Panel Data

Difference in difference
Interrupted time series
Regression discontinuity
Synthetic control

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Soc 212B
Winter 2025

8 Nov 2022

Learning goals for today

At the end of class, you will be able to:

1. Recognize the promises and pitfalls of four methods to study the effects of treatments that turn on once
   1.1 Difference in difference (DID)
   1.2 Interrupted time series (ITS)
   1.3 Regression discontinuity (RD)
   1.4 Synthetic control (SC)

Card, D., & Krueger, A. B. (1994).
Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania.
The American Economic Review, 84(4), 772-793.

Economic theory

When the minimum wage rises, how might employment change?

Economic theory

When the minimum wage rises, how might employment change?

- employees cost more

Economic theory

When the minimum wage rises, how might employment change?

- employees cost more
- employers might get by with fewer employees

The setting

The setting

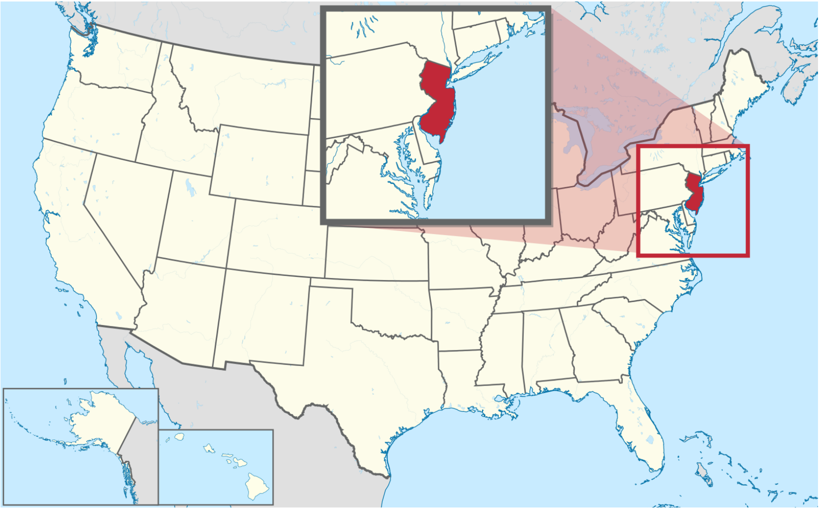
- Federal minimum wage
  - \$3.80 on April 1, 1990
  - \$4.25 on April 1, 1991

The setting

- Federal minimum wage
  - \$3.80 on April 1, 1990
  - \$4.25 on April 1, 1991
- New Jersey minimum wage
  - \$5.05 on April 1, 1992

NJ introduces a high minimum wage.
How would you study the effect on employment?

Source: Wikimedia




New Jersey
Minimum Wage
Rose


New Jersey
Minimum Wage
Rose
Pennsylvania
No change

171 stores	80 stores	99 stores	60 stores

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Phone interview: Feb-Mar 1992 before minimum wage rose
Nov-Dec 1992 after minimum wage rose

171 stores	80 stores	99 stores	60 stores

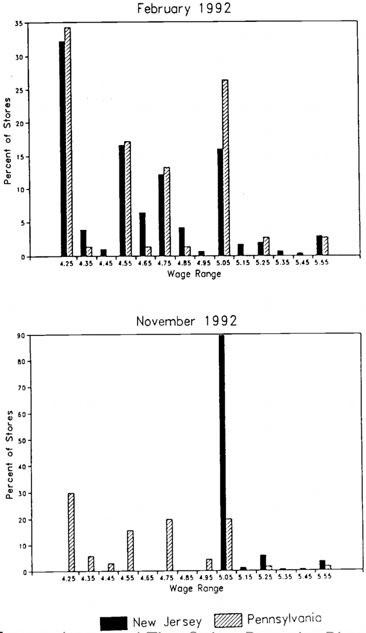
Phone interview: Feb-Mar 1992 before minimum wage rose

Nov-Dec 1992 after minimum wage rose

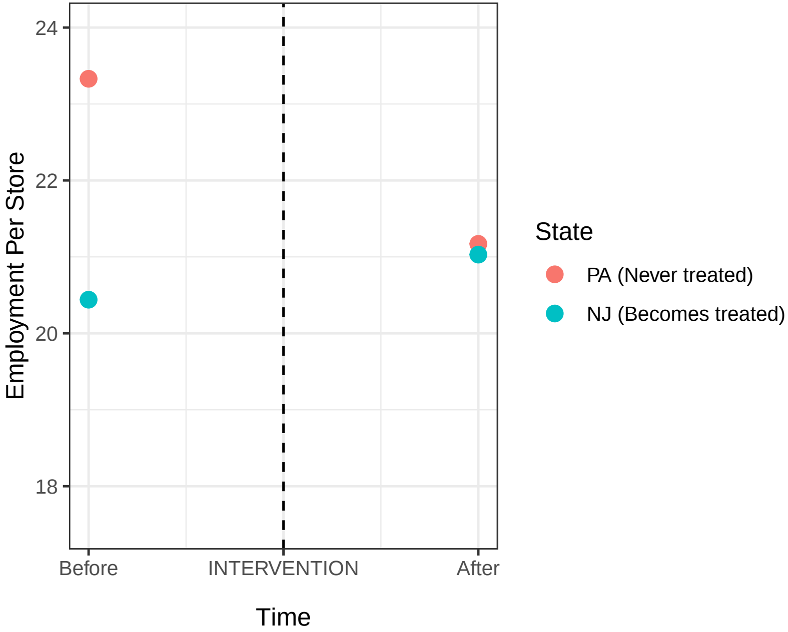
Recorded: How many full-time equivalent employees?

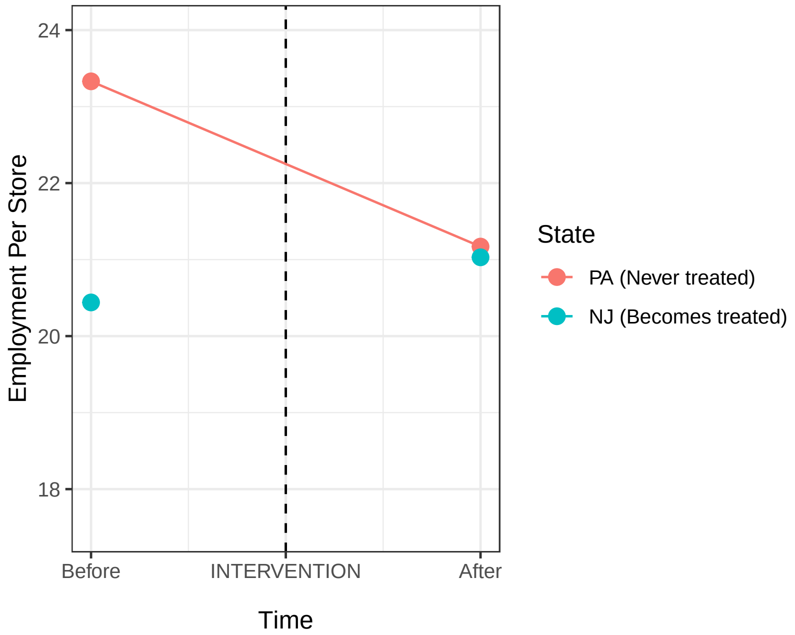
Did starting wages rise in NJ?

Figure 1
Distribution of Starting Wage Rates



How did employment change?


Employment Per Store
24
22
20
18
Before
INTERVENTION
After
Time
State
PA (Never treated)
NJ (Becomes treated)



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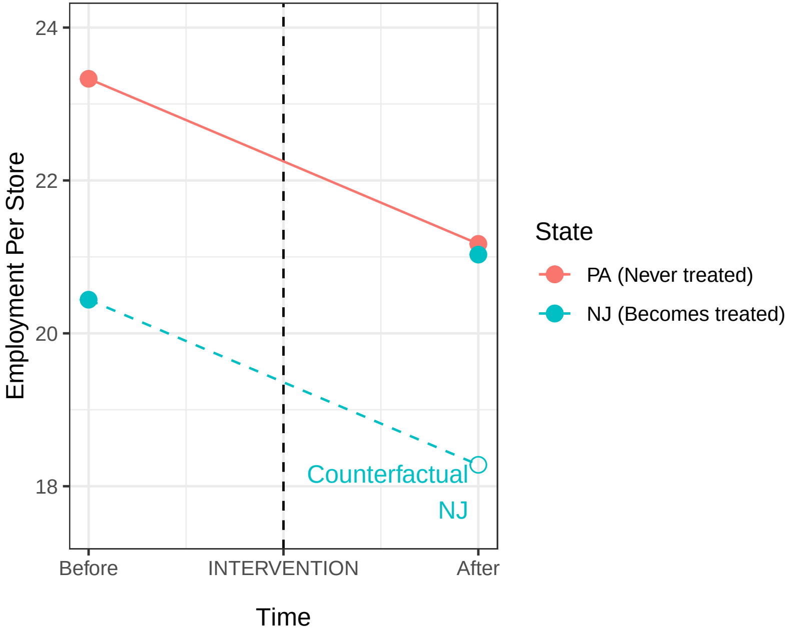
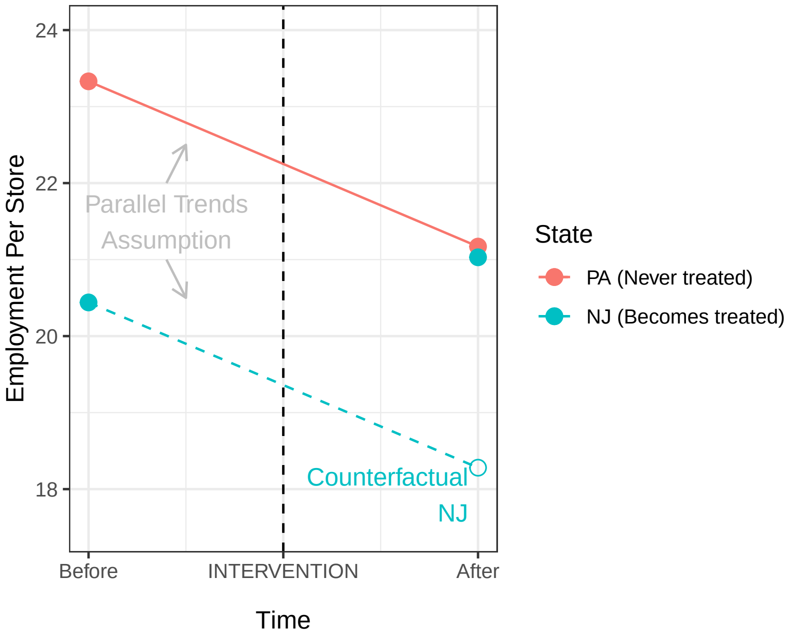
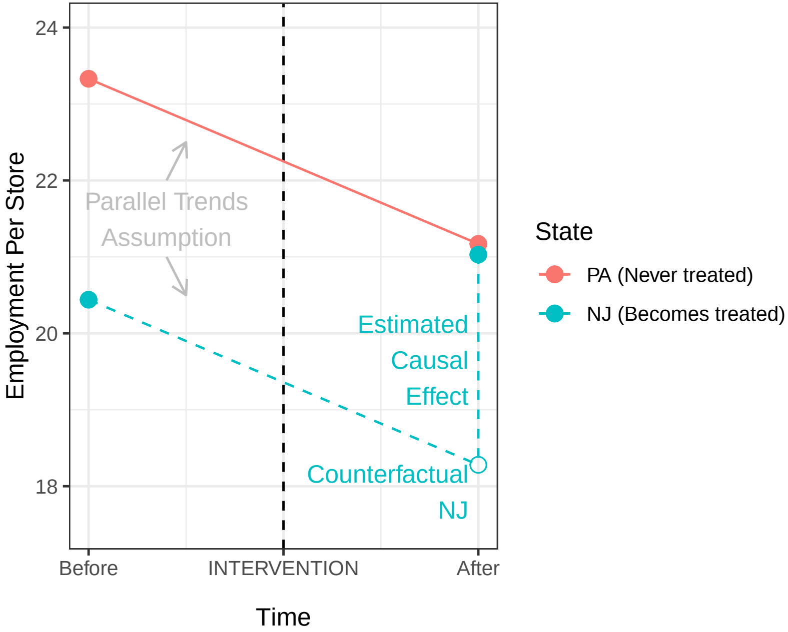


24
22
20
18
Employment Per Store
Counterfactual
NJ
Before
INTERVENTION
After
Time
State
PA (Never treated)
NJ (Becomes treated)



24
22
20
18
Employment Per Store
Parallel Trends
Assumption
Counterfactual
NJ
Before
INTERVENTION
After
Time
State
PA (Never treated)
NJ (Becomes treated)


Employment Per Store
24
22
20
18
Parallel Trends
Assumption
Estimated
Causal
Effect
Counterfactual
NJ
Before
INTERVENTION
After
Time
State
PA (Never treated)
NJ (Becomes treated)

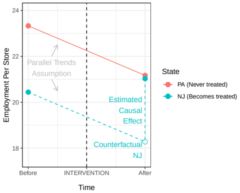
Variable	Stores by state PA (i)	NJ (ii)	Difference, NJ – PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	–2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	–0.14 (1.07)
3. Change in mean FTE employment	–2.16 (1.25)	0.59 (0.54)	2.76 (1.36)

"Contrary to the central prediction of the textbook model of the minimum wage,...we find no evidence that the rise in New Jersey's minimum wage reduced employment at fast-food restaurants in the state."

Card & Krueger 1994, p. 792

- simple study
- well-executed
- upended conventional wisdom

Key assumption: Parallel trends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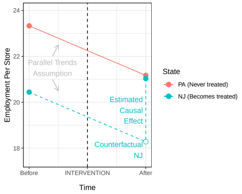
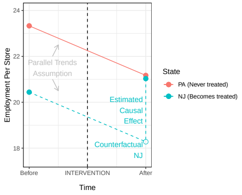
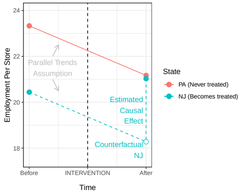


Parallel trends assumption:

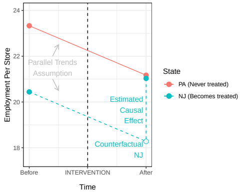


Parallel trends assumption: If no law had taken effect, then



Parallel trends assumption: If no law had taken effect, then

$$\overbrace{Y^0_{\text{NJ,After}} - Y^0_{\text{NJ,Before}}}^{\text{the trend in NJ}} \overbrace{=}^{\text{would equal}} \overbrace{Y^0_{\text{PA,After}} - Y^0_{\text{PA,Before}}}^{\text{the trend in PA}}$$



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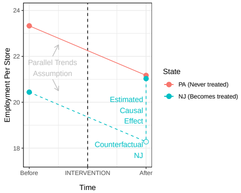
Rearranging yields a formula for the counterfactual outcome



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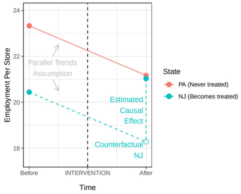
$$\overbrace{Y^0_{\text{NJ,After}} - Y^0_{\text{NJ,Before}}}^{\text{the trend in NJ}} \overbrace{=}^{\text{would equal}} \overbrace{Y^0_{\text{PA,After}} - Y^0_{\text{PA,Before}}}^{\text{the trend in PA}}$$

Rearranging yields a formula for the counterfactual outcome

$$\underbrace{Y^0_{\text{NJ,After}}}_{\text{Counterfactual}} \overbrace{=}^{\text{By Assumption}}$$



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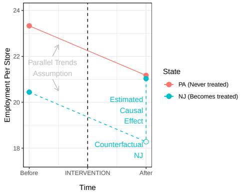
$$\underbrace{Y^0_{\text{NJ,After}}}_{\text{Counterfactual}} \overbrace{=}^{\text{By Assumption}} \underbrace{Y^0_{\text{NJ,Before}} + Y^0_{\text{PA,After}} - Y^0_{\text{PA,Before}}}_{\text{Factual}}$$



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$$\underbrace{Y^0_{\text{NJ,After}}}_{\text{Counterfactual}} \overbrace{=}^{\text{By Assumption}} \underbrace{Y^0_{\text{NJ,Before}} + Y^0_{\text{PA,After}} - Y^0_{\text{PA,Before}}}_{\text{Factual}}$$

$$\text{Effect in NJ} = \underbrace{Y^1_{\text{NJ,After}}}_{\text{Observed}} - \underbrace{Y^0_{\text{NJ,After}}}_{\text{Estimated by Above}}$$

Can we test the parallel trends assumption?

$$\overbrace{Y^0_{\text{NJ,After}} - Y^0_{\text{NJ,Before}}}^{\text{Assumption: The trend in NJ}} \overbrace{=}^{\text{would equal}} \overbrace{Y^0_{\text{PA,After}} - Y^0_{\text{PA,Before}}}^{\text{the trend in PA}}$$

Can we test the parallel trends assumption?

No.

$$\overbrace{\underbrace{Y^0_{\text{NJ,After}}}_{\text{Not Observable}} - Y^0_{\text{NJ,Before}}}^{\text{Assumption: The trend in NJ}} \overbrace{=}^{\text{would equal}} \overbrace{Y^0_{\text{PA,After}} - Y^0_{\text{PA,Before}}}^{\text{the trend in PA}}$$

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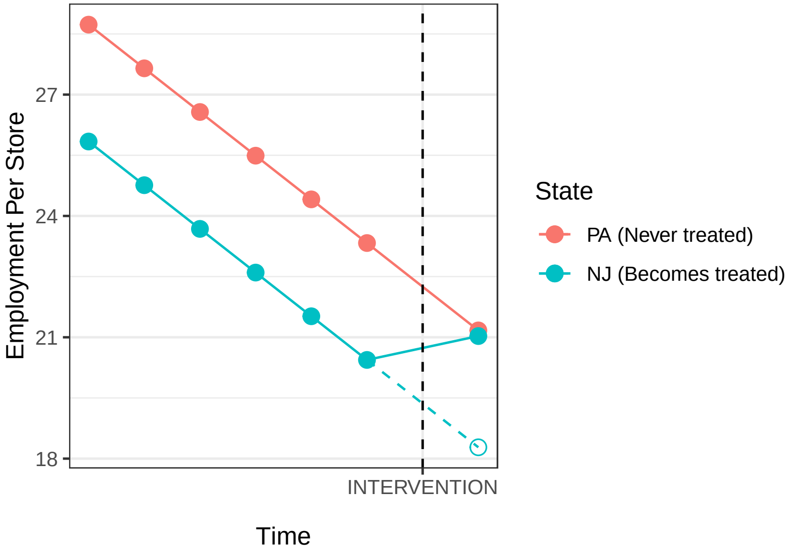
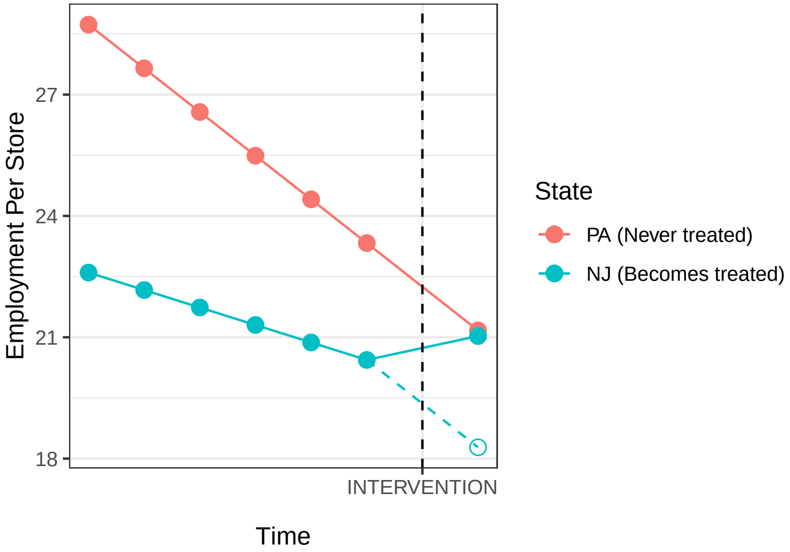
You can make it credible by looking at many pre-treatment periods

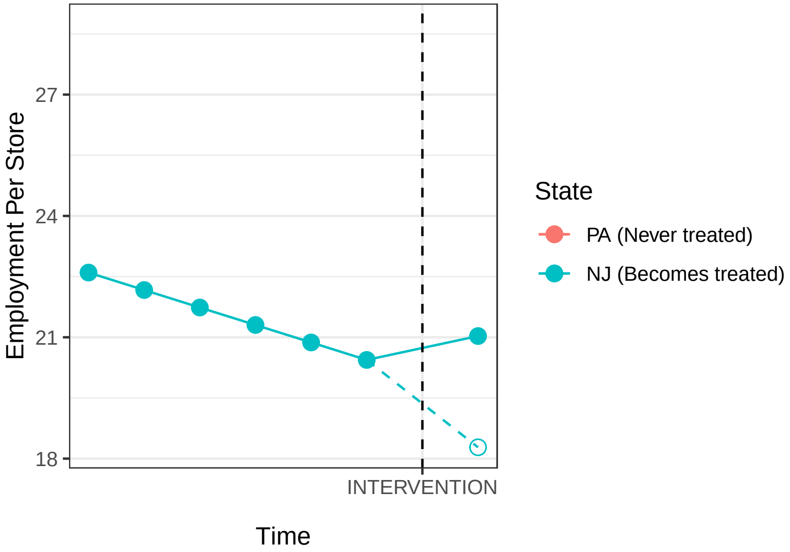
DID would be very credible



DID would be very doubtful



DID would be very doubtful



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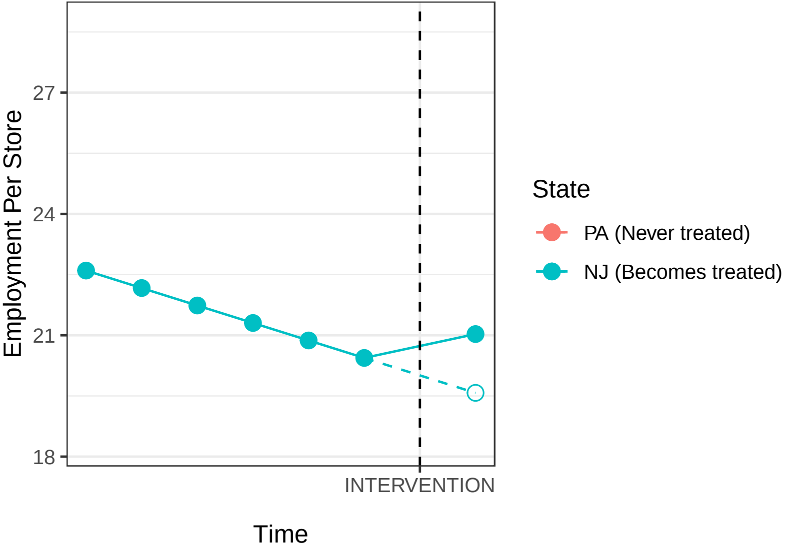
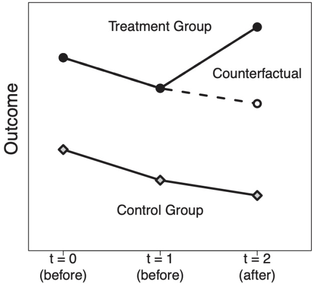
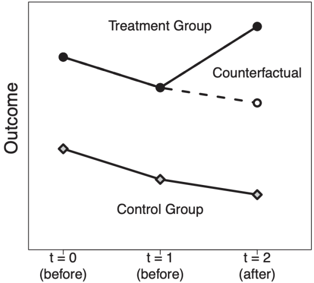


Egami, N., & Yamauchi, S. (2023).
Using multiple pretreatment periods to improve difference-in-differences and staggered adoption designs.
Political Analysis, 31(2), 195-212.

Difference in difference



Difference in difference

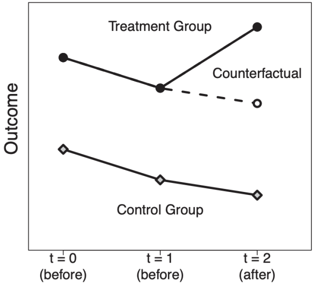


Notation

$Y^{\text{treatment value}}_{\text{(unit)(time)}}$

Example: Y^0_{i1}
is unit i at time 1
under treatment 0

Difference in difference



Parallel Trends Assumption (untestable)

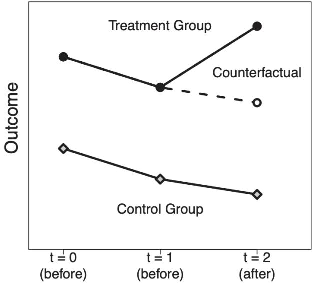
$$E(Y^0_{\text{Treated},2} - Y^0_{\text{Treated},1}) = E(Y^0_{\text{Control},2} - Y^0_{\text{Control},1})$$

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Example: Y^0_{i1} is unit i at time 1 under treatment 0

Difference in difference



Notation

$$Y_{\text{(unit)(time)}}^{\text{treatment value}}$$

Example: Y_{i1}^{0}
is unit i at time 1
under treatment 0

Parallel Trends Assumption (untestable)

$$E(Y^0_{\text{Treated},2} - Y^0_{\text{Treated},1}) = E(Y^0_{\text{Control},2} - Y^0_{\text{Control},1})$$

Extended Parallel Trends (testable)

$$E(Y^0_{\text{Treated},1} - Y^0_{\text{Treated},0}) = E(Y^0_{\text{Control},1} - Y^0_{\text{Control},0})$$

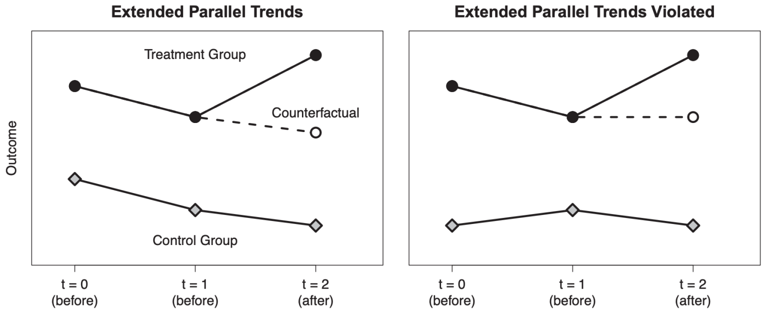
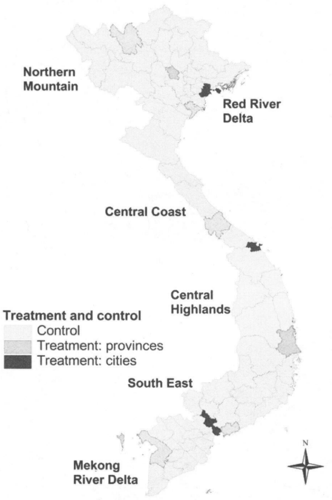

Extended Parallel Trends
Treatment Group
Counterfactual
Control Group
Outcome
t = 0
(before)
t = 1
(before)
t = 2
(after)
Extended Parallel Trends Violated
t = 0
(before)
t = 1
(before)
t = 2
(after)

FIGURE 2. Map of Treatment Provinces and National-Level Cities



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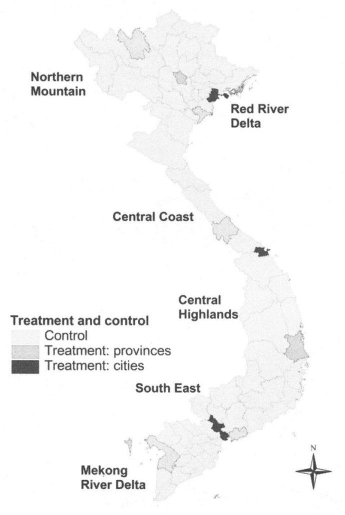
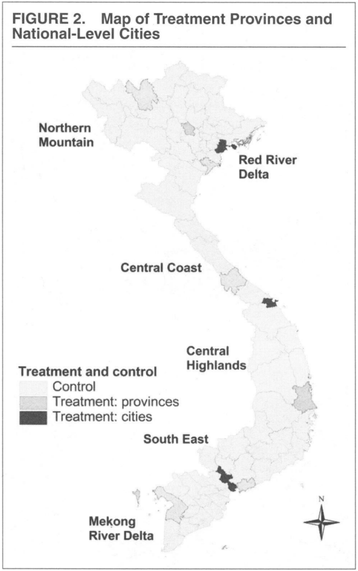


Outcome 1
Education and cultural programs

Is there the following project in the commune?

> Investment on culture and education



Outcome 2
Tap water

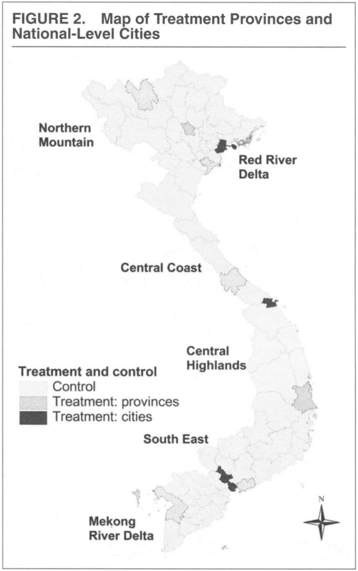
Is there the following project in the commune?

Coded 1

Indoor private piped water
Outdoor private piped water
Public piped water

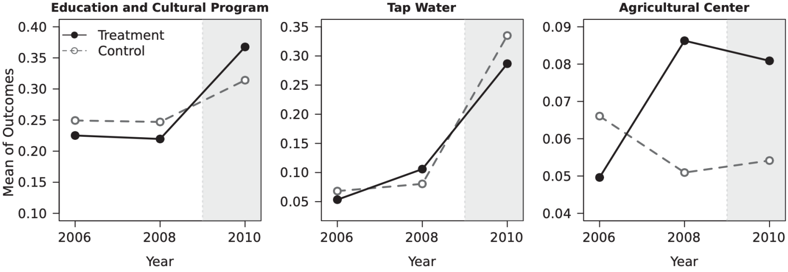
Coded 0

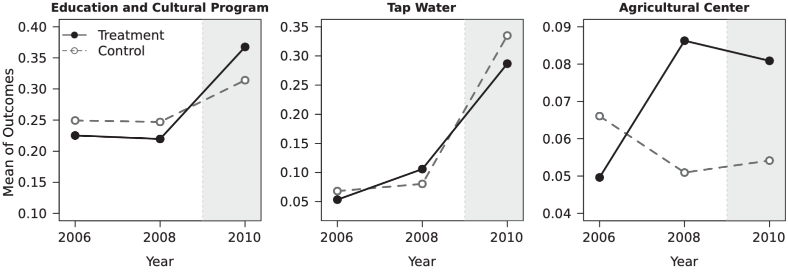
Well water
Well with protection walls
Well without protection walls
Stream water with protection
Stream water without protection
Rainwater
Bottled water
Water brought by pedicab
Tank water
river lake pond



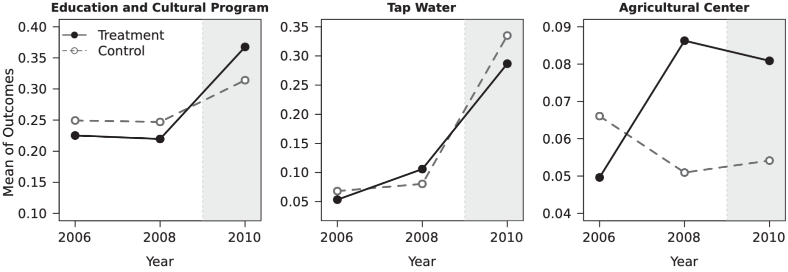
Outcome 3
Agricultural center

Is there any agriculture extension center in this commune?


Education and Cultural Program
Tap Water
Agricultural Center
Treatment
Control
Mean of Outcomes
Year
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.09
0.08
0.07
0.06
0.05
0.04
2006
2008
2010



In each case, do you believe parallel trends?



In each case, do you believe parallel trends?

Table 2. Assessing underlying assumptions using the pretreatment outcomes.

	Estimate	Std. error	*p*-value	95% Std. equivalence CI
Education and cultural program	−0.007	0.096	0.940	[−0.166, 0.166]
Tap water	0.166	0.083	0.045	[−0.302, 0.302]
Agricultural center	0.198	0.082	0.015	[−0.332, 0.332]

Benefit 1: Assessing assumptions

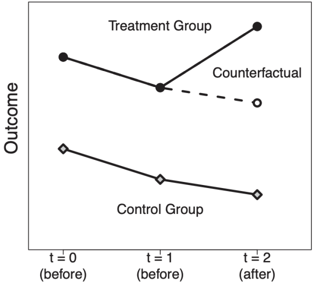
Pre-treatment periods enable us to
assess underlying ssumptions

Parallel trends is untestable, but being parallel
in the pre-treatment period builds confidence

Benefit 2: Improving efficiency

Pre-treatment periods also enable us to
improve estimation accuracy
when parallel trends holds

Benefit 2: Improving efficiency



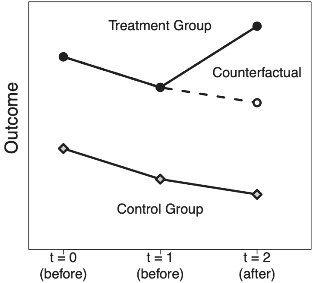
Estimator 1

Estimator 2

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Benefit 2: Improving efficiency



Estimator 1

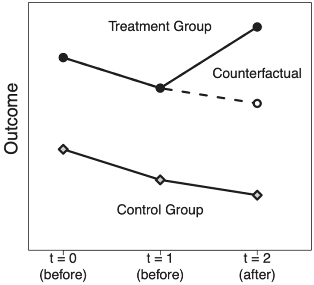
$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T1})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 1}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C1})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 1}}}$$

Estimator 2

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Benefit 2: Improving efficiency



Estimator 1

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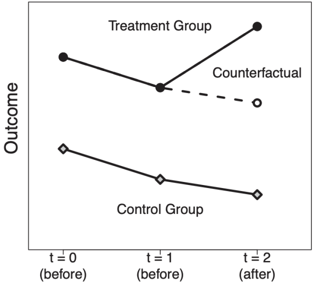
Estimator 2

$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T0})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 0}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C0})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 0}}}$$

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Benefit 2: Improving efficiency



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Estimator 2

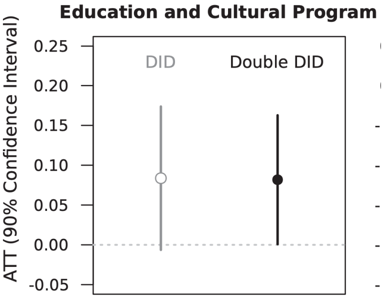
$$\underbrace{(\bar{Y}^1_{T2} - \bar{Y}^0_{T0})}_{\substack{\text{Treatment Group} \\ \text{Time 2 - Time 0}}} - \underbrace{(\bar{Y}^0_{C2} - \bar{Y}^0_{C0})}_{\substack{\text{Control Group} \\ \text{Time 2 - Time 0}}}$$

Notation

$$Y^{\text{treatment value}}_{\text{(unit)(time)}}$$

Pooled estimator:
Average the two!

Benefit 2: Improving efficiency

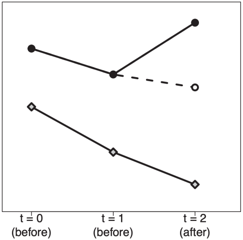


Benefit 3: A more flexible assumption

Pre-treatment periods make it possible to
allow for a more flexible parallel trends assumption

Benefit 3: A more flexible assumption

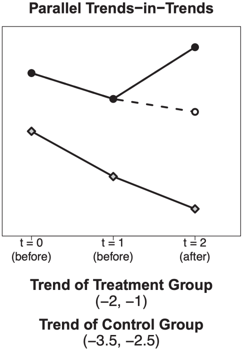
Parallel Trends−in−Trends



Trend of Treatment Group
(−2, −1)

Trend of Control Group
(−3.5, −2.5)

Benefit 3: A more flexible assumption

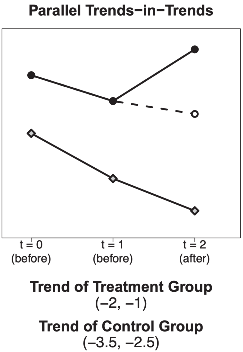


ASSUMPTION 3 (Parallel Trends-in-Trends)

$$\underbrace{\{\mathbb{E}[Y_{i2}(0) \mid G_i = 1] - \mathbb{E}[Y_{i1}(0) \mid G_i = 1]\}}_{\text{Trend of the treatment group from } t=1 \text{ to } t=2} - \underbrace{\{\mathbb{E}[Y_{i1}(0) \mid G_i = 1] - \mathbb{E}[Y_{i0}(0) \mid G_i = 1]\}}_{\text{Trend of the treatment group from } t=0 \text{ to } t=1}$$

$$= \underbrace{\{\mathbb{E}[Y_{i2}(0) \mid G_i = 0] - \mathbb{E}[Y_{i1}(0) \mid G_i = 0]\}}_{\text{Trend of the control group from } t=1 \text{ to } t=2} - \underbrace{\{\mathbb{E}[Y_{i1}(0) \mid G_i = 0] - \mathbb{E}[Y_{i0}(0) \mid G_i = 0]\}}_{\text{Trend of the control group from } t=0 \text{ to } t=1}.$$

Benefit 3: A more flexible assumption



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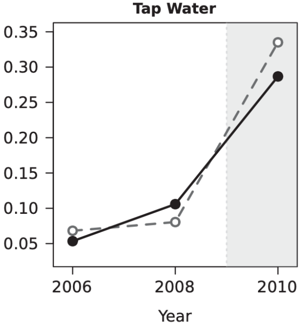
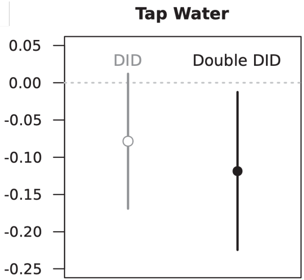
Sequential DID Estimator

$$\widehat{\tau}_{\texttt{s-DID}} = \left\{\left(\frac{\sum_{i:\, G_i=1} Y_{i2}}{n_{12}} - \frac{\sum_{i:\, G_i=1} Y_{i1}}{n_{11}}\right) - \left(\frac{\sum_{i:\, G_i=0} Y_{i2}}{n_{02}} - \frac{\sum_{i:\, G_i=0} Y_{i1}}{n_{01}}\right)\right\}$$
$$- \left\{\left(\frac{\sum_{i:\, G_i=1} Y_{i1}}{n_{11}} - \frac{\sum_{i:\, G_i=1} Y_{i0}}{n_{10}}\right) - \left(\frac{\sum_{i:\, G_i=0} Y_{i1}}{n_{01}} - \frac{\sum_{i:\, G_i=0} Y_{i0}}{n_{00}}\right)\right\},$$

Benefit 3: A more flexible assumption





Benefits of multiple pre-treatment periods

1. assess underlying assumptions
2. improve estimation accuracy
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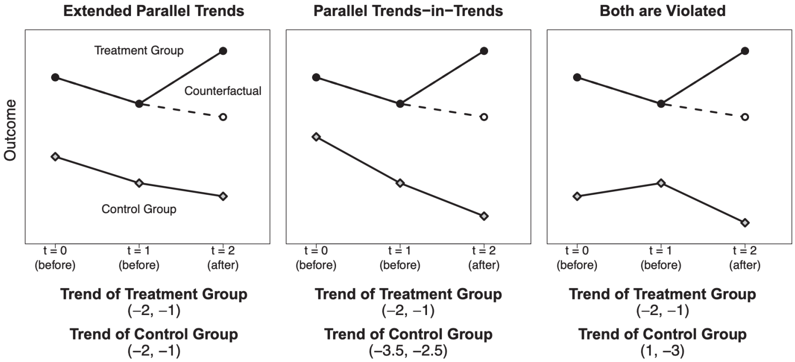

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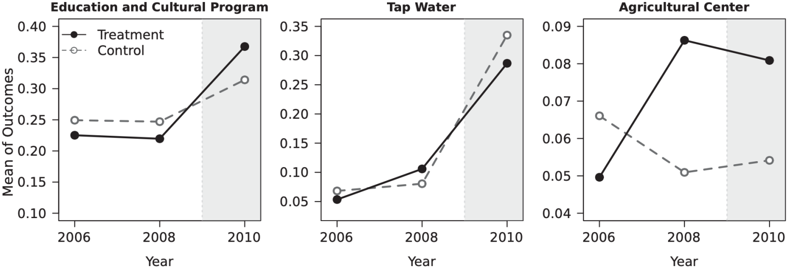
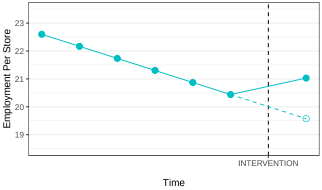


Interrupted time series[1]

[1] Bernal, J. L., Cummins, S., & Gasparrini, A. (2017). Interrupted time series regression for the evaluation of public health interventions: A tutorial. International Journal of Epidemiology, 46(1), 348-355.

Interrupted time series[1]

You study one unit. It is untreated. Then it is treated.



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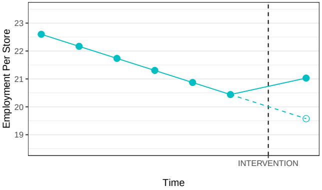

In what settings does this work well?

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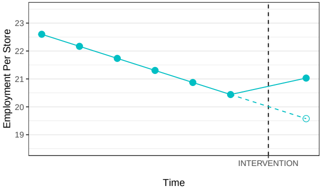

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- When you have a strong pre-treatment trend to forecast Y_t^0

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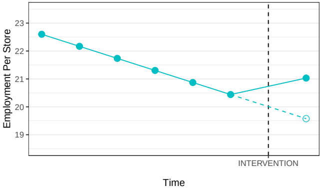
You study one unit. It is untreated. Then it is treated.



In what settings does this work well?

- When you have a strong pre-treatment trend to forecast Y_t^0
- When you don't have a comparable unit that is never treated

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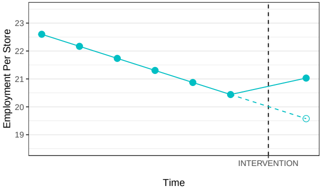

Theoretical Estimand

$$\mathrm{E}(Y^1 - Y^0 \mid T > t_{\text{Intervention}})$$

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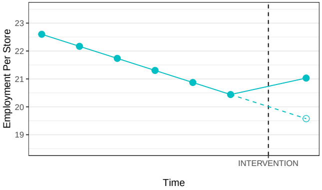

Identifying Assumption

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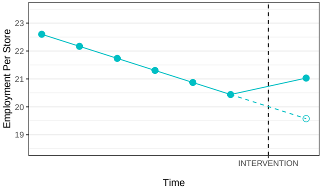

Identifying Assumption

- In the absence of the intervention, the pre-intervention trend in Y^0 would have continued

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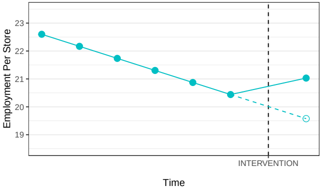

Concrete steps:

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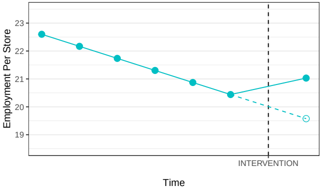

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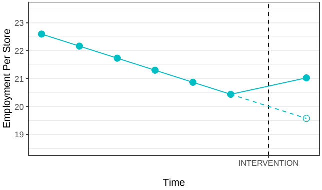

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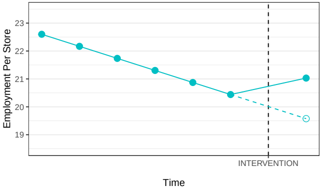

Concrete steps:

1. Learn a model on the pre-treatment period
   - Evaluation metric: Forecast within the pre-treatment period
2. Forecast Y^0 for the post-treatment period

[1] Bernal, J. L., Cummins, S., & Gasparrini, A. (2017). Interrupted time series regression for the evaluation of public health interventions: A tutorial. International Journal of Epidemiology, 46(1), 348-355.

Interrupted time series: When it becomes doubtful

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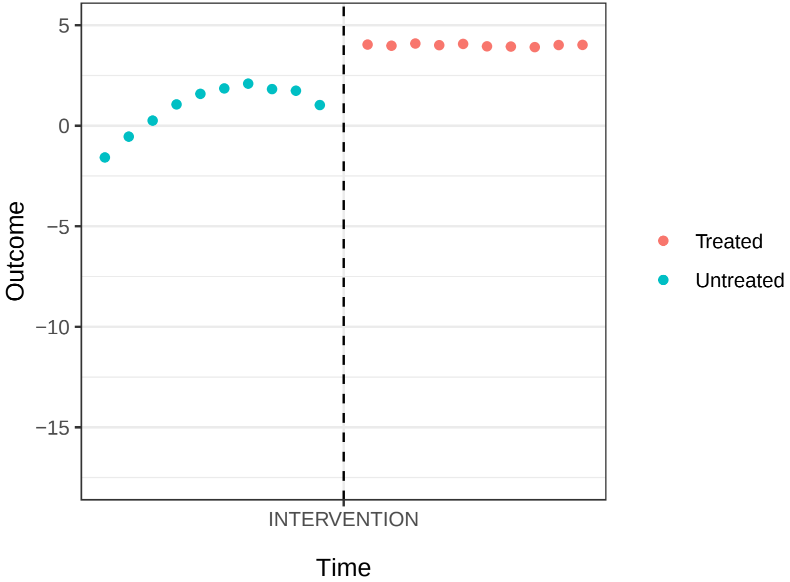

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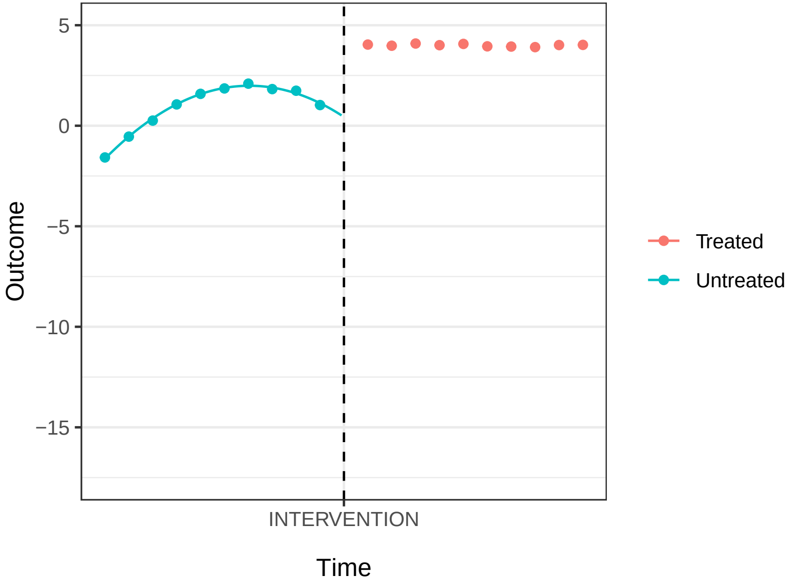

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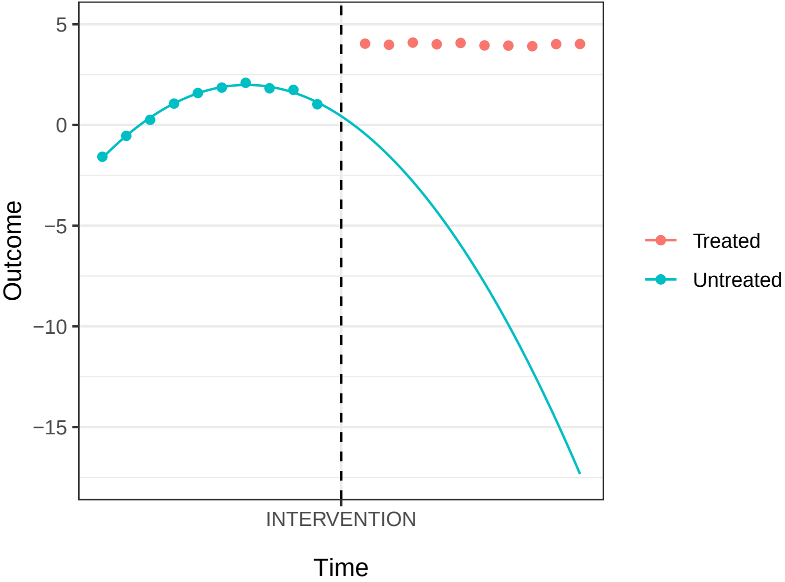

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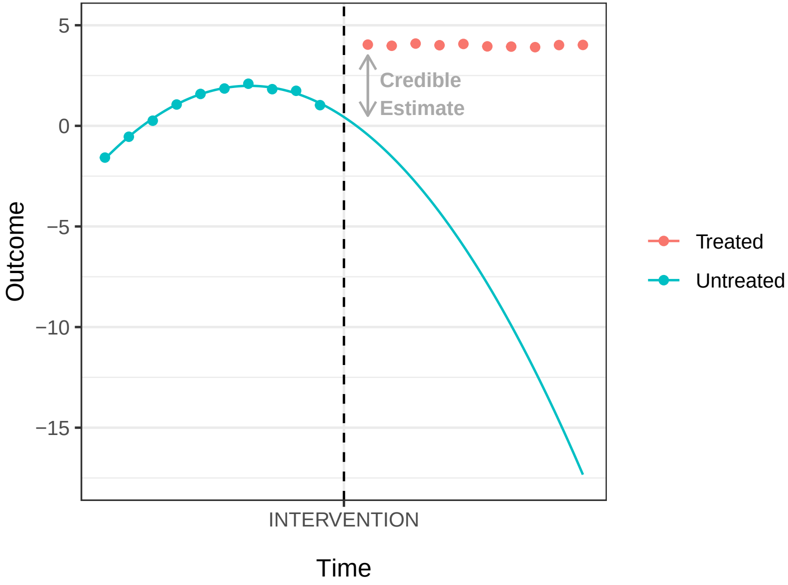

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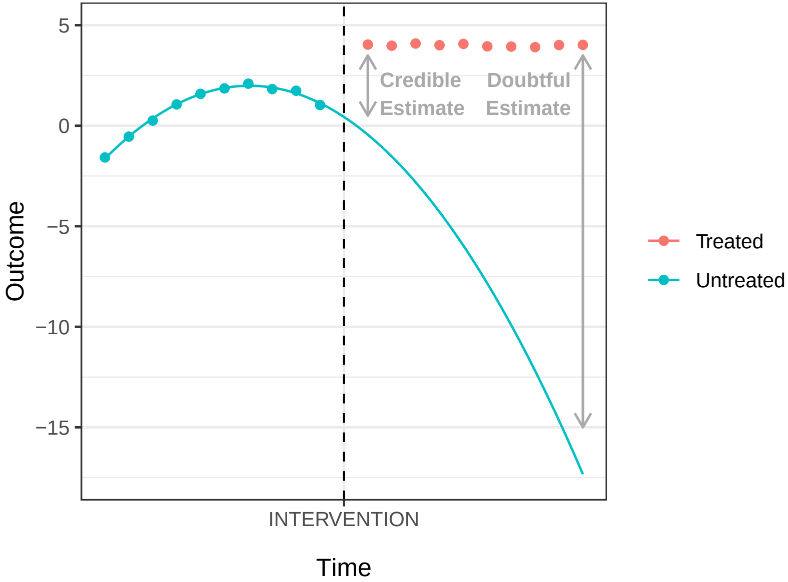


Interrupted time series: Recap

- ITS applies when treatment turns on at one time for all units
- ITS requires a parametric model to extrapolate
- ITS is most credible near the time when treatment turns on

When to use each method

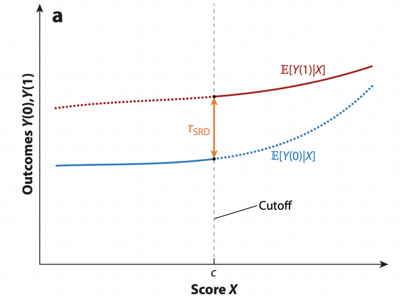
- Difference in difference
  - One unit becomes treated — New Jersey
  - One unit never becomes treated — Pennsylvania
  - The trends in Y^0 are parallel
- Interrupted time series
  - Everyone becomes treated at $X = c$ — New drug
  - You believe you can forecast Y^0 from $X < c$ to $X > c$ — Deaths would have been stable

Regression discontinuity[2]

[2] Cattaneo, M. D., & Titiunik, R. (2022). Regression discontinuity designs. Annual Review of Economics, 14, 821-851.

Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



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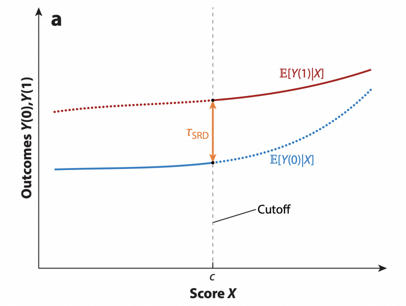

Examples

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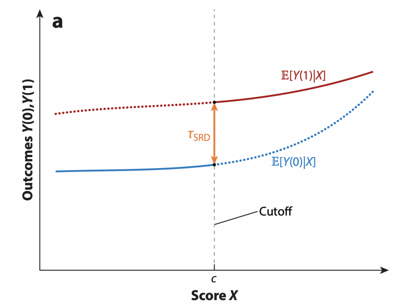
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



Examples

X is PSAT test score
c is a score cutoff
A is National Merit Scholarship

(Thistlewaite & Campbell 1960)

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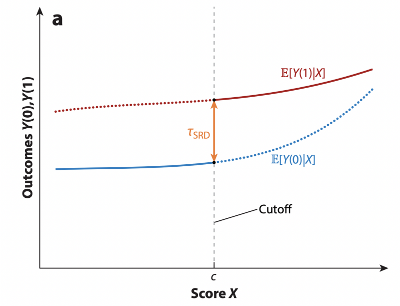
Cattaneo & Titiunik 2022 Fig 1a



Examples

X is PSAT test score
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(Thistlewaite & Campbell 1960)

X is vote share
c is 50%
A is winning the election

(De la Cuesta & Imai 2016)

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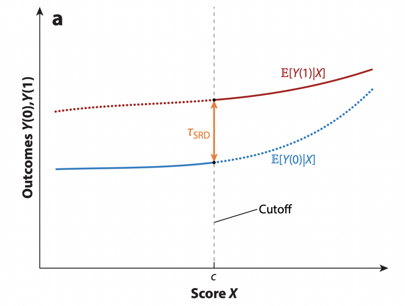
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



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(Thistlewaite & Campbell 1960)

X is vote share
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(De la Cuesta & Imai 2016)

X is date
c is 2am Nov 6 2022
A is hours of PM darkness

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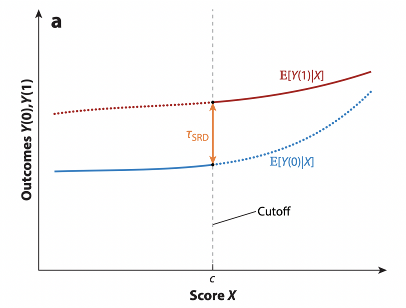
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



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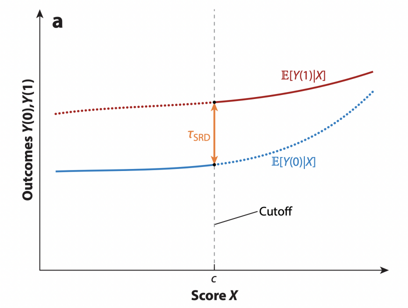
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



Theoretical Estimand

$$E(Y(1) - Y(0) \mid X = c)$$

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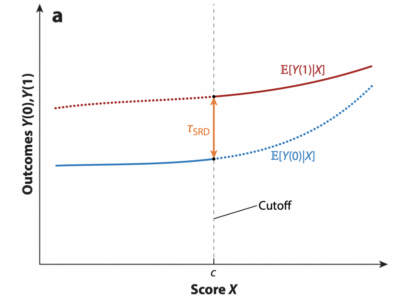
Cattaneo & Titiunik 2022 Fig 1a



Theoretical Estimand

$$E(Y(1) - Y(0) \mid X = c)$$

Empirical Estimand

$$\lim_{x \downarrow c} E(Y \mid X = x)$$

$$-$$

$$\lim_{x \uparrow c} E(Y \mid X = x)$$

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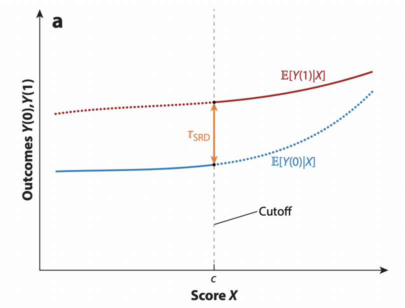

Theoretical Estimand

$\mathrm{E}(Y(1) - Y(0) \mid X = c)$

Empirical Estimand

$$\lim_{x \downarrow c} \mathrm{E}(Y \mid X = x)$$
$$-$$
$$\lim_{x \uparrow c} \mathrm{E}(Y \mid X = x)$$

Identifying Assumptions

$\mathrm{E}(Y(1) \mid X = x)$ and
$\mathrm{E}(Y(0) \mid X = x)$ are
continuous at $x = c$

and $f_X(x) > 0$ for x near c

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Regression discontinuity[2]

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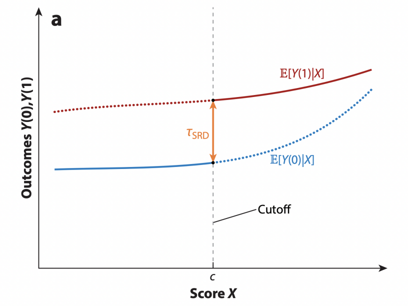

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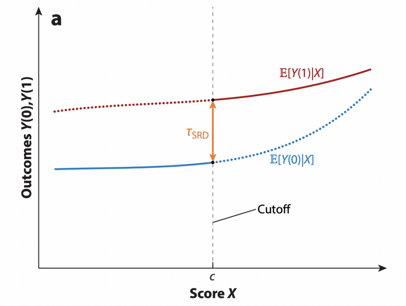

Promises of RD

Drawbacks of RD

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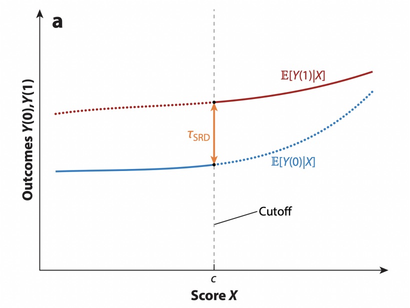
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



Promises of RD

— Highly credible

Drawbacks of RD

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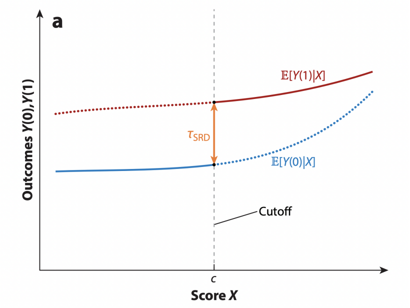
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



Promises of RD

— Highly credible

— Easy to visualize

Drawbacks of RD

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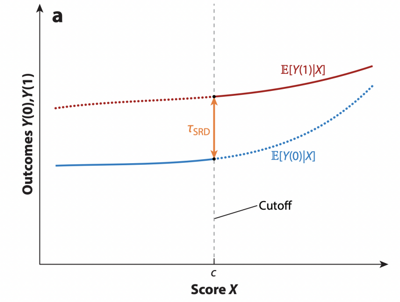
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



Promises of RD

— Highly credible

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Drawbacks of RD

— Local to $X = c$

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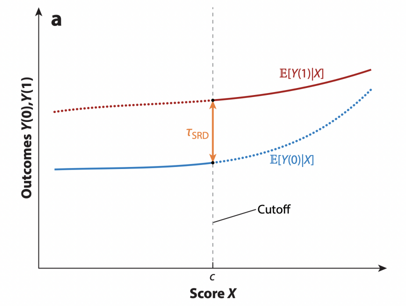
Regression discontinuity[2]

Cattaneo & Titiunik 2022 Fig 1a



Promises of RD

— Highly credible

— Easy to visualize

Drawbacks of RD

— Local to $X = c$

— Sensitive to sorting

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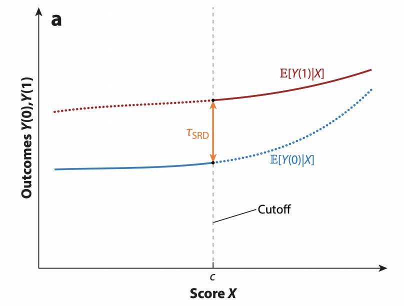
Cattaneo & Titiunik 2022 Fig 1a



Promises of RD

— Highly credible

— Easy to visualize

Drawbacks of RD

— Local to $X = c$

— Sensitive to sorting

(people moving strategically over the cutoff)

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When to use each method

- Difference in difference
  - One unit becomes treated — New Jersey
  - One unit never becomes treated — Pennsylvania
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- Interrupted time series
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- Regression discontinuity
  - Everyone becomes treated at $X = c$ — Win the election
  - You want a local estimate $\text{E}(Y^1 - Y^0 \mid X = c)$ at the cutoff — Close elections
  - Y^0 and Y^1 are continuous at $X = c$

Synthetic control[3]

[3] Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. Journal of the American Statistical Association, 105(490), 493-505.

Synthetic control[3]

In 1988, California implemented a tobacco control program

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Synthetic control[3]

In 1988, California implemented a tobacco control program

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- Money earmarked for smoking-reduction programs

How much did it reduce CA cigarette sales in 1990? 1995? 2000?

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Synthetic control[4]

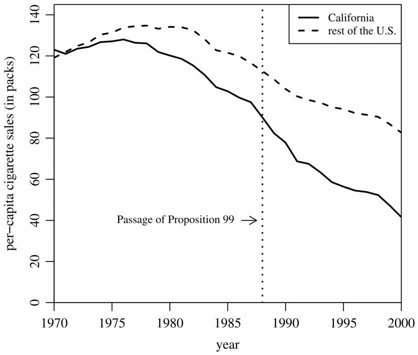

Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

Can't use RD

— Effect at 1988 not of interest

Can't use ITS

— Hard to extrapolate Y^0 trend

Can't use DID

— No other state like CA

Idea: Create a **synthetic CA** to estimate $Y^0_{\mathrm{CA},t}$ for $t \geq 1988$

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Synthetic control[5]

Synthetic CA as a weighted average of other states

Table 1. Cigarette sales predictor means

Variables	California Real	California Synthetic	Average of 38 control states
Ln(GDP per capita)	10.08	9.86	9.86
Percent aged 15–24	17.40	17.40	17.29
Retail price	89.42	89.41	87.27
Beer consumption per capita	24.28	24.20	23.75
Cigarette sales per capita 1988	90.10	91.62	114.20
Cigarette sales per capita 1980	120.20	120.43	136.58
Cigarette sales per capita 1975	127.10	126.99	132.81

NOTE: All variables except lagged cigarette sales are averaged for the 1980–1988 period (beer consumption is averaged 1984–1988). GDP per capita is measured in 1997 dollars, retail prices are measured in cents, beer consumption is measured in gallons, and cigarette sales are measured in packs.

[5] Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. Journal of the American Statistical Association, 105(490), 493-505.

Synthetic control[6]

Synthetic CA as a weighted average of other states

$$\text{Theoretical Estimand:} \quad \tau(t) = Y^1_{\mathrm{CA},t} - Y^0_{\mathrm{CA},t} \qquad t \geq 1988$$

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Synthetic control[6]

Synthetic CA as a weighted average of other states

Theoretical Estimand: $\tau(t) = Y^1_{\text{CA},t} - Y^0_{\text{CA},t} \qquad t \geq 1988$

Empirical Estimand: $\theta(t) = Y^1_{\text{CA},t} - Y^0_{\text{SyntheticCA},t} \qquad t \geq 1988$

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Synthetic control[6]

Synthetic CA as a weighted average of other states

$$\text{Theoretical Estimand:} \quad \tau(t) = Y^1_{\text{CA},t} - Y^0_{\text{CA},t} \qquad t \geq 1988$$

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$$\text{Identifying Assumption:} \quad \underbrace{Y^0_{\text{CA},t}}_{\text{Counterfactual}} = \underbrace{Y^0_{\text{SyntheticCA},t}}_{\text{Factual}} \qquad t \geq 1988$$

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Synthetic control[7]

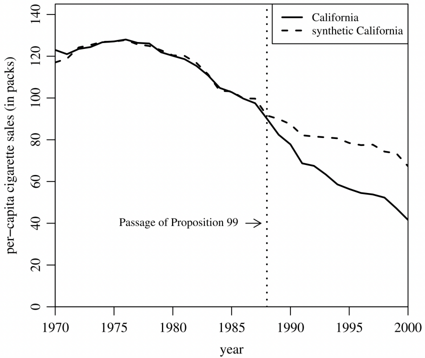

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

[7] Abadie, A., Diamond, A., & Hainmueller, J. (2010). Synthetic control methods for comparative case studies: Estimating the effect of California's tobacco control program. Journal of the American Statistical Association, 105(490), 493-505.

When to use each method

- Difference in difference
  - One unit becomes treated — New Jersey
  - One unit never becomes treated — Pennsylvania
  - The trends in Y^0 are parallel
- Interrupted time series
  - Everyone becomes treated at $X = c$ — New drug
  - You believe you can forecast Y^0 from $X < c$ to $X > c$ — Deaths would have been stable
- Regression discontinuity
  - Everyone becomes treated at $X = c$ — Win the election
  - You want a local estimate $\text{E}(Y^1 - Y^0 \mid X = c)$ at the cutoff — Close elections
  - Y^0 and Y^1 are continuous at $X = c$
- Synthetic control
  - One unit becomes treated — California
  - Many units are never treated — Other states
  - You want to extrapolate far from the cutoff — $1988 \rightarrow 2000$

Learning goals for today

At the end of class, you will be able to:

1. Recognize the promises and pitfalls of four methods to study the effects of treatments that turn on once
    1.1 Difference in difference (DID)
    1.2 Interrupted time series (ITS)
    1.3 Regression discontinuity (RD)
    1.4 Synthetic control (SC)