Y. B. Jun[1], A. Iampan[2]

COMPARATIVE AND ALLIED UP-FILTERS

[1]Department of Mathematics Education, Gyeongsang National University, Jinju 52828, Korea, [2]Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand

E-mail address: skywine@gmail.com, aiyared.ia@up.ac.th

Received date

Abstract. The notions of a comparative UP-filter and an allied UP-filter are introduced, and related properties are investigated. Relations between a UP-filter, an implicative UP-filter and a comparative UP-filter are discussed. Conditions for a UP-filter to be a comparative UP-filter are displayed. Conditions for a comparative UP-filter to be an implicative UP-filter are considered. We show that comparative UP-filters and implicative UP-filters coincide in a meet-commutative UP-algebra X satisfying the condition

$$(\forall x, y, z \in X)\left(\ x \cdot (y \cdot z) = y \cdot (x \cdot z)\ \right).$$

Characterizations of a comparative UP-filter are stated. An extension property for comparative UP-filter is established. Conditions for a UP-filter to be an x-allied UP-filter for given $x \in X$ are provided.

1. Introduction

Prabpayak and Leerawat [3] introduced the notion of a KU-algebra. As a generalization of a KU-algebra, Iampan [1] introduced a new algebraic structure, called a UP-algebra, He studied a UP-subalgebra and a UP-ideal. Somjanta et al. [5] introduced the notion of a UP-filter, and discussed the fuzzy set theory of a UP-subalgebra, a UP-ideal and a UP-filter. Jun and Iampan [2] introduced the concept of an implicative UP-filter, and investigated several properties. They discussed the relation between a UP-filter and an implicative UP-filter, conditions for a

---

2010 Mathematical Subject Classification. 03G25, 06F35.

Key words and phrases. UP-algebra, (implicative, comparative, allied) UP-filter.

UP-filter to be an implicative UP-filter, and characterizations of a (implicative) UP-filter.

In this paper, we introduce the notions of a comparative UP-filter and an allied UP-filter, and investigate related properties. We discuss relations between a UP-filter, an implicative UP-filter and a comparative UP-filter. We state conditions for a UP-filter to be a comparative UP-filter, and for a comparative UP-filter to be an implicative UP-filter. We show that comparative UP-filters and implicative UP-filters coincide in a meet-commutative UP-algebra X satisfying the condition

$$(\forall x, y, z \in X)\left(\ x \cdot (y \cdot z) = y \cdot (x \cdot z) \ \right).$$

We consider characterizations of a comparative UP-filter, and provide conditions for a UP-filter to be an x-allied UP-filter for given $x \in X$. We establish an extension property for comparative UP-filter.

2. Preliminaries

An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* (see [1]) if it satisfies the following conditions.

$$(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0), \tag{1}$$

$$(\forall x \in X)(0 \cdot x = x), \tag{2}$$

$$(\forall x \in X)(x \cdot 0 = 0), \tag{3}$$

$$(\forall x, y \in X)(x \cdot y = 0 = y \cdot x \ \Rightarrow \ x = y). \tag{4}$$

We define a binary relation $\leq$ on a UP-algebra X as follows:

$$(\forall x, y \in X)\left(\ x \leq y \ \Leftrightarrow \ x \cdot y = 0 \ \right). \tag{5}$$

In a UP-algebra X, the following assertions are valid (see [1]).

$$(\forall x \in X)(x \cdot x = 0), \tag{6}$$

$$(\forall x, y, z \in X)(x \cdot y = 0, \ y \cdot z = 0 \ \Rightarrow \ x \cdot z = 0), \tag{7}$$

$$(\forall x, y \in X)(x \cdot y = 0 \ \Rightarrow \ (z \cdot x) \cdot (z \cdot y) = 0), \tag{8}$$

$$(\forall x, y \in X)(x \cdot y = 0 \ \Rightarrow \ (y \cdot z) \cdot (x \cdot z) = 0), \tag{9}$$

$$(\forall x, y \in X)(x \cdot (y \cdot x) = 0), \tag{10}$$

$$(\forall x, y \in X)((y \cdot x) \cdot x = 0 \ \Leftrightarrow \ x = y \cdot x), \tag{11}$$

$$(\forall x, y \in X)(x \cdot (y \cdot y) = 0). \tag{12}$$

A subset F of X is called a *UP-filter* of X (see [5]) if it satisfies the following conditions.

$$0 \in F, \tag{13}$$

$$(\forall x, y \in X)(x \in F,\ x \cdot y \in F \ \Rightarrow\ y \in F). \tag{14}$$

3. Comparative UP-filters

In what follows let X denote a UP-algebra unless otherwise.

Definition 1 ([2]). *A subset F of X is called an implicative UP-filter of X if it satisfies the conditions* (13) *and*

$$(\forall x, y, z \in X)\left(\ x \cdot (y \cdot z) \in F,\ x \cdot y \in F \ \Rightarrow\ x \cdot z \in F\ \right). \tag{15}$$

Definition 2. *A subset F of X is called a comparative UP-filter of X if it satisfies the conditions* (13) *and*

$$(\forall x, y, z \in X)\left(\ x \cdot ((y \cdot z) \cdot y) \in F,\ x \in F \ \Rightarrow\ y \in F\ \right). \tag{16}$$

Example 1. *Consider a UP-algebra $X = \{0, 1, 2, 3\}$ with the binary operation "$\cdot$" which is given in Table 1.*

Table 1. Tabular representation of the binary operation "$\cdot$"

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	2
2	0	0	0	2
3	0	0	0	0

Then $\{0, 1, 2\}$ is a comparative UP-filter of X.

Theorem 1. *Every comparative UP-filter is a UP-filter.*

Proof. Let F be a comparative UP-filter of X and let $x, y \in X$ be such that $x \cdot y \in F$ and $x \in F$. Putting $z = y$ in (16) implies that

$$x \cdot ((y \cdot y) \cdot y) = x \cdot y \in F.$$

It follows from (16) that $y \in F$. Hence F is a UP-filter of X. $\square$

The converse of Theorem 1 is not true in general as seen in the following example.

Example 2. *Let $X = \{0, 1, 2, 3\}$ be the UP-algebra in Example 1. Then $\{0\}$ is a UP-filter of X, but it is not a comparative UP-filter of X since $0 \cdot ((1 \cdot 2) \cdot 1) = 0 \cdot (1 \cdot 1) = 0 \cdot 0 = 0 \in \{0\}$ and $0 \in \{0\}$ but $1 \notin \{0\}$.*

Theorem 2. *Let F be a UP-filter of X. Then F is a comparative UP-filter of X if and only if the following implication is valid.*

$$(\forall x, y \in X)\left((x \cdot y) \cdot x \in F \;\Rightarrow\; x \in F \right). \tag{17}$$

Proof. Assume that F is a comparative UP-filter of X and let $x, y \in X$ be such that $(x \cdot y) \cdot x \in F$. Then

$$0 \cdot ((x \cdot y) \cdot x) = (x \cdot y) \cdot x \in F$$

by (2). Since $0 \in F$, it follows from (16) that $x \in F$. Therefore (17) is valid.

Conversely, let F be a UP-filter of X that satisfies (17). Let $x, y, z \in X$ be such that $x \cdot ((y \cdot z) \cdot y) \in F$ and $x \in F$. Then $(y \cdot z) \cdot y \in F$ by (14), and so $y \in F$ by (17). Therefore F is a comparative UP-filter of X. □

In Example 1, we know that the comparative UP-filter $\{0, 1, 2\}$ is not an implicative UP-filter since $0 \cdot (1 \cdot 3) = 0 \cdot 2 = 2 \in \{0, 1, 2\}$ and $0 \cdot 1 = 1 \in \{0, 1, 2\}$, but $0 \cdot 3 = 3 \notin \{0, 1, 2\}$.

We will consider conditions for a comparative UP-filter to be an implicative UP-filter.

Lemma 1 ([2]). *Let X be a UP-algebra satisfying the following condition*

$$(\forall x, y, z \in X)\left(x \cdot (y \cdot z) = y \cdot (x \cdot z) \right). \tag{18}$$

Then a nonempty subset F of X is a UP-filter of X if and only if the following assertion is valid.

$$(\forall x, y \in F)(\forall z \in X)\left(x \leq y \cdot z \;\Rightarrow\; z \in F \right). \tag{19}$$

In a UP-algebra X, consider the following condition.

$$(\forall x, y \in X)\left((x \cdot y) \cdot y = (y \cdot x) \cdot x \right). \tag{20}$$

The following example shows that a UP-algebra does not satisfy the condition (20) in general.

Example 3. *Let $X = \{0, 1, 2, 3\}$ be a set with the binary operation "$\cdot$" which is given in Table 2.*
Then X is a UP-algebra which does not satisfy the condition (20) *since* $(2 \cdot 3) \cdot 3 = 1 \cdot 3 = 1 \neq 2 = 0 \cdot 2 = (3 \cdot 2) \cdot 2$.

Definition 3 ([4]). *A UP-algebra X is said to be meet-commutative if it satisfies the condition* (20).

Table 2. Tabular representation of the binary operation "·"

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	1
2	0	0	0	1
3	0	0	0	0

Lemma 2 ([2])**.** *Let X be a UP-algebra which satisfies the condition* (18)*. Then every UP-filter is an implicative UP-filter if and only if the following assertion is valid.*

$$(\forall x, y \in X)\left(\ x \cdot (x \cdot y) = x \cdot y\ \right). \tag{21}$$

Theorem 3. *In a meet-commutative UP-algebra X satisfying the condition* (18)*, comparative UP-filters and implicative UP-filters coincide.*

Proof. Let F be a comparative UP-filter of X and let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in F$ and $x \cdot y \in F$. Using (1) and (18), we have

$$x \cdot (y \cdot z) = y \cdot (x \cdot z) \leq (x \cdot y) \cdot (x \cdot (x \cdot z)).$$

Note that F is a UP-filter of X (see Theorem 1), and so $x \cdot (x \cdot z) \in F$ by Lemma 1. Using (18), (20), (6), (3) and (2), we have

$$\begin{aligned}((x \cdot z) \cdot z) \cdot (x \cdot z) &= x \cdot (((x \cdot z) \cdot z) \cdot z)\\ &= x \cdot ((z \cdot (x \cdot z)) \cdot (x \cdot z))\\ &= x \cdot ((x \cdot (z \cdot z)) \cdot (x \cdot z))\\ &= x \cdot ((x \cdot 0) \cdot (x \cdot z))\\ &= x \cdot (0 \cdot (x \cdot z))\\ &= x \cdot (x \cdot z) \in F.\end{aligned}$$

It follows from (17) that $x \cdot z \in F$. Therefore F is an implicative UP-filter of X. Now, let F be an implicative UP-filter of X. Then F is a UP-filter of X (see [2]). Also $x \cdot (x \cdot y) = x \cdot y$ for all $x, y \in X$ by Lemma 2, which implies from (11) that $(x \cdot (x \cdot y)) \cdot (x \cdot y) = 0$. Let $x, y \in X$ be such that $(x \cdot y) \cdot x \in F$. Since X is meet-commutative, we have

$$((x \cdot y) \cdot x) \cdot x = (x \cdot (x \cdot y)) \cdot (x \cdot y) = 0,$$

and so $x = (x \cdot y) \cdot x \in F$ by (11). Thus F is a comparative UP-filter of X by Theorem 2. □

In general, any implicative UP-filter may not be a comparative UP-filter as seen in the following example.

Example 4 ([2]). *Let $X = \{0, a, b, c\}$ be a set with the binary operation "$\cdot$" which is given in Table 3.*

Table 3. Tabular representation of the binary operation "$\cdot$"

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	0	0
b	0	a	0	c
c	0	a	b	0

Then X is a UP-algebra (see [1]), and it is routine to verify that $\{0\}$, $\{0, b\}$, $\{0, c\}$, and $\{0, b, c\}$ are implicative UP-filters of X (see [2]). On the other hand, $\{0, b\}$ is not a comparative UP-filter of X since $b \cdot ((c \cdot a) \cdot c) = b \cdot (a \cdot c) = b \cdot 0 = 0 \in \{0, b\}$ and $b \in \{0, b\}$ but $c \notin \{0, b\}$.

Lemma 3 ([2]). *Every UP-filter F of X has the following assertion.*

$$(\forall x, y \in X)\left(x \leq y,\ x \in F \ \Rightarrow \ y \in F \right). \tag{22}$$

Lemma 4 ([2]). *Let X be a UP-algebra satisfying the condition (18). Given a UP-filter F of X, the following are equivalent.*

(1) *F is an implicative UP-filter of X.*
(2) *F satisfies the following condition.*

$$(\forall x, y \in X)\left(x \cdot (x \cdot y) \in F \ \Rightarrow \ x \cdot y \in F \right). \tag{23}$$

(3) *F satisfies the following condition.*

$$(\forall x, y, z \in X)\left(x \cdot (y \cdot z) \in F \ \Rightarrow \ (x \cdot y) \cdot (x \cdot z) \in F \right). \tag{24}$$

Theorem 4. *Let X be a UP-algebra satisfying the condition (18). For any implicative UP-filter F of X, the following are equivalent.*

(1) *F is a comparative UP-filter of X.*
(2) *F satisfies the following condition.*

$$(\forall x, y \in X)\left((x \cdot y) \cdot y \in F \ \Rightarrow \ (y \cdot x) \cdot x \in F \right). \tag{25}$$

Proof. Assume that F is a comparative UP-filter of X and let $x, y \in X$ be such that $(x \cdot y) \cdot y \in F$. Note that

$$x \cdot ((y \cdot x) \cdot x) = (y \cdot x) \cdot (x \cdot x) = (y \cdot x) \cdot 0 = 0,$$

that is, $x \leq (y \cdot x) \cdot x$, and so $((y \cdot x) \cdot x) \cdot y \leq x \cdot y$. It follows that

$$\begin{aligned}(x \cdot y) \cdot y &\leq (y \cdot x) \cdot ((x \cdot y) \cdot x) = (x \cdot y)((y \cdot x) \cdot x)\\ &\leq (((y \cdot x) \cdot x) \cdot y)((y \cdot x) \cdot x).\end{aligned}$$

Since $(x \cdot y) \cdot y \in F$ and F is a UP-filter of X by Theorem 1, we have

$$0 \cdot ((((y \cdot x) \cdot x) \cdot y)((y \cdot x) \cdot x)) = (((y \cdot x) \cdot x) \cdot y)((y \cdot x) \cdot x) \in F \tag{26}$$

by Lemma 3 and (2), which implies from (16) that $(y \cdot x) \cdot x \in F$.

Conversely, suppose that F satisfies the condition (25). Let $x, y, z \in X$ be such that $x \cdot ((y \cdot z) \cdot y) \in F$ and $x \in F$. Then $(y \cdot z) \cdot y \in F$ since F is a UP-filter. Note that

$$(y \cdot z) \cdot y \leq (y \cdot z) \cdot ((y \cdot z) \cdot z),$$

which implies from Lemma 3 that $(y \cdot z) \cdot ((y \cdot z) \cdot z) \in F$. Hence $(y \cdot z) \cdot z \in F$ by (23), and so $(z \cdot y) \cdot y \in F$ by (25). Note that

$$(y \cdot z) \cdot y \leq z \cdot y \leq x \cdot (z \cdot y)$$

by (9) and (10). It follows from (22) then $x \cdot (z \cdot y) \in F$, and so that $z \cdot y \in F$. Hence $y \in F$, and therefore F is a comparative UP-filter of X. □

Lemma 5 ([2])**.** (Extension property for implicative UP-filter) *Let X be a UP-algebra satisfying the condition* (18)*. Given an implicative UP-filter F of X, if a UP-filter G of X contains F, then G is an implicative UP-filter of X.*

Theorem 5. (Extension property for comparative UP-filter) *In a meet-commutative UP-algebra X satisfying the condition* (18)*, every UP-filter containing a comparative UP-filter is a comparative UP-filter.*

Proof. The proof is straightforward by Theorem 3 and Lemma 5. However, we provide the numerical process. Given a comparative UP-filter F of X, let G be a UP-filter of X which contains F. Then F is an implicative UP-filter of X, and so G is an implicative UP-filter of X by Lemma 5. Let $x, y \in X$ be such that $(y \cdot x) \cdot x \in G$. If we put $u := (y \cdot x) \cdot x$, then $u \cdot ((y \cdot x) \cdot x) = 0 \in F$. Hence, by (18) and (24), we have

$$(y \cdot (u \cdot x)) \cdot (u \cdot x) = (u \cdot (y \cdot x)) \cdot (u \cdot x) \in F.$$

It follows from (25) that $((u \cdot x) \cdot y) \cdot y \in F \subseteq G$. Note that

$$\begin{aligned}(y \cdot x) \cdot x &\leq (((y \cdot x) \cdot x) \cdot x) \cdot x = (u \cdot x) \cdot x \\ &\leq (x \cdot y)((u \cdot x) \cdot y) \\ &\leq (((u \cdot x) \cdot y) \cdot y) \cdot ((x \cdot y) \cdot y).\end{aligned}$$

Using (22), we have $(((u \cdot x) \cdot y) \cdot y) \cdot ((x \cdot y) \cdot y) \in G$, and so $(x \cdot y) \cdot y \in G$. Therefore G is a comparative UP-filter of X by Theorem 4. □

Definition 4. *Let x be a fixed element of X. A subset F of X is called an allied UP-filter of X with respect to x (briefly, x-allied UP-filter of X) if it satisfies the condition* (13) *and*

$$(\forall y, z \in X)\left(\ x \cdot (y \cdot z) \in F,\ x \cdot y \in F\ \Rightarrow\ z \in F\ \right). \tag{27}$$

By an *allied UP-filter*, we mean an x-allied UP-filter of X for all x in X.

Example 5. *Let $X = \{0, 1, 2, 3\}$ be a set with the binary operation "$\cdot$" which is given in Table 4.*

TABLE 4. Tabular representation of the binary operation "$\cdot$"

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	0	0	0

Then $\{0\}$ is a 0-allied UP-filter of X. The set $\{0, 1, 2\}$ is an allied UP-filter of X with respect to 0, 1 and 2. But $\{0, 1, 2\}$ is not a 3-allied UP-filter of X since $3 \cdot (3 \cdot 3) = 3 \cdot 0 = 0 \in \{0\}$ and $3 \cdot 3 = 0 \in \{0\}$ but $3 \notin \{0\}$.

Proposition 1. *For any $x \in X$, every x-allied UP-filter contains x itself.*

Proof. For any $x \in X$, let F be an x-allied UP-filter of X. Note that $x \cdot (0 \cdot x) = x \cdot x = 0 \in F$ and $x \cdot 0 = 0 \in F$. Hence $x \in F$ by (27). □

Remark 1. *Proposition 1 shows that there are no proper allied UP-filters in UP-algebras.*

Theorem 6. *For any $x \in X$, if F is a UP-filter of X which contains x, then it is an x-allied UP-filter of X.*

Proof. Let $y, z \in X$ be such that $x \cdot (y \cdot z) \in F$ and $x \cdot y \in F$. Since F is a UP-filter of X, it follows that $y \cdot z \in F$ and $y \in F$ and so that $z \in F$. Therefore F is an x-allied UP-filter of X. □

Theorem 7. *Every 0-allied UP-filter is a UP-filter, and vice versa.*

Proof. Let F be a 0-allied UP-filter of X and let $x, y \in X$ be such that $x \cdot y \in F$ and $x \in F$. Then $0 \cdot x = x \in F$ and $0 \cdot (x \cdot y) = x \cdot y \in F$. It follows from (27) that $y \in F$. Thus F is a UP-filter of X. Conversely, let F be a UP-filter of X. Assume that $0 \cdot (x \cdot y) \in F$ and $0 \cdot x \in F$ for all $x, y \in X$. Then $x \cdot y \in F$ and $x \in F$, which imply that $y \in F$. Thus F is a 0-allied UP-filter of X. □

Theorem 8. *Let F be a UP-filter of X and let $a, b \in X$ be such that $a \leq b$. If F is an a-allied UP-filter of X, then it is a b-allied UP-filter of X.*

Proof. Let $a, b \in X$ be such that $a \leq b$ and let F be an a-allied UP-filter of X. Let $y, z \in X$ be such that $b \cdot (y \cdot z) \in F$ and $b \cdot y \in F$. Since $a \leq b$, it follows from (9) that $b \cdot (y \cdot z) \leq a \cdot (y \cdot z)$ and $b \cdot y \leq a \cdot y$. Using Lemma 3, we have $a \cdot (y \cdot z) \in F$ and $a \cdot y \in F$, which imply from (27) that $z \in F$. Therefore F is a b-allied UP-filter of X. □

The converse of Theorem 8 is not true in general. In fact, in Example 5, note that $2 \leq 0$ and $\{0\}$ is 0-allied UP-filter of X. But $\{0\}$ is not a 2-allied UP-filter of X since $2 \cdot (2 \cdot 1) = 2 \cdot 0 = 0 \in \{0\}$ and $2 \cdot 2 = 0 \in \{0\}$ but $1 \notin \{0\}$.

The following example shows that there exists $a(\neq 0) \in X$ such that any UP-filter is not an a-allied UP-filter.

Example 6. *In the UP-algebra X in Example 5, $\{0\}$ is a UP-filter of X. But it is not a 3-allied UP-filter of X since $3 \cdot (3 \cdot 3) = 3 \cdot 0 = 0 \in \{0\}$ and $3 \cdot 3 = 0 \in \{0\}$ but $3 \notin \{0\}$.*

We now provide conditions for a UP-filter to be an x-allied UP-filter for given $x \in X$.

Theorem 9. *Let X be a UP-algebra satisfying the condition (18). For a UP-filter F of X and given $x(\neq 0) \in X$, the following assertions are equivalent.*

(1) *F is an x-allied UP-filter of X.*

(2) *F satisfies the following condition.*

$$(\forall y, z \in X) \left(x \cdot (y \cdot z) \in F \Rightarrow (x \cdot y) \cdot z \in F \right). \tag{28}$$

(3) *F satisfies the following condition.*

$$(\forall y \in X) \left(x \cdot (x \cdot y) \in F \Rightarrow y \in F \right). \tag{29}$$

Proof. Assume that F is an x-allied UP-filter of X. Let $y, z \in X$ be such that $x \cdot (y \cdot z) \in F$. Using (18) and (1), we have

$$\begin{aligned} x \cdot ((y \cdot z) \cdot ((x \cdot y) \cdot z)) &= (y \cdot z) \cdot (x \cdot ((x \cdot y) \cdot z)) \\ &= (y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) \\ &= 0 \in F. \end{aligned}$$

Hence $(x\cdot y)\cdot z \in F$ by (27). If F is a UP-filter of X satisfying the condition (28), then it is clear that F is an x-allied UP-filter of X. Therefore (1) and (2) are equivalent. Let F be a UP-filter of X that satisfies the condition (29). Let $y, z \in X$ be such that $x \cdot (y \cdot z) \in F$. Using (1), (2), (6), (8), (9) and (18) we have

$$\begin{aligned}
0 &= (x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) \\
&\leq (x \cdot (y \cdot z)) \cdot (((y \cdot z) \cdot (x \cdot ((x \cdot y) \cdot z))) \cdot (x \cdot (x \cdot ((x \cdot y) \cdot z)))) \\
&= ((y \cdot z) \cdot (x \cdot ((x \cdot y) \cdot z))) \cdot ((x \cdot (y \cdot z)) \cdot (x \cdot (x \cdot ((x \cdot y) \cdot z)))) \\
&= ((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z))) \cdot ((x \cdot (y \cdot z)) \cdot (x \cdot (x \cdot ((x \cdot y) \cdot z)))) \\
&= 0 \cdot ((x \cdot (y \cdot z)) \cdot (x \cdot (x \cdot ((x \cdot y) \cdot z)))) \\
&= (x \cdot (y \cdot z)) \cdot (x \cdot (x \cdot ((x \cdot y) \cdot z)))
\end{aligned}$$

which implies that

$$(x \cdot (y \cdot z)) \cdot (x \cdot (x \cdot ((x \cdot y) \cdot z))) = 0 \in F.$$

Since $x \cdot (y \cdot z) \in F$ and F is a UP-filter of X, it follows from (14) that

$$x \cdot (x \cdot ((x \cdot y) \cdot z)) \in F.$$

Hence $(x\cdot y)\cdot z) \in F$ by (29). This shows that (3) $\Rightarrow$ (2) is valid. Suppose that a UP-filter F of X satisfies the condition (28). Putting $y = x$ and $z = y$ in (28) induces (29) by (6) and (2). This completes the proof. $\square$

Conclusion

we have introduced the notions of a comparative UP-filter and an allied UP-filter, and have investigated related properties. We have discussed relations between a UP-filter, an implicative UP-filter and a comparative UP-filter. We have stated conditions for a UP-filter to be a comparative UP-filter, and for a comparative UP-filter to be an implicative UP-filter. We have shown that comparative UP-filters and implicative UP-filters coincide in a meet-commutative UP-algebra X satisfying the condition

$$(\forall x, y, z \in X)\left(\ x \cdot (y \cdot z) = y \cdot (x \cdot z)\ \right).$$

We have considered characterizations of a comparative UP-filter, and have provided conditions for a UP-filter to be an x-allied UP-filter for given $x \in X$. We have established an extension property for comparative UP-filter. In further study, we will apply this notion/results to other type of UP-filters. Also, we will study the fuzzy and soft set theory of comparative UP-filters and allied UP-filters.

References

[1] A. Iampan, J. Algebra Relat. Top. **5** (1), 35–54 (2017).

[2] Y. B. Jun and A. Iampan, Afrika Matematika (submitted).

[3] C. Prabpayak and U. Leerawat, Scientia Magna **5** (1), 54–57 (2009).

[4] K. Sawika, R. Intasan, A. Kaewwasri and A. Iampan, Korean J. Math. **24** (3), 345–367 (2016).

[5] J. Somjanta, N. Thuekaew, P. Kumpeangkeaw and A. Iampan, Annal. Fuzzy Math. Inform. **12** (6), 739–756 (2016).