(M, N)-Soft intersection nearsemirings and (M, N)-α-inclusion along with its algebraic applications

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Abstract—In this note, we introduce (M, N)-soft intersection nearsemirings (abbreviate as (M, N)-SI-nearsemirings) by utilizing the intersection operation of sets. We study the set theoretic characteristics of (M, N)-Soft intersection nearsemirings with the effects of different types of sets operations. (M, N)-SI-subnearsemirings, (M, N)-SI-ideals, and (M, N)-SI-c-ideals are also introduced and discussed. Furthermore, we introduce the notions of (M, N)-α-inclusion, soft uni-int c-products, soft uni-int c-sums and study (M, N)-SI-nearsemirings by using these operations. We also inter-relate (M, N)-SI-nearsemirings and classical nearsemirings by utilizing (M, N)-α-inclusion.

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1. INTRODUCTION AND PRELIMINARIES

Theory of soft set was introduced by Molodtsov [16] and is proved an efficient tool to handle the complications of modeling uncertain data in many fields as compared to the classical methods. To increase the usefulness of soft sets, numerous operations have been introduced and applied in the literature (e.g., [2, 18] etc). Subsequently, N. Cagmen et al. have redefined the various operations of Molodtsov's soft sets which proved more efficient for the development of number of new results in soft set theory. Number of algebraic soft structures have been explored such as; soft groups [3], soft rings [1], soft semirings [8], soft nearrings [17]. Recently, we (first author with A. Rehman) have introduced and discussed soft nearsemirings [10]. Further to this, soft intersection group (soft int group) [5], soft intersection rings (soft int-rings) [6], soft intersection near-rings (soft int nearrings) [19], soft k-int-ideals of semirings [7], soft intersection h-ideals of hemirings [14] are also introduced and discussed in the literature. Recently, the notion of (M, N)-SI-h-ideals have been discussed in ([15]). In this note, we introduce (M, N)-soft intersection nearsemirings (in short, (M, N)-SI-nearsemirings), (M, N)-SI-ideals, and (M, N)-SI-c-ideals. In due course, we introduced (M, N)-α inclusion and some structural characterizations of (M, N)-soft intersection nearsemirings have been obtained. Finally, We also provide the algebraic applications of (M, N)-SI-nearsemirings and its substructures towards classical nearsemirings. The outline of this paper is as follows: In this (first) section, we provide introductory and useful material about nearsemirings and necessary discussions about soft set theory. In second section, we introduce (M, N)-SI-nearsemirings, (M,N)-SI-ideals and apply several operations to these algebraic soft structures. In the third section, we introduce (M, N)-SI-c-ideals and discuss few of its algebraic characteristics. In the fourth section, we introduce the notion of $(M,$

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N)-α-inclusion and discuss few of its algebraic applications.

Nearsemiring has its own importance in algebra and modern technologies. Among the other applications of nearsemiring, it plays an important role in the theory of automata and computer languages. In 2005, Krishna and Chatterjee [13], discussed the condition of minimality of generalized linear sequential machines using the theory of nearsemirings.

We call a system $(R, +, .)$ is a (right) nearsemiring if, (i) $(R, +)$ is a monoid, (ii) $(R, .)$ is a semigroup, (iii) $(a + b)\ .c = a.c + b.c$, for all a, b, c in R. Nearsemiring is said to be a zero-symmetric, if there exist $0 \in R$ such that $0 + a = a + 0 = a$ and $0.a = a.0 = 0$. Some authors use the term of seminearrings for zero symmetric nearsemirings. A nonempty subset I of a nearsemiring $(R, +, .)$ is said to be a left (right) ideal if (i) for all x, $y \in I$, $x + y \in I$, and (ii) for all $r \in R$, and $x \in I$, $rx \in I$ $(xr \in I)$. Following [9], an ideal I of a nearsemiring $(R, +, .)$ is called a c-ideal of R if for any a, $b \in I$, there exist x, $y \in R$ such that $x + y + a = b + y + x$. Equivalently, we call an ideal I of a nearsemiring $(R, +, .)$ a c-ideal of R if for any a, $b \in I$, there exist x, $y \in R$ such that $x + y + a = b + y + x$, implies $x \in I$ [9]. A subnearsemiring T (left ideal, right ideal, ideal) of R is said to be a c-subnearsmiring (left c-ideal, right c-ideal, c-ideal) of R, if for any x, $y \in R$, and a, $b \in T$ (resp., a, $b \in I$), such that $x + y + a = b + y + x$, it follows $x \in T$ (resp., $x \in I$).
Let R and R' be two nearsemirings, a mapping $\phi : R \to R'$ is said to be a nearsemiring homomorphism if for all a, $b \in R$; $\phi(a + b) = \phi(a)+ \phi(b)$, $\phi(ab) = \phi(a)\ \phi(b)$, $\phi(0) = 0$. On the other hand, if R and R' be two nearsemirings, a mapping $\psi : R \to R'$ is said to be a nearsemiring anti-homomorphism if for all a, $b \in R$; $\phi(a + b) = \phi(a) + \phi(b)$, $\phi(ab) = \phi(b)\phi(a)$, $\phi(0) = 0$. For further discussions and standard terminologies about nearsemirings, we refer ([12, 21, 22]).

Throughout, by R we mean a nearsemiring unless otherwise stated, U the initial universe, E the possible parameters associated with the objects in U, $\wp(U)$ the family of subsets of U, $S(U)$ be the collection of all soft sets over U.

Definition 1. *[4]*A soft set F_A over the universe U is a set defined by ordered pairs $F_A = \{(x, f_A) : x \in E,\ f_A(x) \in \wp(U)\}$, where f_A represents a mapping, $f_A : E \to \wp(U)$ such that $f_A(x) = \emptyset$, if $x \notin A$.

Definition 2. *[4] Let $f_A, f_B \in S(U)$. Then,*

(1) *f_A is a soft subset of f_B i.e., $f_A \tilde{\subset} f_B$ if $f_A(x) \subset f_B(x)$ for each $x \in E$. If $f_A(x) = f_B(x)$ for all $x \in E$, we call f_A and f_B are soft equal.*

(2) the complement of f_A denoted by $f_A^{\tilde{c}}$ is a soft set defined $f_{A^{\tilde{c}}} = f_A^c(x)$ for all $x \in E$, where $f_A^c(x)$ is the set $f_A^c(x) = U - f_A(x)$ for all $x \in E$.

(3) their intersection $f_A \tilde{\cap} f_B$ and union $f_A \tilde{\cup} f_B$ will be $f_{A \tilde{\cap} B}(x) = f_A(x) \cap f_B(x)$ and $f_{A \tilde{\cup} B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$, respectively .

(4) the $\wedge$-product of f_A and f_B i.e., $f_A \wedge f_B$ is defined as, $f_{A\wedge B} : E \times E \to \wp(U)$, and $f_{A\wedge B}(x, y) = f_A(x) \cap f_B(y)$.

(5) $\vee$-product of f_A and f_B is defined by $f_{A\vee B} : E \times E \to \wp(U)$ where $f_{A\vee B}(x, y) = f_A(x) \cup f_B(y)$.

Definition 3. *[5] Let G* be a group and $f_G \in S(U)$ is said to be a soft intersection groupoid over U, if $f_G(xy) \supseteq f_G(x) \cap f_G(y)$ for all $x, y \in G$. And if f_G satisfies $f_G(x^{-1}) = f_G(x)$ for all $x \in G$ then we say that f_G is a soft intersection group over U .

Definition 4. *[5]* Let Ψ be a function from A to B and $f_A, f_B \in S(U)$. The soft subsets $\Psi(f_A) \in S(U)$ and $\Psi^{-1}(f_B) \in S(U)$ defined by,

$$\begin{aligned}\Psi(f_A)(y) &= \cup\{f_A(x) : x \in A,\ \Psi(x) = y\},\ \text{if } y \in \Psi(A)\\ &= \emptyset,\ \text{if } y \notin \Psi(A)\end{aligned}$$

for all $y \in B$ and $\Psi^{-1}(f_B)(x) = f_B(\Psi(x))$ for all $x \in A$. Then $\Psi(f_A)$ is called a soft image of f_A and $\Psi^{-1}(f_B)$ is called the soft pre-image (or soft inverse image) of f_B under Ψ .

Definition 5. *[5]* Let f_R be a soft set over U and α be a subset of U. Then, α-inclusion of soft set f_R is defined by $f_R^{\supseteq\alpha} = \{x \in R : f_R(x) \supseteq \alpha\}$, for all $x \in R$.

Definition 6. *[10]* Let (η, A) be the soft right nearsemiring over right nearsemiring R_1 and (ζ, B) be the soft left nearsemiring over the left nearsemiring R_2. Suppose $f : R_1 \to R_2$ and $g : A \to B$ be the two mappings. Then the pair (f, g) is called a soft nearsemiring anti-homomorphism if it satisfies the following conditions.

(1) f is an anti-epimorphism of nearsemirings.

(2) g is a surjective mapping.

(3) $f(\eta(x)) = \zeta(g(x))$ for all $x \in A$.

If f is an anti-isomorphism and g is bijective then we call (f, g) a soft nearsemiring anti-isomorphism .

Definition 7. *[11]* Let R be a nearsemiring and f_R be a soft set over U. Then,

(1) f_R is called a soft int nearsemiring (*SI-nearsemiring*) over U, if it satisfies the following properties.

(i) $f_R(x + y) \supseteq f_R(x) \cap f_R(y)$,

(ii) $f_R(xy) \supseteq f_R(x) \cap f_R(y)$ for all $x, y \in R$.

(2) f_R is a soft left (right) int-ideal over U if it satisfies the following conditions.

(i) $f_R(x + y) \supseteq f_R(x) \cap f_R(y)$

(ii) $f_R(xy) \supseteq f_R(y)$ $(f_R(xy) \supseteq f_R(x))$ for all $x, y \in R$.

Definition 8. *[6]* If f_R and h_R are two soft sets over U. Then,

(1) their soft intersection product is defined by $(f_R \odot h_R)(r) = \bigcup\limits_{r=xy} \{f_R(x) \cap h_R(y)\}$, if there exist $x,\ y \in R$ such that $r = xy$ otherwise $(f_R \odot h_R)(r) = \emptyset$.

(2) their soft intersection sum is defined by $(f_R \oplus h_R)(r) = \bigcup\limits_{r=x+y} \{f_R(x) \cap h_R(y)\}$, if there exist $x, y \in R$ such that $r = x + y$ otherwise $(f_R \oplus h_R)(r) = \emptyset$

2. (M, N)-SOFT INTERSECTION NEARSEMIRINGS AND (M, N)-SI-IDEALS

In this section, we introduce the notions of (M, N)-Soft intersection nearsemirings ((M, N)-SI-nearsemirings) and (M, N)-SI ideals with illustrative examples. We apply different soft set operations to these newly established soft structures for the sake of investigations. Throughout, we assume that $\emptyset \subseteq M \subset N \subseteq U$.

Definition 9. Let R be a nearsemiring and f_R be a soft set over U. Then, f_R is called an (M, N)-soft intersection nearsemiring ((M, N)-SI- nearsemiring) of a nearsemiring R over U, if it satisfies the following properties.

(1) $f_R(x+y) \cup M \supseteq f_R(x) \cap f_R(y) \cap N$

(2) $f_R(xy) \cup M \supseteq f_R(x) \cap f_R(y) \cap N$, for all $x, y \in R$.

Example 1. Let us consider $U = D_6 = \{e, a, a^2, b, ab, a^2b\}$ and a (right) nearsemiring $R = \{0, x, y\}$ defined by the following tables.

$+$	0	x	y
0	0	x	y
x	x	x	y
y	y	y	y

$.$	0	x	y
0	0	x	y
x	x	x	y
y	y	x	y

We assume that R is the set of parameters and $U = D_6$. We construct a soft set over U as, $f_R(0) = \{e, a, b, ab\}$, $f_R(x) = \{a, b, ab\}$, $f_R(y) = \{a, b\}$. Let us take $M = \{e, a\}$ and $N = U$. Then, it is easy to verify that the soft set f_R is an (M, N)-SI-nearsemiring.

Remark 1. If f_R is a soft intersection nearsemiring over U, then f_R is an $(\emptyset, U)$-SI-nearsemiring of R over U. Thus, every SI-nearsemiring is an (M, N)-SI-nearsemiring. However, the converse is not true, in general.

Example 2. Let us consider that $U = D_6 = \{e, a, a^2, b, ab, a^2b\}$ and a (left) nearsemiring $R = \{0, x, y, z\}$ defined by the following tables.

$+$	0	x	y	z
0	0	x	y	z
x	x	x	y	z
y	y	y	y	z
z	z	z	z	z

$.$	0	x	y	z
0	0	0	0	0
x	0	0	0	y
y	0	0	0	0
z	0	y	0	x

Let us consider that R is a set of parameters and $U = D_6$ is the universal set. Let us take $M = \{e, a\}$ and $N = U$. We construct a soft set over U as, $f_R(0) = \{e, a^2, b, a^2b\}$, $f_R(x) = \{a, b, ab\}$, $f_R(y) = \{a, a^2, b\}$, and $f_R(z) = \{a^2, a^2b\}$. Then, one can easily prove that f_R is an (M, N)-SI-nearsemiring of R over U. However, $f_R(xy) = f_R(0) = \{e, a^2, b, a^2b\} \not\supseteq f_R(a) \cap f_R(b) = \{a, b\}$. Thus f_R is not a soft int nearsemiring over U.

Corollary 1. It is easy to verify that if $f_R(x) = U$ for all $x \in R$, then f_R is a (M, N)-SI-nearsemiring. We will abbreviate such (M, N)-SI- nearsemiring by $\widetilde{\mathbb{R}}$.

The following results can be easy to verify.

Proposition 1. (1) *If f_R and h_R are two $(M,\, N)$-SI- nearsemirings of R over U, then $f_R \widetilde{\cap} h_R$ is also an $(M,\, N)$-SI- nearsemiring U.*
(2) *If f_R and g_R are two $(M,\, N)$-SI- nearsemirings of R over U, then $f_R \wedge g_R$ is also an $(M,\, N)$-SI- nearsemiring.*
(3) *If f_R, h_T are two $(M,\, N)$-SI- nearsemirings of R over the same universal set U. Then, $f_R \times h_T$ is also a soft an are two $(M,\, N)$-SI- nearsemirings of R over the same universal set $U \times U$.*

However, if f_R and f_T are two $(M,\, N)$-SI- nearsemirings of R over the same universal set U, then $f_R\vee\, f_T$ need not be a $(M,\, N)$-SI- nearsemiring, we demonstrate this in example 3.

Example 3. Let us consider $U = D_8 = \{e,\, a,\, a^2,\, a^3,\, b,\, ab,\, a^2b,\, a^3b\}$ and two nearsemirings $R = \{0,\, x,\, y\}$ and $T = \{0,\, x',\, y',\, z',\, t'\}$ defined by the following tables.

Table 1

$+$	0	x	y
0	0	x	y
x	x	x	y
y	y	y	y

$.$	0	x	y
0	0	x	y
x	x	x	y
y	y	x	y

Table 2

$+$	$0'$	x'	y'	z'	t'
0	$0'$	x'	y'	z'	t'
x'	x'	x'	y'	t'	t'
y'	y'	y'	y'	t'	t'
z'	z'	t'	t'	z'	t'
t'	t'	t'	t'	t'	t'

$.$	$0'$	x'	y'	z'	t'
0	$0'$	0	$0'$	$0'$	$0'$
x'	$0'$	x'	x'	x'	x'
y'	$0'$	x'	y'	y'	t'
z'	$0'$	x'	y'	y'	t'
t'	$0'$	x'	y'	y'	t'

We assume that R and T are the set of parameters, $U = D_8$, and $M = \{a,\, b\}$, $N = U$. We construct $f_R(0) = \{e,\, a,\, b,\, ab\}$, $f_R(x) = \{a,\, b,\, ab\}$, and $f_R(y) = \{a,\, b\}$. Then it is easy to verify that f_R is an $(M,\, N)$-SI- nearsemiring U. Similarly, consider a nearsemiring $T = \{0',\, x',\, y',\, z',\, t'\}$, and we construct $f_T(0') = D_8$, $f_T(x') = \{e,\, a,\, b,\, ab\}$, $f_T(y') = \{e,\, a^2,\, b,\, a^2b\}$, $f_T(z') = \{e,\, b\}$, and $f_T(t') = \{e,\, a,\, a^2,\, b\}$. Clearly, f_T is an $(M,\, N)$-SI- nearsemiring. Let $(x,\, z')$, $(y,\, 0') \in R \times T$. Then, $f_{R\vee T}((x,\, z') +\, (y,\, 0')) \cup M = f_{R\vee T}(y,\, z') \cup M = f_R(y) \cup f_T(z') \cup M = \{a,\ b\} \cup \{e,\, b\} \cup \{a,\, b\} = \{e,\, a,\, b\} \not\supseteq f_{R\vee T}(x,\, z') \cap f_{R\vee T}(y, 0') \cap N = (f_R(x) \cup f_T(z')) \cap\ (f_R(y) \cup f_T(0')) \cap N = [\{a, b, ab\} \cup \{e, b\}] \cap [\{a,\, b\} \cup D_8] \cap U = \{a, b, ab\} \cap D_8 \cap U = \{a, b, ab\} \cap U = \{a, b, ab\}$. Hence $f_R\vee\, f_T$ is not an $(M,\, N)$-SI- nearsemiring.

The following lemma is obvious.

Lemma 1. (1) If $r \in R$, such that $r = \sum\limits_{i=1}^{n} a_i b_i$ and f_R be an $(M,\, N)$-SI- nearsemirings of R over U, then $f_R(r) \cup M = f_R(\sum\limits_{i=1}^{n} a_i b_i) \cup M \supseteq f_R(a_i) \cap f_R(b_i) \cap N$ for all $1 \leq i \leq n$.
(2) If R is an additively commutaive nearsemiring and $r = \sum\limits_{i=1}^{n} a_i + b_i$ such that f_R is an $(M,\, N)$-SI- nearsemirings of R over U. Then, $f_R(r) \cup M = f_R\ (\sum\limits_{i=1}^{n} a_i + b_i) \supseteq f_R(a_i) \cap f_R(b_i) \cap N$ for all $1 \leq i \leq n$.

Here we present the definition of soft uni-int product introduced in [20] and discuss it in the context of nearsemiring .

Definition 10. *[20]* Let f_R and h_R be the soft set over the common universe U. Then, soft union intersection (uni-int) product $f_R \star h_R$ is defined by

$$(f_R \star h_R)(r) = \begin{cases} \bigcup\limits_{r=\sum\limits_{i=1}^{n} a_i b_i} \{f_R(a_i) \cap h_R(b_i)\}, \text{ if } r = \sum\limits_{i=1}^{n} a_i b_i \text{ and } a_i b_i \neq 0, \text{ for all } 1 \leq i \leq n \\ = \emptyset, \text{ otherwise} \end{cases}$$

It is worth noting that if a nearsemiring R is distributively generated and having multiplicative identity then, $(f_R \star h_R)(r) \neq \emptyset$.

Theorem 1. *Let R be a commutative, distributively generated nearsemiring with multiplicative identity and f_R be the soft set over U. Then, f_R is an (M, N)-SI-nearsemiring of R over U iff $f_R(r_1 + r_2) \cup M \supseteq f_R(r_1) \cap f_R(r_2) \cap N$ and $f_R \cup M \supseteq (f_R \star f_R) \cap N$ for all $r_1, r_2 \in R$.*

Proof. We assume that f_R be an (M, N)-SI-nearsemiring of R over U. Then, $f_R(r_1 + r_2) \cup M \supseteq (f_R(r_1) \cap f_R(r_2)) \cap N$. Let $r = \sum\limits_{i=1}^{n} x_i y_i$ such that $(f_R \star f_R)(r) = \emptyset$, then it is easy to verify. Let $(f_R \star f_R)(r) \neq \emptyset$. Then,

$$\begin{aligned} (f_R \star f_R)(r) \cap N &= \bigcup_{r=\sum\limits_{i=1}^{n} x_i y_i} (f_R(x_i) \cap f_R(y_i)) \cap N \\ &\subseteq \bigcup_{r=\sum\limits_{i=1}^{n} x_i y_i} (f_R(x_i y_i)) \cup M \\ &= \bigcup_{r=\sum\limits_{i=1}^{n} x_i y_i} (f_R)(r) \cup M \\ &= ((f_R)(r)) \cup M \end{aligned}$$

Hence, $f_R \cup M \supseteq (f_R \star f_R) \cap N$.
Conversely, let $f_R(r_1 + r_2) \cup M \supseteq f_R(r_1) \cap f_R(r_2) \cap N$ and $f_R \cup M \supseteq (f_R \star f_R) \cap N$ for all $r_1, r_2 \in R$. Then,

$$\begin{aligned} f_R(r_1 r_2) \cup M &\supseteq (f_R \star f_R)(r_1 r_2) \cap N \\ &= \bigcup_{r_1 r_2=\sum\limits_{i=1}^{n} x_i y_i} (f_R(x_i) \cap f_R(y_i)) \cap N \\ &= (f_R(r_1) \cap f_R(r_2)) \cap N. \end{aligned}$$

Hence, f_R is an (M, N)-SI-nearsemiring of R over U. □

Let R be an additively commutative nearsemiring, then we have the following.

Theorem 2. *Let f_R be an (M, N)-SI-nearsemiring of R over U. Then, $f_R \cup M \supseteq (f_R \oplus \widetilde{\mathbb{R}}) \cap N$.*

Proof. Let f_R be an (M, N)-SI-nearsemiring of R over U. Let $r = x + y$ such that $(f_R \oplus \widetilde{\mathbb{R}})(r) \neq \emptyset$, for all $x \in R$. Then,

$$\begin{aligned} (f_R \oplus \widetilde{\mathbb{R}})(r) \cap N &= \bigcup_{r=y+z} \{f_R(y) \cap \widetilde{\mathbb{R}}(z)\} \cap N \\ &\subseteq (f_R(y+z) \cap R) \cup M \\ &= f_R(r) \cup M. \end{aligned}$$

Hence, $f_R \cup M \supseteq (\, f_R \oplus \widetilde{\mathbb{R}}) \cap N$. $\square$

Definition 11. Let f_R be an $(M,\, N)$-SI-nearsemiring of R over U and f_S be a nonempty soft subset of f_R over U. If f_S is itself an $(M,\, N)$-SI- nearsemiring of R over U. Then, we call f_S an $(M,\, N)$-SI-subnearsemiring of R over U and is denoted by $f_S \,\tilde{\le}_i^{(M,N)}\, f_R$.

Example 4. Let us consider $U = D_8 = \{e,\, a,\, a^2,\, a^3,\, b,\, ab,\, a^2b,\, a^3b\}$ and a (right) nearsemiring $R = \{0,$ $x,\, y,\, z,\, t\}$ defined by the following tables.

Table 3

$+$	0	x	y	z	t
0	0	x	y	z	t
x	x	x	y	t	t
y	y	y	y	t	t
z	z	t	t	z	t
t	t	t	t	t	t

$.$	0	x	y	z	t
0	0	0	0	0	0
x	0	x	x	x	x
y	0	x	y	y	t
z	0	x	y	y	t
t	0	x	y	y	t

Here we assume that R is the set of parameters, $U = D_8$ and $M = \{e,\, b\}$, $N = U$. We construct $f_R(0) = D_8$, $f_R(x) = \{e,\ a,\ a^2,\, a^3,\ b,\ ab,\ \ a^2b\}$, $f_R(y) = \{e,\ a^2,\ b,\ a^2b\} = f_R(t)$, $f_R(z) = \{e,\ b\}$. Clearly, f_R is an $(M,\, N)$-SI-nearsemiring of R over U. Following the operations defined in table3, $S = \{0,\, y,\, z,\, t\}$ is a right sub-nearsemiring of R. We take S as a set of parameters $E = S$, $U = D_8$ and $M = \{e,\, b\}$, $N = U$. Then, we construct $f_S(0) = D_8$, $f_S(y) = f_S(t) = \{e,\, a^2,\, b\}$ and $f_S(z) = \{e,\, b\}$, which is an $(M,\, N)$-SI- nearsemiring of R over U. Evidently, $f_S \,\tilde{\le}_i^{(M,N)}\, f_R$.

Proposition 2. *Let f_R is an $(M,\, N)$-SI-nearsemiring over U. If $f_S \,\tilde{\le}_i^{(M,N)}\, f_R$ and $f_T \,\tilde{\le}_i^{(M,N)}\, f_R$ over U, then $f_S \,\tilde{\cap}\, f_T \,\tilde{\le}_i^{(M,N)}\, f_R$ over U.*

Definition 12. Let R be a nearsemiring and f_R be a soft set over U. Then, f_R is an $(M,\, N)$-SI-left (right) ideal of R over U, if it satisfies the following conditions.

(1) $f_R(x+y) \cup M \supseteq f_R(x) \cap f_R(y) \cap N$

(2) $f_R(xy) \cup M \supseteq f_R(y) \cap N$ (resp., $f_R(xy) \cup M \supseteq f_R(x) \cap N$) for all $x,\, y \in R$.

If f_R is an $(M,\, N)$-SI-left and right ideal of R over U, then we call a f_R is an $(M,\, N)$-SI-ideal of R over U. However, we may also define a two sided $(M,\, N)$-SI-ideal of R over U as follows.

Definition 13. Let R be a nearsemiring and f_R be a soft set over U. Then, f_R is an $(M,\, N)$-SI-ideal of R over U, if it satisfies the following properties.

(1) $f_R(x+y) \cup M \supseteq f_R(x) \cap f_R(y) \cap N$

(2) $f_R(xy) \cup M \supseteq f_R(x) \cup f_R(y) \cap N$, for all $x,\, y \in R$.

We highlight the relationship between definitions [12 & 13] in examples [5 & 6].

Example 5. Let us consider $U = D_8 = \{e,\, a,\, a^2,\, a^3,\, b,\, ab,\, a^2b,\, a^3b\}$ and a right nearsemirings $R = \{0,$ $a,\, b,\, c,\, d\}$ defined in the tables.

+	0	a	b	c	d
0	0	a	b	c	d
a	a	a	b	d	d
b	b	b	b	d	d
c	c	d	d	c	d
d	d	d	d	d	d

.	0	a	b	c	d
0	0	0	0	0	0
a	0	a	a	a	a
b	0	b	b	b	d
c	0	a	b	b	d
d	0	d	d	d	d

We assume that R is the set of parameters, $U = D_8$ and $M = \{e, b\}$, $N = U$. We construct, $f_R(0) = D_8$, $f_R(a) = \{e, a, a^2, a^3, b, ab, a^2b\}$, $f_R(b) = f_R(d) = \{e, a^2, b, a^2b\}$ and $f_R(c) = \{e, b\}$. Then, it is easy to verify that f_R is an (M, N)-SI-right ideal of R over U. However, it is not an (M, N)-SI-left ideal of R over U. For this, let $a, b \in R$ and $f_R(b.a) \cup M = f_R(b) \cup M = \{e, a^2, b, a^2b\} \not\supseteq f_R(a) \cap N = \{e, a, a^2, a^3, b, ab, a^2b\} \cap U = \{e, a, a^2, a^3, b, ab, a^2b\}$. Since f_R is not an (M, N)-SI-left ideal of R over U, we have $f_R(b.a) = f_R(b) = \{e, a^2, b, a^2b\} \not\supseteq f_R(b) \cup f_R(a) = f_R(a) = \{e, a, a^2, a^3, b, ab, a^2b\}$. Evidently, f_R is not two sided (M, N)-SI-ideal of R over U.

Example 6. Let us consider $U = D_8$ and a (right) nearsemiring $R = \{0, x, y, z, t\}$ defined in tables of Example *4*. We assume that R is the set of parameters, $U = D_8$ and $M = \{e, b\}$, $N = U$. We construct $f_R(0) = D_8$, $f_R(x) = \{e, a, a^2, a^3, b, ab, a^2b\}$, $f_R(y) = \{e, a^2, b, a^2b\} = f_R(t)$, $f_R(z) = \{e, b\}$. Clearly, f_R is (M, N)-SI-left and right ideal over U. Thus, by doing simple calculations one can easily verify that $f_R(xy) \supseteq f_R(x) \cup f_R(y)$, for all $x, y \in R$.

Corollary 2. It is easy to verify that if $f_R(x) = U$ for all $x \in R$, then f_R is clearly an (M, N)-SI-ideal of R over U. We will abbreviate such a soft int-ideal by $f_{\tilde{R}}$.

Proposition 3. *If R is a left seminearring (i.e., zero-symmetric nearsemiring) and f_R be an (M, N)-SI-right-ideal of R over U and h_R be an (M, N)-SI-left-ideal of R over U. Then, $f_R \odot h_R \subseteq f_R \tilde{\cap} h_R$.*

3. (M, N)-SI-c-IDEALS

In this section, we introduce the concept of an (M, N)-SI-c-ideal of nearsemiring R. We also investigate the basic properties and some characterizations of (M, N)-SI-c-ideals. Finally, we introduce the notions of soft uni-int-c-product and soft uni-int-c-sum of two (M, N)-SI-c-ideals and also utilize them.

Definition 14. Let f_R be a soft set over U. Then, f_R is said to be an (M, N)-soft intersection left (right) c-ideal (in short, (M, N)-SI-left (right) c-ideal of *)* R over U, if it satisfies the following properties.

(1) $f(x + y) \cup M \supseteq f(x) \cap f(y) \cap N$,

(2) $f(xy) \cup M \supseteq f(y) \cap N$ ($f(xy) \cup M \supseteq f(x) \cap N$),

(3) $f(x) \cup M \supseteq f(a) \cap f(b) \cap N$ with $x + y + a = b + y + x$, for all $a, b, x, y \in R$.

A soft set over U is called an (M, N)-SI-c-ideal of R, if it is both left and right (M, N)-SI-c-ideal.

Definition 15. Let f_R be an (M, N)-SI-left (right)-ideal of R over U. Then, f_R is said to be an (M, N)-SI-left (right) c-ideals of R over U, if $f_R(x) \cup M \supseteq f_R(a) \cap f_R(b) \cap N$ with $x + y + a = b + y + x$, for all $a, b \in I$ and $x, y \in R$.

By extending Example 5, we provide an example of an (M, N)-SI-right c-ideal of a nearsemiring R over U.

Example 7. Let us consider $U = D_8 = \{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ and a right nearsemiring $R = \{0, a, b, c, d\}$ defined in tables of Example 5. Let $M = \{e, b\}$ *and* $N = U$. We construct f_R as, $f_R(0) = D_8$, $f_R(a) = \{e, a, a^2, a^3, b, ab, a^2b\}$, $f_R(b) = f_R(d) = \{e, a^2, b, a^2b\}$ and $f_R(c) = \{e, b\}$. Clearly, f_R is an (M, N)-SI-right ideal over U. Furthermore, to prove f_R is an *(M, N)-SI-right c-ideal*, we have to show it satisfies condition (iii) of Definition *14*. For this, let $f_R(0) \cup M = D_8 \supseteq (f_R(b) \cap f_R(c)) \cap N = \{e, a^2, b, a^2b\} \cap \{e, b\} \cap U = \{e, b\}$, with $0 + d + b = c + d + 0 \Rightarrow d = d$ for $0, b, c, d \in R$. Similarly, we can prove for all $a, b, x, y \in R$.

The following results are very easy to prove.

Proposition 4. (1) *Let f_R and g_R be the two (M, N)-SI-left $(right)$ c-ideals of R over U. Then $f_R \wedge g_T$ is a soft left (right) c-int-ideal over U.* (2) *Let f_R and h_R be two (M, N)-SI-left $(right)$ c-ideals of nearsemiring R over U. Then, $f_R \tilde{\cap} h_R$ is also (M, N)-SI-left $(right)$ c-ideal of R over U.*

Definition 16. Let f_R, g_R be the two soft sets over U. We define soft intersection c-product of f_R and g_R as follows.

$$(f_R \circ_c h_R)(r) = \begin{cases} \bigcap\limits_{x+y+a_1b_1=a_2b_2+y+x} \{f_R(a_i) \cap g_R(b_i) : i = 1, 2\}, \\ \text{if } x \text{ is expressible as } x + y + a_1b_1 = a_2b_2 + y + x \\ \qquad = \emptyset, \text{ otherwise} \end{cases}$$

Theorem 3. *Let f_R be an (M, N)-SI-right-c-ideal and g_R be an (M, N)-SI-left-c-ideal over U. Then, $(f_R \tilde{\cap} g_R) \cup M \tilde{\supset} (f_R \circ_c g_R) \cap N$.*

Proof. Let $x \in R$, if $(f_R \circ_c g_R)(x) = \emptyset$, then trivially completes the proof. Let us consider $(f_R \circ_c g_R)(x) \neq \emptyset$. Since f_R is an (M, N)-SI-right-c- ideal,

$$\begin{aligned} f_R(x) \cup M \ &\supseteq \ f_R(a_1a_2) \cap f_R(b_1b_2) \cap N \\ &\supseteq \ f_R(a_1) \cap f_R(b_1) \cap N \end{aligned}$$

for each $a_i, b_i \in R, i = 1, 2$, satisfying $x + y + a_1b_1 = a_2b_2 + y + x$. Similarly, we can easily prove that $g_R(x) \supseteq g_R(a_2) \cap g_R(b_2)$. Finally,

$$\begin{aligned} (f_R \circ_c g_R)(x) \cap N \ &= \bigcap_{x+y+a_i=b_i+y+x} \{f_R(a_i) \cap g_R(b_i) : i = 1,\ 2\} \cap N \\ &\subseteq \ f_R(x) \cap g_R(x) \cup M \\ &= \ (f_R \tilde{\cap} g_R)(x) \cup M. \end{aligned}$$

Hence, $(f_R \tilde{\cap} g_R) \cup M \tilde{\supset} (f_R \circ_c g_R) \cap N$ □

Definition 17. (1) Let f_R, g_R be the two soft sets over U. Then, the soft uni-int c-sum of f_R and g_R is defined as follows.

$$(f_R \oplus_c g_R)(x) = \begin{cases} \bigcup\limits_{x+y+a_i=b_i+y+x} \{f_R(a_i) \cap f_R(b_i)\}, \text{ if } x \text{ is expressible as } x + y + a_i = b_i + y + x \\ \qquad = \emptyset, \text{ if } x \text{ is not expressible as } x + y + a_i = b_i + y + x \end{cases} \tag{2}$$

Let f_R and g_R be the soft set over the common universe U. Then, soft uni-int c-product $f_R \star_c g_R$ of f_R

and g_R is defined by

$$(f_R \star_c h_R)(x) = \begin{cases} \bigcup\limits_{x+y+\sum\limits_{i=1}^{n} a_i b_i = \sum\limits_{j=1}^{m} a'_j b'_j + y + x} \{f_R(a_i) \cap f_R(b_i) \cap (h_R(a'_j) \cap h_R(b'_j)\}, \\ \text{for all } 1 \leq i \leq n,\ 1 \leq j \leq m \\ = \emptyset, \text{ if x cannot be expressed as } x + y + \sum\limits_{i=1}^{n} a_i b_i = \sum\limits_{j=1}^{m} a'_j b'_j + y + x \end{cases}$$

Theorem 4. *Let f_R be an (M, N)-SI-c-ideal over U. Then, $f_R \cup M \supseteq (f_R \oplus_c \mathbb{R}) \cap N$.*

Proof. Let f_R be an (M, N)-SI-c-ideal over U. Assume that x is expressible as $x + y + a_i = b_i + y + x$ such that $(f_R \oplus \mathbb{R})(x) \neq \emptyset$, for all $x \in R$. Then,

$$\begin{aligned} (f_R \oplus_c \mathbb{R})(x) \cap N &= \bigcup_{x+y+a_i=b_i+y+x} \{f_R(a_i) \cap \mathbb{R}\,(b_i)\} \cap N \\ &= \bigcup_{x+y+a_i=b_i+y+x} \{f_R(a_i) \cap\ \mathbb{R}(b_i)\} \cap N \\ &\subseteq f_R(a_i + b_i) \cup M \\ &= f_R(x) \cup M. \end{aligned}$$

Hence, $f_R \cup M \supseteq (f_R \oplus_c \mathbb{R}) \cap N$. □

4. (M, N)-α-INCLUSION AND ITS APPLICATIONS

In this section, we introduce the notion of (M, N)-α-inclusion and investigate it with respect to (M, N)-SI-nearsemirings along with its algebraic applications. We also examine soft image, soft pre-image of (M, N)-SI-nearsemirings, (M, N)-SI-ideals and (M, N)-SI-c-ideals.

Definition 18. Let f_R be a soft set over U. Assume α be any subset of U and $\emptyset \subseteq M \subset N \subseteq U$. Then, (M, N)-α-inclusion of soft set f_R is defined by $f_R^{\supseteq (M,\, N)-\alpha} = \{x \in R : f_R(x) \cup M \supseteq \alpha \cap N\}$, for all $x \in R$.

If $\alpha = \emptyset$, then the set $f_R^{\supseteq (M,\, N)-\emptyset} = \{x \in R : f_R(x) \cup M \supset \emptyset\}$, for all $x \in R$ is said to be a support of f_R.

Corollary 3. Let f_R be a soft set over U. Then every (M, N)-α-inclusion of f_R is an $(\emptyset, U)$-α-inclusion.

Example 8. Let us conisider $U = \mathbb{Z}$ as a universal set, $E = \{1, 2, 3, 4\}$ the subset of set of parameters and $\alpha = \{2, 4, 6\}$ be a subset of U. We define the soft set f_R by

$$\begin{aligned} f_R(1) &= \{1, 2, 5, 6, 7\} \\ f_R(2) &= \{2, 3, 4, 5, 6, 8\} \\ f_R(3) &= \{-4, -2, 0, 1, 2, 3, 4, 6\} \\ f_R(4) &= \emptyset \end{aligned}$$

It is worth noting that $f_R^{\supseteq \alpha} = \{2, 3\}$ and $f_R^{\emptyset} = \{1, 2, 3\}$. Now we discuss different cases of $f_R^{\supseteq (M,\, N)-\alpha}$.
Case1 : If $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Suppose $M = \{2, 3, 4, 5, 6\}$ and $N = \{2, 3, 4, 5, 6, 7\}$. Then, one can easily see that $f_R^{\supseteq (M,\, N)-\alpha} = \{1, 2, 3\}$ and $f_R^{\supseteq (M,\, N)-\emptyset} = \{1, 2, 3, 4\}$. Thus, if $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$, then $f_R^{\supseteq \alpha} \subset f_R^{\supseteq (M,\, N)-\alpha}$.
Case 2 : If $\emptyset \subseteq M \subset \alpha \subset N \subseteq U$. Let $M = \{1\}$ and $N = \{2, 3, 4, 5, 6, 7\}$. Then, one can easily check that $f_R^{\supseteq \alpha} = \{2, 3\}$ and $f_R^{\emptyset} = \{1, 2, 3\}$. However, $f_R^{\supseteq (M,\, N)-\alpha}(1) = f_R(1) \cup M \not\supseteq \alpha \cap N$, and hence $f_R^{\supseteq (M,\, N)-\alpha} = f_R^{\supseteq \alpha}$.

Case 3 : If $\emptyset \subseteq M \subset N \subset \alpha \subseteq U$. For this let $M = \{1\}$, and $N = \{1, 4\}$. Then, $f_R^{\supseteq(M,N)-\alpha}(1) = f_R(1) \cup M \not\supseteq \alpha \cap N$, $f_R^{\supseteq(M,N)-\alpha}(2) = f_R(2) \cup M \supseteq \alpha \cap N$, $f_R^{\supseteq(M,N)-\alpha}(3) = f_R(3) \cup M \supseteq \alpha \cap N$, $f_R^{\supseteq(M,N)-\alpha}(4) = f_R(4) \cup M \not\supseteq \alpha \cap N$. Hence, $f_R^{\supseteq(M,N)-\alpha} \subset f_R^{\supseteq\alpha}$.

We have observed that (M, N)-α-inclusion contracts (as in case3) and extends α-inclusion (as in case1). But here, we will discuss its structural characteristics.

Remark 2. Let $f_A, f_B \in S(U)$ be two soft sets over U and $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Then,
(1) $f_A^{\supseteq(M,N)\text{-}\alpha} \cup f_B^{\supseteq(M,N)\text{-}\alpha} \subseteq (f_A \tilde{\cup} f_A)^{\supseteq(M,N)\text{-}\alpha}$
(2) $f_A^{\supseteq(M,N)\text{-}\alpha} \cap f_B^{\supseteq(M,N)\text{-}\alpha} \subseteq (f_A \tilde{\cap} f_A)^{\supseteq(M,N)\text{-}\alpha}$.

Proposition 5. *Let f_R be an (M, N)-SI-nearsemiring over U and $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Then, $f_R^{\supseteq(M,N)\text{-}\alpha}$ is a sub-nearsemiring of a nearsemiring R, whenever it is non-empty.*

Proof. Let $f_R^{\supseteq(M,N)\text{-}\alpha} \neq \emptyset$. Let $x, y \in f_R^{\supseteq(M,N)\text{-}\alpha}$, then $f_R(x) \cup M \supseteq \alpha \cap N$, $f_R(y) \cup M \supseteq \alpha \cap N$ and

$$\begin{aligned} f_R(x+y) \cup M &\supseteq (f_R(x) \cap f_R(y)) \cap N \\ &= (f_R(x) \cap N) \cap (f_R(y) \cap N) \\ &\supseteq (\alpha \cap N) \cap (\alpha \cap N) \\ &= \alpha \cap N \end{aligned}$$

Thus, $x + y \in f_R^{\supseteq(M,N)\text{-}\alpha}$. Similarly, one can easily show that $f_R(xy) \cup M \supseteq (f_R(x) \cap f_R(y)) \cap N$, it follows that $xy \in f_R^{\supseteq(M,N)\text{-}\alpha}$. Thus, $f_R^{\supseteq(M,N)\text{-}\alpha}$ is a sub-nearsemiring. □

Theorem 5. *Let f_R be a soft set over U and $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Then, f_R is an (M, N)-SI-left (right) ideal of R over U iff $f_R^{\supseteq(M,N)-\alpha}$ is a left (right) ideal of R such that $f_R^{\supseteq(M,N)-\alpha} \neq \emptyset$.*

Proof. Assume that $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Let f_R is an (M, N)-SI-left ideal of R over U. Consider $a, b \in f_R^{\supseteq(M,N)-\alpha} \Rightarrow f_R(a) \cup M \supseteq \alpha \cap N$ and $f_R(b) \cup M \supseteq \alpha \cap N$. It follows that, $f_R(a+b) \cup M \supseteq f_R(a) \cap f_R(b) \cap N \supseteq \alpha \cap N$. Hence $a + b \in f_R^{\supseteq(M,N)_\alpha}$. Again consider, $a \in f_R^{\supseteq(M,N)_\alpha}$ which implies $f_R(a) \cup M \supseteq \alpha \cap N$ and $r \in R$, it follows that $f_R(ra) \cup M \supseteq f_R(a) \cap N \supseteq \alpha \cap N$. Thus $ra \in f_R^{\supseteq(M,N)-\alpha}$, and hence $f_R^{\supseteq(M,N)-\alpha}$ is a left ideal of a nearsemiring R such that $f_R^{\supseteq(M,N)-\alpha} \neq \emptyset$.
Conversely, suppose that $f_R^{\supseteq(M,N)-\alpha}$ is a left ideal of R, such that $f_R^{\supseteq(M,N)-\alpha} \neq \emptyset$. Let $a, b \in f_R^{\supseteq(M,N)-\beta}$ such that $\beta \cap N = f_R(a) \cap f_R(b) \cap N$ for each $a, b \in R \Rightarrow a + b \in f_R^{\supseteq(M,N)-\beta}$. Thus, $f_R(a+b) \cup M \supseteq \beta \cap N = f_R(a) \cap f_R(b) \cap N$. Also, for each $a \in f_R^{\supseteq(M,N)-\gamma}$ such that $\gamma = f_R(a)$, we get $ra \in f_R^{\supseteq(M,N)-\gamma} \Rightarrow f_R(ra) \cup M \supseteq f_R(a) \cap N$. Thus, f_R is an (M, N)-SI-left ideal of R over U. □

Theorem 6. *Let f_R be a soft set over U and $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Then, f_R is an (M, N)-SI-left (right) c-ideal of R over U, if and only if $f_R^{\supseteq(M,N)-\beta}$ is a left (right) c-ideal of R such that $f_R^{\supseteq(M,N)-\beta} \neq \emptyset$.*

Proof. Assume that $\emptyset \subseteq \alpha \subset M \subset N \subseteq U$. Refer to Theorem 5, in which we have proved that f_R is an (M, N)-SI-left (right) ideal of R over U if and only if $f_R^{\supseteq(M,N)-\alpha}$ is a left (right) ideal of R for any $\alpha \in \wp(U)$ such that $f_R^{\supseteq(M,N)_\alpha} \neq \emptyset$. Suppose that f_R is an (M, N)-SI-left (right) ideal of R over U. Let $a, b \in f_R^{\supseteq(M,N)-\alpha}$ and $x, y \in R$, such that $x + y + a = b + y + x$. Since $a, b \in f_R^{\supseteq(M,N)-\alpha}$ implies that $f_R(a) \cup M \supseteq \alpha \cap N$ and $f_R(b) \cup M \supseteq \alpha \cap N$. Also we have $f_R(x) \cup M \supseteq f_R(a) \cap f_R(b) \cap N$. Then $f_R(x) \cup M \supseteq \alpha \cap N$, so $x \in f_R^{\supseteq(M,N)-\alpha}$. Hence $f_R^{\supseteq(M,N)_\alpha}$ is a left c-ideal of R.

Conversely, let $f_R^{\supseteq(M,\,N)-\alpha}$ be the left c-ideal of R, such that $f_R^{\supseteq(M,\,N)-\alpha} \neq \emptyset$. Let $a,\, b,\, x,\, y \in R$, such that $x + y + a = b + y + x$. Suppose $f_R(a) = \alpha_1,\ f_R(b) = \alpha_2$ and $\alpha_1 \cap \alpha_2 \cap N = \alpha$, where α, $\alpha_i \in \wp(U)$. Then, $a \in f_R^{\supseteq(M,\,N)-\alpha}$ and $b \in f_R^{\supseteq(M,\,N)-\alpha}$. Since $f_R^{\supseteq(M,\,N)-\alpha}$ is a left c-ideal of R, we have $x \in f_R^{\supseteq(M,\,N)-\alpha}$ i.e., $f_R(x) \cup M \supseteq f_R(a) \cap f_R(b) \cap N$. Hence, $(M,\,N)$-SI-left c-ideal of R over U. □

Theorem 7. *Let f_R and f_T be soft sets over U and Ψ be a nearsemiring anti-isomorphism from R to T. If f_R is an $(M,\,N)$-SI-left ideal of R over U, then $\Psi(f_R)$ is an $(M,\,N)$-SI-right ideal of T over U.*

Proof. Let $t_1,\ t_2 \in T$. Since Ψ is surjective, then there exist $r_1,\ r_2 \in R$ such that $\Psi(r_1) = t_1$, $\Psi(r_2) = t_2$. Then we have,

$$(\Psi(f_R))(t_1+t_2) \cup M = \bigcup\{f_R(r) : r \in R,\ \Psi(r) = t_1 + t_2\} \cup M$$
$$= \bigcup\{f_R(r) : r \in R,\ r = \Psi^{-1}(t_1+t_2)\} \cup M$$
$$= \bigcup\{f_R(r) : r \in R,\ r = \Psi^{-1}(\Psi(r_1+r_2)) = r_1 + r_2\} \cup M$$
$$= \bigcup\{f_R(r_1+r_2) : r_i \in R,\ \Psi(r_i) = t_i,\ i = 1,\, 2\} \cup M$$
$$\supseteq \bigcup\{f_R(r_1) \cap f_R(r_2) : r_i \in R,\ \Psi(r_i) = t_i,\ i = 1,\, 2\} \cap N$$
$$= (\bigcup\{f_R(r_1) : r_1 \in R,\ \Psi(r_1) = t_1\}) \cap (\cup\{f_R(r_2) : r_2 \in R,\ \Psi(r_2) = t_2\}) \cap N$$

$= (\Psi(f_R))(t_1)) \cap (\Psi(f_R))(t_2) \cap N$. Similarly, it is easy to prove that $(\Psi(f_R))(t_1t_2) \cup M \supseteq \Psi(f_R))(t_1) \cap \Psi(f_R))(t_2) \cap N$. Moreover,

$$\begin{aligned}
(\Psi(f_R))(t_1t_2) \cup M &= \bigcup\{f_R(r) : r \in R,\ \Psi(r) = t_1t_2\} \cup M \\
&= \bigcup\{f_R(r) : r \in R,\ r = \Psi^{-1}(t_1t_2)\} \cup M \\
&= \bigcup\{f_R(r) : r \in R,\ r = \Psi^{-1}(\Psi(r_2r_1)) = r_2r_1\} \cup M \\
&= \bigcup\{f_R(r_2r_1) : r_i \in R,\ \Psi(r_i) = t_i,\ i = 1,\, 2\} \cup M \\
&\supseteq \bigcup\{f_R(r_1) : r_1 \in R,\ \Psi(r_1) = t_1\} \cap N \\
&= (\Psi(f_R))(t_1)) \cap N.
\end{aligned}$$

Thus, $\Psi(f_R)$ is an $(M,\,N)$-SI-right ideal of T over U. □

Similarly, one can verify that if f_R is an $(M,\,N)$-SI-right ideal of R over U, then $\Psi(f_R)$ is an $(M,\,N)$-SI-left ideal of T over U.

Theorem 8. *Let f_R and f_T be soft sets over U and Ψ be a nearsemiring anti-homomorphism from R to T. If f_T is an $(M,\,N)$-SI-left ideal of T over U, then $\Psi^{-1}(f_T)$ is an $(M,\,N)$-SI-right ideal of R over U.*

Proof. Let $r_1, r_2 \in R$. Then,

$$\begin{aligned}
(\Psi^{-1}(f_T))(r_1+r_2) \cup M &= f_T(\Psi(r_1+r_2) \cap M \\
&= f_T((\Psi(r_1) + (\Psi(r_1)) \cup M \\
&\supseteq f_T((\Psi(r_1) \cap (\Psi(r_1)) \cap N \\
&= (\Psi^{-1}(f_T))(r_1) \cap (\Psi^{-1}(f_T))(r_2) \cap N.
\end{aligned}$$

Similarly, it is easy to verify that $(\Psi^{-1}(f_T))(r_1r_2)\cup M\supseteq\Psi^{-1}(f_T))(r_1)\cap\Psi^{-1}(f_T))(r_2)\cap N$. Also,

$$\begin{aligned}(\Psi^{-1}(f_T))(r_1r_2)\cup M &= f_T(\Psi(r_1r_2)\cup M\\ &= f_T(\Psi(r_2)\Psi(r_1))\cup M\\ &\supseteq f_T(\Psi(r_1))\cap N\\ &= (\Psi^{-1}(f_T))(r_1)\cap N.\end{aligned}$$

Hence $\Psi^{-1}(f_T)$ is an $(M,\,N)$-SI-right ideal of R over U. □

Conclusion

In this work, we have introduced $(M,\,N)$-SI-nearsemirings by using soft sets along with intersection and union operation of sets. We have also introduced $(M,\,N)$-SI-subnearsemirings, $(M,\,N)$-SI-left ideal and $(M,\,N)$-SI-c-ideals of a nearsemirings with illustrative examples. We also introduced the notion of $(M,\,N)$-α-inclusion of soft sets and applied to $(M,\,N)$-SI-ideals. To extend this study, one can further study the other algebraic structures on this pattern.

Conflict of interest. The authors declare that they have no conflict of interest.

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