This week, you need time to write ideas for the research proposal (remember—due Oct 31 at 5pm!). The problem set therefore involves only conceptual questions and no coding. You should submit a PDF.

1 (40 points) Controlled and natural direct effects

You are studying a treatment A and a mediator M. If you use causal notation other than $Y^{a,m}$ and M^a, then define your notation.

1.1. (10 points) Write a mathematical statement of the following quantity: the controlled direct effect of treatment (1 vs 0) under an intervention to hold the mediator at the value 1.

1.2. (10 points) Write a mathematical statement of the following quantity: the natural direct effect of treatment (1 vs 0) under an intervention to hold the mediator at the value which would have been realized under treatment $A = 0$.

We will consider two settings where you might study these estimands.
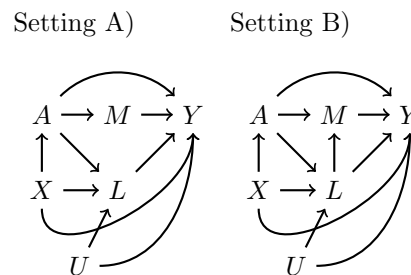
1.3. (10 points) Can the controlled direct effect be identified in Setting A, Setting B, neither, or both Settings A and B?

1.4. (10 points) Can the natural direct effect be identified in Setting A, Setting B, neither, or both Settings A and B?

2 (10 points) A common mediation error

A political scientist randomizes a treatment A: some registered voters get cards in the mail telling them the importance of voting, while others do not. All individuals in the study then receive a mail survey collecting data on a mediator M: whether they are interested in political affairs (yes or no). Finally, using administrative records, the researcher measures an outcome Y: whether the individual votes in the next election. For simplicity, assume we get data for (A, M, Y) for all sampled individuals (no non-response).

The researcher asks a research question involving mediation: is there a direct effect of the treatment on voting that does not operate through an effect on interest in political affairs? To answer this question, the researcher estimates an OLS model for Y as a function of A and M, and looks at the coefficient on M.[1]

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[1] I've added some details, but this general example comes from p. 203 of a good methods paper: Green, D. P., Ha, S. E., Bullock, J. G. (2010). Enough already about "black box" experiments: Studying mediation is more difficult than most scholars suppose. The Annals of the American Academy of Political and Social Science, 628(1), 200-208.

The DAG below illustrates this setting.

$$U$$

Randomized
card encouraging $A \longrightarrow M \longrightarrow Y$ Votes
voting

Interest in
political
affairs
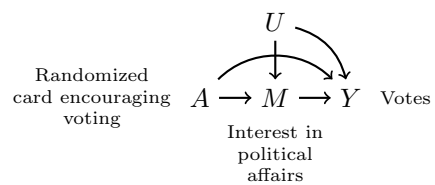
2.1. (10 points) Explain to the political scientist how their attempt at mediation has undermined the validity of their experiment. You might refer to the unobserved U.