Doubly-Robust Estimation

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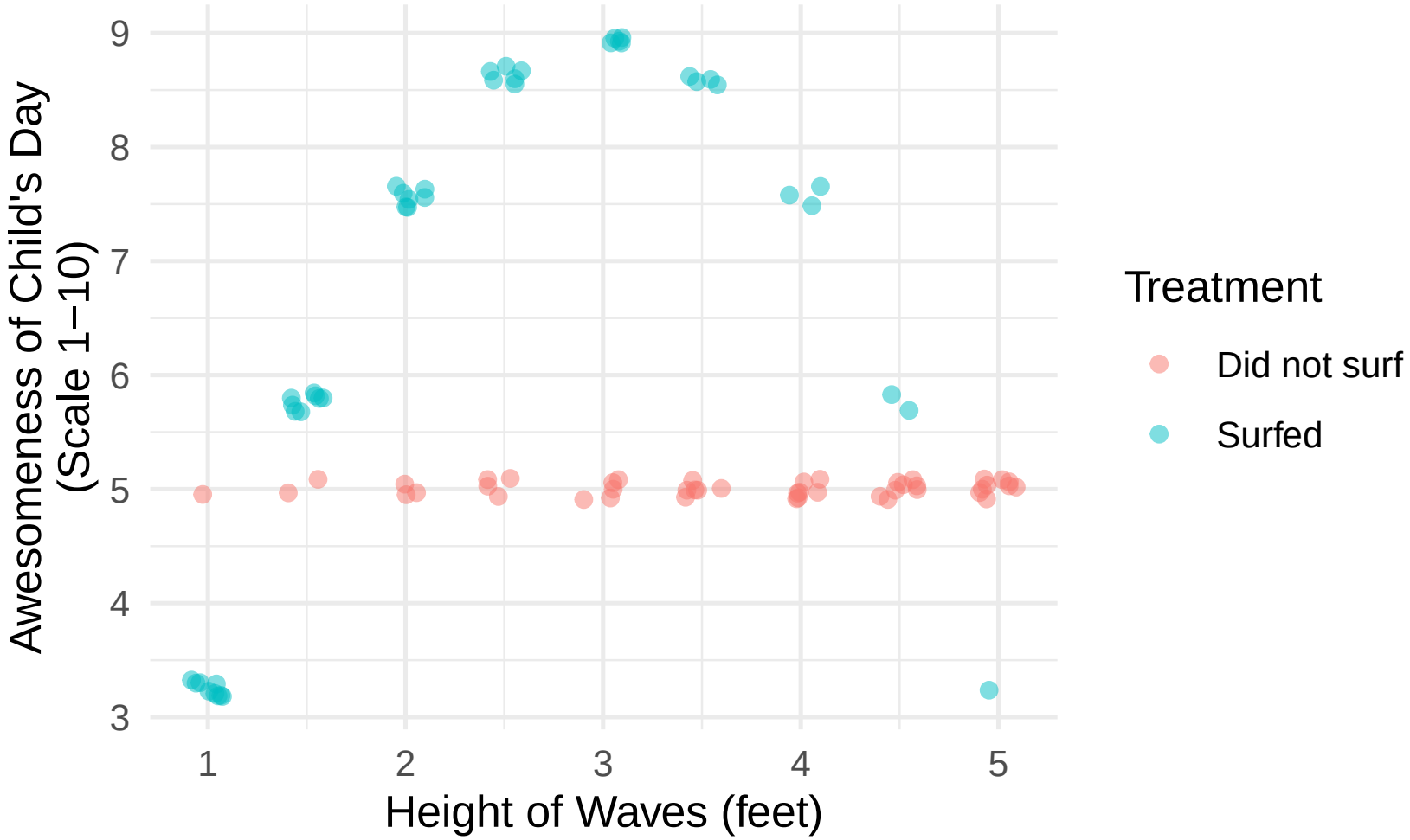
Winter 2025

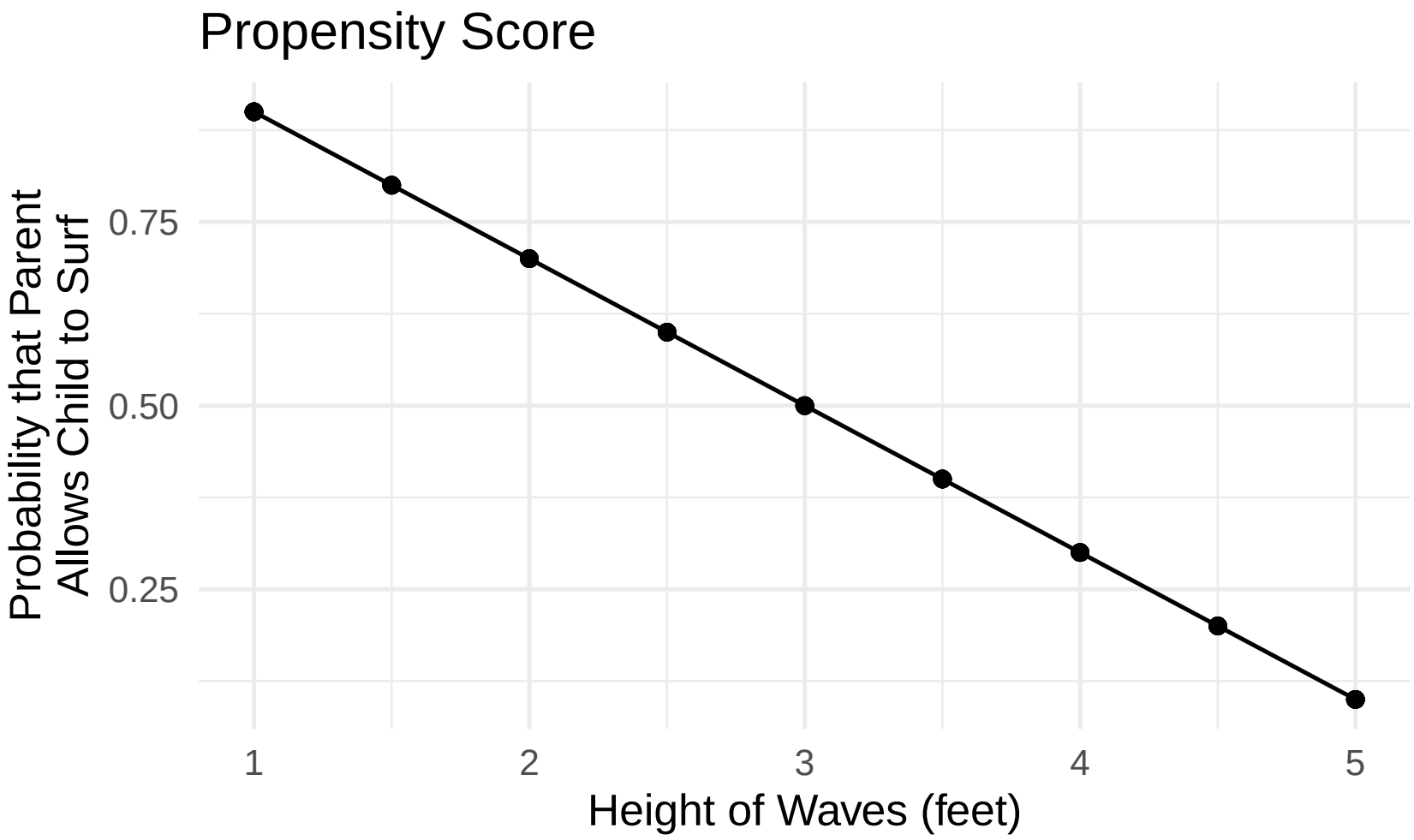
Learning goals for today

At the end of class, you will be able to estimate average causal effects by modeling treatment assignment probabilities.

Optional reading:

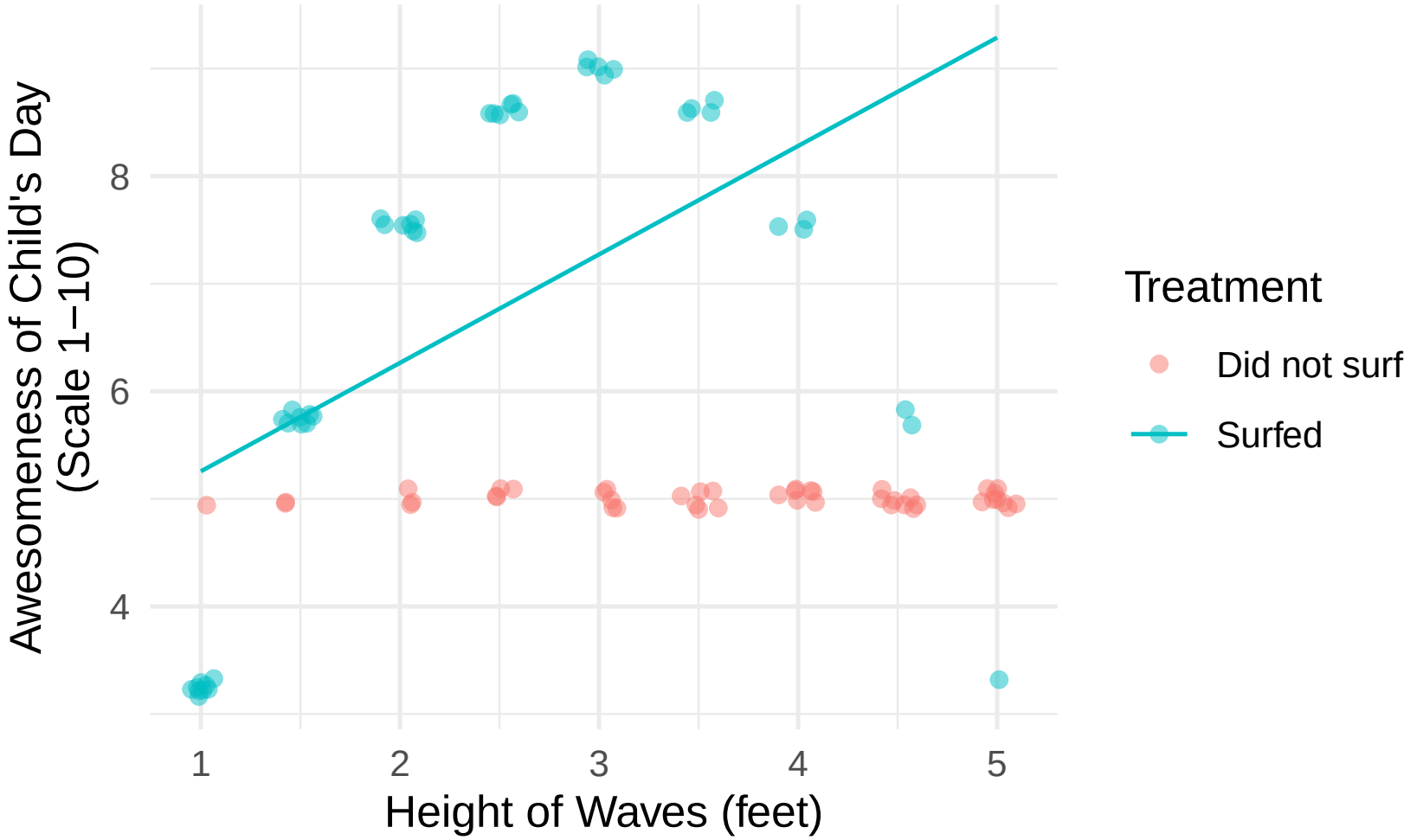
- Hernán and Robins 2020 Chapter 12.1–12.5, 13, 15.1


Awesomeness of Child's Day
(Scale 1–10)
9
8
7
6
5
4
3
1
2
3
4
5
Height of Waves (feet)
Treatment
Did not surf
Surfed

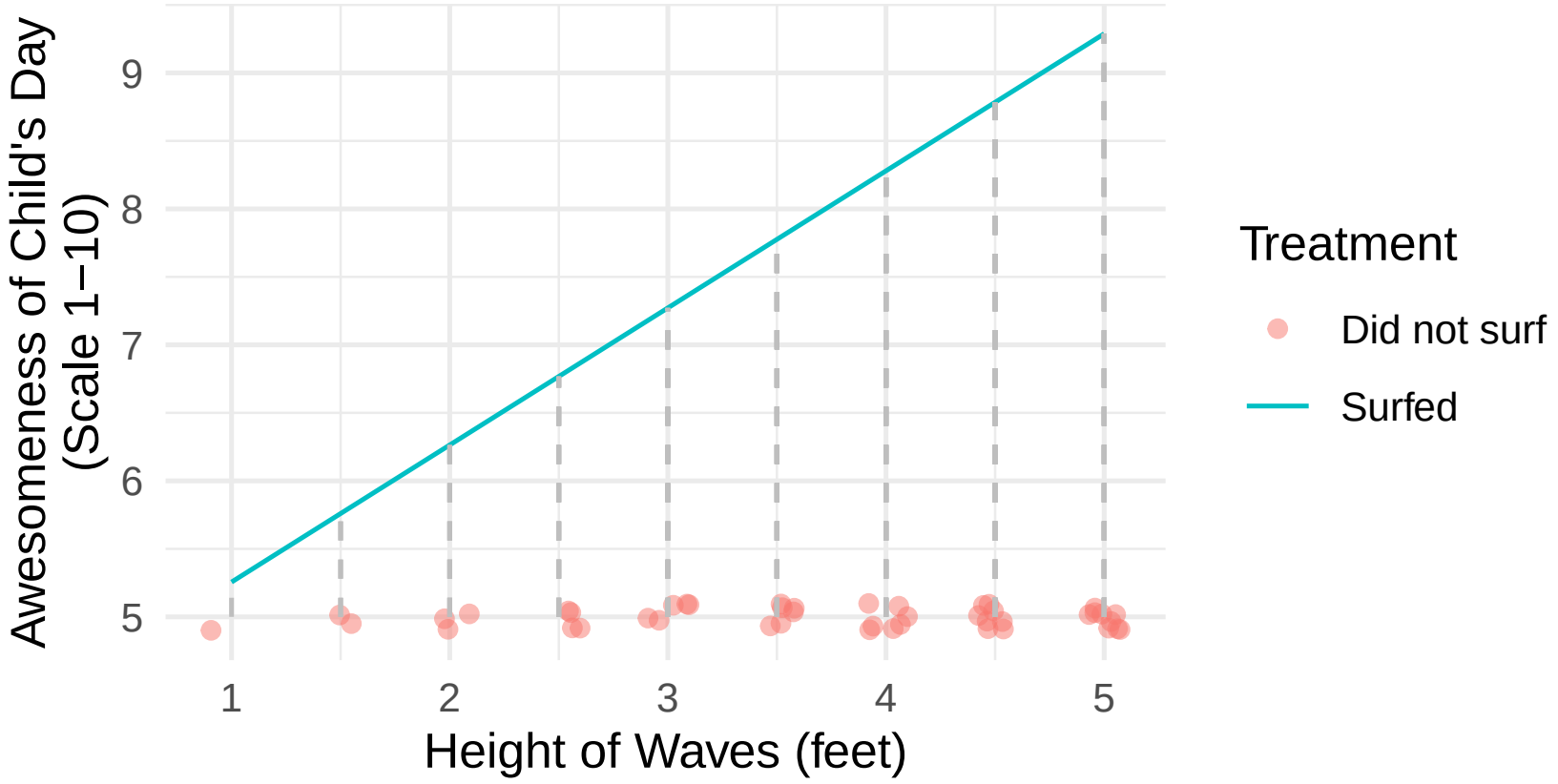

Propensity Score
Probability that Parent Allows Child to Surf
0.75
0.50
0.25
1
2
3
4
5
Height of Waves (feet)

Child:
How much more awesome would my day have been if I had surfed on the days when my parents didn't let me?

$$\text{ATC} = \frac{1}{n_0} \sum_{i:A_i=0} \left(Y_i^1 - Y_i^0\right)$$

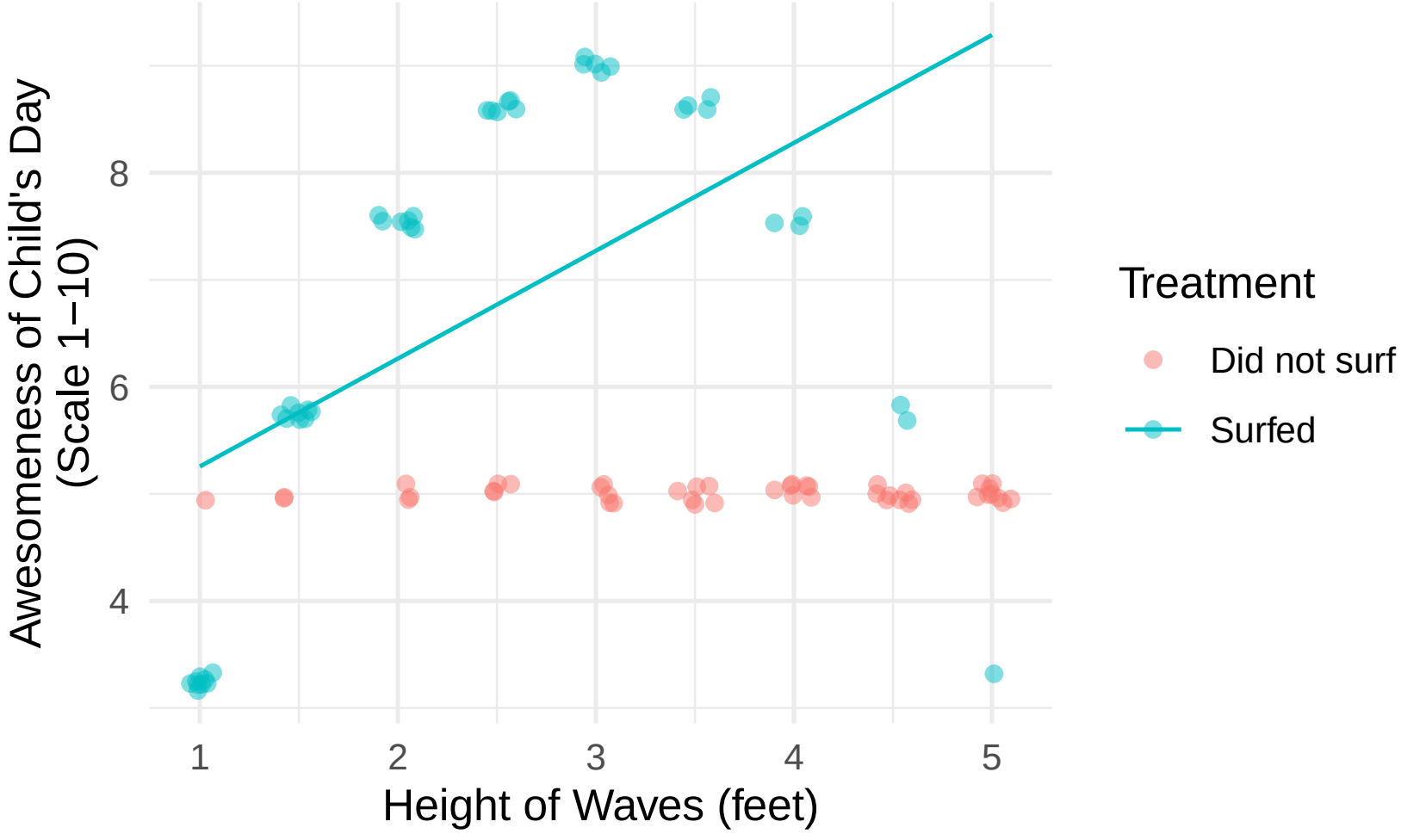

Awesomeness of Child's Day
(Scale 1–10)
8
6
4
1
2
3
4
5
Height of Waves (feet)
Treatment
Did not surf
Surfed

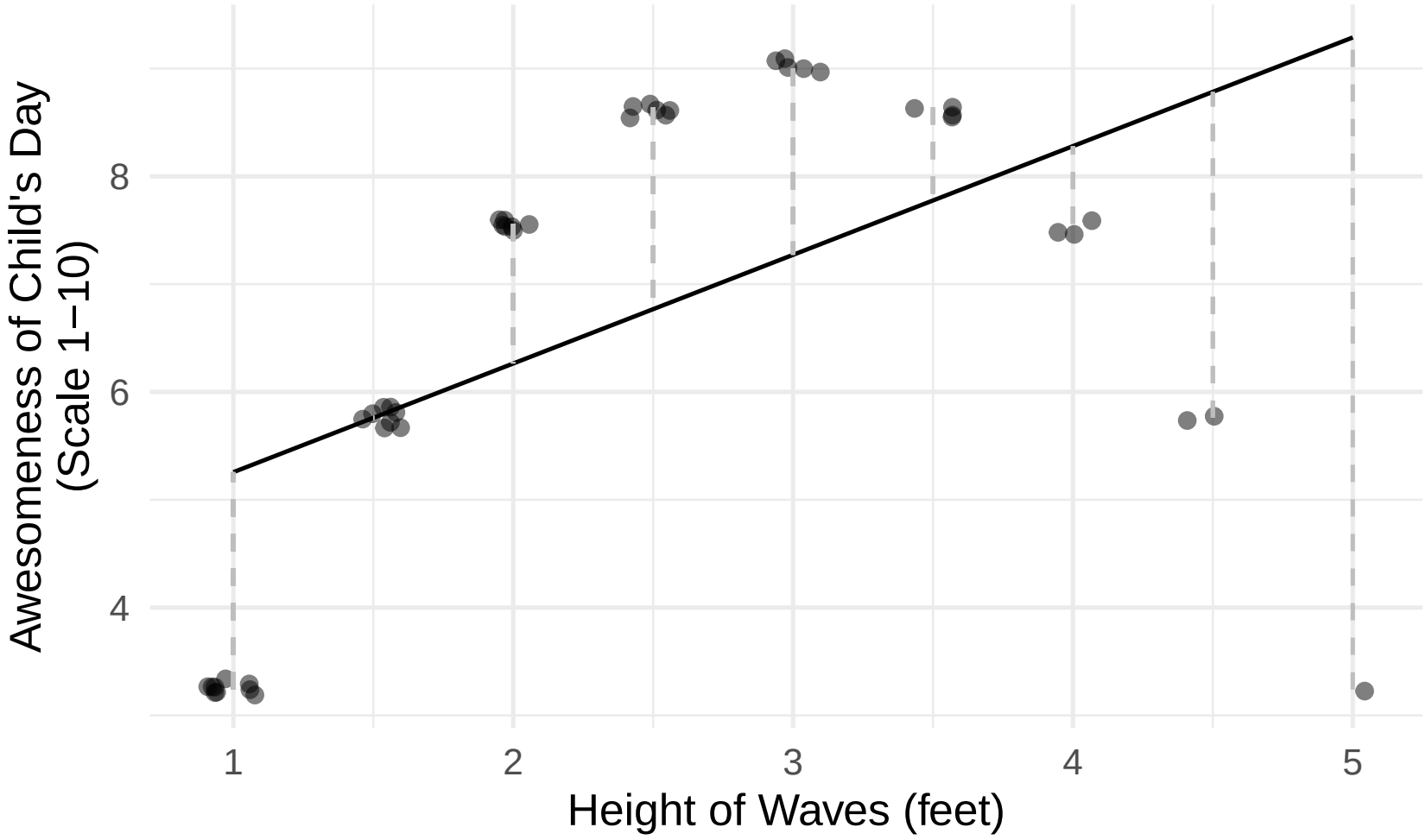
ATC: On average, awesomness would increase by 2.94 if I had surfed on the days I wasn't allowed.

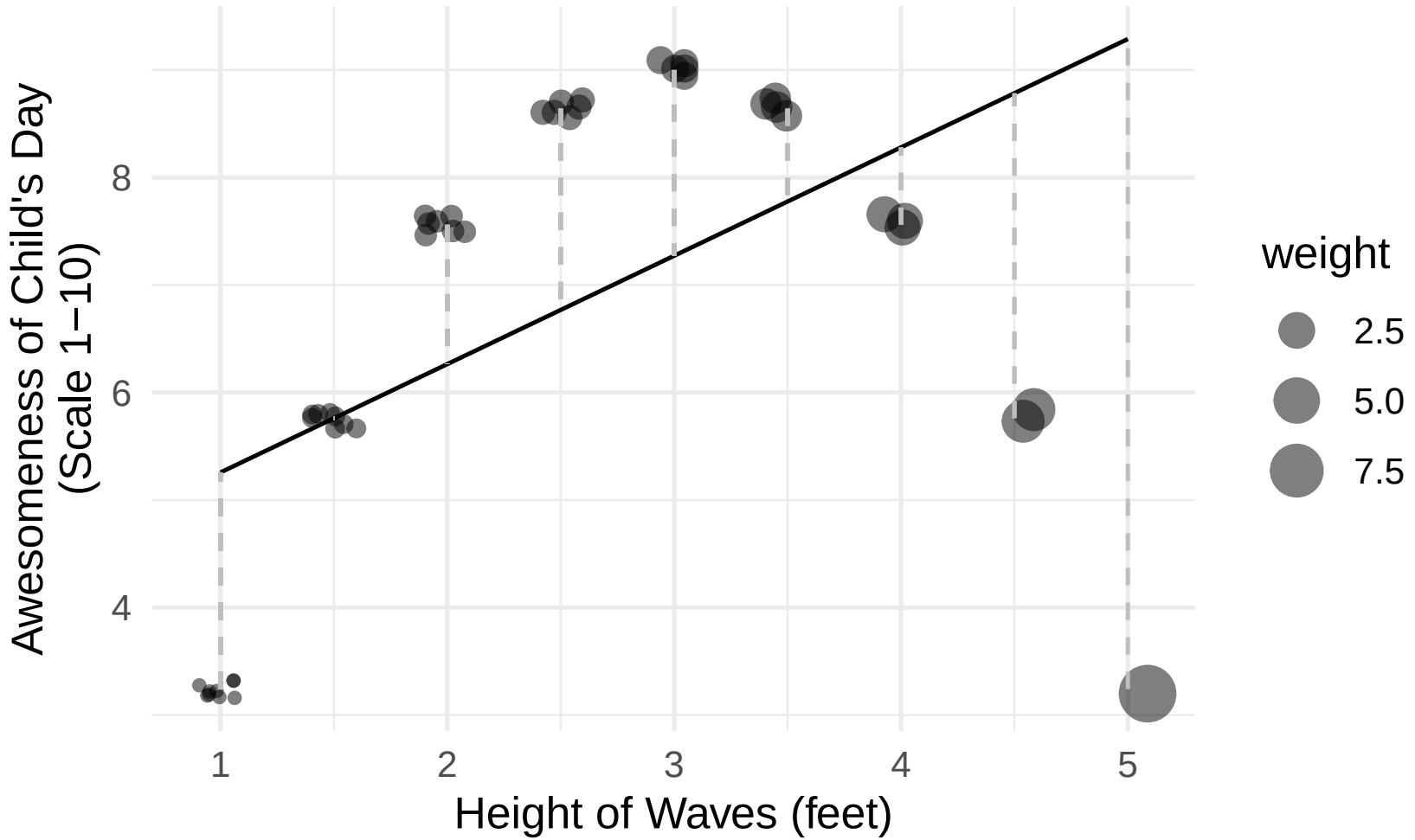


To discuss:

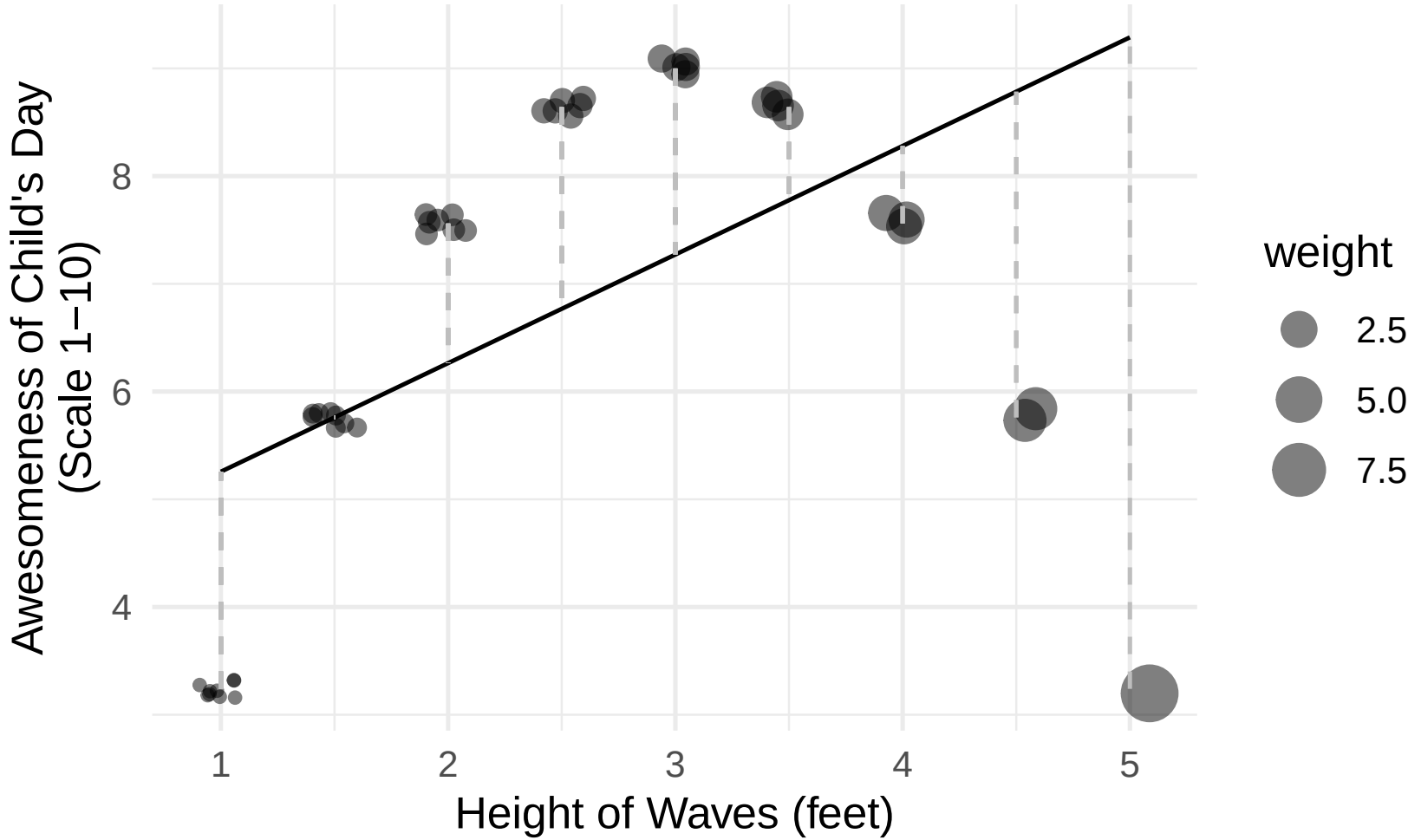
- In what sense is this line best-fit to the wrong goal?
- How important is the error at each x-value?



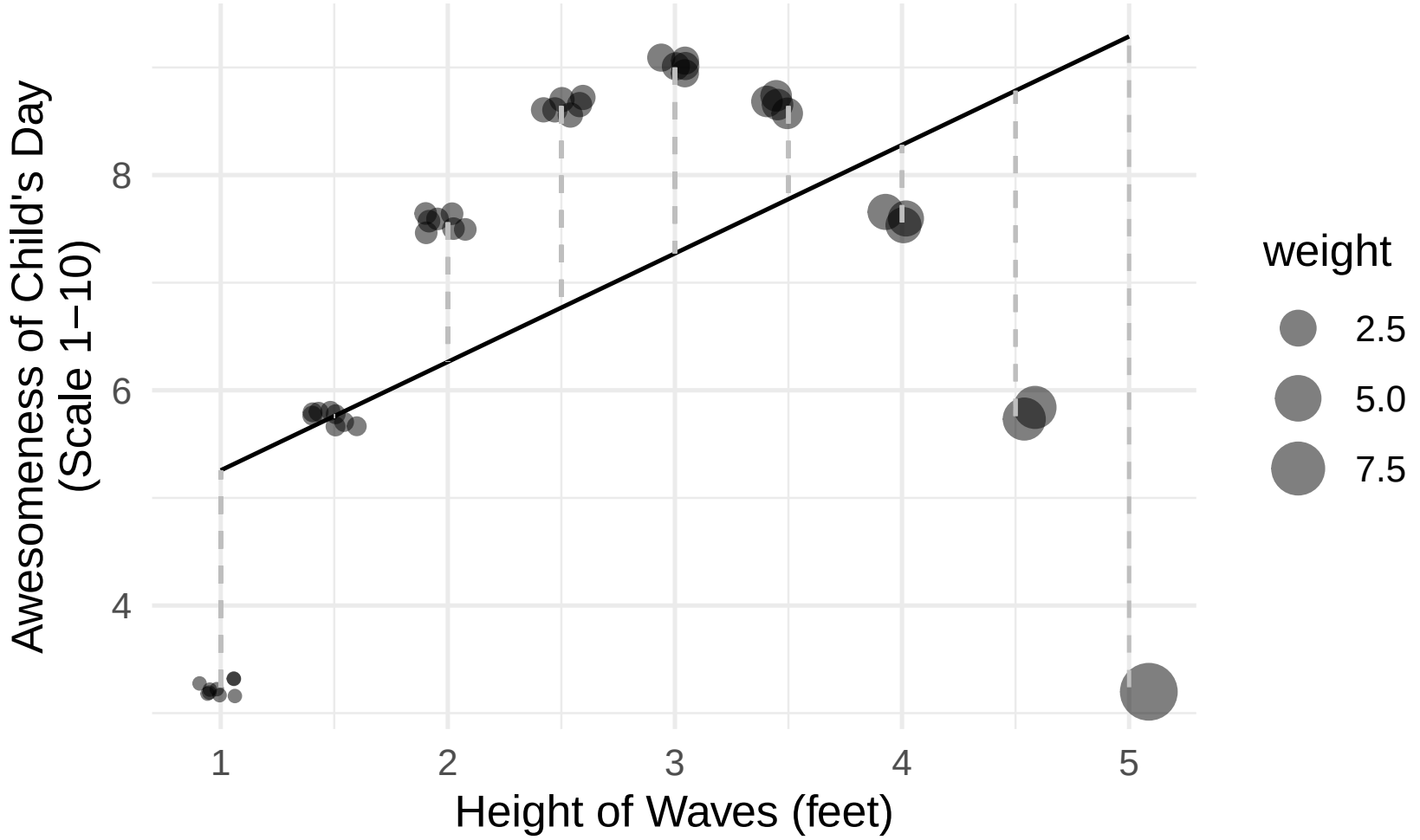

Awesomeness of Child's Day
(Scale 1–10)
8
6
4
1
2
3
4
5
Height of Waves (feet)


Awesomeness of Child's Day
(Scale 1–10)
8
6
4
1
2
3
4
5
Height of Waves (feet)
weight
2.5
5.0
7.5

Weighted average error: 1.34.



Weighted average error: 1.34.
Corrected estimate: 2.94 - 1.34 = 1.60



Doubly-robust estimation: Summary

For the ATC:

- Predict $\hat{Y}^1$
- Among treated cases,
  - Weight by $\frac{\hat{P}(A{=}1)}{\hat{\mathrm{P}}(A{=}0)}$
  - Take weighted average error: $\hat{Y}^1 - Y$
  - This is a bias correction:
    model was fit at x-values of treated cases,
    target to predict is x-values of untreated cases
- Among untreated cases, take average $\hat{Y}^1$
- Then subtract the bias correction

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