Causal Inference 2: Directed Acyclic Graphs

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Learning goals for today

- fork structures
- collider structures
- causal reasoning and statistical independence

A hypothetical experiment in two population subgroups

A hypothetical experiment in two population subgroups

People who like exercise	People who don't like exercise

A hypothetical experiment in two population subgroups

People who like exercise	People who don't like exercise
Treatment 75% assigned an exercise coach for 1 month	**Treatment** 25% assigned an exercise coach for 1 month

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People who like exercise

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75% assigned an exercise coach for 1 month

People who don't like exercise

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Outcome: How many pull-ups can they do?

A hypothetical experiment in two population subgroups

People who like exercise	People who don't like exercise
Treatment 75% assigned an exercise coach for 1 month	**Treatment** 25% assigned an exercise coach for 1 month

Outcome: How many pull-ups can they do?

Question for you:
Give 2 reasons why those assigned a coach can do more pull-ups

A hypothetical experiment in two population subgroups

$$\begin{tikzcd}
\text{Assigned a Coach} \arrow[r] & \text{Pull-Ups} \\
\text{Likes Exercise} \arrow[u] \arrow[ur] &
\end{tikzcd}$$

A hypothetical experiment in two population subgroups

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Nodes are random variables. **Edges** ($\rightarrow$) are causal relations

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The graph links causal assumptions to statistical dependence

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In this graph, (Assigned a Coach) and (Pull-Ups) are statistically dependent because of two open paths:

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- (Assigned a Coach) $\rightarrow$ (Pull-Ups)
    - a causal path: all arrows go one direction

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Nodes are random variables. **Edges** ($\to$) are causal relations

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In this graph, (Assigned a Coach) and (Pull-Ups) are statistically dependent because of two open paths:

- (Assigned a Coach) $\to$ (Pull-Ups)
  - a causal path: all arrows go one direction
- (Assigned a Coach) $\leftarrow$ (Likes Exercise) $\to$ (Pull-Ups)
  - a backdoor path containing a fork

A hypothetical experiment in two population subgroups

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How to study the causal effect (Assigned a Coach) $\rightarrow$ (Pull-Ups)?

A hypothetical experiment in two population subgroups

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- split into two subgroups: likes exercise and don't

A hypothetical experiment in two population subgroups

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- split into two subgroups: likes exercise and don't
- we have a simple randomized experiment within each subgroup

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- estimate within each subgroup
- pool the two estimates for an average causal effect estimate

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- estimate within each subgroup
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Terminology: Identify by **conditioning** on (Likes Exercise)

Colliders: The sprinkler example

Example from Pearl, J. (1988). Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference.

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- I set my sprinklers to turn on at random times
- It rains at random times

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- I set my sprinklers to turn on at random times
- It rains at random times
- (Sprinklers) or (Rain) can make the grass wet

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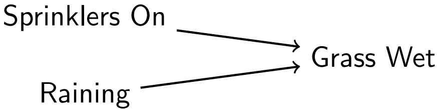

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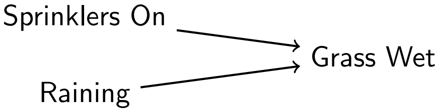

Questions for you:

- Are (Sprinklers On) and (Raining) statistically dependent?
- Are (Sprinklers On) and (Raining) statistically dependent once I restrict to times when the (Grass Wet = `TRUE`)?

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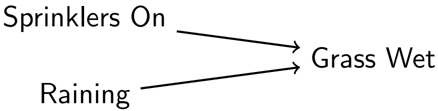

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$$\begin{tikzcd} \text{Sprinklers On} \arrow[dr] & \\ & \text{Grass Wet} \\ \text{Raining} \arrow[ur] & \end{tikzcd}$$

- (Grass Wet) is a **collider** (arrows collide $\rightarrow\leftarrow$)

Colliders: The sprinkler example

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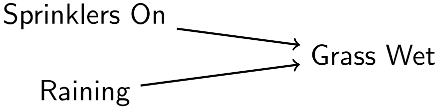

- (Grass Wet) is a **collider** (arrows collide $\rightarrow\leftarrow$)
- A collider blocks a path
  - marginal independence of (Sprinklers On) and (Raining)

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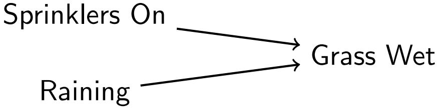

- (Grass Wet) is a **collider** (arrows collide $\rightarrow\leftarrow$)
- A collider blocks a path
  - marginal independence of (Sprinklers On) and (Raining)
- Conditioning on a collider opens the path
  - conditional dependence of (Sprinklers On) and (Raining) when restricting to times when (Grass Wet $=$ `True`)

Using DAGs to identify causal effects: Game plan

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1. Draw a DAG

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   - Create nodes for treatment and outcome

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1. Draw a DAG
   - Create nodes for treatment and outcome
   - Add other nodes

Using DAGs to identify causal effects: Game plan

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     - anything causally related to any two nodes in the graph

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2. List all paths between treatment and outcome

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   variables that jointly block all non-causal paths

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      (and no descendant of that collider is adjusted)

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   - a path is blocked if it contains an adjusted non-collider
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     (and no descendant of that collider is adjusted)
   - otherwise unblocked

Practice

To what extent does completing a 4-year college degree affect a person's future earnings?

Effect of a 4-year degree on future earnings

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degree earnings

Effect of a 4-year degree on future earnings

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degree $\longrightarrow$ earnings

Effect of a 4-year degree on future earnings

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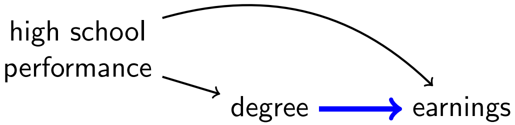

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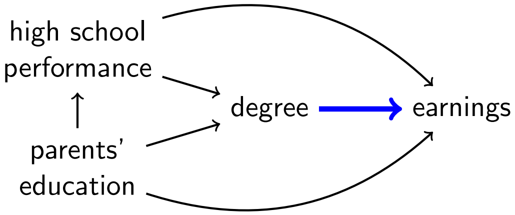
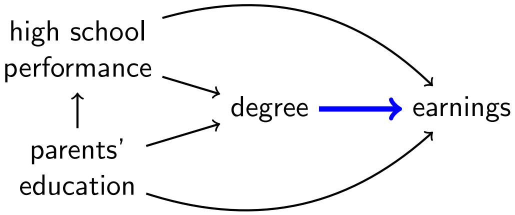


Effect of a 4-year degree on future earnings

2) List all paths between the treatment and outcome



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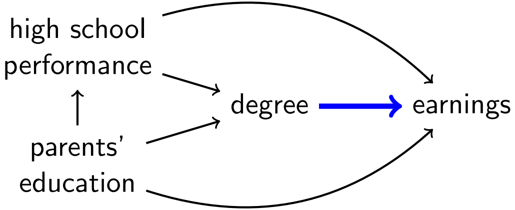

Causal paths

(degree) $\rightarrow$ (earnings)

Effect of a 4-year degree on future earnings

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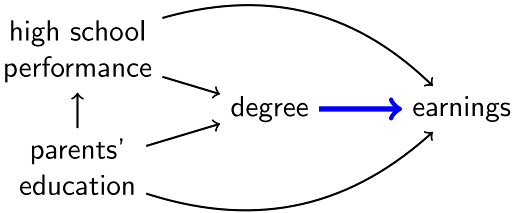


Causal paths

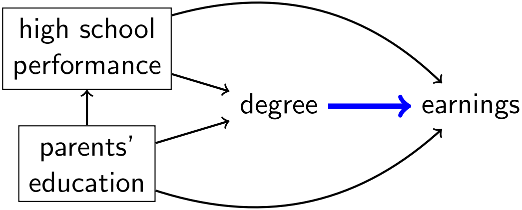
(degree) $\rightarrow$ (earnings)

Backdoor paths

(degree) $\leftarrow$ (high school performance) $\rightarrow$ (earnings)
(degree) $\leftarrow$ (parents' education) $\rightarrow$ (earnings)
(degree) $\leftarrow$ (high school performance) $\leftarrow$ (parents' education) $\rightarrow$ (earnings)

Effect of a 4-year degree on future earnings

3) Choose a sufficient adjustment set
{high school performance, parents' education}



Causal paths

(degree) → (earnings)

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(degree) ← $\boxed{\text{high school performance}}$ ← $\boxed{\text{parents' education}}$ → (earnings)

DAGs: A promising path

- DAGs connect causal theories to statistical dependence
- Statistical dependence arises through causal paths
- Paths may contain two key structures
  - forks: $A \leftarrow B \rightarrow C$
    (A and C dependent if B unadjusted)
  - colliders: $A \rightarrow B \leftarrow C$
    (A and C dependent if B adjusted)
- Causal identification goal:
  choose a sufficient adjustment set so only the causal path of interest remains open
- Experimental analog:
  Among units who are identical on the sufficient adjustment set, we have a simple randomized experiment

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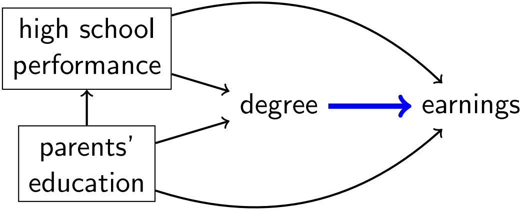

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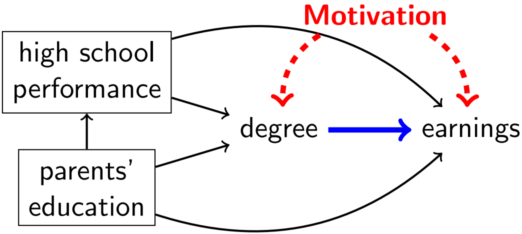

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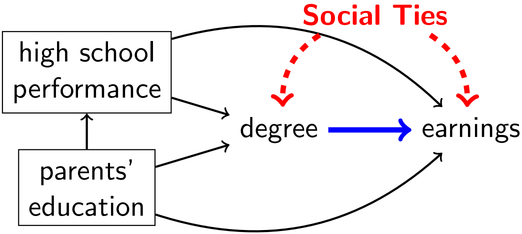

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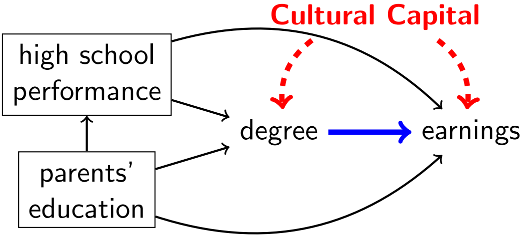

Resources to learn more

- Hernán, M.A., & J.M. Robins. 2020.
  Causal Inference: What If?
  Boca Raton: Chapman & Hall / CRC.
- Pearl, J., & Mackenzie, D. (2018).
  The Book of Why: The New Science of Cause and Effect.
  Basic Books.
- Pearl, J., Glymour, M., & Jewell, N. P. (2016).
  Causal Inference in Statistics: A Primer.
  John Wiley & Sons.
- Pearl, J. (2000).
  Causality.
  Cambridge University Press.