Causal Inference: Potential Outcomes

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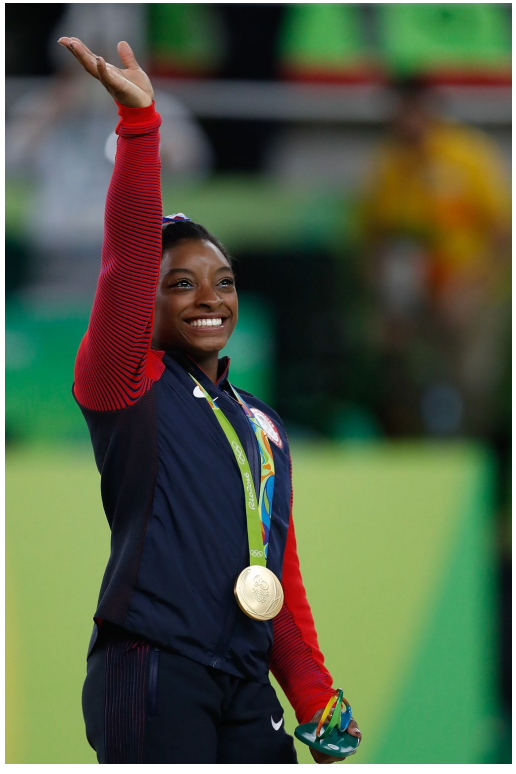
Learning goals for today

- fundamental problem of causal inference
- potential outcomes
- recall mathematical concepts from probability
  - random variables
  - expectation
  - conditional expectation

Causal claims hinge on arguments, not on data



Left photo: By Fernando Frazão/Agência Brasil - http://agenciabrasil.ebc.com.br/sites/_agenciabrasil2013/files/fotos/1035034-_mg_0802_04.08.16.jpg, CCBY3.0br, https://commons.wikimedia.org/w/index.php?curid=50548410
Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, https://commons.wikimedia.org/w/index.php?curid=50584648

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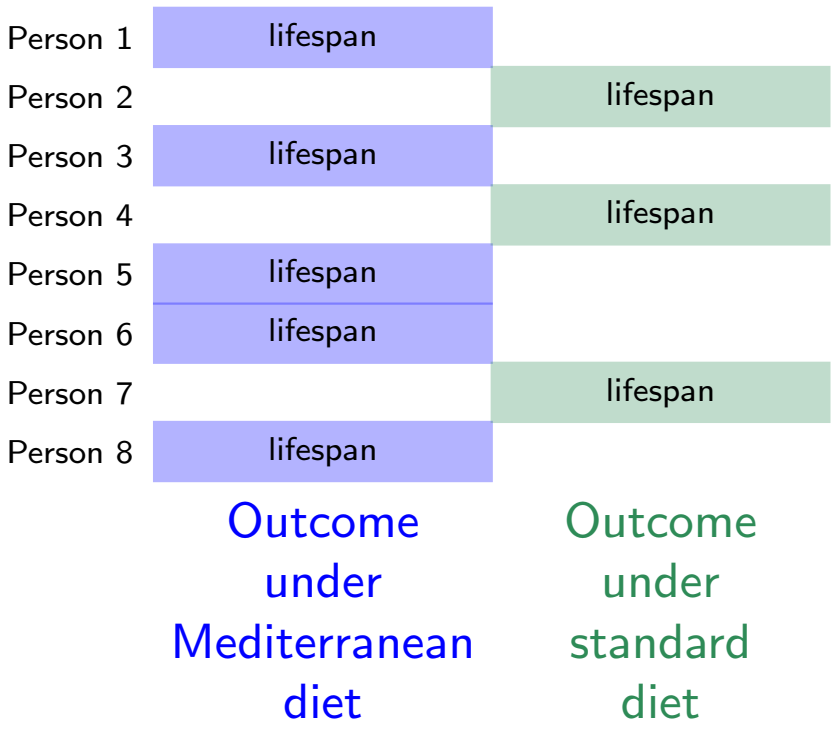
Fundamental problem of causal inference

Holland 1986

Descriptive evidence

average lifespan — average lifespan



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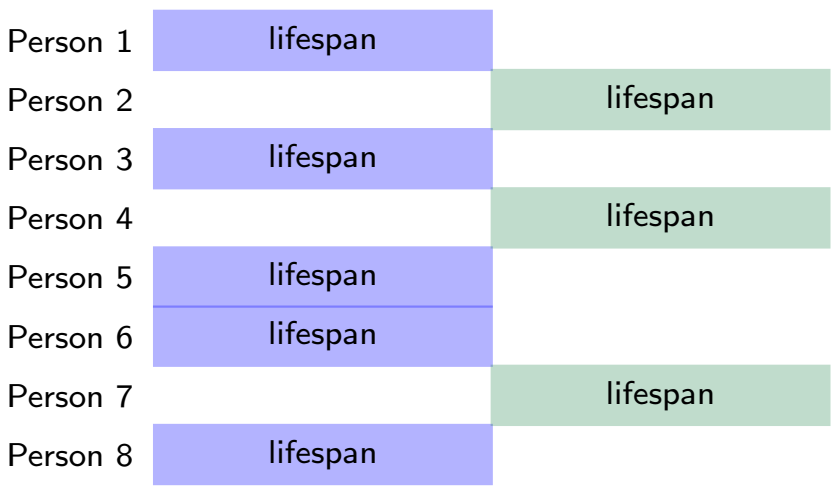

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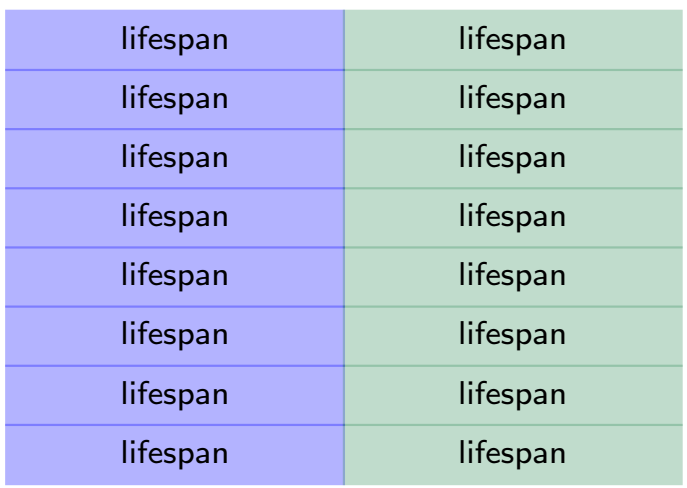

Fundamental problem of causal inference

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	Outcome under Mediterranean diet	Outcome under standard diet
Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

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Causal inference is a **missing data** problem

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$Y_{\text{Ian}}^{\text{MediterraneanDiet}}$	$= \texttt{survived}$	Ian would survive on a Mediterranean diet

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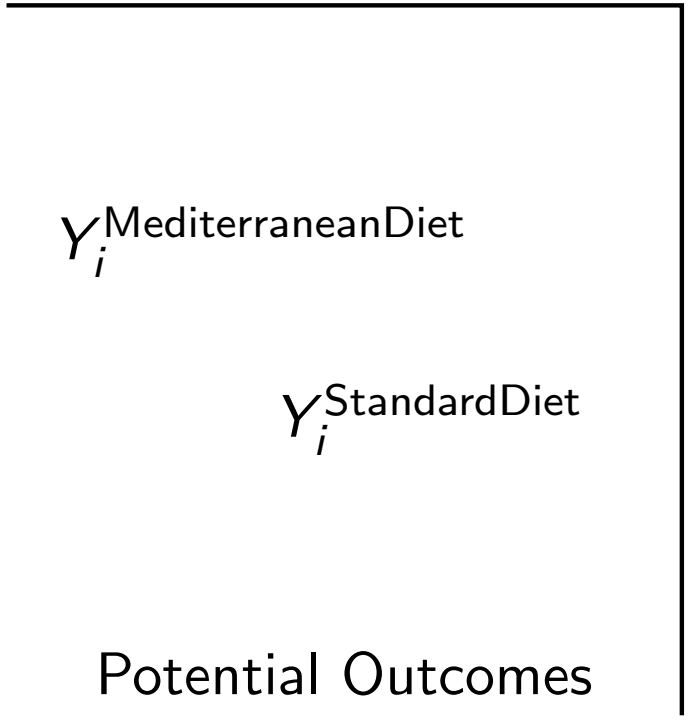
Discuss.

Which potential outcome is observed?

Which is counterfactual?

The consistency assumption

The consistency assumption



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Consistency Assumption

$$Y_i^{A_i} = Y_i$$

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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Check for understanding:

Does it make sense to write $\text{V}(Y_i^a)$? How about $\text{V}(Y^a)$

Notation: Expectation operator

The **expectation operator** E() denotes the population mean

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The quantity Y^a inside the expectation must be a random variable

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A **conditional expectation** is denoted with a vertical bar

$$\mathrm{E}(Y \mid A = a) = \frac{1}{n_a}\sum_{i:A_i=a} Y_i$$

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

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    - On average, a degree causes higher earnings

Practice: How would you write this in math?

1. On average, students who do the homework learn more than those who don't

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Resources to learn more

- Hernán, M.A., & J.M. Robins. 2020.
  Causal Inference: What If?
  Boca Raton: Chapman & Hall / CRC.
- Imbens, G. W., & Rubin, D. B. 2015.
  Causal Inference in Statistics, Social, and Biomedical Sciences.
  Cambridge University Press.
- Brand, J. E. 2023.
  Overcoming the Odds: The Benefits of Completing College for Unlikely Graduates.
  Russell Sage Foundation.