Model Predicted Control for COVID-19 using *Sensitivity Analisys Toolbox*

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***Abstract*—The spread of COVID-19 virus and the strategies implemented by governments to contain the emergency have generated a debate on the optimal lockdown strategy to consider both health conditions and economic losses. In this project we implement a COVID-19 Model following [2] and a cost function following [1]; the goal is to realize an optimal strategy to prevent the diffusion of COVID-19 epidemic using lockdown as control input for the model. An optimal lockdown, a first approach, is obtained by minimizing a cost function using Matlab Optimization Toolbox. Afterwards is presented a cyclic lockdown optimization in which once control of the epidemic is reached, the government imposes reopening cycles in order not to irreparably damage the economy. Then is presented the wise government problem. In section VIII a closed-loop Model Predicted Control is realized using the Sensitivity Analisys Toolbox to evaluate the most significant parameters which are used to determine the Real Model. A series of ODE is used to add realism. The model does not provide the possibility of obtaining an effective cure for COVID-19, nor the presence of vaccines. The results of our model support, at least in an initial phase of the first three months, a strong lockdown policy that decreases gradually its intensity during the time horizon considered.**

I. Introduction

SINCE December 2019, in China, precisely in Wuhan, cases of a new virus called Covid-19 have been recorded; the RNA virus causes multi-organ pathologies, in the most serious cases it leads to interstitial fibrotic pneumonia with severe respiratory insufficiency, often irreversible and with thrombotic phenomena of the various systems of human body. Due to the high contagiousness and the globalized society, the pathogen has spread rapidly throughout the world, registering, as of March 25, 2020, 74.386 cases in Italy, one of the countries most affected by the epidemic. Numerous models have been developed to simulate the progress of the infection and introduce effective containment policies waiting for a cure or vaccine to be found. Among these studies, there are Alvarez et al [1], who study lockdown strategies based on a SIR model, and Gatto et al [2] who propose a more complex model based on the SEPIA model.

The COVID-19 Model used in this work, is different from the Gatto model [2] because the differential equations (1) is modified to introduce lockdown policy. To obtain an optimal lockdown, a cost function is realized taking inspiration from Alvarez et al model [1]; the objective function is divided in two parts: the first weigh the economic losses, the second consider the human lives.

II. COVID-19 Model

The core of model is thus termed SEPIA and includes the following compartments:

$$\begin{aligned}
\dot{S}(t) &= -\lambda \cdot S \cdot (1-\theta \cdot L)^2 \\
\dot{E}(t) &= \lambda \cdot S \cdot (1-\theta \cdot L)^2 - \delta_E \cdot E \\
\dot{P}(t) &= \delta_E \cdot E - \delta_P \cdot P \\
\dot{I}(t) &= \sigma \cdot \delta_P \cdot P - (\eta + \gamma_I + \alpha_I) \cdot I \\
\dot{A}(t) &= (1-\sigma) \cdot \delta_P \cdot P - \gamma_A \cdot A \\
\dot{H}(t) &= (1-\xi) \cdot \eta \cdot I - (\gamma_H + \alpha_H) \cdot H \\
\dot{Q}(t) &= \xi \cdot \eta \cdot I - \gamma_Q \cdot Q \\
\dot{R}(t) &= \gamma_I \cdot I + \gamma_H \cdot H + \gamma_A \cdot A + \gamma_Q \cdot Q \\
\dot{D}(t) &= \alpha_I \cdot I + \alpha_H \cdot H
\end{aligned} \tag{1}$$

where S are susceptible individuals, that become exposed to the viral agent upon contact with infectious individuals, assumed to be those in the presymptomatic P, heavily symptomatic I, or asymptomatic/mildly symptomatic classes A. To describe the exposed individuals that are latently infected but still not contagious is used E, until they enter in the presymptom stage and only then become infectious. Hospitalized individuals H may either recover from infection or die because of it , while home-isolated individuals Q leave their compartment upon recovery . People who recover from infection or die because of COVID-19 populate the class of recovered R and dead D individuals, respectively, independently of their epidemiological compartment of origin [2]. The planner can decide to lockdown a fraction $L(t) \in [0, \bar{L}]$ of those susceptible and those infected, where $\bar{L} \leq 1$ allows us to realistically consider that even in a disaster scenario some economic activity such as energy and basic food production have to continue. We assume that the lockdown is only partially effective in eliminating the transmission of the virus. $\theta \in (0, 1]$ is a measure of the lockdown effectiveness. If $\theta = 1$, the policy is fully effective in curbing the diffusion, but since some contacts will still happen in the population even under a full economic lockdown, we allow $\theta < 1$ [1].

$$\lambda = \frac{\beta_P \cdot P + \beta_I \cdot I + \beta_A \cdot A}{S + E + P + I + A + R} \tag{2}$$

To calculate the force of infection, it is used 2, where β_P, β_I, and β_A are the specific transmission rates of the three infectious classes; to evaluate this rates an $R_0 = 3,6$ is evaluate and also, knowing the β ratios, a system of three equations with three unknowns is calculated (in 3).

$$\begin{cases} \frac{\beta_A}{\beta_P} = 0.033 \\ \frac{\beta_I}{\beta_A} = 1.03 \\ \frac{\beta_P}{\delta_P} + \sigma \cdot \frac{\beta_I}{\eta+\alpha_I+\gamma_I} + (1 - \sigma \cdot \frac{\beta_A}{\gamma_A}) = R_0 \end{cases} \tag{3}$$

III. Free response of a Epidemiological Model

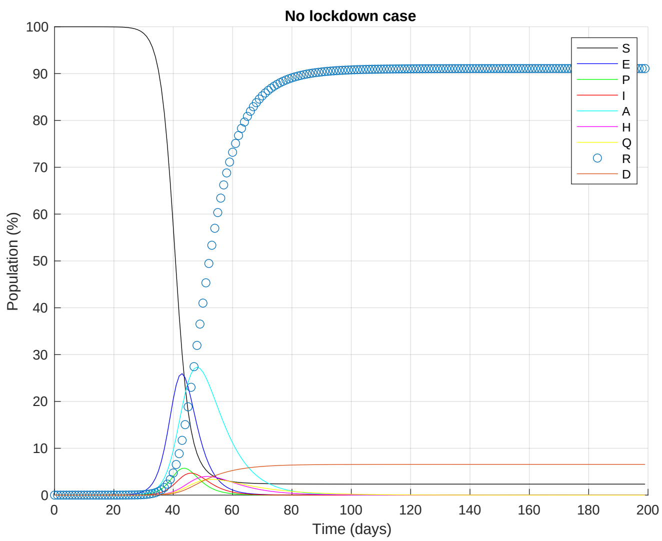

Fig. 1. Free Evolution of Gatto Model

The free evolution of the Gatto et al model [2] without a lockdown strategy ($L = 0$) shows an exponential trend in the spread of the epidemic. For the simulation the fixed step *ode3 solver* is used, and initial condition of about 5 exposed individuals are considered while all the rest susceptible.

The graphs of E,P,I,A, classes grow very quickly reaching a peak among 40/50 days; this implies a saturation of the health system which makes difficult an effective management of the health emergency. After the peak there is a rapid decline due to the exhaustion of the susceptible class; deaths settle around a value of 6.2% corresponding to approximately 3.7 mln people.

This model takes into consideration, for the estimation of its parameters, the situation of March 2020 in the Lombardy region, also considering a spatial distribution of the contagion that includes a network of 107 nodes representative of closely monitored Italian provinces and metropolitan areas. The parameters are evaluated considering some containment measures such as the limitation in movement, but do not consider the presence of further measures introduced later, such as the obligation to wear a mask and a more effective screening using swabs to identify asymptomatic infectious people. In the following paragraphs, the aim will be to avoid the health emergency situation described, but also considering the economic losses through optimal lockdown strategies. Thanks to a reduction in the infection curves, it is possible to guarantee treatment for the entire population, but in the absence of effective drugs and vaccines against the virus, lockdown policies must last longer over time.

IV. R0 Evaluation

a) Analytic R0 Evaluation:: For Analytic R0 (basic reproduction number) Evaluation the spectral radius of the matrix K_L is calculated, as suggested in the Supporting Information of [2].

K_L is calculated as:

$$K_L = -T \cdot \Sigma^{-1} \tag{4}$$

Where T is transmission matrix and Σ is transition matrix, that satisfy the relation:

$$J_0 = T + \Sigma \tag{5}$$

Where J_0 is the Jacobian Matrix of the infection subsystem E, P, I, A. With this approach, $R0$ is equal to 3.6 with a confidence interval of $[3.49, 3.84]$.

b) Operative $R0$ Evaluation:: The operative Evaluation, deals with calculate the values of $R0$ that verify a doubling time between 2.5 and 3.5 days, which is the doubling time estimated in the Italian case based on ISS data. Values of $R0$ ranging between 2 and 4 with step 0.1, return doubling times in the range considered valid if $R0$ is in the interval $[2.2; 2.7]$. The $R0$ value is significantly lower than that reported in the article [2], but in line with the estimates provided by national data. Given that $R0$ calculated in the [2] refers to a spatial model of the beginning of the epidemic, while the model that will be considered from now not include explicit spatial dynamics, to add realism and consider parameters closer to national data, in the simulations an $R0$ equal to 2.7, is considered.

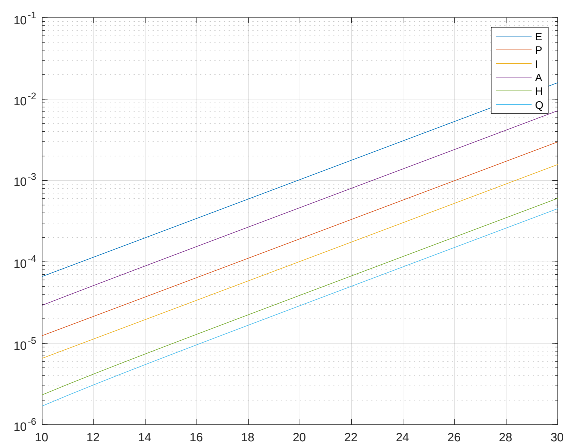

Fig. 2. *Operative R0 Evaluation (Time on x-axis, LogScale on y-axis) The doubling time is extracted from the trend of the exposed, as it is one of the characterizing classes of the infectious dynamic*

c) theoretical $R0$ Evaluation: : A Theoretical $R0$ Evaluation is calculated using the following equation:

$$R_0 = \frac{\lambda \cdot S_0}{\delta_E \cdot E_0} \tag{6}$$

To be consistent with the calculation of the operating $R0$, the first instant of contagion is considered on the $5th$ day from the beginning of the epidemic, and as β values those present in the work [2]. This result, approximately equal to 2.3, is comparable with the values obtained in the Operational calculation (IV-0b). In this work it will consider an $R0$ equal to 2.7 which represents an ideal compromise among the results obtained.

V. Optimal lockdown using *Simulated Annealing algorithm*

*a) **Introduction**:* The optimal lockdown problem involves identifying a lockdown policy strategy that is an ideal compromise between direct costs (the loss of human lives), and the indirect costs (economic losses). The work does not consider the possibility of an effective cure or a vaccine appear, so the only way to keep the epidemic under control is through the containment measures. In order to add realism,it is considered first 14 days without lockdown control. Also in this case initial condition is about 5 exposed individuals and all the rest are considered susceptible. The algorithm used to solve the optimal lockdown problem is the Simulated Annealing, an algorithm that intends to simulate the metal annealing process starting from an initial condition on the metal temperature and searching iteratively for the optimal solution.

*b) **Cost Function**:* The cost function introduced considers, for the optimization algorithms used, the relationship between direct costs and indirect costs. The cost function adopted to obtain an optimal lockdown is:

$$C(t) = \int_0^T (u \cdot PIL \cdot L(t) \cdot (S(t) + E(t) + P(t)+ \\ ... + I(t) + A(t)) + (1-u) \cdot D(t) \cdot VSL) \cdot dt; \quad (7)$$

Where $u \in (0, 1)$ is a weight for economic losses and $(1 - u)$ is weight for cost of human lives. PIL indicate the *GDP per capita daily* equal to 82.12€ [3] and VSL is a *Value Statistical Lives* calculated as twenty times GDP per capita daily. Basically the cost function is divided in two parts: First is referred to economic losses due to lockdown. The other is the cost of fatalities evaluated using *VSL*. Using the sum of classes S, E, P, I, A have been identified those individuals who, affected by lockdown, have the biggest influence on country's economy. In the cost function are considered only the deaths (direct costs) and the economic losses due to the lockdown (indirect costs) by evaluating the number of people who don't work because of the closure of the activities.

Additional direct and indirect costs could be included, explicitly or through the *u* weight, such as the presence of hospitalized patients which weighs economically on the health system (SSN), health problems due to the lack of preventive visits, that in the medium-long term will further weigh on the SSN, social and mental health problems.

*c) **Lockdown Strategy and Control**:* The optimal lockdown is implemented in *Matlab* using Simulated Annealing; a lower and upper bound have been chosen to be closed to the reality, and also an initial guess is taken to address the minimization algorithm.

First fourteen days of epidemic are without control, later starts the algorithm for a control horizon of two hundred days.

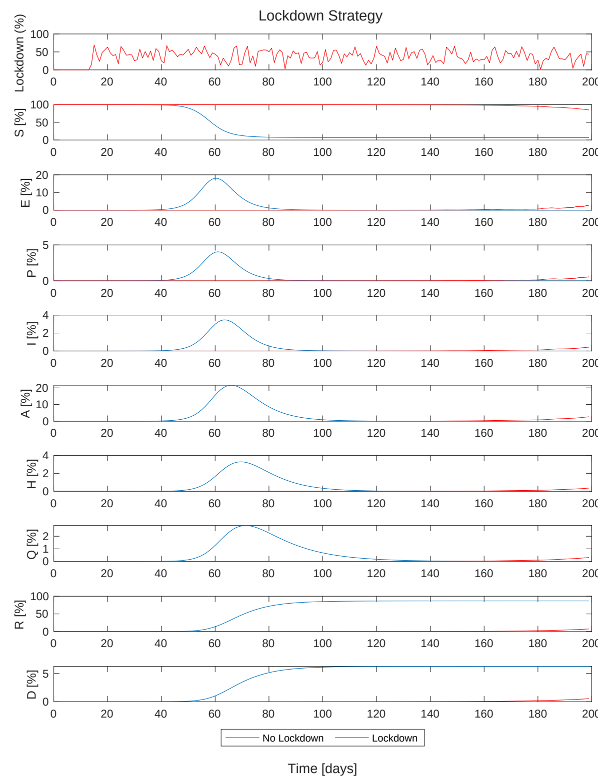

Fig. 3. *optimal lockdown with S.A.*

The control action of the lockdown has an average around 40% with a non-linear jagged trend. A slow increase in the number of infected is inevitable, thus causing an increase in deaths (equal to about 0.5% of the Italian population). Comparing the model in free evolution 1 and that subject to optimal lockdown 3, there is a drastic decrease of the deaths and of the infected; this also implies a greater number of susceptible (about 85% of the Italian population). The optimal Lockdown curve has a jagged trend due to the minimization algorithm used to search for the minimum of the cost function; since *Simulated Annealing* is an algorithm closely linked to the initial conditions, to improve this trend, a U0 more conforming to reality was chosen to obtain a more realistic curve in Fig.6.

In Fig. 4, as the weights of the cost function vary, are obtained curves that weigh more human lives or economic losses.

Using $u = 0.1$ (red curve) human lives are weighed more, maintaining a stronger lockdown curve, reaching a percentage value of susceptible (at day 200) compared to the Italian population of 93%, the infected settle at 0.21% and the deaths

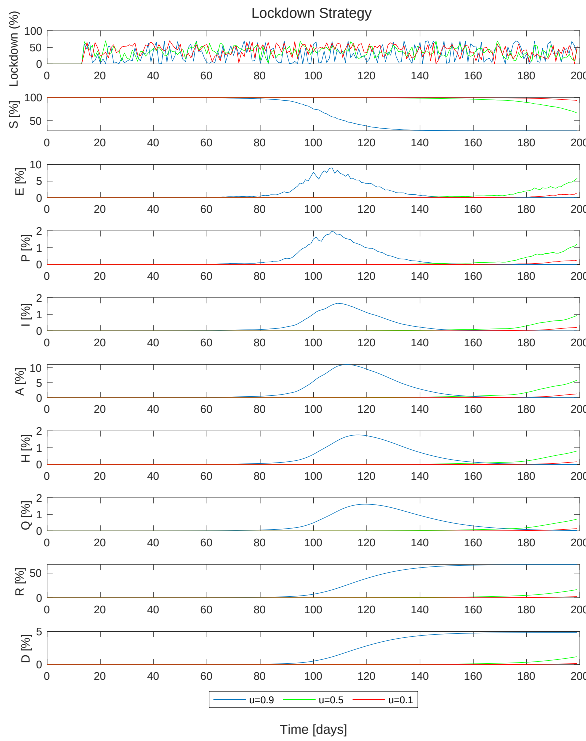

Fig. 4. *Comparison between different values of weight **u**:*
u = 0.1 (red curve) u = 0.5 (green curve) u = 0.9 (blue curve)

at 0.18% , thus obtaining a control that minimizes human losses to the detriment of the country's economy.

Using $u = 0.5$ (green curve) a compromise is obtained between the cost of lives lost and economic losses, reaching a percentage value of the susceptible of 66%, the infected settle at 0.93% and the deaths at 1.21%, there is an increase in the infected due to loosening of lockdown measures.

Using $u = 0.9$ (blue curve), the effects on the country's economy are weighed more heavily, causing a significant increase in deaths which settle at 4.8% and the infected reach a peak, on the 110th day, of 1.66%. As expected, susceptibility also drastically decrease, reaching a population percentage of 28%.

*d) **MultiStart Algorithm**:* The MultiStart algorithm is used to derive an optimal lockdown vector through minimization techniques to search for global solutions to problems containing multiple points of maximum or minimum. The obtained vector is then considered as initial guess to the Simulated Annealing algorithm.

As shown in Fig. 5, the optimal solution involves a value of 70% of lockdown for about 40 days. Then the curve goes down, in a non-linear way, to a low values around the 180th day. The Fig. 6 shows the optimal lockdown strategy using Simulated Annealing with the MultiStart algorithm solution as initial guess. The decreasing trend of the lockdown is shown, starting from a value around 60% maintained for the first two months, after which there is a gradual loosening of the measures until their elimination. This result depends significantly on the initial conditions provided to the algorithm, and allows to keep the epidemic under control. The susceptible

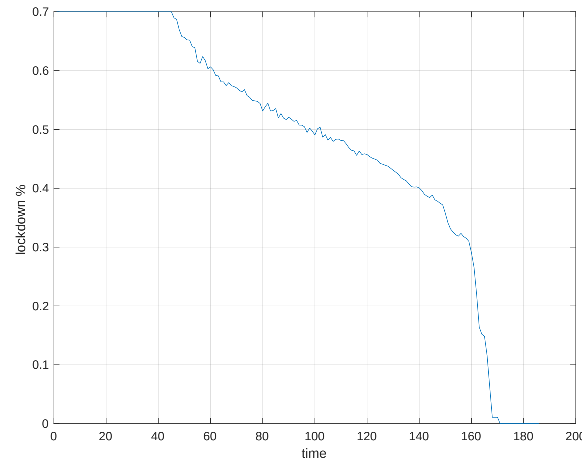

Fig. 5. *Optimal Lockdown with **MultiStart Algorithm** used as Initial Guess in **Simulated Annealing***

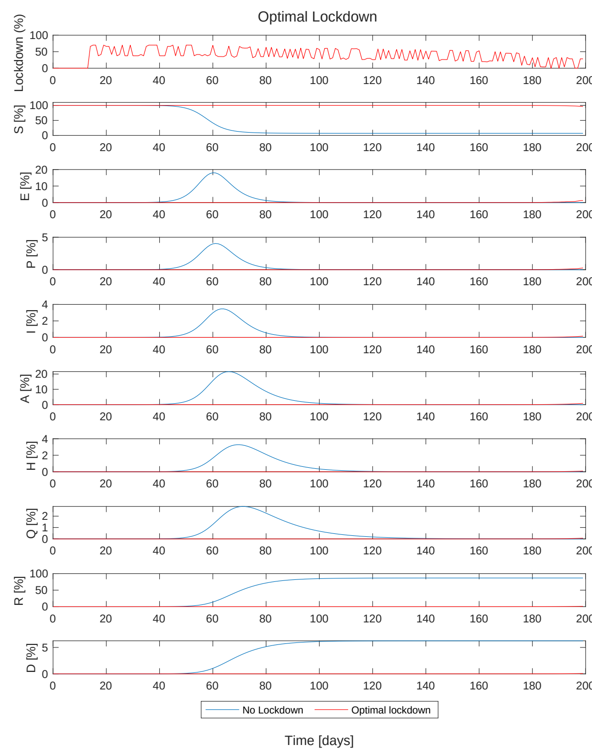

Fig. 6. *Optimal Lockdown Strategy with **Simulated Annealing** using as Initial Guess the lockdown curve in Fig 5*

individuals at day 200 settle at 96%, while the deaths at 0.05% compared to the case without lockdown, which obtains values of 7% and 6.2% respectively. The health system is slightly affected, while overall there is a saving in human lives of around $3.600.000$ individuals. Progressive loosening allows for specific support measures to be provided for activities and families, allowing effective emergency management and consistent communication of the measures imposed by politicians. However, this strategy does not allow to eliminate the virus, which continues to circulate in a limited way, therefore the infections return to grow significantly when the lockdown is completely eliminated.

VI. Cyclic Lockdown Using Simulated Annealing Algorithm

The problem of the cyclic lockdown consists in identifying the possibility of optimal cycles of closures and reopening in which once control of the epidemic is reached, politicians decide to reopen in order not to irreparably damage the economy, causing the epidemic restart in the long run. This approach has been followed by several governments to deal with the emergency considering the problems due to the decline in GDP and also social problems, that were created in the various countries. These policies can lead in the long run to create distrust of the politicians, who do not seem to follow a precise strategy, but pursue the spread of the contagion without trying to anticipate it.

Simulated Annealing is used to choose the amplitude of lockdown control among three reference values 70%, 20% and 0% of lockdown intensity, the choice of epidemic containment measures are evaluate in 14-day steps.

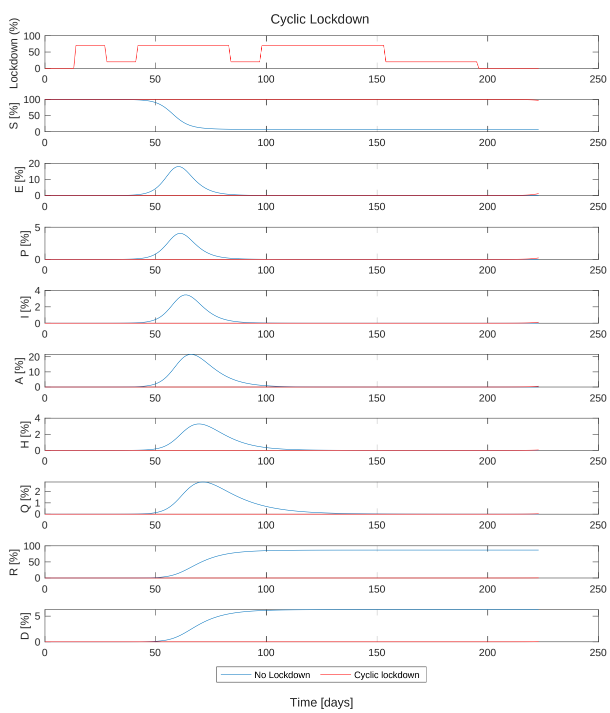

Fig. 7. *Cyclic Optimal Lockdown using Simulated Annealing*

Fig.7 shows the cyclical nature of the lockdown policies; after a short opening period it becomes necessary to impose a stronger lockdown for a longer period of time.

Thanks to this policy the S, E, I, H, D trends (most sensitive classes) assume an optimal development to make sure to limit the human losses and maximize the susceptible. In the first months , however, reopening never lasts more than 14 days; this does not allow an effective economic recovery but there is a restart of the infections every time.

A politician who have to follow strategies in the interest of his country with foresight, also have to take these aspects into consideration to plan effective and sustainable policies to support and assist activities and citizens.

VII. Wise Government Consideration

The problem of the so-called **wise government** consists, in our opinion, in identifying the best lockdown strategy to be implemented in order not to consistently affect the susceptible. Among the results obtained with the different approaches, the most effective epidemic containment policy is the one identified using the vector obtained as a result of the *MultiStart algorithm* as initial guess to Simulated Annealing Algorithm. In this case, the lockdown, initially around 60%, gradually drops around 10% over the 200th day time window, keeping the epidemic under control. Referred to Fig.6.

This strategy makes it possible to plan in advance a gradual reopening of activities and a gradual loosening of containment measures without the risk of emergency closures, further damaging the country's economy. Furthermore, in this way it is possible to program targeted support measures for businesses and citizens.

VIII. Model Predicted Control

*a) **Introduction**:* Model Predicted Control is a control strategy that use a prediction of the behavior of the process over a time horizon, taking a dynamic model and the measures available on the real system (model based). A process model is used to predict the current values of the output variables. The residuals, the difference between the actual and the predicted outputs, serve as feedback signal to a prediction block. For each sampling instant the control law is redetermined and the predictions are updated starting from the new measurements.

The future control signal is predicted by optimizing a cost function on a prediction horizon that may differ from the control horizon. For closed loop predicted control, **Genetic Algorithms** are used, these allow to evaluate different starting solutions by recombining them and introducing elements of disorder (similarly to random genetic mutations). Doing that, they produce new solutions that are evaluated by choosing the best ones in an attempt to converge towards optimal solutions. The real model is built by varying the most significant parameters found through the **Sensitivity Analysis Toolbox**.

*b) **Sensitivity Analysis Toolbox**:* Through this, the correlation of the model parameters for the specified targets is evaluated, both using linear and non-linear methods, i.e. maximization of susceptible and minimization of deaths and infected.

As shown in Fig.8, the parameters that have the greatest influence on the model's output are $1/\delta_P$ (rate at which *presymptomatics* become *seriously infected*), $R0$, $1/\eta$ (rate at which *serious infected* are isolated), β_I/β_A.

To build the real model and implement the closed loop predicted control, we decided to vary $R0$ and $\beta_I, \beta_A, \beta_P$ parameters because they are the most uncertain from the point of view of their estimation; in fact we supposed that the transition parameters among the classes are known with high reliability. The β_i $(i = I, A, P)$ represent the specific transmission rates

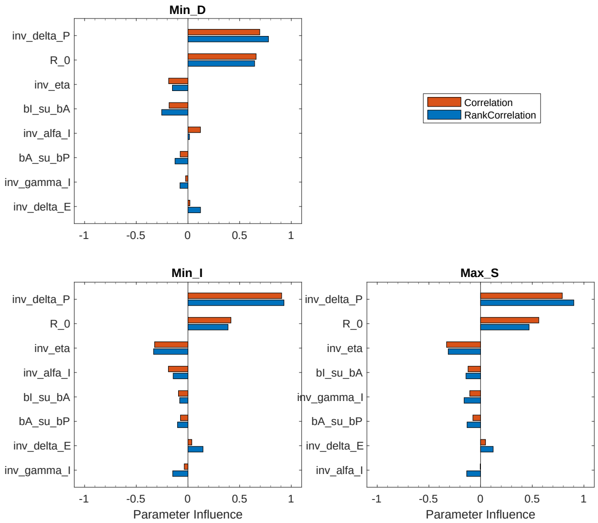

Fig. 8. *Correlation of the model parameters for the specified targets*

of the classes I, A, P and define λ, dependent on the contact and displacement coefficients of the population.

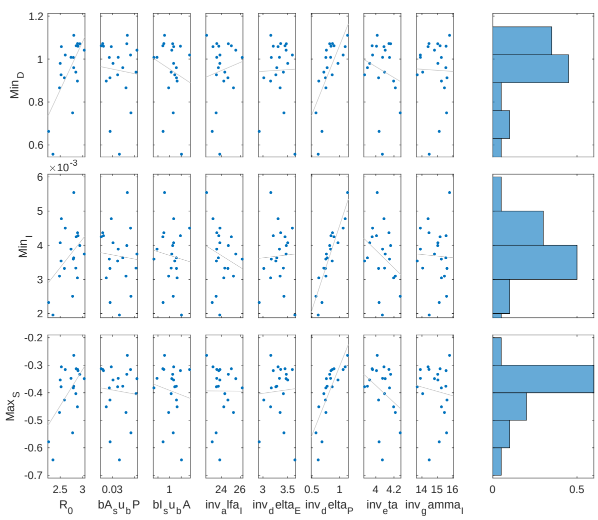

Fig. 9. *Scatter Plot of the variation of model parameters for maximization of susceptible and minimization of deaths and infected*

In Fig.9 a linear dependence of the parameter $R0$ and $1/\delta_P$ for the specified targets is evident, while in Fig. 10 is shown the difference between the real model, obtained using the new parameters, and the nominal model.

*c) **Model Predicted Control**:* The Closed Loop MPC is implemented following the diagram shows in Fig.11.

The **Optimizer (Control action)** evaluates the *future input* for **Model**, using a prediction horizon of 60 and 90 days, a control horizon of 200 days and a time interval for control evaluation of 14 days.

The *Errors* input of **Optimizer** block are evaluated as difference between *Reference trajectory* (Output of The Real Model) and *Predicted Output* (Output of Gatto Model). Through the use of an extended kalman filter (EKF), the matching between

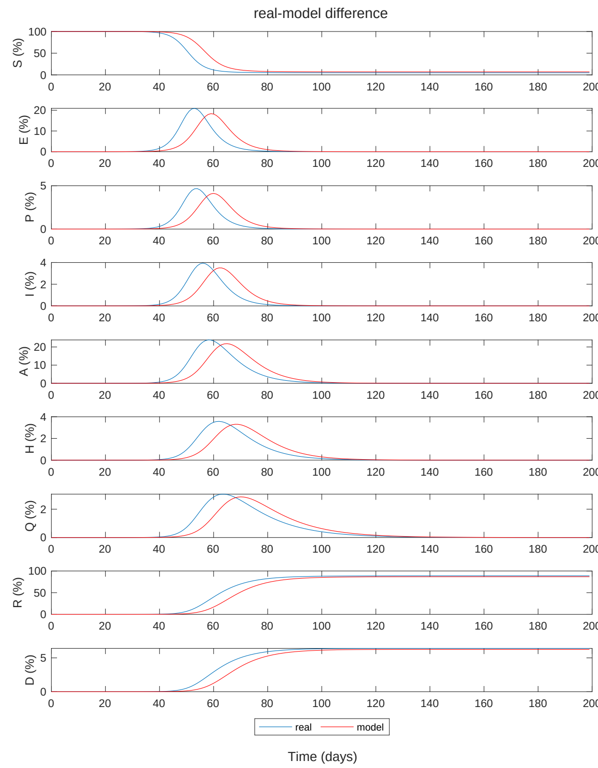

Fig. 10. *Difference between Real Model and Gatto Model*

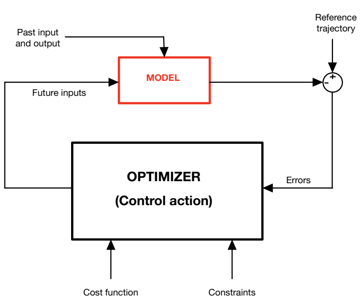

Fig. 11. *Block Diagram of M.P.C. [4]*

the two models is carried out every 14 days, then a lockdown strategy is evaluated on a chosen prediction horizon.

The control action is calculated minimizing the cost function and taking into account of constraint on the class of infected; the average growth over the next two weeks in this class should not exceed the threshold of 0.0005% of infected individuals.

As shown in Fig.12 the predictive control technique, with a prediction horizon of 60 days, allows to obtain results similar to those found previously as regards the control of the epidemic. The lockdown curve does not have a decreasing trend, this can represent a limit due to the increase in problems related to the country's economy and the sustainability of the containment measures adopted.

In Fig. 13 the difference between real model and nominal model shown how the EKF works. The filter performs a matching between all the observable states S, I, H, Q, R, D of the two models, thus excluding the remaining classes. A

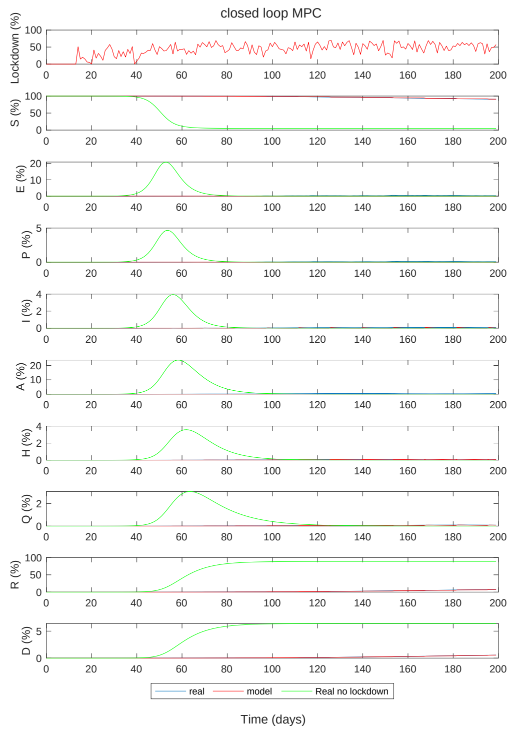

Fig. 12. *MPC Strategy with a predicted horizon of* 60 *days*

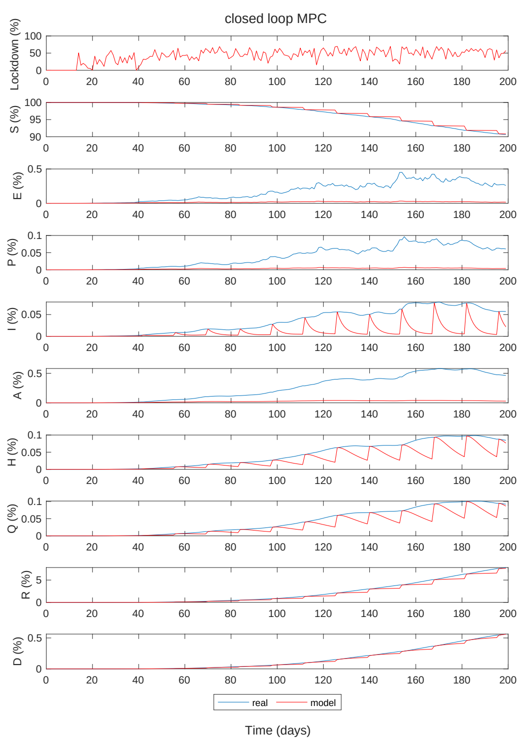

Fig. 13. *Comparison between Real Model and Nominal Model with a predicted horizon of* 60 *days*

sawtooth-like trend is obtained from the pursuit of the real model. The classes E, P, A are difficult to trace and therefore are considered unobservable.

Comparing the Fig.13 and Fig.14 can be observed that

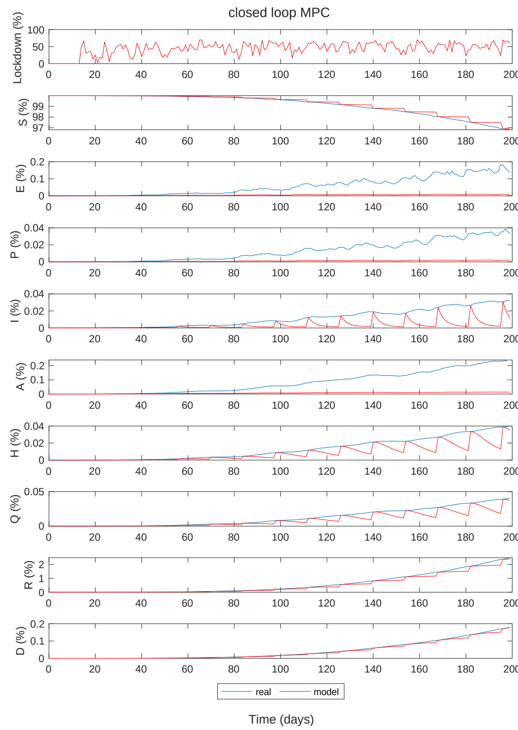

Fig. 14. *Comparison between Real Model and Nominal Model with a predicted horizon of* 90 *days*

with a longer prediction horizon (specifically 30 days more), an improvement in the trend of the curves is obtained. In particular, on the 200th day, deaths drop from 0.55%, with a prediction horizon of 60 days, to 0.17%, with a prediction horizon of 90 days, which translates into a decrease of about 200.000 fewer deaths. Also it is observed an increase of about 6% in the susceptible, which go from 90% in the case of a 60 days horizon, to approximately 96% in the case of a 90 days horizon.

This control strategy turns out to be optimal for epidemic control, allowing to maximize the susceptible population thus avoiding saturating the nation's health system. The expense of computational cost grows as the prediction horizon increases.

IX. Adding Realism with ODE series

To add realism to the discussion, since the model has a maximum probability of exiting the infectious class in the first phase after the entering into these compartments, auxiliary compartments are used, thus building a sequential stages model. By doing that it is possible to mitigate the exponential trends creating a curve which describes the exit from the infectious classes in a more likely way, so it is necessary to divide the infectious classes into at least 3 compartments. The choice of classes is also based on the value of the transition coefficients δ from one class to another, for this reason classes E, P, I, A are chosen.

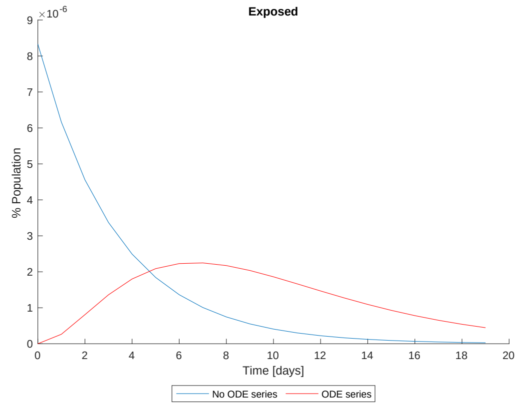

Fig. 15. *Difference between Ode series and no Ode series for percentage of Exposed Individuals*

Gatto Model with Ode series

$$
\begin{aligned}
\dot{S}(t) &= -\lambda \cdot S \cdot (1-\theta \cdot L)^2 \\
\dot{E}_1(t) &= \lambda \cdot S \cdot (1-\theta \cdot L)^2 - \delta_E \cdot E_1 \\
\dot{E}_2(t) &= \delta_E \cdot E_1 - \delta_E \cdot E_2 \\
\dot{E}_3(t) &= \delta_E \cdot E_2 - \delta_E \cdot E_3 \\
\dot{P}_1(t) &= \delta_E \cdot E_3 - \delta_P \cdot P_1 \\
\dot{P}_2(t) &= \delta_P \cdot P_1 - \delta_P \cdot P_2 \\
\dot{P}_3(t) &= \delta_P \cdot P_2 - \delta_P \cdot P_3 \\
\dot{P}_4(t) &= \delta_P \cdot P_3 - \delta_P \cdot P_4 \\
\dot{I}_1(t) &= \sigma \cdot \delta_P \cdot P_4 - (\eta + \gamma_I + \alpha_I) \cdot I_1 \\
\dot{I}_2(t) &= (\eta + \gamma_I + \alpha_I) \cdot I_1 - (\eta + \gamma_I + \alpha_I) \cdot I_2 \\
\dot{I}_3(t) &= (\eta + \gamma_I + \alpha_I) \cdot I_2 - (\eta + \gamma_I + \alpha_I) \cdot I_3 \\
\dot{A}_1(t) &= (1-\sigma) \cdot \delta_P \cdot P_4 - \gamma_A \cdot A_1 \\
\dot{A}_2(t) &= \gamma_A \cdot A_1 - \gamma_A \cdot A_2 \\
\dot{A}_3(t) &= \gamma_A \cdot A_2 - \gamma_A \cdot A_3 \\
\dot{H}(t) &= (1-\xi) \cdot \eta \cdot I_3 - (\gamma_H + \alpha_H) \cdot H \\
\dot{Q}(t) &= \xi \cdot \eta \cdot I_3 - \gamma_Q \cdot Q \\
\dot{R}(t) &= \gamma_I \cdot I_3 + \gamma_H \cdot H + \gamma_A \cdot A_3 + \gamma_Q \cdot Q \\
\dot{D}(t) &= \alpha_I \cdot I_3 + \alpha_H \cdot H
\end{aligned}
\tag{8}
$$

In (8) are shown the differential equation of [2] model with *Ode* series. Thanks to this approach it is possible to obtain a much more realistic behavior, as shown in Fig.15, Fig.16 and Fig.17

Using the *Ode* series approach, by imposing a constant initial guess at 70% with the Simulated Annealing algorithm, a lockdown strategy is obtained approximately constant around 70% for the entire time window of 200 days. From this analysis it is obtained a slowdown of the epidemic.

As can be seen from the Fig.18 the susceptible reach, on the $200th$ day, about 78% of the Italian population while the dead are around 0.4%.

The results in Fig.18 show a behavior similar to the cases analyzed above which allows to control the epidemic by maximizing the susceptible and minimizing the deaths. To obtain these results, however, it is necessary to maintain a fixed lockdown at 70% for the entire simulation time, damaging the country's economy.

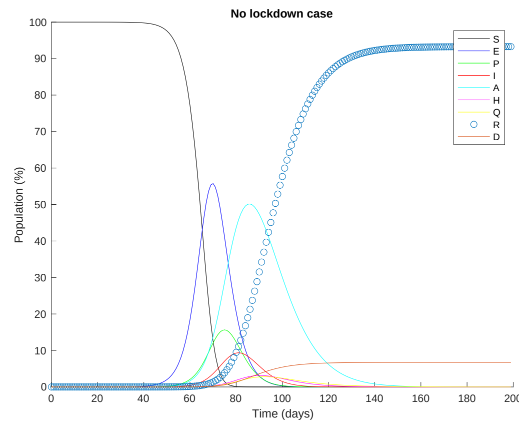

Fig. 16. *Free Response with Nominal Model*

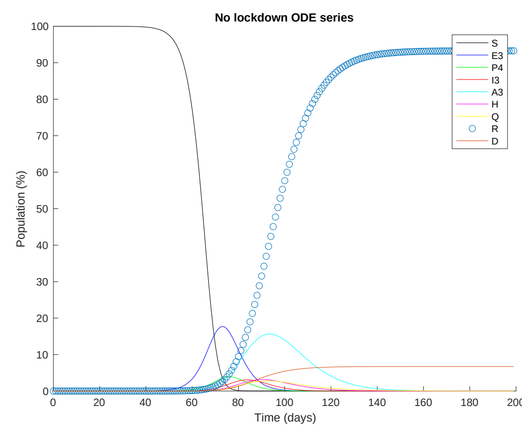

Fig. 17. *Free Response with Ode series Model*

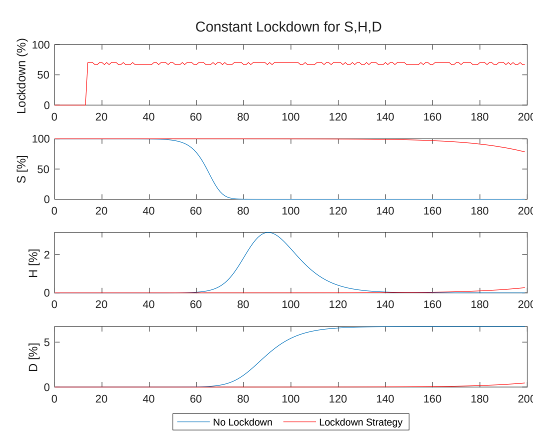

Fig. 18. *Constant Lockdown with Ode series*

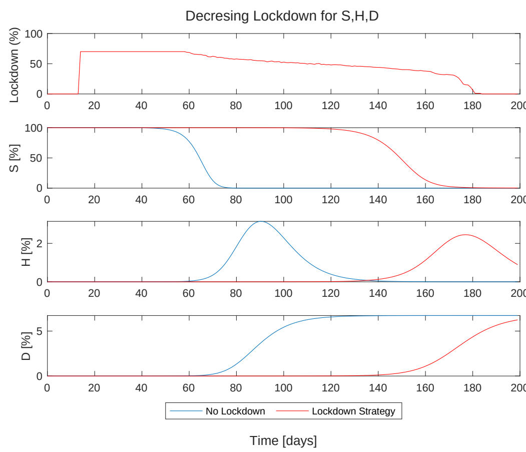

Fig. 19. *Decresing Lockdown with Ode series*

By adding the economic problem to the discussion, using a decreasing initial guess, as previously done for wise government, it is not possible to keep the epidemic under control, only delaying the natural dynamics of the infection.

X. Conclusions

a) ***Gatto Model****:* The Gatto model is an 9-state model that aims to model the dynamics of the Covid-19 epidemic, a good compromise between computational complexity and the search for a realistic dynamics. The model, while simulating plausible scenarios and in line with expectations, presents some critical issues: the presence of further containment measures such as social distancing, the obligation to wear a mask, the sanitation of the hands and the contingency of spaces can be explicitly considered through an analysis of displacements or by modifying the θ coefficient of effectiveness of the lockdown, which we considered equal to 0.8. However, it is necessary to underline that the model suffers from a redundancy problem. In fact, the class of Presymptomatics could, for greater clarity, be joined to other classes. We also observed that the hospitalized class lacks a parameter that takes into account the receptive capacity of the national health system. It is in fact well known that one of the main problems of this epidemic period in Italy was that of avoiding the saturation of the hospital system. This phenomenon could be modeled using a variable transition parameter from class H to class D which grow its value as the percentage of the hospitalized population increased. In order to add realism to the economic dynamics, it could be thought of adding a variable economic cost that took into account the fluctuations in GDP due to the presence of the restrictions.

For the Italian case, consideration could also be given to the capacity of the regional health systems, very different from each other in the resources available, which could suggest a lockdown management based on this type of indicators. This strategy was excluded because in our opinion regional measures, considering such small spatial compartments, lead in the long run to a spread of the virus in alternating phases without the possibility of breaking down the contagion curve in a definitive way and causing strong inhomogeneities among the territories. Any emergency measures to be implemented to improve the management of the epidemic, such as the construction of new hospitals, the addition of beds in intensive care units and the use of resources in the search for a cure or vaccine, are not included in the model. To add realism to the model, it was possible to consider a distribution of social contacts that was not stochastic but more in line with national statistics, for example by age group, or considering the national demographic distribution. This approach may suggest simulating a management of lockdown by age group, limiting the movements of individuals who are more likely to become asymptomatic (and therefore vectors of diffusion more difficult to trace) or limiting contacts with individuals most at risk. It has been assumed that all individuals have infinite life, except for the possibility of dying due to the virus; this simplification is acceptable given the short time horizon considered.

b) ***Algorithms****:* The algorithms chosen for implementation are: *Simulated Annealing*, *Genetic Algorithms* and *MultiStart*. *Simulated Annealing* has been used to search for optimal control on lockdown, since it is very linked to Initial Guess, it does not provide a result applicable in reality, caused by its intrinsic nature that generates a jagged trend, in fact it is suggested to take the values of the lockdown trend as a qualitative reference. An algorithm that is certainly more applicable in practice is the *fmincon*, in fact it was decided to use a *MultiStart Algorithm*, which is based on the *fmincon*. *MultiStart* is a *Global Search algorithm* that has allowed us to obtain an optimal Initial Guess to be provided to *Simulated Annealing*. The performance of the curves improves significantly, smoothing the dynamics. A negative aspect of the *MultiStart Algorithm* is its high computational cost, which required a considerable simulation time; it was also necessary to add simplifying "options" to be able to converge the algorithm. For the design of MPC the *Genetic Algorithms* were used since, together with the *fmincon*, they are the only minimization algorithms that accept non-linear constraints; also in this case, as in the *MultiStart*, the simulation require more time, we decided to add simplifications that allowed us to reduce the computation cost; in particular, in order to respect the non-linear constraints, it was necessary to use a different approach to solve this problem, i.e. through the use of the *Penalty algorithm* which unlike the *Augmented Lagrangian Genetic Algorithm* (ALGA), in its evaluation of the fitness of an individual, it computes a penalty value as follows:

- If the individual is feasible, the penalty function is the fitness function.
- If the individual is infeasible, the penalty function is the maximum fitness function among feasible members of the population, plus a sum of the constraint violations of the (infeasible) individual.

In the end in order to respect the non-linear constraints and minimize the cost function, the above result were obtained VIII-0c.

Thanks to the Sensitivity Analysis Toolbox it was possible to obtain the fundamental parameters for the design of a model as real as possible, through which it was possible to design the

Model Predicted Control. The positive aspect of the Toolbox is the possibility to visualize, through plots, the correlation between the various parameters of the model and how these influence the dynamics of the system. The biggest drawback encountered is the implementation of the entire model in Simulink, due to the use of the aforementioned toolbox and having to explicitly express every single parameter that you wanted to vary. Reserves the right to a sensitivity analysis through the exclusive use of Matlab.

c) ***Conclusion****:* The results obtained with our approach, which tries to undermine the susceptible in a not very significant way but also inserting a weight that considers the economic aspects, support, at least in an initial phase of the first three months, a strong lockdown policy that it immediately abates the rates of contagion, and then gradually decreases its intensity. However, this type of policy is obtained by considering a closed system, and to evaluate the possibility of its use, it is also necessary to consider the policies implemented by neighboring countries, including border controls for travelers and workers in the models. A further aspect necessary to assess the impact of a very intense lockdown is the sustainability of the welfare system; activities that cannot work, workers forced into quarantine and families in difficulty need support to ensure that they are not arise disorders of a social nature. The lack of preventive visits and the increase in physical and mental health problems cause additional costs in the long term, approximately proportional to the duration of the lockdown, which will weigh on the country's economic system.

Finally, we underline again how our work is vitiated by the hypothesis of the absence of effective cure and vaccines, which force us to control the epidemic exclusively with restrictive measures on the population. The model has a strong versatility which allows it to be used also for different epidemic scenarios, simply by acting on the parameters. Through the use of the cost function, we are able to include economic and social problems in the discussion as well as those of an epidemiological nature. The control techniques implemented in our work are well suited to an early epidemic scenario, i.e. when the lockdown is the only possible way for containing the virus, managing to return various and valid control strategies.

Thanks to our techniques we are able to guarantee the normal functioning of the health care system and to drastically reduce the number of deaths, which through the use of MPC, amount to 0.17% of the population against 6.2% in the case of a free evolution, avoiding the death for 3.6 million people.

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