Solutions to selected exercises of Chapters 7–11

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This document contains solutions to the following exercises in the book [1]:

7.2(c), 8.2(e), 8.7(c), 8.9(b),(d), 9.6(b),11.4(c),11.6.

We **strongly** advise you to first try all these exercises by yourself, before looking at all at the solutions below. There is not a lot of variation possible in the way solutions to exercises should be written down. So if your solution in one way or another deviates from a solution below, then consider discussing the differences with your instructor.

7.2 (c) The proposition is not valid for all abstract propositions P, Q and R. To see this, let a and b are distinct propositional variables and let $P = a \wedge b$, $Q = a$ and $R = b$. Then both $P \stackrel{val}{\models} Q$ and $P \stackrel{val}{\models} R$ hold (by $\wedge$-$\vee$-weakening), but $Q \stackrel{val}{\not=} R$.

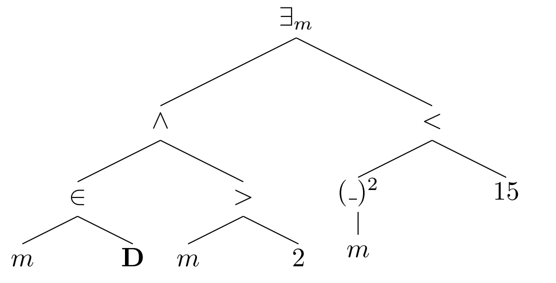
8.2 (e)

$\exists_x[x \in \mathbf{M}:$	There is a person that is
$\quad \mathit{Younger}(x, \text{Bernard}) \wedge$	younger than Bernard and
$\quad \mathit{Man}(x) \wedge$	male and
$\quad \forall_y[y \in \mathbf{M} : \mathit{Child}(x, y) \Leftrightarrow \mathit{Child}(\text{Bernard}, y)]]$	with the same parents.

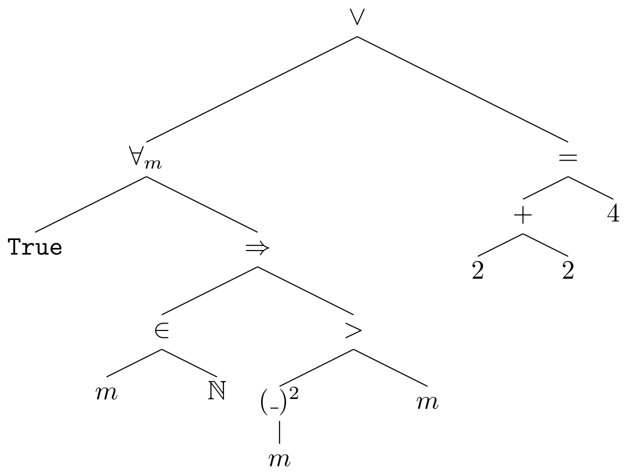
Comment: The formula above expresses the predicate "x is a sibling of y" as $\forall_z[z \in \mathbf{M} : \mathit{Child}(x, z) \Leftrightarrow \mathit{Child}(y, z)]$. This formalisation is based on the interpretation that x and y are siblings if they have the same parents, and (implicitly) assumes that every person has parents (for: people without parents are siblings according to this formula!). An alternative interpretation of "x is a sibling of y" could be "x and y have a parent in common", which can be formalised as $\exists_z[z \in \mathbf{M} : \mathit{Child}(x, z) \wedge \mathit{Child}(y, z)]$. Note that this formulation does imply that x and y are siblings if they only have one parent in common (i.e., they are actually 'half-siblings').

8.7 (c) $\exists_x[x \in \mathbf{D} : \forall_y[y \in \mathbf{D} : x = y]]$.

8.9 (b) The tree associated with $\exists_m[(m \in \mathbf{D}) \wedge (m > 2) : m^2 < 15]$ is:



(d) The tree associated with $\forall_m[m \in \mathbb{N} \Rightarrow m^2 > m] \vee (2 + 2 = 4)$ is



9.6 (b) To show that $\exists_k[P : Q] \wedge \exists_k[P : R] \overset{val}{\not=} \exists_k[P : Q \wedge R]$ we need to find a counterexample, i.e., concrete predicates P, Q and R for which the equivalence does not hold.

Let $P = (k \in \mathbb{Z})$, let $Q = (k > 0)$ and let $R = (k < 0)$. Then, since $1 \in \mathbb{Z}$ and $1 > 0$, the proposition $\exists_k[P : Q]$ is true, and since $-1 \in \mathbb{Z}$ and $-1 < 0$, the proposition $\exists_k[P : R]$ is true. But $\exists_k[P : Q \wedge R]$ is not true, for there does not exist an integer that is both positive and negative.

11.4 (c) The proposition is true, for 29 is a prime number that is 1 plus a multiple of 7 ($29 = 1 + 4 \cdot 7$).

11.6 We need to prove that the square of an odd integer is 1 plus a multiple of 8.

To this end, let n be the square of an odd integer. Then there exists $x \in \mathbb{Z}$ such that $n = (2x + 1)^2 = 4x^2 + 4x + 1$. Clearly, it now remains to establish that $4x^2 + 4x$ is a multiple of eight; we distinguish two cases:

(a) If x is even, then there exists $y \in \mathbb{Z}$ such that $x = 2y$, so

$$4x^2 + 4x = 4(2y)^2 + 4 \cdot 2y = 16y^2 + 8y = 8(2y^2 + y) \ .$$

(b) If x is odd, then there exists $y \in \mathbb{Z}$ such that $x = 2y + 1$, so

$$\begin{aligned} 4x^2 + 4x = 4(2y+1)^2 + 4(2y+1) &= 4(4y^2 + 4y + 1) + 4(2y + 1) \\ &= 16y^2 + 24y + 8 = 8(2y^2 + 3y + 1) \ . \end{aligned}$$

In both cases it is clear that $4x^2 + 4x$ is indeed a multiple of 8.

References

[1] Rob Nederpelt and Fairouz Kamareddine. *Logical Reasoning: A First Course*, volume 3 of *Texts in Computing*. King's College Publications, second revised edition edition, 2011.