15. Treatments in many time periods.
The problem.

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

13 Oct 2022

Learning goals for today

At the end of class, you will be able to:

1. Present treatments that unfold over time in DAGs
2. Recognize the difficulties of treatment-induced confounding

Treatments in many time periods: Motivation

Suppose you teach second grade

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Task: Draw this in a DAG

Treatments in many time periods: Motivation

Child cannot
sound out
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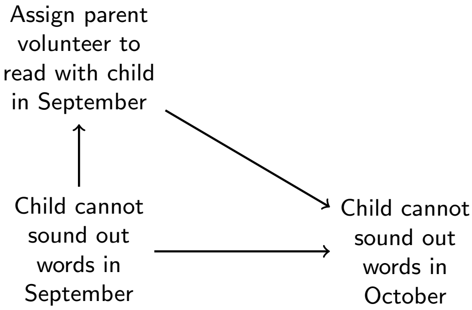
Treatments in many time periods: Motivation

Assign parent volunteer to read with child in September

$\uparrow$

Child cannot sound out words in September

Treatments in many time periods: Motivation



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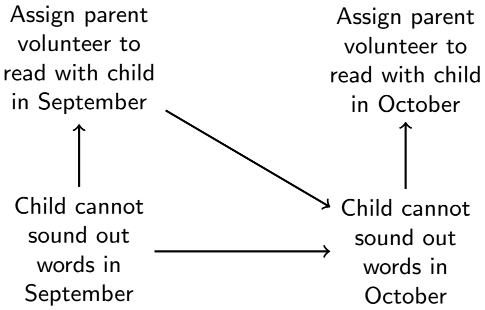

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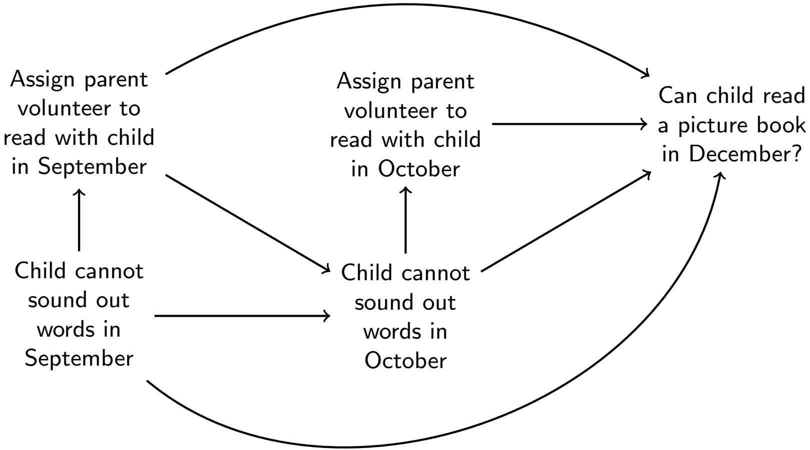

Treatments in many time periods: A general problem

$$
\begin{tikzcd}
A_0 \arrow[r] \arrow[rr, bend left] \arrow[dr] & A_1 \arrow[r] & Y \\
L_0 \arrow[u] \arrow[r] \arrow[urr, bend right] & L_1 \arrow[u] \arrow[ur] &
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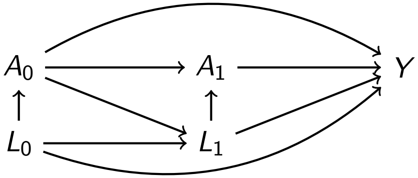

This causal structure occurs

- when a policymaker targets treatment A_k at time k given confounders L_k measured at that time
- in observational settings where treatments unfold over time

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Goal: Study the outcome Y would be realized on average if $A_0, \ldots, A_k$ are set to the values $a_0, \ldots, a_k$.

Treatments in many time periods: The curse of dimensionality

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Each A_k is binary. How many potential outcomes are there?

- $\bar{a} = (0, 0)$: No reading with a parent
- $\bar{a} = (1, 0)$: Read in September, not October
- $\bar{a} = (0, 1)$: Read in October, not September
- $\bar{a} = (1, 1)$: Always read with a parent

Treatments in many time periods: The curse of dimensionality

Suppose the teacher can assign (or not) a parent volunteer to read with a child in each of 9 months in the school year

$$A_0, \dots, A_8$$

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This is why we focus on **treatment strategies**

Treatment strategy

A treatment strategy is a counterfactual policy rule $g()$ for assigning the treatment

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This involves **many treatments**, but only **one strategy**.

Treatment strategy: Exercise

Use math to define the following strategy:

Assign a parent volunteer to read with a child $A_k = 1$ if and only if the child struggles sounding out words $L_k = 0$ and the child did not receive this support last month $A_{k-1} = 0$

Treatment strategy: Exercise

Use math to define the following strategy:

Assign a parent volunteer to read with a child $A_k = 1$ if and only if the child struggles sounding out words $L_k = 0$ and the child did not receive this support last month $A_{k-1} = 0$

$$g(L_k, A_{k-1}) = \mathbb{I}(L_k = 0, A_{k-1} = 0)$$

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A **static** strategy assigns treatments in advance

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A **static** strategy assigns treatments in advance

- Example: Always treat. $g() = 1$

A **dynamic** strategy assigns treatments as a function of the changing values of confounding variables

- Example: Treat if has difficulty sounding out words. $g(L_k) = \mathbb{I}(L_k = 0)$

Identification: The adjustment set

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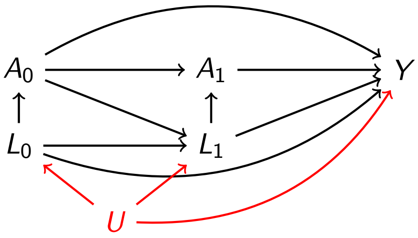

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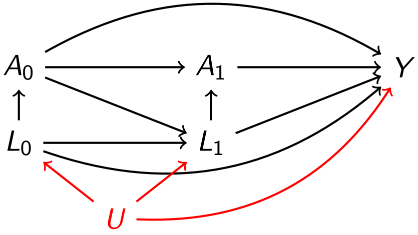

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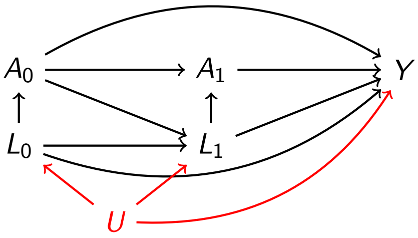

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   a) The total effect of A_0 on Y

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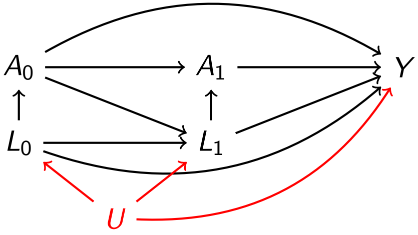

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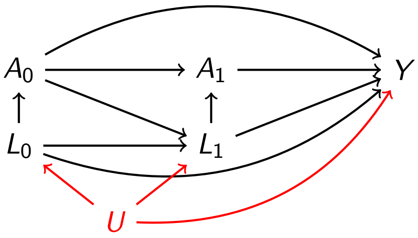

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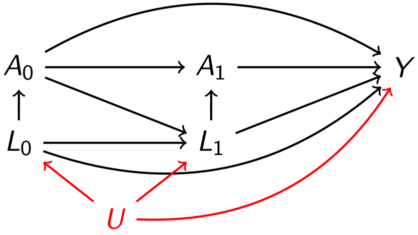

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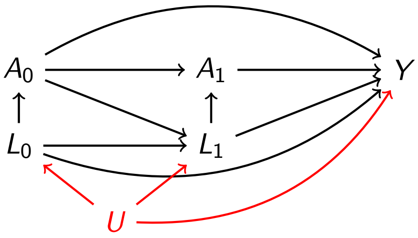

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U is unobserved and cannot be part of the adjustment set

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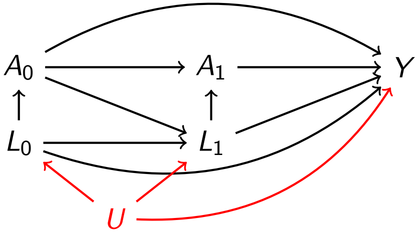

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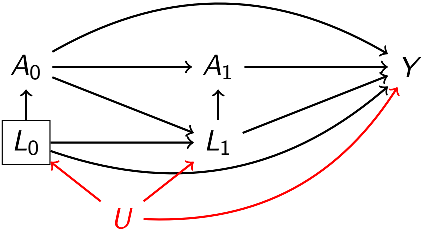

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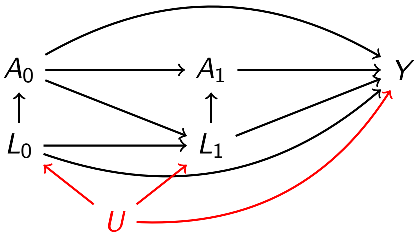

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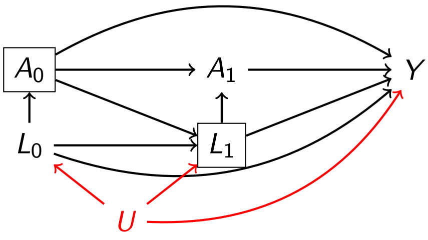

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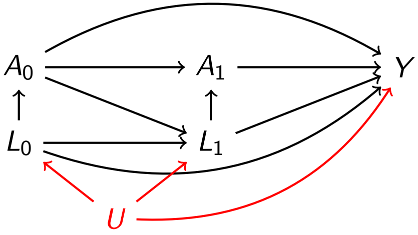

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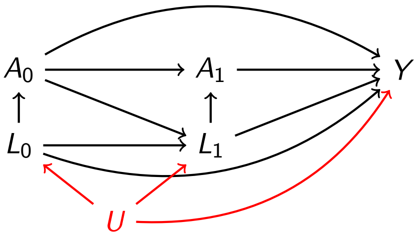

(2) has no solution!

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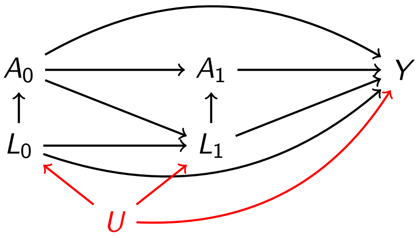

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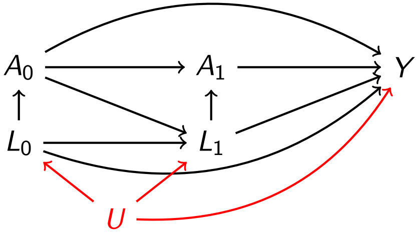

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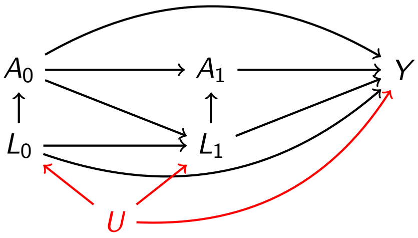

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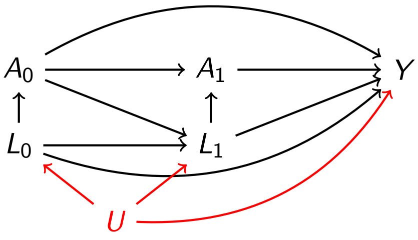

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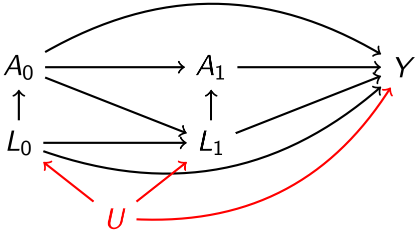

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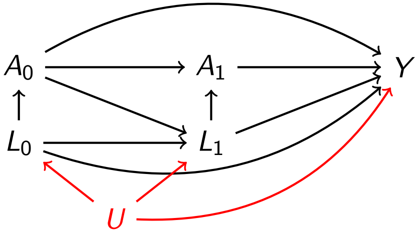

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Next class: How to correctly adjust for treatment-induced confounding

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Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!