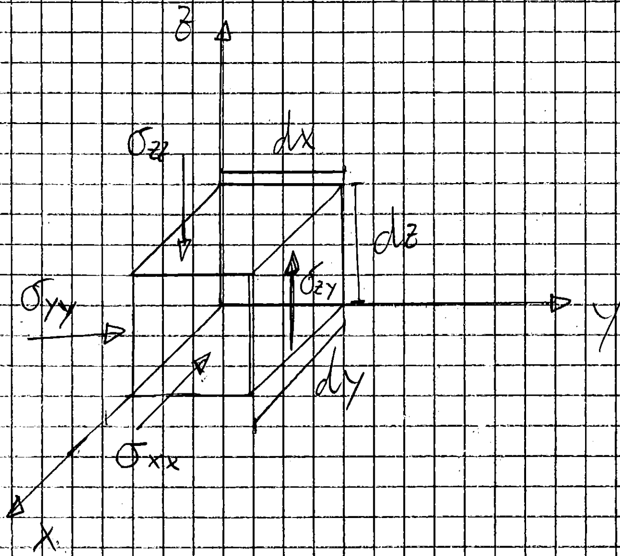
## MODELLO MATEMATICO

Thanks to Isaia Belladelli AA2014-2015

Seismic methods use elastic waves to obtain images.
Each medium has seismic elastic properties.
We consider elastic media (eg rocks, water, air).




$\sigma_{ij}$ = STRESS [Pa]

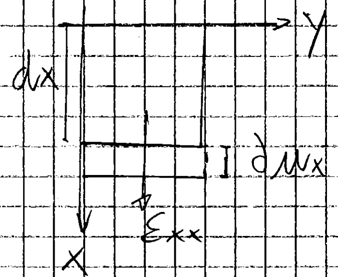
$i$ = direction, $j$ = normal to the face

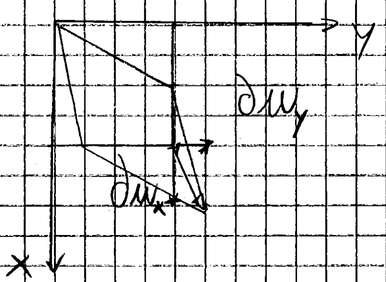
(la prendiamo è sempre applicata anche dalla parte opposta $\sigma \rightarrow \square \leftarrow \sigma$)

$i, j, k, l \in \{x, y, z\}$

$\varepsilon_{kl}$ = STRAIN (or relative deformation) $= \frac{1}{2}\left(\frac{\partial u_k}{\partial l} + \frac{\partial u_l}{\partial k}\right)$ [-]

dove $\vec{u}$ = vector of displacement $= u_x \bar{i}_x + u_y \bar{i}_y + u_z \bar{i}_z$




$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$

deformation along x

$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$

deformation of the shape due to parallel stress

**HOOKE'S LAW**

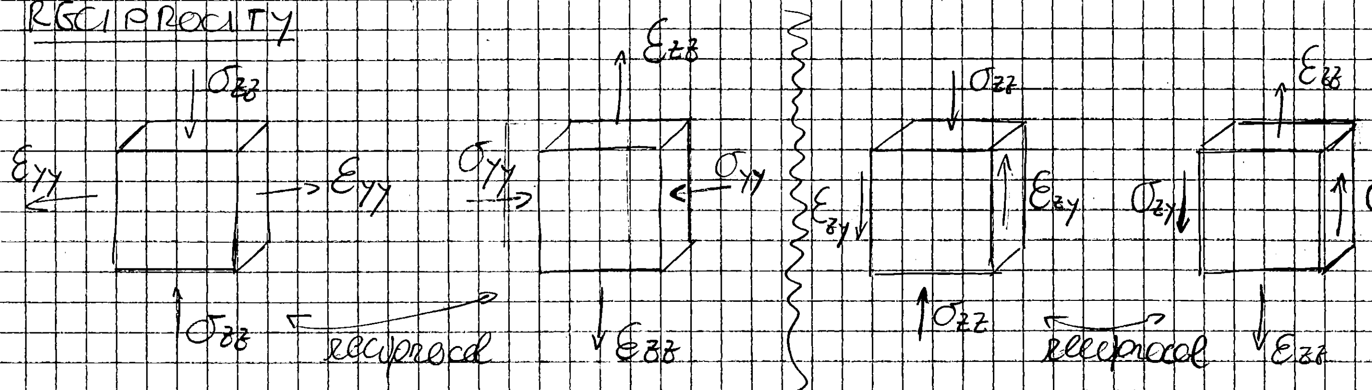
If we consider an elastic medium:

$\underline{\sigma_{ij}} = \underline{C_{ijkl}} \cdot \underline{\varepsilon_{kl}}$ where $C_{ijkl}$ is the TENSOR (matrix)

the TENSOR matrix contains 81 elastic constants

$C_{ijkl}$ [Pa] $(10^9 - 10^{11}$ Pa$)$

## RECIPROCITY



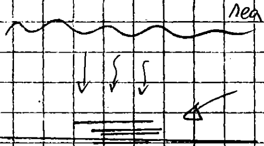

$C_{zz,yy} = C_{yy,zz}$ $\qquad$ $C_{zz,zy} = C_{zy,zz}$

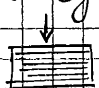
due to reciprocity, we pass from 81 to 21 indip. elastic constants

## ISOTROPY

If the elastic properties do not depend on the direction, the medium is called ISOTROPIC. In this case, we pass from 21 to 2 indip. elastic constants.

example of ANISOTROPIC MEDIA:

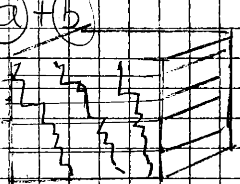
a) 


sedimentary — we have 1 axis of simmetry that is anisotropic , we have 5 indip. el. const

(vertical) TRANSVERSE ISOTROPY

b) 

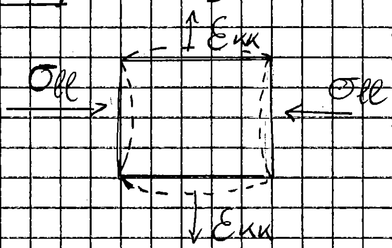
parallel fractures

c) = a + b 

ORTOROMBIC MEDIA

## POISSON's RATIO (or COEFFICIENT) [-]

$$\nu = -\frac{\varepsilon_{kk}}{\varepsilon_{ll}} = -\frac{\dfrac{\partial w_k}{dk}}{\dfrac{\partial w_l}{dl}}$$




$-1 < \nu < 0,5$

$-1$ → shape invariant (auxetic materials)

$0,5$ → totally incompressible (constant volume)

| material | $\nu$ |
|---|---|
| RUBBER | 0,5 |
| CORK | 0 |
| STEEL | 0,28 |
| PAPER | -1 |

$\nu = f(V)$ → velocity of elastic wave

## YOUNG's MODULUS [Pa]

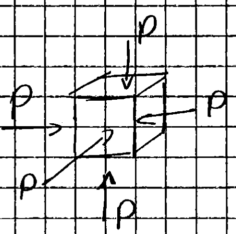
$$E = \frac{\sigma_{ii}}{\varepsilon_{kk}} \quad \text{with } i = k \longrightarrow E = C_{ii,kk} \quad \text{with } i = k$$

es $E_{steel} = 2 \cdot 1 \cdot 10^{11}$ Pa

## BULK's MODULUS [Pa]

$$K = \frac{p}{\frac{\Delta V}{V}}$$

relative volume change

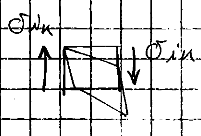



(pressure from all directions).

| material | K |
|---|---|
| STEEL | $1,6 \cdot 10^{11}$ [Pa] |
| WATER | $2,2 \cdot 10^{9}$ " |
| AIR | $1,01 \cdot 10^{5}$ " |

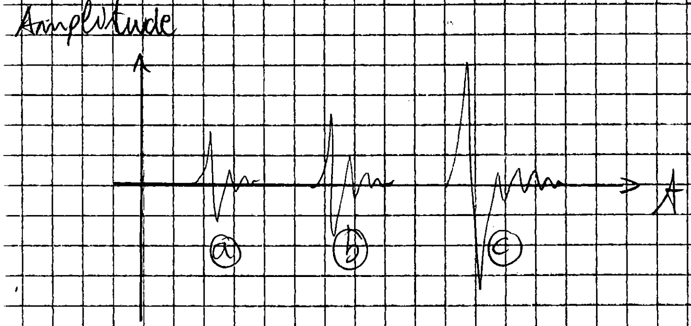
## LAME's PARAMETERS [Pa]

SHEAR RIGIDITY: $\mu = \dfrac{\sigma_{ik}}{\varepsilon_{ik}}$ with $i \neq k$ → $\mu = C_{ik,ik}$



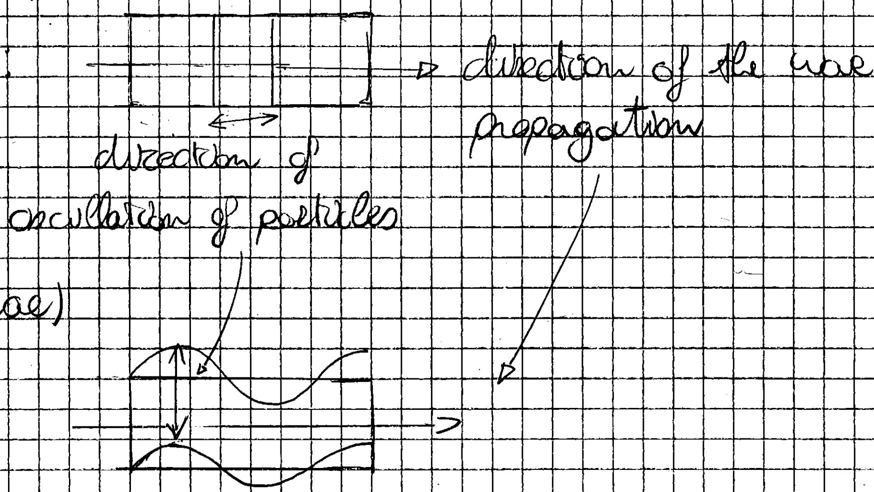

COMPRESSION RIGIDITY: $\lambda = \dfrac{\nu \cdot E}{(1+\nu)(1-2\nu)}$

# EARTHQUAKE




(a) p-waves (primae)

compressional waves:

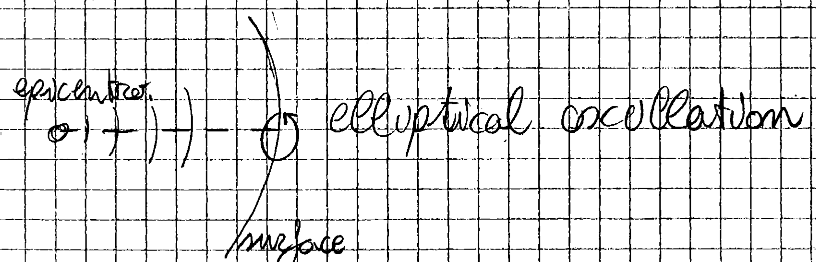



(b) S-waves (secundae)

shear waves:

(c) m-waves (maximae)

surface waves: si propagano dall'epicentro in maniera circolare e non sferica, così facendo si disperde meno energia (ampiezza più ampia)




es.

ELASTIC MEDIA: after the stress they return to their original shape.

PLASTIC MEDIA: " " do not return " (es. pongo).

la perdita dell'energia di un onda è dovuta sia alla geometria di propagazione sia all'assorbimento dei materiali che incontra ($\propto \frac{1}{Q}$): es. le alte frequenze della radio della macchina vengono più assorbite dalla carrozzeria della macchina rispetto a quelle basse. Di solito se $\rho \uparrow$ $Q \uparrow$ (ass. $\downarrow$)

## ELASTIC WAVE EQUATIONS ?

$$\begin{cases} \sigma_{ij}(t) = C_{ijkl}\,\varepsilon_{kl} \\ \vec{F} = m \cdot \vec{a} \quad \text{dove } \vec{F} \propto \sigma, \; m = \underset{\text{density}}{\rho} \cdot \text{Volume}, \; \vec{a} = \dfrac{\partial^2 \vec{u}}{\partial t^2} \end{cases}$$

risolvendo il sistema otteniamo

### (a) COMPRESSION WAVE (P-WAVE)

$$\nabla^2 \phi = \frac{\rho}{(\lambda+\mu)} \cdot \frac{\partial^2 \phi}{\partial t^2} \qquad \text{con} \quad \underset{\text{velocity of prop.}}{V_p} = \sqrt{\frac{\overbrace{(\lambda+2\mu)}^{\text{rigidity}}}{\underbrace{\rho}_{\text{density}}}}$$

### (b) SHEAR WAVE (S-WAVE)

$$\nabla^2 \psi = \frac{\rho}{\mu} \cdot \frac{\partial^2 \psi}{\partial t^2} \qquad \text{con} \quad V_s = \sqrt{\frac{\mu}{\rho}}$$

in un mezzo omogeneo (le proprietà elastiche non cambiano con la posizione) e isotropo ( " " con la direzione) valgono le equazioni delle onde (a) e (b). page

ACOUSTIC MEDIA → FLUIDS (gases and liquids)

in these materials $\mu = 0 \rightarrow V_s = 0$
so the shear waves do not propagate.

$$\vec{u}(x,y,z,t) = \nabla \phi + \nabla \wedge \vec{\psi}$$

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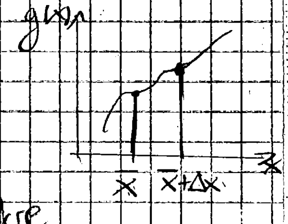
### DERIVATIVE OVERVIEW

$$\frac{\partial g(x)}{\partial x} = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}$$

finite difference implementation of the $1^{st}$ derivative




$$\frac{\partial^2 g(x)}{\partial x^2} = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - 2g(x) + g(x-\Delta x)}{\Delta x^2}$$

" of the $2^{nd}$ derivative

$$g(t) \xrightarrow{\mathcal{F}} G(\omega)$$

$$\frac{\partial g(t)}{\partial t} \xrightarrow{\mathcal{F}} j\,\omega \cdot G(\omega)$$

uso l'approssimazione

$$\frac{g(t) - g(t-\Delta t)}{\Delta t} \xrightarrow{\mathcal{F}} \frac{G(\omega) - G(\omega)e^{-j\omega\Delta t}}{\Delta t} =$$

$$= \frac{1}{\Delta t} \cdot e^{-j\omega\frac{\Delta t}{2}} \left(G(\omega) e^{j\omega\frac{\Delta t}{2}} - G(\omega) e^{-j\omega\frac{\Delta t}{2}}\right) =$$

$$= \frac{1}{\Delta t} e^{-j\omega\frac{\Delta t}{2}} \cdot G(\omega) \cdot j\, 2 \operatorname{sen}\left(\omega \frac{\Delta t}{2}\right) =$$

$\left(\operatorname{sen}(\alpha) \text{ con } \alpha \ll 1 \approx \alpha \quad \text{es. } \alpha < \frac{\pi}{20}\right)$

$$= j\, e^{-j\omega\frac{\Delta t}{2}} \frac{2}{\Delta t} G(\omega) \cdot \omega \frac{\Delta t}{2} =$$

$$= \underbrace{j\,\omega\, G(\omega)}_{\text{uguale a prima}} \; e^{-j\omega\frac{\Delta t}{2}} \text{ delay}$$

$\omega = 2\pi f \rightarrow 2\pi f_{max}$

$\omega \cdot \frac{\Delta t}{2} < \frac{\pi}{20} \longrightarrow \Delta t < \frac{1}{f_{max}} \cdot \frac{1}{20}$

passo di campionamento

es. vibrazione indotta con un martello

$f_{max} = 1\text{ kHz} \qquad \Delta t < 50\ \mu s$

$\lambda = \frac{v}{f} \longrightarrow \lambda_{min} = \frac{v_{min}}{f_{max}}$ → frequenza massima dello spettro del segnale propagante

$\Delta x, \Delta y, \Delta z < \frac{\lambda_{min}}{20}$ distanza massima per il campionamento $(x_n, x_n + \Delta x)$ per una buona approssimaz.

FRIEDRICH - COURHANT - LEVY
CONDITION FOR STABILITY

$\Delta x > v_{max} \cdot \Delta t$

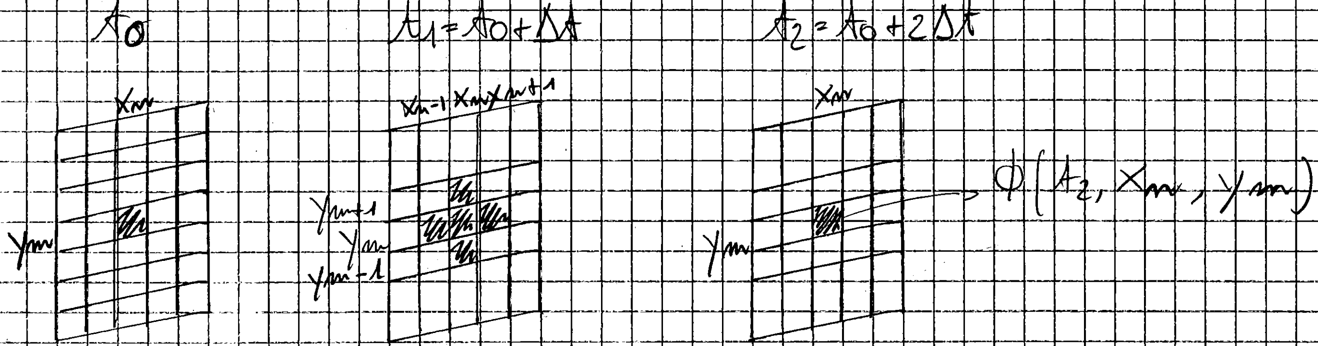
# SOLUTION OF THE P-WAVE EQUATION

$$\phi(t+\Delta t, x_n, y_m) =$$

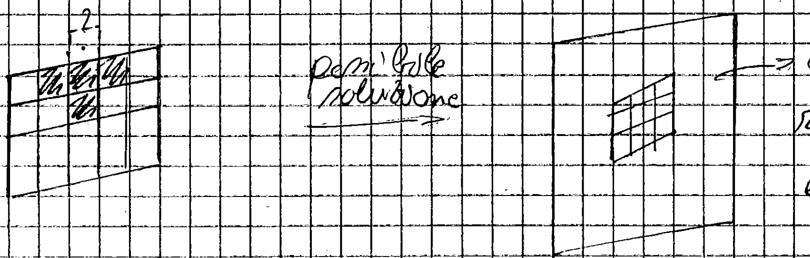
$$= \Delta t^2 \, V^2(x_n, y_m) \left[ \frac{\phi(t,x_{n+1},y_m) - 2\phi(t,x_n,y_m) + \phi(t,x_{n-1},y_m)}{\Delta x^2} + \frac{\phi(t,x_n,y_{m+1}) - 2\phi(t,x_n,y_m) + \phi(t,x_n,y_{m-1})}{\Delta y^2} \right]$$

possible non homogeneous

$$+ 2\phi(t, x_n, y_m) - \phi(t-\Delta t, x_n, y_m).$$

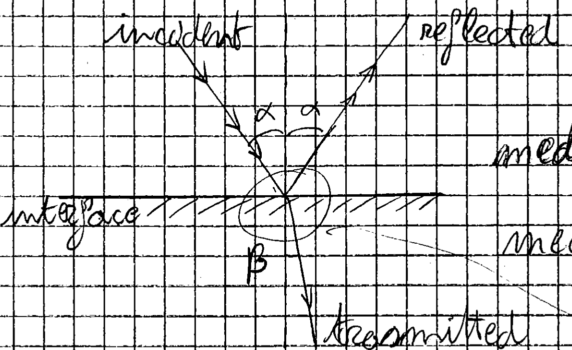



problema delle discontinuità e dei bordi




→ absorbing media, la riflessione sulla discontinuità è zero.

## DISCONTINUITA', RIFLESSIONE E TRASMISSIONE



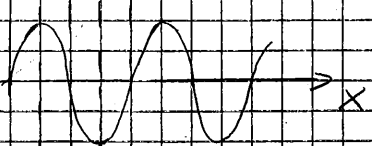

medium 1 ($V_{p1}, V_{s1}, \rho_1$)

medium 2 ($V_{p2}, V_{s2}, \rho_2$)

we have continuity of displacement and stresses.

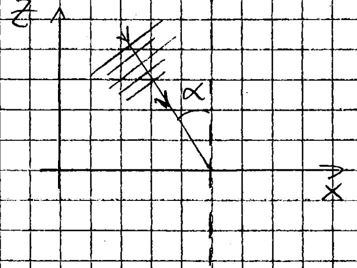
## OVERVIEW

$\Phi_1$ wave fronts




$$\phi(x,t) = \cos\left(2\pi f \frac{x}{V} - 2\pi f t\right)$$

più il "campione" è lontano dalla sorgente ($x=0$) più sarà stato "generato" prima nel tempo




$$\Phi(x,z,t) = \cos\left(2\pi f \frac{x}{V} \operatorname{sen}(\alpha) - 2\pi f t\right) + \cos\left(2\pi f \frac{z}{V} \cos(\alpha) - 2\pi f t\right)$$

numero d'onda $= \dfrac{\omega}{V} = \dfrac{2\pi}{\lambda} = k$

$$\left.\begin{aligned} \xi &= \text{vertical slowness} = \frac{\cos\alpha}{V} \\ p &= \text{horizontal slowness} = \frac{\operatorname{sen}\alpha}{V} \end{aligned}\right\} \sqrt{\xi^2 + p^2} = \frac{1}{V}$$

$p \cdot \omega = \operatorname{sen}\alpha \cdot k$ ; $\xi \cdot \omega = \cos\alpha \cdot k$

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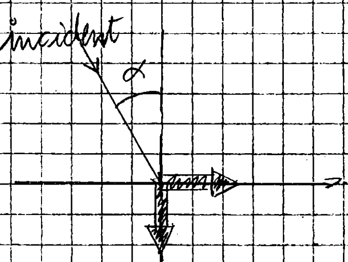
$$\underbrace{\vec{u} = \nabla\phi}_{\text{p-wave}} + \underbrace{\nabla \wedge \vec{\psi}}_{\text{s-wave}}$$

coefficiente di Riflessione $= R_{ij} = \dfrac{I \text{ reflected}}{J \text{ incident}}$ con $i,j \in \{P,S\}$ e $I,J \in \{\phi,\psi\}$

es. $R_{PP} = \dfrac{\phi \text{ refl}}{\phi \text{ inc}}$ $\quad R_{PS} = \dfrac{\psi \text{ refl}}{\phi \text{ inc}}$ mode conversion (una parte dell'energia dell'onda p viene portata all'onda S).

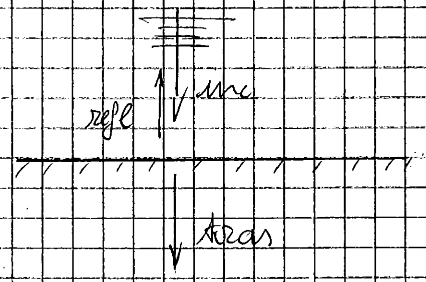
coefficiente di Trasmissione $T_{ij} = \dfrac{I \text{ transmitted}}{J \text{ incident}}$ con $i,j \in \{P,S\}$ e $I,J \in \{\phi,\psi\}$

$$\begin{bmatrix} \phi \\ \psi \end{bmatrix}_{refl} = \underbrace{\begin{bmatrix} R_{PP} & R_{SP} \\ R_{PS} & R_{SS} \end{bmatrix}}_{\text{scattering matrix for potentials}} \begin{bmatrix} \phi \\ \psi \end{bmatrix}_{inc} \qquad \begin{bmatrix} \phi \\ \psi \end{bmatrix}_{transm} = \begin{bmatrix} T_{PP} & T_{SP} \\ T_{PS} & T_{SS} \end{bmatrix} \begin{bmatrix} \phi \\ \psi \end{bmatrix}_{inc}$$




due to tilted direction, there is a generation of both compression and shear stress.
(Es. un onda sonora (solo p-wave) che incide su un muro genera sia un onda p che S).

# ACOUSTIC MEDIA




acoustic medium 1

acoustic medium 2

we consider a normal ($\alpha = 0$) incident p-wave

$R_{pp} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$ where $Z_i$ = acoustic impedance = $V_i \cdot \rho_i$

$T_{pp} = \frac{2 Z_1}{Z_2 + Z_1}$ $(= 1 - R_{pp})$

ESEMPI

(a)

AIR

SOLID (es concrete)

$Z_{air} = 340 \, m/s \cdot 1 \, \frac{kg}{m^3}$

$Z_{concrete} = 4000 \, m/s \cdot 2000 \, \frac{kg}{m^3}$

$\Big\} \; Z_2 \gg Z_1$

$R_{pp} \simeq 1$ $\qquad T_{pp} \approx 0$

es parlare con un muro in mezzo, si sente poco.

(b)

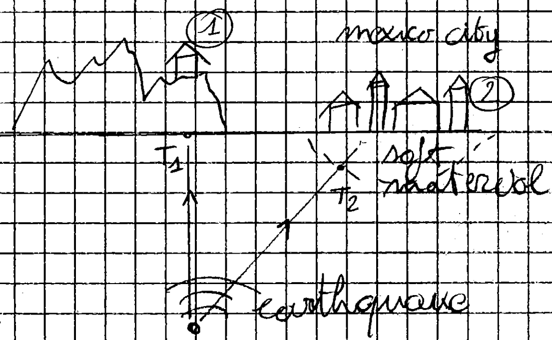
SOLID

AIR

$R_{pp} \approx -1$ $\qquad T_{pp} \approx 2$

es. bussare anche piano, si sente comunque

oppure




essendo $T_2 > T_1$, il terremoto sarà più forte in (2) che in (1)

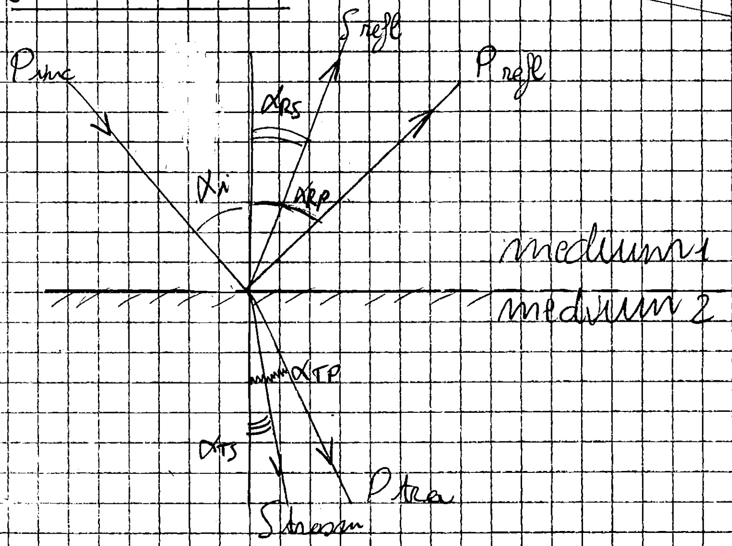
## PROPAGATION VELOCITY

$$V_{p\text{-wave}} = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad V_{S\text{-wave}} = \sqrt{\frac{\mu}{\rho}}$$

rule of thumb: $V_p \approx \sqrt{3}\, V_S \longrightarrow 0 \leq V_S < \frac{V_p}{\sqrt{2}}$

| material | $V_p$ |
| --- | --- |
| FILL NO COMPACTED | 200 m/s |
| AIR | 340 " |
| WATER | 1500 " |
| SANDSTONES | 2000 " |
| GRANITE, BASALT | 6000 " |

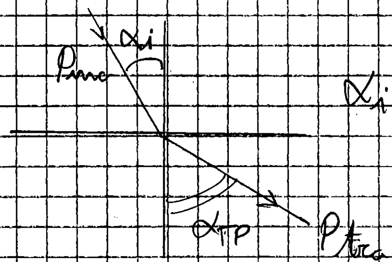
## SNELL'S LAW




the horizontal wave number of the reflected/transmitted waves must be equal to the incident wave.

$$p = \frac{\text{sen}\,\alpha_i}{V_{p1}} = \frac{\text{sen}\,\alpha_{RP}}{V_{p1}} = \frac{\text{sen}\,\alpha_{RS}}{V_{S1}} = \frac{\text{sen}\,\alpha_{TP}}{V_{p2}} = \frac{\text{sen}\,\alpha_{TS}}{V_{S2}}$$

es: acoustic media, $V_{p2} > V_{p1}$

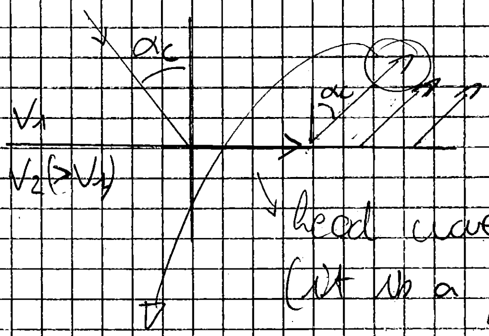



$\alpha_i < \alpha_{TP}$

# SURFACES WAVES

## ① HEAD WAVE

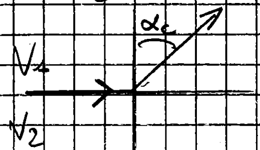
If $V_2 > V_1$, there is an incident angle $\alpha_c$, called critical angle, for which $\alpha_T = \frac{\pi}{2}$.



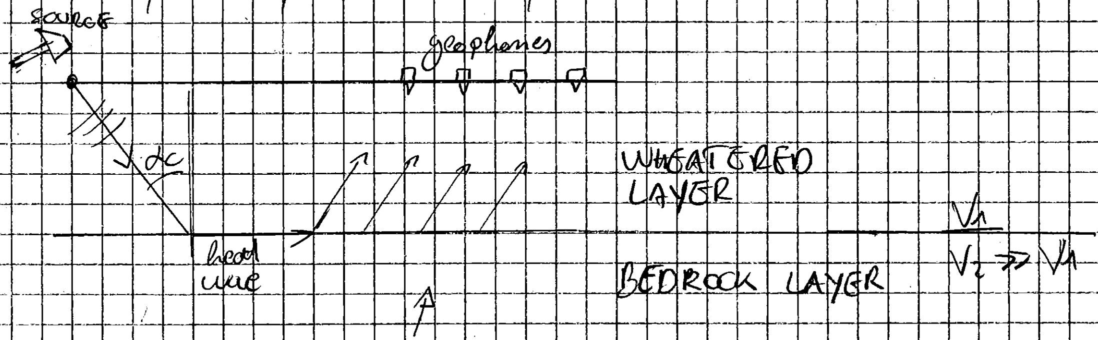

$$\alpha_c = \arcsen\left(\frac{V_1}{V_2}\right)$$

head wave or critical wave or refracted wave
(it is a particular case of surface wave)

Then there is the generation of returning waves with angle $\alpha_c$:

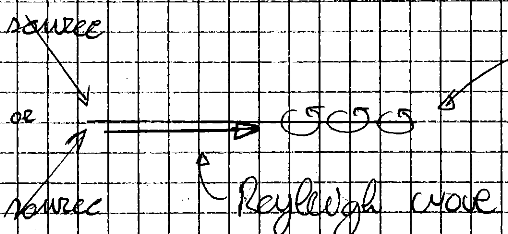



esempio: pianura padana




REFRACTION SEISMIC: ottengo informazioni su $V_1$, $V_2$ e sulla conformazione del layer bedrock

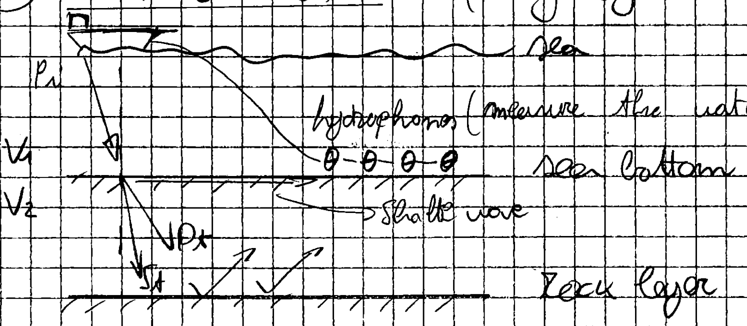
## ② RAYLEIGH WAVE




elliptical movement of particles (dangerous for buildings)

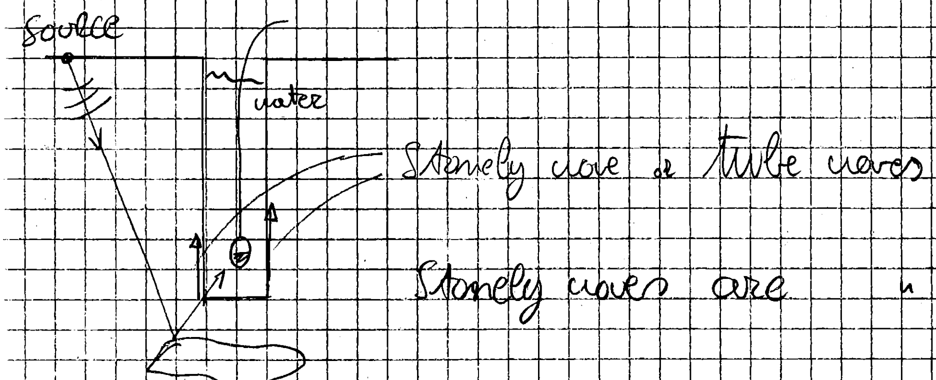
$V_R < V_S < V_P$

## ③ SHALTE WAVE (Rayleigh wave in sea bottom)




Shalte wave is a source of noise for the seismic imaging.

## ④ STONELEY WAVE



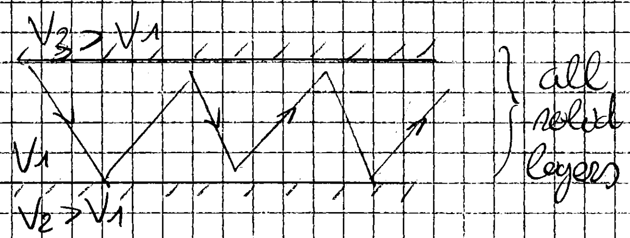

Stonely waves are " .

## 5) CONTINUOS REFLECTIONS (GUIDED WAVES)


If $\alpha > \alpha_c$, we have no transmission.

leaky mode ($\alpha < \alpha_c$)

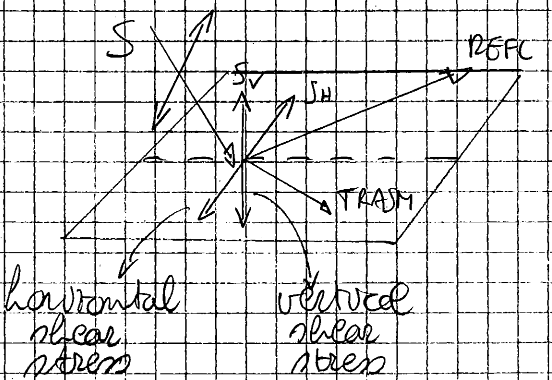
non leaky mode ($\alpha > \alpha_c$)




pseudo Rayleigh waves (leaky or non leaky)

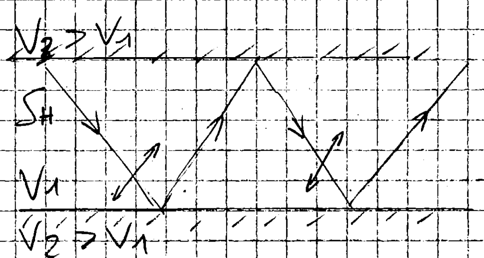
they are a combination of p-wave + S-wave.

## ⑥ LOVE WAVE




$S_V$ create $\left\langle \begin{array}{ll} S_{V,\text{refl}} (\updownarrow), & P_{\text{refl}} \\ S_{V,\text{tra}}, & P_{\text{tra}} \end{array} \right\}$ mode conversion

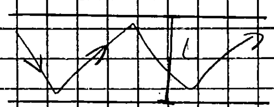
$S_H$ " $\left\langle \begin{array}{l} S_{H},\text{ refl} (\leftrightarrow) \\ S_{H},\text{ tra} \end{array} \right.$




→ Love wave

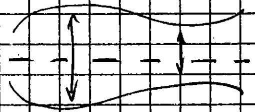
⑦ LAMB WAVE

Lamb wave is a mix of p-wave and s-wave that propagates in thin layer.

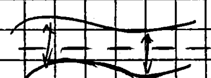


$L \leq \lambda$

Lamb waves produce deformation in the medium they propagate.



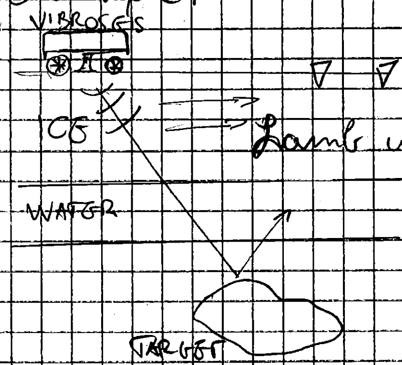
symmetric lamb mode



antysymmetric lamb mode

} Lamb waves have modal propagation (as light in fiber optics)

example:




→ Lamb waves → big source of noise.

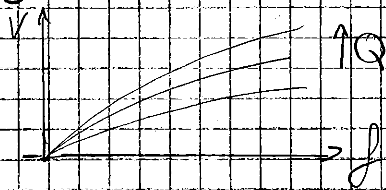
DISPERSION

phase velocity

Surface waves are highly dispersive, i.e. the velocity changes with frequency. Conversely, Body waves (p and s-waves) are a little dispersive.

A causa della dispersione, durante la propagazione la forma del segnale trasmesso cambia.

$V = V(f, Q)$



## SHOCK WAVE

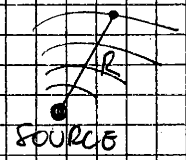
Le shock waves sono prodotte quando vengono utilizzate sorgenti che producono una pressione molto elevata, cioè vicina alla static pressure (es. cose ad altissimo volume).
La propagazione di queste onde presenta non-linearità, sono quindi difficili da analizzare.

# ATTENUATION

attenuation means amplitude decay

There are 3 source of attenuation:

## (1) GEOMETRICAL SPREADING




The energy is spread over a sphere surface ($A_S = 4\pi R^2$)

$$R(t) = V \cdot t$$

energy decay $\propto \frac{1}{R^2} \propto \frac{1}{t^2}$ (amplitude decay $\propto \frac{1}{t}$)

## (2) ABSORPTION

conversion of the elastic energy to heat due to internal friction

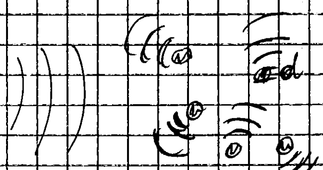
higher frequency waves are more attenuated than lower, because they have more attrition (compression for example) in the same period.

$E(t) \propto e^{-\alpha \cdot d}$ dove $\alpha = \frac{\pi}{\lambda Q} = \frac{f}{V} \cdot \frac{\pi}{Q}$ e $d = V \cdot t$

Q = QUALITY FACTOR (depends on the medium)

good value ≈ 1000 - 500 (es. water, granite) → low absorption
bad value ≈ 10 (es. weathered (uncompacted) rock) → high absorption

## (3) MIE SCATTERING



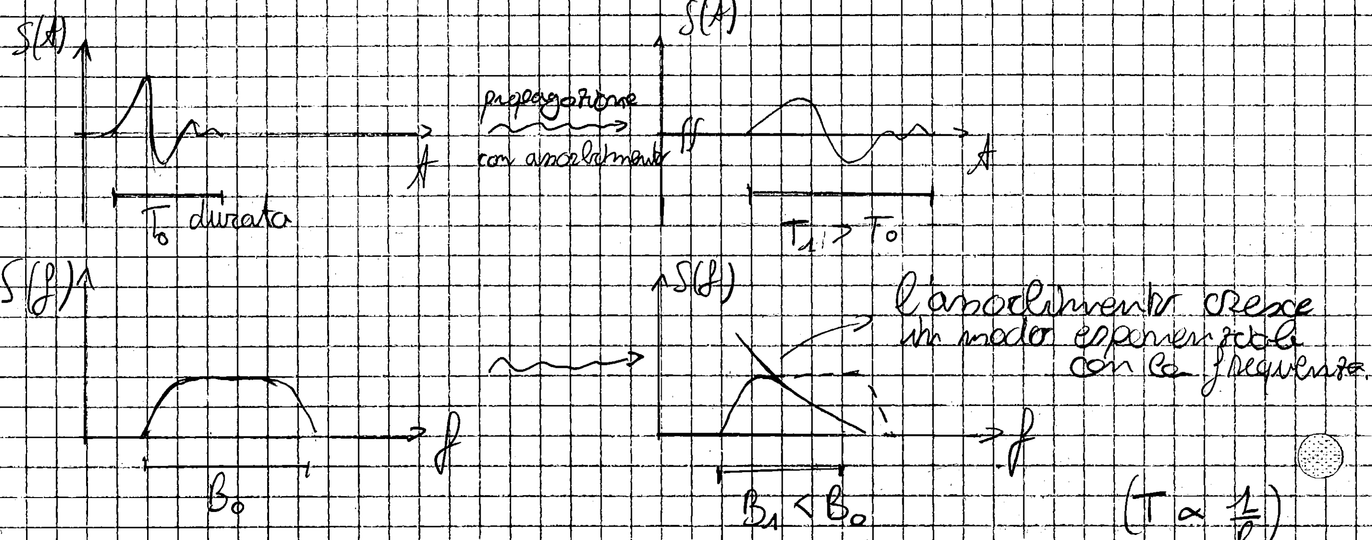
small heterogeneities (es. iron bars, stones, tube, ecc).

if $d \ll \lambda$ → high scattering (isotropic)

## ② ABSORPTION

Durante la propagazione le frequenze più alte vengono attenuate di più e questo comporta cambiamenti nello spettro e quindi cambiamenti (es. durata, forma) del segnale trasmesso.

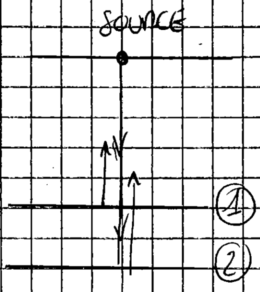
Es.

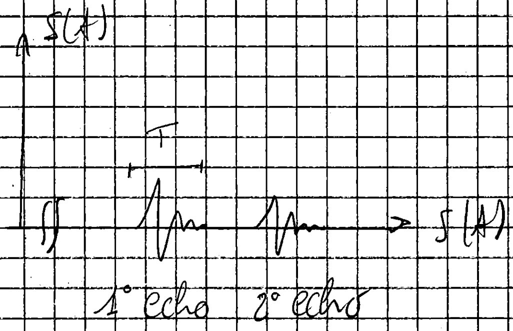



$\left(T \propto \frac{1}{B}\right)$

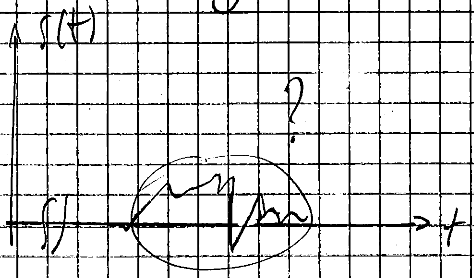
## RESOLUTION (seismic case) [m]

The resolution is the minimum distance at which it is possible to distinguish two different targets.

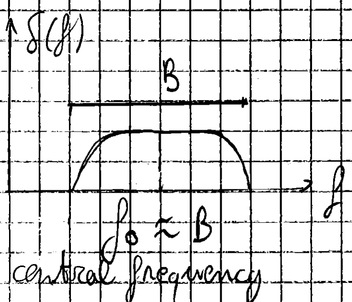


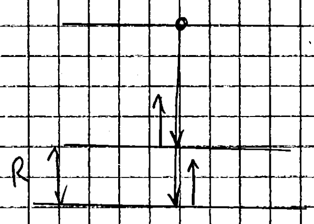




If the duration of the transmitted signal is too long, we can have overlapping of the returning echos (reflected signals) and the different echos could not be distinguished.




So the ideal source could be an impulse ($T \to 0$) which is a signal with infinite bandwidth. (not practical)
Let's now derive the resolution for a signal of bandwidth B:




$\Delta t$ = time between different reflections

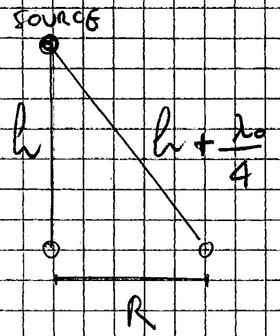
$$\Delta t = \frac{2R}{V} = \frac{T}{2} = \frac{1}{2B} \approx \frac{1}{2f_0}$$

nonostante un piccolo overlap si riesce comunque a distinguere echi diversi

$$R = \frac{V}{4f_0} \longrightarrow \boxed{R = \frac{\lambda_0}{4}}$$ VERTICAL RESOLUTION

valori tipici: sismica profonda, $f = 10-100$ Hz $\to R \approx 50-5$ m
" superficiale, $f = 100-1000$ Hz $\to R \approx 5-0.5$ m

# HORIZONTAL RESOLUTION




$$R = \sqrt{\left(h + \frac{\lambda_0}{4}\right)^2 - h^2} = \sqrt{\frac{\lambda_0 h}{2} + \frac{\lambda_0^2}{16}}$$

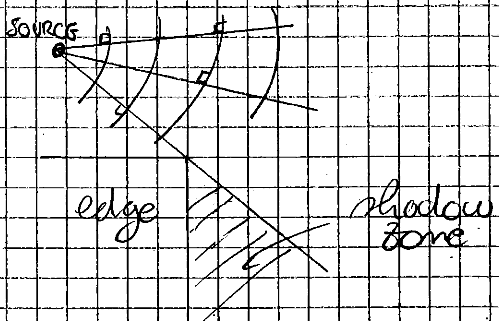
if $h >> \lambda_0$ : $R \approx \sqrt{\frac{\lambda_0 \cdot h}{2}}$

quindi: se $f \uparrow (\lambda \downarrow)$ $R \downarrow$ (higher resolution)

$h \uparrow$ $R \uparrow$ (lower resolution).

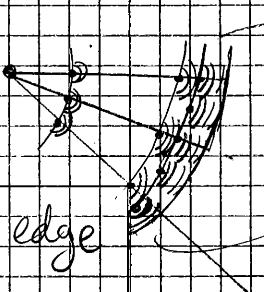
## RAYS

The rays representation is an approximation of the wave equations in which we consider an infinite bandwidth.
The rays are always normal w.r.t. the wavefront




## HUYGENS PRINCIPLE

Every point in a wavefront is an elementary source of spherical waves in the same direction of the rays.




→ the new wavefront is the envelope of the spherical waves.

→ huygens sources near the edge fill progressively the shadow zone.

Let's consider the acoustic wave equation

$$\frac{\partial^2 \phi}{\partial t^2} = V^2 \nabla^2 \phi \qquad \text{con } \phi = \phi(x, y, z, t) \quad \propto \text{pressure}$$

$$\Downarrow \mathcal{F}$$

$$-\omega^2 \phi(x, y, z, \omega) = V^2 \nabla^2 \phi(x, y, z, \omega)$$

hypothesis $\phi(x, y, z, \omega) = \underbrace{A(x, y, z, \omega)}_{\text{amplitude}} \cdot e^{\overbrace{-j\omega T(x, y, z, \omega)}^{\text{phase}}}$

delay $\left[ \mathcal{F}(s(t-\tau)) = S(\omega) \cdot e^{-j\omega\tau} \right]$

hp: $T \neq T(\omega)$ → medium is not dispersive $(V \neq V(\omega))$

this hypothesis is almost true for body waves, not for surface waves we obtain:

$$\underbrace{\nabla^2 A - \omega^2 A\cdot|\nabla T|^2}_{\text{real}} \underbrace{- j\omega\left(2\nabla A\,\nabla T - A\nabla^2 T\right)}_{\text{imaginary}} = \underbrace{-\frac{A\,\omega^2}{V^2}}_{\text{real}}$$

$$\begin{cases} j\omega(\ -\ ) = 0 \\ \nabla^2 A - \omega^2 A\cdot|\nabla T|^2 = -\dfrac{A\omega^2}{V^2} \end{cases}$$

transport equation

$$\cancel{\frac{\nabla^2 A}{A\omega^2}} - |\nabla T|^2 = -\frac{1}{V^2}$$

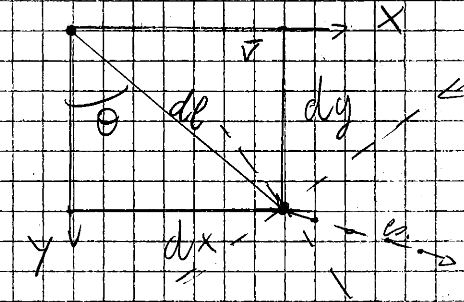
If we consider infinite bandwidth: $\omega \to \infty$

$$|\nabla T|^2 = \frac{1}{V^2}\left(= \overset{\text{slowness}}{S^2}\right)$$

EIKONAL EQUATION

questa equazione ci dà la traiettoria del raggio.

RAY TRACING ALGORITHM (2-D)




Fictitious local interface (normal to the derivative of the velocity $V(x,y)$).
At this interface we apply the Snell law.

$$\left.\begin{aligned} dx &= dl\cdot \operatorname{sen}\theta \\ dy &= dl\cdot\cos\theta \end{aligned}\right\}$$ arrival / starting point

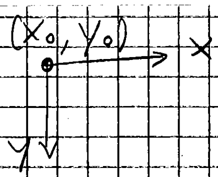
$$dT = \frac{dl}{V} = dl\cdot S$$ incremental time

$$\left.\begin{aligned} p_x &= \frac{\operatorname{sen}\theta}{V} = \operatorname{sen}\theta\cdot S \quad \text{horizontal slowness} \\ p_y &= \frac{\cos\theta}{V} = \cos\theta\cdot S \quad \text{vertical slowness} \end{aligned}\right\} \sqrt{p_x^2+p_y^2} = \frac{1}{V} = S$$

$$\left.\begin{aligned} dp_x &= \frac{\partial S}{\partial x}\cdot dl = S_x\cdot dl \\ dp_y &= \frac{\partial S}{\partial y}\cdot dl = S_y\cdot dl \end{aligned}\right\}$$ indicate ray direction

$$\begin{cases} dx = dl\cdot\operatorname{sen}\theta \\ p_x = \operatorname{sen}\theta\cdot S \end{cases} \Rightarrow dx = dl\cdot\frac{p_x}{S}$$

2-D example:

data: 

; $\underbrace{px_0, py_0}_{\text{angle of departure}}$ ; $T_0 = 0$ ; $dl = \Delta l$ (user choice)

$V_0 = \frac{1}{S_0}$

$$dx = x_1 - x_0 = \Delta l \cdot \frac{px_0}{S_0} \longrightarrow x_1 = \Delta l \frac{px_0}{S_0} + x_0$$

$$dy = y_1 - y_0 = \Delta l \frac{py_0}{S_0}$$

$$dT = T_1 - T_0 = \Delta l \cdot S_0$$

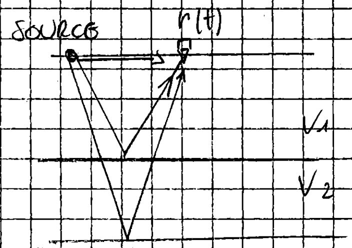
$$dpx = px_1 - px_0 = Sx_0 \cdot \Delta l \quad \text{dove} \quad Sx_0 = \frac{S(x + \overset{\text{diverso da } dx}{\Delta x}, y) - S(x)}{\Delta x}$$

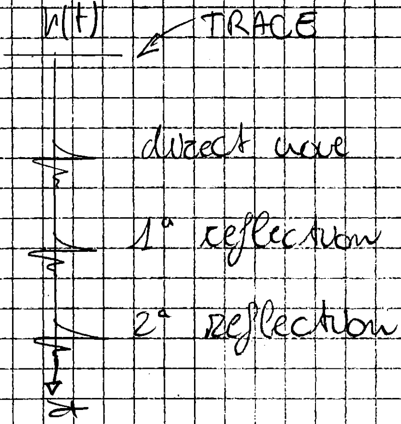
$$dpy = py_1 - py_0 = Sy_0 \cdot \Delta l$$

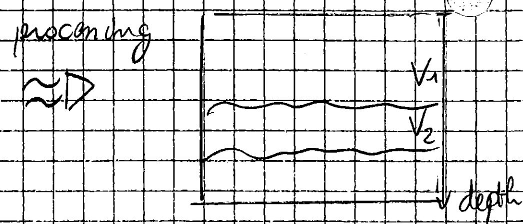
conclusione:

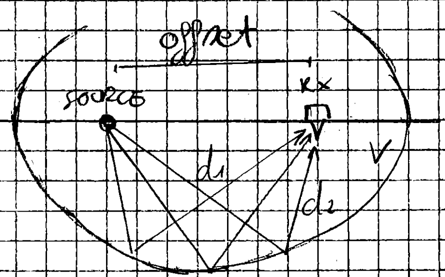
per tracciare il raggio applico la legge di Snell a livello infinitesimale ( )

# HOW TO USE SEISMIC WAVE



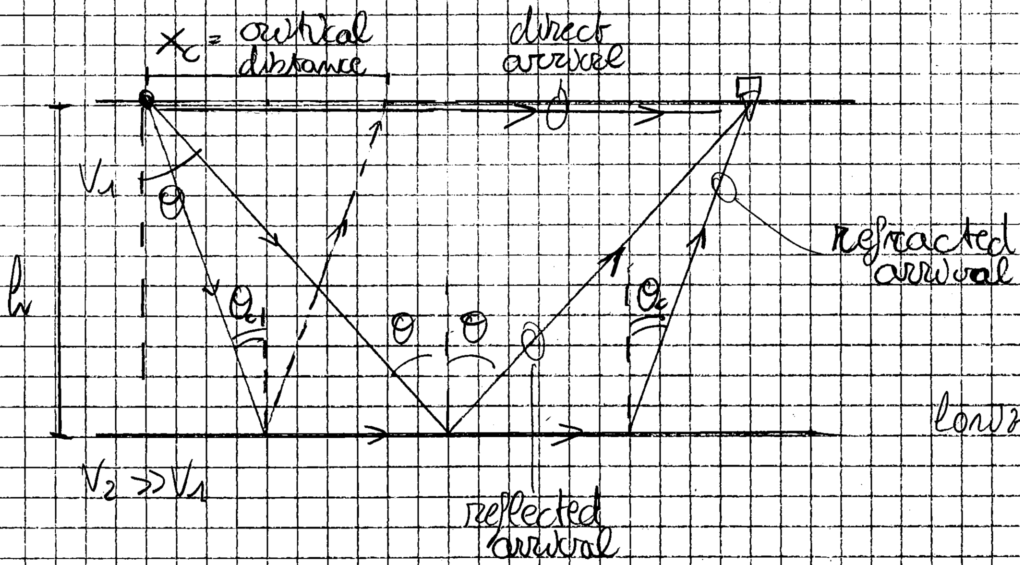

$$\tau = \frac{d}{V}$$

percorso del segnale

tempo di arrivo

velocità

essendo $d = d_1 + d_2$, dato $\tau$ e $V$, il luogo dei punti dove potrebbe trovarsi il bersaglio è un ellisse avente come fuochi • (source) e ▽ (Rx).




$$\theta_c = \arcsin\left(\frac{V_1}{V_2}\right)$$

horizontal plane reflector

$$\tau\ \text{direct}(x) = \frac{x}{V_1}$$

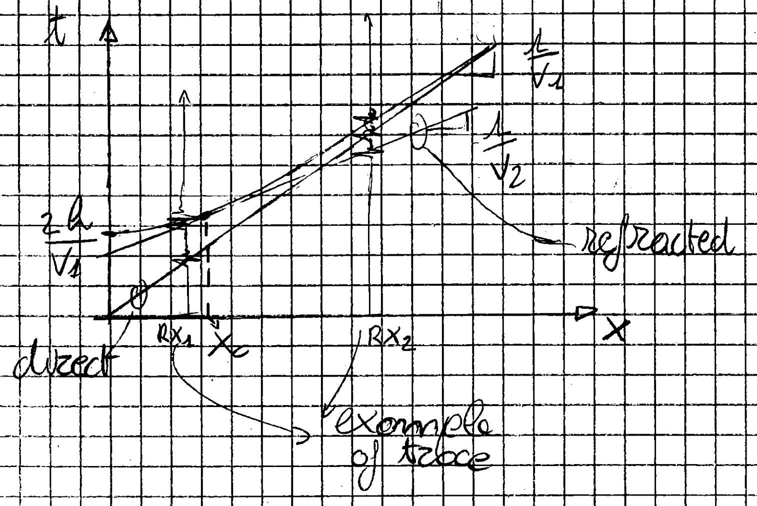
$$\tau\ \text{reflected}(x) = 2\frac{h}{\cos\theta}\cdot\frac{1}{V_1} \qquad \text{for } x \to \infty \quad \tau\ \text{reflected} \to \frac{x}{V_1}$$

$$\tau\ \text{refracted}(x) = \tau\ \text{reflected}(x_c) + \frac{x - x_c}{V_2} = 2\frac{h}{\cos\theta_c}\cdot\frac{1}{V_1} + \frac{x}{V_2} - 2\frac{h\tan\theta_c}{V_2}$$

$$\left(V_2 = \frac{V_1}{\operatorname{sen}\theta_c}\right) \qquad = 2\frac{h}{\cos\theta_c}\cdot\frac{1}{V_1} + \frac{x}{V_2} - 2\frac{h}{V_1}\cdot\operatorname{sen}\theta_c\frac{\operatorname{sen}\theta_c}{\cos\theta_c} =$$

$$= 2\frac{h\cos\theta_c}{V_1} + \frac{x}{V_2}$$

for $x = x_c$ $\qquad \tau_{refracted} = \tau_{reflected}$




COMMON-SHOT GATHER

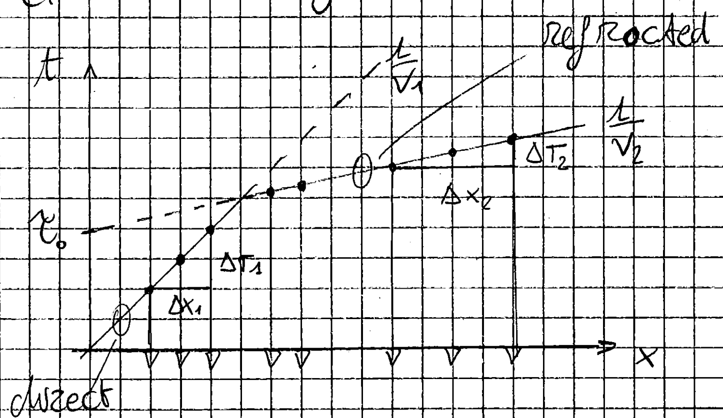
? $t_{refracted}(0)$

## REFRACTION SEISMIC METHOD

### (1) INTERSECTION TIME METHOD

hp: $V_2 > > V_1$ , horizontal plane interface

1) collect data from all geophones for one common shot;
2) for every trace, identify the time of first arrival (picks identification).




GATHER WITH COMPUTED TRAVELTIME CURVES

3) calculate the slope (so $\frac{1}{V_1}$) of direct arrival curve:

$$\frac{1}{V_1} = \frac{\Delta T_1}{\Delta X_1}$$

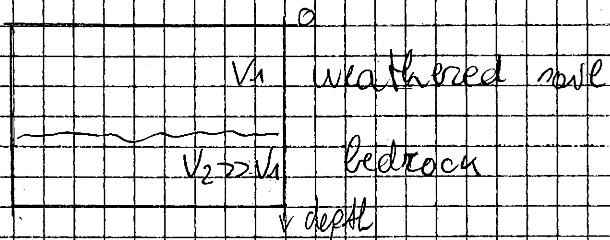
4) " " " (so $\frac{1}{V_2}$) " refracted " "

$$\frac{1}{V_2} = \frac{\Delta T_2}{\Delta X_2}$$

5) compute the height ~~of the height~~ extending the refracted curve down to $x = 0$.

$$\tau_0 = \frac{2 \cdot h \cos\theta_c}{V_1} \longrightarrow h = \frac{V_1}{2} \cdot \frac{\tau_0}{\cos\theta_c} \quad \propto \frac{V_1}{V_2}$$

This method is used for the detection of bedrock layer




Rayleigh waves are a big source of noise (ground roll noise).

## SIGNAL SOURCES

IDEAL SOURCES
- minimum duration
  - "picking" reflection time
  - high resolution
- high amplitude
  - more powerfull source
  - * stack many acquisition

## * STACK MANY ACQUISITION

Consider N acquisition for a single receiver ( $s(t)$ $r(t)$ )

$r_1(t) = s(t) + n_1(t)$

$r_2(t) = s(t) + n_2(t)$ → stationary

$\vdots$

$r_N(t) = s(t) + n_N(t)$

$$\sum_{i=1}^{N} r_i(t) = N \cdot s(t) + \sum_{i=1}^{N} n_i(t) \qquad \text{STACK}$$

$\Rightarrow E_{s,\,STACK} = N^2 \cdot E_s$

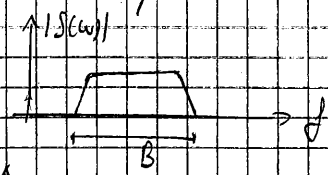
$\Rightarrow P_{noise,\,STACK} = N \cdot P_n$

$$SNR_{\text{single experiment}} = \frac{E_s}{P_n}$$

$$SNR_{\text{stack}} = \frac{N^{\cancel{2}} E_s}{\cancel{N} \cdot P_n} = N \cdot \frac{E_s}{P_n} = N \cdot SNR_{\text{single exp.}}$$

## WAVELET FORM and CHOICE

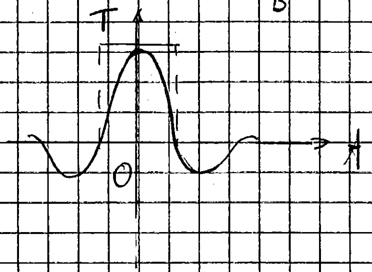
There are infinite wavelets with same spectral amplitude but difference phases:




1) ZERO PHASE WAVELET

$(\sphericalangle S(\omega) = 0)$

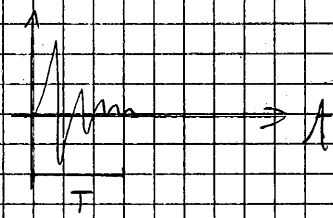
$T = \frac{1}{B}$




2) MINIMUM PHASE WAVELET (es. hammer fall)

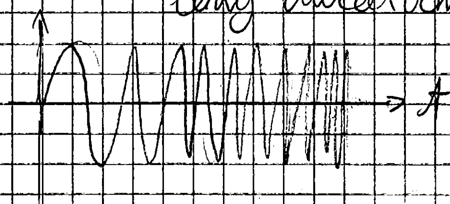
$(\sphericalangle S(\omega) = f(|S(\omega)|)$

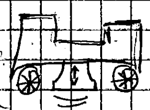
$T \approx \frac{1}{B}$




3) MIXED PHASE WAVELET (es. CHIRP) long duration

frequency changing during time




produced by VIBROSEIS (  )

alla fine l'energia dell'onda è uguale a quella di una esplosione.

### DECONVOLUTION

let's consider $s(t)$ be a chirp and that reflected signals are translated copy of the original, no absorption.

$$r(t) = \sum_{i=1}^{\tilde{N}} A_i \, s(t-\tau_i) \xrightarrow{\mathcal{F}} R(f) = \sum_{i=1}^{\tilde{N}} A_i \cdot S(f) \cdot e^{-j2\pi f \tau_i}$$

ora sostituiamo $s(t)$ (chirp) con una zero phase wavelet $s_0(t)$ tale che $|S_0(f)| = |S(f)|$ (stessa banda in modulo)

$$\underbrace{\hat{R}(f) = \left(\sum_{i=1}^{\tilde{N}} A_i \, e^{-j2\pi f \tau_i}\right) \cdot S(f) \cdot \frac{S_0(f)}{S(f)}}_{\text{ZERO PHASE DECONVOLUTION}} \quad \text{defined only for } f \in B$$

$\rightarrow$ stima di $s(t)$

per avere S(t) (quindi S(f)) posso mettere dei geofoni vicino al trasmettitore, oppure montare un accelerometro sul vibroseis. Logicamente questa sarà una stima $\hat{S}(t) \approx S(t)$

If we use instead a minimum phase wavelet $S_m(t)$ to be substitute to $S(t)$, we obtain:

$$\hat{R}(f) = \frac{R(f)}{\hat{S}(f)} S_m(f)$$ MINIMUM PHASE DECONVOLUTION

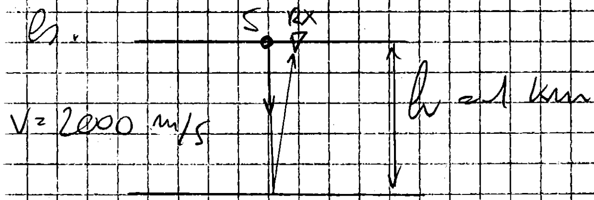
seismic trace → $R(f)$; true wavelet → $\hat{S}(f)$; $S_m(f)$ → desidered wavelet

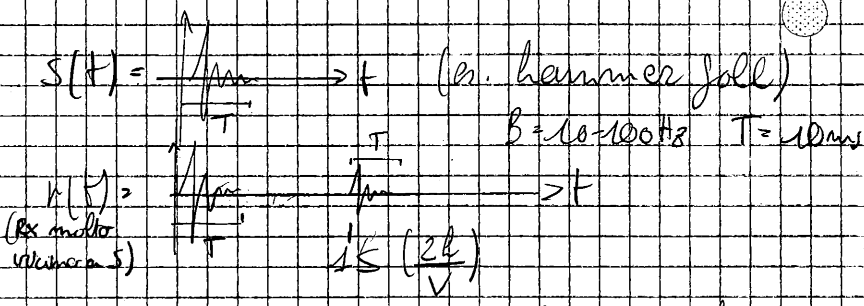
If we use:

$$\hat{R}(f) = \frac{R(f)}{\hat{S}(f)}$$ SPIKE DECONVOLUTION (flat spectrum, zero phase)

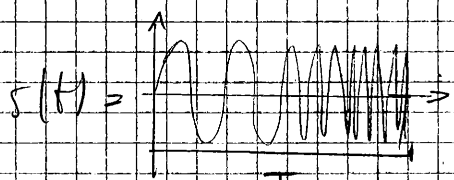
CONCLUSIONE: per la misura utilizziamo una fonte non dannosa per l'ambiente e non pericolosa, facilmente controllabile (vibroseis) che genera un segnale con molta energia e larga banda.
Purtroppo essendo questo segnale lungo al ricevitore avrò sovrapposizioni dovute al segnale diretto e altri echi.
Andando a sostituire quindi matematicamente il segnale trasmesso (chirp) con un altro segnale più breve (es. spike), avrò una traccia migliore e più facilmente interpretabile.

es.

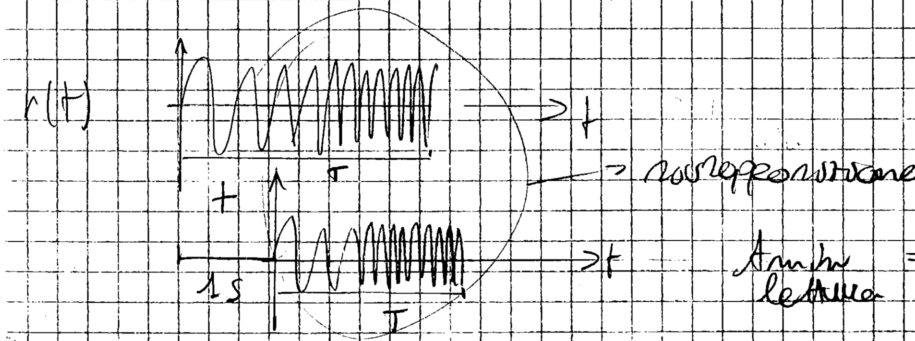


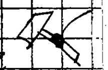
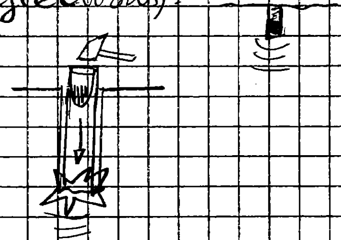
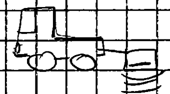
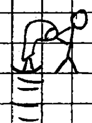

$t_{min\ lettura} = \frac{2h}{V} + T \approx \frac{2h}{V} = 1\,s$




(chirp)
T = 30 s



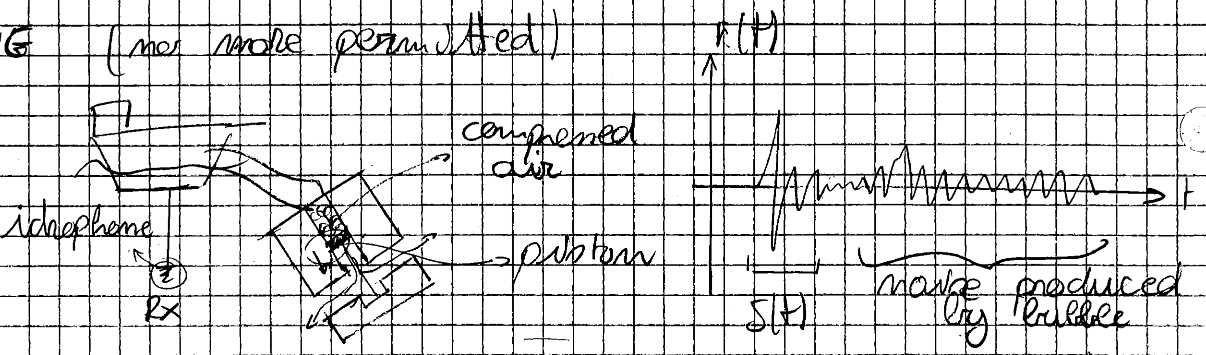

$t_{min\ lettura} = \frac{2h}{V} + T = 31\,s$

| SOURCES | TECHNOLOGIES | IMAGING DEPTH | TYPICAL BANDWIDTH | COMMENTS |
| --- | --- | --- | --- | --- |
| IMPACT (minimum phase) | HAMMER<br>FALLING MASS (up to 1 TON)<br>ACCELERATED MASS (")<br> | SHALLOW - MEDIUM (≈ 200 m) | 10 - 500 Hz | trigger signal = signal that give the start time for the trace, ie bottom putting an accellerometer on the hammer |
| IMPULSIVE (minimum phase) | EXPLOSIVE (the source we put in the media we want to analyze in order to not lose energy due to reflections)<br>GUN<br> | DEEP ≈ 1 km<br>MEDIUM | 10 - 500 Hz | |
| VIBRATIONAL (chirp) | VIBROSEIS (I can obtain as much energy as I want)<br>MINI VIB <br>MINI SOSIE  | DEEP ?<br>MEDIUM<br>SHALLOW | 10 - 500 Hz | |

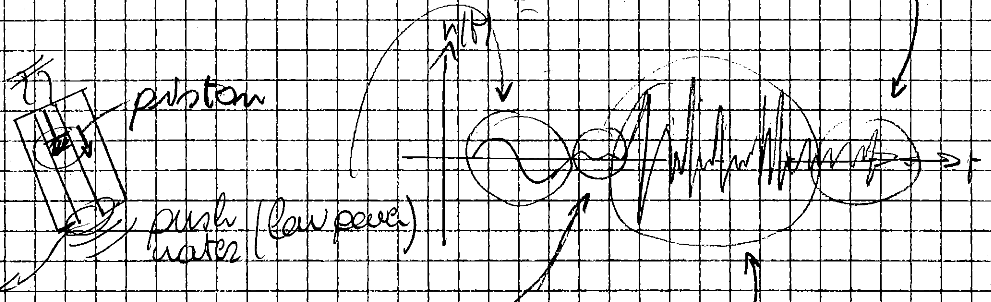
MARINE SOURCES

1) EXPLOSIVE (no more permitted)

2) AIR GUN




3) WATER GUN



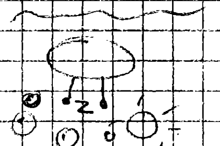

zona di SW che una SOTTOPRESSIONE → fenomeno delle CAVITAZIONE (rilascio di gas e creazione di bolle) → collasso delle bolle create (high power)

HIGH RESOLUTION SOURCES

In order to increase the resolution we have to increase the bandwidth.
- In onshore scenario we cannot use waves with frequency over KHz because they would suffer high attenuation.
- OFFSHORE / MARINE SCENARIO

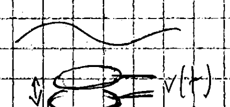
Water presents very low attenuation for elastic waves, instead for e.m. wave it presents high attenuation.

1) SPARKER (scintilla)



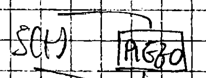
la scintilla vaporizza l'acqua intorno generando bolle; le bolle espandendo generano p-waves (10-5000 Hz)

2) BOOMER




two plates are commanded by an electric signal;
i due piatti avvicinandosi e allontanandosi generano p-waves.

3) SIDE SCAN SONAR PIEZO (sea bottom imaging, fishery, positioning).



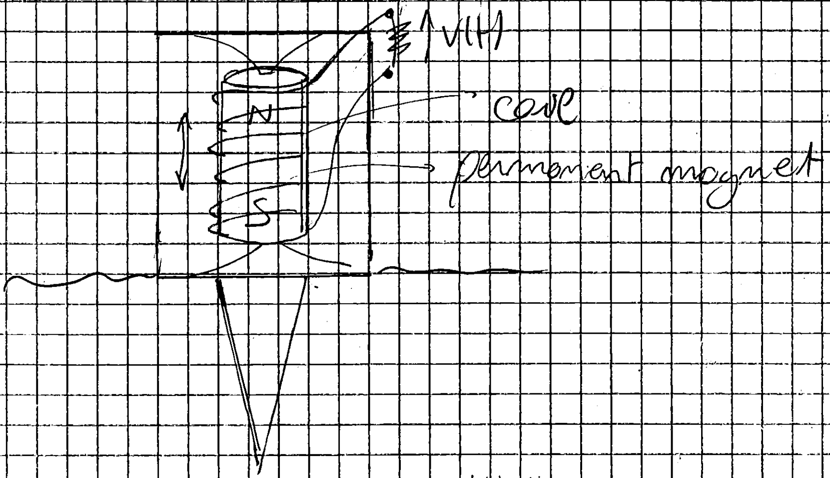

un segnale comanda l'espansione e compressione del materiale piezoelettrico, che genera così p-waves (1 KHz - 1 MHz) → ULTRASOUNDS

chirp signal

# SEISMIC RECEIVER

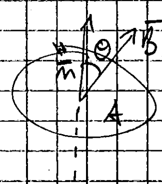
## a) ONSHORE

### ① GEOPHONE (measure the velocity)




muovendosi il magnete perturba il campo magnetico B interno alla spira, così facendo si genera corrente che verrà convertita in tensione sulla resistenza.

$\Phi_B = \vec{B} \cdot \vec{n} A = B \cdot A \cdot \cos\Theta$




$V(t) \propto \frac{d\Phi_B}{dt}$ flusso del campo magnetico $\propto \vec{u}$ spostamento

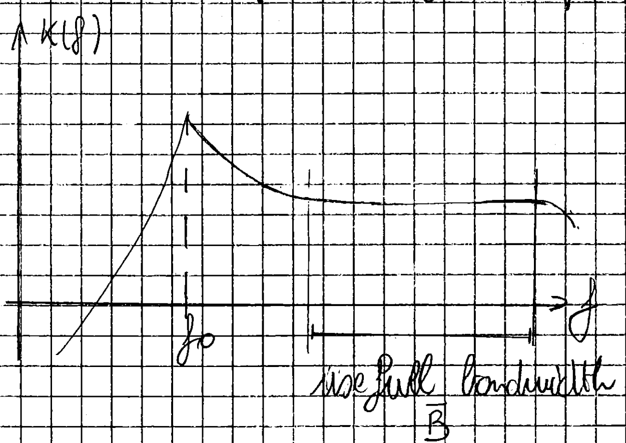
consideriamo la velocità dello spostamento:

$K = SENSITIVITY = \frac{Voltage}{Velocity} \quad \left[ V \cdot \frac{s}{m} \right] \quad \left( es.\ K = 1mV \cdot \frac{s}{m} \right)$

La spira ha una massa, quindi anche lei oscilla su e giù ecc potrebbero essere quindi problemi di risonanza

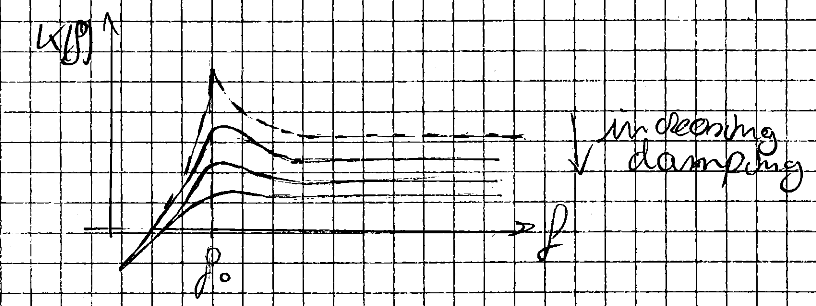
$K = K(f)$, $Vel = Vel(f)$, $m$ = massa spira

$f_0$ = resonant frequency = $2\pi \sqrt{\frac{\sigma}{m}}$ → costante elastica della spira ?

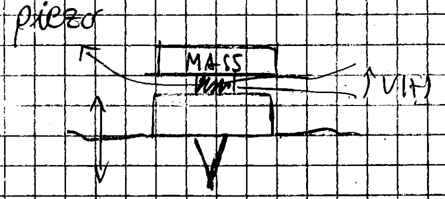



in the $\overline{B}$ we have a linear conversion from velocity to voltage, so no distorsion

In order to reduce the oscillation, we can use a DAMPER (ex. damping resistance)




## 2) ACCELEROMETER (measure acceleration)

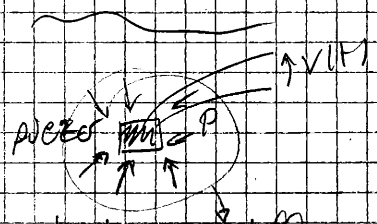



$$V \propto \text{Pressure} = \frac{F}{A} \propto a$$

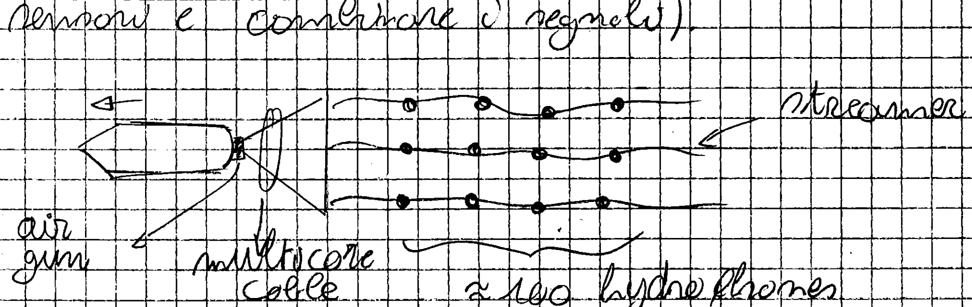
accelerometer has higher sensitivity but lower dynamic range w.r.t. geophone.

## b) OFFSHORE / MARINE RECEIVERS

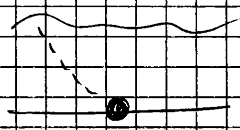
## 1) HYDROPHONE (≈ accelerometer)




esistono metodi di compensazione per la pressione derivante dal movimento della nave tramite i sensori (es. utilizzare due sensori e combinare i segnali).

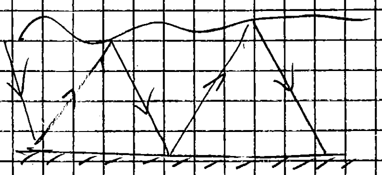


## ② OCEAN BOTTOM SENSORS (OBS)



(w cables or wired)

OBS are used to resolve problems due to multiple reflections that produce ambiguous interpretations.

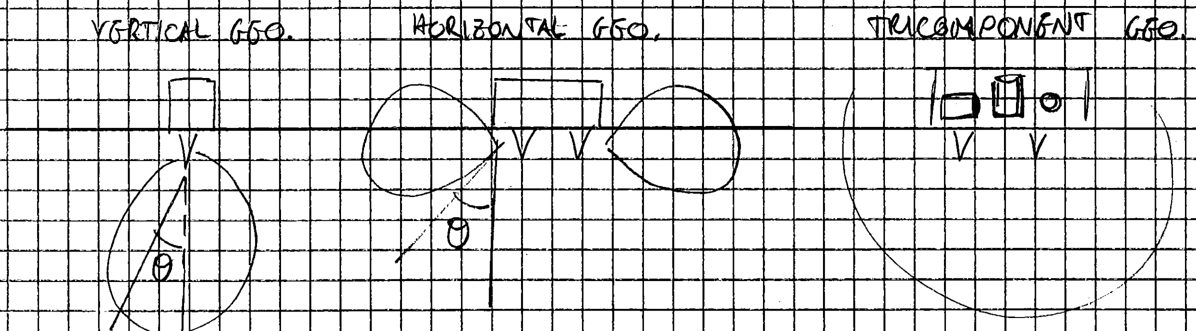


Un'altro modo per contrastare questo problema è utilizzare una particolare configurazione degli idrofoni in modo da capire se l'onda è ascendente o discendente

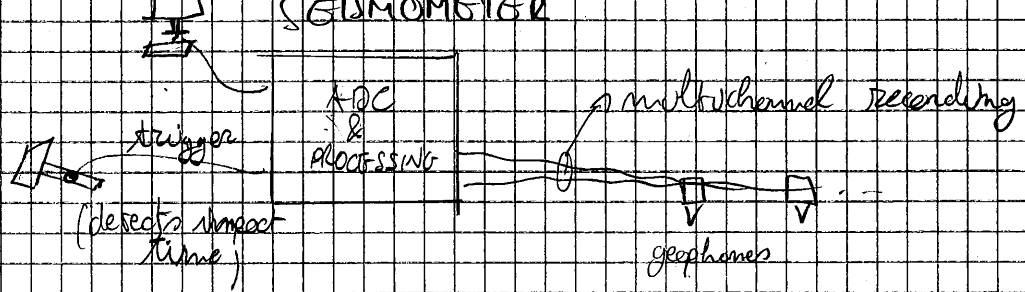
$s_2(t) = s(t-\tau_2)$ con $\tau_2 > \tau_1$

$s_1(t) = s(t-\tau_1)$

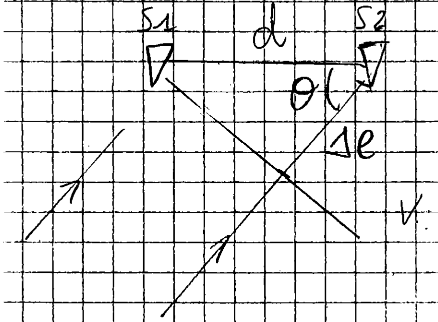
### DIRECTIVITY PATTERNS




$d(\theta) = \cos(\theta)$ $d(\theta) = \text{sen}\,\theta$



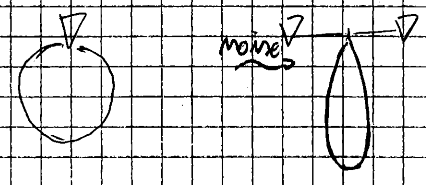
# ARRAYS




$\Delta l = d \cdot \cos\theta$

$\Delta t = \dfrac{\Delta l}{v}$

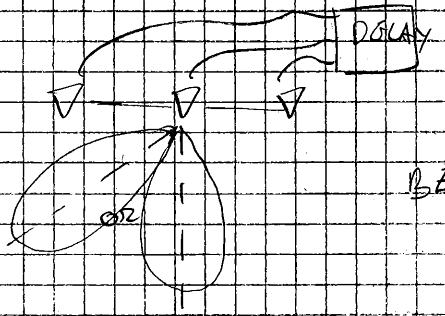
aumenta la direttività (meno sensibile a rumore orizzontale)




if $\Delta l = \lambda$ : $S1 + S2 = 2S1$

$\Delta l = \dfrac{\lambda}{2}$ : $S1 + S2 = 0$

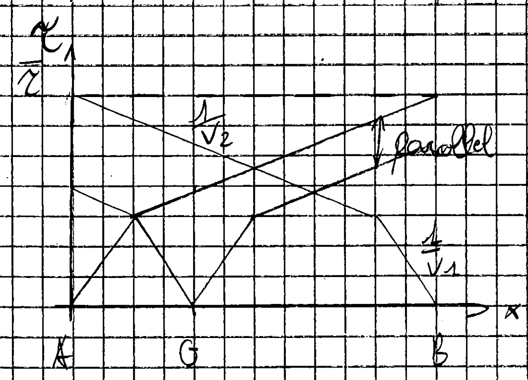
Possiamo anche applicare dei ritardi ad ogni ricevitore in modo da cambiare la direttività dell'array.



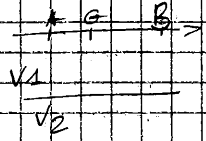

BEAM FORMING

# REFRACTION SEISMIC METHOD

## (2) PLUS-MINUS METHOD

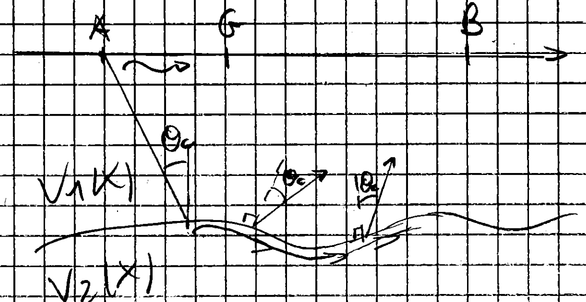


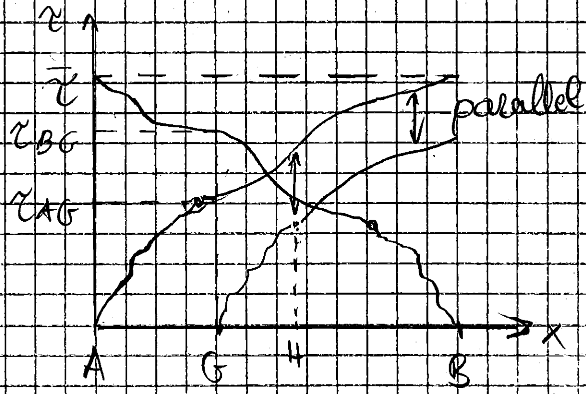

le curve sono parallele perché le onde viaggiano nello stesso mezzo ($V_1, V_2$ cost)




per reciprocità $\tau_{A\to B} = \tau_{B\to A} = \bar{\tau}$

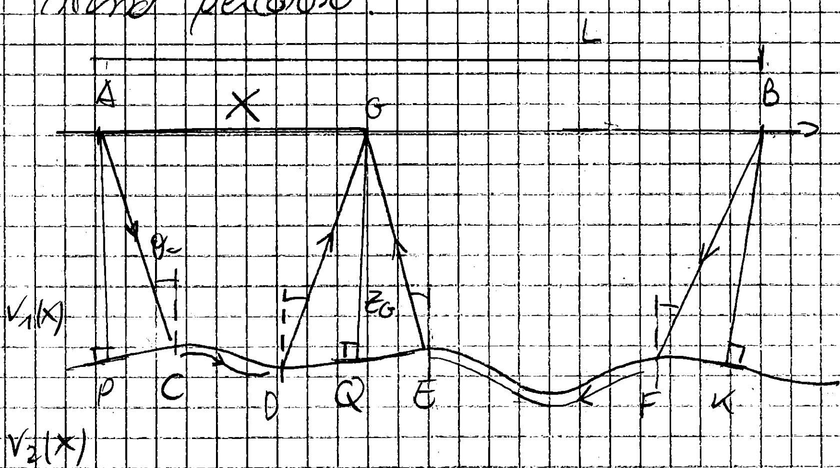
Consideriamo ora invece velocità di cui $V_1(x)$ e $V_2(x)$.




per reciprocità $\tau_{A\to B} = \tau_{B\to A} = \bar{\tau}$

nonostante $V_1$ e $V_2$ variano con x, dal punto H in poi (first arrival è refracted per entrambi) le curve sono comunque parallele perché le onde seguono comunque lo stesso percorso.




$AG = x$

$AB \approx PK$

$QK \approx GB = L - x$

$DG \approx EG$

$$\delta\tau_A = \frac{AC}{V_1} - \frac{PC}{V_2} \qquad \delta\tau_B = \frac{BF}{V_1} - \frac{KF}{V_2}$$

$$\delta\tau_G = \frac{DG}{V_1} - \frac{DQ}{V_2} = \frac{GG}{V_1} - \frac{GQ}{V_2}$$

$$T^- = \frac{\tau_{AG} - \tau_{BG}}{2} = \frac{1}{2}\left[\left(\frac{AC}{V_1} + \frac{CD}{V_2} + \cancel{\frac{DG}{V_1}}\right) - \left(\frac{BF}{V_1} + \frac{FG}{V_2} + \cancel{\frac{GG}{V_1}}\right)\right] =$$

$$= \frac{1}{2}\left(\delta\tau_A + \frac{PC}{V_2} + \frac{CD}{V_2} - \delta\tau_B - \frac{KF}{V_2} - \frac{FG}{V_2}\right) =$$

$$= \frac{1}{2}\left(\underbrace{\delta\tau_A - \delta\tau_B}_{\text{do not depend on } x} + \frac{PD}{V_2} - \frac{KG}{V_2} + \underbrace{\frac{DQ}{V_2} - \frac{GQ}{V_2}}_{0}\right) =$$

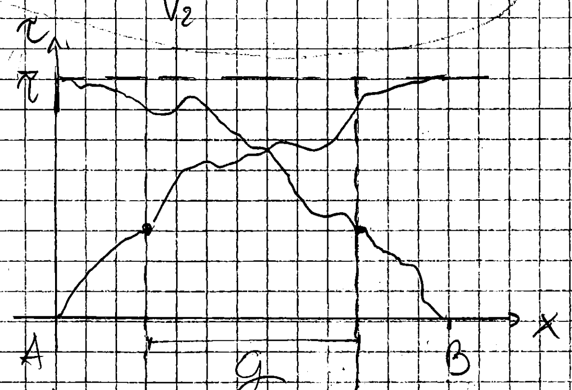
$$= \frac{1}{2}\left(\delta\tau_A - \delta\tau_B + \frac{PG}{V_2} - \frac{KG}{V_2}\right) =$$

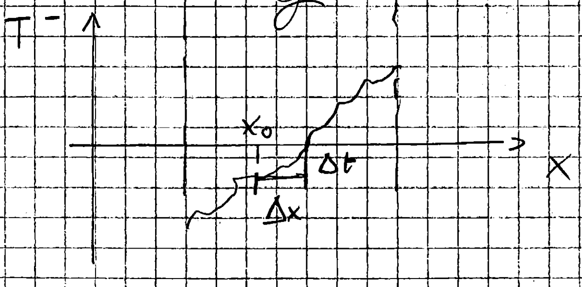
$$= \frac{1}{2}\left(\delta\tau_A - \delta\tau_B + \frac{x}{V_2} - \frac{L-x}{V_2}\right) =$$

$$= \frac{1}{2}\left(\delta\tau_A - \delta\tau_B + \frac{2x}{V_2} - \frac{L}{V_2}\right).$$

Constant

$$T^-(x) = \frac{1}{V_2} \cdot x + \text{const}$$



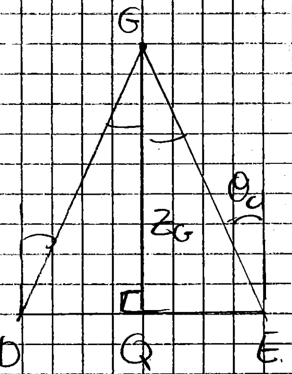

$$\frac{\Delta x}{\Delta t} = V(x_0)$$

LOCAL SLOPE of $T^-(x) = \dfrac{1}{V_2(x)}$

$$T^+ = \frac{1}{2}(\tau_{AG} + \tau_{BG} - \tau_{AB}) =$$

$$= \frac{1}{2}\left[\left(\cancel{\frac{AC}{V_1}} + \cancel{\frac{CD}{V_2}} + \frac{DG}{V_1}\right) + \left(\cancel{\frac{BF}{V_1}} + \cancel{\frac{FE}{V_2}} + \frac{EG}{V_1}\right) - \left(\cancel{\frac{AC}{V_1}} + \cancel{\frac{CD}{V_2}} + \frac{DE}{V_2} + \cancel{\frac{EF}{V_2}} + \cancel{\frac{FB}{V_1}}\right)\right] =$$

$$= \frac{1}{2}\cdot\left(\frac{2DG}{V_1} - \frac{DE}{V_2}\right).$$




$z_G = GQ$ , $\theta_c(x)$

$$DG = \frac{z_G}{\cos\theta_c} , \quad DE = 2\cdot DQ = 2\cdot\tan\theta_c\cdot z_G$$

$$T^+ = \frac{z_G}{\cos\theta_c}\cdot\frac{1}{V_1} - \frac{z_G\tan\theta_c}{V_2}$$

Snell law $\quad \frac{\mathrm{sen}\,\theta_c}{V_1} = \frac{1}{V_2} \qquad V_2 = \frac{V_1}{\mathrm{sen}\,\theta_c}$

$$T^+ = \frac{z_G}{\cos\theta_c}\cdot V_1 - z_G\,\frac{\mathrm{sen}\,\theta_c}{\cos\theta_c}\cdot\frac{\mathrm{sen}\,\theta_c}{V_1} =$$

$$= \frac{z_G}{V_1}\left(\frac{1-\mathrm{sen}^2\theta_c}{\cos\theta_c}\right) = \frac{z_G}{V_1}\left(\frac{\cos^{\cancel{2}}\theta_c}{\cancel{\cos\theta_c}}\right).$$

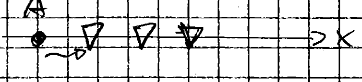
$$T^+(x) = \frac{z_G}{V_1}\cos\theta_c(x)$$

compute all $\tau_{AG}$ and $\tau_{BG}$ for all $G = x \in \mathcal{G}$

compute $T^-(x)$ and $T^+(x)$

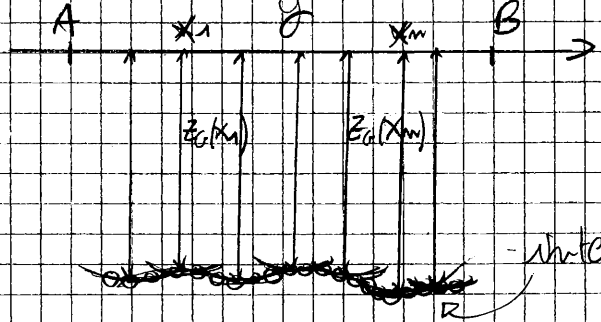
from $T^-(x) \to V_2(x)$

from direct arrivals $\to V_1(x)$



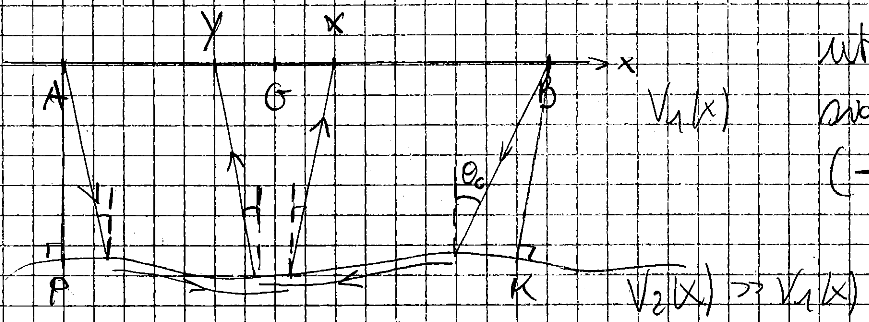

from $V_1(x)$ and $V_2(x) \to \theta_c(x) = \arcsin\left(\frac{V_1(x)}{V_2(x)}\right)$

from $V_1(x)$, $T^+(x)$, $\theta_c(x) \to z_G(x) = T^+(x)\cdot\frac{V_1(x)}{\cos\theta_c(x)}$




we obtain the interface doing the envelope of circonferences.

## 3) GENERALIZED RECIPROCAL METHODS




utilizziamo come sorgente sia il punto A che B (→ reciprocal)

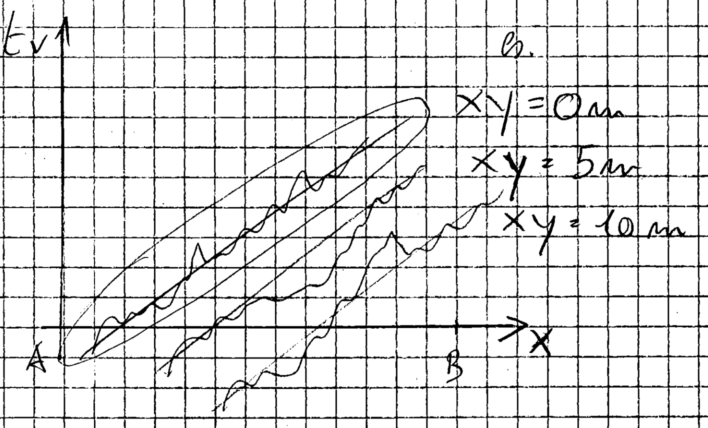
hp: $AB \approx PK$

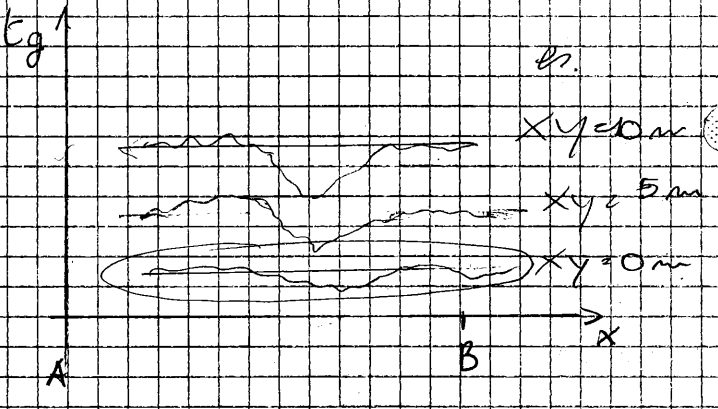
$V_2$ and $V_1$ change more slowly than $z_G(x)$

$t_V = \frac{1}{2}(\tau_{AX} - \tau_{BY} + \tau_{AB})$ VELOCITY ANALYSIS FUNCTION (generalizzazione di $T^-$)

$t_G = \frac{1}{2}(\tau_{AX} + \tau_{BY} - \tau_{AB})$ DEPTH ANALYSIS FUNCTION (generalizzazione di $T^+$)

$t_V$ e $t_G$ sono funzione di XY (segmento) (es. if XY = 10 m, Y —5m— G —5m— X)




Voglio che $V_2$ cambi lentamente quindi seleziono la curva (quindi XY) con l'andamento meno variabile ("più dritta").
in questo caso scelgo XY = 0 m.

Ora applico lo stesso procedimento del plus-minus method:

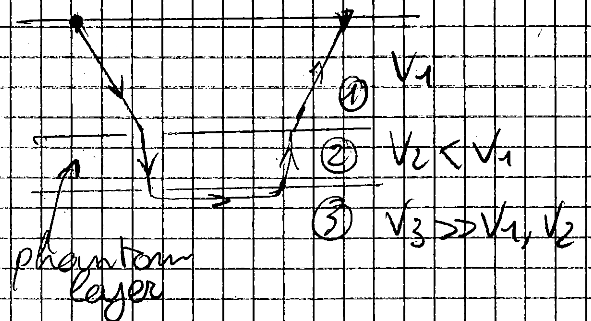
$t_V \rightarrow V_2(x)$

direct arrivals $\rightarrow V_1(x)$

$t_V$ e $t_G \rightarrow \theta_c(x)$

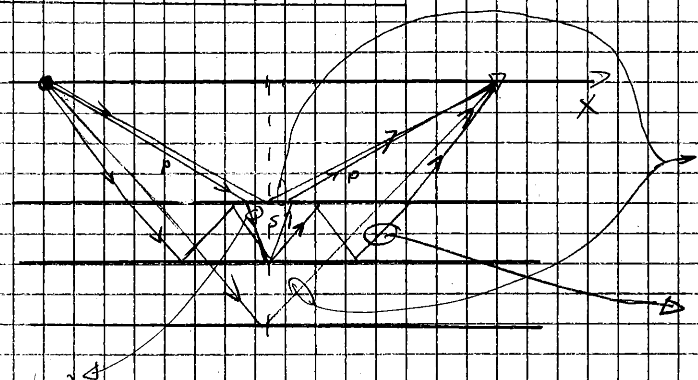
$t_V, t_G, \theta_c \rightarrow z_G(x)$

# ERRORI DOVUTI ALLA CADUTA DELLE IPOTESI




In questo caso $V_2 < V_1$ (hp ~~$V_2 >> V_1$~~) quindi avremo la presenza di un PHANTOM LAYER che provocherà errori nel stima della posizione del layer ③ e delle velocità di propagazione.

# REFLECTION SEISMIC

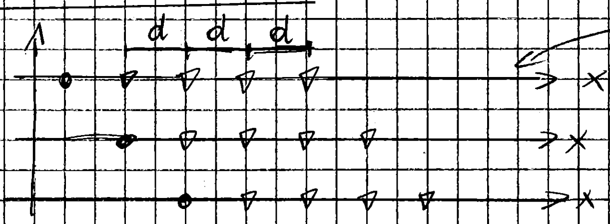



abbiamo più componenti che arrivano al ricevitore:

1) DIRECT REFLECTIONS (primary reflections)
2) MULTIPLES
3) CONVERTED WAVE ARRIVALS (change the type of wave passing through different layers ($P \to S$ or $S \to P$))
4) NOISE

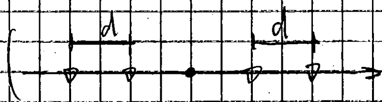
Essendo le p-waves più veloci, consideriamo solo questo tipo di onde e le loro riflessioni dirette.

# DATA-COLLECTION



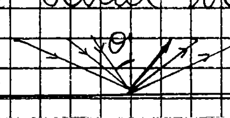

acquisition line in off-line geometry.

multiple coverage

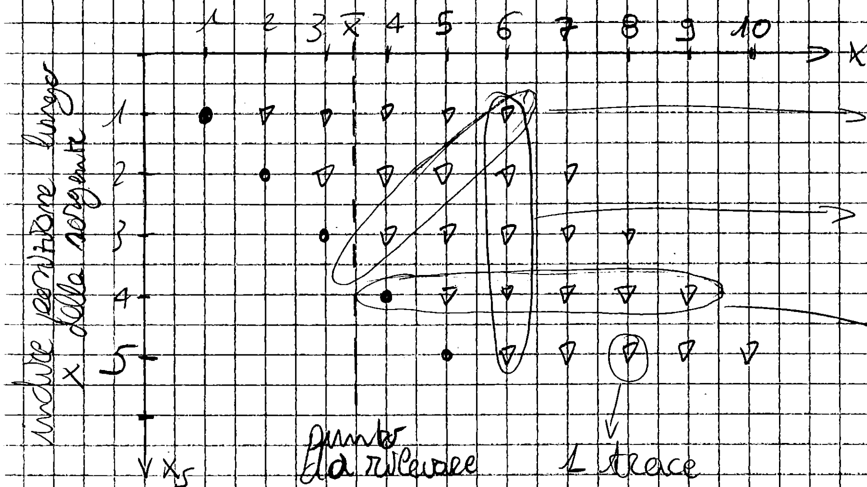
(  strip line geometry)

Se un punto viene studiato N volte, cioè è il punto centrale di N riflessioni, si dice che ha COVERAGE N. (es. N=10)

È utile vedere su uno stesso punto con angoli di incidenza diversi in modo da studiare come varia il coefficiente di attenuazione in funzione di θ:



# STACKING CHART OF A LINE ACQUISITION

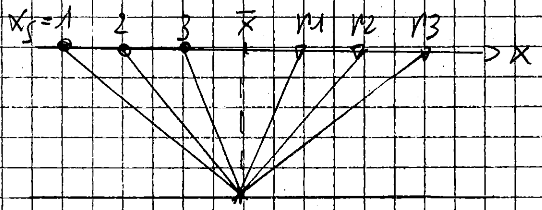



→ COMMON MID POINT GATHER (# traces = coverage ; es. 3)

→ COMMON RECEIVER GATHER

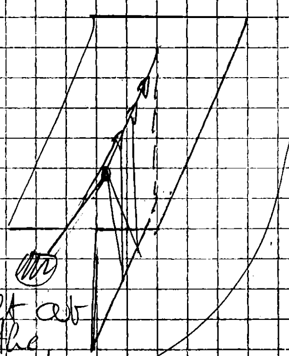
→ COMMON SHOT GATHER

es. COVERAGE $\bar{x}$




COVERAGE $(\bar{x}) = 3$

## 2D - ACQUISITION



→ problem with 3D-effects (phantom targets).

target out of the plane to be studied

## FORMATO DEI SEISMIC DATA

| | | | | | | |
|---|---|---|---|---|---|---|
| REEL | HEADER 1 | TRACE 1 | HEADER 2 | TRACE 2 | | ... |

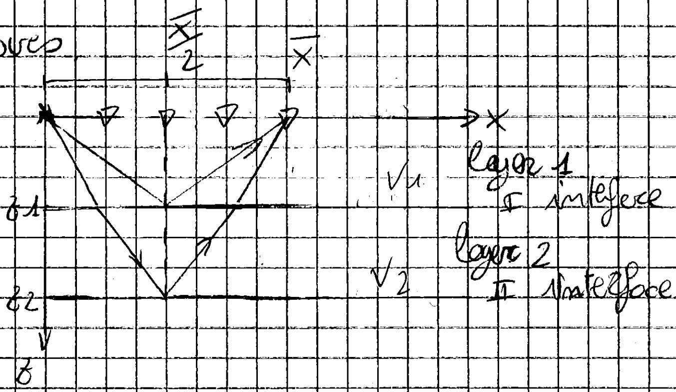
↳ contiene informazioni su un determinato trace (position of geophone and source, trigger time, number of samples, gain, filters, ecc.)

↳ HEADER GENERALE : contiene informazioni generali sulla campagna di misura (location, # geofoni, # sorgenti, ecc.)

# REFLECTION SEISMIC METHOD

we consider only p-waves

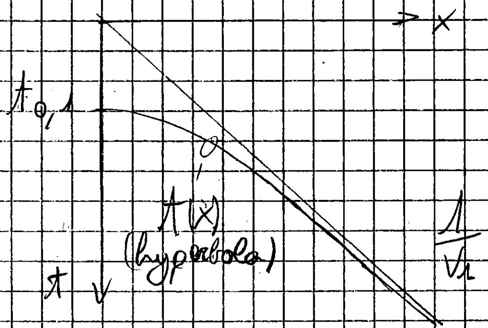
hp: $z_i \gg \bar{x}$




time of arrival of the reflected p-waves on I interface

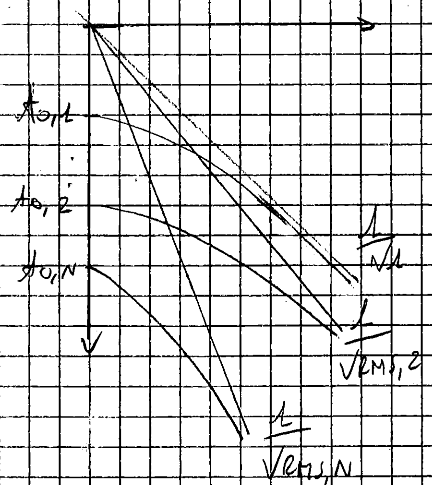
$$t_1(x) = \frac{2}{V_1}\cdot\sqrt{z_1^2 + \left(\frac{x}{2}\right)^2} = \sqrt{\left(\frac{2z_1}{V_1}\right)^2 + \left(\frac{x}{V_1}\right)^2} = \sqrt{t_{0,1}^2 + \left(\frac{x}{V_1}\right)^2}$$

time of normal propagation

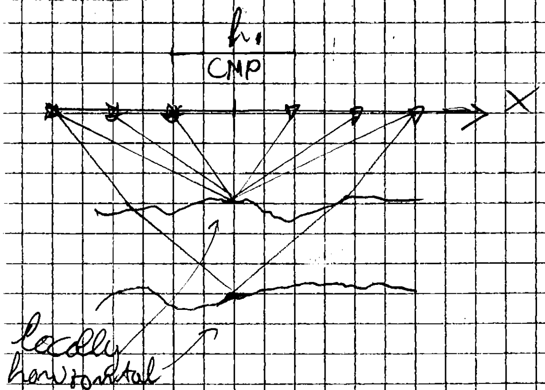



$$t_N(x) = \sqrt{t_{0,N}^2 + \left(\frac{x}{V_{RMS,N}}\right)^2}$$ → approximation

$$V_{RMS,N} = \sqrt{\frac{\sum_{1}^{N} V_i^2 \cdot t_i}{\sum_{i=1}^{N} t_i}}$$ with $t_i$ = total travel time in layer i

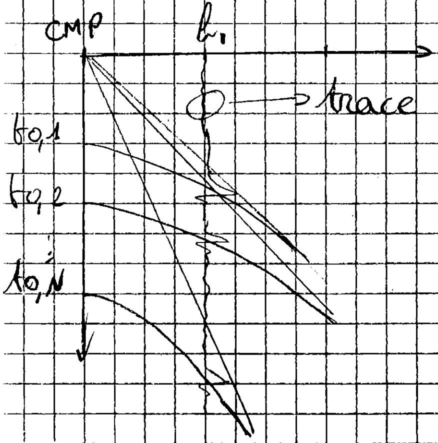


## MULTI COVERAGE ACQUISITION → COMMON MID POINT GATHER

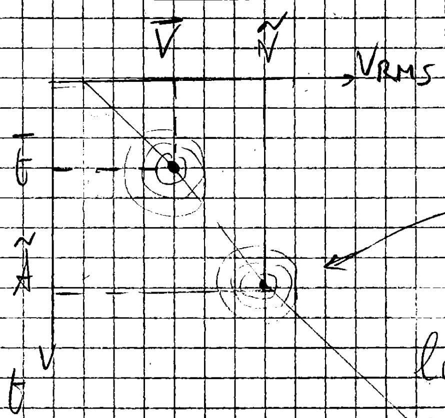



$$t_N(h) = \sqrt{t_{0,N}^2 + \left(\frac{h}{V_{RMS,N}}\right)^2}$$

$h$ → distanza tra TX e RX




## SEMBLANCE GATHER

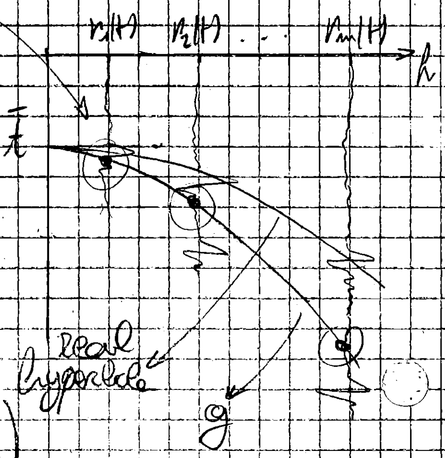



Ogni punto (coppia $(t, V_{RMS})$) è colorato secondo il suo valore di $S$ ($S=1$ … $S=0$).

linea che unisce i massimi → $V_{RMS}(t)$ optimal

$$t(h) = \sqrt{\bar{t}^2 + \left(\frac{h}{\bar{V}}\right)^2} = g \quad \text{TENTATIVE TRAJECTORY (for all possible } (t, V_{RMS}))$$

$$\text{SEMBLANCE} = S = (t, V_{RMS}) = \frac{\left(\sum_{1}^{m} r_i(\bar{t} \circ g)\right)^2}{m^2}$$



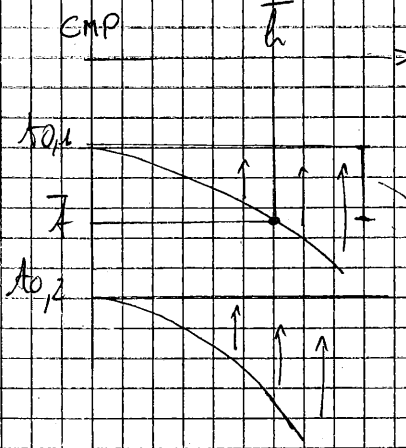

dove $m$ = numero di ricevitori,
$r_i(t)$ = traccia ricevuta dall'i-esimo ricevitore.

Se g coincide con una iperbole reale : $S \to 1$

La traiettoria g ha uno spessore lungo T (upward direction)

g ⊗ t (sommo prima su tutte le righe e poi sommo per tracce).

utilizzando $V_{RMS,opt}(t)$ andiamo a "raddrizzare" le iperbole del CMP gather.




NORMAL MOVE-OUT CORRECTION (NMO)

scelta in base a $\bar{t}$

$$t_{shift}(\bar{t}, \bar{h}) = \sqrt{t_{0,1}^2 + \left(\frac{\bar{h}}{V_{RMS}(\bar{t})}\right)^2} - t_{0,1}$$

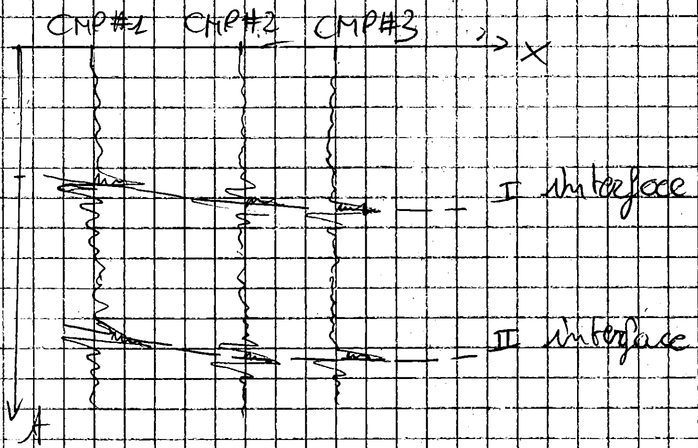
cerco minimo

STACK ALONG OFFSET

$$V_{TOT\,CMP\#1}(t) = \sum_1^m v_i'(t)$$ dove $v_i'(t)$ è il trace $v_i(t)$ shiftato secondo $t_{shift}(t, h)$.

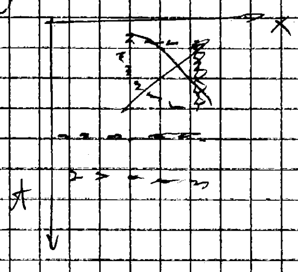
$V_{TOT\,CMP\#1}$ avrà tutti i vantaggi dell'azione di STACK (cioè SNR maggiore)

STACK GATHER (immagine finale)




PROBLEMI

lo step NMO può provocare "stretching", cioè distorsione del segnale iniziale → per migliorare il grafico andiamo a tagliare i "triangoli" formatisi dopo NMO.



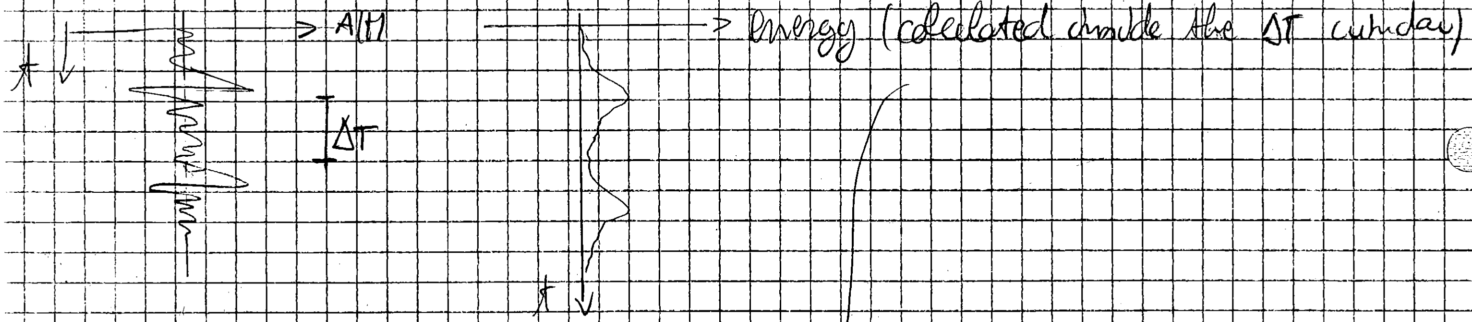
# REFLECTION SEISMIC PROCESSING

1) DEMULTIPLEXING = riordinare i dati dopo l'acquisizione
2) EDITING = Cancelling noise trace (mute) or reversing "reverse geophone" traces.

3) GAIN RECOVERY = amplification of high attenuated parts of traces.

$$\underset{\text{attenuation}}{A} \propto \underset{\text{geom}}{\frac{K_1}{A}}\ \underset{\text{absorption}}{e^{-K_2 t}} \longrightarrow r(t)_{\text{recover}} \approx r(t)\cdot \frac{1}{K_1}\cdot t \cdot e^{+K_2 t}$$

OR AUTOMATIC GAIN RECOVERY (AGR)




$$r(t)_{\text{recover}} = r(t)\cdot \frac{1}{\sqrt{\int_{t-\frac{\Delta T}{2}}^{t+\frac{\Delta T}{2}} |r(t)|^2\, dt}}$$

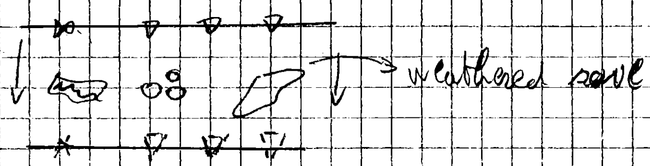
ottengo così un trace senza grossi "sbalzi" in ampiezza.

$\Delta T \approx 2/3\ T$ (wavelet duration) (se troppo piccolo → distorsione segnale
" " grande → non faccio nessuna recovery)

4) GEOMETRY INPUTS = informazioni sulla geometria del sistema

5) APPLICATION OF STATICS

nel primo layer possiamo avere molte disomogeneità (h. acqua, tubi, ecc.), utilizziamo quindi la refraction seismic per avere informazioni su questo layer.
Quindi utilizziamo queste infos per fare una static shift di tutti i traces. → è come se spostassimo l'acquisition line su un layer orizzontale.



6) DECONVOLUTION (vedi prima)

7) FILTERING

Given a source (es. hammer), we must choose rightly

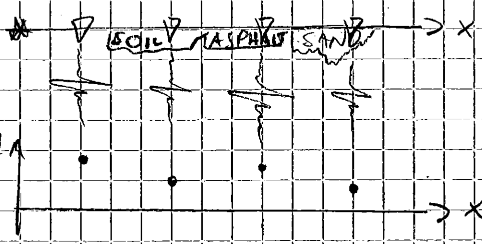
- geophone with $f_{res} < f_{min}$ (es. $f_{res} < 10\,Hz$)

- sampling frequency: $f_{sam} \geq 2 f_{max}$ (es. $f_{sam} \geq 2000\,Hz$) ($\approx 10 \cdot f_{max}$)

filtering of the noise out of signal bandwidth.

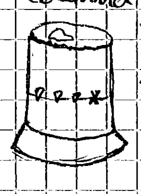
8) TRACE EQUALIZATION

we want to decoupling traces from the ground:

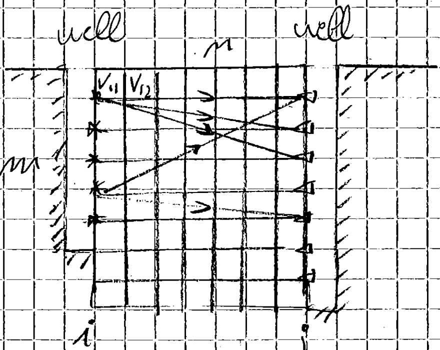



we have to compute the energy of all traces, then multiply each traces for the inverse of his own energy. (like AGR)

8) REAL TIME TOMOGRAPHY

use traveltime to get velocity model of some object (es.  )






$A_{ij}$ = picking of first arrival

$V_{m,n}$ = velocity in cell (m,n)

$$A_{ij} = \sum_{a=\tilde{m}}^{\hat{m}} \sum_{b=1}^{n} \frac{l_{ab}}{V_{ab}} = \sum\sum l_{ab} \cdot s_{ab}$$

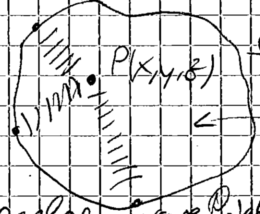
dove:

$s_{ab} = \frac{1}{V_{ab}}$ = slowness $[\frac{s}{m}]$, $l_{ab}$ è la lunghezza del percorso che fa il raggio nella cella (a,b), $\tilde{m}$ e $\hat{m}$ dipendono da quali celle il raggio $(i,j)$ attraversa.

# SEISMIC ACQUISITION

- PRE-PROCESSING
- CMP SORTING
- VEL ANALYSIS ⟶ PRE-STACK (DATA + VELOCITY)
- NMO CORRECTION
- STACK → ARTIFACTS (cmp is not locally horizontal)

## KIRCHOFF INTEGRAL

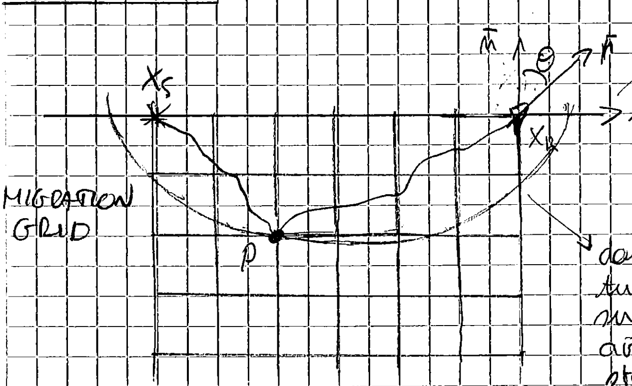



If you know m(Ω), you can draw m in every point of the volume.

$$m(x,y,z) = \int_\Omega m_\Omega \cdot G(\Omega, P)\, d\Omega$$

Green function (describe the propagation between two points.)

## MIGRATION




hp: consideriamo la superficie come chiusa rispetto al volume di interesse.

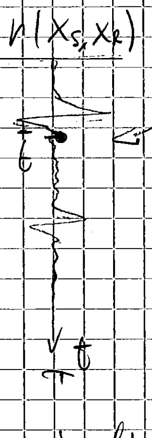
V known

dato $r(x_s, x_R)$ tutti i pti sull'ellisse avranno lo stesso valore.

For each point P on the grid, trace a ray from $x_s$ to $x_R$. Calculate the total travel time $\bar{t}$:

$$\bar{t} = t_{x_s \to P} + t_{P \to x_R}.$$

The couple $(x_s, x_R)$ is related to a certain trace $r(x_s, x_R)$.
For each point, assign to it the value of $r(x_s, x_R)$ given $\bar{t}$.

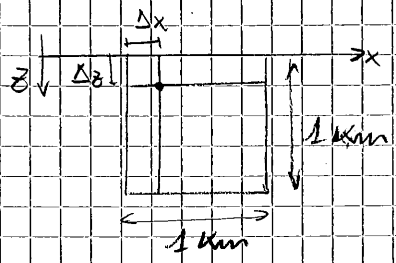



Repeat this process for each trace.
alla fine ogni punto avrà un vettore di valori ognuno corrispondente ad un trace; sommando questi valori e colorando il punto secondo il risultato otterremmo l'immagine sulla superficie.

il risultato può essere anche trovato utilizzando l'integrale di Kirchoff:

$$m(P) = \int_{\omega} \int_{\text{all traces}} \underbrace{\underbrace{A_s \cdot A_R}_{\text{attenuazione da } x_s \text{ a } P} \cdot e^{j\omega(t_{x_sP} + t_{Px_R})}}_{|A_R|^2 \quad \text{green function}} \cdot \underbrace{r(x_s, x_R)}_{\text{recorded field}} \cdot \underbrace{\vec{n} \times \nabla t_{Px_R}}_{\cos\theta}\, dx_R$$

esempio: 100 shots
100 receivers
20 m between receivers → 2 km line




hp: $\Delta x \approx \Delta z$

$\Delta z \approx \frac{\lambda_0}{4} = 2,5\ m$

$B = 100\ Hz$, $v = 1000\ m/s$ → $\lambda = 10\ m$

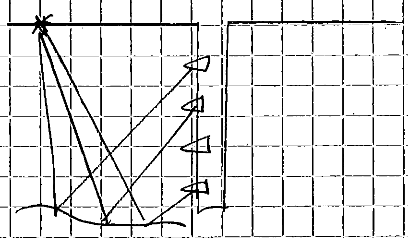
we have: $100 \times 100 = 10^4$ traces

$\frac{10^3}{2,5} \times \frac{10^3}{2,5} = 16 \cdot 10^4$ punti

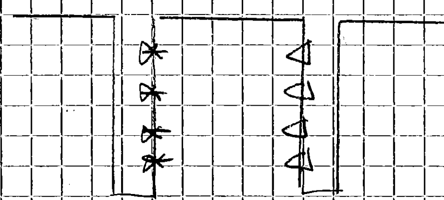
$10^4 \cdot (16 \cdot 10^4) = 16 \cdot 10^8$ computazioni

Logicamente l'ipotesi di una superficie contornante il volume di interesse non è reale, avremmo quindi degli artefatti nell'immagine. ?

VERTICAL SEISMIC PROFILE (VSP)



CROSSWELL

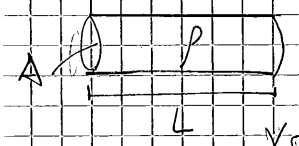


the traces don't "form" ellipses

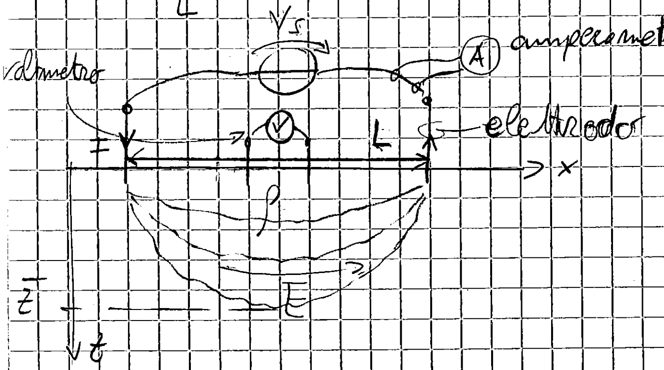
## ELECTRICAL METHOD

- water is a good conductor.
- similar impedance (→ V) among shallow layer in sea bottom

So we exploit electrolytic conduction, i.e. movement of ions (salt dissolved in water) to discover soil resistivity ($\rho$).




$$R = \rho \cdot \frac{L}{A} \rightarrow \rho\ [\Omega\, m]$$




$\bar{z} \approx \frac{1}{3} L$ (quindi se voglio "osservare" più in profondità devo allargare gli elettrodi ($L \uparrow\ \bar{z} \uparrow$))

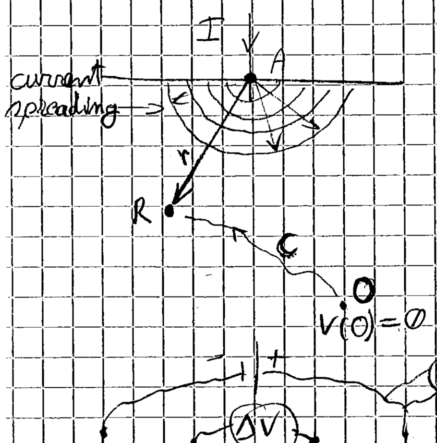
Logicamente attraversando con la corrente più layer, otterrò una $\rho$ fittizia → l'obiettivo è quello di ottenere la vera $\rho(x, z)$.

$$E = E_0 \cdot \mathrm{sen}(2\pi f_0 t) \qquad \frac{dE}{dt} = E_0 \cdot 2\pi f_0 \cos(2\pi f_0 t)$$

$J = \sigma \cdot E + \varepsilon \cdot \frac{dE}{dt}$ dove $\sigma$ = conductivity $= \frac{1}{\rho}$ ($\approx 0{,}1 - 100$), $\varepsilon$ = electric permittivity ($\approx 10^{-12}$)

current density

for low $f_0$ : $J = \sigma \cdot E$
for high $f_0$ : $J \approx \varepsilon \cdot \frac{dE}{dt}$
} $f_{border} \approx 10^7$ Hz

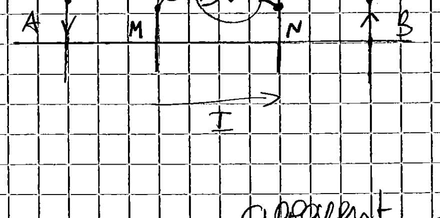



AIR (non conductive)

HALFSPACE

$$V(R) = -\int_C E\, dl = -\int_{\infty}^{R} \frac{I}{2\pi R^2} \cdot \rho\, dl \approx \frac{I}{2\pi R} \cdot \rho$$

$$E = \frac{J}{\sigma} = J \cdot \rho \quad e \quad J(r) = \frac{I}{2\pi r^2}$$



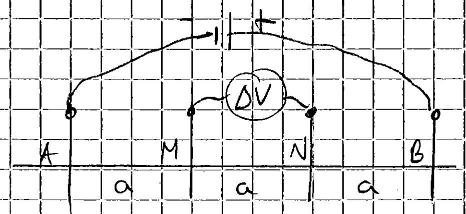

$$V_M - V_N = \Delta V \qquad V_M = \frac{I\rho}{2\pi(AM)} - \frac{I \cdot \rho}{2\pi(MB)} \qquad V_N = \frac{I\rho}{2\pi(AN)} - \frac{I\rho}{2\pi(NB)}$$

$$\Delta V = \frac{I\rho}{2\pi} \cdot \left(\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right)$$

apparent resistivity → $\rho_a = \frac{\Delta V}{I} \cdot \underbrace{2\pi \left( \quad \right)^{-1}}_{\text{geometrical factor} = K}$

# GEOMETRICAL CONFIGURATIONS

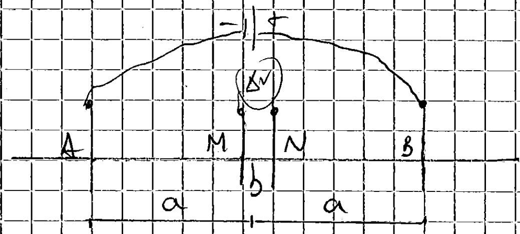
## ① WENNER GEOMETRY




$K = 2\pi a$

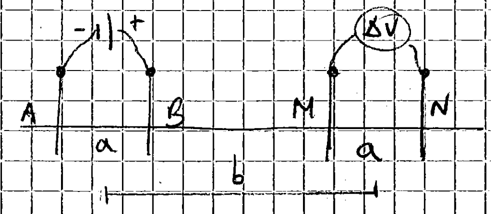
## ② SCHLUMBERGER GEOMETRY

$a \gg b$

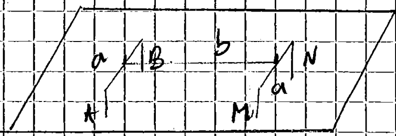



$K = \pi \frac{a}{b} (\quad)$

## ③ DIPOLE-DIPOLE

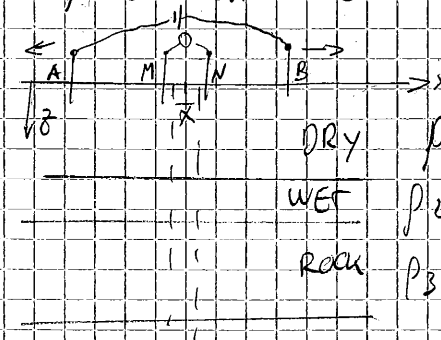



## ④ TRANSVERSE DIPOLE-DIPOLE




We use different geometries to change the sensitivity w.r.t. the points of subsurface:

### 1) VERTICAL ELECTRICAL SOUNDING (VES)




$\rho_1 = 100\ \Omega m$

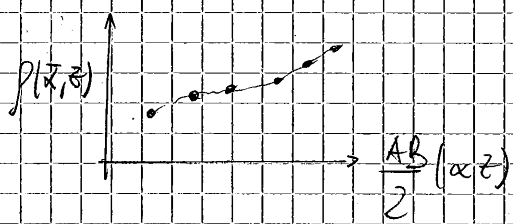
$\rho_2 = 1-10\ \Omega m$

$\rho_3 = 100-1000\ \Omega m$

la geometria Schlumberger è più sensibile nella zona verticale sotto MN.
↳ quindi per stimare $\rho(\bar{x}, z)$ utilizziamo questa geometria.

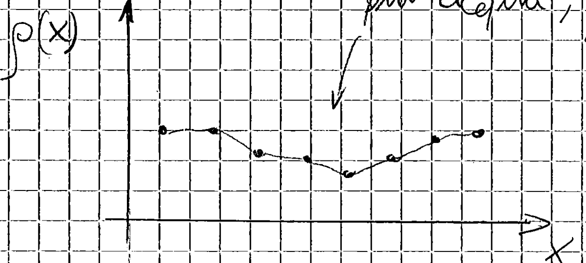
Con l'aumentare della distanza AB ($a\uparrow$) aumenta la profondità della misura

$z \propto \frac{1}{3} AB$

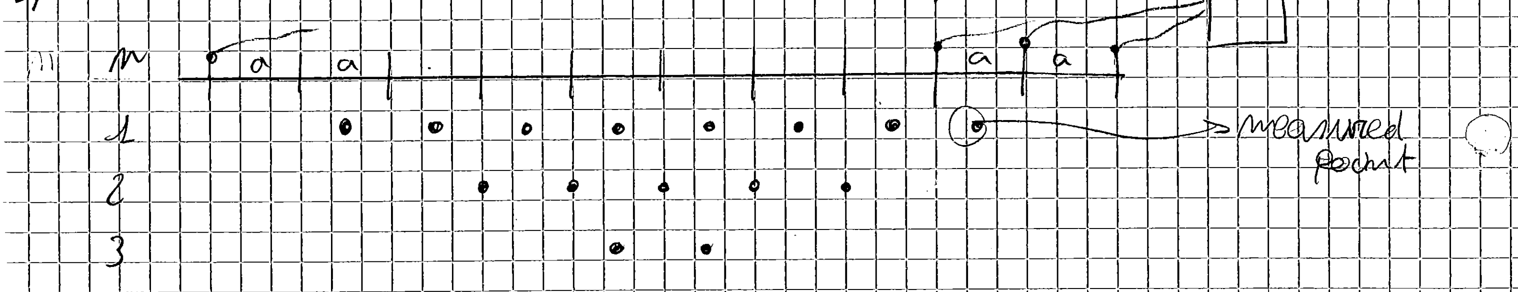



### 2) CONSTANT SEPARATION TRAVERSE (CST)


più acqua, $\rho\downarrow$

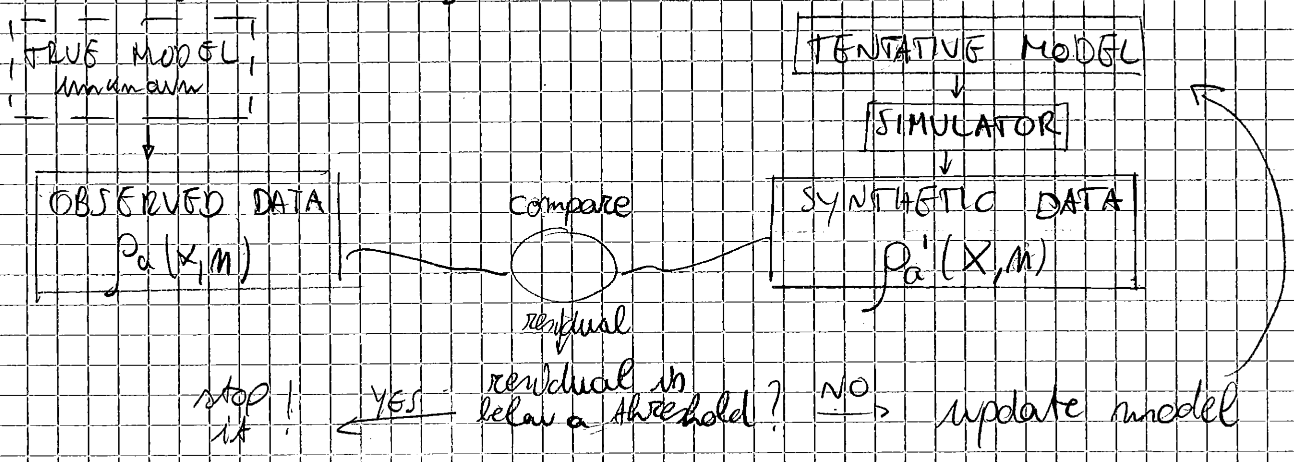


## 3) PSEUDO - SECTION (wenner in this case)




m = number of a section consider as MN (in this case, being wenner, also AM)
il control system sceglie i 4 elettrodi (A, B, M, N) da cui effettuare la misura; in questo modo posso studiare diversi punti senza spostare niente.

## INVERSION (from $\rho_a \rightarrow \rho_{TRUE}$)




problems: inverse problem is ill conditioned → we can have same data ($\rho'$) from different models.

To increase the reliability of the solution, we can use additional information, like seismic refraction (to know the position of the main horizons).

## LIMITS OF ELECTRICAL METHODS

1) DEPTH OF SENSITIVITY

$\bar{z} = \frac{L}{3}$ and max $L \approx 1$ km, so $\bar{z}_{max} \approx 300$ m

2) NEED TO CLOSE CIRCUIT

it doesn't work in dry sand (desert), ice, hard media (no possibility to plant electrodes)

# ELECTRICAL RESISTIVITY TOMOGRAPHY

## ARCHIE'S LAW

Empirical law that relates resistivity and other params for porous rocks (P.S. Clay doesn't obey to Archie's law!)

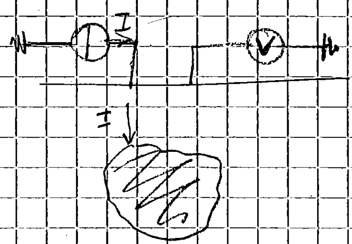
Resistivity in solution with salt changes with temperature and concentration ($\rho$ = 1 - 400 $\Omega m$)

To measure $\rho$ I have to wait few seconds ($\approx$ 10 s) per electrode because ions have time to move.

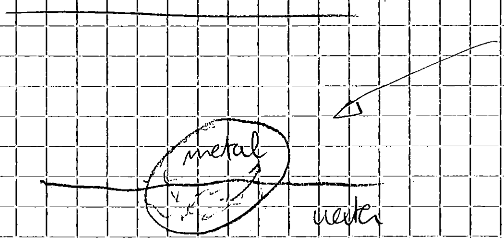
## MISE - A - LA - MASSE METHOD

By injecting current into a conductive underground body it gets equipotentials.

Moving (V) we can measure how big is the body.




## MINERAL - EXPLORATION




due to oxidation we have a "natural" battery. We can perform a mise-a-la-masse without injecting current.

$$J = \sigma \cdot E + \varepsilon \cdot \frac{dE}{dt}$$ so if $\omega \uparrow \rightarrow \rho_A \downarrow$ (due to $\frac{dE}{dt} \uparrow$)

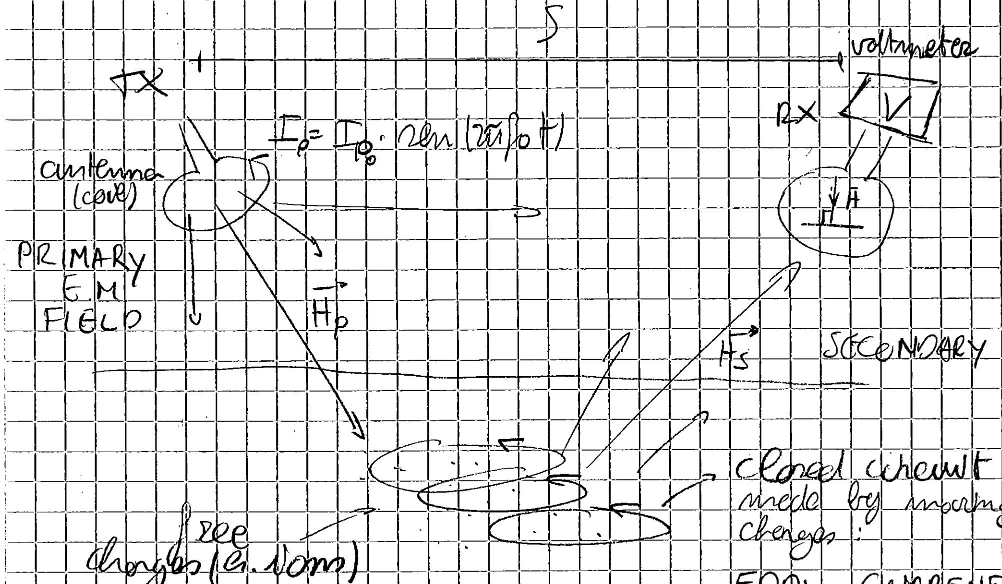

Electromagnetic conductivity is different than electrical conductivity.

# ELECTROMAGNETIC METHOD

$$J = \sigma \cdot E + \varepsilon \frac{dE}{dt} \quad \text{with} \quad E = E_0 \, \mathrm{sen}(2\pi f_0 t)$$

for $f < 10^7 \rightarrow J = \sigma \cdot E$ so we can analyze the conductivity $\sigma$: GROUND CONDUCTIVITY METER

for $f > 10^7 \rightarrow J = \varepsilon \cdot \frac{dE}{dt}$ so " " " electrical permittivity $\varepsilon$: GROUND PENETRATING RADAR (GPR).




at the RX antenna we have the summation of primary and secondary em field.

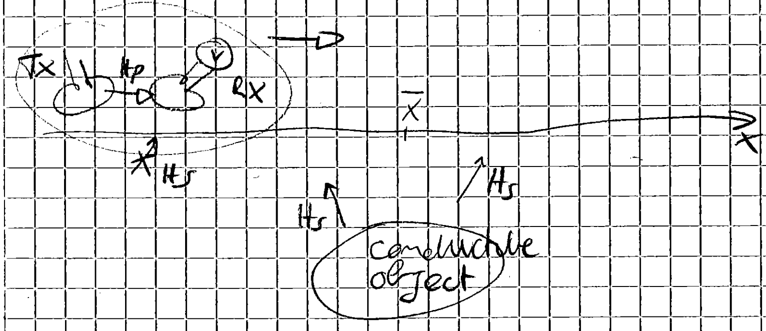
$V \propto \vec{H}$ (normal to the surface)

EDDY CURRENTS (sinusoidal with $f = f_0$) (multidirez)

this method works only if the medium is conductive

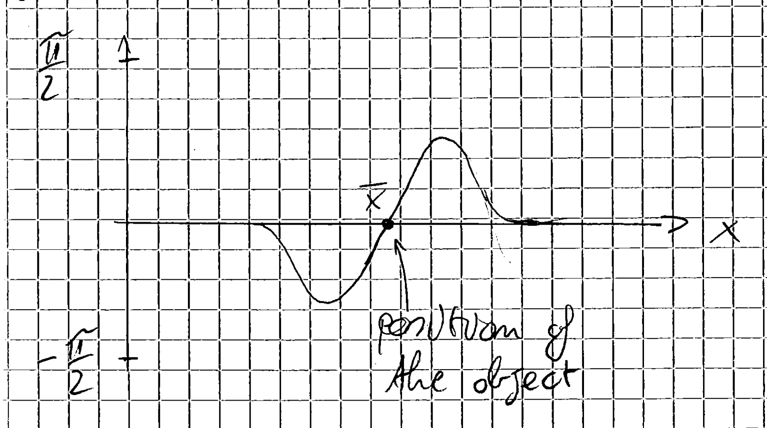
The intensity of those currents is proportional to $\sigma$.

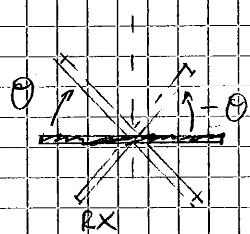
## POLARIZATION ANGLE METHOD



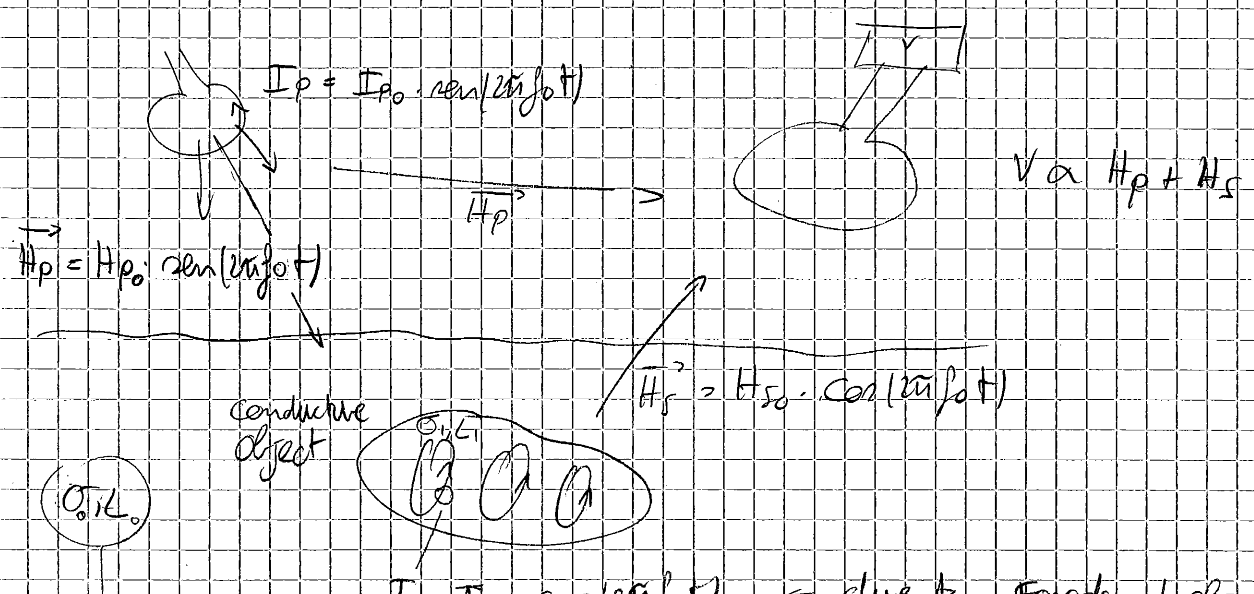

metto il ricevitore in modo tale da avere $V=0$, cioè la componente del campo magnetico $\vec{H} = \vec{H}_p + \vec{H}_s$ normale alla superficie di RX è $= 0$.

$\theta : V = 0$



# GROUND CONDUCTIVITY METER



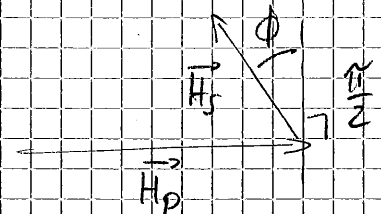

$I_s = I_{s_0} \cdot \cos(2\pi f_0 t)$ ← due to Faraday-Laplace formula:

$$I_{IND} \propto \frac{dH_{INC}}{dt}$$

we can model a medium such as —eeee—MM—
where $L$ = inductance and $R$ = resistance ($\propto \frac{1}{\sigma}$)   $L$   $R$

$$\vec{H_s} = H_{s_0} \cdot sen\Big(\underbrace{2\pi f_0 t - \frac{\pi}{2}}_{\cos(\,)} - \phi\Big) \quad \text{where } \phi = \tan^{-1}\left(\frac{2\pi f_0 L}{R}\right)$$

abbiamo quindi una differenza di fase tra $\vec{H_p}$ e $\vec{H_s}$.




from $\phi$ we obtain an estimation of $\sigma_0$
due to the fact that this estimation is very rough, we use this method to detect anomalyes in the ground (es. metal, rock, ecc.)

in realtà il campo magnetico $= \vec{B} = \mu \cdot \vec{H}$, noi consideriamo $\mu = \mu_r \mu_0$ con $\mu_r \approx 1$ (soil, air, water, rocks, concrete, ecc.)

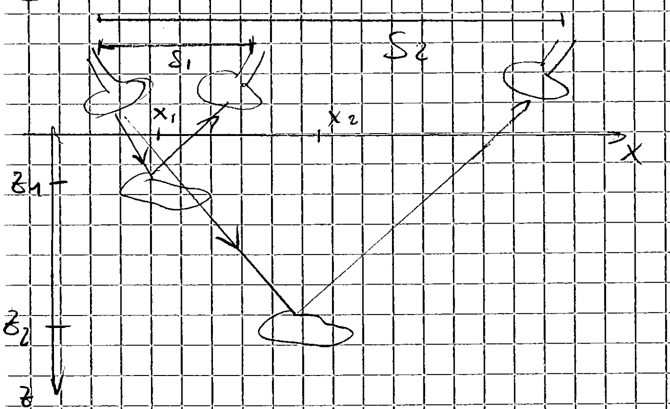
Come facciamo ad aumentare la profondità di rivelazione?

1) FREQUENZA

$$\delta = \text{SKIN DEPTH} = \frac{503}{\sqrt{f \cdot \sigma}} \quad \left(\text{depth when } A = \frac{A_0}{e}\right)$$

if $f \uparrow$ $\delta \downarrow$ quindi il campo penetra meno nel terreno. ☹

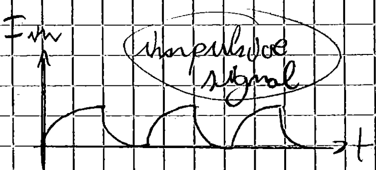
## ② DISTANZA S (distanza tra TX e RX)

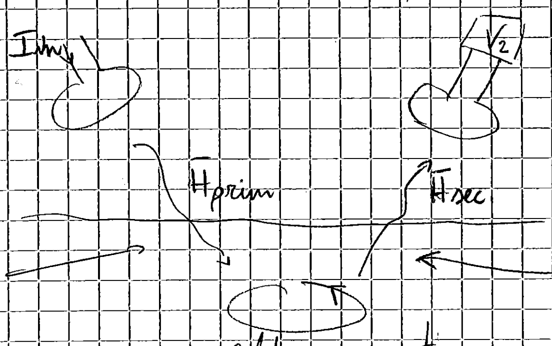



$z_2 > z_1 \rightarrow S_2 > S_1$

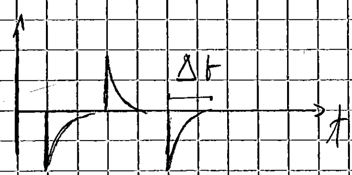
Quindi se uso $10^3 < f_0 < 10^4$, ottengo delle misure su $\sigma(z)$ dove $z$ andrò a sceglierla variando $f$ e $S$.

## TDEM (TIME DOMAIN E.M.)




$V_2$ due to $H_{sec}$




quando $I_{tx}$ viene spenta, $H_{prim} = 0$ molto velocemente

invece $\bar{H}_{sec} \rightarrow 0$ più lentamente a causa del moto degli ioni; questo provoca un $\Delta t \neq 0$ al Rx.

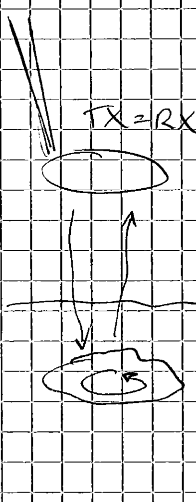
eddy currents

misurando $\Delta t$, si può capire il valore della conduttività del "bersaglio":

se $\Delta t \approx 100$ ns $\rightarrow$ high conductivity (es. metal)

se $\Delta t \approx 0$ s $\rightarrow$ low

es. METAL DETECTOR




rivela qualcosa di conduttivo nel terreno (es. metallo).

VLF (VERY LOW FREQUENCY $f \approx 25$ kHz)

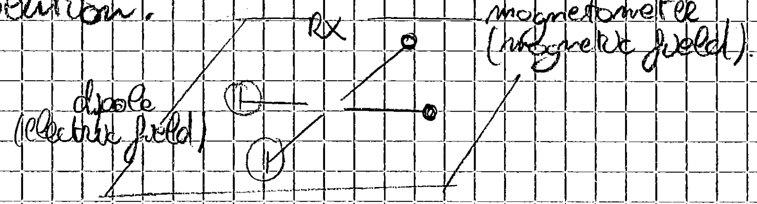
this method uses the already present em field ("natural source")
It reveals big anomalies in the ground (near the surface).

AF MAG (AUDIO FREQUENCY MAGNETIC $f \approx 100$ Hz)

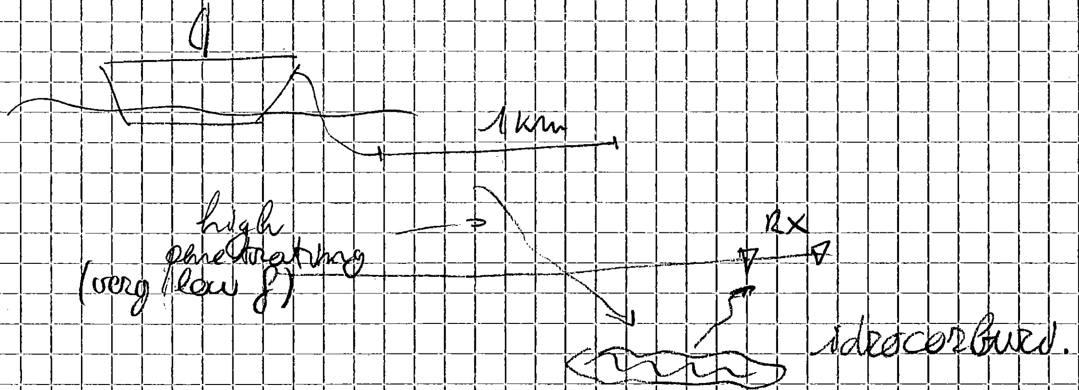
this method uses the em. generated by storms (lightnings)
It reveals "

MAGNETO TELLURIC METHOD ($f \approx 10^{-3} - 10$ Hz)

this method uses the em generated by the solar wind coming from the sun. Being very low frequency, the field can penetrate very deeply into the ground, but at the same time we have very low resolution.




(MARINE) CONTROLLED SOURCE ($f \approx 1$ Hz)



GPR (GROUND PENETRATING RADAR)

$$J = \sigma \cdot E + \varepsilon \cdot \frac{dE}{dt} \xrightarrow{\mathcal{F}} J(\omega) = (\sigma + j\omega\varepsilon) E(\omega)$$

we use $f > 10^7$ Hz $\rightarrow J \approx \varepsilon \cdot \frac{dE}{dt}$

$\varepsilon$ = electrical permettivity $\rightarrow$ dielectric conduction (we don't have particles moving but a reordination of molecules)

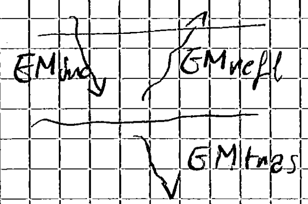
$\varepsilon = \varepsilon_r \cdot \varepsilon_0$ , $\mu = \mu_r \mu_0$

we want to measure $Z$ = electrical impedance = $Z(\varepsilon, \mu, \sigma)$

we make some hypotesis:

- $\mu \approx \mu_0$ (for soil, water, air)
- we study only layer with $\sigma \rightarrow \infty$ (es. dry)

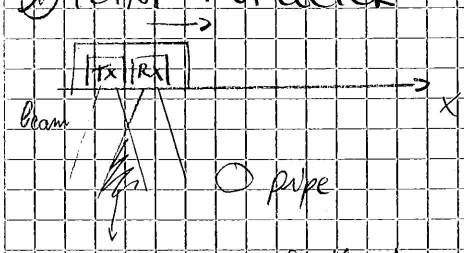
$\rightarrow Z(\varepsilon)$. ?perché



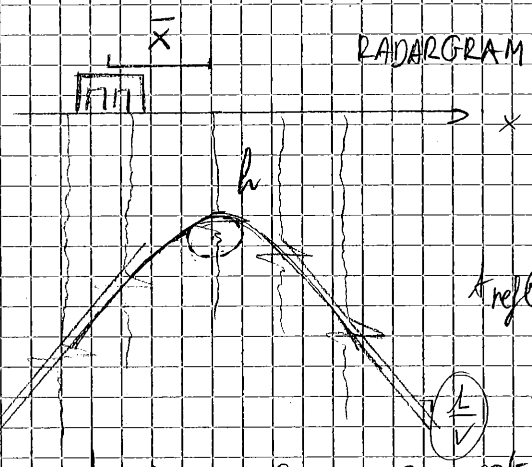

$$V = \frac{c}{\sqrt{\varepsilon_r}}$$

example of possible scenario:

① POINT REFLECTOR




common zone: if the target is inside, I will measure the echo.

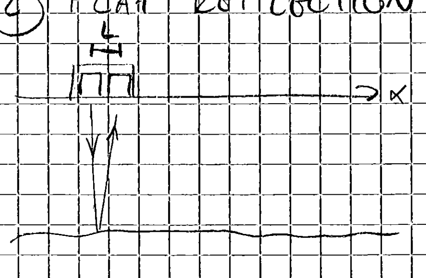


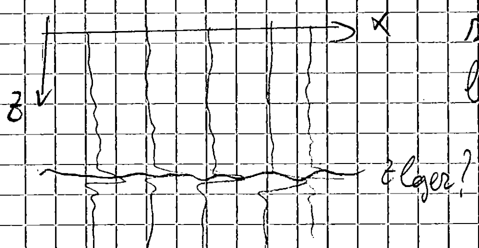

$t_{refl} = \frac{2}{V} \sqrt{h^2 + \bar{x}^2}$

iperbola

from the iperbola calculate $V(x)$ and so we can convert time to depth. OR we can do migration analysis

② FLAT REFLECTION

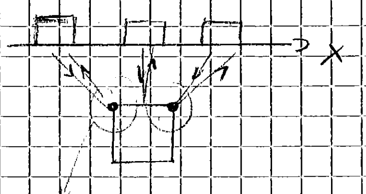



we don't have an iperbola, so we have an ambiguity between $V(x)$ and $z(x)$.

L = OFFSET is fixed, so we can't perform CMP analysis.

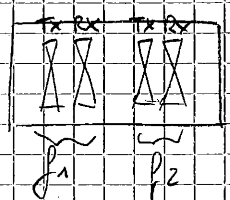
## 3/ RECTANGULAR REFLECTOR

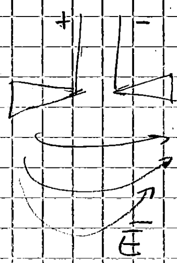



point reflector


In real instrument, there are two pairs of antenna at different frequencies ($f_1$ and $f_2$) ($f_2 > f_1$).

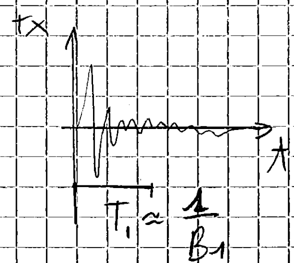
We use $f_1$ for medium/deep revelation and $f_2$ for shallow revelation but with higher resolution.

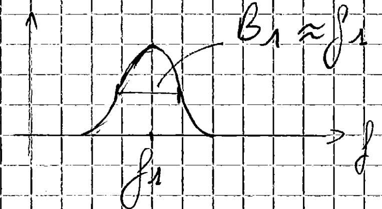



BOWTIE ANTENNA

si utilizza un set di antenne alla volta, in modo da non avere interferenza.




Stacking of N traces in the same condition (position) → SNR↑

$$RES = \frac{\lambda}{4}$$

# E.M. WAVE EQUATIONS

$$\nabla^2 \vec{E} - \sigma\mu \cdot \frac{\partial \vec{E}}{\partial t} + \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$\varepsilon = \underbrace{\varepsilon_0}_{\text{vacuum}} \cdot \varepsilon_r$ = elect. permettivity    $\mu = \underbrace{\mu_0}_{\text{vacuum}} \cdot \mu_r$ = magnetic permeability

$\mu_r \approx 1$ for soil, rock, water, road, ecc.

$\varepsilon_r = 2 - 10$ " "

$\varepsilon_r = 81$ for water

$V = \frac{1}{\sqrt{\varepsilon\mu}} = c \cdot \frac{1}{\sqrt{\varepsilon_r}}$ dove $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

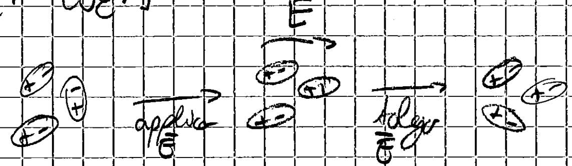
$Z$ = electrical impedance $= \frac{|H|}{|E|} = \sqrt{\frac{\mu}{\varepsilon + j\frac{\sigma}{\omega}}}$

$|E| = E_x e^{-\gamma z}$ dove $\gamma$ = propagation constant $= \alpha + j\beta$ con $\alpha$ = costante di attenuazione

$\beta$ = costante di fase.

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} = j\omega\sqrt{\mu\varepsilon'\left[1 - j\left(\frac{\varepsilon''}{\varepsilon'} + \frac{\sigma}{\omega\varepsilon'}\right)\right]}$$

$\varepsilon \propto$ reorientation of molecules




Dopo che applico e tolgo $E$ le molecole tendono a tornare nello stato iniziale a seconda del materiale e facendo questo si creano delle frizioni che generano calore.

$\varepsilon = \varepsilon' + j\varepsilon''$ dove $\varepsilon'$ = dielectric capacity (quanto tende a tornare allo stato iniziale).

$\varepsilon''$ = absorption effect (quanta energia viene trasformata in calore a causa della frizione tra molecole).

Es. acqua ha alto $\varepsilon''$ → infatti nel microonde si scalda molto.

possibili casi considerando $\varepsilon$ e $\sigma$:

1) $\varepsilon'' = 0$, $\sigma = 0$ → PERFECT DIELECTRIC → $\alpha = 0$

NO ABSORPTION

2) $\varepsilon'' = 0$, $\sigma \neq 0$ e $\sigma \ll \varepsilon'\omega$ → $\alpha = \cancel{\omega}\sqrt{\mu\varepsilon'} \cdot \frac{\sigma}{2\omega\varepsilon'}$  $\alpha \underset{\text{mostly}}{\propto} \sigma$

3) $\varepsilon'' \neq 0$ e $\varepsilon'' \ll \varepsilon'$, $\sigma = 0$ → $\alpha = \omega\sqrt{\mu\varepsilon'} \cdot \frac{\varepsilon''}{2\varepsilon'}$  $\alpha \underset{\text{mostly}}{\propto} f$

4) $\varepsilon'' \neq 0$, $\sigma \neq 0$ → $\alpha \propto \sigma, \varepsilon'', f$

Consideriamo ora $Z$ nei vari casi: $\left(Z = \sqrt{\frac{\mu}{\varepsilon + j\frac{\sigma}{\omega}}}\right)$

1) $Z = \sqrt{\frac{\mu}{\varepsilon'}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{1}{\varepsilon_r}} = Z_0 \cdot \sqrt{\frac{1}{\varepsilon_r}}$

so a change of $Z$ → change in velocity $\left(V \propto \frac{1}{\sqrt{\varepsilon_r}}\right)$

2) $Z = \sqrt{\frac{\mu}{\varepsilon'}} \cdot \left(1 + j\frac{\sigma}{2\omega\varepsilon'}\right)$

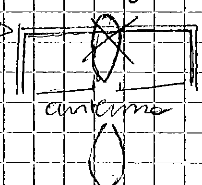
3) $Z = \sqrt{\frac{\mu}{\varepsilon'}} \cdot \left(1 + j\frac{\varepsilon''}{2\varepsilon'}\right)$

---

nei metalli $\sigma$ è grande → alta riflessione $(R \to 1)$

in uno scenario con metal shield non si può usare GPR a causa delle forti riflessioni.

Schermi metallici sono utilizzati per evitare riflessioni provenienti da oggetti sopra l'antenna. →




| $f_0$ [MHz] ($\approx B$), $T = \frac{1}{B}$ | RESOLUTION | | PENETRATION | ANTENNA DIM [cm] |
|---|---|---|---|---|
| 50 | 1 m | big object | 10-30 m | 50×50 |
| 150 | 50 cm | | | : |
| 250 | 27 cm | pipes | 2-6 m | 10×10 |
| | : | | : | : |
| 2000 | 2,5 cm | iron bars in concrete | 0,2-0,6 m | 1×1 |

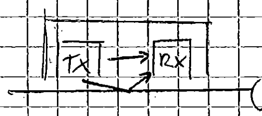
# PROCESSING OF GPR

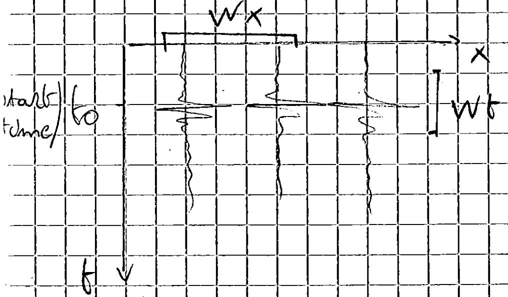
1) REMOVE EM NOISE OUT OF THE SIGNAL BANDWIDTH

use bandpass filter

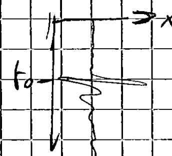
2) REMOVE BACKGROUND NOISE

due to direct air arrival and floor reflections




we use median filter along x direction, i.e. per ogni t ricavo il valore mediano tra n traces, così ottengo un median trace




, ora sottraggo questo trace al trace originale.

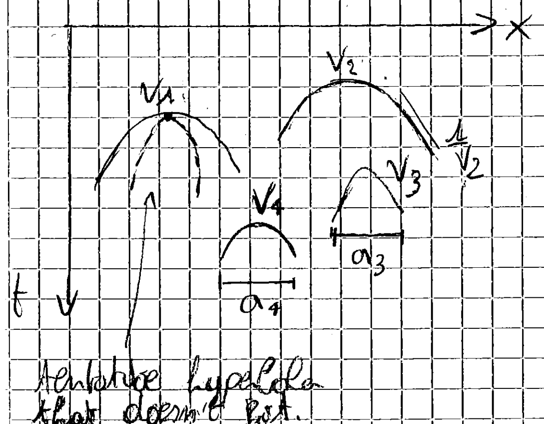
Se utilizzo lunghe linee di acquisizione, il background noise potrebbe variare lungo x, quindi il valore mediano lo calcolo all'interno di finestre Wx.
Per non introdurre il segnale utile, utilizzo il filtro solo all'interno di finestre Wt.

3) ATTENUATION RECOVER

use gain equalization → energy equalization $r(t)\cdot K_1 \frac{1}{t} \cdot e^{+K_2 t}$
→ AGC

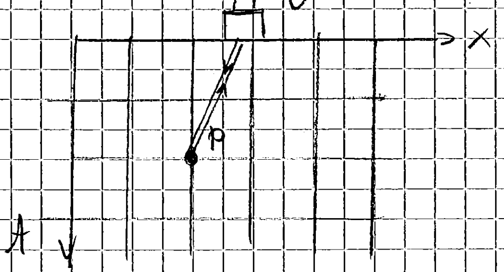
4) VELOCITY ANALYSIS

we look for hyperbole



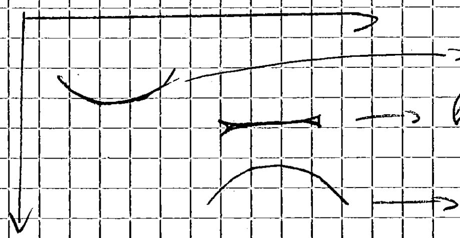

Using tentative hyperbola $g(x,t,v,a)$ ($a$ = apertura iperbole), we find the various velocities, then, interpolating, we obtain $V(x,t)$.

now we apply MIGRATION:



for each point P we sum the values of each trace $x_i(\bar{t})$ given $\bar{t}$ = the travel time of the wave (go and back) according to the distance, $V(x,t)$ and aperture$(x,t)$.

after migration we obtain an image, in which we can see some artifacts due to wrong $V(x,t)$ and aperture$(x,t)$




If there are these artifacts we can change $V(x,t)$ and aperture$(x,t)$ and try again.
The aperture$(x,t)$ tells us how many traces consider for a certain point.

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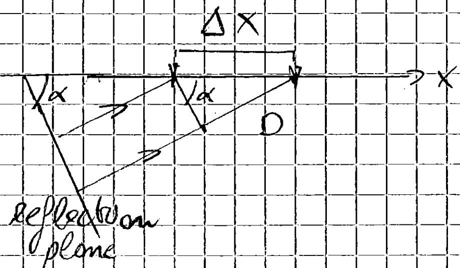
Which is the maximum distance between traces?
in order to avoid aliasing among different hyperbola, the delay between two adjacent acquisitions $(\Delta t)$ must be less than T (wavelet duration).

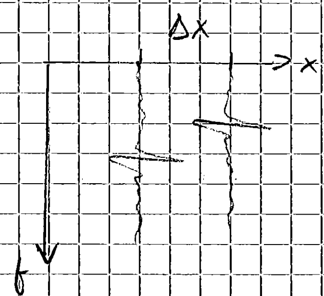



$$D = \Delta x \operatorname{sen}(\alpha)$$

$$\Delta t = \frac{D}{V_{media}} < T = \frac{1}{B}$$

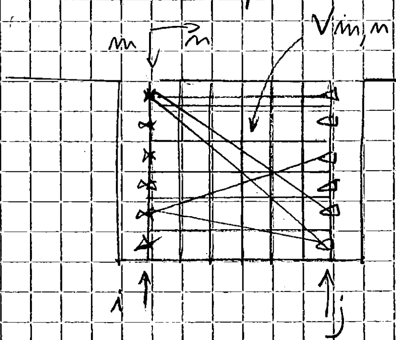
$$\Downarrow$$

$$\Delta x < \frac{\lambda}{\operatorname{sen}\alpha}$$




Es. $f = 250$ MHz $\Delta x \approx 2$-$5$ cm

TOMOGRAPHY




$$A_{i,j} = \sum_{k=\text{cell for ray}} S_k \, \ell_k$$

$p = i \cdot j$ , $q = m \cdot n$

NON-OBSERVABILITY PROBLEM: alcune celle non sono attraversate dai raggi.

we work on a LINEAR MODEL:

$d = G \cdot m$ — data/observations, model

in our case: $d$ = travel time $t_{(m \cdot n) \times 1}$
$m$ = slowness for each cell $\bar{S}$

LEAST SQUARE MINIMIZATION

we want to minimize the error $\varepsilon = d_0 - Gm$ (inverted model)

$\min \|e\|^2 = \min \|d_0 - Gm\|^2$

we compute the derivative versus $m$ and put it to $\emptyset$.

$m = (G^T \cdot G)^{-1} G^T \cdot d_0$ (but we have no control on non-observability)

REGOLARIZATION

we want to minimize: $\min\{\|d_0 - Gm\|^2 + \lambda \|Dm\|^2\}$

where $D$ = regolarization term, $\lambda$ = weight of regolarization

min energy → $m = (G^T G + \lambda \cdot I)^{-1} G^T \cdot d_0$

DUMPED LEAST SQUARE SOLUTION

$m = (G^T \cdot G + \lambda D^T \cdot D)^{-1} G^T \cdot d_0$

we use regolarization to have comparable numbers from different parameters (ex. $\rho$, $V_p$, $\underset{porosity}{\Phi}$)

if we want to minimize $\frac{ds}{dx}$ (to obtain horizontal layers)

so we choose: $D = \begin{bmatrix} -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ \vdots & & & & & \vdots \\ 0 & \dots & & 0 & -1 & 1 \end{bmatrix}$

if " " " $\frac{ds}{dz}$ ( " vertical layers)

cella sotto alla cella x-1

$D = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & \dots \\ 0 & -1 & 0 & 0 & 1 & \dots \end{bmatrix}$

# SINGULAR VALUE DECOMPOSITION

$$\underset{p\times q}{G} = \underset{p\times n}{U}\,\underset{n\times n}{\Lambda}\cdot \underset{n\times q}{V^T}$$

where $U$ = singular vectors in data space,
$V$ = " " " model space,
$\Lambda = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots \sqrt{\lambda_n})$ matrix of singular values ($\sqrt{\lambda_i} \geq 0$ and $\sqrt{\lambda_1} > \sqrt{\lambda_2} \dots > \sqrt{\lambda_n}$)

$$d = G\cdot m = U \Lambda V^T \cdot m$$

↓ inversion

$$m = G^{-1} d = V \Lambda^{-1} U^T \cdot d = V \cdot \underbrace{\begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & & \emptyset \\ & \frac{1}{\sqrt{\lambda_2}} & \\ & & \ddots \\ \emptyset & & & \frac{1}{\sqrt{\lambda_n}} \end{bmatrix}}_{\Lambda^{-1}} \underbrace{\begin{bmatrix} \boxed{\quad\quad} \\ \boxed{\quad\quad} \\ \vdots \\ \boxed{\quad\quad} \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} S_{11} \\ S_{22} \\ \vdots \\ S_{m,n} \end{bmatrix}}_{S}$$

singular vectors

the vectors related to non observable cells will be multiply for the inverse of the lower singular values (so for big numbers).
To avoid this, we put to zero this coefficient (es. $\frac{1}{\sqrt{\lambda_n}} \to \emptyset$) → so we exclude automatly non observable cells → TRUNCATED SVD INVERSION

---

# BAYESIAN INVERSION

we use a priori informations in order to improve the model estimation (Es. V costante in all cell $\pm \Delta V$ con una nota probabilità)

$$m = \left(G^T \cdot C_d^{-1} \cdot G + C_m^{-1}\right)^{-1} \left(G^T C_d^{-1} \cdot d_0 + C_m^{-1} \cdot m_0\right)$$

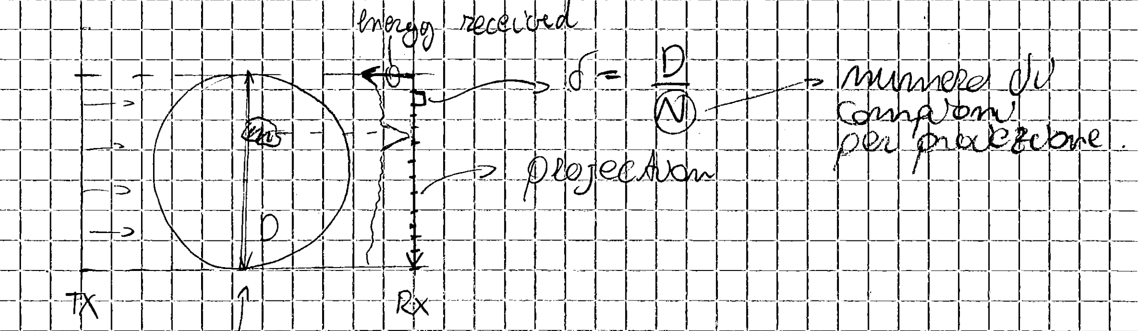
explain the noise in data

starting model

covariance matrix of the a priori model

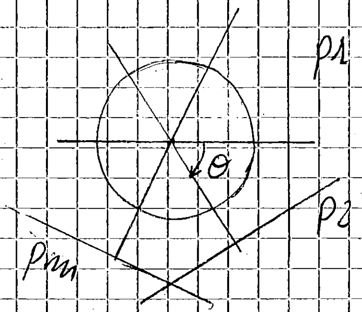
$$\underset{\text{a posteriori}}{C_m} = \left(G^T \cdot C_d^{-1} \cdot G + C_m^{-1}\right)^{-1}$$

# TOMOGRAPHY




Corpo su cui vogliamo operare la tomografia (es. testa con un cancro)

Facciamo delle proiezioni tutte intorno all'oggetto!

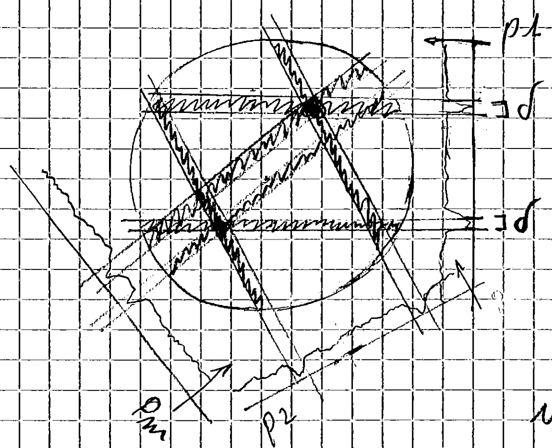



almeno $\theta = \pi$

$M$ = numero di proiezioni

## BACK PROJECTION

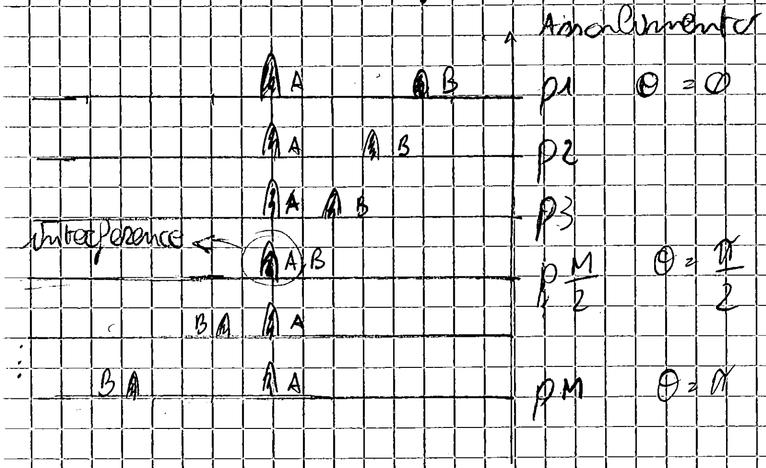
Dalle proiezioni ottenute tracciamo delle linee sull'oggetto in corrispondenza dei picchi di assorbimento misurati; Ognuna di queste linee "contiene" un'energia proporzionale al picco di assorbimento. (spread the energy over the line)



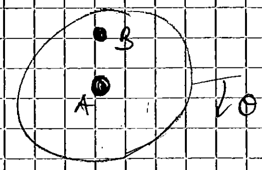

le intersezioni delle linee ci daranno la posizione delle anomalie cercate (es. cancro o bolla d'acqua).

L'immagine ottenuta non sarà uguale all'immagine iniziale, ma sarà un insieme di M "linee" (di spessore $\delta$) con dei picchi in presenza delle anomalie cercate.

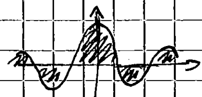
Vogliamo ora capire la vera ampiezza dei vari picchi ed eliminare picchi fittizi:




Es.




Voglio che l'energia di un picco non interferisca con quella di un altro, quindi prima di sommare le varie proiezioni, moltiplico ogni proiezione per una funzione a media nulla (Es. centrata sul picco di interesse in modo che gli altri picchi (moltiplicati per $\pm f$ a seconda della posizione) si cancelleranno tra loro).



somma dei lobi = 0)

In modo che gli altri picchi non vengano moltiplicati sempre per fattori positivi, impongo che:

$$\frac{\pi}{M} = \Delta\theta \qquad \Delta\theta \cdot \frac{D}{2} < \delta$$

Dall'energia così calcolata, capisco di che "materiale" è fatta l'anomalia (Es. cancro, grumo di sangue, osso, ecc.).