

Instrumental variables

Soc 212C

Spring 2025

Instrumental variables

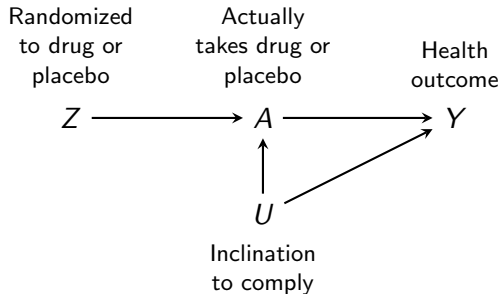
A popular approach to causal inference with unmeasured treatment-outcome confounding

We will discuss

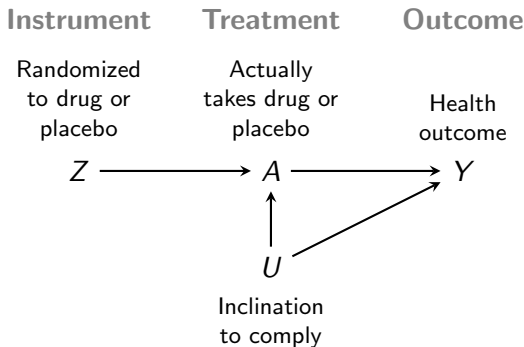
- ▶ IV in randomized experiments with non-compliance
- ▶ IV in observational settings

Like all methods for causal inference from observational data, IV requires untestable assumptions that may be doubtful in many applications

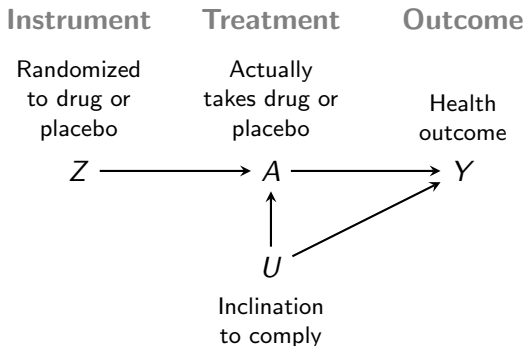
Instrumental variables: Experiment with noncompliance



Instrumental variables: Experiment with noncompliance



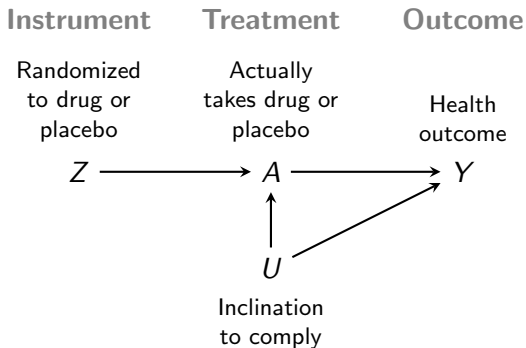
Instrumental variables: Experiment with noncompliance



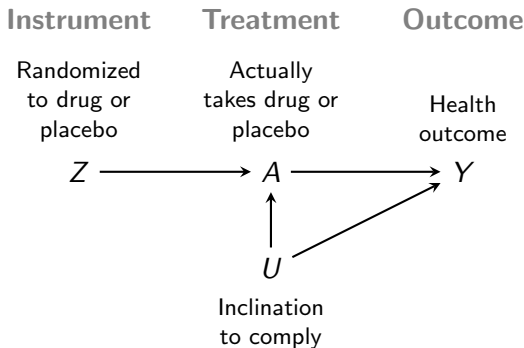
Two ideas

1. Intent to treat effect
2. Average effect among compliers

Instrumental variables: 1) Intent to treat effect

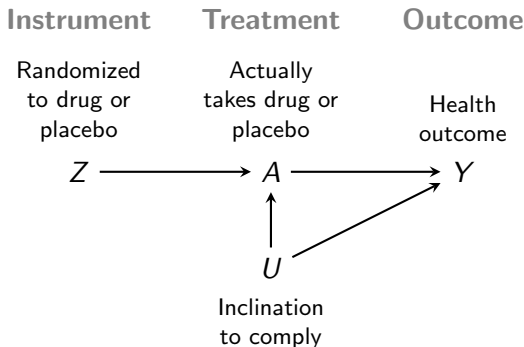


Instrumental variables: 1) Intent to treat effect



Ignore A . What is the effect of Z on Y ?

Instrumental variables: 1) Intent to treat effect



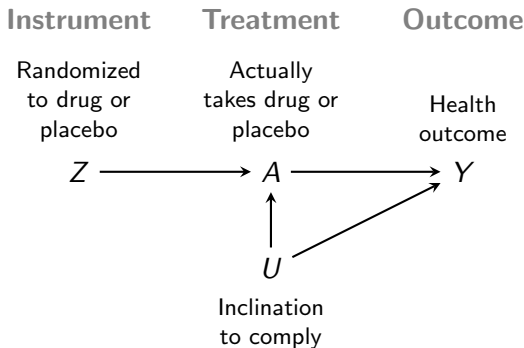
Ignore A . What is the effect of Z on Y ?

$$E(Y^1 - Y^0)$$

$\uparrow \quad \quad \uparrow$

Outcome Outcome
under under
 $Z = 1$ $Z = 0$

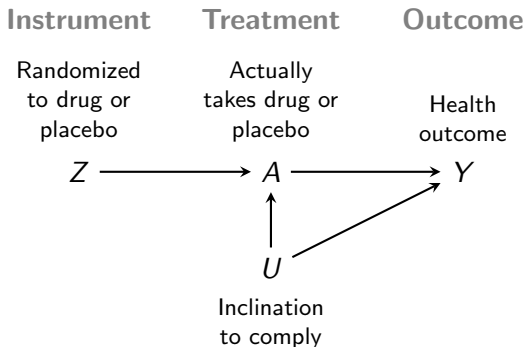
Instrumental variables: 1) Intent to treat effect



Ignore A . What is the effect of Z on Y ?

$$\begin{array}{ccc} E(Y^1 - Y^0) & & = \\ \uparrow & \nwarrow & \uparrow \\ \text{Outcome} & \text{Outcome} & \text{By} \\ \text{under} & \text{under} & \text{Positivity} \\ Z = 1 & Z = 0 & \text{Consistency} \\ & & \text{Exchangeability} \\ & & \text{for } Z \end{array}$$

Instrumental variables: 1) Intent to treat effect



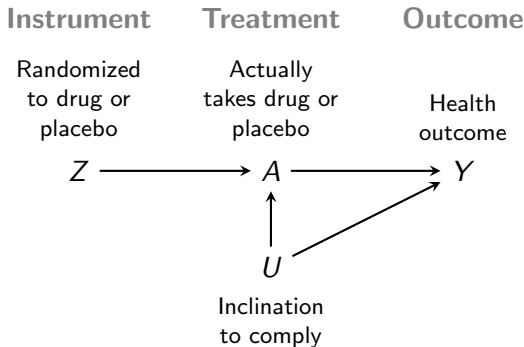
Ignore A . What is the effect of Z on Y ?

$$\begin{array}{ccc} E(Y^1 - Y^0) & = & E(Y \mid Z = 1) - E(Y \mid Z = 0) \\ \uparrow \quad \quad \uparrow & \uparrow & \text{Mean difference in} \\ \text{Outcome under} & \text{By} & \text{observable outcomes} \\ \text{Z = 1} & \text{Positivity} & \\ & \text{Consistency} & \\ & \text{Exchangeability} & \\ & \text{for Z} & \end{array}$$

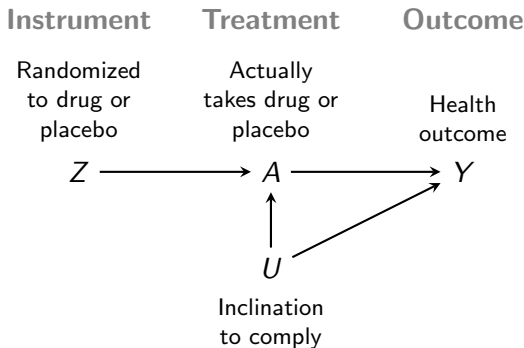
Instrumental variables: 2) Average effect among compliers

Imbens, G., & Angrist, J. (1994). Identification and estimation of local average treatment effects. *Econometrica*, 62(2), 467-475.

Instrumental variables: 2) Average effect among compliers

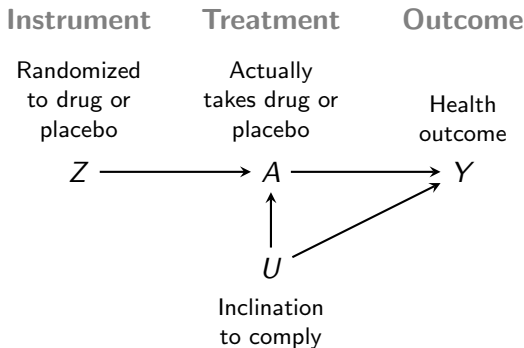


Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

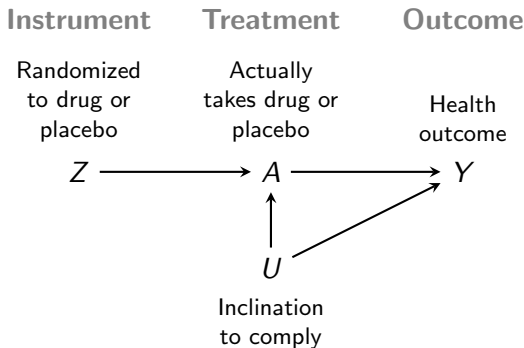
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \rightarrow Y$

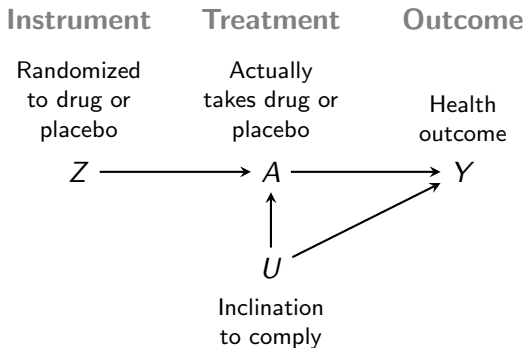
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \rightarrow Y$ (we just did)

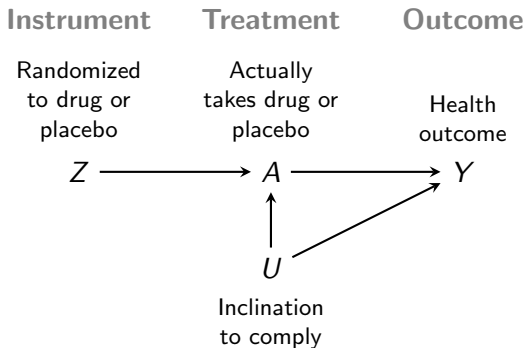
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \rightarrow Y$ (we just did)
2. Study the effect of $Z \rightarrow A$

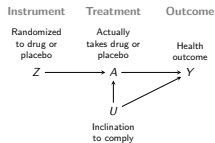
Instrumental variables: 2) Average effect among compliers



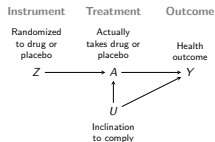
Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \rightarrow Y$ (we just did)
2. Study the effect of $Z \rightarrow A$
3. Learn about $A \rightarrow Y$ since $Z \rightarrow Y$ is $Z \rightarrow A \rightarrow Y$

Instrumental variables: 2) Average effect among compliers

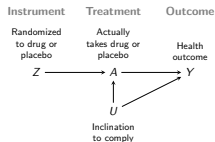


Instrumental variables: 2) Average effect among compliers



The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

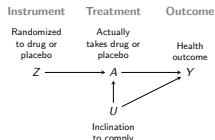
Instrumental variables: 2) Average effect among compliers



The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

Compliers $A^{Z=0} = 0$ $A^{Z=1} = 1$ (follow assignment)

Instrumental variables: 2) Average effect among compliers

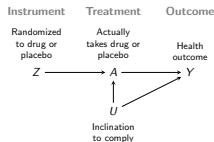


The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

Compliers $A^{Z=0} = 0$ $A^{Z=1} = 1$ (follow assignment)

Always takers $A^{Z=0} = 1$ $A^{Z=1} = 1$ (always take treatment)

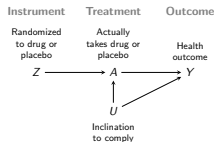
Instrumental variables: 2) Average effect among compliers



The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

Compliers	$A^{Z=0} = 0$	$A^{Z=1} = 1$	(follow assignment)
Always takers	$A^{Z=0} = 1$	$A^{Z=1} = 1$	(always take treatment)
Never takers	$A^{Z=0} = 0$	$A^{Z=1} = 0$	(never take treatment)

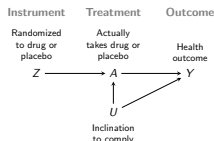
Instrumental variables: 2) Average effect among compliers



The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

Compliers	$A^{Z=0} = 0$	$A^{Z=1} = 1$	(follow assignment)
Always takers	$A^{Z=0} = 1$	$A^{Z=1} = 1$	(always take treatment)
Never takers	$A^{Z=0} = 0$	$A^{Z=1} = 0$	(never take treatment)
Defiers	$A^{Z=0} = 1$	$A^{Z=1} = 0$	(defy assignment)

Instrumental variables: 2) Average effect among compliers

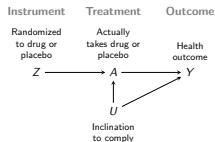


The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

Compliers	$A^{Z=0} = 0$	$A^{Z=1} = 1$	(follow assignment)
Always takers	$A^{Z=0} = 1$	$A^{Z=1} = 1$	(always take treatment)
Never takers	$A^{Z=0} = 0$	$A^{Z=1} = 0$	(never take treatment)
Defiers	$A^{Z=0} = 1$	$A^{Z=1} = 0$	(defy assignment)

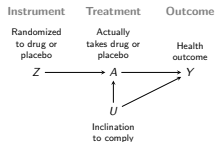
Discuss: In which strata is the effect $Z \rightarrow Y$ zero?

Instrumental variables: 2) Average effect among compliers



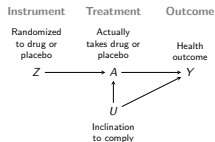
Among **always takers** and **never takers**,

Instrumental variables: 2) Average effect among compliers



Among **always takers** and **never takers**,
 Z does not affect A

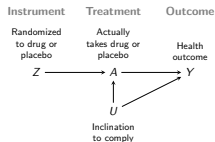
Instrumental variables: 2) Average effect among compliers



Among **always takers** and **never takers**,
 Z does not affect A

Z only affects Y through A

Instrumental variables: 2) Average effect among compliers

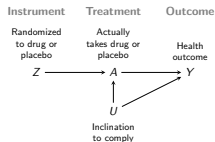


Among **always takers** and **never takers**,
 Z does not affect A

Z only affects Y through A

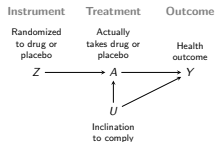
In these strata, Z does not affect Y

Instrumental variables: 2) Average effect among compliers



Among **compliers**,

Instrumental variables: 2) Average effect among compliers

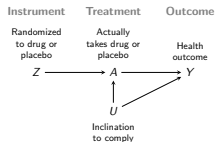


Among **compliers**,

$Z = 1$ implies $A = 1$ and

$Z = 0$ implies $A = 0$

Instrumental variables: 2) Average effect among compliers



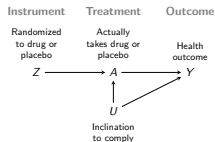
Among **compliers**,

$Z = 1$ implies $A = 1$ and

$Z = 0$ implies $A = 0$

In these strata, $Z = A$

Instrumental variables: 2) Average effect among compliers



Among **compliers**,

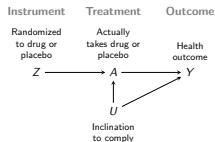
$Z = 1$ implies $A = 1$ and

$Z = 0$ implies $A = 0$

In these strata, $Z = A$

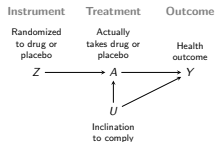
$Z \rightarrow Y$ and $A \rightarrow Y$ are the same

Instrumental variables: 2) Average effect among compliers



Among **defiers**,

Instrumental variables: 2) Average effect among compliers

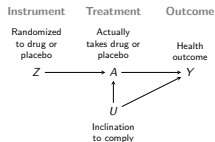


Among **defiers**,

$Z = 1$ implies $A = 0$ and

$Z = 0$ implies $A = 1$

Instrumental variables: 2) Average effect among compliers



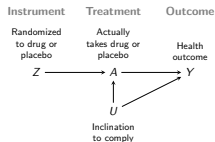
Among **defiers**,

$Z = 1$ implies $A = 0$ and

$Z = 0$ implies $A = 1$

In these strata, $Z = 1 - A$

Instrumental variables: 2) Average effect among compliers



Among **defiers**,

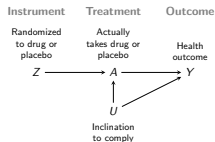
$Z = 1$ implies $A = 0$ and

$Z = 0$ implies $A = 1$

In these strata, $Z = 1 - A$

$Z \rightarrow Y$ and $A \rightarrow Y$ are the same magnitude
but have opposite signs

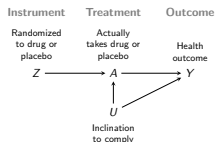
Instrumental variables: 2) Average effect among compliers



Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Defiers	$(Z \rightarrow A) = -1$	$(Z \rightarrow Y) = -(A \rightarrow Y)$

Instrumental variables: 2) Average effect among compliers

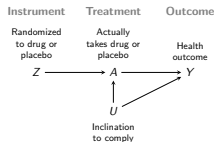


Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Defiers	$(Z \rightarrow A) = -1$	$(Z \rightarrow Y) = -(A \rightarrow Y)$

Assume **no defiers** in the population

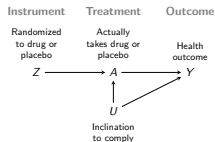
Instrumental variables: 2) Average effect among compliers



Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$

Instrumental variables: 2) Average effect among compliers

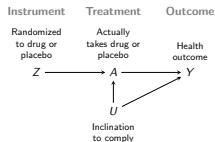


Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$

Discuss a hypothetical.

Instrumental variables: 2) Average effect among compliers



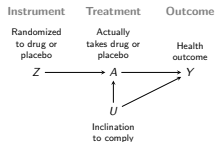
Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$

Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Instrumental variables: 2) Average effect among compliers



Four principal strata

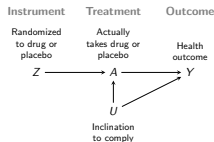
Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$

Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of $Z \rightarrow Y$ among compliers is 0.6

Instrumental variables: 2) Average effect among compliers



Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$

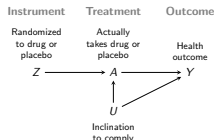
Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of $Z \rightarrow Y$ among compliers is 0.6

What is the average effect of $Z \rightarrow Y$ in the population?

Instrumental variables: 2) Average effect among compliers



Four principal strata

Compliers	$(Z \rightarrow A) = +1$	$(Z \rightarrow Y) = (A \rightarrow Y)$
Always takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$
Never takers	$(Z \rightarrow A) = 0$	$(Z \rightarrow Y) = 0$

Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of $Z \rightarrow Y$ among compliers is 0.6

What is the average effect of $Z \rightarrow Y$ in the population? **0.3**

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$E(Y^{Z=1} - Y^{Z=0})$$

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$\begin{aligned} & E(Y^{Z=1} - Y^{Z=0}) \\ &= \sum_s E(Y^{Z=1} - Y^{Z=0} \mid S = s) \underbrace{P(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \end{aligned}$$

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$\begin{aligned} & E(Y^{Z=1} - Y^{Z=0}) \\ &= \sum_s E(Y^{Z=1} - Y^{Z=0} \mid S = s) \underbrace{P(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\ &= E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}} \\ &\quad + E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Always-Taker})\pi_{\text{Always-Taker}} \\ &\quad + E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Never-Taker})\pi_{\text{Never-Taker}} \\ &\quad + E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Defier})\pi_{\text{Defier}} \end{aligned}$$

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$\begin{aligned} & E(Y^{Z=1} - Y^{Z=0}) \\ &= \sum_s E(Y^{Z=1} - Y^{Z=0} \mid S = s) \underbrace{P(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\ &= E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}} \\ &\quad + E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Always-Taker})\pi_{\text{Always-Taker}} \quad (= 0) \\ &\quad + E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Never-Taker})\pi_{\text{Never-Taker}} \quad (= 0) \\ &\quad + E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Defier})\pi_{\text{Defier}} \quad (= 0) \end{aligned}$$

Instrumental variables: 2) Average effect among compliers

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Instrumental variables: 2) Average effect among compliers

$$\begin{array}{ccccc} \text{Average effect} & = & \text{Average among} & \times & \text{Proportion} \\ \text{of instrument} & & \text{compliers} & & \text{who comply} \end{array}$$

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Instrumental variables: 2) Average effect among compliers

$$\begin{array}{ccccc} \text{Average effect} & = & \text{Average among} & \times & \text{Proportion} \\ \text{of instrument} & & \text{compliers} & & \text{who comply} \end{array}$$

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Among compliers, $(Z \rightarrow Y) = (A \rightarrow Y)$.

Instrumental variables: 2) Average effect among compliers

$$\begin{array}{ccccc} \text{Average effect} & = & \text{Average among} & \times & \text{Proportion} \\ \text{of instrument} & & \text{compliers} & & \text{who comply} \end{array}$$

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Among compliers, $(Z \rightarrow Y) = (A \rightarrow Y)$.

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Instrumental variables: 2) Average effect among compliers

$$\begin{array}{ccccc} \text{Average effect} & = & \text{Average among} & \times & \text{Proportion} \\ \text{of instrument} & & \text{compliers} & & \text{who comply} \end{array}$$

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Among compliers, $(Z \rightarrow Y) = (A \rightarrow Y)$.

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Rearrange to get the complier average treatment effect

Instrumental variables: 2) Average effect among compliers

$$\begin{array}{ccccc} \text{Average effect} & = & \text{Average among} & \times & \text{Proportion} \\ \text{of instrument} & & \text{compliers} & & \text{who comply} \end{array}$$

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Among compliers, $(Z \rightarrow Y) = (A \rightarrow Y)$.

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Rearrange to get the complier average treatment effect

$$E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier}) = \frac{E(Y^{Z=1} - Y^{Z=0})}{\pi_{\text{Complier}}}$$

Instrumental variables: 2) Average effect among compliers

$$\begin{array}{ccccc} \text{Average effect} & = & \text{Average among} & \times & \text{Proportion} \\ \text{of instrument} & & \text{compliers} & & \text{who comply} \end{array}$$

$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Among compliers, $(Z \rightarrow Y) = (A \rightarrow Y)$.

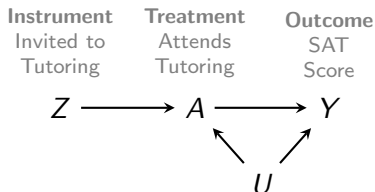
$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Rearrange to get the complier average treatment effect

$$\begin{aligned} E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier}) &= \frac{E(Y^{Z=1} - Y^{Z=0})}{\pi_{\text{Complier}}} \\ &= \frac{E(Y^{Z=1} - Y^{Z=0})}{E(A^{Z=1} - A^{Z=0})} \end{aligned}$$

IV in observational settings

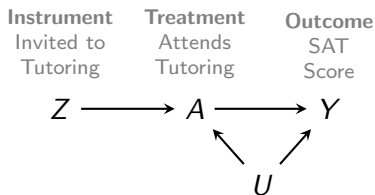
Running example



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

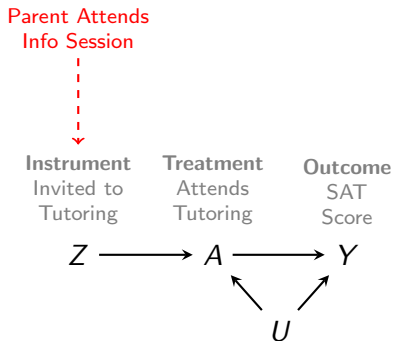
Requirement 1) Instrument exchangeability



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

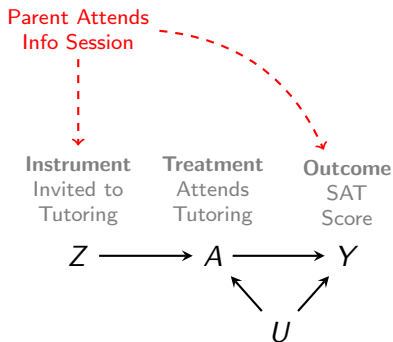
Requirement 1) Instrument exchangeability



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

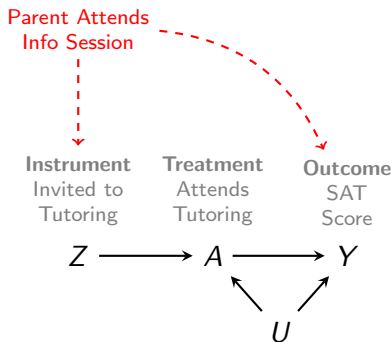
Requirement 1) Instrument exchangeability



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 1) Instrument exchangeability

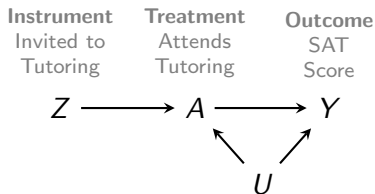


The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

doesn't work!

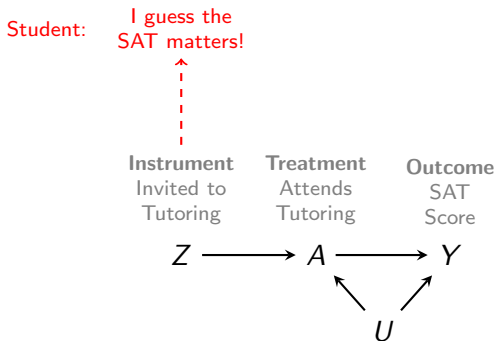
Requirement 2) No direct effect of Z on Y



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

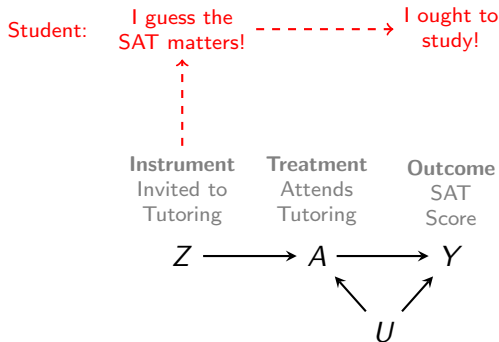
Requirement 2) No direct effect of Z on Y



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

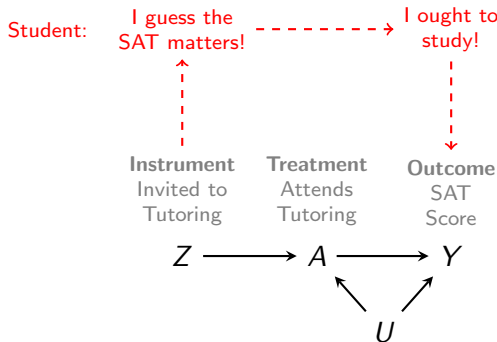
Requirement 2) No direct effect of Z on Y



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

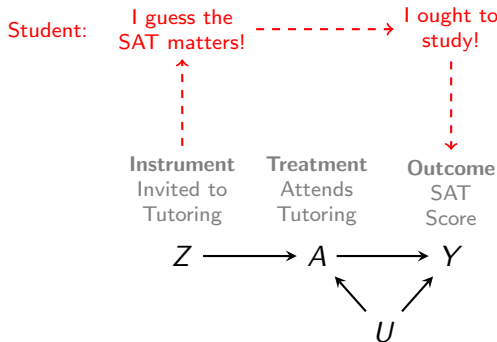
Requirement 2) No direct effect of Z on Y



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 2) No direct effect of Z on Y

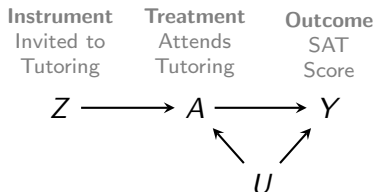


The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

doesn't isolate $A \rightarrow Y$

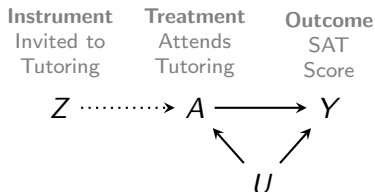
Requirement 3) Relevance



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 3) Relevance

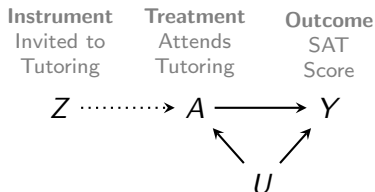


The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

hardly any!

Requirement 3) Relevance

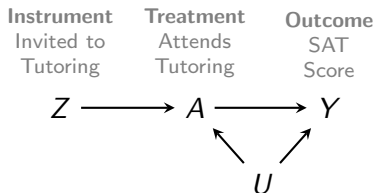


The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

hardly any!
tiny denominator!

Requirement 4a) Homogeneity

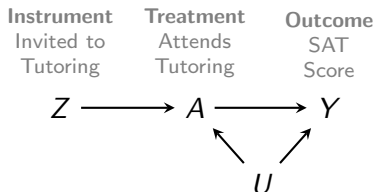


The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 4a) Homogeneity

Want to learn about everyone



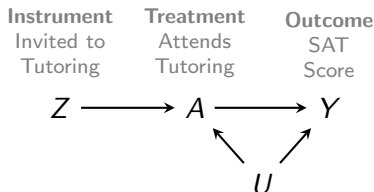
The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 4a) Homogeneity

Want to learn about everyone

But some people do not accept the invitation



The instrumental variable steps

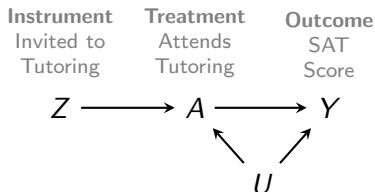
- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 4a) Homogeneity

Want to learn about everyone

But some people do not accept the invitation

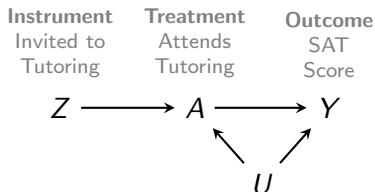
Assume each person's $A \rightarrow Y$ is the same



The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 4b) Monotonicity

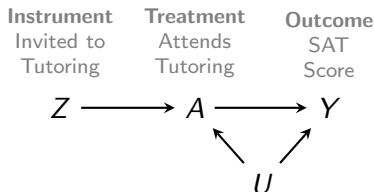


The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 4b) Monotonicity

Assume the invitation never **prevents** tutoring. $A^{z=1} \geq A^{z=0}$



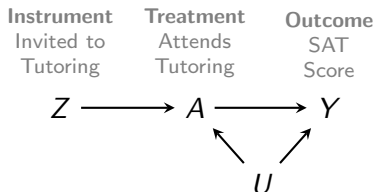
The instrumental variable steps

- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

Requirement 4b) Monotonicity

Assume the invitation never **prevents** tutoring. $A^{z=1} \geq A^{z=0}$

Average effect among the compliers: $E(Y^{a=1} - Y^{a=0} \mid Z^{a=1} = 1, Z^{a=0} = 0)$



The instrumental variable steps

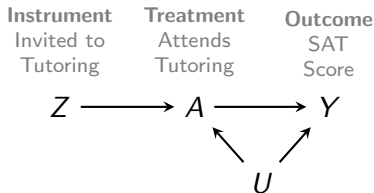
- 1) Estimate effect of Z on Y
- 2) Estimate effect of Z on A
- 3) Divide (1) / (2)

IV requires (1–3) and either (4a) or (4b)

- 1) instrument exchangeability
- 2) no direct $Z \rightarrow Y$ effect
- 3) relevance so that Z is related to A

and either

- 4a) homogeneity such that $A \rightarrow Y$ is constant, or
- 4b) monotonicity, such that $Z \rightarrow A$ is either positive for everyone or negative for everyone



Where to learn more

Felton, C., and B.M. Stewart. 2024. [“Handle with Care: A Sociologist’s Guide to Causal Inference with Instrumental Variables.”](#) Sociological Methods & Research.

An observational settings that is **clean**

1970 RANDOM SELECTION SEQUENCE, BY MONTH AND DAY

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	305	086	108	032	330	249	093	111	225	359	019	129
2	159	144	029	271	298	228	350	045	161	125	034	328
3	251	297	267	083	040	301	115	261	049	244	348	157
4	215	210	275	081	276	020	279	145	232	202	266	165
5	101	214	293	269	364	028	188	054	082	024	310	056
6	224	347	139	253	155	110	327	114	006	087	076	010
7	306	091	122	147	035	085	050	168	008	234	051	012
8	199	181	213	312	321	366	013	048	184	283	097	105
9	194	338	317	219	197	335	277	106	263	342	080	043
10	325	216	323	218	065	206	284	021	071	220	282	041
11	329	150	116	014	037	134	248	324	158	237	046	039
12	221	068	300	346	133	272	015	142	242	072	066	314
13	318	152	259	124	295	069	042	307	175	138	126	163
14	238	004	354	231	178	356	331	198	001	294	127	026
15	017	089	169	273	130	180	322	102	113	171	131	320
16	121	212	166	148	055	274	120	044	207	254	107	096
17	235	189	033	260	112	073	098	154	255	288	143	304
18	140	292	332	090	278	341	190	141	246	005	146	128
19	058	025	200	336	075	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	063	192	185	135
21	186	363	334	062	250	060	027	291	204	243	156	070
22	337	290	265	316	326	247	153	339	160	117	009	053
23	118	057	256	252	319	109	172	116	119	201	182	162
24	059	236	258	002	031	358	023	036	195	196	230	095
25	052	179	343	351	361	137	067	286	149	176	132	084
26	092	365	170	340	357	022	303	245	018	007	309	173
27	355	205	268	074	296	064	289	352	233	264	047	078
28	077	299	223	262	308	222	088	167	257	094	281	123
29	349	285	362	191	226	353	270	061	151	229	099	016
30	164	---	217	208	103	209	287	333	315	038	174	003
31	211	---	030	---	313	---	193	011	---	079	---	100

<https://www.historynet.com/whats-your-number/>



[https://commons.wikimedia.org/wiki/File:
1969_draft_lottery_photo.jpg](https://commons.wikimedia.org/wiki/File:1969_draft_lottery_photo.jpg)

Drafted to serve
in Vietnam

Served in
military

Earnings in 1980s

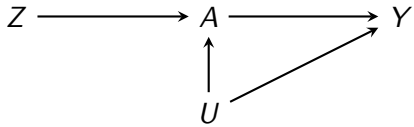
Z

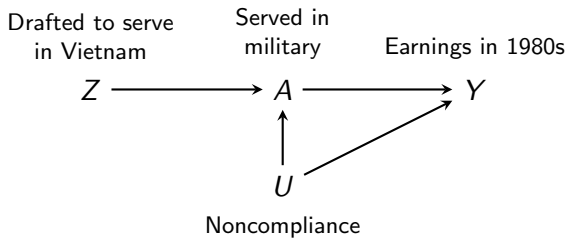
A

Y

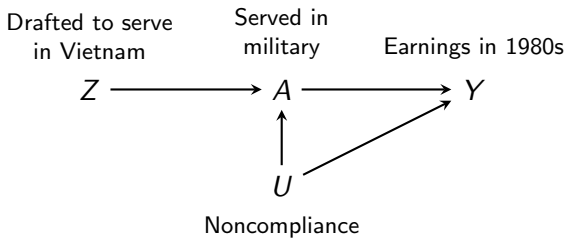
U

Noncompliance



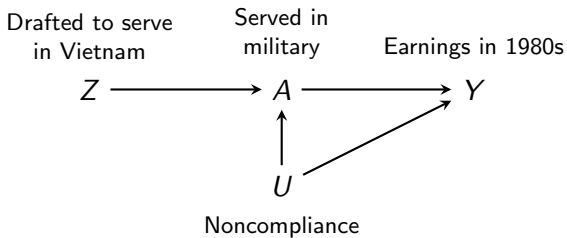


This is credible



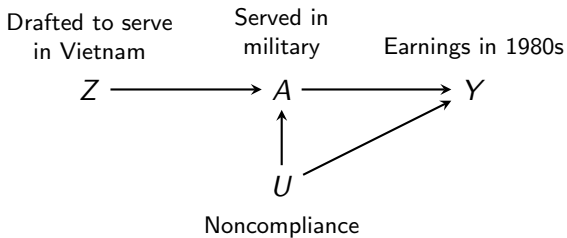
This is credible

- Exogeneity: Randomly selected draft numbers



This is credible

- ▶ Exogeneity: Randomly selected draft numbers
- ▶ Exclusion: Being drafted affects earnings only through service



This is credible

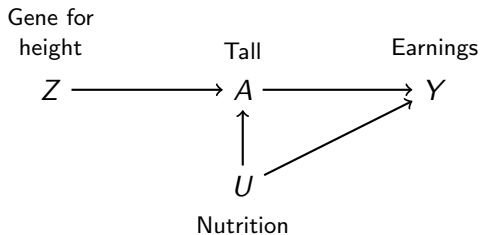
- ▶ Exogeneity: Randomly selected draft numbers
- ▶ Exclusion: Being drafted affects earnings only through service
- ▶ Monotonicity: No one joins the military in defiance of not being drafted

Angrist, J. D. (1990). [Lifetime earnings and the Vietnam era draft lottery: Evidence from Social Security administrative records](#). The American Economic Review, 313-336.

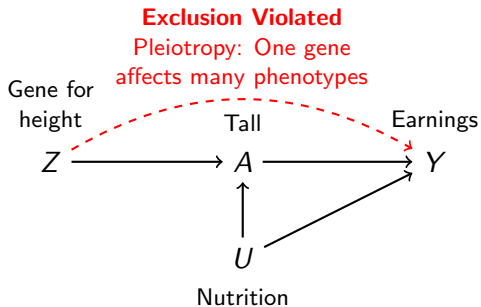
Observational settings that are **less straightforward**

A difficult example: Genes as instruments

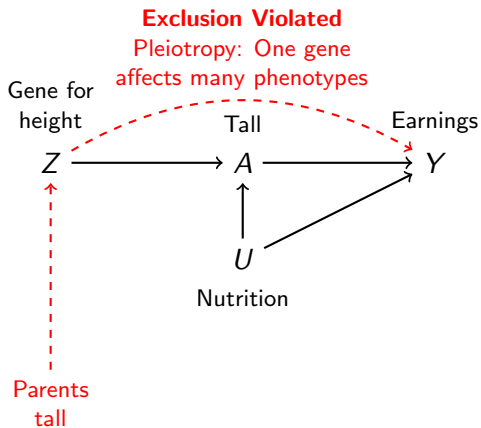
A difficult example: Genes as instruments



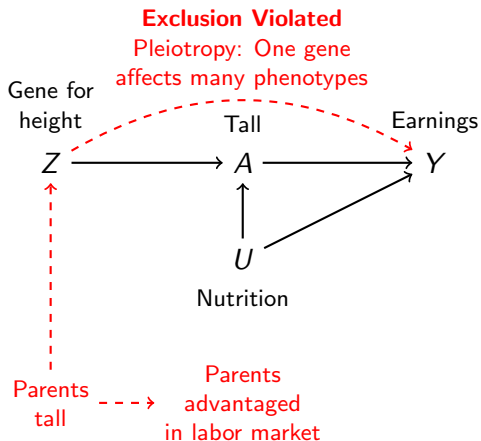
A difficult example: Genes as instruments



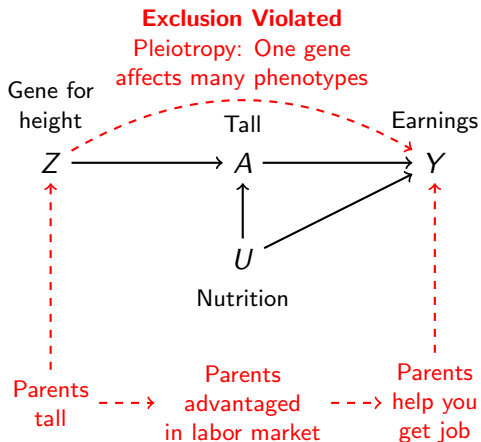
A difficult example: Genes as instruments



A difficult example: Genes as instruments



A difficult example: Genes as instruments



A difficult example: Genes as instruments

