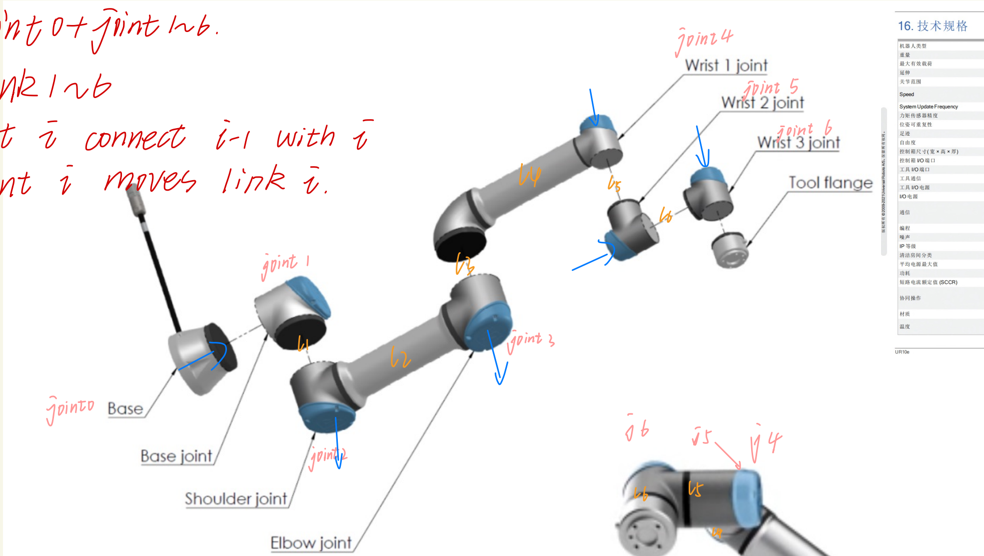
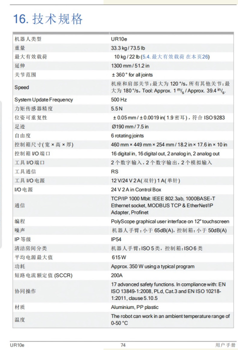
joint 0 + joint 1~6.
link 1~6

joint i connect i-1 with i
joint i moves link i.





16. 技术规格

机器人类型	UR10e
重量	33.3 kg / 73.5 lb
最大有效载荷	10 kg / 22 lb (5.4. 最大有效载荷 在本页26)
延伸	1300 mm / 51.2 in
关节范围	± 360 ° for all joints
Speed	机座和肩部关节：最大为 120 °/s。所有其他关节：最大为 180 °/s。Tool: Approx. 1 m/s / Approx. 39.4 in/s.
System Update Frequency	500 Hz
力矩传感器精度	5.5 N
位姿可重复性	± 0.05 mm / ± 0.0019 in(1.9 密耳)，符合 ISO 9283
足迹	Ø190 mm / 7.5 in
自由度	6 rotating joints
控制箱尺寸(宽 × 高 × 厚)	460 mm × 449 mm × 254 mm / 18.2 in × 17.6 in × 10 in
控制箱 I/O 端口	16 digital in, 16 digital out, 2 analog in, 2 analog out
工具 I/O 端口	2 个数字输入，2 个数字输出，2 个模拟输入
工具通信	RS
工具 I/O 电源	12 V/24 V 2 A(双针) 1 A(单针)
I/O 电源	24 V 2 A in Control Box
通信	TCP/IP 1000 Mbit: IEEE 802.3ab, 1000BASE-T Ethernet socket, MODBUS TCP & EtherNet/IP Adapter, Profinet
编程	PolyScope graphical user interface on 12" touchscreen
噪声	机器人手臂：小于 65dB(A)，控制箱：小于 50dB(A)
IP 等级	IP54
清洁房间分类	机器人手臂：ISO 5类，控制箱：ISO 6 类
平均电源最大值	615 W
功耗	Approx. 350 W using a typical program
短路电流额定值 (SCCR)	200A
协同操作	17 advanced safety functions. In compliance with: EN ISO 13849-1:2008, PLd, Cat.3 and EN ISO 10218-1:2011, clause 5.10.5
材质	Aluminium, PP plastic
温度	The robot can work in an ambient temperature range of 0-50 °C

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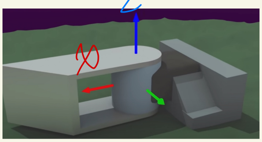
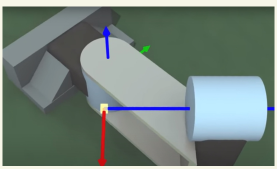
UR10e 74 用户手册

坐标系
选取原则





j0

• 典型刚体—齐次变换矩阵.

$$P^0 = d_1^0 + R_1^0 P^1$$

tran   Rot.

$$\Rightarrow \quad H_1^0 = \left[\begin{array}{c|c} R_1^0 & d_1^0 \\ \hline 0 & 1 \end{array}\right]$$

rot 3x3   P 3x1

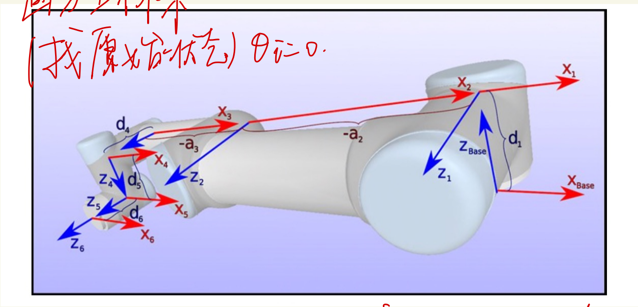
4x4

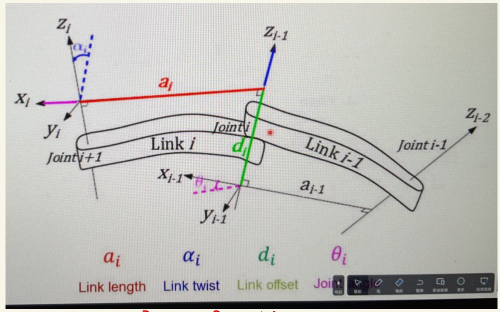
$$H_i^{i-1} = A_i = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$\theta_i \longrightarrow d_i \longrightarrow a_i \longrightarrow \alpha_i$$

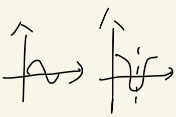
画分坐标系

(找原始状态) $\theta_i = 0$.





按 $\theta_i \to d_i \to a_i \to \alpha_i$ 完成



θ

UR10e							
Kinematics	theta [rad]	a [m]	d [m]	alpha [rad]	**Dynamics**	Mass [kg]	Center of Mass [m]
Joint 1	0	0	0.1807	π/2	Link 1	7.369	[0.021, 0.000, 0.027]
Joint 2	0	-0.6127	0	0	Link 2	13.051	[0.38, 0.000, 0.158]
Joint 3	0	-0.57155	0	0	Link 3	3.989	[0.24, 0.000, 0.068]
Joint 4	0	0	0.17415	π/2	Link 4	2.1	[0.000, 0.007, 0.018]
Joint 5	0	0	0.11985	-π/2	Link 5	1.98	[0.000, 0.007, 0.018]
Joint 6	0	0	0.11655	0	Link 6	0.615	[0, 0, -0.026]

$$H_i^{i-1} = \begin{bmatrix} C_{\theta i} & -S_{\theta i} & 0 & 0 \\ S_{\theta i} & C_{\theta i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\alpha i} & -S_{\alpha i} & 0 \\ 0 & S_{\alpha i} & C_{\alpha i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

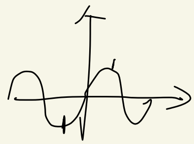
$$= \begin{bmatrix} C_{\theta i} & -S_{\theta i} C_{\alpha i} & S_{\theta i} S_{\alpha i} & a_i C_{\theta i} \\ S_{\theta i} & C_{\theta i} C_{\alpha i} & -C_{\theta i} S_{\alpha i} & a_i S_{\theta i} \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0.1807 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

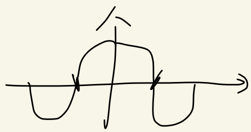
$$H_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & -0.6127\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & -0.6127\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & -0.57155\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & -0.57155\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 0.17415 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



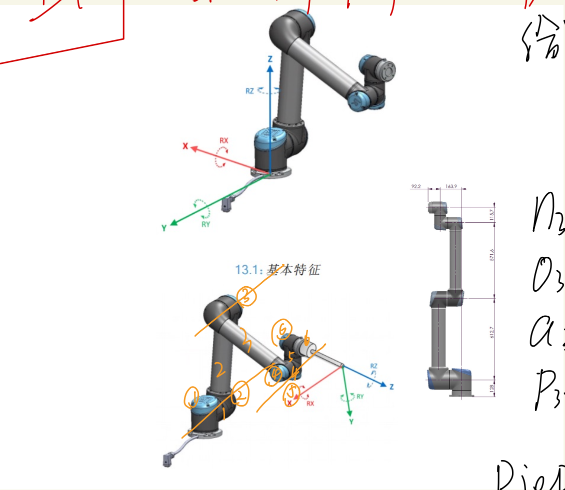
$$H_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0.11985 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$H_6^5 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0.11655 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = T_1^0 \, T_2^1 \, T_3^2 \, T_4^3 \, T_5^4 \, T_6^5$$

IK 目的：解析解(封闭解)



给定 $\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\left.\begin{aligned} n_{3\times1} &= n(\cdots\cdots) \\ o_{3\times1} &= o(\cdots) \\ a_{3\times1} &= a(\cdots) \end{aligned}\right\} 3$$

$$p_{3\times1} = p(\cdots)\ \} 3约束$$

N=6 关节

6个未知数 12个方程

6个独立方程. 6个未知数

超越方程

无解析解

Pieper准则：

三个相邻关节轴交于一点或三轴线平行

存在解析解.

齐次变换矩阵逆运算

$R^{-1}=R^T$

$H^{-1}=\begin{bmatrix} R^T & -R^Td \\ 0 & 1 \end{bmatrix}$

$$T = {}^0_1T\ {}^1_2T\ {}^2_3T\ {}^3_4T\ {}^4_5T\ {}^5_6T$$

$$\therefore\ {}^0_1T^{-1} \cdot T \cdot {}^5_6T^{-1} = {}^1_2T\ {}^2_3T\ {}^3_4T\ {}^4_5T = {}^1_5T$$

$$左 = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ \cos\theta_1 & -\cos\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ & & & \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_6 & S_6 & 0 & 0 \\ -S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & -d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_6(n_xC_1+n_yS_1)-S_6(O_xC_1+O_yS_1) & S_6(n_xC_1+n_yS_1)+C_6(O_xC_1+O_yS_1) & a_xC_1+a_yS_1 & p_xC_1-d_6(a_xC_1+a_yS_1)+p_yS_1 \\ n_xC_6-O_zS_6 & O_zC_6+n_zS_6 & a_z & p_z-d_1-a_zd_6 \\ S_6(O_yC_1-O_xS_1)-C_6(n_yC_1-n_xS_1) & -S_6(n_yC_1-n_xS_1)-C_6(O_yC_1-O_xS_1) & a_xS_1-a_yC_1 & -p_yC_1+d_6(a_yC_1-a_zS_1)+p_xS_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

右 $= {}^{1}_{5}T =$
$$\begin{bmatrix} C_{234}C_5 & -S_{234} & -C_{234}S_5 & a_3C_{23}+a_2C_2+d_5S_{234} \\ S_{234}C_5 & C_{234} & -S_{234}S_5 & a_3S_{23}+a_2S_2-d_5C_{234} \\ S_5 & 0 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{\theta_1:\ \text{第3行第4列相等}}$

$$-P_yC_1+d_6(a_yC_1-a_zS_1)+P_xS_1=d_4$$

$$(d_6a_y-P_y)C_1-(a_xd_6-P_x)S_1=d_4$$

$$mC_1-nS_1=d_4$$

$$\therefore \theta_1=A\tan2(m,n)-A\tan2(d_4,\pm\sqrt{m^2+n^2-d_4^2})$$
$$(m^2+n^2-d_4^2\geq 0)$$

逆运动学 $\theta\in[-\pi,+\pi]$
arctan $[-\frac{\pi}{2},\frac{\pi}{2}]$
使用 Arctan2(x,y)

$$-\sin\theta P_x+\cos\theta P_y=d$$

$P_x=\rho\cos\varphi$ $P_y=\rho\sin\varphi$ $(\rho=\sqrt{x^2+y^2},\ \varphi=A\tan2(P_y,P_x))$

得 $\sin(\varphi-\theta)=\frac{d}{\rho}$

则 $\varphi-\theta=A\tan2(\frac{d}{\rho},\pm\sqrt{1-\frac{d^2}{\rho^2}})$

则 $\theta=A\tan2(P_y,P_x)-A\tan2(d,\pm\sqrt{P_x^2+P_y^2-d^2})$

$(P_x^2+P_y^2-d^2\geq 0)$

$\boxed{\theta_2:\ \text{3行3列}}$

$$a_xS_1-a_yC_1=C_5$$

$$\theta_5=\pm\arccos(a_xS_1-a_yC_1)\qquad (|a_xS_1-a_yC_1|\leq 1)$$

$\boxed{\theta_3:\ \text{3行3列}}$

$$S_6(O_yC_1-O_xS_1)-C_6(n_yC_1-n_xS_1)=S_5$$

$$\begin{cases} n_xS_1-n_yS_1=m \\ O_xS_1-O_yC_1=n \end{cases}$$

则 $m c_6 - n s_6 = s_5$.

$$\theta_6 = A\tan2(m, n) - A\tan2(s_5, \pm\sqrt{m^2+n^2-s_5^2})$$

$$T = {}^0_1T\ {}^1_2T\ {}^2_3T\ {}^3_4T\ {}^4_5T\ {}^5_6T$$

$${}^1_0T^{-1} \cdot T \cdot {}^5_6T^{-1} \cdot {}^4_5T^{-1} = {}^1_2T\ {}^2_3T\ {}^3_4T = {}^1_4T$$

$$d_6 a_y - P_y = m, \quad a_x d_6 - P_x = n$$

$$\theta_1 = A\tan2(m, n) - A\tan2(d_4, \pm\sqrt{m^2+n^2-d_4^2}) \quad (m^2+n^2-d_4^2 \geq 0)$$

$$\theta_5 = \pm\arccos(a_x s_1 - a_y c_1) \quad (|a_x s_1 - a_y c_1| \leq 1)$$

$$n_x s_1 - n_y c_1 = m, \quad O_x s_1 - O_y c_1 = n$$

$$\theta_6 = A\tan2(m, n) - A\tan2(s_5, 0) \quad (s_5 \neq 0)$$

$$\begin{cases} d_5(s_6(n_x c_1 + n_y s_1) + c_6(O_x c_1 + O_y s_1)) - d_5(a_x c_1 + a_y s_1) + P_x c_1 + P_y s_1 = m \\ P_z - d_1 - a_z d_6 + d_5(O_z c_6 + n_z s_6) = n \end{cases}$$

$$\theta_3 = \pm\arccos\left(\frac{m^2+n^2-a_2^2-a_3^2}{2a_2a_3}\right) \quad 其中\ m^2+n^2 \leq (a_2+a_3)^2$$

$$s_2 = \frac{(a_3c_3+a_2)n - a_3s_3m}{a_2^2+a_3^2+2a_2a_3c_3}, \quad c_2 = \frac{m + (a_3s_3s_2)}{a_3c_3+a_2}$$

$$\theta_2 = \mathrm{Atan2}(S_2, C_2).$$

$$\theta_4 = \mathrm{Atan2}\left(-S_6(n_x C_1 + n_y S_1) - C_6(O_x C_1 + O_y S_1),\ O_z C_6 + n_z S_6\right) - \theta_2 - \theta_3$$

奇异位置

$$\begin{bmatrix}
+\theta_{11} & & \theta_{31} & & +\theta_{51} & \theta_{61} \\
+\theta_{11} & & \vdots & & +\theta_{51} & \theta_{61} \\
+\theta_{11} & \text{8种} & \vdots & \text{8种} & -\theta_{52} & \theta_{62} \\
+\theta_{11} & & \vdots & & -\theta_{52} & \theta_{62} \\
-\theta_{12} & & & & +\theta_{53} & \theta_{63} \\
-\theta_{12} & & & & +\theta_{53} & \theta_{63} \\
-\theta_{12} & & & & -\theta_{54} & \theta_{64} \\
-\theta_{12} & & \theta_{38} & & -\theta_{54} & \theta_{64}
\end{bmatrix}$$