5. Exchangeability: Assumptions to block backdoor paths

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

6 Sep 2022

Responding to feedback

- Group exercises are good for learning
- Would be better with written instructions
- Would be better with more time allocated to the hard one

Learning goals for today

At the end of class, you will be able to:

1. Encode causal theories in Directed Acyclic Graphs (DAGs)
2. Identify causal effects by blocking backdoor paths
3. Understand collider variables

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\end{tikzcd}$$

What makes it a DAG? It is directed and acyclic.

- Directed: Every edge has an arrow. Causality flows one way.
- Acyclic: There are no cycles

Not DAGs:

$$\begin{tikzcd}
 & B \arrow[dr, dash] & \\
A \arrow[ur, dash] \arrow[rr, dash] & & C
\end{tikzcd}$$

Undirected

$$\begin{tikzcd}
 & B \arrow[dr] & \\
A \arrow[ur] & & C \arrow[ll]
\end{tikzcd}$$

Cyclic

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3. DAGs are mathematically precise, with formal properties
4. DAGs are intuitive

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Coin Flip

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Job Training

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Recall: Heads and tails are **exchangeable**.

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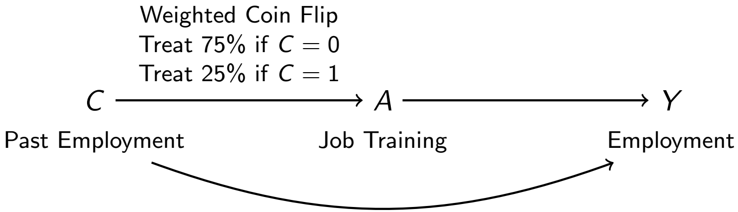

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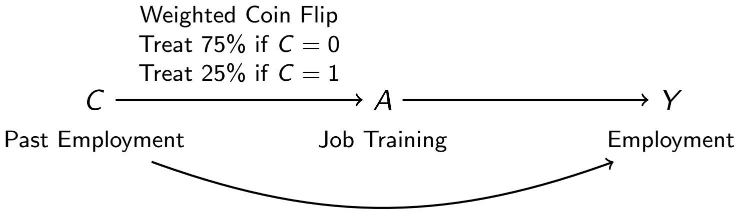

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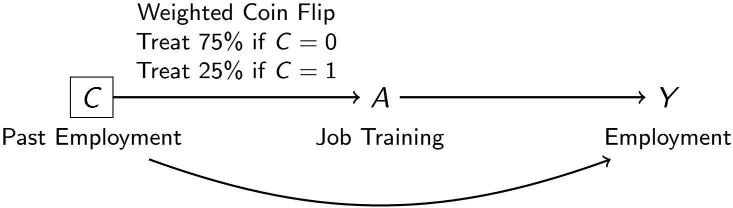

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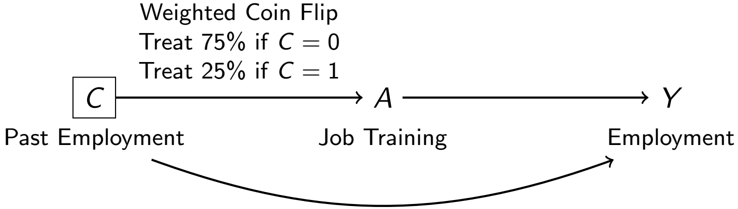

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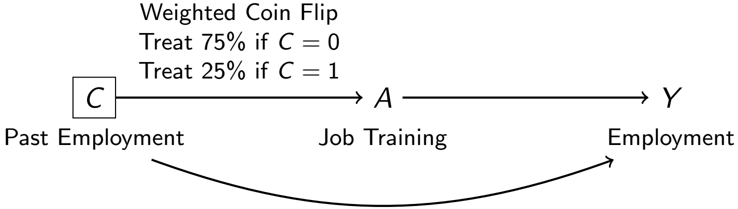

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Next up: Formalize this intuition

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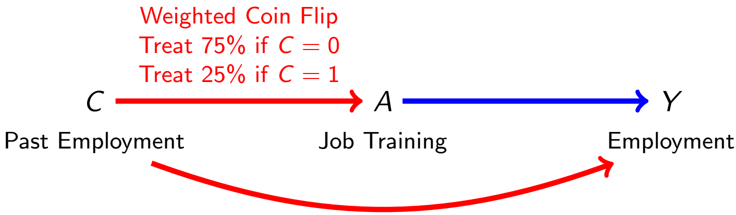

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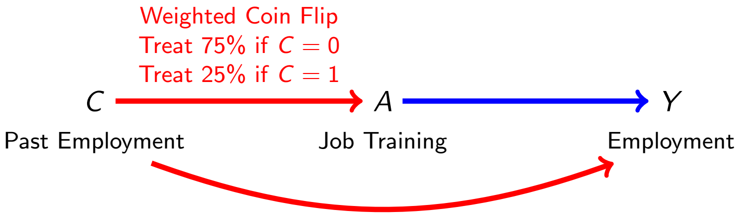

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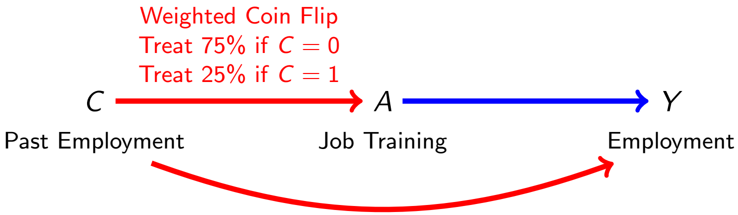

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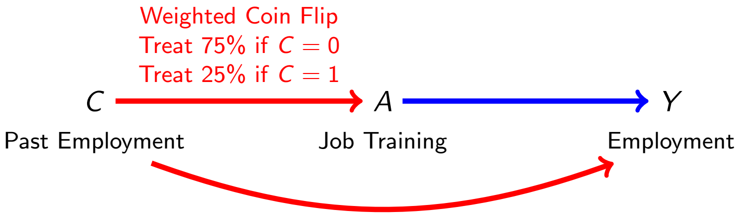

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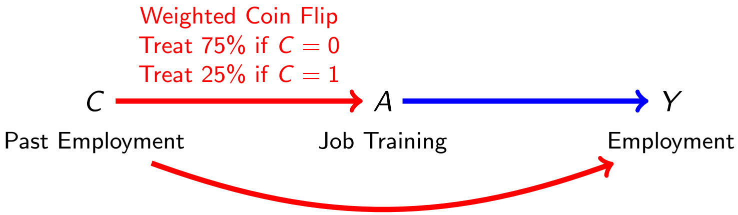

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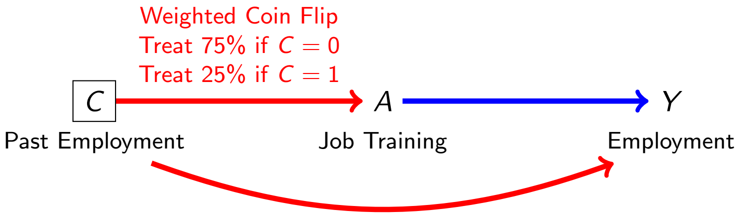

To block the backdoor path, condition on C

- Analyze within strata of C
- Denoted by the box

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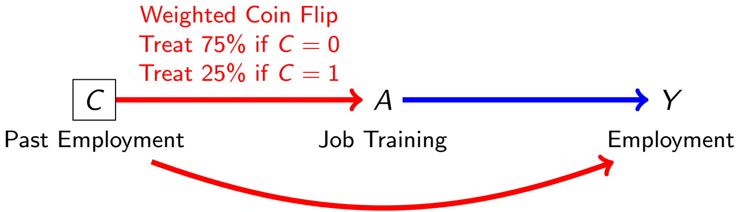
Only difference: In an experiment we know the DAG is true.

Key Concept: Causal Identification



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Causal identification links
causal quantities (involving potential outcomes, e.g. Y^a)
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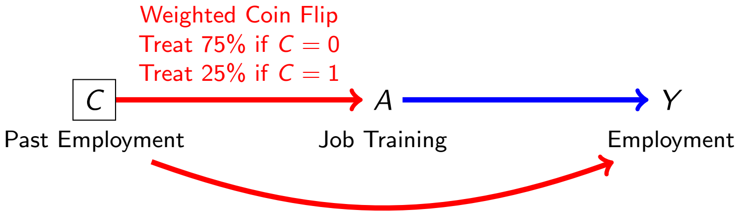

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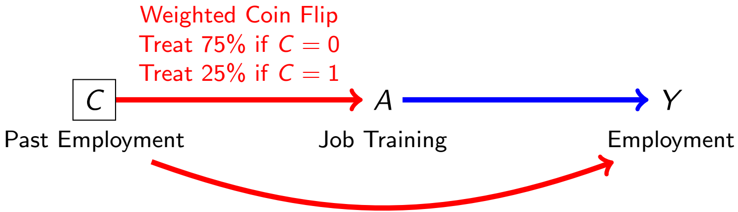

Once we block the backdoor path,

$$\underbrace{\mathrm{E}(Y^a \mid C = c)}_{\text{Causal Quantity}} = \underbrace{\mathrm{E}(Y \mid C = c, A = a)}_{\text{Statistical Quantity}}$$

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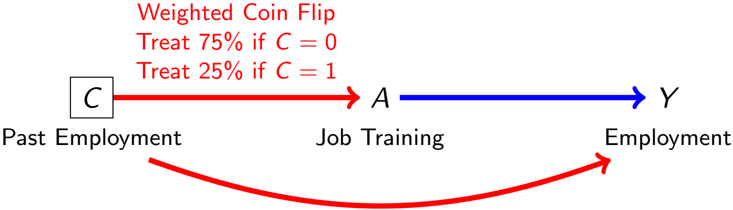

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Also known as **conditional exchangeability** (Hernán & Robins 3.2)

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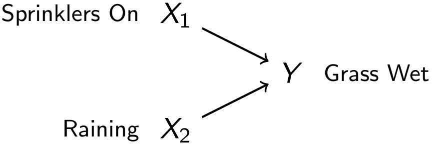

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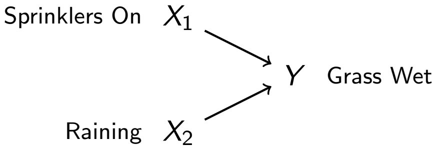

We say Y is a **collider** along the path $X_1 \rightarrow Y \leftarrow X_2$

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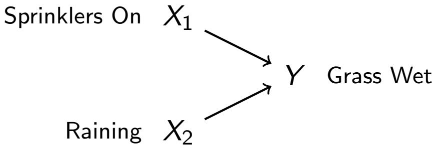

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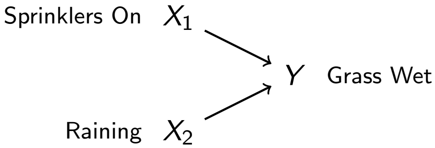

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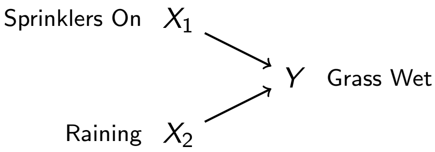

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- X_1 is independent of X_2
  - (Sprinklers On) is uninformative about (Raining)
- Conditioning on Y opens the path
  - If the grass is wet (conditional on $Y = 1$), then either (Sprinklers On) or (Raining)

---

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Ancestors vs. Colliders

Conditioning on an ancestor
closes an open path

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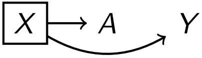

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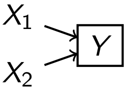

In the population,
A and Y are **related**

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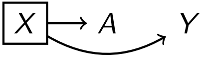

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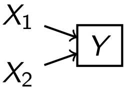

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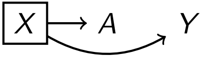

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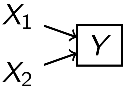

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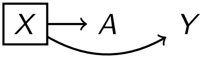

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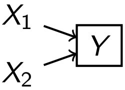

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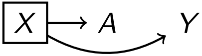

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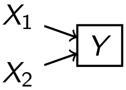

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Example
— X_1 is sprinklers on
— X_2 is rain
— Y is wet grass

Ancestors vs. Colliders[2]

The consequences of conditioning on a descendant differ

[2] A great paper on colliders: Elwert, F., & Winship, C. (2014). Endogenous selection bias: The problem of conditioning on a collider variable. Annual Review of Sociology, 40.

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Conditioning on
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$$
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But conditioning on the descendant of a collider still opens one

$$
\begin{tikzcd}
X_1 \arrow[dr] & & \\
& Y \arrow[r] & \boxed{Z} \\
X_2 \arrow[ur] & &
\end{tikzcd}
$$

[2] A great paper on colliders: Elwert, F., & Winship, C. (2014). Endogenous selection bias: The problem of conditioning on a collider variable. Annual Review of Sociology, 40.

d-separation: Formal definition of blocking backdoor paths

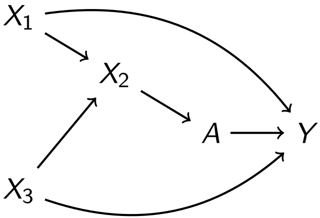
From Greenland, Pearl, & Robins 1999, p. 45:

> *...we say that a set of variables S separates two other sets R and T, or S blocks every path between R and T, if the following criteria are met:*
>
> 1. *Every unblocked path from R to T is intercepted by a variable in S, and*
> 2. *Every unblocked path from R to T generated by adjustment for the variables in S is intercepted by a variable in S*
>
> *(This concept is usually called "d-separation of R and T by S" in the graphical literature, where d stands for "directional."*

Intuition: $R \leftarrow \boxed{S} \rightarrow T$

Exercise

Find 3 sufficient adjustment sets to identify $A \to Y$



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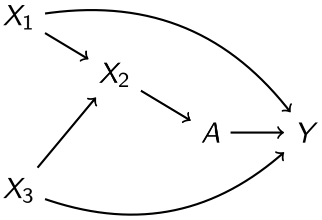

Answer: $\{X_2\}, \{X_1, X_3\}, \{X_1, X_2, X_3\}$

Exercise

What is the smallest adjustment set that identifies $A \to Y$?

$$\begin{tikzcd}
X_1 \arrow[dr] \arrow[ddrrr, bend left] & & & \\
& X_2 & & \\
X_3 \arrow[ur] \arrow[rr] & & A \arrow[r] & Y
\end{tikzcd}$$

Exercise

What is the smallest adjustment set that identifies $A \to Y$?

$$
\begin{tikzcd}
X_1 \arrow[dr] \arrow[ddrrr, bend left] & & & \\
& X_2 & & \\
X_3 \arrow[ur] \arrow[rr] & & A \arrow[r] & Y
\end{tikzcd}
$$

Answer: The empty set! Don't condition on anything.
The collider X_2 already blocks the path.

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> Adjust for all variables that are related to the treatment and related to the outcome.

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Examples where that rule fails:

$$\begin{tikzcd} & M \arrow[dr] & \\ A \arrow[ur] \arrow[rr] & & Y \end{tikzcd}$$

— M is related to A
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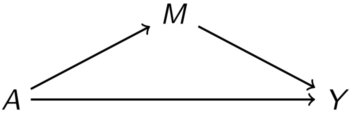

— M is related to A
— M is related to Y
— But conditioning on M blocks a causal path!

Imai, K., Keele, L., Tingley, D., & Yamamoto, T. (2011). Unpacking the black box of causality: Learning about causal mechanisms from experimental and observational studies. American Political Science Review, 105(4), 765-789.

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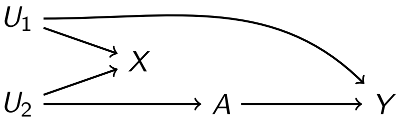

— X is related to A
— X is related to Y

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$$
\begin{tikzcd}
U_1 \arrow[dr] \arrow[drrr, bend left] & & & \\
& X & & \\
U_2 \arrow[ur] \arrow[rr] & & A \arrow[r] & Y
\end{tikzcd}
$$

— X is related to A

— X is related to Y

— But conditioning on X opens the backdoor path!

Greenland, S. (2003). Quantifying biases in causal models: classical confounding vs collider-stratification bias. Epidemiology, 14(3), 300-306.

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Examples where that rule fails:

$$\begin{tikzcd} & & & U \arrow[dl] \arrow[dr] & \\ Z \arrow[rr] & & A \arrow[rr] & & Y \end{tikzcd}$$

— Z is related to A
— Z is related to Y, given A

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$$\begin{tikzcd} & & U \arrow[dl] \arrow[dr] & \\ Z \arrow[r] & A \arrow[rr] & & Y \end{tikzcd}$$

— Z is related to A
— Z is related to Y, given A
— But conditioning on Z amplifies bias from U!

Pearl, J. (2011). Invited commentary: Understanding bias amplification. American Journal of Epidemiology, 174(11), 1223-1227.

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This is why we need a DAG.

Learning goals for today

At the end of class, you will be able to:

1. Encode causal theories in Directed Acyclic Graphs (DAGs)
2. Identify causal effects by blocking backdoor paths
3. Understand collider variables

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!