17. Mediation: Controlled Direct Effects.

Ian Lundberg

Cornell Info 6751: Causal Inference in Observational Settings

Fall 2022

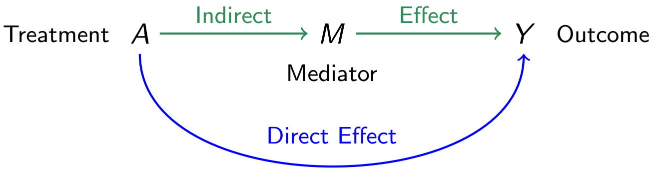
20 Oct 2022

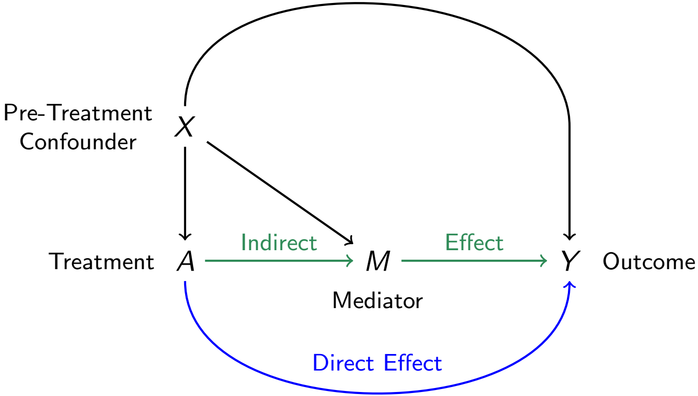
Learning goals for today

At the end of class, you will be able to:

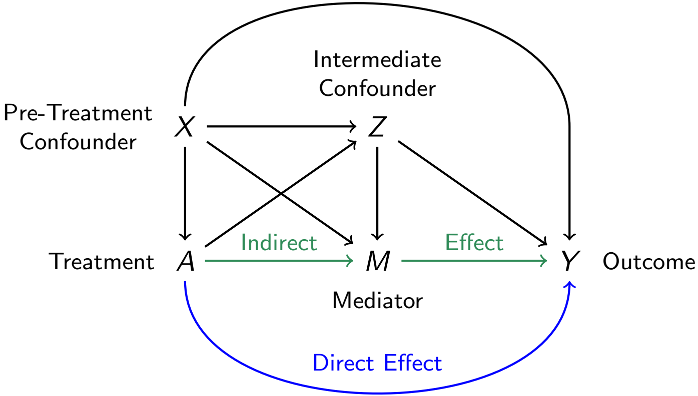
1. Define controlled direct effects
2. Connect them to longitudinal treatments
3. Built intuition for a new estimator: sequential g-estimation

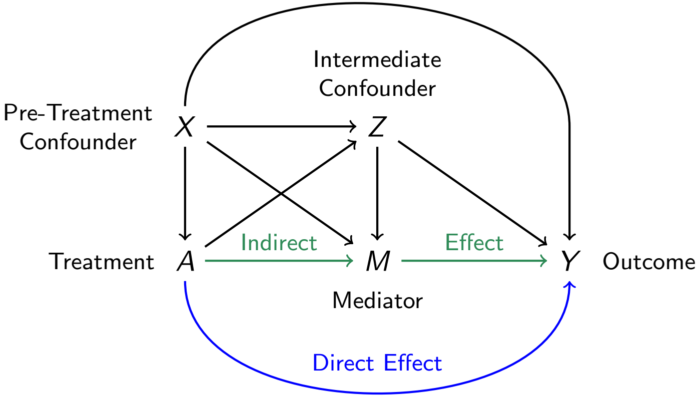
Treatment $A \xrightarrow{\text{Total Effect}} Y$ Outcome


Treatment
A
Indirect
M
Effect
Y
Outcome
Mediator
Direct Effect



Pre-Treatment
Confounder
X
Treatment
A
Indirect
M
Effect
Y
Outcome
Mediator
Direct Effect


Intermediate
Confounder
Pre-Treatment
Confounder
X
Z
Treatment
A
Indirect
M
Effect
Y
Outcome
Mediator
Direct Effect



Before formally defining direct effects, we need a new tool

Single World Intervention Graphs (SWIGs)

Richardson & Robins 2013

Single World Intervention Graphs (SWIGs)

Richardson & Robins 2013

$$
\begin{tikzcd}
A \arrow[r] & Y \\
L \arrow[u] \arrow[ur] &
\end{tikzcd}
$$

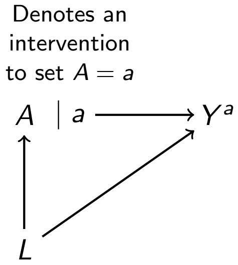
Single World Intervention Graphs (SWIGs)

Richardson & Robins 2013

$$\begin{tikzcd} A \mid a \arrow[r] & Y^a \\ L \arrow[u] \arrow[ur] & \end{tikzcd}$$

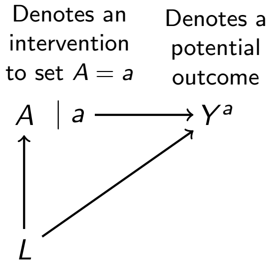
Single World Intervention Graphs (SWIGs)

Richardson & Robins 2013



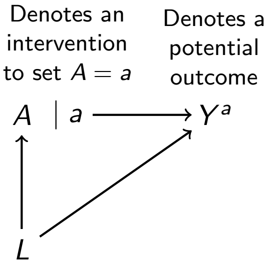
Single World Intervention Graphs (SWIGs)

Richardson & Robins 2013



Single World Intervention Graphs (SWIGs)

Richardson & Robins 2013



SWIGs help in at least two settings:

1. When causal assumptions differ for each potential outcome
2. When we want to focus on a particular intervention

SWIGs help (1): When causal assumptions differ for each potential outcome

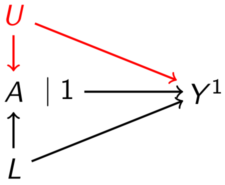
SWIGs help (1): When causal assumptions differ for each potential outcome

Suppose an unobserved U affects the treatment A

SWIGs help (1): When causal assumptions differ for each potential outcome

Suppose an unobserved U affects the treatment A

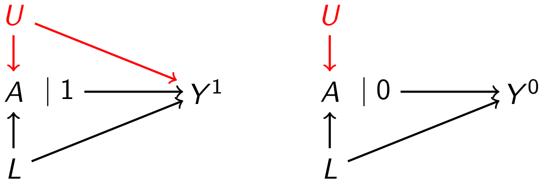
Suppose U affects Y^1



SWIGs help (1): When causal assumptions differ for each potential outcome

Suppose an unobserved U affects the treatment A

Suppose U affects Y^1 But U does not affect Y^0



SWIGs help (1): When causal assumptions differ for each potential outcome

Suppose an unobserved U affects the treatment A

Suppose U affects Y^1

$$\begin{tikzcd}
U \arrow[d] \arrow[drr] & & \\
A \mid 1 \arrow[rr] & & Y^1 \\
L \arrow[u] \arrow[urr] & &
\end{tikzcd}$$

But U does not affect Y^0

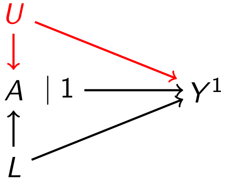
$$\begin{tikzcd}
U \arrow[d] & & \\
A \mid 0 \arrow[rr] & & Y^0 \\
L \arrow[u] \arrow[urr] & &
\end{tikzcd}$$

In this case, $\mathrm{E}(Y^1)$ is not identified but $\mathrm{E}(Y^0)$ is identified.

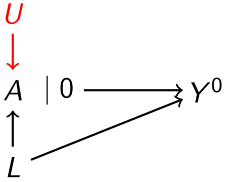
SWIGs help (1): When causal assumptions differ for each potential outcome

Suppose an unobserved U affects the treatment A

Suppose U affects Y^1



But U does not affect Y^0



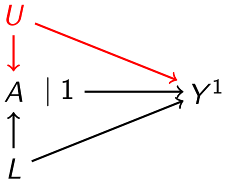
In this case, $\mathrm{E}(Y^1)$ is not identified but $\mathrm{E}(Y^0)$ is identified.

- The ATC $\mathrm{E}(Y^1 - Y \mid A = 0)$ is not identified

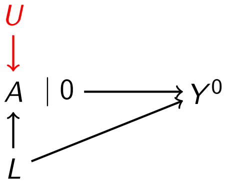
SWIGs help (1): When causal assumptions differ for each potential outcome

Suppose an unobserved U affects the treatment A

Suppose U affects Y^1



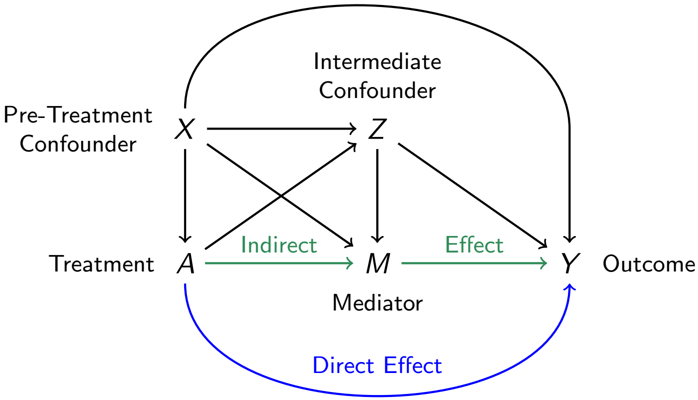
But U does not affect Y^0



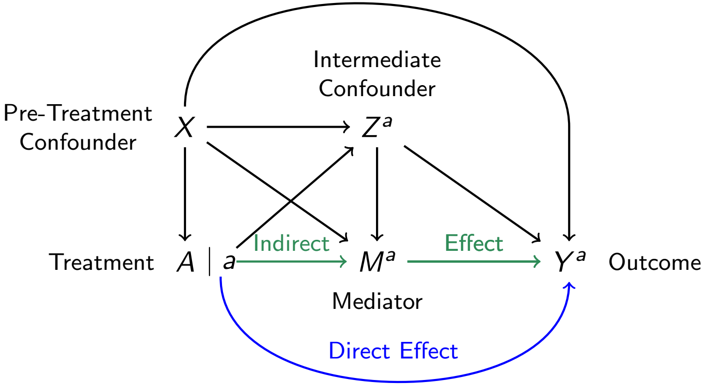
In this case, $\mathrm{E}(Y^1)$ is not identified but $\mathrm{E}(Y^0)$ is identified.

- The ATC $\mathrm{E}(Y^1 - Y \mid A = 0)$ is not identified
- The ATT $\mathrm{E}(Y - Y^0 \mid A = 1)$ is identified

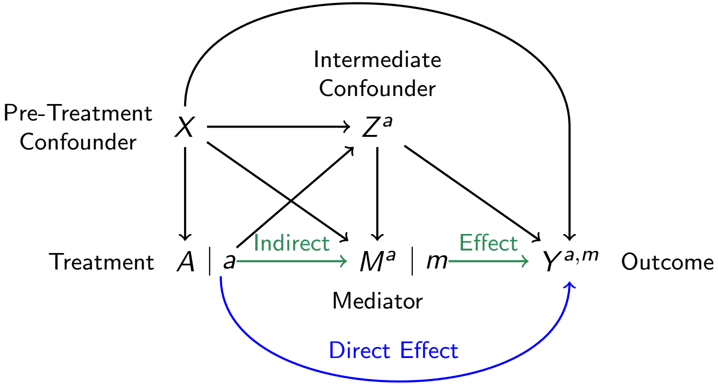
SWIGs help (2): When we want to focus on a particular intervention



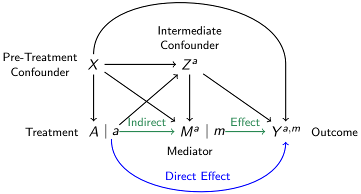
SWIGs help (2): When we want to focus on a particular intervention



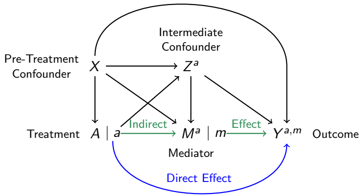
SWIGs help (2): When we want to focus on a particular intervention



Controlled direct effect (CDE)



Controlled direct effect (CDE)



Definition: Controlled Direct Effect

$$\tau(m) = \mathrm{E}\left(Y^{1,m} - Y^{0,m}\right)$$

The effect of an intervention to set treatment $A = 1$ vs $A = 0$ while also intervening to set the mediator to $M = m$

CDE in an experiment

You are an elementary school principal

CDE in an experiment

You are an elementary school principal

Y Kids Read Books

CDE in an experiment

You are an elementary school principal

Librarian Visits Class A

Y Kids Read Books

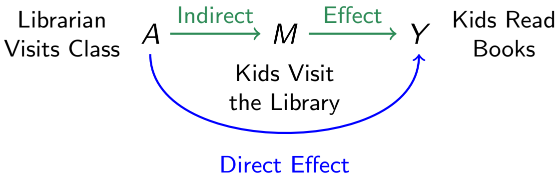
CDE in an experiment

You are an elementary school principal

$$\text{Librarian Visits Class}\quad A \xrightarrow{\text{Indirect}} \underset{\text{Kids Visit the Library}}{M} \xrightarrow{\text{Effect}} Y \quad \text{Kids Read Books}$$

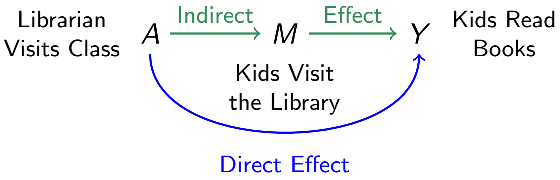
CDE in an experiment

You are an elementary school principal



CDE in an experiment

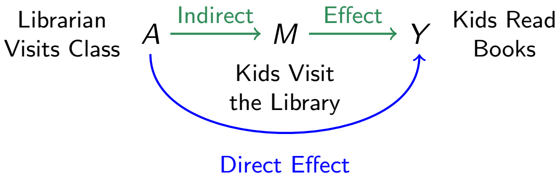
You are an elementary school principal



Experiment for the
Total Effect

CDE in an experiment

You are an elementary school principal

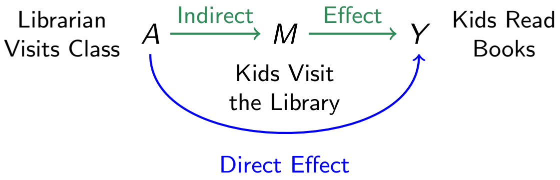


Experiment for the
Total Effect

1) Librarian visits random classes

CDE in an experiment

You are an elementary school principal



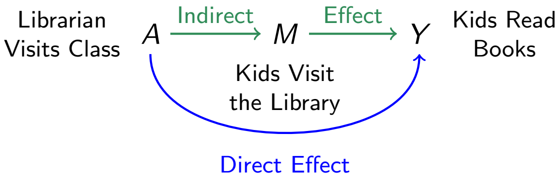
Experiment for the
Total Effect

1) Librarian visits random classes

2) Measure the outcome

CDE in an experiment

You are an elementary school principal

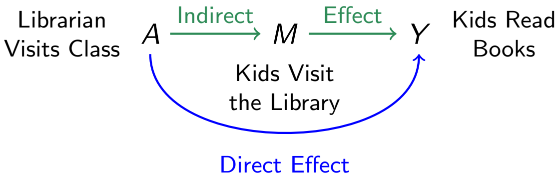


Experiment for the
Direct Effect
$\tau(0) = E\left(Y^{10} - Y^{00}\right)$

Experiment for the
Direct Effect
$\tau(1) = E\left(Y^{11} - Y^{01}\right)$

CDE in an experiment

You are an elementary school principal



Experiment for the
Direct Effect
$\tau(0) = E\left(Y^{10} - Y^{00}\right)$

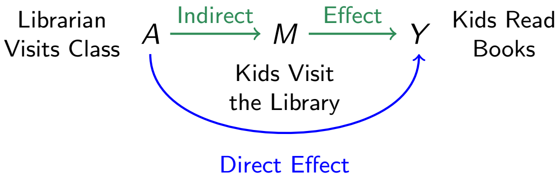
1) Librarian visits random classes

Experiment for the
Direct Effect
$\tau(1) = E\left(Y^{11} - Y^{01}\right)$

1) Librarian visits random classes

CDE in an experiment

You are an elementary school principal



Experiment for the Direct Effect
$\tau(0) = E\left(Y^{10} - Y^{00}\right)$

1) Librarian visits random classes

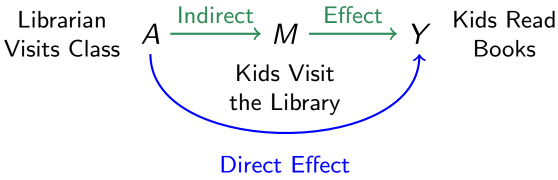
2) You close the school library

Experiment for the Direct Effect
$\tau(1) = E\left(Y^{11} - Y^{01}\right)$

1) Librarian visits random classes

CDE in an experiment

You are an elementary school principal



Experiment for the
Direct Effect
$\tau(0) = E\left(Y^{10} - Y^{00}\right)$

1) Librarian visits random classes

2) You close the school library

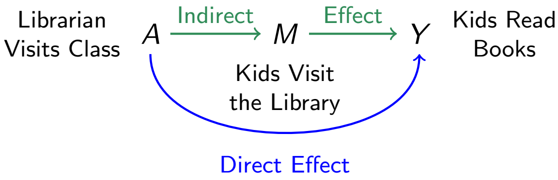
Experiment for the
Direct Effect
$\tau(1) = E\left(Y^{11} - Y^{01}\right)$

1) Librarian visits random classes

2) You make every kid visit the library

CDE in an experiment

You are an elementary school principal



Experiment for the Direct Effect
$\tau(0) = E\left(Y^{10} - Y^{00}\right)$

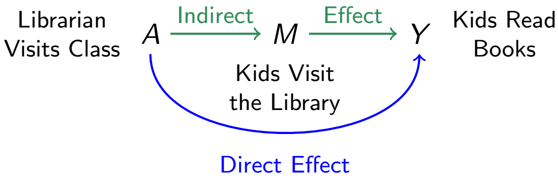
1) Librarian visits random classes
2) You close the school library
3) Measure the outcome

Experiment for the Direct Effect
$\tau(1) = E\left(Y^{11} - Y^{01}\right)$

1) Librarian visits random classes
2) You make every kid visit the library
3) Measure the outcome

CDE in an experiment

You are an elementary school principal



Note

These two estimands are **not** the same.

There are **two** direct effects.

Experiment for the Direct Effect
$\tau(0) = E\left(Y^{10} - Y^{00}\right)$

1) Librarian visits random classes
2) You close the school library
3) Measure the outcome

Experiment for the Direct Effect
$\tau(1) = E\left(Y^{11} - Y^{01}\right)$

1) Librarian visits random classes
2) You make every kid visit the library
3) Measure the outcome

CDE warning: Non-manipulable mediators

It is hard to study mediators that occur inside a person's head

CDE warning: Non-manipulable mediators

It is hard to study mediators that occur inside a person's head

- Psychological stimulus $\rightarrow$ Stress $\rightarrow$ Test performance

CDE warning: Non-manipulable mediators

It is hard to study mediators that occur inside a person's head

- Psychological stimulus $\rightarrow$ Stress $\rightarrow$ Test performance
- Exposure to racial outgroup $\rightarrow$ Racial resentment $\rightarrow$ Voting

CDE warning: Non-manipulable mediators

It is hard to study mediators that occur inside a person's head

- Psychological stimulus $\rightarrow$ Stress $\rightarrow$ Test performance
- Exposure to racial outgroup $\rightarrow$ Racial resentment $\rightarrow$ Voting
- Father incarcerated $\rightarrow$ Mother depressed $\rightarrow$ Child behavior

CDE warning: Non-manipulable mediators

It is hard to study mediators that occur inside a person's head

- Psychological stimulus $\rightarrow$ Stress $\rightarrow$ Test performance
- Exposure to racial outgroup $\rightarrow$ Racial resentment $\rightarrow$ Voting
- Father incarcerated $\rightarrow$ Mother depressed $\rightarrow$ Child behavior

No experiment could manipulate these mediators

CDE warning: Non-manipulable mediators

It is hard to study mediators that occur inside a person's head

- Psychological stimulus $\rightarrow$ Stress $\rightarrow$ Test performance
- Exposure to racial outgroup $\rightarrow$ Racial resentment $\rightarrow$ Voting
- Father incarcerated $\rightarrow$ Mother depressed $\rightarrow$ Child behavior

No experiment could manipulate these mediators

Mediators outside a person's head are easier to study

- Example: Require every kid to visit the school library

CDE identification and estimation in observational studies

Observational example

Zhou, Xiang. 2022. Attendance, Completion, and Heterogeneous Returns to College: A Causal Mediation Approach. Sociological Methods and Research.

Research Question

How do college attendance (A)
and completion (M)
affect earnings (Y)?

Observational example

Zhou, Xiang. 2022. Attendance, Completion, and Heterogeneous Returns to College: A Causal Mediation Approach. Sociological Methods and Research.

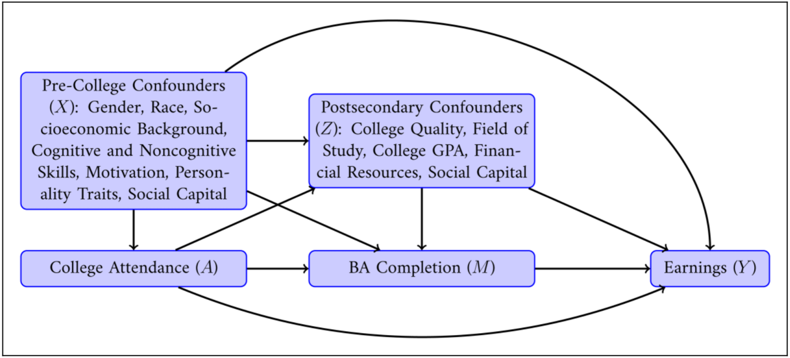


Figure 2. Hypothesized causal relationships in a direct acyclic graph.

Observational example

Zhou, Xiang. 2022. Attendance, Completion, and Heterogeneous Returns to College: A Causal Mediation Approach. Sociological Methods and Research.

The original paper uses Double Machine Learning (complicated)
We will discuss sequential g-estimation (simpler)

Observational example

Zhou, Xiang. 2022. Attendance, Completion, and Heterogeneous Returns to College: A Causal Mediation Approach. Sociological Methods and Research.

The original paper uses Double Machine Learning (complicated)
We will discuss sequential g-estimation (simpler)

A good reference on sequential g-estimation is:

Acharya, A., Blackwell, M., & Sen, M. (2016). Explaining causal findings without bias: Detecting and assessing direct effects. American Political Science Review, 110(3), 512-529.

Estimation by sequential g-estimation

$$
\begin{tikzcd}
X \arrow[r] \arrow[d] \arrow[dr] \arrow[drr, bend left] & Z \arrow[d] \arrow[dr] & \\
A \arrow[r] \arrow[ur] \arrow[rr, bend right] & M \arrow[r] & Y
\end{tikzcd}
$$

High-level overview:

1. Estimate the effect of the mediator
   - Model Y given X, A, Z, M
2. Construct $\tilde{Y}$ with the effect of the mediator removed
   - $\tilde{Y} = Y - \left[\mathsf{E}(Y^M \mid X, A, Z) - \mathsf{E}(Y^0 \mid X, A, Z)\right]$
3. Estimate treatment effect on the de-mediated outcome
   - Model $\tilde{Y}$ given X, A

Estimation by sequential g-estimation

$$\begin{tikzcd}
X \arrow[r] \arrow[d] \arrow[dr] \arrow[drr, bend left] & Z \arrow[d] \arrow[dr] & \\
A \arrow[r] \arrow[ur] \arrow[rr, bend right] & M \arrow[r] & Y
\end{tikzcd}$$

Step 1: What outcome would have been realized at each $M = m$?

$$\mathrm{E}(Y^m \mid X, A, Z) = \mathrm{E}(Y \mid X, A, Z, M = m)$$

because $M \rightarrow Y$ is identifiied given $\{X, A, Z\}$

Estimation by sequential g-estimation

$$\begin{tikzcd}
X \arrow[r] \arrow[d] \arrow[dr] \arrow[drr, bend left] & Z \arrow[d] \arrow[dr] & \\
A \arrow[r] \arrow[ur] \arrow[rr, bend right] & M \arrow[r] & Y
\end{tikzcd}$$

Step 2: Construct a **de-mediated outcome**

$$\tilde{Y} = Y - \gamma(X, A, M)$$

where the de-mediation function γ is

$$\underbrace{\gamma(X, A, M)}_{\substack{\text{Not a function of } Z \\ \text{See below}}} \quad = \underbrace{\mathrm{E}(Y \mid X, A, Z, M) - \mathrm{E}(Y \mid X, A, Z, M = 0)}_{\text{Causal effect of the factual mediator value } M \text{ vs } 0}$$

New assumption: No $Z \times M$ interactions (simplifies estimation)

- The effect $M \rightarrow Y$ does not depend on Z
- By this assumption, γ is not a function of Z

Estimation by sequential g-estimation

$$\begin{tikzcd}
X \arrow[r] \arrow[d] \arrow[dr] \arrow[drr, bend left] & Z \arrow[d] \arrow[dr] & \\
A \arrow[r] \arrow[ur] \arrow[rr, bend right] & M \arrow[r] & Y
\end{tikzcd}$$

Step 3: Estimate the treatment effect on the de-mediated outcome

$$\mathrm{E}(Y^{a,0} \mid X) = \mathrm{E}(\tilde{Y} \mid X, A = a)$$

Estimation by sequential g-estimation

$$
\begin{tikzcd}
X \arrow[r] \arrow[d] \arrow[dr] \arrow[drr, bend left] & Z \arrow[d] \arrow[dr] & \\
A \arrow[r] \arrow[ur] \arrow[rr, bend right] & M \arrow[r] & Y
\end{tikzcd}
$$

High-level overview:

1. Estimate the effect of the mediator
   - Model Y given X, A, Z, M
2. Construct $\tilde{Y}$ with the effect of the mediator removed
   - $\tilde{Y} = Y - \left[\mathsf{E}(Y^M \mid X, A, Z) - \mathsf{E}(Y^0 \mid X, A, Z)\right]$
3. Estimate treatment effect on the de-mediated outcome
   - Model $\tilde{Y}$ given X, A

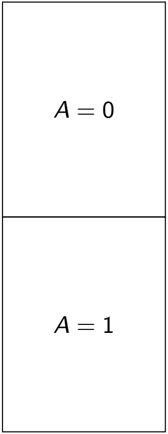
A visual summary: Nonparametric sequential g-estimation

Text here will tell the story for those reading these slides online.

A visual summary: Nonparametric sequential g-estimation

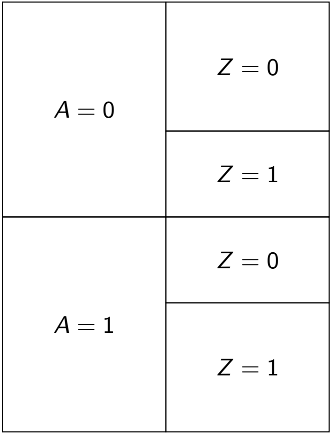
Treatment variable A.

You can think of this as randomized, or you can take this entire story to take place within subgroups of $\vec{X}$ sufficient to yield exchangeability.



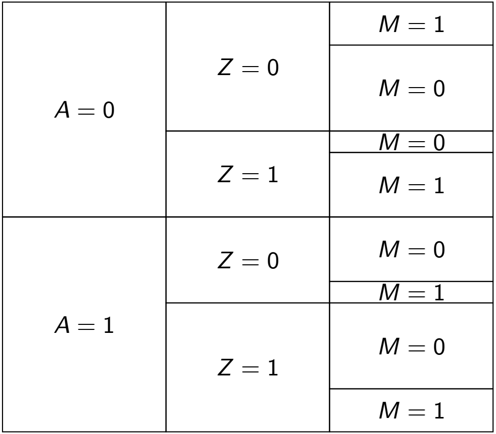
A visual summary: Nonparametric sequential g-estimation

A affects an intermediate confounder Z



A visual summary: Nonparametric sequential g-estimation

Z affects the mediator M



A visual summary: Nonparametric sequential g-estimation

We observe outcome means $\bar{Y}$ in each subgroup.
We can now impute the outcome Y^{A0} under $M = 0$
in each stratum of $\{A, Z\}$.

$A = 0$	$Z = 0$	$M = 1$	$\bar{Y}$
		$M = 0$	$\bar{Y}$
	$Z = 1$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$
$A = 1$	$Z = 0$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$
	$Z = 1$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$

A visual summary: Nonparametric sequential g-estimation

We observe outcome means $\bar{Y}$ in each subgroup.
We can now impute the outcome Y^{A0} under $M = 0$
in each stratum of $\{A, Z\}$.

$A = 0$	$Z = 0$	$\mathrm{E}(Y^{00} \mid A = 0, Z = 0)$	
	$Z = 1$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$
$A = 1$	$Z = 0$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$
	$Z = 1$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$

A visual summary: Nonparametric sequential g-estimation

We observe outcome means $\bar{Y}$ in each subgroup.
We can now impute the outcome Y^{A0} under $M = 0$
in each stratum of $\{A, Z\}$.

$A = 0$	$Z = 0$	$\mathrm{E}(Y^{00} \mid A = 0, Z = 0)$	
	$Z = 1$	$\mathrm{E}(Y^{00} \mid A = 0, Z = 1)$	
$A = 1$	$Z = 0$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$
	$Z = 1$	$M = 0$	$\bar{Y}$
		$M = 1$	$\bar{Y}$

A visual summary: Nonparametric sequential g-estimation

We observe outcome means $\bar{Y}$ in each subgroup.
We can now impute the outcome Y^{A0} under $M = 0$
in each stratum of $\{A, Z\}$.

<table>
<tr><td rowspan="2">$A = 0$</td><td>$Z = 0$</td><td colspan="2">$\mathrm{E}(Y^{00} \mid A = 0, Z = 0)$</td></tr>
<tr><td>$Z = 1$</td><td colspan="2">$\mathrm{E}(Y^{00} \mid A = 0, Z = 1)$</td></tr>
<tr><td rowspan="3">$A = 1$</td><td>$Z = 0$</td><td colspan="2">$\mathrm{E}(Y^{10} \mid A = 1, Z = 0)$</td></tr>
<tr><td rowspan="2">$Z = 1$</td><td>$M = 0$</td><td>$\bar{Y}$</td></tr>
<tr><td>$M = 1$</td><td>$\bar{Y}$</td></tr>
</table>

A visual summary: Nonparametric sequential g-estimation

We observe outcome means $\bar{Y}$ in each subgroup.
We can now impute the outcome Y^{A0} under $M = 0$
in each stratum of $\{A, Z\}$.

$A = 0$	$Z = 0$	$\mathrm{E}(Y^{00} \mid A = 0, Z = 0)$
	$Z = 1$	$\mathrm{E}(Y^{00} \mid A = 0, Z = 1)$
$A = 1$	$Z = 0$	$\mathrm{E}(Y^{10} \mid A = 1, Z = 0)$
	$Z = 1$	$\mathrm{E}(Y^{10} \mid A = 1, Z = 1)$

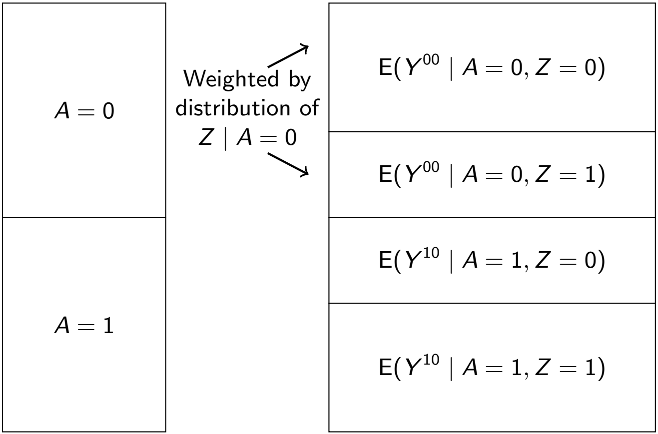
A visual summary: Nonparametric sequential g-estimation

To focus on the effect of A, we now ignore Z.

<table>
<tr><td rowspan="2">$A = 0$</td><td>$\mathrm{E}(Y^{00} \mid A = 0, Z = 0)$</td></tr>
<tr><td>$\mathrm{E}(Y^{00} \mid A = 0, Z = 1)$</td></tr>
<tr><td rowspan="2">$A = 1$</td><td>$\mathrm{E}(Y^{10} \mid A = 1, Z = 0)$</td></tr>
<tr><td>$\mathrm{E}(Y^{10} \mid A = 1, Z = 1)$</td></tr>
</table>

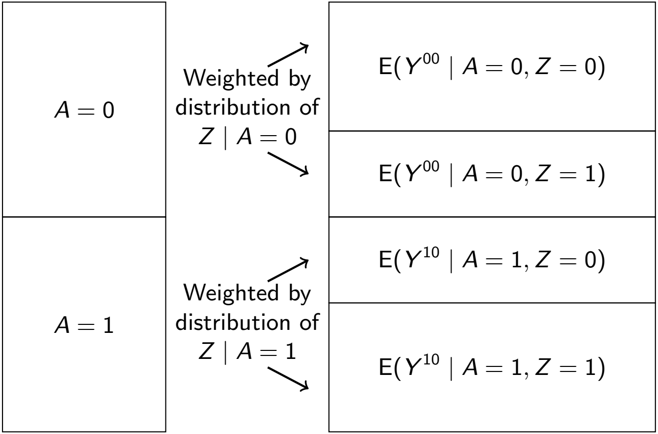
A visual summary: Nonparametric sequential g-estimation

To focus on the effect of A, we now ignore Z.
We have a weighted average over $Z \mid A = a$ for each a.
Because the effect of A is identified, $\underbrace{(Z \mid A = a)}_{\text{Observational}} \quad \sim \quad \underbrace{(Z^a)}_{\text{Interventional}}$



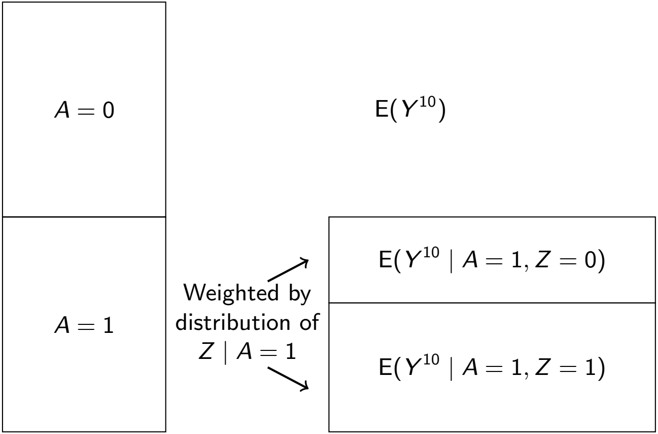
A visual summary: Nonparametric sequential g-estimation

To focus on the effect of A, we now ignore Z.
We have a weighted average over $Z \mid A = a$ for each a.
Because the effect of A is identified, $\underbrace{(Z \mid A = a)}_{\text{Observational}} \quad \sim \quad \underbrace{(Z^a)}_{\text{Interventional}}$



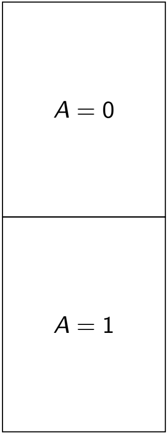
A visual summary: Nonparametric sequential g-estimation

To focus on the effect of A, we now ignore Z.
We have a weighted average over $Z \mid A = a$ for each a.
Because the effect of A is identified, $\underbrace{(Z \mid A = a)}_{\text{Observational}} \sim \underbrace{(Z^a)}_{\text{Interventional}}$



A visual summary: Nonparametric sequential g-estimation

To focus on the effect of A, we now ignore Z.
We have a weighted average over $Z \mid A = a$ for each a.
Because the effect of A is identified, $\underbrace{(Z \mid A = a)}_{\text{Observational}} \sim \underbrace{(Z^a)}_{\text{Interventional}}$

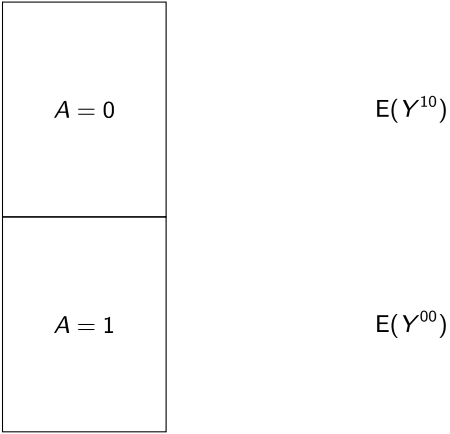


$\mathrm{E}(Y^{10})$

$\mathrm{E}(Y^{00})$

A visual summary: Nonparametric sequential g-estimation

The difference is the CDE $\tau(0)$!



Learning goals for today

At the end of class, you will be able to:

1. Define controlled direct effects
2. Connect them to longitudinal treatments
3. Built intuition for a new estimator: sequential g-estimation

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!