CSE103 Homework 9

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Language being referenced in this homework assignment comes from Canvas under Files → Handouts → CKY Algorithm Notes.pdf

Language $\mathcal{A}$:

$$S \to AB \mid BA \mid SS \mid AC \mid BD$$
$$A \to a$$
$$B \to b$$
$$C \to SB$$
$$D \to SA$$

Problem #1: Is string $babaab \in L(\mathcal{A})$?

6						
5						
4						
3						
2						
1	x					z
	b	a	b	a	a	b

Table 1: Initial Unfilled Table

I will denote each row as an array row[x][y], where x is the row number and y is the index of that array. For example, row[1][0] refers to the bottom left corner marked with an x while row[1][5] is marked with z.

row[1] will contain all possible derivations for the letter right below it.

6						
5						
4						
3						
2						
1	B	A	B	A	A	B
	b	a	b	a	a	b

Table 2: Row 1 filled

row[2] contains all possible derivations for substrings of length 2.

- row[2][0] will look at string ba. ba contains substrings b and a, which are derived from B and A in that order. The cartesian product is BA. BA is found in grammar $\mathcal{A}$ through S. Therefore, row[2][0] $= S$.

- row[2][1] will look at string ab. ab contains substrings a and b, which are derived from A and B in that order. The cartesian product is AB. AB is found in grammar $\mathcal{A}$ through S. Therefore, row[2][1] $= S$.

- row[2][2] will look at string ba. ba contains substrings b and a, which are derived from B and A in that order. The cartesian product is BA. BA is found in grammar $\mathcal{A}$ through S. Therefore, row[2][2] $= S$.

- row[2][3] will look at string aa. aa contains substrings a and a, which are derived from A and A in that order. The cartesian product is AA. AA is found in grammar $\mathcal{A}$ through $\emptyset$. Therefore, row[2][3] $= \emptyset$.

- row[2][4] will look at string ab. ab contains substrings a and b, which are derived from A and B in that order. The cartesian product is AB. AB is found in grammar $\mathcal{A}$ through S. Therefore, row[2][4] $= S$.

6						
5						
4						
3						
2	S	S	S	$\emptyset$	S	
1	B	A	B	A	A	B
	b	a	b	a	a	b

Table 3: Row 2 filled

row[3] will follow similar procedure as row[2]. We will look at substrings of length 3.

- row[3][0] will look at substring bab. This string can be further split up into two substrings, $s_1 = b, ab$ and $s_2 = ba, b$. I will look at s_1 first. b can be derived from B and ab can be derived from S. The cartesian product is BS. BS is not found in grammar $\mathcal{A}$. We will now look at s_2. ba is derived from S and b is derived from B. Cartesian product

SB can be found in grammar $\mathcal{A}$ through C. Therefore, since s_1 cannot be derived, but s_2 can, row[3][0] $= C$.

- row[3][1] will look at substring aba. This string can be further split up into two substrings, $s_1 = a, ba$ and $s_2 = ab, a$. I will look at s_1 first. a can be derived from A and ba can be derived from S. Cartersian product AS is not found in grammar $\mathcal{A}$. I will look at s_2 now. ab is derived from S and a is derived from a. Cartesian product SA is derived through D. Therefore, since s_1 cannot be derived, but s_2 can, row[3][1] $= D$.

- row[3][2] will look at substring baa. This string produces $s_1 = b, aa$ and $s_2 = ba, a$. There are no derivations for substring aa in grammar $\mathcal{A}$ as denoted in row[2][3]. For substring s_2, we can obtain ba from S and a from A. The cartesian product is SA, which can be obtain through D. Since s_1 does not contain a derivation, we do not take that into account and look onto at s_2's derivation. Therefore, row[3][2] $= D$.

- row[3][3] will look at substring aab. This string produces $s_1 = a, ab$ and $s_2 = aa, b$. We can discard taking into consideration s_2 because it does not have a derivation. s_1 can be derived from A and S. The cartesian product is AS, which is not in the grammar $\mathcal{A}$. Therefore, there are no derivations for any of these substrings so row[3][3] is $\emptyset$.

6						
5						
4						
3	C	D	D	$\emptyset$		
2	S	S	S	$\emptyset$	S	
1	B	A	B	A	A	B
	b	a	b	a	a	b

Table 4: Row 3 filled

row[4] will look at substrings of length 4.

- row[4][0] will look at substring $baba$. This string can be broken down into $s_1 = b, aba$, $s_2 = ba, ba$, and $s_3 = bab, a$. For string s_1, b is derived from B and aba is derived from D. Cartersian product is BD. This can be obtained through S. For string s_2, ba can be obtained from S. Since we have two of ba, the cartesian product is SS, which can be obtained through S. For string s_3, bab can be obtained through C and a is obtained through A. Cartesian product is CA, which is not obtained through the grammar at all. Therefore, since s_1 and s_2 can both be derived by S, we say that row[4][0] $= S$.

- row[4][1] will look at substring $abaa$. This string can be split up into: $s_1 = a, baa$, $s_2 = ab, aa$, and $s_3 = aba, a$. For string s_1, a is obtained through A and baa is obtained through D. Cartersian product AD is not in the grammar. For string s_2, aa has no derivation. For string s_3, aba is derived through D and a is derived through A. Carteisan product DA is also not in the grammar. Therefore, row[4][1] $= \emptyset$.

- row[4][2] will look at substring $baab$. This string produces substrings $s_1 = b, aab$, $s_2 = ba, ab$, and $s_3 = baa, b$. For string s_1, aab has no derivation. For string s_2, ba is obtained through S and ab is obtained through S. Carteisan product SS is obtained through S. For string s_3, baa is obtained through D and b is obtained through B. Cartesian product DB is not in the grammar. Therefore, row[4][2] $= S$.

6						
5						
4	S	$\emptyset$	S			
3	C	D	D	$\emptyset$		
2	S	S	S	$\emptyset$	S	
1	B	A	B	A	A	B
	b	a	b	a	a	b

Table 5: Row 4 filled

row[5] will look at substrings of length 5

- row[5][0] will look at substring $babaa$. This string can be split into: $s_1 = b, abaa$, $s_2 = ba, baa$, $s_3 = bab, aa$, and $s_4 = baba, a$. For string s_1, $abaa$ has no derivation. For string s_2, ba is derived from S and baa is derived from D. The cartesian product, SD, is not in the grammar. For string s_3, aa has no derivation. For string s_4, $baba$ is derived through S and a through A. Carteisan product, SA, is found through D. Therefore, row[5][0] = D.

- row[5][1] will look at substring $abaab$. This string produces substrings: $s_1 = a, baab$, $s_2 = ab, aab$, $s_3 = aba, ab$, and $s_4 = abaa, b$. String s_1 can be derived from A and S; however, cartesian product AS is not in the grammar. String s_2 has no derivation because of aab. String s_3 can be derived from D and S; however, DS is not in the grammar. s_4 has no derivation because $abaa$ does not. Therefore, row[5][1] has no derivation.

6						
5	D	$\emptyset$				
4	S	$\emptyset$	S			
3	C	D	D	$\emptyset$		
2	S	S	S	$\emptyset$	S	
1	B	A	B	A	A	B
	b	a	b	a	a	b

Table 6: Row 5 filled

row[6] will look at substrings of length 6

- row[6][0] will look at string $babaab$. We can have substrings as follows: $s_1 = b, abaab$, $s_2 = ba, baab$, $s_3 = bab, aab$, $s_4 = baba, ab$, and $s_5 = babaa, b$. s_1. In s_1, $abaab$ has no derivation. In s_2, you can derivate the string through S and S. Thus the cartesian product SS can be derived from S. In s_3, aab has no derivation. In s_4, you can derive it through S and S. The cartesian product is SS, which is derived through S. In s_4, you derive it through D and B, but DB is not in the grammar. Since s_2 and s_4 both get their derivations from S, then row[6][0] $= S$.

6	S					
5	D	$\emptyset$				
4	S	$\emptyset$	S			
3	C	D	D	$\emptyset$		
2	S	S	S	$\emptyset$	S	
1	B	A	B	A	A	B
	b	a	b	a	a	b

Table 7: Row 6 filled

In conclusion, since row[6] contains the start variable S, then the string $babaab \in L(\mathcal{A})$.

Problem #2: Is string $bababb \in L(\mathcal{A})$?

I will follow the same procedure as I did for problem #1. For the sake of simplicity, I will give only important and relevant information.

6						
5						
4						
3						
2						
1						
	b	a	b	a	b	b

Table 8: Initial Unfilled Table

Again, row[1] contains the derivations of the letters directly below each box.

6						
5						
4						
3						
2						
1	B	A	B	A	B	B
	b	a	b	a	b	b

Table 9: First Row Filled

The following table are as follows:

<table>
<tr><th></th><th>s</th><th>substring
of s</th><th>derivated
from</th><th>Cartesian
Product</th><th>In grammar?
Where?</th></tr>
<tr><td rowspan="2">row[2][0]</td><td rowspan="2">ba</td><td>b</td><td>B</td><td rowspan="2">BA</td><td rowspan="2">{S}</td></tr>
<tr><td>a</td><td>A</td></tr>
<tr><td rowspan="2">row[2][1]</td><td rowspan="2">ab</td><td>a</td><td>A</td><td rowspan="2">AB</td><td rowspan="2">{S}</td></tr>
<tr><td>b</td><td>B</td></tr>
<tr><td rowspan="2">row[2][2]</td><td rowspan="2">ba</td><td>b</td><td>B</td><td rowspan="2">BA</td><td rowspan="2">{S}</td></tr>
<tr><td>a</td><td>A</td></tr>
<tr><td rowspan="2">row[2][3]</td><td rowspan="2">ab</td><td>a</td><td>A</td><td rowspan="2">AB</td><td rowspan="2">{S}</td></tr>
<tr><td>b</td><td>B</td></tr>
<tr><td rowspan="2">row[2][4]</td><td rowspan="2">bb</td><td>b</td><td>B</td><td rowspan="2">BB</td><td rowspan="2">$\emptyset$</td></tr>
<tr><td>b</td><td>B</td></tr>
</table>

<table>
<tr><th></th><th>s</th><th>substring
of s</th><th>derivated
from</th><th>Cartesian
Product</th><th>In grammar?
Where?</th></tr>
<tr><td rowspan="2">row[3][0]</td><td rowspan="2">bab</td><td>b, ab</td><td>B, S</td><td>{BS}</td><td rowspan="2">{C}</td></tr>
<tr><td>ba, b</td><td>S, B</td><td>{SB}</td></tr>
<tr><td rowspan="2">row[3][1]</td><td rowspan="2">aba</td><td>a, ba</td><td>A, S</td><td>{AS}</td><td rowspan="2">{D}</td></tr>
<tr><td>ab, a</td><td>S, A</td><td>{SA}</td></tr>
<tr><td rowspan="2">row[3][2]</td><td rowspan="2">bab</td><td>b, ab</td><td>B, S</td><td>{BS}</td><td rowspan="2">{C}</td></tr>
<tr><td>ba, b</td><td>S, B</td><td>{SB}</td></tr>
<tr><td rowspan="2">row[3][3]</td><td rowspan="2">abb</td><td>a, bb</td><td>A, $\emptyset$</td><td>$\emptyset$</td><td rowspan="2">{C}</td></tr>
<tr><td>ab, b</td><td>S, B</td><td>{SB}</td></tr>
</table>

	s	substring of s	derivated from	Cartesian Product	In grammar?	Where?
row[4][0]	baba	b, aba	B, D	{BD}	{S}	{S}
		ba, ba	S, S	{SS}	{S}	
		bab, a	C, A	{CA}	{∅}	
row[4][1]	abab	a, bab	A, C	{AC}	{S}	{S}
		ab, ab	S, S	{SS}	{S}	
		aba, b	D, B	{∅}	{∅}	
row[4][2]	babb	b, abb	B, C	{BC}	{∅}	{∅}
		ba, bb	S, ∅	{∅}	{∅}	
		bab, b	C, A	{CA}	{∅}	

	s	substrings of s	derivated from	Carteisan Product	In Grammar?	Final
row[5][0]	babab	b, abab	B, S	{BS}	{∅}	C
		ba, bab	S, C	{SC}	{∅}	
		bab, ab	C, S	{CS}	{∅}	
		baba, b	S, B	{SB}	{C}	
row[5][1]	ababb	a, babb	A, ∅	{∅}	{∅}	C
		ab, abb	S, C	{SC}	{∅}	
		aba, bb	D, ∅	{∅}	{∅}	
		abab, b	S, B	{SB}	{C}	

	s	substrings of s	derivated from	Carteisan Product	In Grammar?	Final
row[6][0]	bababb	b, ababb	B, C	{BC}	{∅}	{∅}
		ba, babb	S, ∅	{∅}	{∅}	
		bab, abb	C, C	{CC}	{∅}	
		baba, bb	S, ∅	{∅}	{∅}	
		babab, b	C, B	{CB}	{∅}	

6	$\emptyset$					
5	C	C				
4	S	S	$\emptyset$			
3	C	D	C	C		
2	S	S	S	S	$\emptyset$	
1	B	A	B	A	B	B
	b	a	b	a	b	b

Table 10: Final Table

In conclusion, since row[6] does not contain start variable S, then string $bababb \notin L(\mathcal{A})$.

Problem #3:

Give Turing machine transition table for a Turing Machine that accepts language:

$\{x \in \{0,1\}^* \mid x$ begins with 0 and has as many 1 to 0 transitions as 0 to 1 transitions$\}$

	0	1	B
p_0	R	p_1R	Accept
p_1	p_0R	R	Reject

This transition table shows that the turing machine will read each symbol and move states only when there is a change in input. It will accept if there is never a transition or if there is transition from 0-to-1 and then from 1-to-0. If there is a transition from 0-to-1 and the string ends there, the machine rejects such string.