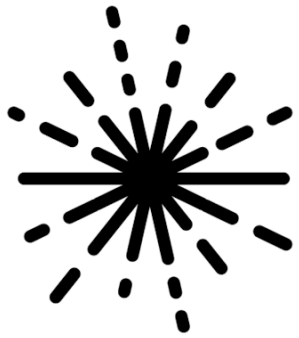

CS208: Applied Privacy for Data Science
Reconstruction Attacks

School of Engineering & Applied Sciences
Harvard University

February 3, 2025

Cohen & Nissim

Linear Program Reconstruction in Practice

- Use queries of sums over random subsets to reconstruct individual data.
- Importantly, the members of the subset are reported in each sum.
- Received the Aircloak Bounty ($5000) for reidentifying challenge data in the *Diffix* commercial system.

`https://journalprivacyconfidentiality.org/index.php/jpc/article/view/711`

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  - Thm [Dinur-Nissim `03]: given $m = n$ uniformly random sets S_j and answers a_j s.t. $\left|a_j - q_{S_j}(x)\right| \leq E = o(\sqrt{n})$, whp adversary can reconstruct $1 - o(1)$ fraction of the bits x_i.

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Regression Based Reconstruction

From CS109:

True vs. Statistical Model

We will assume that the response variable, Y, relates to the predictors, X, through some unknown function expressed generally as:

$$Y = f(X) + \varepsilon$$

Regression Based Reconstruction

Find $\hat{x_1}, \ldots, \hat{x_N}$ s.t.:

$$\hat{x} = \underset{\hat{x}}{\text{argmin}}[\sum_{j=1}^{m}(a_j - \sum_{i \in S_j} \hat{x_i})^2]$$

$$\hat{x} = \underset{\hat{x}}{\text{argmin}}[\sum_{j=1}^{m}(a_j - \sum_{i=1}^{n} \hat{x_i}\ s_{j,i})^2]$$

$$\hat{x} = \underset{\hat{x}}{\text{argmin}}[\sum_{j=1}^{m}(a_j - \hat{a_j})^2]$$

In R see:
`lm()`
In Python see for example:
`linear_model.LinearRegression()`
from scikit-learn.

Regression Based Reconstruction

$$a_j = x_1 s_{1,j} + x_2 s_{2,j} + \ldots + x_n s_{n,j} + e_j$$

Here:

n	is the number of people in the database
m	is the number of queries
i	is a person index
j	is query index
a_j	is j-th query release
$s_{i,j}$	is a $\{0,1\}$-indicator $i \in S_j$
x_h	is h's sensitive data
e_i	is the residual/error of the i-th prediction

Regression Based Reconstruction

$$a_j = x_1 s_{1,j} + x_2 s_{2,j} + \ldots + x_n s_{n,j} + e_j$$
$$7 = 1 \cdot 1 + 0 \cdot 1 + 1 \cdot 0 + 0 \cdot 0 + \ldots + 0 \cdot 1 + 2$$
$$4 = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 + 0 \cdot 1 + \ldots + 0 \cdot 1 + (-1)$$
$$6 = 1 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 + \ldots + 0 \cdot 0 + 3$$

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Regression Based Reconstruction

$$a_j = \hat{x}_1 s_{1,j} + \hat{x}_2 s_{2,j} + \ldots + \hat{x}_n s_{n,j} + e_j$$

Here:

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Regression Based Reconstruction

$$a_j = \hat{x}_1 s_{1,j} + \hat{x}_2 s_{2,j} + \ldots + \hat{x}_n s_{n,j} + e_j$$

$$7 = 0.92 \cdot 1 + 0.11 \cdot 1 + 1.07 \cdot 0 + -0.08 \cdot 0 + \ldots + 0.07 \cdot 1 + 5.71$$

$$4 = 0.92 \cdot 0 + 0.11 \cdot 1 + 1.07 \cdot 1 + -0.08 \cdot 1 + \ldots + 0.07 \cdot 1 + 2.31$$

$$6 = 0.92 \cdot 0 + 0.11 \cdot 0 + 1.07 \cdot 0 + -0.08 \cdot 1 + \ldots + 0.07 \cdot 0 - 1.04$$

Here:

n	is the number of people in the database
m	is the number of queries
i	is a person index
j	is query index
a_j	is j-th query release
$s_{i,j}$	is a $\{0,1\}$-indicator $i \in S_j$
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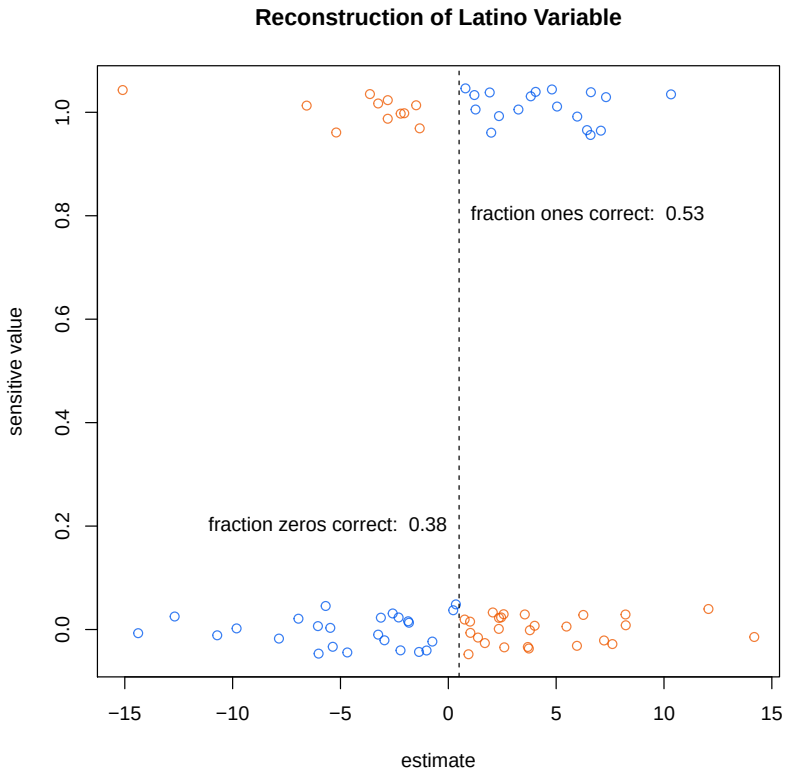
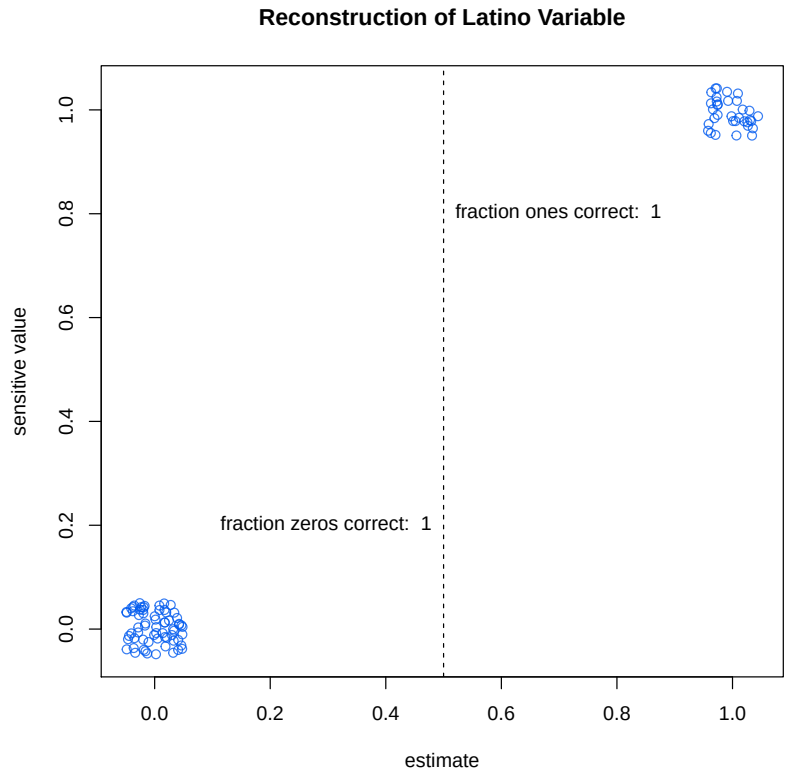
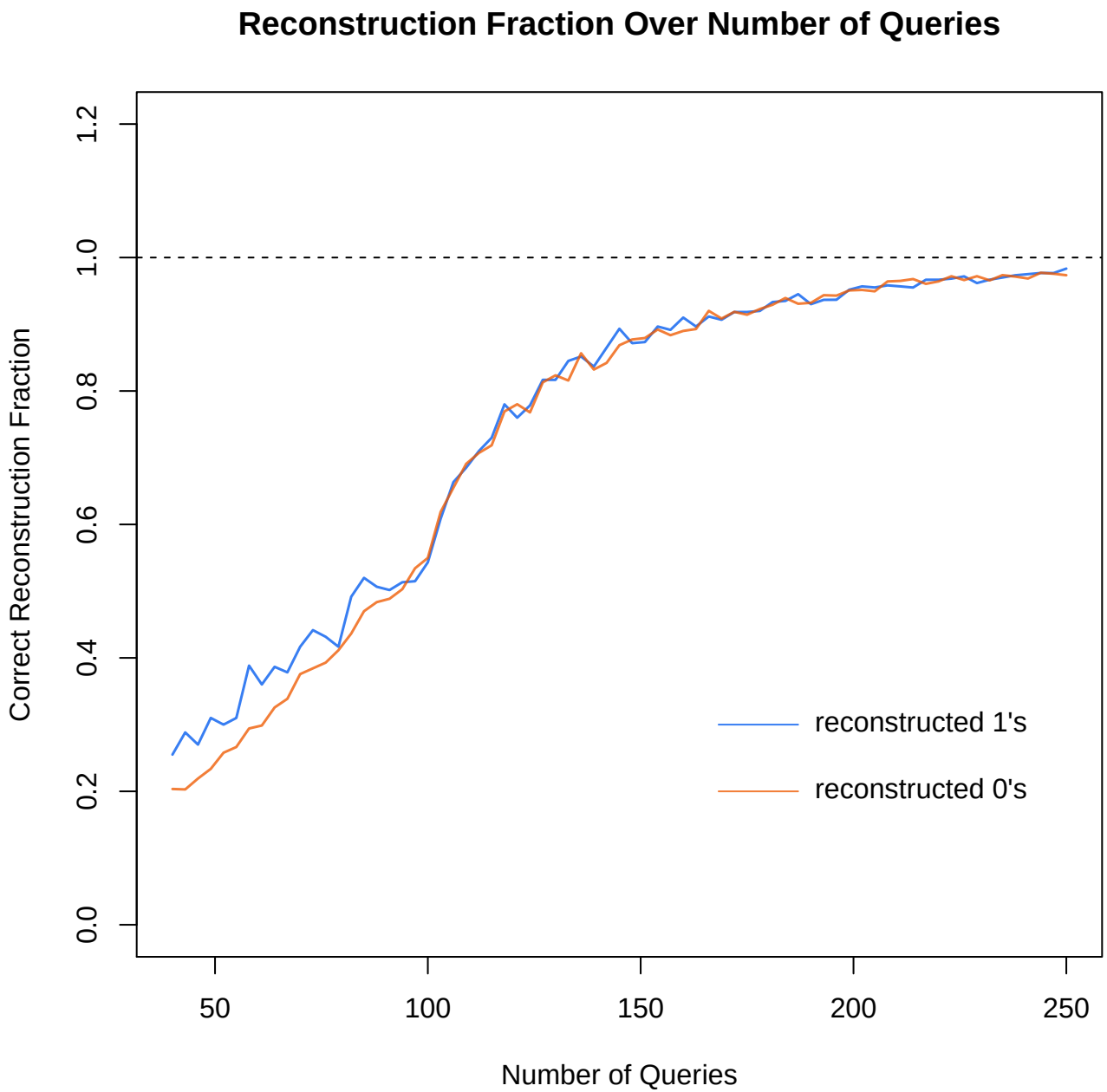
Example

From `wk2_regression_attack.ipynb`:

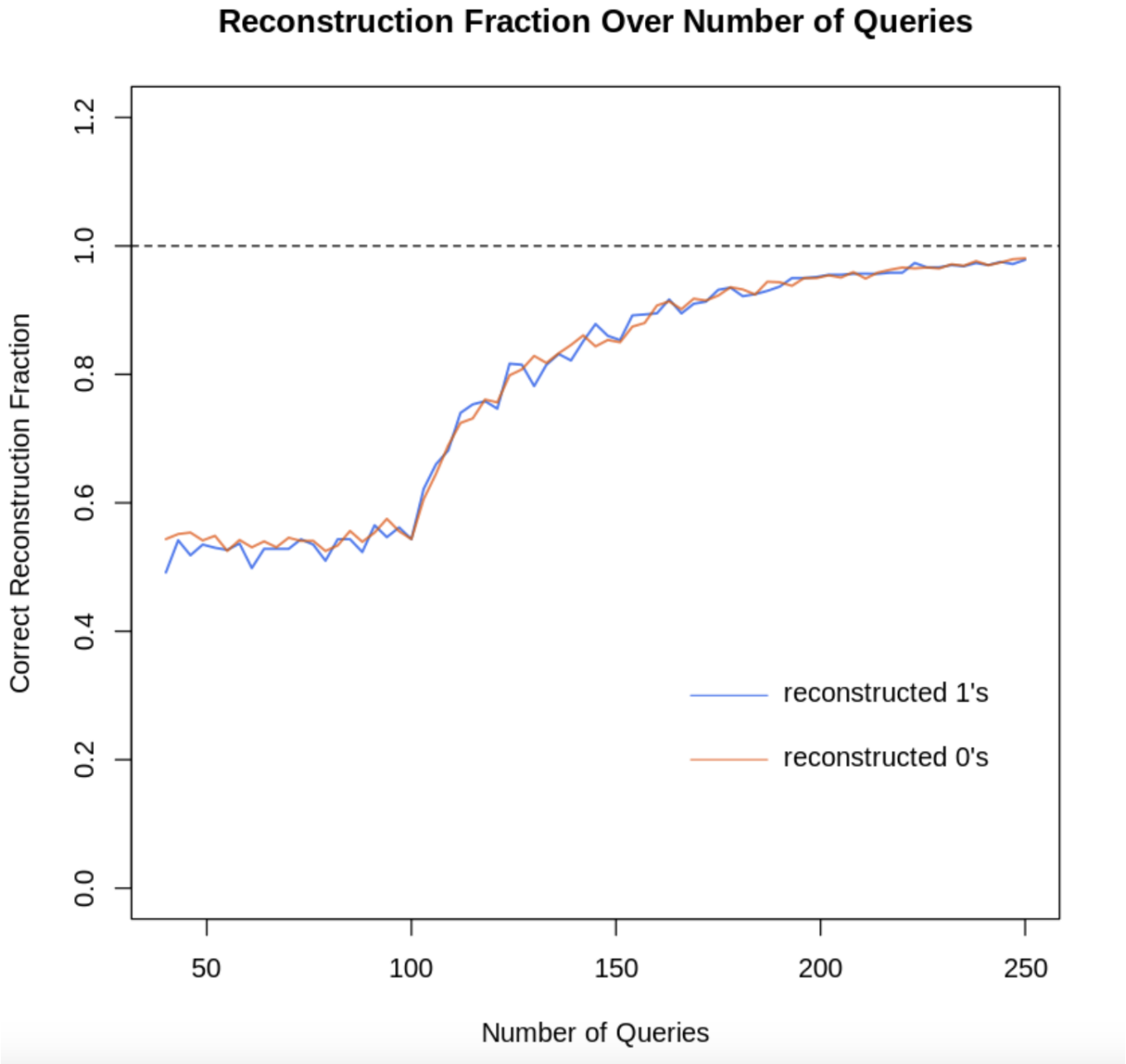




Example: Rounding to Nearest 5



Example: Rounding to Nearest 5 w/ Priors



Example: Normal Errors