Marginal Structural Models in One Period

Ian Lundberg

Learning goals for today

At the end of class, you will be able to:

1. Gain efficiency with marginal structural models
2. Recognize how that gain comes through information sharing
3. Understand stabilized weights

From $A \in \{0, 1\}$ to $A \in \{1, 2, 3, 4, 5\}$

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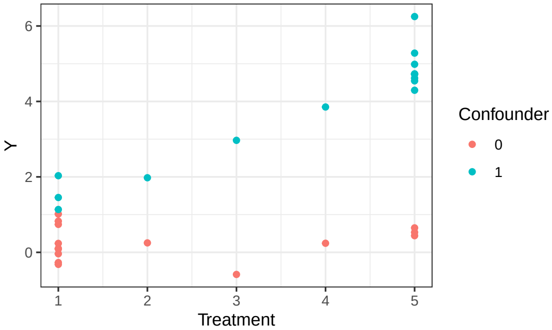

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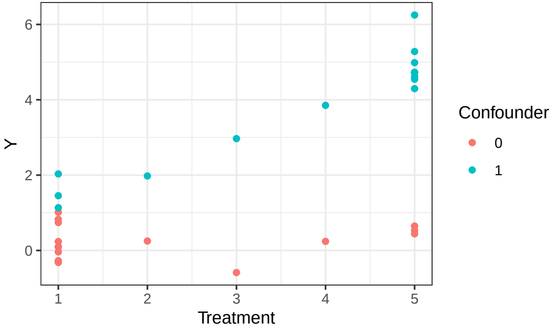
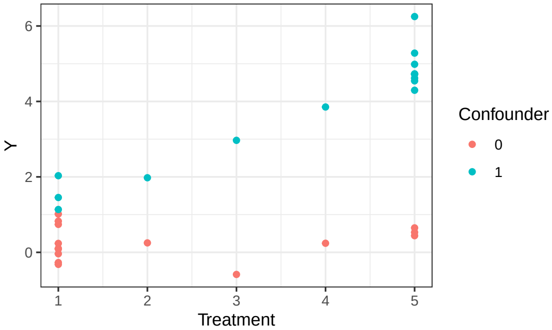


How to estimate $E(Y^3)$?

From $A \in \{0, 1\}$ to $A \in \{1, 2, 3, 4, 5\}$



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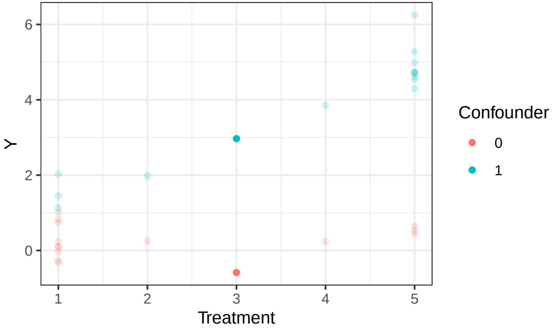

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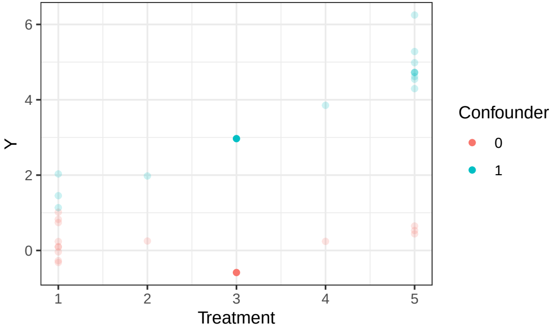

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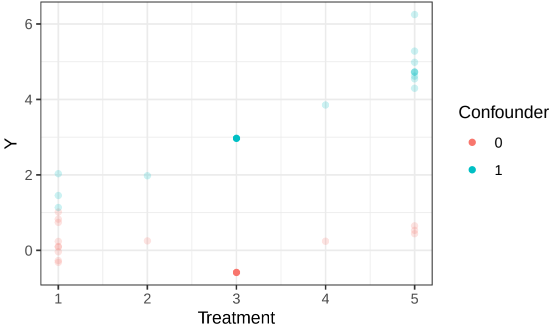

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Inverse probability weighting

But only 2 units! High variance!

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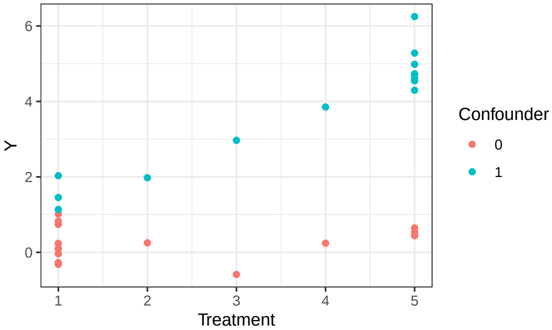

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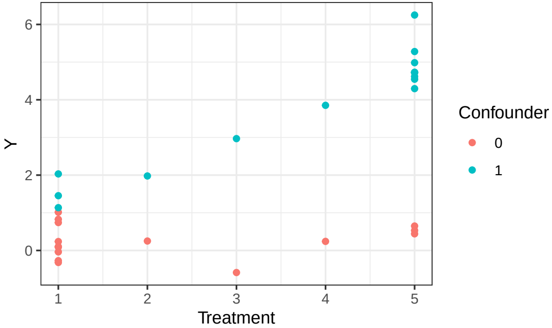

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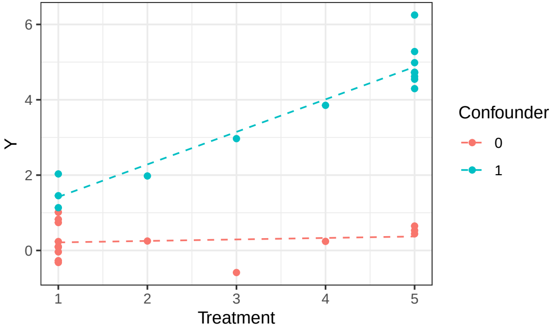

How to estimate $\text{E}(Y^3)$?

Outcome modeling

1) Fit a model for $\text{E}(Y \mid A, L)$

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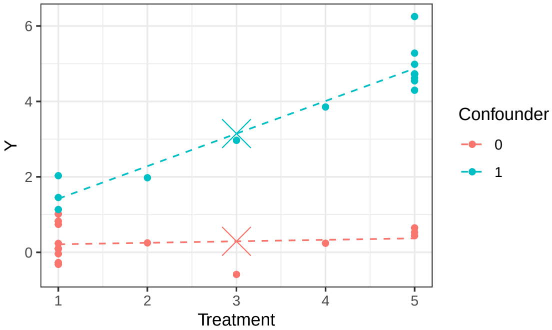

How to estimate $\mathrm{E}(Y^3)$?

Outcome modeling

1) Fit a model for $\mathrm{E}(Y \mid A, L)$
2) Predict at $A = 3$ in each group

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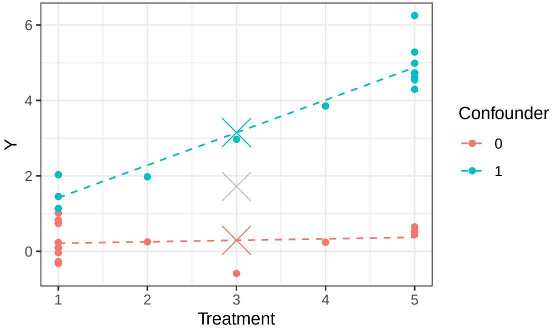

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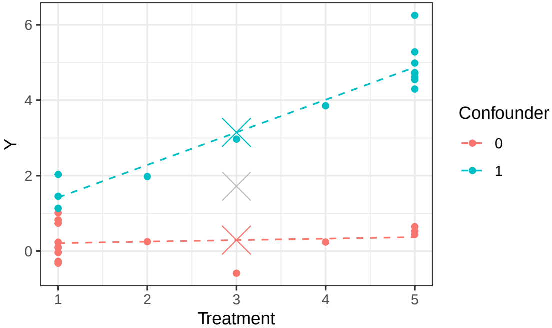

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But so much modeling!

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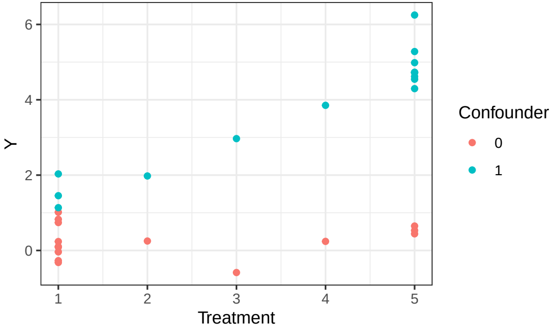

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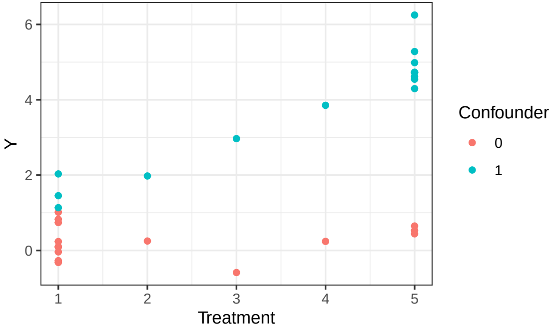

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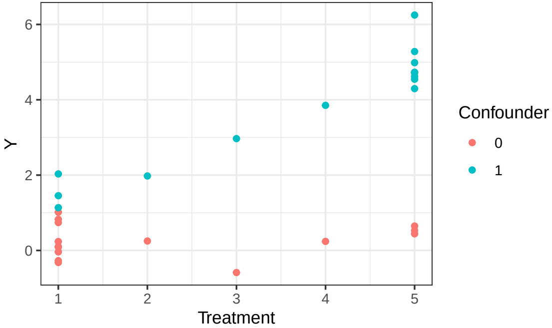

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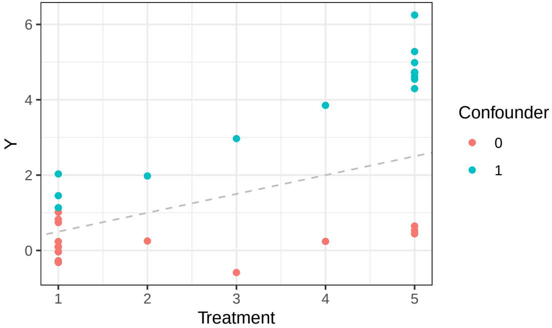

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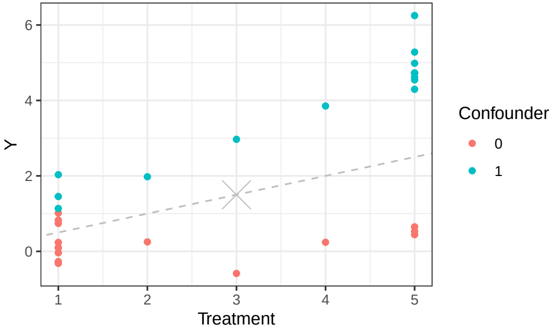

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1) Reweight to a pseudo-population (inverse probability weights)
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Reweight to a pseudo-population

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Reweight to a pseudo-population

$$\vec{L} \dashrightarrow A \longrightarrow Y$$

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- Effectively: Remove the dashed edge

In our pseudo-population, the mean given $A = a$ equals the expected outcome under an intervention to set $A = a$

$$\text{E}_{\text{PseudoPopulation}}(Y \mid A = a) = \text{E}(Y^a)$$

Marginal structural models

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To estimate:

$$E(Y^a) = E_{\text{PsuedoPopulation}}(Y \mid A = a) = \alpha + \beta a$$

This is OLS weighted to the pseudo-population

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$$\text{E}(Y^a) = f(a) \quad \text{for some simple function } f()$$

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4. Estimate $\hat{\text{E}}(Y^a)$: Weighted regression of Y on A, using $\hat{w}$

Stabilized weights

Standard weights can be high-variance: denominator is small

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This yields efficiency gains only for when the model for $\mathrm{E}(Y^a)$ is not saturated (Hernán & Robins p. 158)

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When A is continuous, pooling information over A is appealing.

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See Hernán & Robins 12.4.

Reading

Hernán & Robins 12.4 on marginal structural models

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