(2) 1. Determine whether the following abstract proposition is a tautology, a contradiction, or a contingency:

$$(a \Rightarrow b \wedge c) \wedge (d \Leftrightarrow b \vee c) \wedge (a \wedge \neg d) \ .$$

Motivate your answer.

2. Let A be the set of all airplanes, and let P be the set of all pilots. Furthermore, let N be a predicate on $P \times A$, and let F be a predicate on A, with the following interpretations:

$N(p, a)$ means 'p navigates a', and

$F(a)$ means 'a is flying'.

Give formulas of predicate logic that express the following statements:

(1) (a) If every pilot navigates an airplane, then every airplane is flying.

(1) (b) Every flying airplane is navigated by at least two pilots.

(3) 3. Prove with a *derivation* (i.e., using the methods described in *Part II* of the book) that the formula

$$(\forall_x[P(x) : \neg Q(x)] \wedge \forall_y[\neg R(y)]) \Rightarrow \neg\exists_z[P(z) : Q(z) \vee R(z)]$$

is a tautology.

(2) 4. Determine whether the following formula holds for all sets A, B and C. If so, then give a proof; if not, then give a counterexample.

$$B \times C \subseteq A \times A \ \Rightarrow \ A \backslash (B \cap C) = A \ .$$

5. Consider the mapping $F : \mathbb{R} \to \mathbb{R}$ defined for all $x \in \mathbb{R}$ by

$$F(x) = 2x^2 + 3 \ .$$

(1) (a) Determine $F(\{-1, 0, 1\})$.

(1) (b) Is F a bijection? (Motivate your answer with a proof or a counterexample.)

(2) 6. Let $V = \mathcal{P}(\{1, 2, 3\}) \backslash \mathcal{P}(\{3\})$. Draw the Hasse diagram of $\langle V, \subseteq \rangle$.

7. Let S be the binary relation on $\mathbb{R}$ defined, for all $x, y \in \mathbb{R}$, by

$$x \ S \ y \text{ if, and only if, } y - x \in \mathbb{N} \ .$$

(3) (a) Prove that S is a *reflexive ordering*.

(1) (b) Is S a linear ordering? (Motivate your answer with a proof or a counterexample.)

(3) 8. Prove with induction that $4n^3 - 4n$ is divisible by 3 for all $n \in \mathbb{N}$.