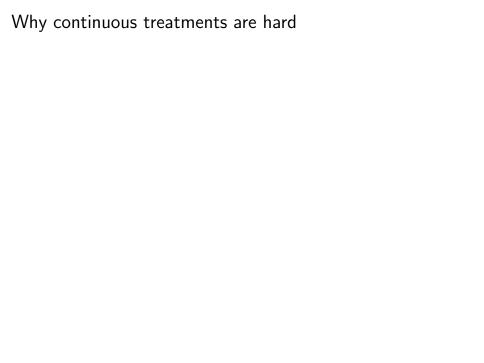
Continuous treatments: Brief introduction

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Learning goals

- 1. Define causal effects with continuous treatments
- 2. Understand an outcome modeling estimator
- 3. Select a causal estimand involving credible counterfactuals



Person 1

Person 2

Person 3

Person 4

| | Outcome under treatment value | | | | |
|----------|-------------------------------|---|--|--|--|
| | Untreated Treated | | | | |
| Person 1 | 0 | 0 | | | |
| Person 2 | 0 | 0 | | | |
| Person 3 | 0 | 0 | | | |
| Person 4 | 0 | 0 | | | |

| | Factual | Outcome under treatment value | | | |
|----------|-----------|-------------------------------|---------|--|--|
| | treatment | Untreated | Treated | | |
| Person 1 | Untreated | 0 | 0 | | |
| Person 2 | Treated | 0 | 0 | | |
| Person 3 | Treated | 0 | 0 | | |
| Person 4 | Untreated | 0 | 0 | | |

| | Factual | Outcome under treatment value | | | |
|----------|-----------|-------------------------------|---------|--|--|
| | treatment | Untreated | Treated | | |
| Person 1 | Untreated | • | 0 | | |
| Person 2 | Treated | 0 | • | | |
| Person 3 | Treated | 0 | • | | |
| Person 4 | Untreated | • | 0 | | |

| | Factual | Outcome under treatment value | | | | | |
|----------|-----------|-------------------------------|---|---|---|---|---|
| | treatment | 1 | 2 | 3 | 4 | 5 | |
| Person 1 | 3 | 0 | 0 | • | 0 | 0 | 0 |
| Person 2 | 2 | 0 | • | 0 | 0 | 0 | 0 |
| Person 3 | 5 | 0 | 0 | 0 | 0 | • | 0 |
| Person 4 | 4 | 0 | 0 | 0 | • | 0 | 0 |

| | Factual | Outcome under treatment value | | | | | |
|----------|-----------|-------------------------------|------|------|------|------|-----|
| | treatment | 1 | 2 | 3 | 4 | 5 | |
| Person 1 | 3 | 000 | 0000 | 0000 | 0000 | 0000 | 000 |
| Person 2 | 2 | 000 | 0000 | 0000 | 0000 | 0000 | 000 |
| Person 3 | 5 | 000 | 0000 | 0000 | 0000 | 0000 | 000 |
| Person 4 | 4 | 000 | 0000 | 0000 | 0000 | 0000 | 000 |

Solution: Parametric outcome model

$$\mathsf{E}(Y^a \mid \vec{X}) = \mathsf{E}(Y \mid A = a, \vec{X}) \qquad \text{by causal assumptions} \qquad (1)$$
$$= \alpha + \beta \mathbf{a} + \vec{X}' \vec{\gamma} \qquad \text{by a statistical model} \qquad (2)$$

Procedure:

- ► Model Y given A and \vec{X}
- ► Set *A* to the value of interest *a*
- ► Predict for all units
- ▶ Average to estimate $E(Y^a)$

Additive shift esitmands for credible counterfactuals

For some units, some treatment values are implausible

- $ightharpoonup ec{X} = {\sf child}$ has two parents with college degrees
- ► A = family income of \$10,000 per year
- ► *A* never occurs given $\vec{X} = \vec{x}$

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Additive shift estimands are plausible:

$$\tau_i = \mathsf{E}(Y^{A_i + \delta} - Y^{A_i})$$

Predict counterfactual outcome if treatment increases by δ

Recap: Continuous treatments are

- the same as categorical treatments in these ways
 - ► assume conditional exchangeability
 - ightharpoonup model Y given A, \vec{X}
 - ightharpoonup predict under counterfactual A = a
- ▶ different from categorical treatments in these ways
 - ▶ huge number of treatment values and thus potential outcomes
 - may require careful choice of a credible counterfactual