Continuous treatments: Brief introduction

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Soc 212C

Learning goals

1. Define causal effects with continuous treatments
2. Understand an outcome modeling estimator
3. Select a causal estimand involving credible counterfactuals

Why continuous treatments are hard

Why continuous treatments are hard

Person 1

Person 2

Person 3

Person 4

Why continuous treatments are hard

	Outcome under treatment value	
	Untreated	Treated
Person 1	$\circ$	$\circ$
Person 2	$\circ$	$\circ$
Person 3	$\circ$	$\circ$
Person 4	$\circ$	$\circ$

Why continuous treatments are hard

	Factual treatment	Outcome under treatment value	
		Untreated	Treated
Person 1	Untreated	$\circ$	$\circ$
Person 2	Treated	$\circ$	$\circ$
Person 3	Treated	$\circ$	$\circ$
Person 4	Untreated	$\circ$	$\circ$

Why continuous treatments are hard

	Factual treatment	Outcome under treatment value	
		Untreated	Treated
Person 1	Untreated	•	○
Person 2	Treated	○	•
Person 3	Treated	○	•
Person 4	Untreated	•	○

Why continuous treatments are hard

	Factual treatment	Outcome under treatment value					
		1	2	3	4	5	...
Person 1	3	○	○	●	○	○	○
Person 2	2	○	●	○	○	○	○
Person 3	5	○	○	○	○	●	○
Person 4	4	○	○	○	●	○	○

Why continuous treatments are hard

	Factual treatment	Outcome under treatment value					
		1	2	3	4	5	...
Person 1	3	○○○○○○○○●○○○○○○○○○○○○					
Person 2	2	○○○○●○○○○○○○○○○○○○○○○					
Person 3	5	○○○○○○○○○○○○○○○○●○○○○					
Person 4	4	○○○○○○○○○○○○●○○○○○○○○					

Solution: Parametric outcome model

$$
\begin{aligned}
\mathrm{E}(Y^a \mid \vec{X}) &= \mathrm{E}(Y \mid A = a, \vec{X}) && \text{by causal assumptions} && (1) \\
&= \alpha + \beta a + \vec{X}'\vec{\gamma} && \text{by a statistical model} && (2)
\end{aligned}
$$

Procedure:

- Model Y given A and $\vec{X}$
- Set A to the value of interest a
- Predict for all units
- Average to estimate $\mathrm{E}(Y^a)$

Additive shift esitmands for credible counterfactuals

For some units, some treatment values are implausible

- $\vec{X} =$ child has two parents with college degrees
- $A =$ family income of \$10,000 per year
- A never occurs given $\vec{X} = \vec{x}$

Additive shift esitmands for credible counterfactuals

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- $\vec{X}$ = child has two parents with college degrees
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Additive shift estimands are plausible:

$$\tau_i = \text{E}(Y^{A_i+\delta} - Y^{A_i})$$

Predict counterfactual outcome if treatment increases by δ

Recap: Continuous treatments are

- the same as categorical treatments in these ways
  - assume conditional exchangeability
  - model Y given $A, \vec{X}$
  - predict under counterfactual $A = a$
- different from categorical treatments in these ways
  - huge number of treatment values and thus potential outcomes
  - may require careful choice of a credible counterfactual