Marginal Structural Models

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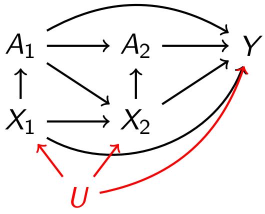
Learning goals for today

1. define causal estimands
2. make causal assumptions with Directed Acyclic Graphs
3. estimate with inverse probability weights
   - in one period
   - in two periods
4. estimate with a marginal structural model

Running example

- at time 1,
  - a student receives a midterm test score X_1
  - they decide whether to study more A_1
- at time 2,
  - a student receives a midterm test score X_2
  - they decide whether to study more A_2
- they get a final exam score Y



Defining causal effects

Effect of studying at time 2

Expected outcome $E(Y^{a_2})$ if assigned to the value a_2 for studying at time 2

Student	$Y^{a_2=0}$	$Y^{a_2=1}$
1	80	95
2	70	95
3	90	90
$\vdots$	$\vdots$	$\vdots$

Defining causal effects

Effect of studying at time 2

Expected outcome $E(Y^{a_2})$ if assigned to the value a_2 for studying at time 2

Student	$Y^{a_2=0}$	$Y^{a_2=1}$
1	80	?
2	70	?
3	?	90
$\vdots$	$\vdots$	$\vdots$

Defining causal effects

Effect of studying at time 1 and 2

Expected outcome $E(Y^{a_1,a_2})$ if assigned
to the value (a_1, a_2) for studying at time 1 and time 2

Student	$Y^{a_1=0,a_2=0}$	$Y^{a_1=0,a_2=1}$	$Y^{a_1=1,a_2=0}$	$Y^{a_1=1,a_2=1}$
1	70	80	75	90
2	80	95	80	95
3	90	90	90	90
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Defining causal effects

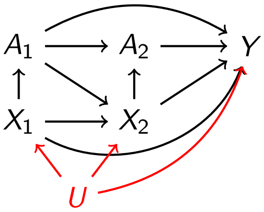
Effect of studying at time 1 and 2

Expected outcome $E(Y^{a_1,a_2})$ if assigned
to the value (a_1, a_2) for studying at time 1 and time 2

Student	$Y^{a_1=0,a_2=0}$	$Y^{a_1=0,a_2=1}$	$Y^{a_1=1,a_2=0}$	$Y^{a_1=1,a_2=1}$
1	?	?	?	90
2	?	95	?	?
3	?	?	90	?
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

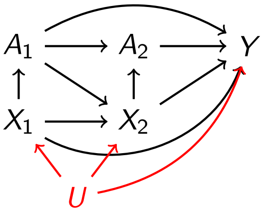
Causal assumptions

We observe data from this model

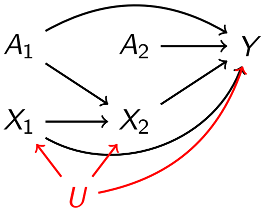


Causal assumptions

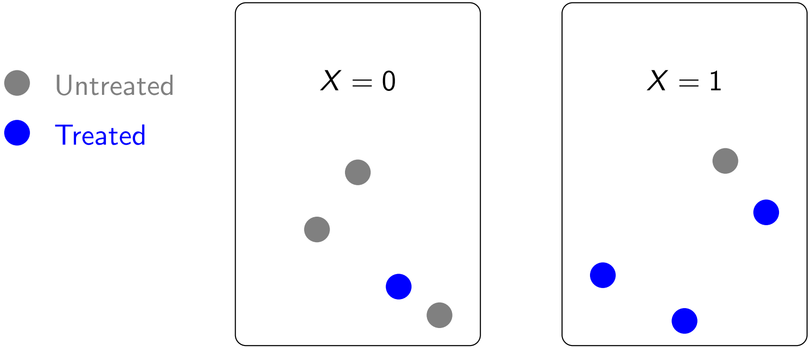
We observe data from this model



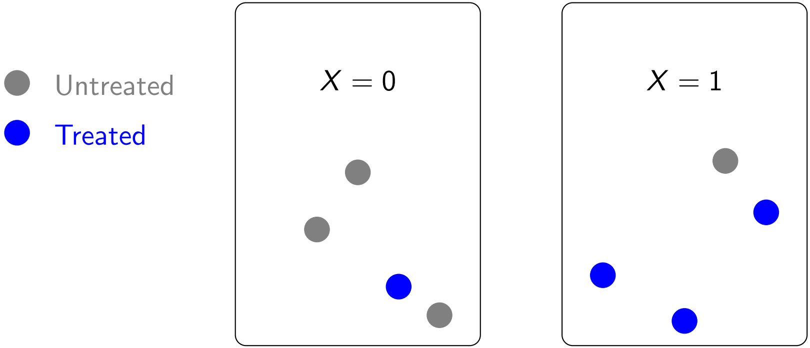
We want this



Inverse probability of treatment weighting

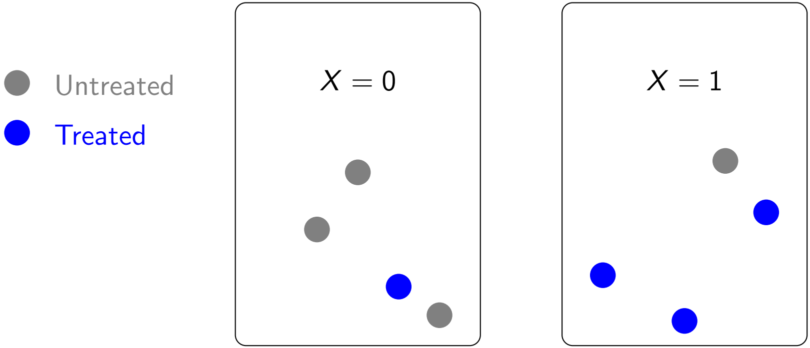


Inverse probability of treatment weighting



Propensity score: $\pi_i = \mathrm{P}(A = A_i \mid X = X_i)$

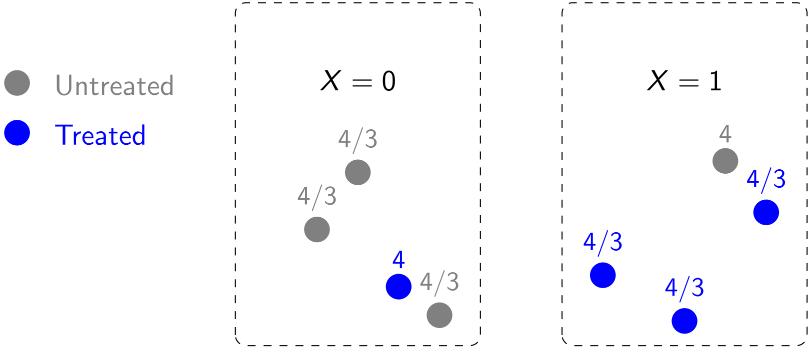
Inverse probability of treatment weighting



Propensity score: $\pi_i = \mathrm{P}(A = A_i \mid X = X_i)$

Inverse probability weight: $w_i = \frac{1}{\pi_i}$

Inverse probability of treatment weighting



Propensity score: $\pi_i = \mathrm{P}(A = A_i \mid X = X_i)$

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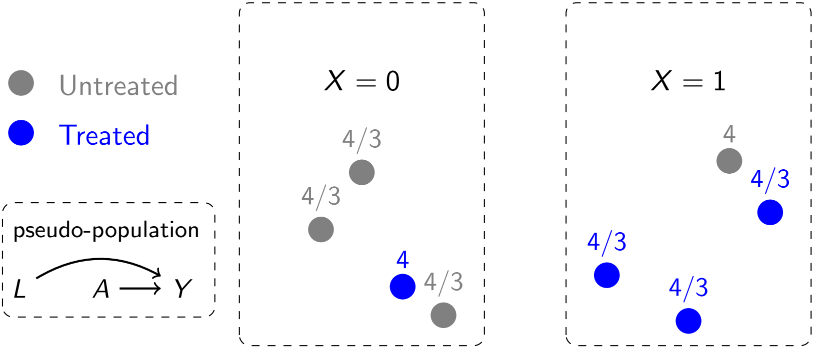
Inverse probability of treatment weighting



Propensity score: $\pi_i = \mathrm{P}(A = A_i \mid X = X_i)$

Inverse probability weight: $w_i = \frac{1}{\pi_i}$

Inverse probability weighting

At every time t, define an inverse probability of treatment weight given the measured past confounders and treatments

$$W^{A_t} = \frac{1}{\mathsf{P}(A_t \mid \bar{A}_{t-1}, \bar{L}_t)}$$

Inverse probability weighting

At every time t, define an inverse probability of treatment weight given the measured past confounders and treatments

$$W^{A_t} = \frac{1}{\mathrm{P}(A_t \mid \bar{A}_{t-1}, \bar{L}_t)}$$

Define the overall weight as the product

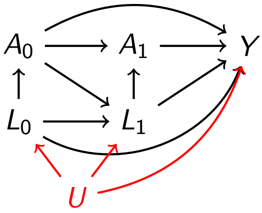
$$W^{\bar{A}} = \prod_{k=1}^{K} \frac{1}{\mathrm{P}(A_k \mid \bar{A}_{k-1}, \bar{L}_k)}$$

Inverse probability weighting

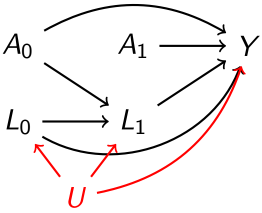
The weight

$$W^{\bar{A}} = \prod_{k=1}^{K} \frac{1}{\mathrm{P}(A_k \mid \bar{A}_{k-1}, \bar{L}_k)}$$

Takes us from this



to this pseudo-population



Marginal structural models

- inverse probability weighting estimates by weighted means

$$E(Y^{a_1=1,a_2=1}) = \frac{1}{\sum_{i:\vec{A}=1} w_i} \sum_{i:\vec{A}=1} Y_i w_i$$

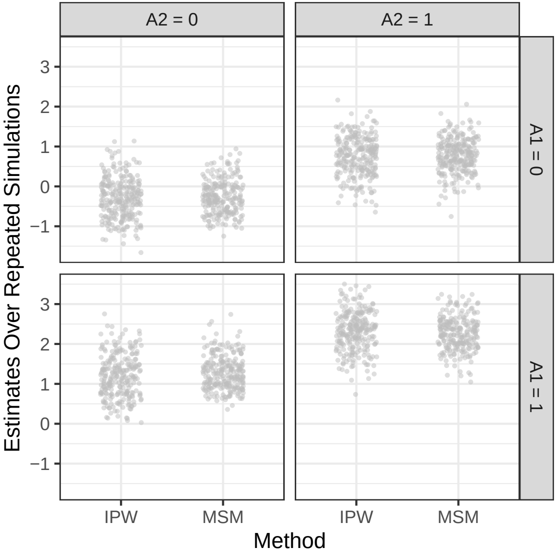
- marginal structural model estimates by a weighted regression

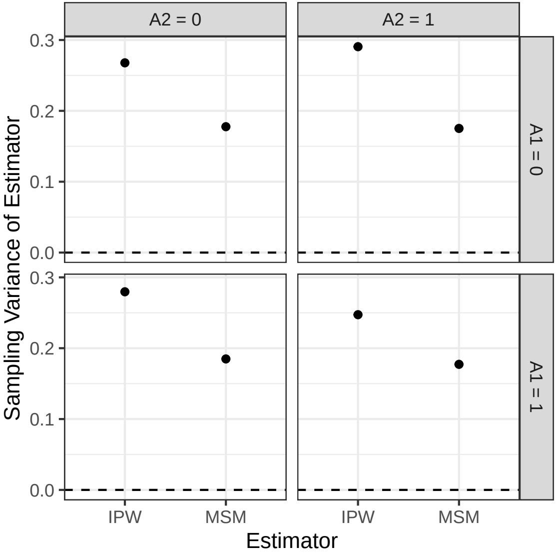
$$E(Y^{a_1,a_2}) = \beta_0 + \beta_1 a_1 + \beta_2 a_2$$

Let's try it

- logistic regression for treatment at each time
- predict the propensity score
- create inverse probability weights
- estimate by IPW and MSM

A1	A2	estimate
0	0	$\hat{\mathrm{E}}(Y^{00})$
0	1	$\hat{\mathrm{E}}(Y^{01})$
1	0	$\hat{\mathrm{E}}(Y^{10})$
1	1	$\hat{\mathrm{E}}(Y^{11})$


A2 = 0
A2 = 1
A1 = 0
A1 = 1
3
2
1
0
−1
Estimates Over Repeated Simulations
IPW
MSM
Method


A2 = 0
A2 = 1
A1 = 0
A1 = 1
0.3
0.2
0.1
0.0
Sampling Variance of Estimator
IPW
MSM
Estimator