3. Consistency

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

30 Aug 2022

Learning goals for today

At the end of class, you will be able to:

1. Reason about when one unit's treatment affects another unit's outcome (interference)
2. Reason about treatments that hide distinct versions
3. Formalize the assumption of treatment variation irrelevance

Hernán (2016): Does Water Kill?

London cholera epidemic, 1854.
John Snow deduced that the water was the cause of death.

Source: Wikimedia Commons

Hernán (2016): Does Water Kill?

Does drinking water kill?

Hernán (2016): Does Water Kill?

Does drinking **fresh** water kill?

Hernán (2016): Does Water Kill?

Does drinking **a swig of** fresh water kill?

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Does drinking a swig of fresh water **from the Broad Street pump** kill?

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Does drinking a swig of fresh water from the Broad Street pump **between August 31 and September 10** kill?

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Does drinking a swig of fresh water from the Broad Street pump between August 31 and September 10 kill

compared with drinking all your water from other pumps?

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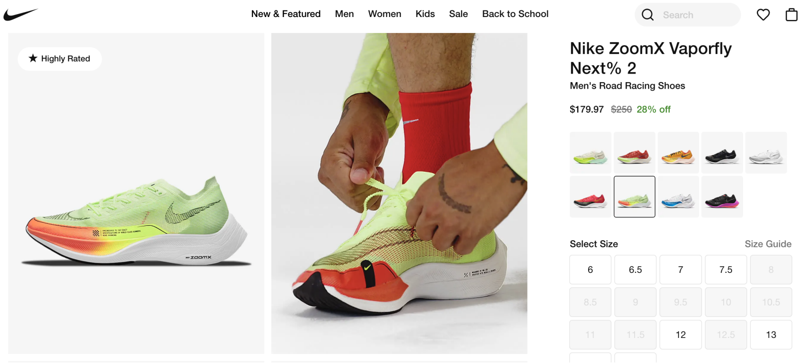
Recommendation: Specify versions
“until no meaningful vagueness remains,”
(Hernan 2016)

Interference

Interference Example 1: Springy Shoes



[1]

Image source: Nike[1]

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- Suppose two closely-matched people run a race.
- Would the Nike Vaporfly affect the outcome?

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We will focus on the outcome for person 1.

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Potential outcomes depend on the treatments of both units (sometimes termed **interference**)

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What if each potential outcome depends on the whole population's treatments?

Interference Example 2: College

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Human Capital Story: It makes you more productive.

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Human Capital Story: It makes you more productive.

Sorting Story: It helps you jump the line to a better job

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In words again: Unit i's outcome depends only on unit i's treatment

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Part of the consistency assumption.
Sometimes called the Stable Unit Treatment Value Assumption (SUTVA)

Multiple versions of treatment

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Earnings go up from a **Cornell** degree.

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Then how should we define the treatment?

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Let A_i be a detailed treatment. Values of A_i include

- No BA
- Cornell BA
- UPenn BA
- ITT Tech BA
- (many more)

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OK if all BAs have equal effects
Otherwise, this definition is problematic

Treatment variation irrelevance (Vanderweele 2009)[2]

Within the analyzed treatments,
all variations of the treatment
have the same effect

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In Vanderweele's notation,

- x is the treatment
- k_x and k'_x are versions of the treatment x
- Potential outcomes are in parentheses

$$Y_i(x, k_x) = Y_i(x, k'_x) \quad \text{for all} \quad k_x, k'_x$$

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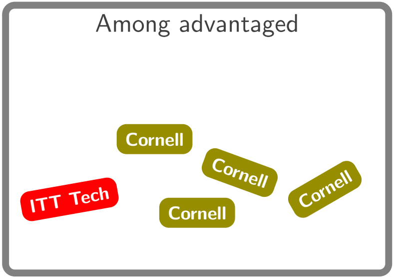

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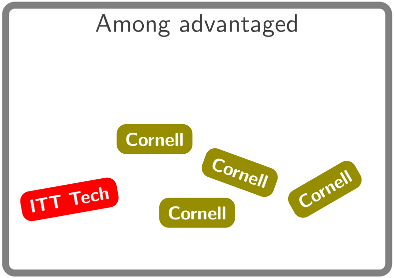

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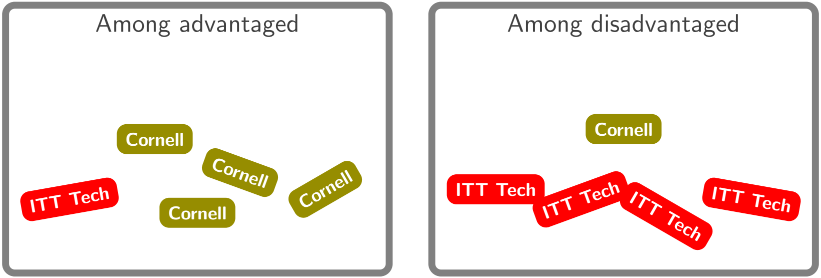

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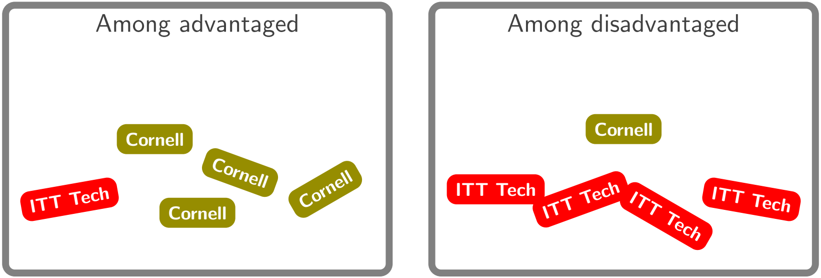

Mistaken claim: College is more helpful to the advantaged.
Better claim: The advantaged disproportionately take the effective treatment.

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   - Requires great care to specify the estimand here

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More research on these questions!

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Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!