CS208: Applied Privacy for Data Science
Machine Learning under DP

School of Engineering & Applied Sciences
Harvard University

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Following slides from:

Practical Method to Reduce Privacy Loss when Disclosing Statistics Based on Small Samples

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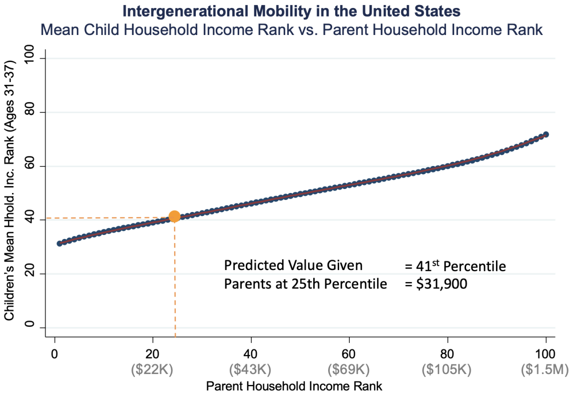
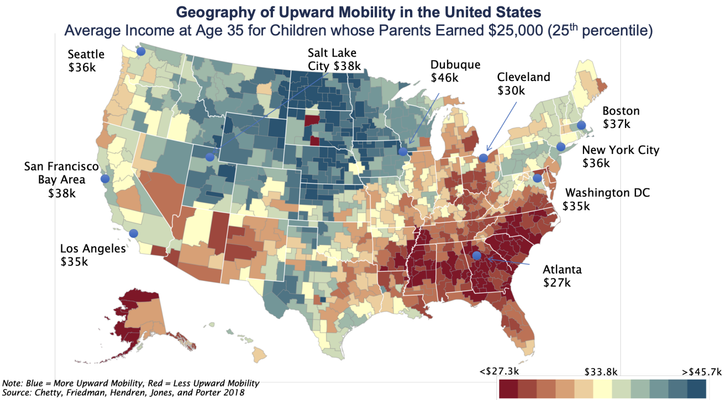
Publishing Statistics Based on Small Cells

- Social scientists increasingly use confidential data to publish statistics based on cells with a small number of observations
- Causal effects of schools or hospitals [e.g., Angrist et al. 2013, Hull 2018]
- Local area statistics on health outcomes or income mobility [e.g., Cooper et al. 2015, Chetty et al. 2018]



Source: Chetty, Friedman, Hendren, Jones, Porter (2018)



Geography of Upward Mobility in the United States
Average Income at Age 35 for Children whose Parents Earned $25,000 (25th percentile)
Seattle $36k
Salt Lake City $38k
Dubuque $46k
Cleveland $30k
Boston $37k
New York City $36k
San Francisco Bay Area $38k
Washington DC $35k
Los Angeles $35k
Atlanta $27k
<$27.3k
$33.8k
>$45.7k
Note: Blue = More Upward Mobility, Red = Less Upward Mobility
Source: Chetty, Friedman, Hendren, Jones, and Porter 2018

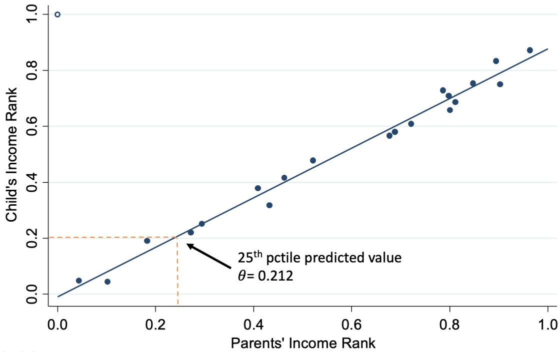
Controlling Privacy Loss

- Problem with releasing such estimates at smaller geographies (e.g., Census tract): risk of disclosing an individual's data
- Literature on differential privacy has developed practical methods to protect privacy for simple statistics such as means and counts [Dwork 2006, Dwork et al. 2006]
- But methods for disclosing more complex estimates, e.g. regression or quasiexperimental estimates, are not feasible for many social science applications [Dwork and Lei 2009, Smith 2011, Kifer et al. 2012]

This Paper: A Practical Method to Reduce Privacy Loss

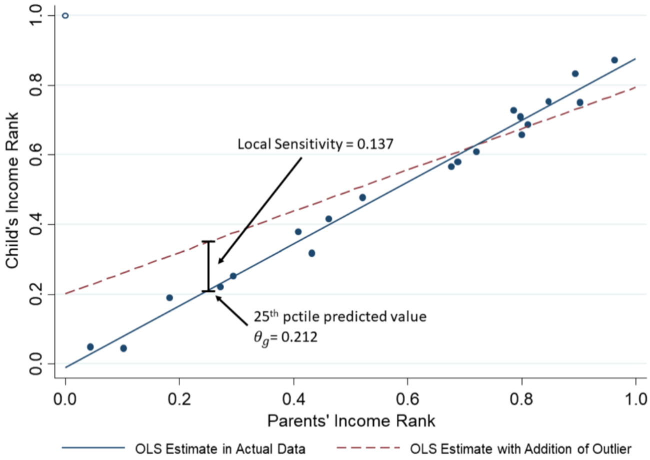
- We develop and implement a simple method of controlling privacy loss when disclosing arbitrarily complex statistics in small samples
  - The “Maximum Observed Sensitivity” (MOS) algorithm
- Method outperforms widely used methods such as cell suppression both in terms of privacy loss and statistical accuracy
  - Does not offer a formal guarantee of privacy, but potential risks occur only at more aggregated levels (e.g., the state level)

Example Regression from One Small Cell



Source: Authors' simulations.

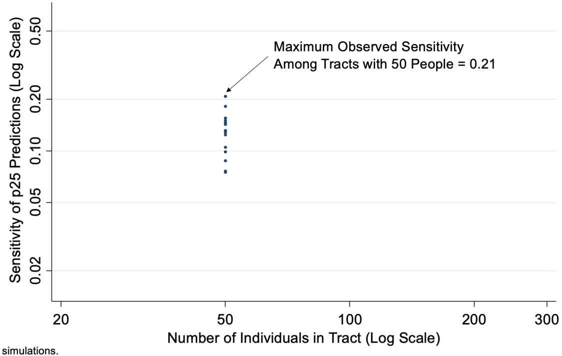
Figure 1: Calculation of local sensitivity



Maximum Observed Sensitivity

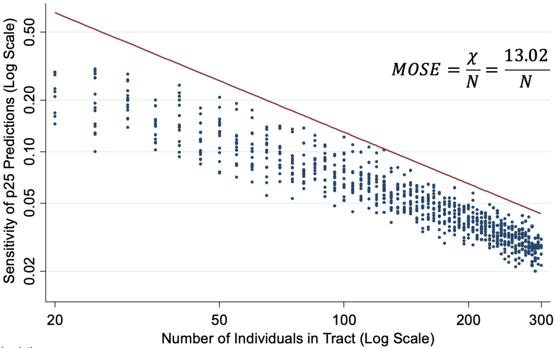
- Our method: use the maximum observed local sensitivity across all cells in the data
  - In geography of opportunity application, calculate local sensitivity in every tract
  - Then use the maximum observed sensitivity (MOS) across all tracts within a given state as the sensitivity parameter for every tract in that state
- Analogous to Empirical Bayes approach of using actual data to construct prior on possible realizations rather than considering all possible priors

Maximum Observed Sensitivity Envelope



Source: Authors' simulations.

Computing Maximum Observed Sensitivity



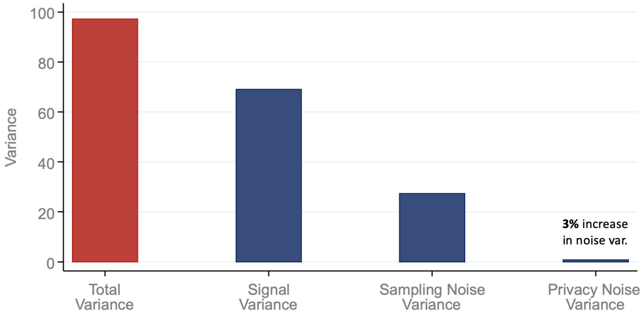
Source: Authors' simulations.

Producing Noise-Infused Estimates for Public Release

- Main lesson: tools from differential privacy literature can be adapted to control privacy loss while improving statistical inference
  - Opportunity Atlas has been used by half a million people, by housing authorities to help families move to better neighborhoods, and in downstream research [Creating Moves to Opportunity Project; Morris et al. 2018]
  - The MOS algorithm can be practically applied to any empirical estimate
- Example: difference-in-differences or regression discontinuity
  - Even when there is only one quasi-experiment, pretend that a similar change occurred in other cells of the data and compute MOS across all cells

Variance Decomposition for Tract-Level Estimates

Teenage Birth Rate For Black Women With Parents at 25th Percentile



Source: Chetty, Friedman, Hendren, Jones, Porter (2018)

Conclusion

- Use max observed sensitivity χ, tract counts, and exogenously specified privacy parameter ϵ to add noise and construct public estimates:

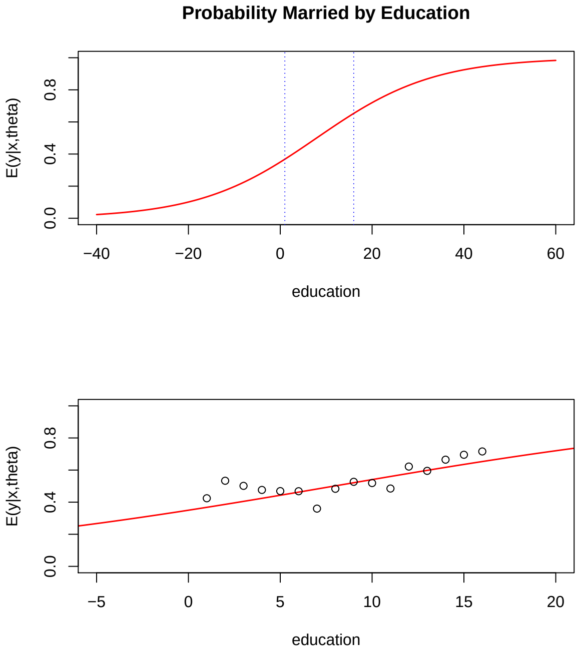
$$\tilde{\theta}_g = \theta_g + L\left(0, \frac{\chi}{\epsilon N_g}\right) \quad \tilde{N}_g = N_g + L\left(0, \frac{1}{\epsilon}\right)$$

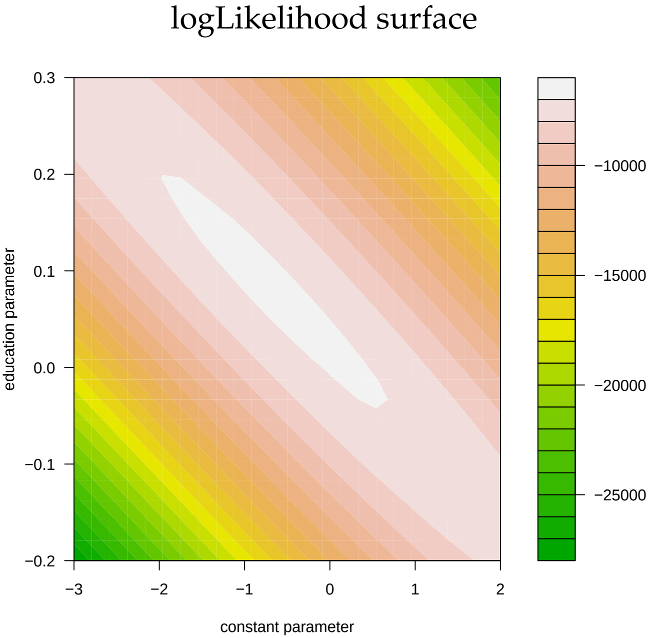
  - This method not "provably private," but it reduces privacy risk to release of the single max observed sensitivity parameter (!)
  - Privacy loss from release of regression statistics themselves is controlled below risk tolerance threshold ϵ.
- Critically, χ can be computed at a sufficiently aggregated level that disclosure risks are considered minimal ex-ante

DP Optimization of Complex Models

Logit Model

$$
\begin{aligned}
logL(y|x,\theta) &= \sum_{i=1}^{N} y_i \log(\pi_i) + (1-y_i)\log(1-\pi_i), \\
\pi_i &= \frac{1}{1+e^{-\beta_0-\beta_1 x_i}}.
\end{aligned}
$$


Probability Married by Education
E(y|x,theta)
0.0
0.4
0.8
−40
−20
0
20
40
60
education
E(y|x,theta)
0.0
0.4
0.8
−5
0
5
10
15
20
education


logLikelihood surface
education parameter
constant parameter
0.3
0.2
0.1
0.0
−0.1
−0.2
−3
−2
−1
0
1
2
−10000
−15000
−20000
−25000

Algorithm 1 Differentially private SGD (Outline)

Input: Examples $\{x_1, \ldots, x_N\}$, loss function $\mathcal{L}(\theta) = \frac{1}{N}\sum_i \mathcal{L}(\theta, x_i)$. Parameters: learning rate η_t, noise scale σ, group size L, gradient norm bound C.

Initialize θ_0 randomly

for $t \in [T]$ **do**

Take a random sample L_t with sampling probability L/N

Compute gradient

For each $i \in L_t$, compute $\mathbf{g}_t(x_i) \leftarrow \nabla_{\theta_t}\mathcal{L}(\theta_t, x_i)$

Clip gradient

$\bar{\mathbf{g}}_t(x_i) \leftarrow \mathbf{g}_t(x_i)/\max\left(1, \frac{\|\mathbf{g}_t(x_i)\|_2}{C}\right)$

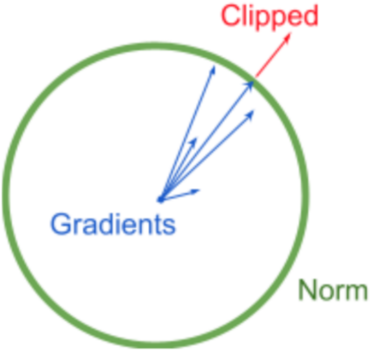
Add noise

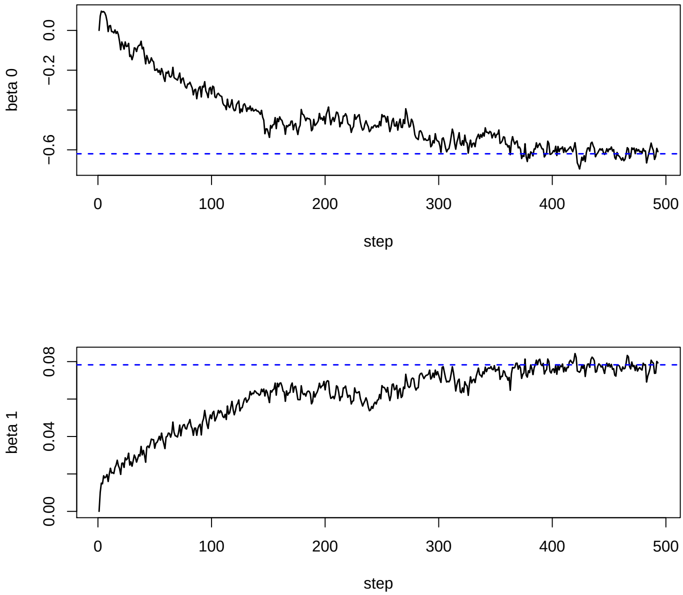
$\tilde{\mathbf{g}}_t \leftarrow \frac{1}{L}\left(\sum_i \bar{\mathbf{g}}_t(x_i) + \mathcal{N}(0, \sigma^2 C^2 \mathbf{I})\right)$

Descent

$\theta_{t+1} \leftarrow \theta_t - \eta_t \tilde{\mathbf{g}}_t$

Output θ_T and compute the overall privacy cost (ε, δ) using a privacy accounting method.


Clipped
Gradients
Norm


beta 0
0.0
−0.2
−0.6
0
100
200
300
400
500
step
beta 1
0.00
0.04
0.08
0
100
200
300
400
500
step