17/04/17 (1)

GEOPHYSICAL IMAGING

METHOD 1: physics → processing → application (geophones, metaldetector) → software

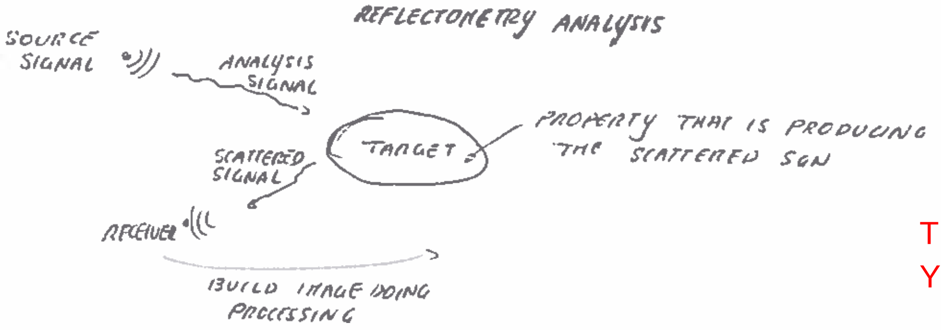
Geophysics: obtain measures of some physical values of earth.
We have a radar sending sign to earth and this gets reflected (method to look below the surface).

Subsurface: can be the well
We can find an obstacle in a metallic pipe for oil using acoustic waves

ACOUSTIC RADAR

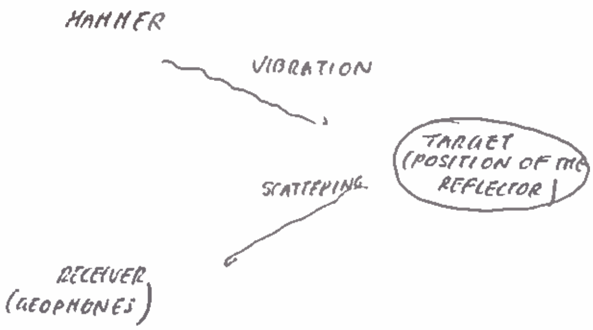
Physics here is using acoustic waves to have an image of the pipe

REFLECTOMETRY ANALYSIS

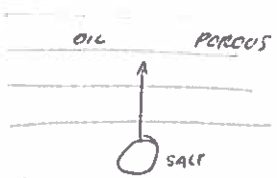


Thanks to
Yasmin Sarbaoui

SEISMIC METHODS



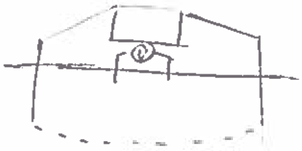
An example of common situation is a target like salt body
We know that a lower density material tries to get out from the surface



The positions of PINCH OUT are good candidates to have oil if the rock is porous. The oil is up in the pinch out position because it floats over water.

We have two methods: elastic method and electrical method and electromagnetic waves method

ELECTRICAL METHOD



Capacity of the subspace to propagate the current
Why we can have a current flow in the subsurface?
If we have water → good conductivity. So we normally use the electr. method in presence of water. It can be used to check if we have contaminants going out of a landfield.
Water is conductive while oils and gas are not

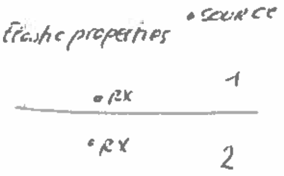
ELECTROMAGNETIC WAVES

We will see:
- Low frequency EM: METAL DETECTOR (10kHz)
- High frequency EM (>10⁶Hz): GROUND PENETRATING RADAR

We have two problems: resolution and observability. Some applications are impossible.
With the elastic method we have no conductivity, with GPR we cannot go below 100m

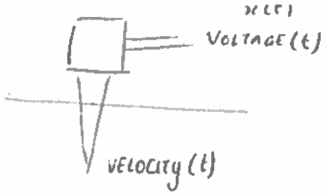
What if we have two mediums?

Elastic properties



⇒ DA FARE A CASA

22/09/17 ②



ANALOGIC SGN → FILE ON PC (SAMPLED VERSION)

x(nT) T: time interval

1/T: sampling freq → how many samples you take per second

Analogic world ⇒ Digital world

cassette audio, vinyl disc

CD, MP3 → The data lasts forever unless you destroy the file

In digital world we have to deal with several processes like copy, communication, life, reproducibility, one way (PC), processing, while in analogic world we have velocity, wave field and electronics.

x(t) will be the physical parameter we are measuring and t the independent parameter

$x_1(t)$, $x_2(t)$, $x_3(t)$ } $X(t)$ This is the multichannel signal (eg. a set of geophones or sensors)

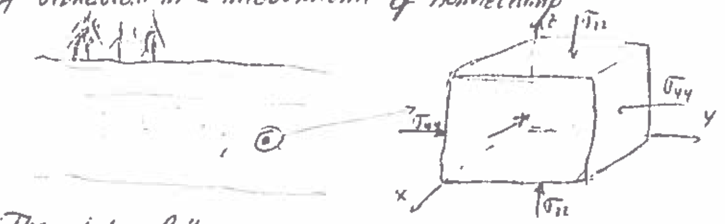
$X(t, space)$ → where the space is the dimension

ELASTIC MEDIA

We use the vibrations to investigate the subsurface.

The signal coming back gives information on the target.
We will talk about vibrations, ie ELASTIC WAVES.

We will talk now about ELASTIC MEDIA.
A vibration is a movement of something



We consider an equilibrium situation
we have same forces acting on the cube's faces
to keep the equilibrium.
This forces can provoque a deformation

The state of the FORCES ~~are~~ is the STRESS, ie the composition of all the forces that are acting on the faces
We can have a normal stress (like compression) or shear stress that is trying to apply a deformation
I divide everything by the area: $\frac{Nw}{m^2} = P_a$

I define $k, l = \{x, y, z\}$ with k direction of the stress and l the face where the stress is applied ($\perp k$)

σ_{kl} → e.g if i have σ_{zz} means that i have the force going like z axis on the face ⊥ to z axis.

- For $k = l$ the force is perpendicular to the face so we are compressing or expanding
- For $k \neq l$ we have a shear stress

STRAIN

So we define the strain (deformation) by the displacement of the vector

Displacement: $\vec{u} = u_x \vec{1}_x + u_y \vec{1}_y + u_z \vec{1}_z = [m]$

We want to define the new position of the vertex, a percentage displacement.

$$\boxed{\varepsilon_{mn} = \frac{1}{2}\left(\boxed{\frac{du_m}{dn}} + \frac{du_n}{dm}\right)} \qquad m, n = x, y, z \Rightarrow \text{STRAIN}$$

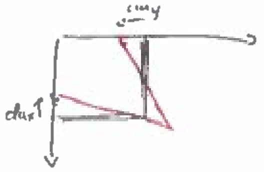
Incrementation on the m axis with respect to n axis

Example

$\varepsilon_{xx} = \frac{du_x}{dx}$ → displacement along the x direction with respect to the original total dimension

Example 2

$\varepsilon_{xy} = \left(\frac{du_x}{dy} + \frac{du_y}{dx}\right)$



- For $m \neq n$ we have an angular deformation
- For $m = n$ we have an angle-conserving compression

HOOKE'S LAW

There is a linear link between stress and strain

$\boxed{\sigma_{kl} = C_{klmn} \varepsilon_{mn}}$ C_{klmn} is a matrix of constants where $klmn = \{x, y, z\}$

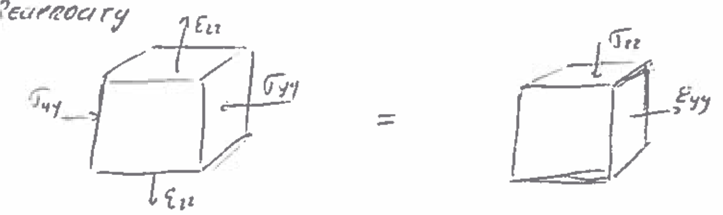
So how many combinations do we have? $klmn \rightarrow 81$ comb.

This combination is true for small stresses.

The bodies that don't have an elastic behaviour are PLASTIC MEDIA (eg plasticine) and they don't go back to the original shape. An earthquake can produce a plastic deformation

To lower the possible combination we notice that there is reciprocity

Reciprocity



$\sigma_{yy} = C_{yyzz} \varepsilon_{zz}$ $\sigma_{zz} = C_{zzyy} \varepsilon_{yy}$ $\Longrightarrow$

$C_{yyzz} = C_{zzyy}$

Thanks to this we can reduce the combinations to 21.

It exists a medium with 21 numbers (molecular cristal)

If we have an ISOTROPIC MEDIUM (elastic properties don't change with direction) we have that:

$C_{xxxx} = C_{yyyy} = C_{zzzz}$ and this means that the elastic constants become Two

This numbers are is telling the behaviour of the body with respect to compression and shear.

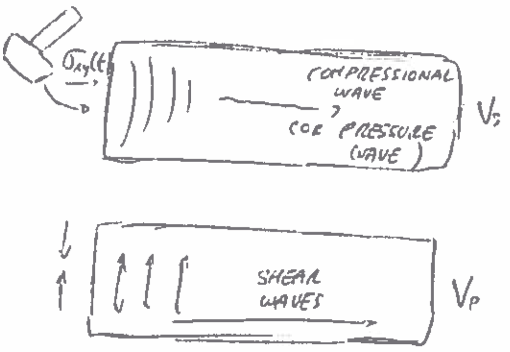
Normally the mediums of real world are ANISOTROPIC. Due to sedimentation we have horizontal layers (thin layers) so the vertical behaviour is different by the horizontal one.

We can talk about TRANSVERSE ISOTROPY and we have 5 elastic constants

HOMOGENEOUS MEDIUM (elastic)

A medium is homogeneous if the properties don't change with the position

PROPAGATION OF STRESS

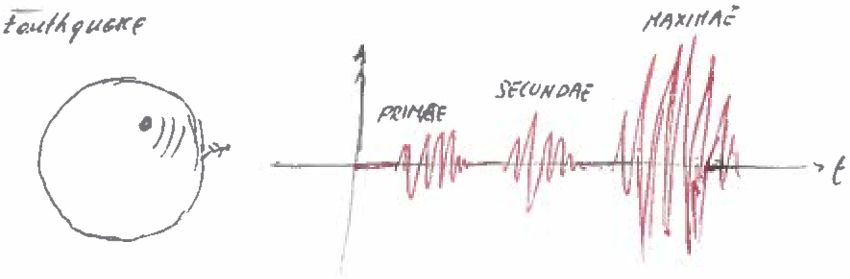


Both of this waves exist and this was discovered by a chinese studying earth quakes

The pressure waves are the primae, the fastest, the shear are the secundae and the maximae is another solution of the wave equation.

In the graphic it's shown how normally an earthquake arrives.

Earthquake



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$\sigma_{ik} = \overset{\uparrow B}{C_{ikmn}} \underset{\downarrow \text{ADIMENSIONAL}}{\varepsilon_{mn}}$ → valid for a small stress

↓ Force pushing the face → STRESS : F/AREA = PRESSURE = PASCAL

To fill the matrix constant we need 21 numbers for a general medium and 2 for an isotropic medium. We are gonna see how to obtain this two numbers (compression and shear)

C is small for a sponge and big for steel (hard to deform)

EXPERIMENT 1 To obtain a number related to c

Take a sample, compress and measure the ~~distortion on~~ deformation wrt 1 the other directions (usually it happens when u compress in one direction you get an enlargement in the other direction.

So we define the poisson coeff.

- **POISSON COEFFICIENT** : $\boxed{\nu = - \frac{\frac{\partial u_y}{\partial y}}{\frac{\partial u_x}{\partial x}}}$

→ This can be any ⊥ direction ②
it can be also z

→ applied force ①

- If the body maintains the volume, if i compress by one direction, i have the same expansion in the other directions so $\boxed{\nu = 0.5}$
- If the body get compressed by one direction without expanding in the others (sponge) $\boxed{\nu = 0}$

So we get $\boxed{0 \le \nu \le \frac{1}{2}}$

ν is not defined for fluids (gas and liquid)

ν can go down to −1, means you compress in one direction and get a compression in the others. This happens with some cristals that has to maintain the angles. (AUXETIC materials)

So $-1 \le \nu \le 0.5$.

ν is adimensional and it's related with the elastic constants.

- EXPERIMENT 2 **YOUNG MODULUS**

Young modulus: $E = \frac{\sigma_{xx}}{\varepsilon_{xx}}$ ratio between stress in one dimension and deformation in the same direction

We notice that $E = \frac{\sigma_{xx}}{\varepsilon_{xx}} = C_{xxxx} \underset{\substack{\uparrow \\ \text{FOR ISOTROPIC} \\ \text{MEDIUM}}}{=} \frac{\sigma_{yy}}{\varepsilon_{yy}} = \frac{\sigma_{zz}}{\varepsilon_{zz}} = C_{yyyy} = C_{zzzz}$ [Pa]

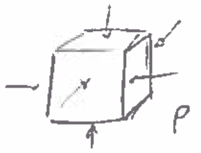
So this is the elastic constant responsible for compression

For steel we have $2.1 \cdot 10^{11}$ Pa

This can't be used for liquids

- For liquids there is another quantity called the **BULK MODULUS**

Related to a sample that is getting forces from all the directions.



we take a container and apply pressure on all the faces

We measure the Bulk Modulus:

$\boxed{K = \frac{P}{\Delta V / V}}$

Ratio between pressure applied and the volume deformation where V is the volume

For steel : $K = 160 \cdot 10^{9}$ Pa

There is a relation between ν, E and K since we only need two parameters

$\boxed{K = \frac{1}{3} \frac{E}{1 - 2\nu}}$

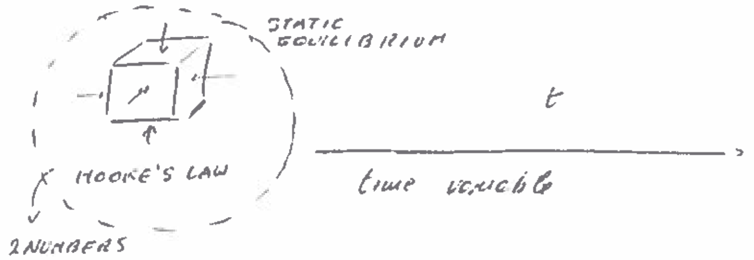
- So the two common parameters are the **LAMÉ PARAMETERS**

$\boxed{\mu = \frac{\sigma_{ik}}{\varepsilon_{ik}}}$ (with $i \neq k$) = $\boxed{C_{ikik}}$ → Elastic constant for shear stress

$\boxed{\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}}$ [Pa]   rigidity of the volume

For all fluids $\mu = 0$ → $C_{ikik} = 0$ So we need only one elastic constant. These are called **ACOUSTIC MEDIA** that are part of the elastic media (simpler). Fluids are air, liquids etc.

Now everything is defined for the static equilibrium of the cube



DINAMIC EQUILIBRIUM

$$\frac{F}{A} \rightarrow \begin{cases} \sigma_{ik}(t) = C_{ikmn}\,\varepsilon_{mn}(t) & \text{HOOKE'S LAW} \\ \vec{F} = m\vec{a} \end{cases}$$

ε_{mn} ↳ displacement

m: V·density $\vec{a}$ → $\frac{d^2u}{dt^2}$

↳ STARTING EQUATIONS

Going from static to dinamic we see that we have a dependency on density

So there is a link between forces (t), displacement (t).

Our parameters become two elastic constants + density; so 3 parameters

The density is $\rho = \left[\frac{Kg}{m^3}\right] = \left[\frac{g}{cm^3}\right]$. $\rho_{H_2O} = 1000\ Kg/m^3$ $\rho_{AIR} = 1 Kg/m^3$ $\rho_{SAND\ STONE} = 2000\ Kg/m^3$

$\rho_{BASALT\ GRANITE} = 6000\ kg/m^3$ $\rho_{oil} = 800\ kg/m^3$

WAVE EQUATION

Link between displacement of forces and elastic constants

$\boxed{V^2 \nabla f = \rho \frac{\partial^2 f}{\partial t^2}}$ (similar to wave eq of EM field that has ∇ der in space and ∂ der in time domain)

Where:

V is the velocity and it contains the elastic constants

f is something like the pressure

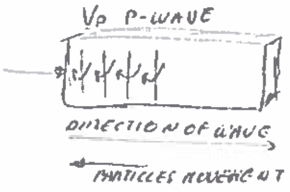
We will then use a finite approximation of the derivaty like $\frac{df}{dt} = \frac{\Delta f}{\Delta t}$

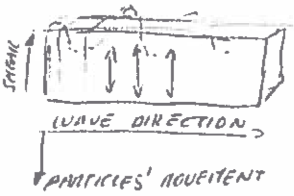
For a **HOMOGENEOUS MEDIUM** we have two solutions of the wave equation:

- one is travelling with velocity V_p that is the velocity of the pressure wave
- one is travelling with velocity V_s that is equivalent to a vibration that is propagating like a shear wave.

Those are linked to the elastic constants

$\boxed{V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}}$ $\boxed{V_s = \sqrt{\frac{\mu}{\rho}}}$



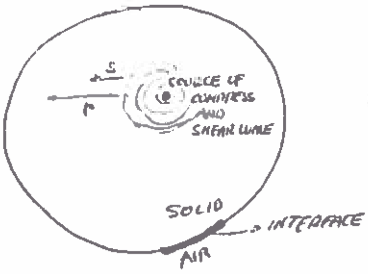


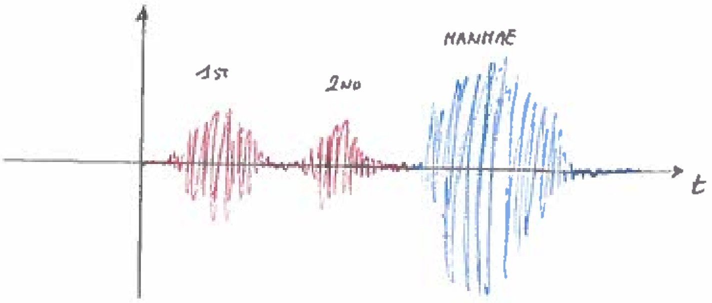
For $\mu > 0$ and $\lambda > 0$ ⇒ $V_p > V_s$

Acoustic media doesn't support shear waves ($\mu = 0$), so $V_s = 0$. They only support pressure wave

$V_p = \sqrt{\frac{\lambda}{\rho}}$

P and S waves travel in the volume of the rock





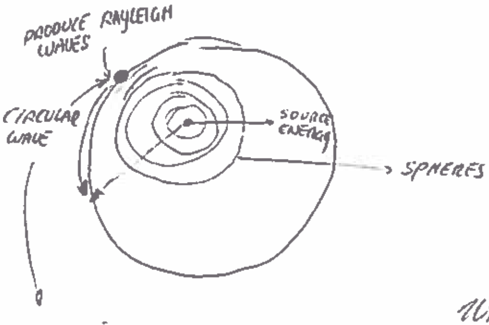
For every position in earth we have the first arrival (P-WAVE) and second arrival (S wave) and they are BODY (or VOLUME) WAVES since they propagate within the volume.

Then we have a third wave (MAXIMAE) that is produced at the interface. These are the M-WAVES i.e SURFACE WAVES that are the solution of the wave equation at the contact between two media (e.g solid and air)

SURFACE WAVES

RAILEIGH WAVE is one particular solution at the solid-air interface $\frac{AIR}{SOLID}$.

This solution shows that the velocity is $V_R < V_S$ and it propagates in the solid media



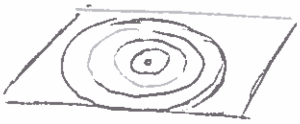
We have some source with an initial energy. The waveforms are spheres with increasing radius. The same energy is distributed on the spheres.

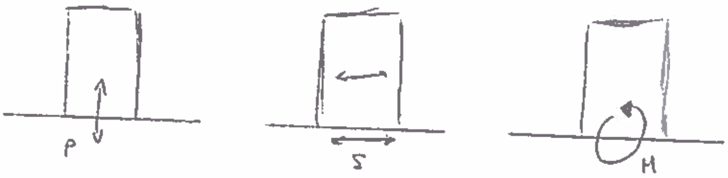
The energy at one point of the sphere is $\frac{ENERGY}{RADIUS^2}$ ↳ ÷ SURFACE OF SPHERE

The energy decays like the distance square.

When the wave reaches the surface we don't have volumes anymore, but circles so the energy decays with the radius $\left(\frac{ENERGY}{RADIUS}\right)$ and not with its square anymore.

That's why the maximae are due to Rayleigh wave.





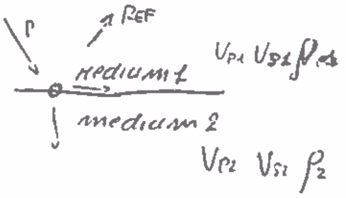
J. we have a wave equation by which we can simulate propagations

WAVE PROPAGATION

↓

SIMULATE PROPAGATION OF VIBRATIONS → V_P, V_S, ρ

We will see what happens with discontinuity



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WAVE EQUATION: $V^2\nabla^2\Phi = \frac{d^2\Phi}{dt}$ The velocity depends on two elastic constants + density

$$V_P = \sqrt{\frac{\lambda+2\mu}{\rho}} \qquad V_S = \sqrt{\frac{\mu}{\rho}}$$

By mixing all the constants we obtain $\boxed{\frac{V_P}{V_S} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}}$ with $0 < \nu < \frac{1}{2}$ poisson coefficient

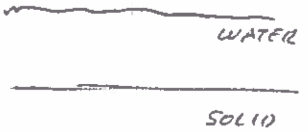
We see that $\boxed{\sqrt{2} < \frac{V_P}{V_S} < \infty}$

By the THUMB RULE we have that $V_P \simeq \sqrt{3}V_S$ or $V_S = \frac{V_P}{\sqrt{3}}$ while for the fluids $V_S = 0$, $\frac{V_P}{V_S} = \infty$

Rayleigh waves have an elliptical movement.

OTHER SURFACE WAVES

STONELEY - SCHOLTE WAVES

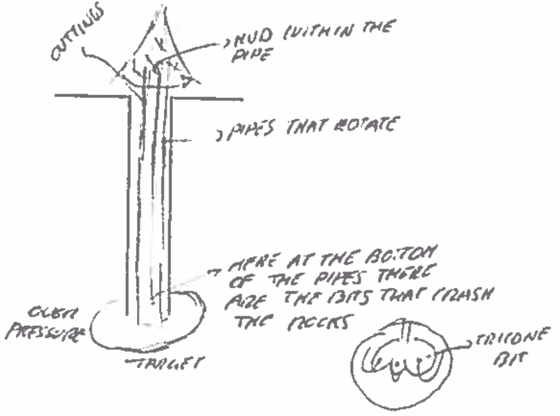


A common wave is the solution of the wave equation at the water and solid interface

This is not a dangerous situation but it can be useful for some measurements if you have sensors.

We see some applications of this waves

- Measurements in a well (pozzo)



The mud helps removing the cuttings and the overpressure.

If you remove a pipe from a ~~pl~~ high pressure place you have "coca cola" effect

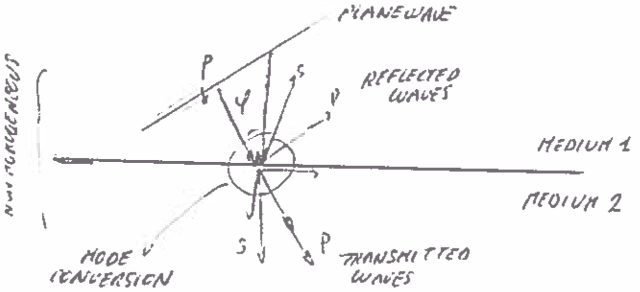
It can be useful to take some acquisition while getting to the target like vibration analysis

To during the perforations it's a common use to do the VERTICAL SEISMIC PROFILE and it helps to get a better image of the target.

To couple the geophone we can use water so that we will have water-rock situation => STONELEY

Here we will have TUBE WAVES

PLANE WAVES AND INTERFACES



We define the reflection and transmission ~~parameter~~ coeff that are functions of the elastic parameters of the two media

V_{P1} V_{S1} ρ_1

V_{P2} V_{S2} ρ_2 φ seven numbers

Reflection coefficient $\boxed{R_{PP} = \frac{\text{Amplitude of reflected } P}{\text{Amplitude of incident } P}}$ where R_{PP} ← scattered, ↑ incident

We can also have R_{PS} R_{SS} R_{SP}

Transmission coefficient $T_{PP} = \frac{\text{AMP TRANSMITTED } P}{\text{AMP INCIDENT } P}$, T_{PS}, T_{SS}, T_{SP}

Eight numbers that completely define the scatter properties of the interface

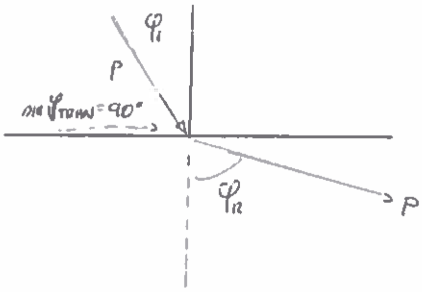
We have to impose the continuity of displacement and stress at the interface to be able to define the formula to obtain these numbers

To define the angle (direction) we introduce the SNELL LAW: $\boxed{\frac{\sin\varphi}{\text{Velocity}} = \text{constant}}$

From it we get: $\frac{\sin \varphi_{incident}}{V_{P\,incident}} = \frac{\sin \varphi_{reflected}}{V_{P\,reflected}}$

$$\frac{\sin \varphi_{INCIDENT\,P}}{V_{P1}} = \frac{\sin \varphi_{REF\,S}}{V_{S1}} \qquad V_{P1} > V_{S1} \qquad \varphi_{INC\,P} > \varphi_{REF\,S}$$

The slower the wave, the smaller the angle



$\frac{\sin \varphi_{INC\,P}}{V_{P1}} = \frac{\sin \varphi_{TRANS\,P}}{V_{P2}}$ It depends on velocities.

- $V_{P2} > V_{P1}$ (very usual situation)

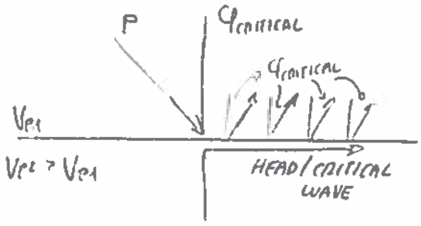
we have the special case where the ~~incident~~ transmitted angle is 90°

$\frac{\sin \varphi_{INC\,P}}{V_{P1}} = \frac{\sin \pi/2}{V_{P2}}$ This is called CRITICAL ANGLE. So φ_{INC} is $\varphi_{CRITICAL}$

And we have $\varphi_{CRITICAL}$ when the transmitted wave has $\varphi_{TRANS} = 90°$

This can happen only if $V_2 > V_1$

We have a wave travelling along the interface on the second layer called CRITICAL WAVE (or HEAD WAVE) a very fast wave

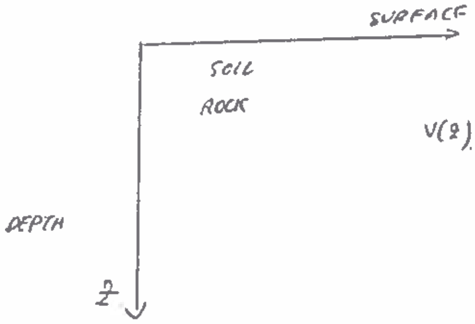


VELOCITIES OF P AND S WAVE DEPENDING ON PROPERTIES OF MEDIUM (Porosity, Depth, Pressure ...)

$V = \sqrt{\frac{RIGIDITY}{\rho}}$

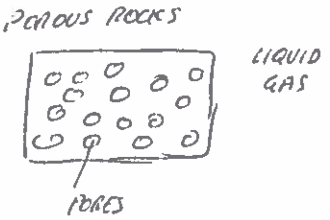
Steel: V_P = 5000 m/s
Soil: V_P = 500 m/s
Water: V_P = 1500 m/s
Air: V_P = 340 m/s (sound)
Sandstone: V_P = 2000 m/s
Basalt: V_P = 6000 m/s

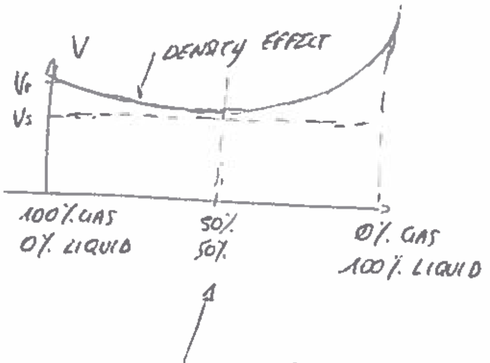
Increasing density we increase the rigidity (that grows faster than density). So we can still say that the higher the density the faster the velocity even if the formula is saying the opposite



↑DEPTH ⇒ ↑PRESSURE ⇒ ↑COMPACTION ⇒ ↑DENSITY ⇒ ↑RIGIDITY

We have an increase of velocity with depth





V_S: shear → not sensitive to the liquid part

V_P → sensitive to liquids. At the beginning the rock becomes more dense than liquid when filling with water. Then it becomes more rigid and velocity goes up.

We can detect the pressure with this shape. By the changing of velocity we can detect over-pressure zones

SCATTER COEFFICIENT

Let's consider acoustic media

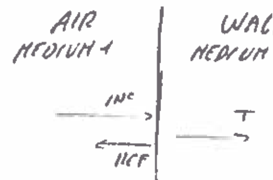
ACOUSTIC MEDIA (Normal incidence)

R_{PP} and T_{PP} and we consider normal incidence (angle = 0)

$$\boxed{R = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$ where Z is the acoustic impedance $Z = V \cdot \rho$

$$\boxed{T = \frac{2Z_1}{Z_2 + Z_1}}$$

Suppose two really different media :



$V_1 = 340 m/s$ | $V_2 = 3000 m/s$

$\rho_1 = 1 kg/m^3$ | $\rho_2 = 3000 kg/m^3$

$Z_1 = V_1 \rho_1$ | $Z_2 = V_2 \rho_2$

$Z_2 >> Z_1$

If Z_1 is negligible i have that $R = \frac{Z_2 - \cancel{Z_1}}{Z_2 + \cancel{Z_1}} \cong 1$

$T = \frac{2Z_1}{Z_2} \approx 0$

If i speak against the wall the voice is not going ahead

For $Z_1 >> Z_2$ i have $R = \frac{-Z_1}{Z_1} = -1$ $T = 2$ I hear a sound that is the double of the sound comming from the wall (like earthquake)

RIGUARDA VIDEO YOUTUBE $\boxed{3/10/2017}$

Pressure

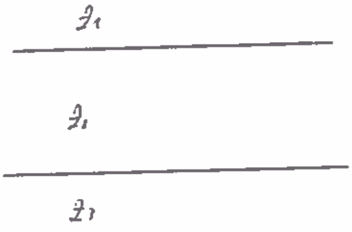
$V^2 \nabla^2 \Phi = \frac{d^2\Phi}{dt^2}$ with Φ displacement pressure. In volume homogeneous isotropic $\rightleftharpoons$ V_P V_S

$\frac{AIR}{SOLID}$ => Rayleigh Wave $\frac{WATER}{SOLID}$ => STONELEY - SCHOLTE (TUBE) WAVE

If we have a big difference between the impedance of the two interfaces we get to the two extremes :

- $Z_1 >> Z_2$ wave totally reflected eg

GUIDED WAVES

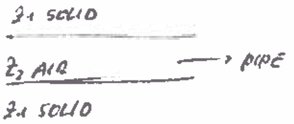


PSEUDO RAYLEIGH WAVE

$Z_1, Z_1 >> Z_2$

In this case we have a completely guided wave

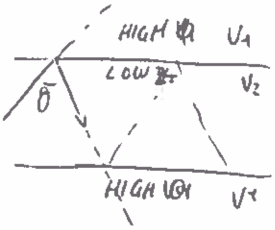
An example of pseudo-Rayleigh wave is :



Big jump between Z_1 and Z_2. The wave remains within the tube and so the energy

We have two kinds of pseudo Rayleigh wave
- Leaky: Every time that the wave hits the interface we have an energy loss
- Non leaky

Those are waves that remain stucked in the middle due to big difference of the materials

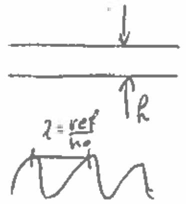


$\theta < \theta_c$ Leaky

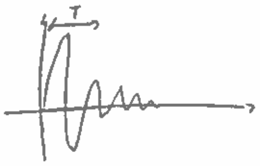
$\theta > \theta_c$ non-Leaky

$V_1 >> V_2$

For guided waves we have Pseudo Rayleigh and another way

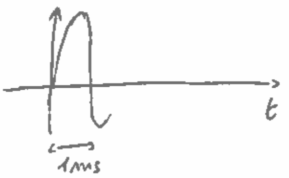


$h \ll$ wavelength
it makes sens to talk about wavelengths for sinusoidal materials
$h \ll \bar{\lambda}$
↳ wavelet length

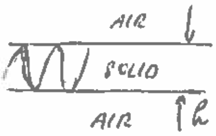


Duration of the main lobe

Wavelet length

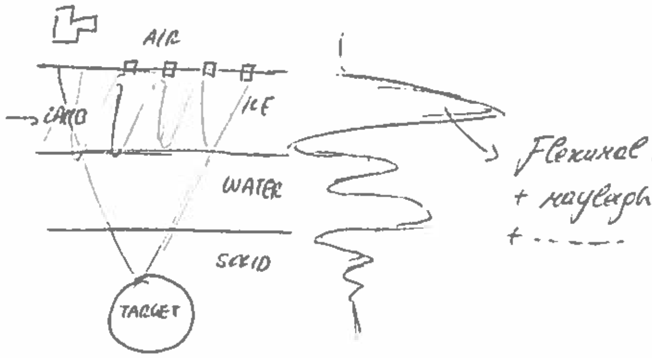


$\lambda = T \cdot V$ where T is the duration of the wavelet
How long does the wavelet become in a material at a certain velocity?



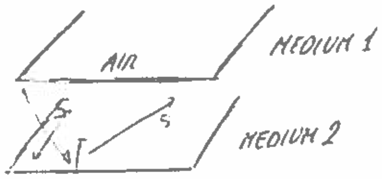
$h \ll$ wavelength
LAMB WAVES (PLATE WAVES; FLEXURAL WAVE)
Two types symmetric

Anti symmetric



Flexural wave
+ rayleigh
+ - - - - -

Abbiamo diversi tipi d'onda che mascherano l'onda riflessa generata dal target.
La soluzione è battere una sorta di tubo fino alla parte solida e sotto generare l'onda



I don't have compressions because medium 2 keeps on moving

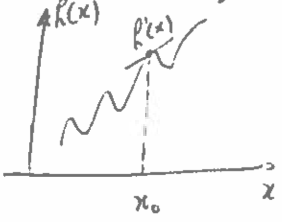
$$V^2_{(x,y,z)} \nabla^2 \Phi_{(x,y,z,t)} = \frac{d^2 \Phi_{(x,y,z,t)}}{dt^2}$$

$\Phi = \Phi(x,y,z,t) \rightarrow$ pressure of the field

$V = V(x,y,z) = V_e$
↳ Velocità dell'onda

Utilizzeremo questa approssimazione per poterla utilizzare nei software

$$\nabla^2 \phi = \frac{d\Phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2}$$

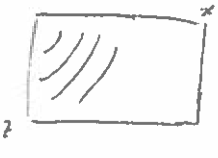


$$h'(x_0) = \frac{h(x_0 + dx) - h(x_0)}{dx} \quad dx \rightarrow 0$$

dx lo sostituiamo con Δx

$$h'(x_0) = \frac{h(x_0 + \Delta x) - h(x_0)}{\Delta x \rightarrow \text{finite}}$$ → finite different approximation of derivative

$$f''(x_0) \cdot \left[\frac{f'(x_0 + \Delta x) - f(x_0)}{\Delta x} - \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x} \right] \frac{1}{\Delta x} \cong \frac{f(x_0 + \Delta x) - 2f(x_0) + f(x_0 - \Delta x)}{\Delta x^2}$$



x, z variable (in y everything is constant) $V^2(x,z) \left[\frac{d^2\Phi(x,z,t)}{dx^2} + \frac{d^2\phi(x,z,t)}{dt^2} \right] = \frac{d^2\Phi(x,z,t)}{dt^2}$

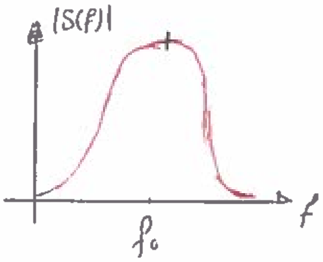
$$V^2(x_0,z_0) \left[\frac{\Phi(x_0 + \Delta x, z_0, t_0) - 2\Phi(x_0, z_0, t_0) + \Phi(x_0 - \Delta x, z_0, t_0)}{\Delta x} + \frac{\Phi(x_0, z_0 + \Delta z, t) - 2\Phi(x_0, t_0, t_0) + \Phi(x_0, z_0 + \Delta z}{\Delta z^2} \right.$$

$$\overset{!}{=} \Phi(x_0, z_0, t_0 + \Delta t) - 2\Phi(x_0, z_0, t_0) + \Phi(x_0, z_0, t_0 - \Delta t)$$

6/10/17 ⑦

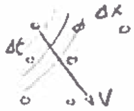
We have this model: elastic media + source where we produce a variation

$s(t) \Longleftrightarrow S(f)$ f_{max}: sampling theorem → $f_{sampling} = \frac{1}{\Delta t} > 2 f_{max}$



We use the RICKER WAVELET that is completely described by the central frequency f_0

$f_{max} \simeq 3\text{-}4 f_0 \Longrightarrow$ This criteria says that $f_{sampling} \geq 2 f_{max} = 2(3\text{-}4 f_0) \overset{!}{=} 6\text{-}8 f_0$



The second criteria describes the discretization of the space.

We want $\boxed{\Delta t \cdot V < \Delta x}$ in order to visit all the points

$\frac{\Delta t \cdot V}{\Delta x} < C$ Condition of Courant, Friedrichs Lewy. In practice $\boxed{\frac{\Delta t \cdot V}{\Delta x} < 1}$

Recall the approximation $\frac{d\phi}{dt} \simeq \frac{\phi(t+\Delta t) - \phi(t)}{\Delta t}$ We analyse the Fourier transform

$$\xrightarrow{\mathcal{F}} j2\pi f \Phi(f) = \frac{\Phi}{\Delta t}[e^{-j2\pi f \Delta t} - 1] = \frac{\Phi(f)}{\Delta t} e^{-j2\pi f \frac{\Delta t}{2}} \left[e^{-j2\pi f \frac{\Delta t}{2}} - e^{j2\pi f \frac{\Delta t}{2}}\right] = \frac{\Phi(f)}{\Delta t} e^{j2\pi f \frac{\Delta t}{2}} j\sin\left(2\pi f \frac{\Delta t}{2}\right)$$

If $\pi f \Delta t < 10°$ we have that $\sin\alpha = \alpha$ so we can cancel the sinusoid

$$\overset{!}{=} \frac{\Phi(f)}{\Delta t} e^{j2\pi f \frac{\Delta t}{2}} j\sin\left(2\pi f \frac{\Delta t}{2}\right) = \frac{\Phi(f)}{\Delta t} e^{j2\pi f \frac{\Delta t}{2}} j 2\pi f \frac{\Delta t}{2} = j2\pi f \Phi(f) \underbrace{e^{j2\pi f \frac{\Delta t}{2}}}_{\to \text{ small delay}}$$

$\pi f \Delta t < 10$ and this should be true for all the frequencies of the signal

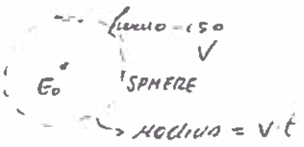
ATTENUATION

We use the acoustic wave equation to talk about attenuation

During propagation the amplitude decays with the distance

There are 3 phenomena produced with this decay:

- GEOMETRICAL SPREADING



Energy in one point: $E = \frac{E_0}{\underbrace{4\pi \, radius^2}_{SURFACE}} \div \frac{E_0}{(vt)^2} - \frac{E_0}{t^2} - \frac{E_0}{distance^2}$

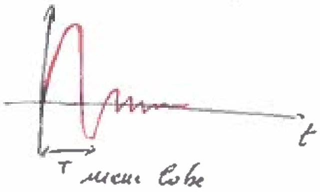
It decays very quickly with the distance. This is not losing energy, it's just distributing it over a bigger area

- BI-SCATTERING

In reality we have a lot of small eterogeneities, so the wave that is going in one direction is scattered in all directions

The total energy that arrives to the surface is lower and the scattered signals are like noise.

Small means small wrt the wavelet length



Wavelet length: T·V > dimension of heterogeneities

- ABSORPTION

A wave is a displacement of the particles wrt the original position

Due to FRICTION some elastic energy goes to heat (this cause in reality the medium is not fully elastic i.e rock)

The higher the frequency the higher the absorption and it's LINEAR relation

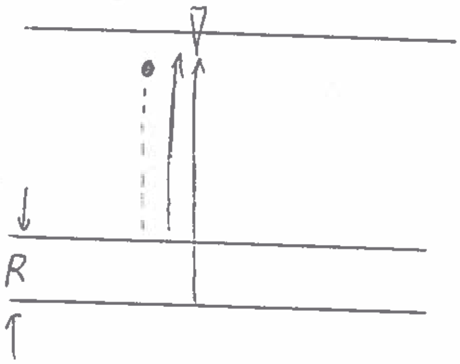
! This way we are changing the shape of the signal: $E_0 e^{-\alpha \, distance}$

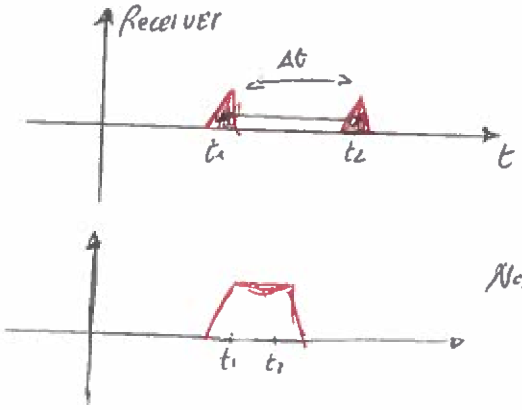
α = absorption coeff $\div$ $f/Q \to$ Quality factor of the rock

Question for exam: Please tell me why a wave during a propagation attenuates
↳ you have 5 phenomena: Geometrical, Biscott and absorption and the only one that is changing the shape is absorption due to multiplic. with exponential

There is another phenomena that is connected to absorption and it's <u>DISPERSION</u> for which the V changes with the frequency

We want to know the resolution we can have by reflections



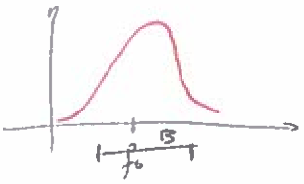


Not enough resolution

Resolution is ability to detect 2 targets → measuring distance between two targets so that you can distinguish the two targets

$\Delta t = \frac{2R}{V} \overset{\text{distance between 2 targets}}{>}$ duration of wavelet (main lobe duration T)

$T = \frac{1}{B}$ $\qquad \frac{2R}{V} > \frac{1}{2B}$



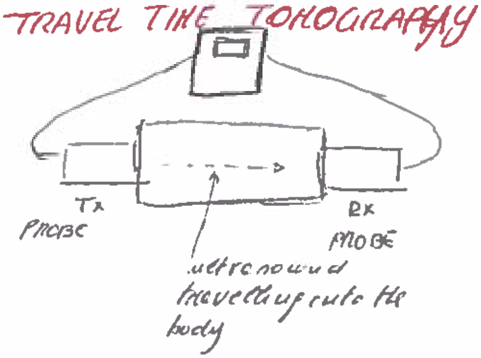
In many systems $B \simeq f_0$ central freq (eg radar

$$\boxed{R \cdot \frac{\lambda}{4}}$$

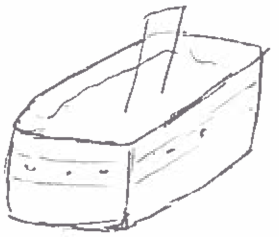
HAMMER HIT: 100 Hz = $B \simeq f_0$
↳ on soft soil
$V = 500\,m/s$
$\lambda_0 = \frac{V}{f_0} = 5m$ $\qquad R \simeq 1m$ $\qquad \nearrow V \quad \searrow R$

TRAVEL TIME TOMOGRAPHY

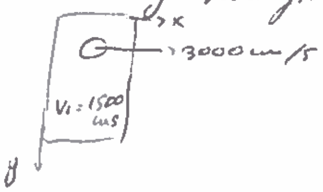
10/10/2017 ⑧



System to measure the travelling time of an acoustic wave.
If i know the distance and measure the time i can get the velocity



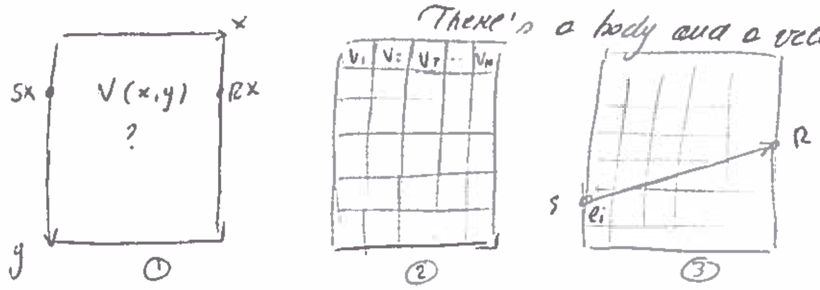
Plastic container. It's full of water and inside i put some anomaly. Seen from the top



Through the travel time i want to get the velocity

Problem of deriving the velocity

There's a body and a velocity field



1) N cells, N_i, N_R receivers $(x_R\ y_R)$, N_S sources $(x_S\ y_S)$

2) We approximate the trajectory from S to R as a straight line.

$\boxed{t_{K,m}} = \sum_{\text{along cells}} \longrightarrow \boxed{\sum_{i = \text{cells crossing the ray}} l_i / V_i}$ $\qquad \frac{1}{V} = s =$ Slowness $[S/m]$

l is the length of cell i

$= \sum_i s_i l_i \rightarrow$ linear

If we have 10 sources and 10 receivers we have 100 equations

$$t_{K,m} = \sum_i \frac{l_i}{V_i} = \sum_i \textcircled{s}\, l_i$$

$t_{K,m}$ ↑ measured

s ↑ UNKNOWN

l_i ↑ computed

So in our system of equations we are looking for s

0 = cells that are not crossed

NUMBER OF CELLS

$$\underbrace{\begin{bmatrix} t_{11} \\ \\ \\ \\ \\ \end{bmatrix}}_{T} = \underbrace{\begin{bmatrix} 0 & l_2 & l_3 & 0 & 0 & \cdots & l_N \\ & & & & & & \\ & & N_R\,(\text{rows}) & N\,(\text{columns}) & & & \\ & & & & & & \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_N \end{bmatrix}}_{S} \; N_{\text{rows}}$$

Each row of this matrix contains the history of the ray.

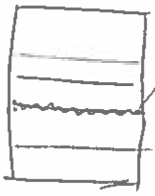
N T ~~[illegible]~~ $(N_R \cdot N_S \cdot 1)$ — dimension of vector T

$T = A \cdot S$

Compute A^{-1}, then $\Rightarrow A^{-1}T = A^{-1}AS = S$

We can have ~~two~~ some complications:

- It's complicated to calculate the inverse of a matrix
- If we have a lot of cells we can have many of them that are not crossed by any ray. A has a null column. This mean the solution is inconditional (in the inversion problem)
- For some reason we have two rows which are the same



In this case two rows are the same that wouldn't be a real problem with no noise

If we have big grids we operate with the average.

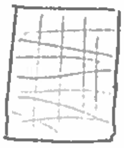
⟶ To fight against singularity we can also operate with the REGULARIZATION

I want to maximize the derivative of velocity versus x. We chose a smooth model with no big jumps

Suppose we have already computed one solution $s(x,y)$ but we don't have an initial choice but I have straight rays which I DON'T WANT

So what I can do is to iterate.

So from this first model I do a new ray tracing



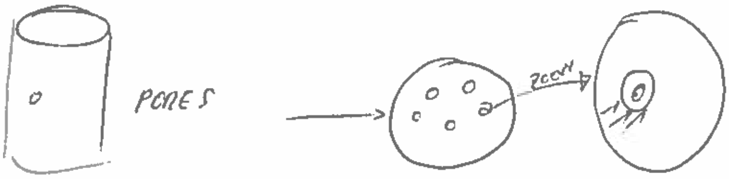
So we have curved rays and here we compute the values of s.

We do this up to the S_i of the m model

With ultra sounds we have one more problem: we are at very high frequency

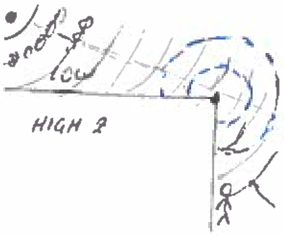
$$\text{Resolution} = \frac{\lambda_0}{4} = \frac{V}{f_0}\frac{1}{4}$$

In a body with pores

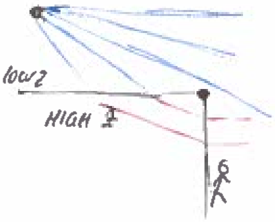


There is a scattering due to air bubbles

DEFINITION OF RAY WRT WAVE EQUATION



Can i hear the person speaking? Yes for the HUYGEN PRINCIPLE:
Each point on the waveform is the source of small spherical waveforms.
So the new waveform is the envelope of all this spherical waves
The point that is on the wedge is filling with spherical waveforms the space behind the corner



If we do ray tracing we have straight lines as long as we are on the same medium.
When we hit the high z zone we have the Snell law
Rays don't contain all the informations of the wave equation. We have some approximations.

WAVE EQUATION (ACOUSTIC): $V^2\nabla^2\Phi = \frac{d\Phi}{dt}$

We approximate the derivatives with finite difference; that is not an approximation if we properly chose $\Delta x, \Delta y, \Delta z, \Delta t$

If we pass to frequency domain we have $x,y,z,t \longrightarrow x,y,z,f$, $\Phi \Rightarrow \underline{\Phi}$, $\boxed{V^2\nabla^2\underline{\Phi} = -\omega^2\underline{\Phi}}$

Then we suppose there is a solution of the form $\boxed{\underline{\Phi}(x,y,z,f) = A(x,y,z,f)e^{-j\omega T(x,y,z)}}$

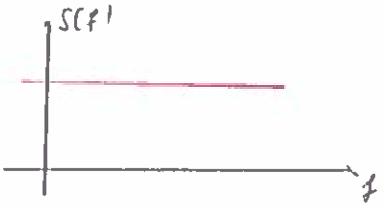
If we put this into the wave equation we obtain a form like $Re(\;) + Im(\;) = 0$

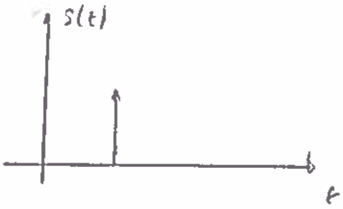
This implies $Re(\;) = 0$ and $Im(\;) = 0$

$Re(\;) \longrightarrow \frac{\nabla^2 A}{A\omega^2} - |\nabla^2 T| = \frac{1}{V^2}$ for $\omega \to \infty$ (The source signal has infinite values). From this approx we get the EIKONAL EQUATION: $\boxed{\nabla T = \frac{1}{V} = S}$ Ray equation that gives me the trajectories

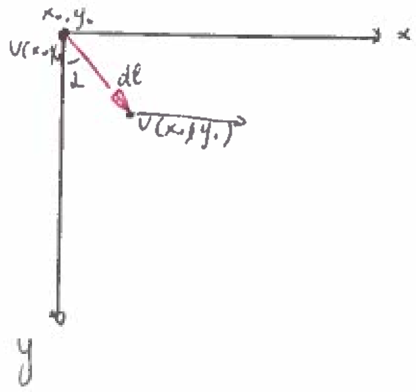
$Im(\;) \longrightarrow 2\nabla A \cdot \nabla T + A\nabla^2 T = 0$ ↳ TRANSPORT EQUATION

What does it mean that a source has an infinite bandwidth?





Impulse. It has a resolution equal to zero



The vector is travelling with velocity $v(x_0,y_0)$
Then for the movement we have to derive the position

$$\boxed{dl < \begin{matrix} dx = dl\sin\alpha \\ dy = dl\cos\alpha \end{matrix} \qquad d\tau = \frac{dl}{V} \qquad dp_x = S_x dl \qquad dp_y = S_y dl}$$

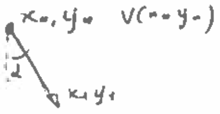
V_x, V_y derivatives

Locally we have the snell law

$P_x = \frac{\sin\alpha}{V}$ $P_y = \frac{\cos\alpha}{V}$ Where α is the direction of the ray

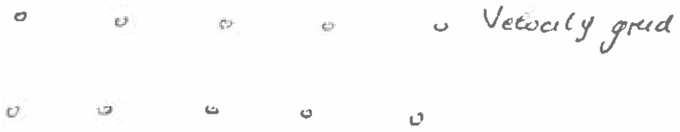
$P_x^2 + P_y^2 = \frac{1}{V^2} = S^2$ So S_x is the derivative of the slowness $S_x = \frac{dS}{dx}$

In this equations we can trace rayes

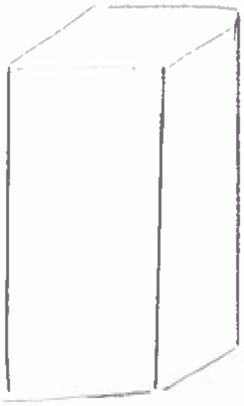


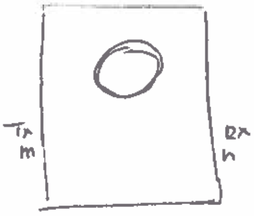
x_0, y_0 : Starting point

$$\begin{cases} x_1 = x_0 + dx = x_0 + dl \sin\alpha \\ y_1 = y_0 + dy = y_0 + dl \cos\alpha \\ T_1 = T_0 + \dfrac{dl}{V(x_0 y_0)} \\ P_{x_1} = P_{x_0} + dP_x = S_x(x_0 y_0) dl + P_x \\ P_{y_1} = P_{y_0} + dP_y = S_y(x_0 y_0) dl + P_y \end{cases}$$



TRAVEL TIME TOMOGRAPHY



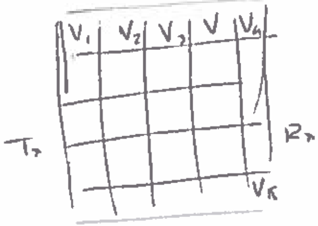


We are simulating a portion of column with an anomaly

We are sending an elastic vibration from Tx to Rx

m and n are the positions of Tx and Rx

t_{mn} travelling time. I will get a collection of travelling time



K cells

V_i?

$$t_{mn} = \sum_{\substack{\text{All the cells}\\ \text{crossed by the}\\ \text{ray}}} \frac{l_i}{V_i} = \sum_i l_i \cdot S_i \quad \text{where} \quad S_i = \frac{1}{V}\ [s/m]$$

$$\begin{bmatrix} t_{11} \\ \\ t_{mn} \\ T \\ \\ \end{bmatrix}_{N_{Rx}\cdot N_{Tx}} = \begin{bmatrix} \overset{\text{FIRST RAY}}{l_1 \cdots l_s \cdots 0 \; 0 \cdots} \\ 0 \\ 0 \\ 0 \\ A \quad \vdots \\ 0 \end{bmatrix} \overset{(K\times 1)}{\begin{bmatrix} \\ \\ S_i \\ \\ \\ \end{bmatrix}}$$

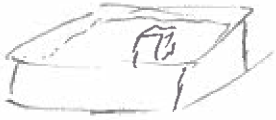
$$T = A \cdot S \rightarrow A^{-1}T = A^{-1}AS \rightarrow \boxed{S = A^{-1}T}$$

In many cases this relation doesn't work eg a cell isn't covered by any ray => no ray
no solution

If you are not measuring the first arrival, the threshold is triggered by some other sgn.

See experiment min 30 youtube

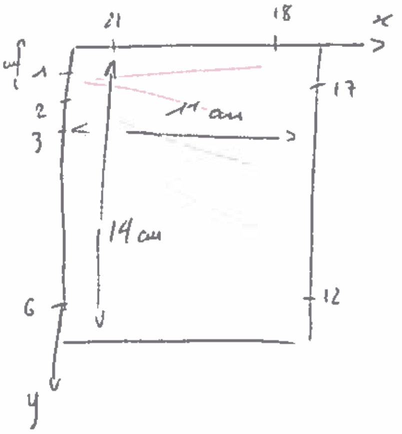
Box of water with some anomaly inside
it produced strange results.



TIME TOMOGRAPHY

Use this time hot glue with an anomaly inside
We map the velocity of the body by measuring the travelling time from all the points to all the points

See experiment on youtube



RECEIVER

SOURCE	12	13	14	15	16	17
1						
2	104	94.7	92.1	80	71.5	70.3
3	75.2	100	77.9	73	66.6	69.3
4	86	91	72.2	66.8	65.1	66.4
5	81.8	78.8	66.1	69.9	66	76.7
6	74	71	74.1	74.3	79.2	82.5
18	74	72	75.2	75.5	85.5	85
19						
20						
21						

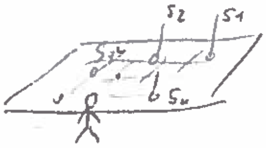
μs

Wax : $\frac{3.8 cm}{19.8 \mu s} = 1910 m/s$

Hot glue : $\frac{4.6 cm}{29.1 \mu s} = 1580 m/s$

20/10/2017

AUTOLOCALIZATION



One at a time produce a sound and at the end there will be autolocalization.
So there is a self positioning procedure with geophones
SOS TEAM is the name of this procedure

Suppose we have a pipe with an anomaly, and we try within the echo to localize the anomaly

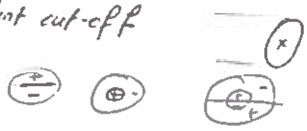
This is a guided wave (acoustic) while in the free space we'd have spikes
There is no geometrical divergency and no MIE SCATTERING. There is low absorption.

So the responsible of the attenuation is the air and the shell
Within the pipe each piece of air is producing pressure. 
There is also some friction that happens due to the interaction with the pipe shell

Same pressure in different areas. I have more energy in the second
So the attenuation factor $e^{-\alpha distance}$ → α becomes proportional to the $\frac{1}{distance}$ (radius)

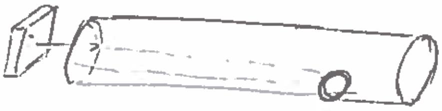
For radius 5cm → 100 m
50cm → 10 m

For the mode (0,0) → $0 < f <$ first cut-off
mode (0,1), (1,0) ...



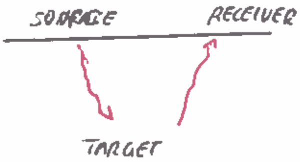
A lot of different modes

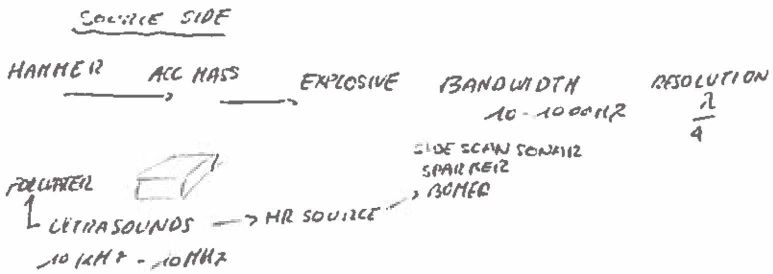
Let's suppose we have a speaker



We will generate a CHIRP (sinusoid $\sin(2\pi f(t)t)$) 
So we produce a chirp $s(t) \rightarrow S(f)$
We do $S(f) \cdot S^*(f)$

We have what produce vibrations and what records vibrations





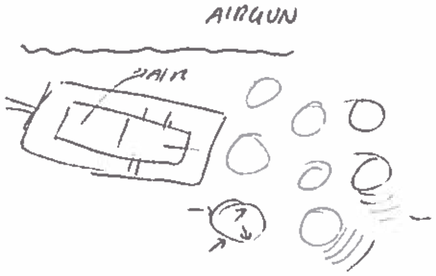
Due to the bandwidth all the sources ~~are bandlimited~~ have a limited resolution

When you put the wave in the medium you can measure the time

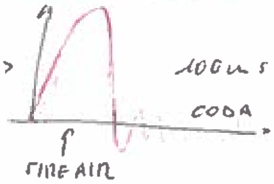
To have a higher resolution we need a different source. We can use ultrasounds for example but it's not working in all the mediums.

SOURCE SIDE

For marine environment we use **AIR GUN** that in practice find air under the water



The bubbles are getting bigger and smaller producing this way a series of P-waves. So the AIRGUN causes the socalled **BUBBLE EFFECT**

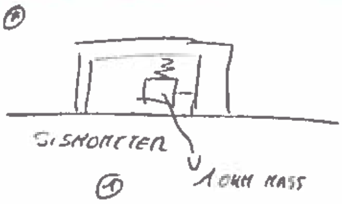


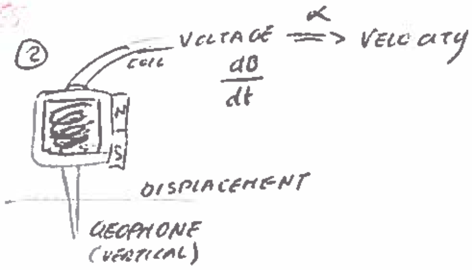
The coda is an effect we don't want

To solve the problem we can find more then one airgun at a time (GI AIRGUN)

RECEIVER SIDE

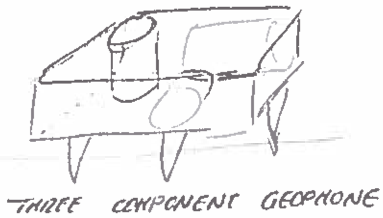
GEOPHONES





VOLTAGE $\overset{\propto}{\Longrightarrow}$ VELOCITY

$\frac{dB}{dt}$



We are using something similar to a sismometer (1st pic). The mass is oscillating for inertia and writes on a paper

We use something similar 2. We have something that can oscillate, a moving mass suspended within a magnet, then we have something we can connect to earth, and the coils that have connectors to external world. Coil moving (magnet moving) produce voltage. The displaceme is producing magnetic field that variating in time produce a voltage that is proportional to velocity. This is called **GEOPHONE**. Due to directivity this is sensitive only to vertical movements so it's the vertical geophone. For the horizontal field we have to tilt the same system.

For both we need a three compouent geophone.

Geophones are cheap and not sensitive to acceleration

* ACCELEROMETER

More sensitive for small variations

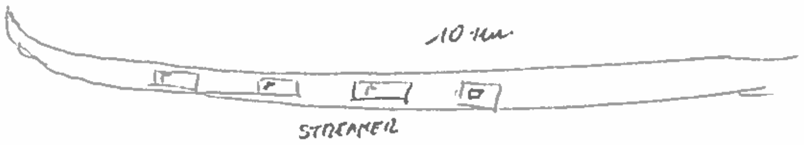
The mass gives the inertia to the system => The bigger the mass the more it's sensitive to low freq.

It produces a voltage $\propto$ to pressure => $\frac{Force}{Air} = \frac{m\,\boxed{a}}{Air}$

A geophone can catch both small and high amplitudes

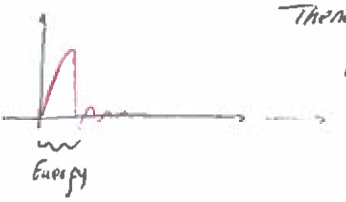
* HYDROPHONE

Piece of plastic that has elastic properties similar to water
This pipes are made in a plastic pipe called STREAMER filled with oil



So hydrophones measure the pressure in the water → The pipe must be in the water

We want to dimension another source.



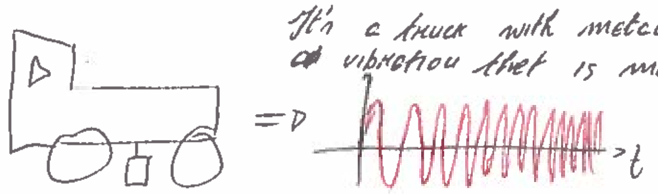
There are impulsive sources when the main energy is in the main lobe

$E = \int |s(t)|^2 dt$ and we can increase it by increasing time ~~and~~ or amplitude (chirp)

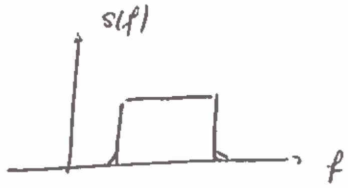
→ SPECTRUM OF THE HAMMER.

The idea is to use a long duration signal that doesn't produce damages to buildings. This signal can be a chirp and we can produce something like this with a <u>VIBROSEIS</u>

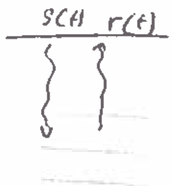
It's a truck with metal foot pressing against earth and it produces a ~~a~~ vibration that is moving like a chirp



The signal is very long and the chirp can go from 10 Hz to 100 Hz



The difference with the Hammer spectrum is in the phase. Hammer is minimum phase while here is MIXED PHASE



So what we do is to send the signal and receive all the signals + echoes back:

$r(t) = \sum_i A_i s(t-\tau_i)$ with τ delay and A amplitude

$R(f) = \sum_i A_i S(f) e^{-j\omega\tau}$

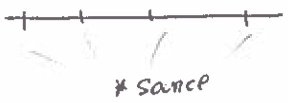
We can do this: → Run the signal on the computer and transform it in the equivalent short duration ⟹ DECONVOLUTION (~~minimum phase~~ wave replacement,

$r(t) \rightarrow \frac{R(f)}{S(f)} \overset{\nearrow \text{Hammer}}{\bar{S}(f)} \Longrightarrow \sum_i A_i \bar{S}(f) e^{-j2\pi f\tau_i}$ ⟶ Minimum phase deconvolution (or wave replacement) because we are passing to a minimum phase signal

↑ This way we have the response of the subspace and we can compute a hypothetical feedback we'd have with an explosion

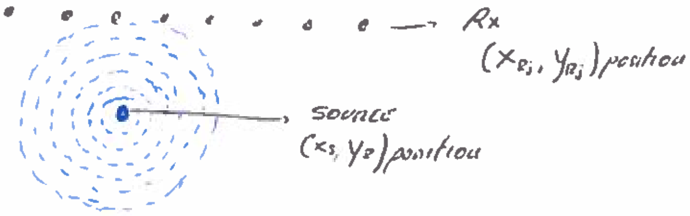
Next time → How to detect the position of a source

Two different ways:
- processing time of arrival (TOA) or differential TOA
- detecting the energy



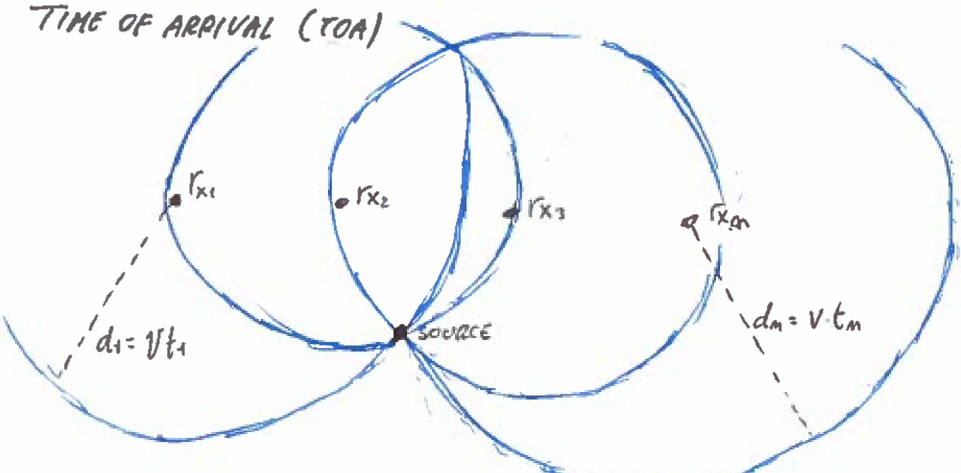
3/11/2017 (15)

Seismic source localization → Application of seismic "sensing". A source can be airgun, dynamite.
h_p = known and constant velocity.



We will see three different methods to find the position (x_s, y_s) of the source

TIME OF ARRIVAL (TOA)



We know the exact moment in which the sound was emitted by the source.
Knowing that the velocity v is constant and knowing the time of arrival to each receiver we can track several circumferences of ray $d_i = vt_i$
L'intersezione di tutte le circonferences will correspond to the source position.

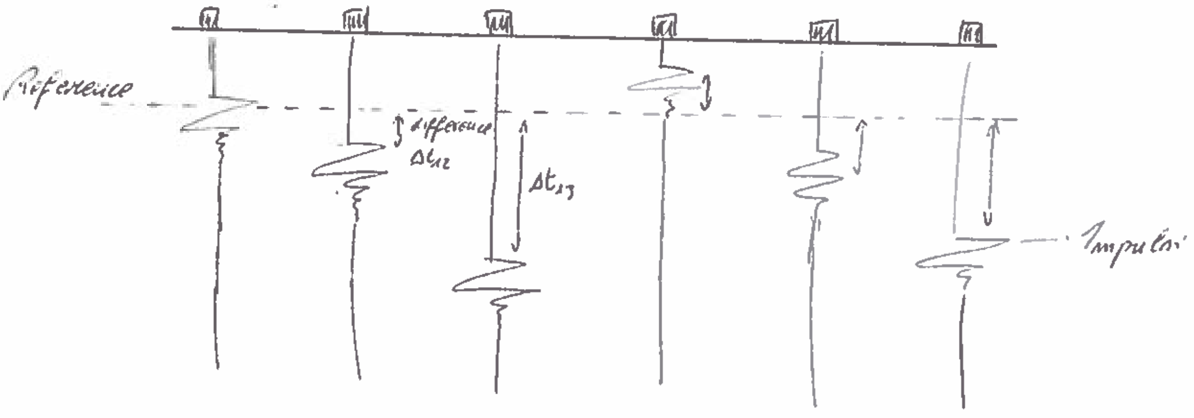
Receivers

$$\left\{ \sqrt{(x_{R_j} - x_s)^2 + (y_{R_j} - y_s)^2} \cdot \frac{1}{V} = t_j \right. \quad \vdots \quad n$$

→ circumference equation
we will have n equations corresponding to the number of receivers
unknown (x_s, y_s)

DIFFERENTIAL TIME OF ARRIVAL (DTOA)

The instant of emission of the signal is unknown. So DTOA is the difference between the time of arrival detected (taken as reference time) and the other times of arrival



$$\left\{ \frac{\sqrt{(x_j - x_s)^2 + (y_j - y_s)^2}}{v} - \frac{\sqrt{(x_i - x_s)^2 + (y_i - y_s)^2}}{v} = \Delta t_j \right.$$

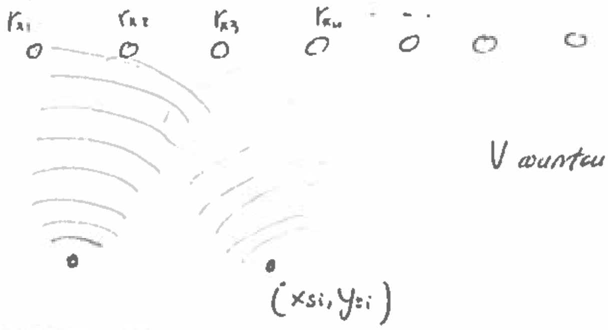
n-1 linear equations where the first one doesn't add any extra information

TDOA
TOA → They work well when we have one source at a time (one signal at a time)

Let's suppose we have a continuous source (not impulsive). We can correlate the signals

$S_1 * S_2$ → max in t_0
$S_1 * S_2$ → max in $t_2 - t_1$

MANY CONTINUOUS SOURCES



V constant

Robust performance in advers acoustic environments. The algorithm can be interpreted as a beamforming-based approach that for the candidate position that maximizes the output.

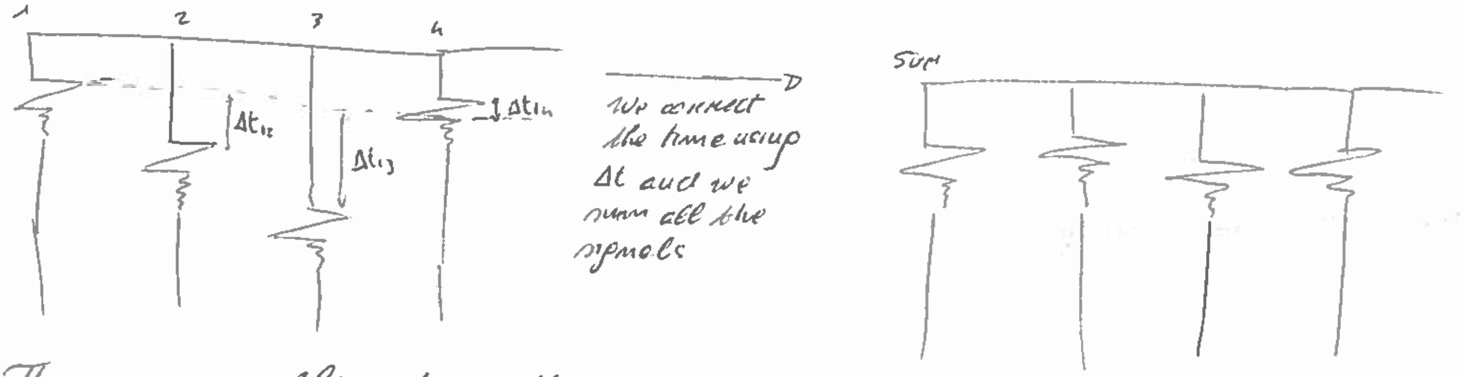
I rotate the receiver in such a way to find the maximum received power.

$S_j(t)$ signal in a certain position

S_1 reference signal $\longrightarrow$ $\Delta t_{12}, \Delta t_{13}, \Delta t_{14}$

$$\Delta t_{12} = \frac{d_1}{V} - \frac{d_2}{V}$$

We record up



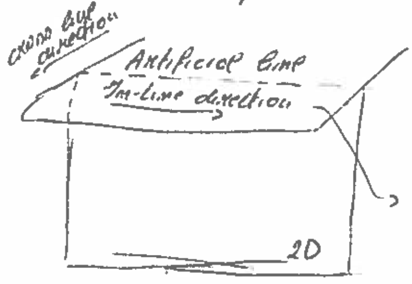
We correct the time using Δt and we sum all the signals

This way we amplify N times the same signal

We obtain a <u>SUPERTRACE</u> that contains all the signals and different receivers

2D SEISMIC

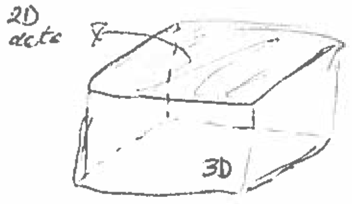
If we have one receiver and one source, it becomes very hard to find the right position of the source. Seismic acquisition is made by multiple sources and receivers.



2D seismic → Data collected on a line → Image on the vertical slice below the line

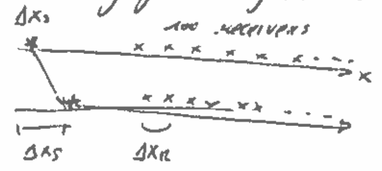
This works if also the subsurface is more or less 2D

→ Constant features

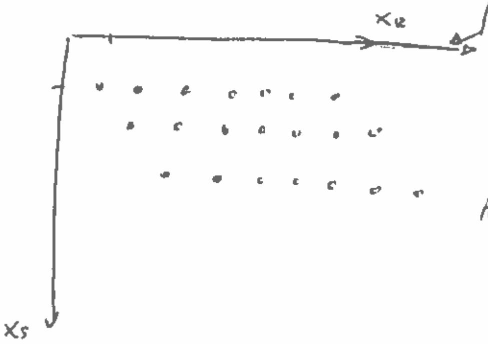


If i want the full volume of the subsurface (3-D seismic)

Normally for every source we have hundreds of receivers. When i move the source i also move the receiver

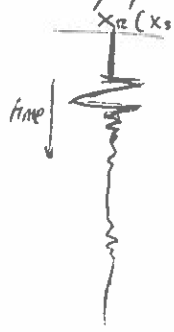


STACKING CHART Defines the position of the sources and the receiver



We move step by step the first receiver to the last position

Every point (receiver) is a seismic trace



$X_R(X_s)$ → position of a receiver wrt a specific source

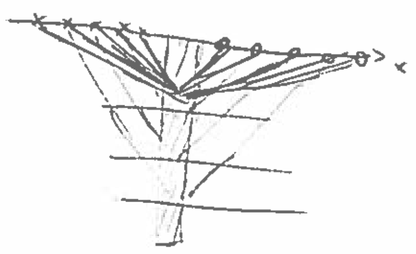
If we take all the data along one horizontal line it's called **COMMON SHOT GATHER**

If we take all the data along a vertical line it's called **COMMON RECEIVER GATHER**

If we take those on the diagonal we get the **COMMON OFFSET GATHER**

Offset is the distance between shot and receiver $x_R - x_s$

The data on the anti-diagonal are **COMMON MID-POINT GATHER**



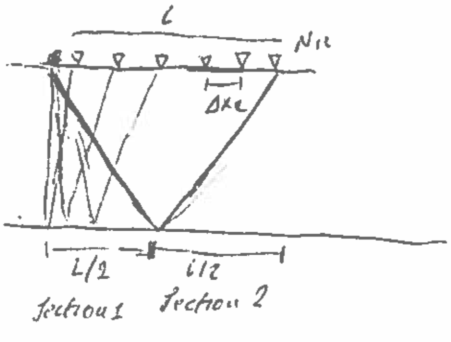
Common mid-point

If layers are flat horizontal, all this traces are giving information of the same reflection. So the common mid-point is also the common depth point ⇒ Multi Coverage (more traces with diff. angles)

We define the **COVERAGE** as the number of traces CMP gather

100% · NR = % coverage

Suppose flat and horizontal earth



For an extension of L, we are illuminating a portion of the subsurface long L/2

If we move everything by $\Delta X_s = L/2$ we take a picture of section 2

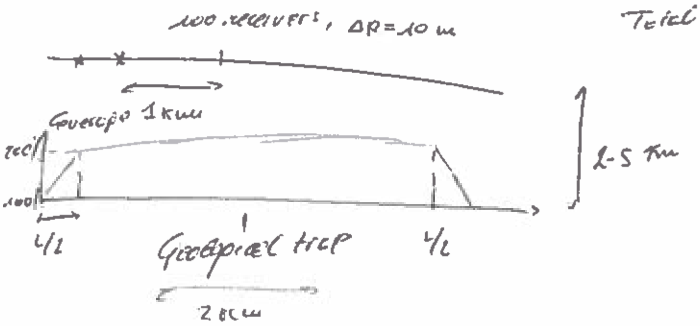
If $\Delta X_s = L/2$ → CMP = 1 TRACE ⇒ Simple coverage (100%)

If $\Delta X_s = \Delta X_R = \frac{L}{N_R}$ → Coverage $= \frac{N_R}{2}$

How to chose Δx_R?

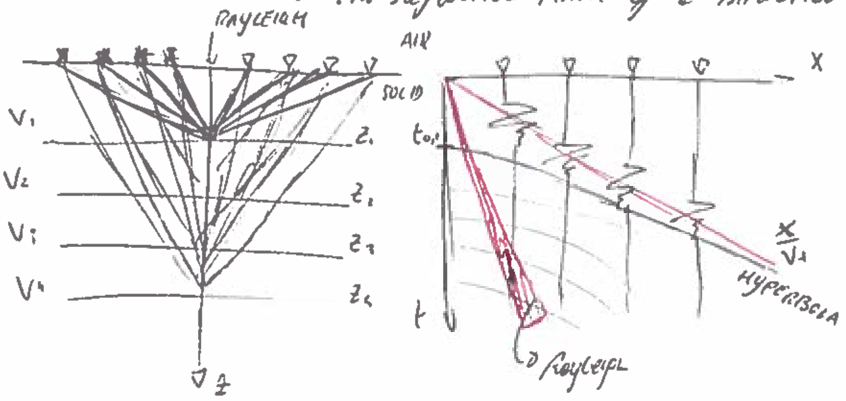
$\Delta X_R \simeq$ wavelet length

Total line = 10 km → 100 shots → 100'000 traces.



In the common mid-point gather we see the same point many times → this is helpful

We are interested to the reflection time of a situation like this:



$$\boxed{t_{direct} = \frac{x}{V_1}}$$

$$\boxed{t_{reflected,1}} = \frac{2}{V_1}\sqrt{z_1^2 + \left(\frac{x}{2}\right)^2} = \frac{1}{V_1}\sqrt{(2z_1)^2 + x^2}$$

$$= \sqrt{\frac{(2z_1)^2}{V_1^2} + \frac{x^2}{V_1^2}}$$

vertical time of line

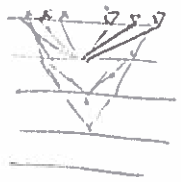
$$\boxed{t_{reflected,1} = \sqrt{t_{0,1}^2 + \left(\frac{x}{V_1}\right)^2}}$$

I want to check if there's a law describing the continuity of the reflection of the same point on different receivers

We see that $\lim_{x \to \infty} t_{R,1}(x) = \frac{x}{V_1}$

We have Rayleigh waves at AIR-SOLID interface. They travel really slowly

We have to consider also the reflections given by the other layers



$$t_{reflected,j}(x) = \sqrt{t_{0,j}^2 + \left(\frac{x^2}{V_{RMS,j}^2}\right)}$$

↳ weighted average of V_i with $i = 1 \div j$

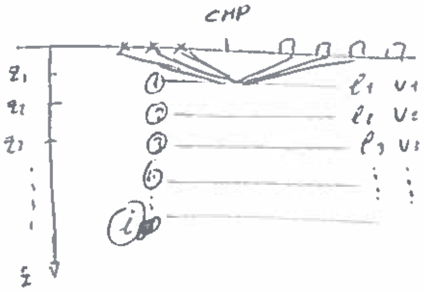
We still have hyperbolas

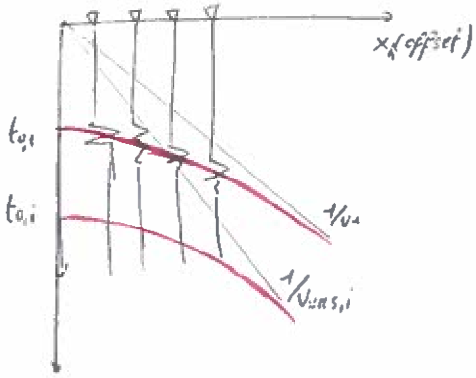
13/11/2017

REFLECTION SEISMIC

Seismic waves to have an image of the subsurface

2D Aquisition → multicoverage data.





$x_h = x_R - x_S$ = offset

$$\boxed{t_1(x_h) = \sqrt{t_{0,1}^2 + \left(\frac{x_h}{V_1}\right)^2}} \to \text{hyperbola}$$

Where $t_{0,1}$ is the two way travel time to the interface

$$t_i(x_h) = \sqrt{t_{0,i}^2 + \left(\frac{x_h}{V_{RMS,i}}\right)^2}$$

$V_{RMS,i}$ → mean of all the velocities up to the layer i

$$V_{RMS,i} = \sqrt{\frac{\sum_{k=1..i} V_k^2 \cdot T_k}{\sum_{k=1..i} T_k}}$$

T_k → two way travel time within the layer k

More or less the hyperbola are converging to Vrms. From the shape of the hyperbola we can get info about the velocity → This way we get the depth

It's very complicate to detect hyperbola very close to each other.

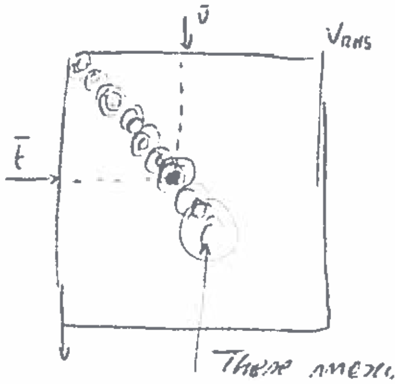
From $V_{RMS,i}$ we can get the DIX FORMULA but it's really unstable (too much sensitive to error)
↳ $V_i = f(V_{RMS,i})$

We need to recognize hyperbola possibly in an automatic way

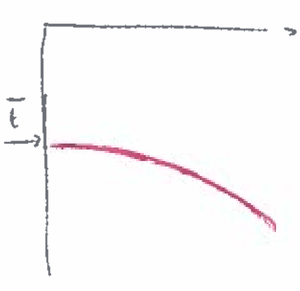
SEMBIANCE GATHER (AUTOMATIC HYPERBOLA DETECTOR)

We need two parameters:

$t(x) = \sqrt{\bar{t}^2 + \left(\frac{x}{\bar{v}}\right)^2}$ $\bar{t}, \bar{v}$ → two numbers that define our hyperbola



These maxima are the hyperbola



$$S(\bar{t},\bar{v}) = \left|\sum^{\text{along hyperbola } t} d(x,t)\right|^2 \Big/ \sum_{\text{along hyperbola}} |d(x,t)|^2$$

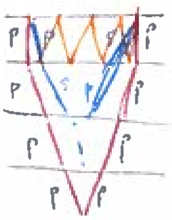
We are testing if this hyperbola exists in the data

We will have a big value if it's following a true hyperbola

$0 \le S \le 1$

In image processing it's called HALF INTEGRAL

We can have more hyperbola at the same time:



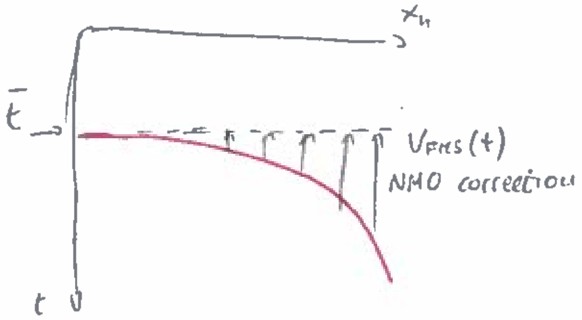
- direct p arrival
- More conversions (converted arrivels)
- Multiples

We have to chose which one is the Vrms we are looking for: we chose the HIGHEST Vrms for each t to get the true Vrms(t) of direct p arrivels (fastest)

At the end of this process we have done the VELOCITY ANALYSIS

① Multi coverage acquisition
② Select one CMP (one position along x)
③ Compute semblance (random transform)
④ On semblance follow the maxima and highest Vrms
⑤ ⇓ $V_{rms}(t)$ VELOCITY ANALYSIS

We are trying to sum all the true hyperbolas → we make the hyperboles flat and sum horizontally: **NORMAL MOVEOUT CORRECTION**

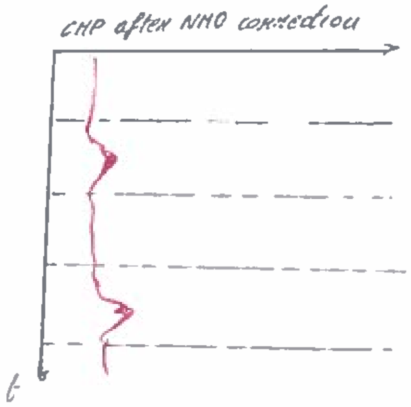


$$t \rightarrow \boxed{\sqrt{\bar{t}^2+\left(\frac{x_h}{V_{rms}(t)}\right)^2} - \bar{t} = t_{NMO}(x_h,\bar{t})}$$

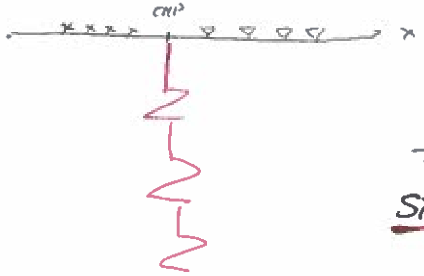
For every time we draw the hyperbole and move up in the flat version

For high offsets we see wavelet distortion.

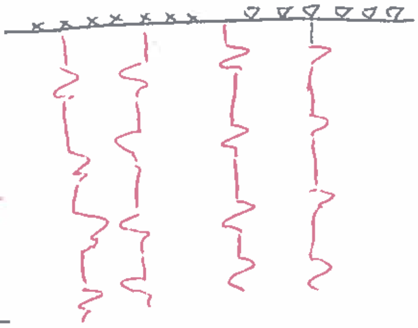
So in the graphic we will see the CMP NMO correction



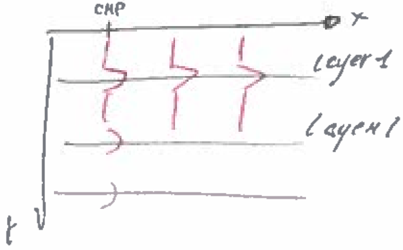
The operation of summation along the x axis is called STACK OF CMP along x_h. The image of the subsurface that we get is:



We get for all the CMPs ⟹ **STACK GATHER**

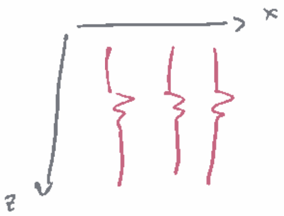


What we get is that the interface will be the wavelet and duration ÷ $\frac{1}{\text{Bandwidth}}$



The duration is proportional to the resolution

From this we define the time to depth to get to the DEPTH STACK GATHER



19/11/19 (18)
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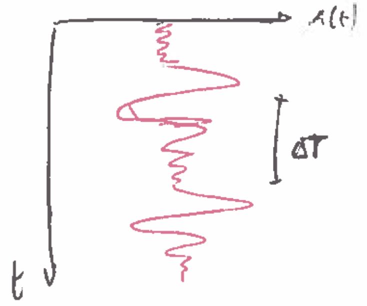
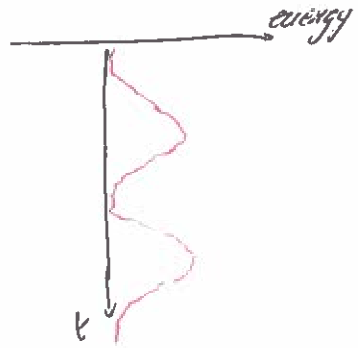
REFLECTION SEISMIC PROCESSING

- Demultiplexing: re-order data after acquisition
- Golgrip: noise trace cancelling (mute) on "reverse geophone" traces
- Gain recovery: amplification of high attenuated parts of traces

$$\underset{\text{Attenuation}}{A} \propto \frac{K_1}{t} e^{-K_2 t} \longrightarrow \underset{\text{recover}}{r(t)} \simeq r(t) \frac{1}{K_1} t e^{K \cdot t}$$

or AUTOMATIC GAIN RECOVERY (AGR)





$$\underset{\text{recover}}{r(t)} = r(t) \frac{1}{\sqrt{\int_{t-\frac{\Delta T}{2}}^{t+\frac{\Delta T}{2}} |r(t)|^2 dt}}$$

This way i obtain traces without big jumps in amplitude

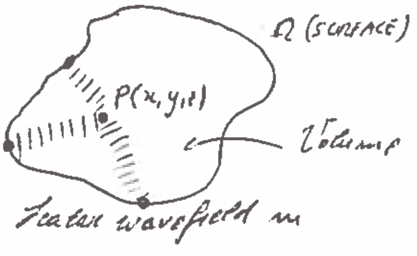
$$\Delta T = 2/3 \cdot \underset{\substack{\text{wavelet}\\\text{duration}}}{T}$$ ⟶ If it's too small we have signal distortion. If it's too big we do no recovery

- Geometry inputs: informations on the geometry of the system
- Application of statics
- Deconvolution
- Filtering: Given a source we have to chose the right geophone and the right sampling freq ($\geq 2 f_{max}$) filtering the noise outto the bandwidth
- Trace equalization
- Real time tomography: use travel time to get velocity model of an object

SEISMIC ACQUISITION

- Pre-processing
- CMP Sorting
- VEL Analysis ⟶ pre stack (data + velocity)
- NMO correction
- Stack ⟶ artifacts (CMP is not locally horizontal)

KIRCHÖFF INTEGRAL

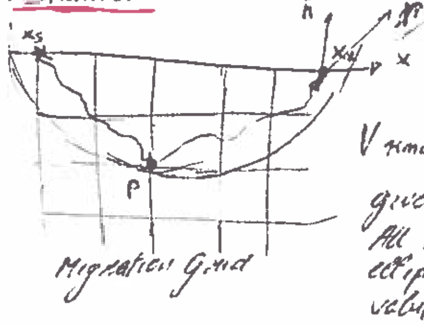


If you know $m(\Omega)$ you can derive m in any position of the volume

$$m(x,y,z) = \int_{\Omega} m_{\Omega} \underbrace{G(\Omega,P)}_{\text{Green function}}\, d\Omega$$

Green function (it describes the propagation between two points)

MIGRATION



V known

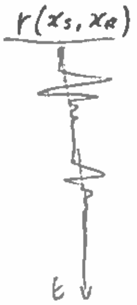
given $r(x_r, x_s)$
All the points on the ellipse will have the same value

Hp: we consider a close surface with respect to the volume of interest.

For each point P on the grid, trace a ray from x_s to x_r and calculate the total travel time $\bar{t}$

$$\bar{t} = t_{x_s \to p} + t_{p \to x_r}$$

The couple (x_S, x_R) is related to a certain trace $r(x_S, x_R)$. For each point, assign to it the value of $r(x_S, x_R)$ given t:



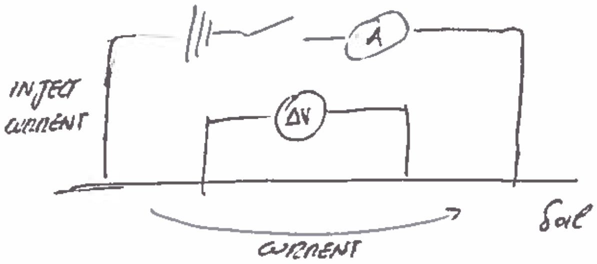
Repeat this process for each trace. Each point will have a vector of values, each one corresponding to one trace; summing this values and coloring the point wrt the result we'll obtain the image of the subsurface

We can also find the result by using the Kirchhof integral

$$m(P) = \int_{\omega} \int_{\text{All Traces}} \underbrace{\underset{\downarrow \ \text{Attenuation } x_S \to P}{A_S}\, A_R \cdot e^{j\omega(t_{x_S P} + t_{P x_R})}}_{\text{Green}} \ \underbrace{r(x_S, x_R)}_{\text{Recorded Field}} \cdot \underbrace{\vec{n} \times \nabla t_P\, x_R \, dx_R}_{\cos\vartheta}$$

21/11/2017

ELECTRICAL METHOD



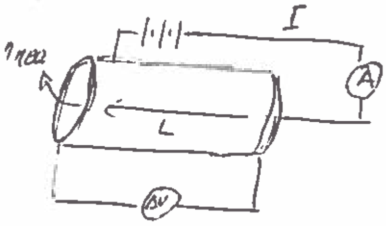
Current → moving charges
Metal → electronic conduction

① ELECTRONIC CONDUCTION

Ions → charged atoms / charged molecules
Soil → contains a lot of salt
"The ions are transmitted to the soil and they "close" the circuit

② ELECTROLYTIC CONDUTION

Ohm law: $R = \frac{\Delta V}{I}$



High resistivity: the current has difficulty to ~~the~~ flow due to the presence of few ions

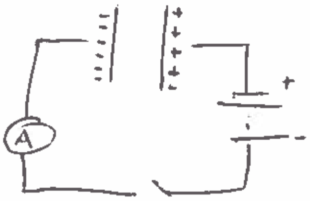
$R \div \frac{L}{Area}$

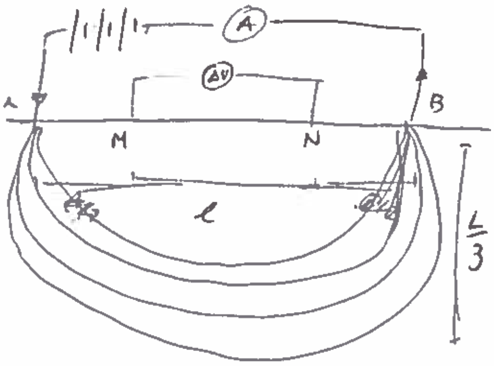
We introduce the resistivity ρ $\boxed{R = \rho \frac{L}{AREA}}$

↓ It defines how hard or easy is for the current to flow into a body

$\rho = R \frac{AREA}{L} \, [\Omega \cdot m]$

Metal resistivity: $10^{-12} \div 10^{-5}$ Ω m
Electrolytic conduction $10^{-2} \div 10^{7}$ Ω·m
Polarization conduction → insulator $\rho \sim 10^{10}$





A, B current electrodes → one injects current, the other pulls it out

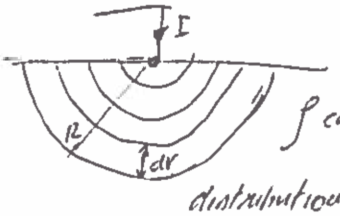
M, N measure a potential

Bigger ℓ, bigger depth

$\rho(x)$ constant separator transverse (used to visualize the lateral variation of resistivity)

$\rho(z)$ Vertical electrical sounding (estimation of electrical conductivity or resistivity of the medium)

$\rho(x,z)$ → PSEUDOSECTION

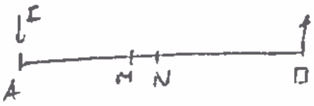


ρ constant $\frac{dV}{dr} = -\rho \bar{J}$

distribution of the current

$$E = \rho \bar{J}$$

$$V(r) = \int_0^r -\rho \frac{I}{2\pi r^2} dr \qquad V(r) = \rho \frac{I}{2\pi r}$$



$$V_M = \rho \frac{I}{2\pi} \left(\frac{1}{AM} - \frac{1}{BM} \right) \qquad V_N = \rho \frac{I}{2\pi} \left(\frac{1}{AN} - \frac{1}{NB} \right)$$

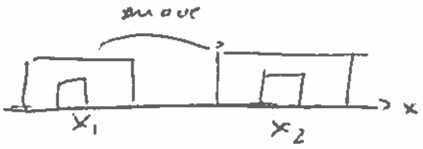
↳ distance between A and M

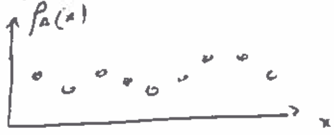
$$\Delta V = V_M - V_N = \rho \frac{I}{2\pi} \left(\frac{1}{AM} - \frac{1}{BM} - \frac{1}{AN} + \frac{1}{NB} \right)$$ → geometric factor K^{-1}

$$\boxed{\rho = \frac{\Delta V}{I} K \ (\Omega \cdot m)}$$ For a constant resistivity

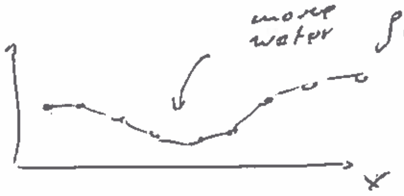
The one we obtain in the real experiments is the apparent resistivity ρ_a

① CONSTANT SEPARATION TRAVERSE (CST)



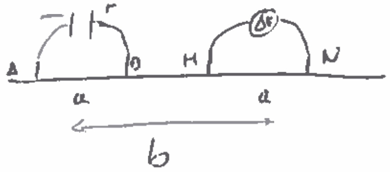


eg

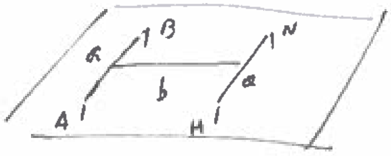


I move everything on left and right to obtain the resistivity variation along x

DIPOLE - DIPOLE

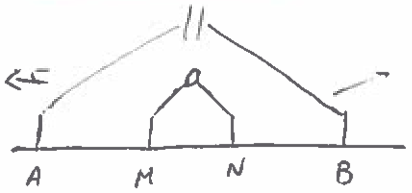


TRANSVERSE DIPOLE DIPOLE



We can get different sensitivities by changing the geometries:

② VERTICAL ELECTRICAL SOUNDING (VES)

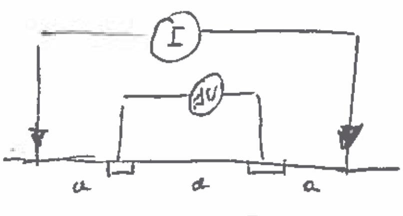


Here distance between A and B to go deeper

Two common arrays are used for VES:

- Wenner
- Schlumberger

↳ WENNER ARRAY



$K = 2a\pi$

For sounding measurements the electrodes in a Wenner array are expanded around a center point by equally increasing the spacing

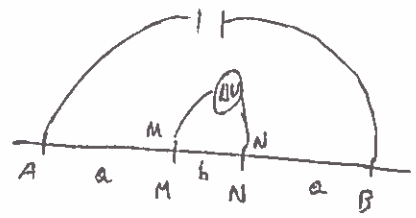
Vantages:

- high SNR
- good resolution of horizontal layers
- good depth sensitivity

Disadvantages:

- not good in determining the lateral location of deep inhomogeneities since the large a-spacing degrades lateral resolution

↳ SCHLUMBERGER



$a >> b$

$K \simeq \pi \frac{a}{b}$

Vantages:
- High SNR
- Good resolution of horizontal layers
- Good depth sensitivity
- Faster than Wenner

③ **PSEUDO-SECTION**

Obtained by VES/CST
Apparent resistivity obtained by measuring

The inverse problem is ill conditioned. We can enrich the infos by other methods

LIMITS OF ELECTRICAL METHOD

① Depth of sensitivity $\boxed{z = \frac{L}{3}}$

② Need to close the circuit
It doesn't work in dry sand, ice, hard media (no possibility to plant electrodes)

ELECTRICAL RESISTIVITY TOMOGRAPHY

ARCHIE'S LAW

It relates the electrical conductivity of a sedimentary rock with its porosity

$$C_t = \frac{1}{a} C_w \phi^m S_w^n$$

C_w electrical conductivity ϕ: porosity
C_t electrical conductivity of fluid saturated rock

w: water saturation   m: cementation exponent of the rock   n: saturation exponent
a: tortuosity factor

sistivity in solution with salt changes with temperature and concentration

$$R_t = a \phi^{-m} S_w^{-n} R_w$$

↑ luid saturated
ck resistivity

↑ Brine resistivity

a, m, n = constant depend on the location

$0,5 < a < 2,5$
$1,3 < m < 2,5$
$n \simeq 2$

to use the formula in practice to get the porosity

$$\rho = a \phi^{-m} S_w^{-n} \rho_w$$

↑ Ricavato tramite inverso

↑ porosity

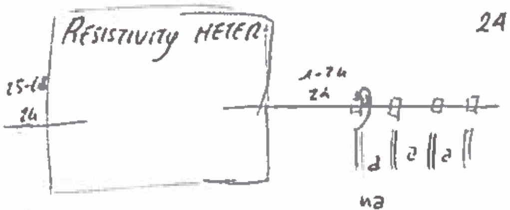
↑ to be measured

MISE A LA MASSE

Electrical no resistivity method that has been used in the mining industry since 1926 for delineating conductive subsurface or bodies. By injecting current into a conductive underground body it gets

Moving Ⓥ we can measure how big is the body

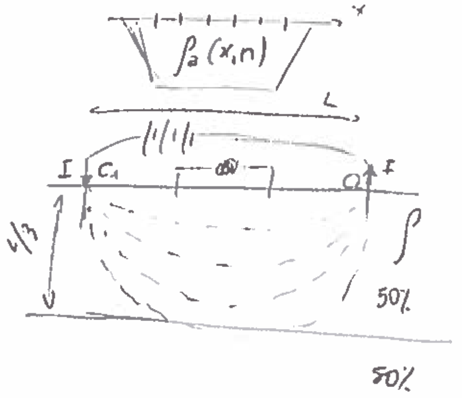
ELECTRICAL METHOD



24 + 48 cables

System choosing 5 electrodes at a time that measure ΔV, ρ_a etc
We'll have many measurements for every 4 electrodes

PSEUDO SECTION



We call apparent resistivity since we get an average of the anomalies

We can measure how deep is the area

If we go $\frac{L}{3}$ deep, we have 50% of the current flowing in the first $\frac{L}{3}$ part.

We are sensitive only to this first part

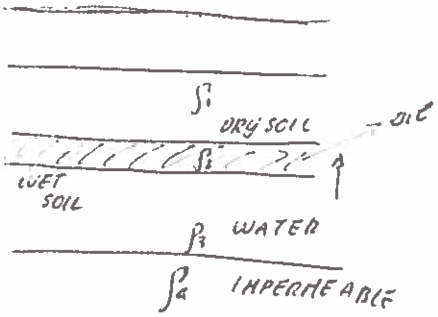
L : distance between C₁ and C₂ ⟶ depth = $\frac{L}{3}$

There are a lot of currents flowing and bodies producing parasitic currents ⟹ **NOISE**
To limit the noise we have to limit the distance

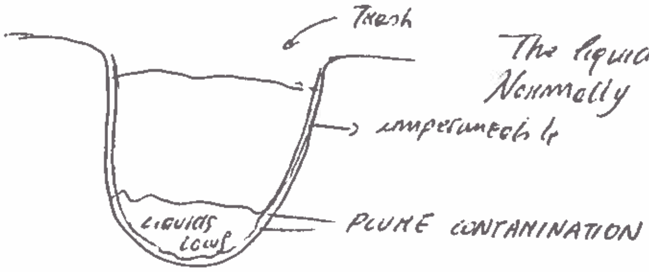
$$\boxed{\begin{array}{l} 0 < L < 1000\,m \\ 0 < depth < 300\,m \end{array}}$$

The resistivity is associated to the presence of **VOIDS** (tomb, cavern, tunnel), **WATER** (dissolves salts),
low ρ anomaly
AQUIFER
SALT WATER (is more conductive so lower resistivity), **DISTILLED WATER** (highly resistive)
APPLICATIONS: looking for water, quality of water, oil, hydrocarbures
⟶ **CONTAMINANTS**

Materials like oil/diesel/fuel are highly resistant ($\rho = 1000\,\Omega m$). $\boxed{LNAPL}$ → It floats over the water
Light Non Aqueous Phase Liquids

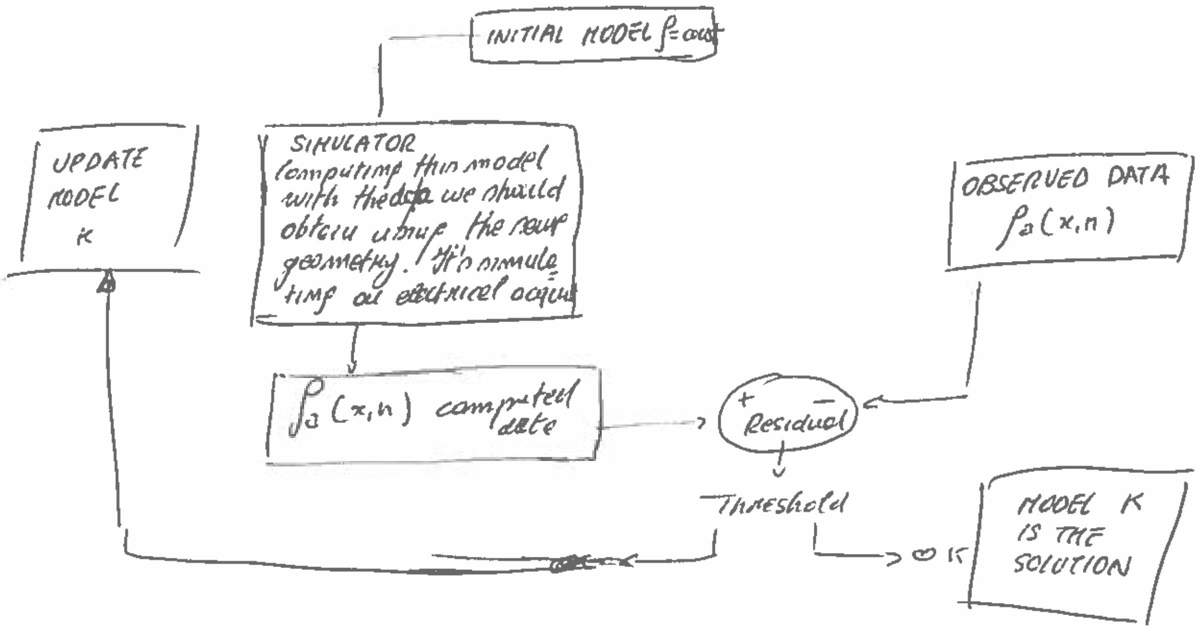


This kind of liquids are not water contaminants



The liquids produced by rubbish have a high resistivity
Normally they put an impermeable layer

How to pass from apparent resistivity $\rho_a(x,n)$ to the real one $\rho(x,z)$.
It's made by a software
We have the **OBSERVED DATA** $\rho_a(x,n) \longrightarrow$ REAL $\rho(x,z)$?



The electrical problem is ILL CONDITIONED → We have many solutions totally equivalent from the point of view of the residual.

So if we have A PRIORI INFO (that comes from other methods) i can add it to make a better choice

model $\rho(x,z)$

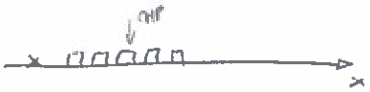
Softwares · RES2DMO → $\rho_a(x,n)$ $\xrightarrow[\rho_a(x,n)]{\text{Acquis Geom}}$ RES2INV → $\rho(x,z)$

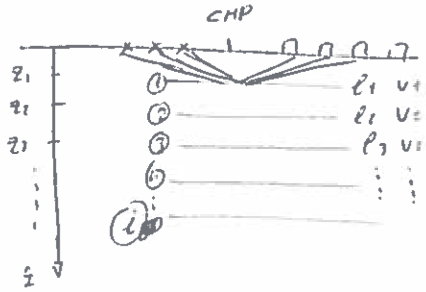
Pseudosection

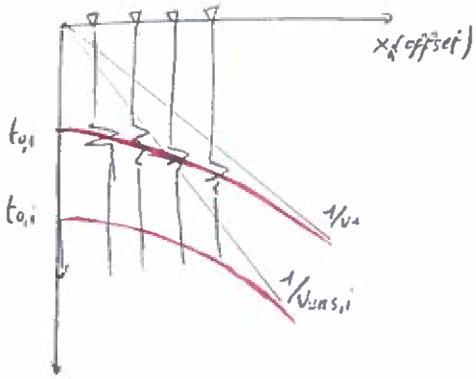
REFLECTION SEISMIC

Seismic waves to have an image of the subsurface

2D Aquisition → multicoverage data.







$x_h = x_R - x_S$ = offset

$$\boxed{t_1(x_h) = \sqrt{t_{0,1}^2 + \left(\frac{x_h}{V_1}\right)^2}} \rightarrow \text{hyperbola}$$

Where $t_{0,1}$ is the two way travel time to the interface

$$t_i(x_h) = \sqrt{t_{0,i}^2 + \left(\frac{x_h}{V_{rms,i}}\right)^2}$$

$V_{rms,i}$ → mean of all the velocities up to the layer i

$$V_{rms,i} = \sqrt{\frac{\sum_{k=1\ldots i} V_k^2 \cdot T_k}{\sum_{k=1\ldots i} T_k}}$$

T_k → two way travel time within the layer k

More or less the hyperbole are converging to Vrms. From the shape of the hyperbola we can get info about the velocity → This way we get the depth

It's very complicate to detect hyperbola very close to each other.

From $V_{rms,i}$ we can get the DIX FORMULA but it's really unstable (too much sensitive to error)
↳ $V_i = f(V_{rms,i})$

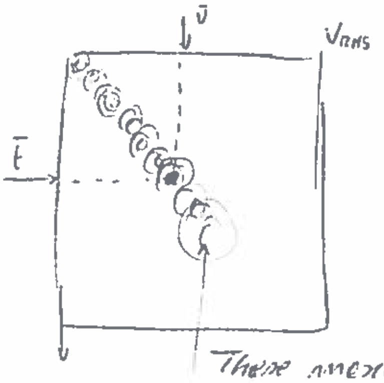
We need to recognize hyperbola possibly in an automatic way

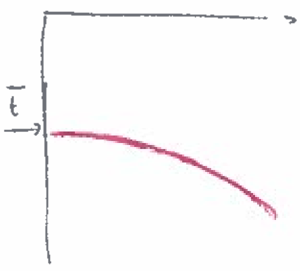
SEMBIANCE GATHER (AUTOMATIC HYPERBOLA DETECTOR)

We need two parameters:

$$t(x) = \sqrt{\bar{t}^2 + \left(\frac{x}{\bar{v}}\right)^2}$$

$\bar{t}, \bar{v}$ → two numbers that define one hyperbola





$$S(\bar{t},\bar{v}) = \left|\sum^{\text{along hyperbola } \bar{t}} d(x,t)\right|^2 \Big/ \sum_{\text{along hyperbola}} \left|d(x,t)\right|^2$$

We are looking if this hyperbola exists in the data

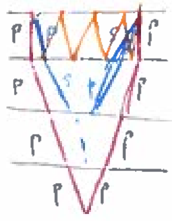
We will have a big value if it's following a true hyperbola

These maxima are the hyperbola

$0 \leq S \leq 1$

In image processing it's called **HALF INTEGRAL**

We can have more hyperbola at the same time:



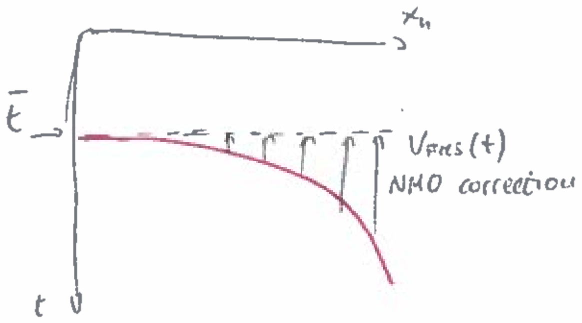
▬ direct p arrival
▬ More conversions (converted arrivals)
ʍʍ Multiples

We have to chose which one is the Vrms we are looking for: we chose the HIGHEST Vrms for each t to get the true Vrms(t) of direct p arrivals (fastest)

At the end of this process we have done the VELOCITY ANALYSIS

① Multi coverage acquisition
① Select one CMP (one position along x)
③ Compute semblance (random transform)
④ On semblance follow the maxima and highest Vrms
⑤ $\Downarrow$ $V_{rms}(t)$ VELOCITY ANALYSIS

We are trying to sum all the true hyperboles → we make the hyperboles flat and sum horizontally: NORMAL MOVEOUT CORRECTION

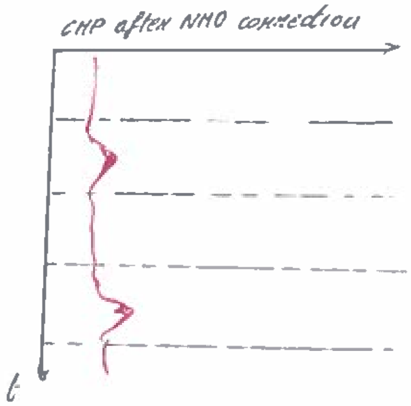


$$t \longrightarrow \boxed{\sqrt{\bar{t}^2 + \left(\frac{x_h}{V_{rms}(t)}\right)^2} - \bar{t} = t_{NMO}(x_h, \bar{t})}$$

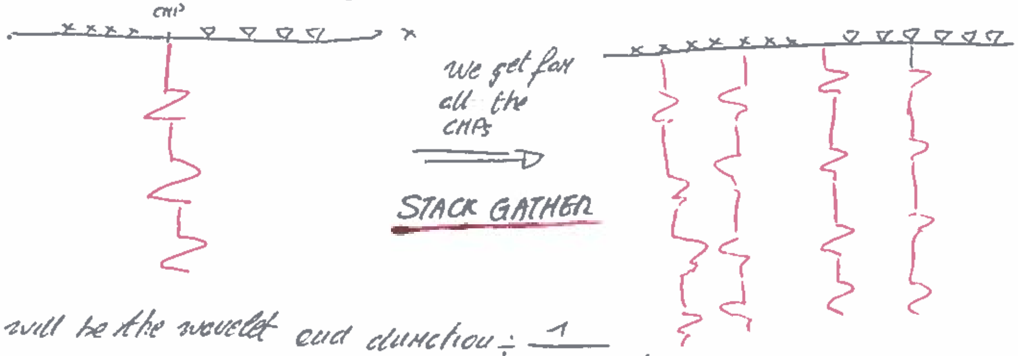
For every time we draw the hyperbole and move up to the flat version

For high offsets we see wavelet distortion.

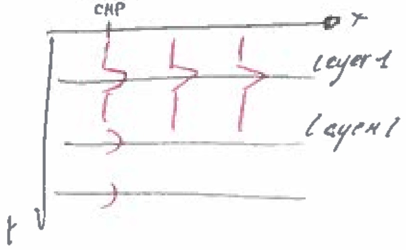
So in the graphic we will see the CMP NMO correction



The operation of summation along the x axis is called STACK OF CMP along x_h. The image of the subsurface that we get is:

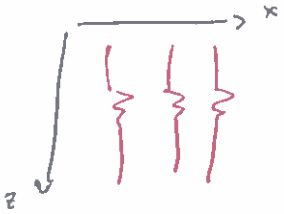


What we get is that the interface will be the wavelet and duration $\div \frac{1}{Bandwidth}$



The duration is proportional to the resolution

From this we define the time to depth to get to the DEPTH STACK GATHER



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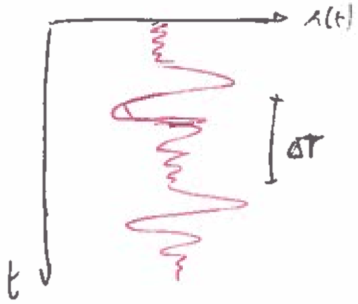
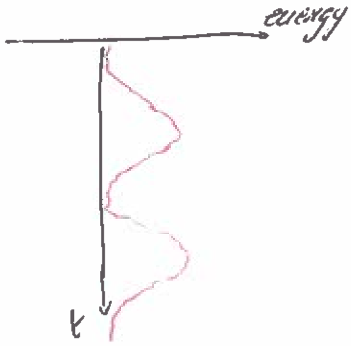
REFLECTION SEISMIC PROCESSING

- Demultiplexing: re-order data after acquisition
- Golgrup: noise trace cancelling (mute) or "reverse geophone" traces
- Gain recovery: amplification of high attenuated parts of traces

$$\underset{\text{Attenuation}}{A} \propto \frac{K_1}{t} e^{-K_2 t} \longrightarrow \underset{\text{recover}}{x(t)} \simeq r(t) \cdot \frac{1}{K_1} t e^{K \cdot t}$$

or AUTOMATIC GAIN RECOVERY (AGR)





$$\underset{\text{recover}}{x(t)} = r(t) \frac{1}{\sqrt{\int_{t-\frac{\Delta T}{2}}^{t+\frac{\Delta T}{2}} |r(t)|^2 dt}}$$

This way i obtain traces without big jumps in amplitude

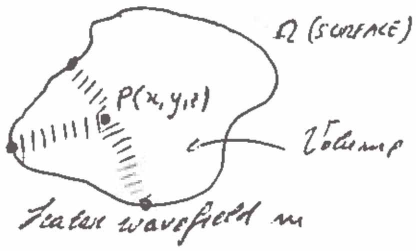
$\Delta T = 2/3 \cdot \underset{\text{wavelet duration}}{T}$ → If it's too small we have signal distortion. If it's too big we do no recovery

- Geometry inputs: informations on the geometry of the system
- Application of statics
- Deconvolution
- Filtering: Given a source we have to chose the right geophone and the right sampling freq ($\geq 2 f_{max}$) filtering the noise outside the bandwidth
- Trace equalization
- Real time tomography: use travel time to get velocity model of an object

SEISMIC ACQUISITION

- Pre-processing
- CMP Sorting
- Vel Analysis → pre stack (data + velocity)
- NMO correction
- Stack → artifacts (CMP is not locally horizontal)

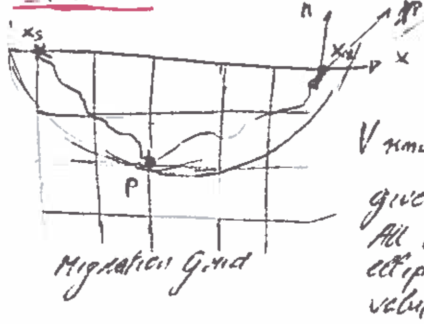
KIRCHÖFF INTEGRAL



If you know m(Ω) you can derive m in any position of the volume

$$m(x,y,z) = \int_\Omega m_\Omega \underbrace{G(\Omega, P)}_{\text{Green function (it describes the propagation between two points)}} d\Omega$$

MIGRATION



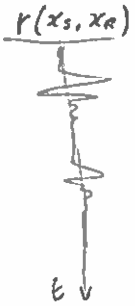
+ Hp: we consider a close surface with respect to the volume of interest.

For each point P on the grid, trace a ray from x_s to x_R and calculate the total travel time $\bar{t}$

V known $\bar{t} = t_{x_s \to P} + t_{P \to x_R}$

given $r(x_R, x_s)$
All the points on the ellipse will have the same value

The couple (x_s, x_R) is related to a certain trace $r(x_s, x_R)$. For each point, assign to it the value of $r(x_s, x_R)$ given $\bar{t}$:



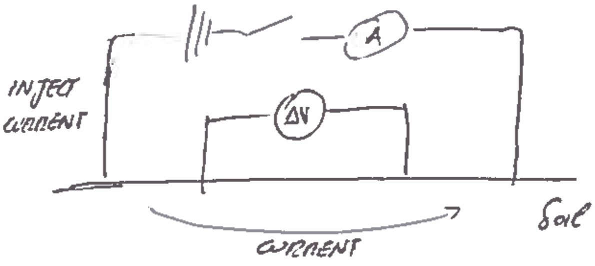
Repeat this process for each trace. Each point will have a vector of values, each one corresponding to one trace; summing this values and colouring the point with the result we'll obtain the image of the subsurface

We can also find the result by using the Kirchhof integral

$$m(P) = \int_{\omega} \int_{\text{All Traces}} \underbrace{\underbrace{A_S}_{\text{Attenuation } x_s \to P} A_R \cdot e^{j\omega(t_{x_sP} + t_{Px_R})}}_{\text{Green}} \underbrace{r(x_s, x_R)}_{\text{Recorded Field}} \cdot \underbrace{\vec{n} \times \nabla t_P x_R}_{\cos\vartheta} \, dx_R$$

ELECTRICAL METHOD

21/11/2017



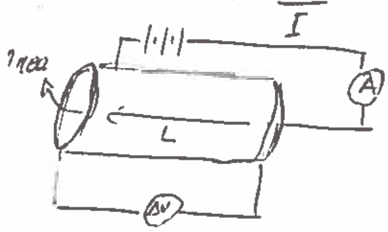
Current → moving charges
Metal → electronic conduction

① ELECTRONIC CONDUCTION

Ions → charged atoms / charged molecules
Soil → contains a lot of salt
The ions are transmitted to the soil and they "close" the circuit

② ELECTROLYTIC CONDUTION

Ohm law: $R = \frac{\Delta V}{I}$



High resistivity: the current has difficulty to ~~the~~ flow due to the presence of few ions

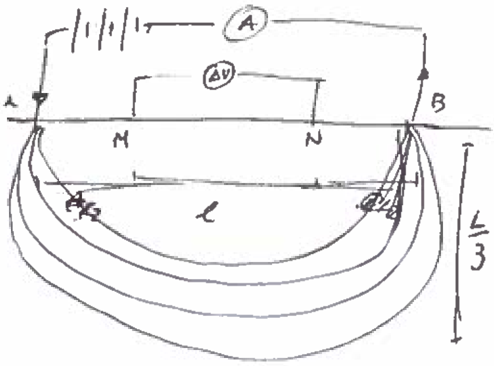
$R \div \frac{L}{AREA}$

We introduce the resistivity ρ $\boxed{R = \rho \frac{L}{AREA}}$

↓ It defines how hard or easy is for the current to flow into a body

$\rho = R \frac{AREA}{L}$ $[\Omega \cdot m]$

Metal resistivity: $10^{-12} \div 10^{-5}$ $\Omega \cdot m$
Electrolytic conduction $10^{-2} \div 10^{3}$ $\Omega \cdot m$
Polarization conduction → insulator $\rho \sim 10^{10}$



A, B current electrons → one injects current, the other puts it out

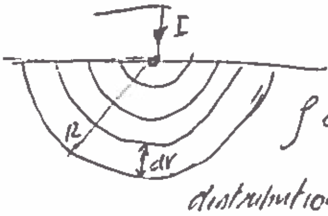
M, N measure a potential

Bigger ℓ, bigger depth

$p(x)$ constant separator transverse (used to visualize the lateral variations of resistivity)

$p(z)$ vertical electrical sounding (estimation of electrical conductivity or resistivity of the medium)

$\rho(x,z)$ → PSEUDOSECTION

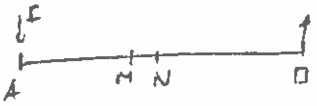


ρ constant $\frac{dV}{dr} = -\rho \bar{J}$

distribution of the current

$$E = \rho \bar{r}$$

$$V(r) = \int_0^r -\rho \frac{I}{2\pi r^2} dr \qquad V(r) = \rho \frac{I}{2\pi r}$$



$$V_M = \rho \frac{I}{2\pi}\left(\frac{1}{AM} - \frac{1}{BM}\right) \qquad V_N = \rho \frac{I}{2\pi}\left(\frac{1}{AN} - \frac{1}{NB}\right)$$

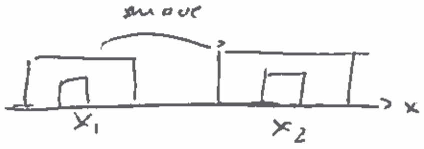
↳ distance between A and M

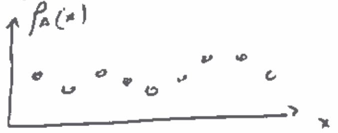
$$\Delta V = V_M - V_N = \rho \underbrace{\frac{I}{2\pi}\left(\frac{1}{AM} - \frac{1}{BM} - \frac{1}{AN} + \frac{1}{NB}\right)}_{} \xrightarrow{} \text{geometric factor } K^{-1}$$

$$\boxed{\rho = \frac{\Delta V}{I} K \ (\Omega \cdot m)}$$ For a constant resistivity

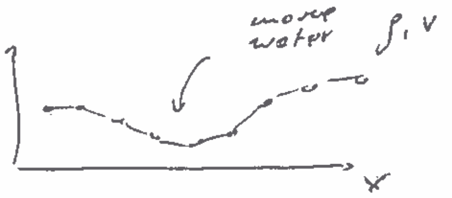
The one we obtain in the real experiments is the apparent resistivity ρ_a

① CONSTANT SEPARATION TRAVERSE (CST)



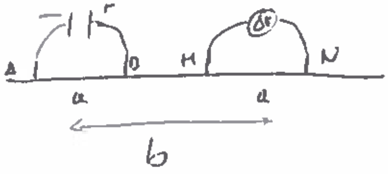


eg

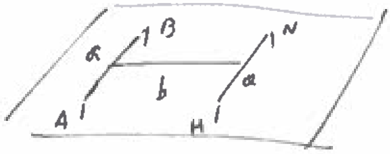


I move everything on left and right to obtain the sensitivity variations along x

DIPOLE-DIPOLE

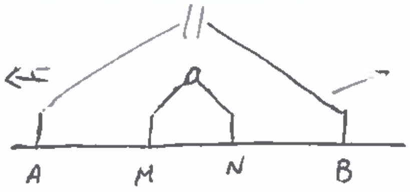


TRANSVERSE DIPOLE DIPOLE



We can get different sensitivities by changing the geometries:

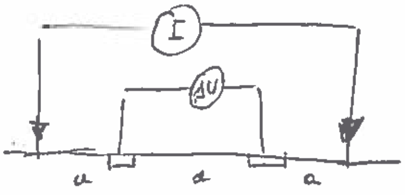
② VERTICAL ELECTRICAL SOUNDING (VES)



Here distance between A and B to go deeper

Two common arrays are used for VES:
- Wenner
- Schlumberger

↳ WENNER ARRAY



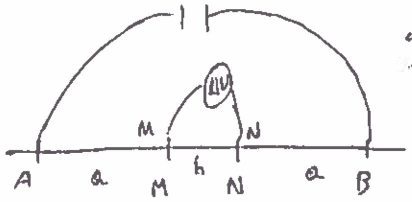
$$K = 2a\pi$$

For sounding measurements the electrodes in a Wenner array are expanded around a center point by equally increasing the spacing

Vantages:
- high SNR
- good resolution of horizontal layers
- good depth sensitivity

Disadvantages:
- not good in determining the lateral location of deep inhomogeneities since the large a-spacing degrades lateral resolution

↳ SCHLUMBERGER



$a >> b$

$K \simeq \pi \frac{a}{b}$

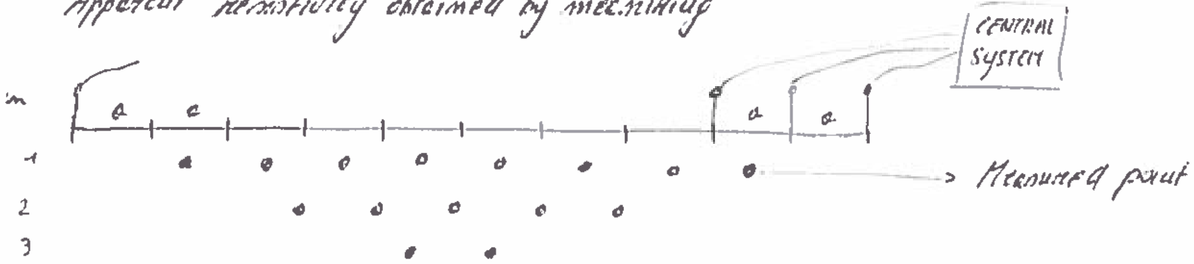
Vantages:

- High SNR
- good resolution of horizontal layers
- good depth sensitivity
- Faster then wenner

③ PSEUDO-SECTION

Obtained by VES / CST

Apparent resistivity obtained by measuring



The inverse problem is ill conditioned. We can enrich the infor by other methods

LIMITS OF ELECTRICAL METHOD

① Depth of sensitivity $\boxed{z = \frac{L}{3}}$

② Need to close the circuit
It doesn't work in dry sand, ice, hard media (no possibility to plant electrodes)

ELECTRICAL RESISTIVITY TOMOGRAPHY

ARCHIE'S LAW

It relates the electrical CONDUCTIVITY of a sedimentary rock with its porosity

$C_t = \frac{1}{a} C_w \phi^m S_w^n$

C_w electrical conductivity

ϕ: porosity

C_t electrical conductivity of fluid saturated rock

S_w: water saturation

m: cementation exponent of the rock

n: saturation exponent

a: tortuosity factor

Resistivity in solution with [illegible] salt changes with temperature and concentration

$R_t = a \phi^{-m} S_w^{-n} R_w$

Fluid saturated rock resistivity

Brine resistivity

a, m, n = constant depend on the location

$0{,}5 < a < 2{,}5$

$1{,}3 < m < 2{,}5$

$n \simeq 2$

How to use the formula in practice to get the porosity

$\rho = a \phi^{-m} S_w^{-n} \rho_w$

Ricavato tramite inverso

porosity

to be measured

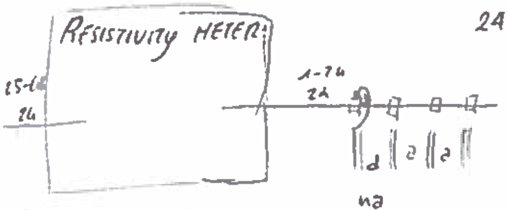
MISE A LA MASSE

Electrical no resistivity method that has been used in the mining industry since 1926 for delineating conductive subsurface or bodies. By injecting current into a conductive underground body it gets

Moving Ⓥ we can measure how big is the body

24/11/2017

ELECTRICAL METHOD



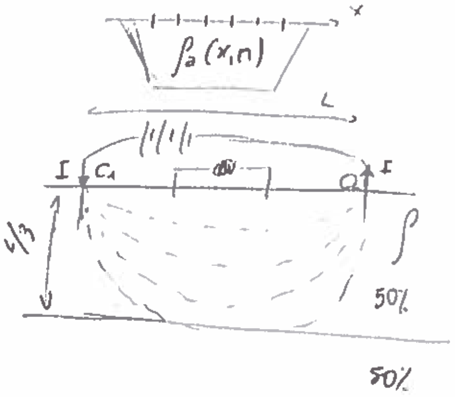
24 + 48 cables

System choosing 5 electrodes at a time that measure ΔV, ρ_a etc
We'll have many measurements for every 4 electrodes

We call apparent resistivity since we get an average of the anomalies

We can measure how deep is the area

PSEUDO SECTION



If we go $\frac{L}{3}$ deep, we have 50% of the current flowing in the first $\frac{L}{3}$ part.
We are sensitive only to this first part
L: distance between C_1 and C_2 $\longrightarrow$ depth $= \frac{L}{3}$

There are a lot of currents flowing and bodies producing parassitic currents => **NOISE**
To limit the noise we have to limit the distance

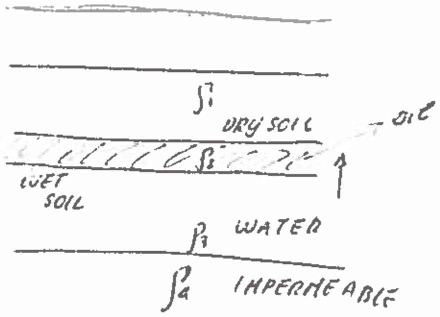
$0 < L < 1000\,m$
$0 < \text{depth} < 300\,m$

The resistivity is associated to the presence of **VOIDS** (tomb, cavern, tunnel), **WATER** (dissolved salts), **SALT WATER** (is more conductive so lower resistivity), **DISTILLED WATER** (highly resistive)

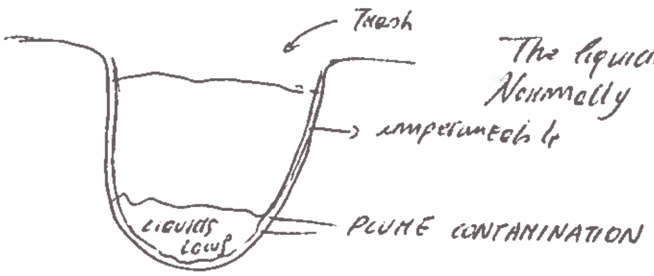
low ρ anomaly
AQUIFER

APPLICATIONS: looking for water, quality of water, oil, hydrocarbures
→ **CONTAMINANTS**

Materials like oil/diesel/fuel are highly resistant ($\rho = 1000\,\Omega m$). **LNAPL** → It floats over the water
Light Non Aqueous Phase Liquids

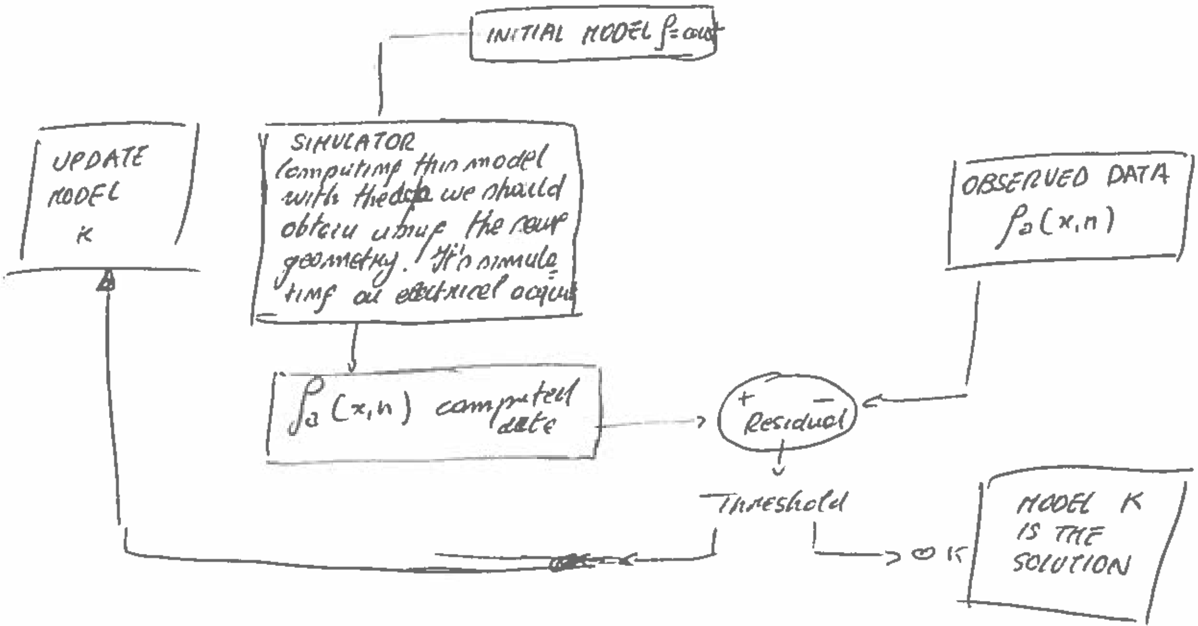


This kind of liquids are not water contaminants



The liquids produced by rubbish have a high resistivity
Normally they put an impermeable layer

How to pass from apparent resistivity $\rho_a(x,n)$ to the real one $\rho(x,z)$.
It's made by a software
We have the **OBSERVED DATA** $\rho_a(x,n) \longrightarrow$ REAL $\rho(x,z)$?



The electrical problem is ILL CONDITIONED → We have many solutions totally equivalent from the point of view of the residual.

So if we have A PRIORI INFO (that comes from other methods) i can add it to make a better choice

$$\downarrow \ \text{Softwares}\cdot\ \overset{\text{model } \rho(x,z)}{\text{RES2DMO}} \longrightarrow \underset{\text{Pseudosection}}{\rho_a(x,n)} \xrightarrow{\text{Acquis Geom } \rho_a(x,n)} \text{RES2INV} \longrightarrow \rho(x,z)$$