P}{\partial t} + \mathbf{v}_E \cdot \nabla P = 0, \quad (2)$$

$$\frac{\partial A_\|}{\partial t} = -\nabla_\| \Phi + \frac{\eta}{\mu_0} \nabla_\perp^2 A_\| - \frac{\eta_H}{\mu_0} \nabla_\perp^4 A_\|, \quad (3)$$

$$\varpi = \frac{n_0 M_i}{B_0}\left(\nabla_\perp^2 \phi + \frac{1}{n_0 Z_i e} \nabla_\perp^2 P\right), \qquad \Phi = \phi + \Phi_0,$$

$$J_\| = J_{\|0} - \frac{1}{\mu_0} \nabla_\perp^2 A_\|, \qquad v_E = \frac{1}{B_0}(b_0 \times \nabla_\perp \Phi). \quad (4)$$

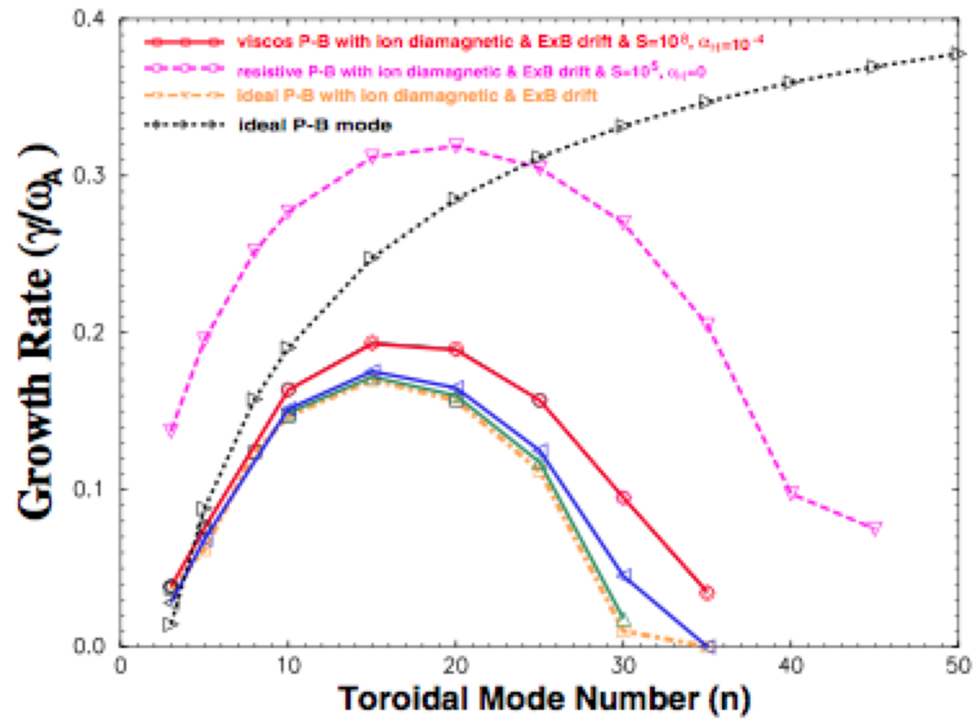


FIG. 1 (color online). Toroidal mode spectrum of the first equilibrium as calculated by BOUT++ for following cases: ideal MHD (black dotted line), ideal MHD with $E \times B$ and diamagnetic drift (yellow square), $S = 10^5$ and $S_H = \infty$ (pink inverted triangle), $S = 10^8$ and $\alpha_H = 10^{-4}$ (red open circle), $S = 10^8$ and $\alpha_H = 10^{-5}$ (blue right triangle), $S = 10^8$ and $\alpha_H = 10^{-6}$ (green triangle). The growth rates are normalized to the Alfvén frequency ω_A.

- **Linear stability is not strongly affected by η_H**

$$\alpha_H = \eta_H / R^2 \eta_R$$

$$S = V_A R / \eta_R$$

*X. Q. Xu, B. Dudson, P. B. Snyder, et al., Phys. Rev. Lett. **105**, 175005 (2010)

Recent simulations of ELMs using the BOUT++ code show that μ_e has a strong effect on ELM size*

- **ELM size is strongly reduced**
- **by η_H**

TABLE II. Elm sizes vs hyper-Lundquist number S_H

α_H ($S_H = S/\alpha_H$)	10^{-4}	6×10^{-5}	10^{-5}	5×10^{-6}
Case 1 $S = 10^7$	11.59%	8.45%	5.04%	
Case 1a $S = 10^7$	21.68%	17.97%	10.7%	
Case 2 $S = 10^8$	5.94%		0.22%	0.14%
Case 2a $S = 10^8$	11%		1.47%	1.5%

$$S_H = V_A R^3 / \eta_H$$

$$S = V_A R / \eta_R$$

- **Less reconnection appears to be generated by the ELM event**

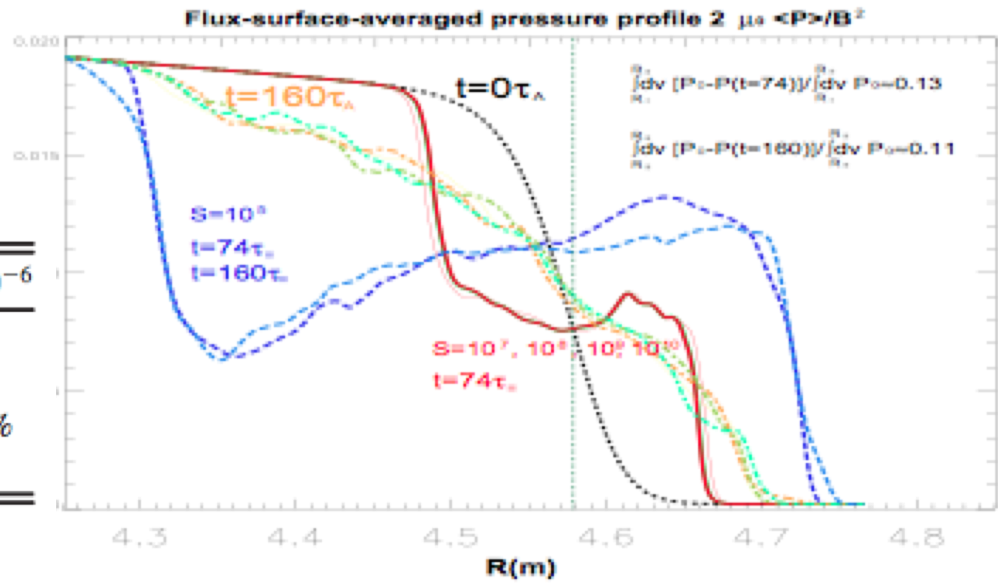


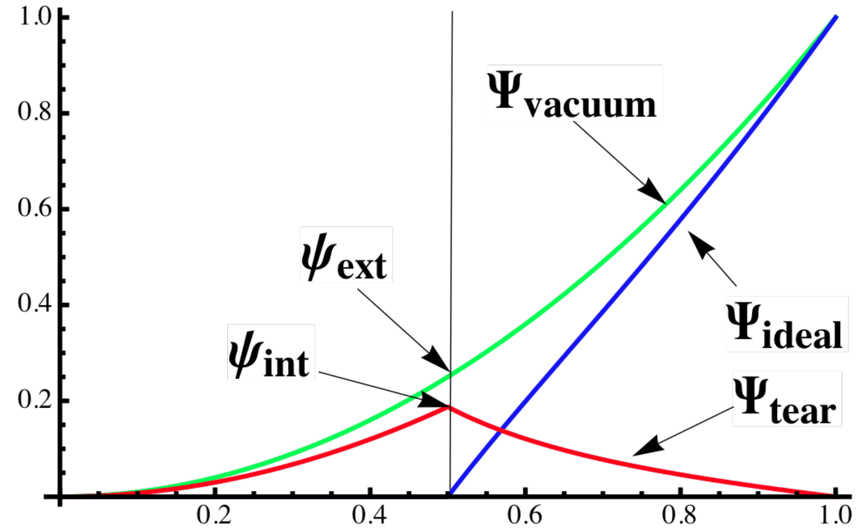
FIG. 2 (color online). Radial pressure profiles at several different Lunquist numbers S and time slices ($t = 0$, 74, $160\tau_A$) for the first equilibrium: The black dotted line for $t = 0$; blue dashed group lines for $S = 10^5$ at $t = 74\tau_A$ and $160\tau_A$; red solid group lines for $S \geq 10^7$ at $t = 74\tau_A$; yellow dotted-dashed group lines for $S \geq 10^7$ at $t = 160\tau_A$. The vertical line indicates the position of peak pressure gradient. Here $S_H = 10^{12}$.

Linear boundary layer reconnection theory

- At low collisionality, all perturbations are nearly ideal, except in the vicinity of a rational surface $k_{\parallel} = 0$.

- Far from the rational surface, the solution appears to be part ideal shear Alfven wave and part tearing mode

$$A_z = \psi_s A_{tear}(r_s) + \psi_a A_{ideal}(r_s)$$
$$A_{tear}(r_s) = 1, \qquad A_{tear}(a) = 0$$
$$A_{ideal}(r_s) = 0, \qquad A_{ideal}(a) = 1$$



- Non-ideal physics determines the plasma response inside the “linear layer”

$$\Delta_{layer} = -\tfrac{4\pi}{c}\int J_z dx / \psi_s \propto \left(i\omega\tau_A\right)^{\alpha} \qquad \Delta = \left[\frac{r}{A_z}\frac{dA_z}{dr}\right] = \left[\frac{rB_y}{A_z}\right]$$

- Plasma rotation generally acts to shield the reconnected flux

$$\Delta_{layer} = \Delta_{tear} + \Delta_{ideal}(\psi_a/\psi_s) \qquad \frac{\psi_s}{\psi_a} = \frac{\Delta_{ideal}}{\Delta_{layer} - \Delta_{tear}} \propto \frac{1}{\left(i\omega\tau_A\right)^{\alpha}}$$

Linear single-fluid reconnection theory: visco-resistive MHD

reviewed in Fitzpatrick PoP (1998)

- **2-field plasma model** (at low beta)

$$\partial_t A + \nabla_{\parallel}\phi = \eta\nabla_{\perp}^2 A$$

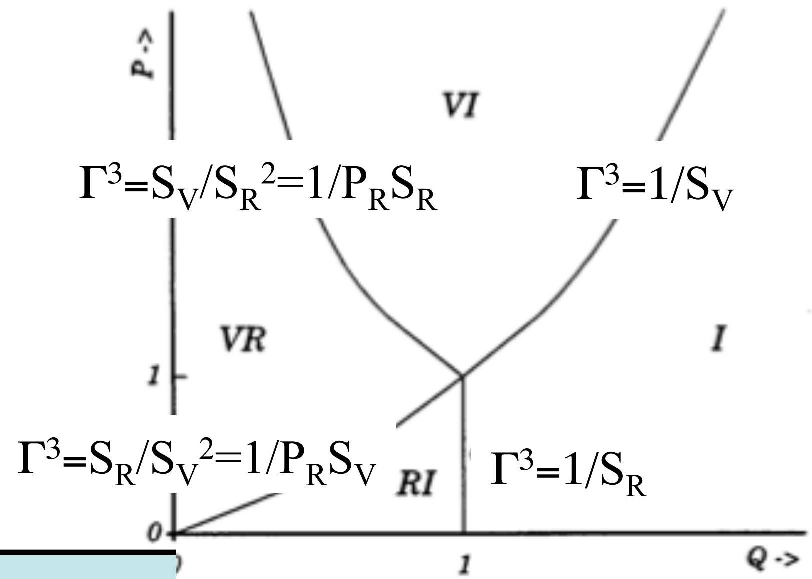
$$\left(\partial_t + v_E\cdot\nabla\right)\nabla_{\perp}^2\phi = \tfrac{v_A^2}{c}\nabla_{\parallel}\nabla_{\perp}^2 A + \mu_i\nabla_{\perp}^4\phi$$

$$\mu_s \rightarrow \mu_s / m_s n_s$$

$$\eta \rightarrow \eta_{\parallel} / \mu_0$$

- **2 reconnecting cases:** constant ψ

  2 ideal cases: not constant $\psi(r_s)=0$



1. A schematic diagram showing the extent of the four linear response ...es in normalized viscosity, P, versus normalized "slip frequency," Q. ... The four regimes are the *visco-resistive* regime (VR), the *resistive-*...*ial* regime (RI), the *visco-inertial* regime (VI), and the *inertial* regime

Regime	Δ	δ	$\Gamma=\omega\tau_A$
RI: inertia + resistivity	$2.1 S_R^{3/4}\Gamma^{5/4}$	$\left(\Gamma/S_R\right)^{1/4}$	$S_R^{-3/5}$
I: inertia + inductance	$-\pi/\Gamma$	$1/\Gamma$	$1/\pi$
VR: viscosity + resistivity	$2.1 S_R^{5/6} S_V^{-1/6}\Gamma$	$S_R^{-1/6} S_V^{-1/6}$	$S_R^{-5/6} S_V^{1/6}$
VI: viscosity + inductance	$-4.6\left(S_V/\Gamma\right)^{1/4}$	$\left(\Gamma/S_V\right)^{1/4}$	S_V

- **Dimensionless #'s**

$$S_R = \tau_R/\tau_A = L v_A/\eta$$

$$S_V = \tau_V/\tau_A = L v_A/\mu_i$$

$$P_R = \tau_R/\tau_V = \mu_i/\eta$$

$$\Gamma = \omega\tau_A \qquad Q_R = \omega\tau_A S_R^{1/3}$$

Linear single-fluid reconnection theory: with hyper-resistivity/electron viscosity

- **2-field plasma model** (at low beta)

$$\partial_t A + \nabla_\parallel \phi = \eta_\parallel \nabla_\perp^2 A - \mu_e d_e^2 \nabla_\perp^4 A$$

$$\left(\partial_t + v_E \cdot \nabla\right)\nabla_\perp^2 \phi = \tfrac{v_A^2}{c} \nabla_\parallel \nabla_\perp^2 A + \mu_i \nabla_\perp^4 \phi$$

$$d_e = c/\omega_{pe}$$

$$\mu_s \to \mu_s / m_s n_s$$

$$\eta \to \eta_\parallel / \mu_0$$

- **2 new reconnecting cases:** constant ψ

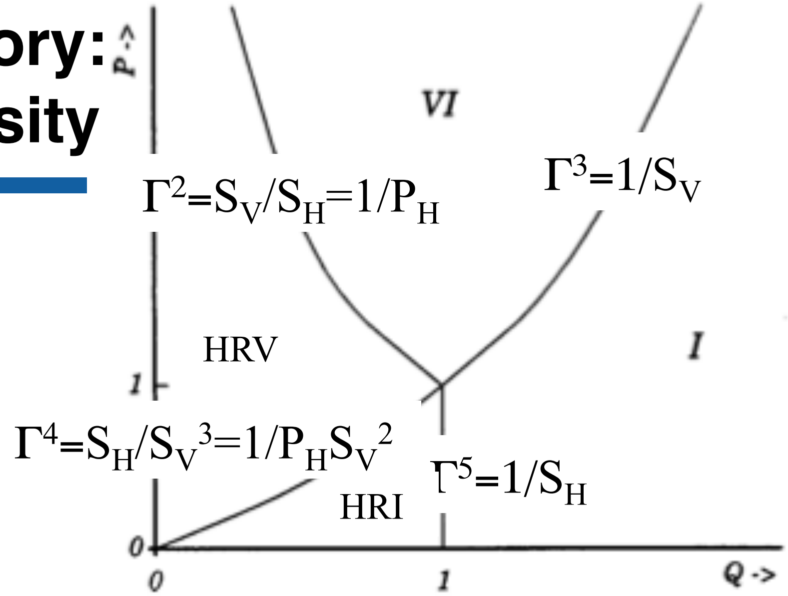


FIG. 1. A schematic diagram showing the extent of the four linear response regimes in normalized viscosity, *P*, versus normalized "slip frequency," *Q*, space. The four regimes are the *visco-resistive* regime (VR), the *resistive-inertial* regime (RI), the *visco-inertial* regime (VI), and the *inertial* regime (I).

Regime	Δ	δ	$\Gamma = \omega\tau_A$
HRI: inertia + hyper-resistivity	$0.79 S_H^{1/2} \Gamma^{3/2}$	$\left(\Gamma / S_H\right)^{1/6}$	$S_H^{-1/3}$
I: inertia + inductance	$-\pi / \Gamma$	$1/\Gamma$	$1/\pi$
HRV: viscosity + hyper-resistivity	$3.2 S_H^{5/8} S_V^{-3/8} \Gamma$	$S_H^{-1/8} S_V^{-1/8}$	$S_H^{-5/8} S_V^{3/8}$
VI: viscosity + inductance	$-4.6\left(S_V / \Gamma\right)^{1/4}$	$\left(\Gamma / S_V\right)^{1/4}$	S_V

- **Dimensionless #'s**

$$S_R = \tau_R / \tau_A = L v_A / \eta$$

$$S_V = \tau_V / \tau_A = L v_A / \mu_i$$

$$\Gamma = \omega \tau_A \qquad Q_R = \omega \tau_A S_R^{1/3}$$

- **Hyper-Lundquist #**

$$S_H = \tau_H / \tau_A = L^3 v_A / \mu_e d_e^2$$

$$P_H = \tau_H / \tau_V = \mu_e d_e^2 / \mu_i L^2$$

$$Q_H = \omega \tau_A S_H^{1/4}$$

Nonlinear MHD reconnection is known to be SLOW: Sweet-Parker & hyper-Sweet-Parker

- Reconnection rate determined by balance between resistivity & convection

$$v/L \sim \eta/L^2$$

- At low collisionality, resistivity can only compete if the reconnection zone contracts to a narrow layer of width δ

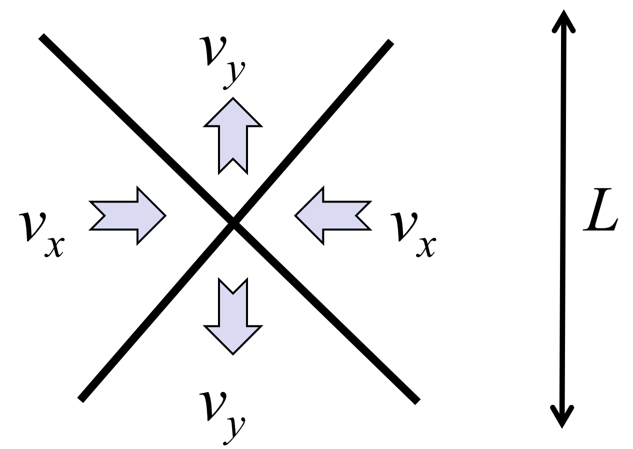
$$v_y/L \sim \eta_R/\delta^2 \sim \eta_H/\delta^4$$

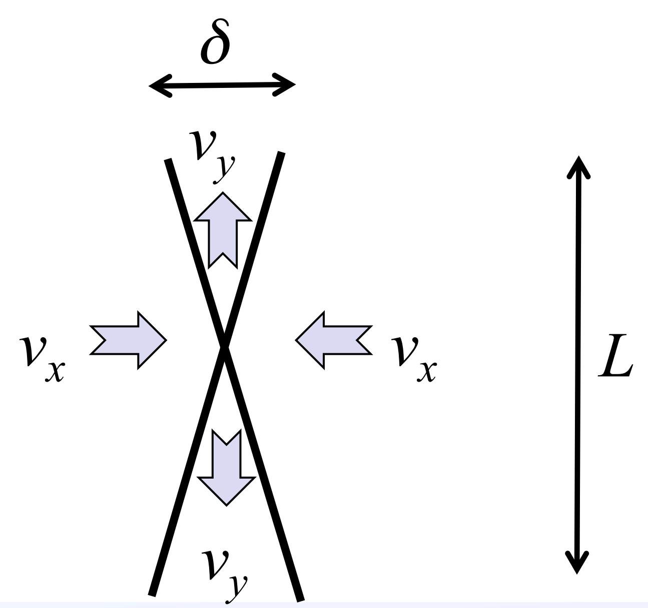
- Nonlinear forcing generates Alfvenic outflows $v_y \sim v_A$ and small widths

$$\delta_R/L \sim \left(\eta_R/v_A L\right)^{1/2} = S_R^{-1/2}$$

$$\delta_H/L \sim \left(\eta_H/v_A L^3\right)^{1/4} = S_H^{-1/4}$$

- But the reconnection rate is limited by the aspect ratio
  - Conservation of mass $\quad \gamma L/v_A \sim v_x/v_y \sim \delta/L$





Nonlinear MHD reconnection can lead to a secondary tearing instability that is FAST: the "plasmoid" instability

- The thin current sheet can become unstable to tearing itself and begin to copiously generate multiple island substructures that break up the sheet
  - N. Louriero et al Phys. Plasmas 14 100703 (2007)

- The maximum growth rate of the tearing mode is the Sweet-Parker rate applied to the sheet itself

$$S_\delta = v_A \delta / \eta_R = S_R^{1/2}$$

$$\gamma \delta / v_A \sim S_\delta^{-1/2} \sim S_R^{-1/4}$$

- For the most unstable modes

$$k_x \delta \sim S_\delta^{\ 1/2} \qquad k_y \delta \sim S_\delta^{\ -1/4}$$

$$k_x L \sim S_R^{3/4} \qquad k_y L \sim S_R^{\ 1/4}$$

- **Relative to the large scales, the resistive plasmoid instability is Super-Alfvenic!**

$$\gamma L / v_A \sim S_\delta^{\ 1/2} \sim S_R^{\ 1/4}$$

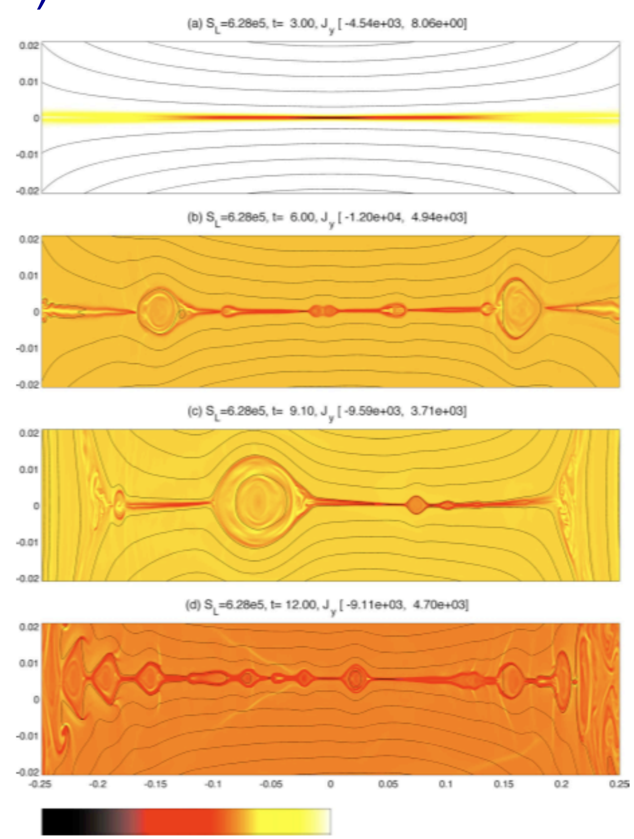


FIG. 1. (Color online) Time-sequence of the nonlinear evolution of the current density J_y of a Sweet–Parker current sheet in a large system of Lundquist number $S_L = 6.28 \times 10^5$. The black lines represent surfaces of constant ψ.

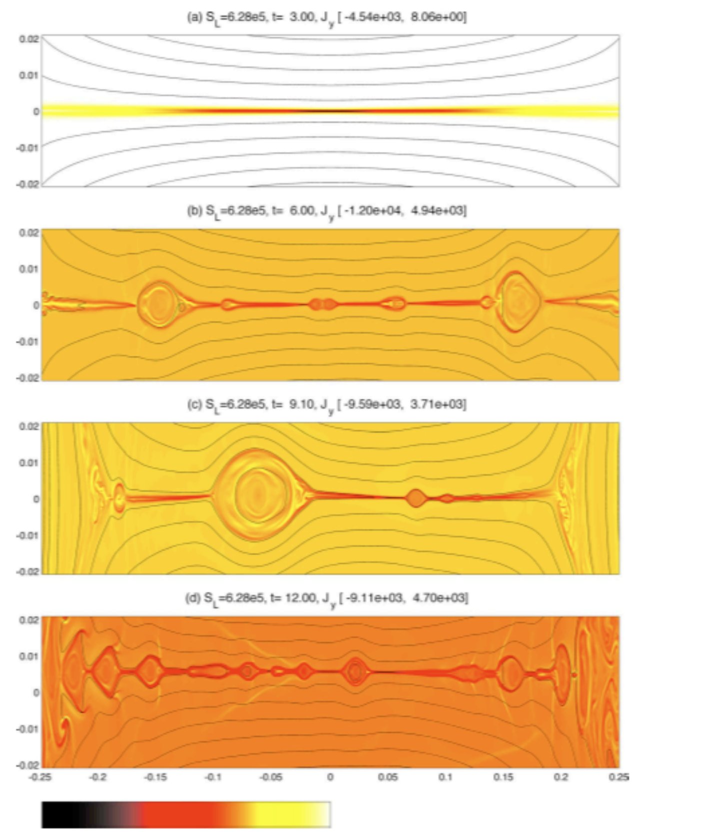
A. Bhattacharjee, et al.
Phys. Plasmas 16 112102 (2009)

Hyper-resistive plasmoid instability is FAST too!

- The maximum growth rate of the hyper-resistive tearing mode is the hyper-Sweet-Parker rate

$$S_\delta = v_A \delta^3 / \eta_H = S_H^{1/4}$$

$$\gamma\delta / v_A \sim S_\delta^{-1/4} \sim S_H^{-1/16}$$



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- For the most unstable modes

$$k_x\delta \sim S_\delta^{\;1/4} \qquad k_y\delta \sim S_\delta^{\;1/4}$$

$$k_x L \sim S_H^{5/16}$$

- **The hyper-resistive plasmoid instability is also super-Alfvenic!**

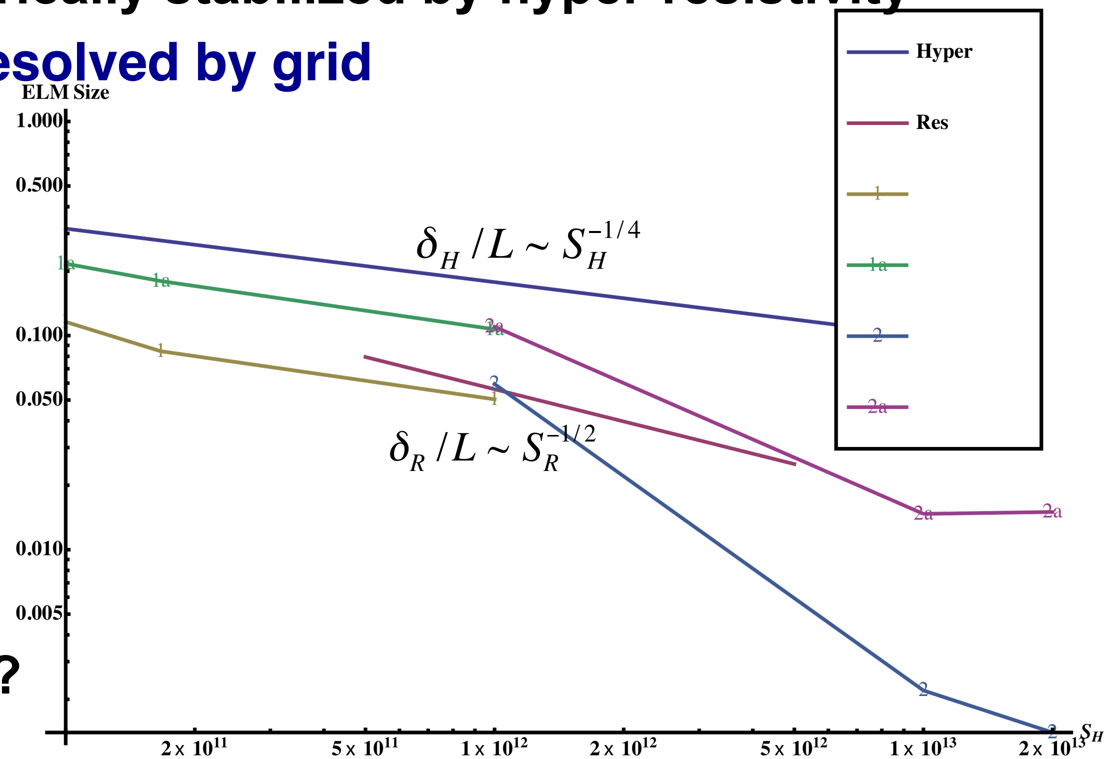
$$\gamma L / v_A \sim S_\delta^{3/8} \sim S_H^{3/16}$$

If hyper-resistivity aids reconnection, why does it reduce ELM size in BOUT++ simulations?

- **Large $S_R > 10^5$ runs are numerically stabilized by hyper-resistivity**
  - **Smallest scales can be resolved by grid**

- **Predicted ELM size increases with both resistivity and hyper-resistivity**
  - decrease with both S_R,S_H

- **Is reconnection responsible?**
  - **Cases 1,1a:** appear to follow hyper-resistive scaling
  - **Cases 2, 2a:** much faster scaling
    - Transition to another regime?



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Conclusions of hyper-resistive reconnection

- If set by anomalous electron viscosity, hyper-resistive transport may be a big player in setting the characteristics of ELMs & turbulent transport
  – Increases reconnection rates and scales

- Hyper-resistivity changes reconnection physics quantitatively
  – Changes linear layer widths and reconnection times
  – Nonlinear Sweet-Parker: $\gamma_H \sim \delta_H / L \sim S_H^{-1/4}$ vs $\gamma_R \sim \delta_R / L \sim S_R^{-1/2}$

- Hyper-resistivity changes reconnection physics quantitatively
  – Changes the border between ideal physics & reconnection & it's scaling

- Hyper-resistivity can be used numerically to make the layer widths large enough to be computationally resolved
  – However, ideal modes should not be allowed to tear prematurely
    - Should treat different regions as ideal or hyper-resistive based on internal plasma parameters