16. Treatments in many time periods. What to do.

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

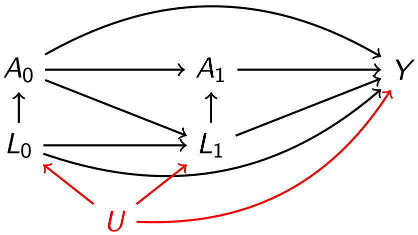
18 Oct 2022

Learning goals for today

At the end of class, you will be able to:

1. Reason about the sequential ignorability assumption
2. Apply inverse probability weighting to treatments over time

Identification: The adjustment set



A joint adjustment set for $\bar{A}$ is doomed

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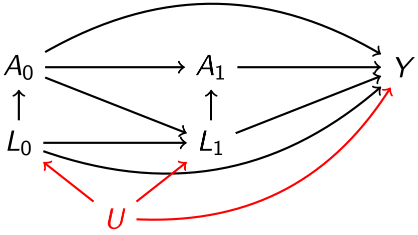

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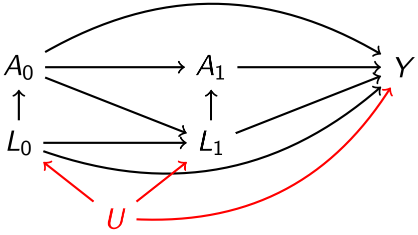

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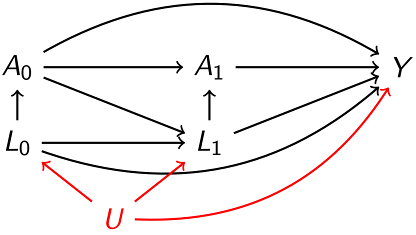

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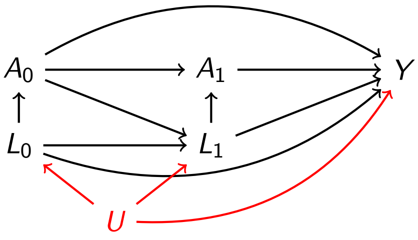

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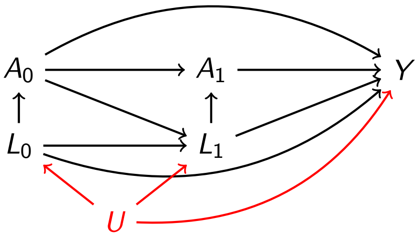

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To proceed, we need a different adjustment set in each time period

Notation

- $\bar{A}_k = (A_0, A_1, \ldots, A_k)$ treatments up to time k
- $\bar{L}_k = (L_0, L_1, \ldots, L_k)$ confounders up to time k
- $g()$ treatment strategy
- Y^g potential outcome under that strategy

Identification: Sequential ignorability

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Potential outcome under assignment rule g	is independent of	treatment at time k	given	treatments up to $k-1$ followed rule g	and confounders up to time k
Y^g	$\perp\!\!\!\perp$	A_k	$\mid$	$\bar{A}_{k-1} = g(\bar{A}_{k-2}, \bar{L}_{k-1}),$	$\bar{L}_k$

for all assignment rules g and time periods $k = 1, \ldots, K$

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\begin{tikzcd}
A_0 \arrow[r] \arrow[rr, bend left] \arrow[dr] & A_1 \arrow[r] & Y \\
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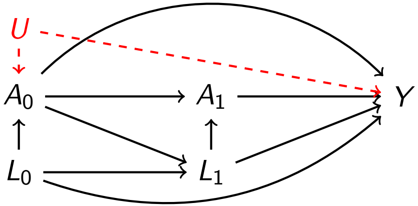

Identification: Sequential ignorability

$$\overset{\text{Potential outcome under assignment rule } g}{Y^g} \quad \overset{\text{is independent of}}{\perp\!\!\!\perp} \quad \overset{\text{treatment at time } k}{A_k} \quad \overset{\text{given}}{\Big|} \quad \overset{\text{treatments up to } k-1 \text{ followed rule } g}{\bar{A}_{k-1} = g(\bar{A}_{k-2}, \bar{L}_{k-1}),} \quad \overset{\text{and confounders up to time } k}{\bar{L}_k}$$

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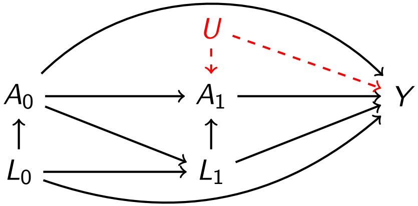

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Estimation: Two strategies

1. Inverse probability weighting ($+$ marginal structural models)
2. Structural nested mean models (coming next class)

Inverse probability weighting: DAG motivation

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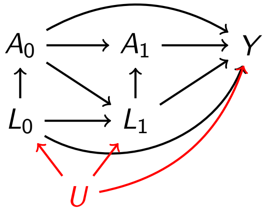
We observe data from this model



Inverse probability weighting: DAG motivation

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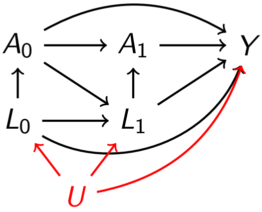

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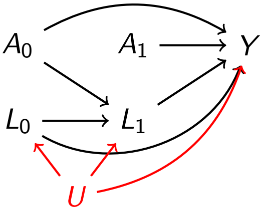

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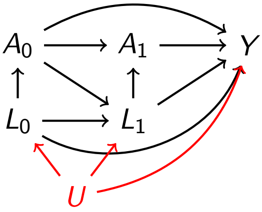

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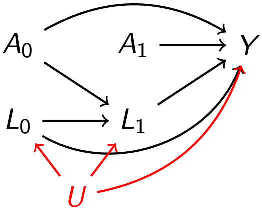

1. How would you weight to estimate the effect of A_0?
2. How would you weight to estimate the effect of A_1?

We will combine these

Inverse probability weighting

In time 0, define an inverse probability of treatment weight

$$W^{A_0} = \frac{1}{\mathrm{P}(A_0 \mid L_0)}$$

such that A_0 does not depend on L_0 after weighting

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Continue through all time periods.

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Continue through all time periods.

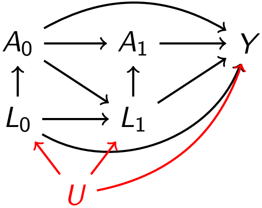
Define the overall weight as the product

$$W^{\bar{A}} = \prod_{k=0}^{K} \frac{1}{\mathrm{P}(A_k \mid \bar{A}_{k-1}, \bar{L}_k)}$$

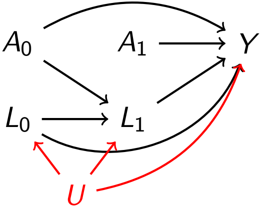
Inverse probability weighting

$$W^{\bar{A}} = \prod_{k=0}^{K} \frac{1}{\mathrm{P}(A_k \mid \bar{A}_{k-1}, \bar{L}_k)}$$

Takes us from this



to this pseudo-population



Inverse probability weighting with marginal structural models

Finally, we can put a model on top of the weighting.

$$\mathrm{E}(Y^{\bar{a}}) = \mathrm{E}(Y \mid \bar{A} = \bar{a}) = h(\bar{a})$$

for some function $h()$ that pools information.

Example: Outcomes depend on the proportion of periods treated

$$h(\bar{a}) = \frac{1}{K+1}\sum_{k=0}^{K} a_k$$

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Real example: Neighborhood disadvantage

Wodtke, G. T., Harding, D. J., & Elwert, F. (2011). Neighborhood effects in temporal perspective: The impact of long-term exposure to concentrated disadvantage on high school graduation. American Sociological Review, 76(5), 713-736.

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Wodtke et al. 2011

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  - poverty
  - unemployment
  - welfare receipt
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  - education
  - occupational structure

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This 5-value treatment is "neighborhood disadvantage"

Real example: Neighborhood disadvantage

Wodtke et al. 2011

Neighborhoods are experienced over time:

$$\bar{a}$$

is a trajectory of neighborhood disadvantage over ages $2, 3, \ldots, 17$

The authors study the effect of neighborhood disadvantage,

$$\begin{aligned} E(Y_{\bar{a}} - Y_{\bar{a}'}) &= E(Y_{\bar{a}}) - E(Y_{\bar{a}'}) \\ &= P(Y_{\bar{a}} = 1) - P(Y_{\bar{a}'} = 1), \end{aligned} \qquad (1)$$

Example:
$\bar{a}$ is residence in the most advantaged neighborhood each year
and
$\bar{a}'$ is residence in the most disadvantaged neighborhood each year

Real example: Neighborhood disadvantage

Wodtke et al. 2011

Problem: Neighborhoods A_1 shape family characteristics L_2, which confound where people live in the future A_2

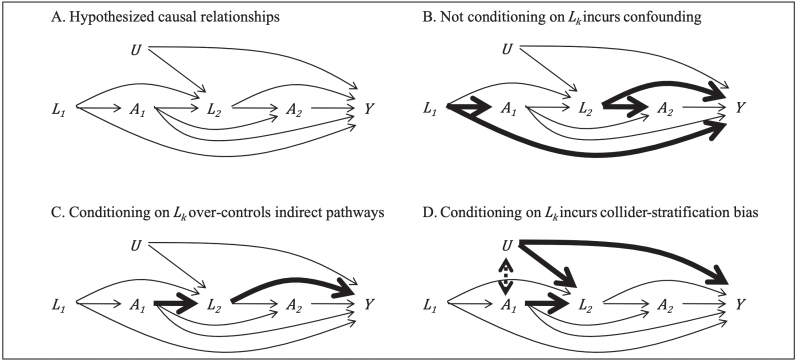

Figure 1. Causal Graphs for Exposure to Disadvantaged Neighborhoods with Two Waves of Follow-up

Note: A_k = neighborhood context, L_k = observed time-varying confounders, U = unobserved factors, Y = outcome.

Table 2. Time-Dependent Sample Characteristics

Variable	Blacks (n = 834)			Nonblacks (n = 1,259)		
	Age 1	Age 10	Age 17	Age 1	Age 10	Age 17
NH disadvantage index, percent						
1st quintile	3.48	3.60	3.48	13.34	19.14	20.65
2nd quintile	3.24	3.72	6.00	19.46	18.67	21.84
3rd quintile	5.28	5.88	7.79	26.13	23.27	22.48
4th quintile	14.87	18.11	18.47	26.13	23.99	21.13
5th quintile	73.14	68.71	64.27	14.93	14.93	13.90
FU head's marital status, percent						
Unmarried	33.93	44.84	52.04	5.88	11.36	15.09
Married	66.07	55.16	47.96	94.12	88.64	84.91
FU head's employment status, percent						
Unemployed	27.22	32.61	33.09	8.10	8.02	9.69
Employed	72.78	67.39	66.91	91.90	91.98	90.31
Public assistance receipt, percent						
Did not receive AFDC	81.06	75.66	82.37	96.27	96.19	97.93
Received AFDC	18.94	24.34	17.63	3.73	3.81	2.07
Homeownership, percent						
Do not own home	69.66	53.48	50.12	40.19	22.32	20.73
Own home	30.34	46.52	49.88	59.81	77.68	79.27
FU income in $1,000s, mean	19.68	25.04	27.45	32.59	46.65	57.50
FU head's work hours, mean	30.08	26.82	27.51	42.65	40.84	40.68
FU size, mean	5.75	5.32	4.81	4.22	4.69	4.33
Cum. residential moves, mean	.32	2.48	3.64	.32	2.16	3.02

Note: NH = neighborhood; FU = family unit. Statistics reported for children not lost to follow-up before age 20 (first imputation dataset).

Real example: Neighborhood disadvantage

Wodtke et al. 2011

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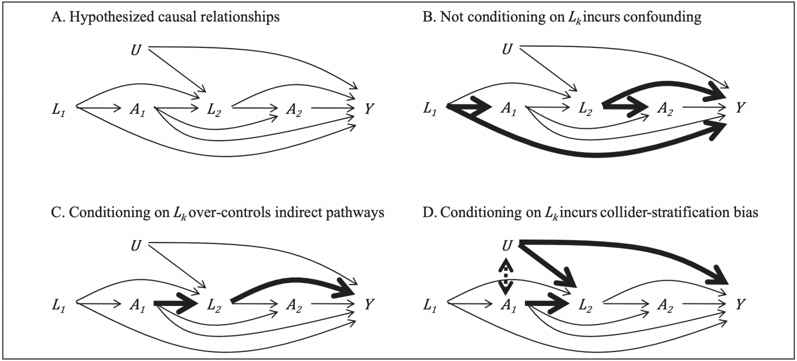

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Solution: MSM-IPW

$$w_i = \prod_{k=1}^{K} \frac{1}{P(A_k = a_{ki} \mid \bar{A}_{k-1} = \bar{a}_{(k-1)i}, \bar{L}_k = \bar{l}_{ki})}. \quad (4)$$

Also with stabilized weights

$$sw_i = \prod_{k=1}^{K} \frac{P(A_k = a_{ki} \mid \bar{A}_{k-1} = \bar{a}_{(k-1)i}, L_0 = l_0)}{P(A_k = a_{ki} \mid \bar{A}_{k-1} = \bar{a}_{(k-1)i}, \bar{L}_k = \bar{l}_{ki})}, \quad (5)$$

Real example: Neighborhood disadvantage

Wodtke et al. 2011

Marginal structural model: Logit

- 5-category treatment entered numerically
- Baseline covariates included due to stabilized weights
- Weights adjust for time-varying confounding

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Wodtke et al. 2011

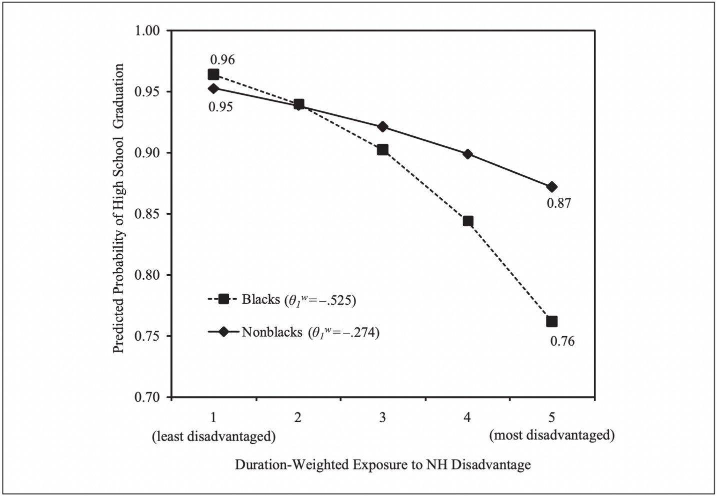

Figure 3. Predicted Probability of High School Graduation by Neighborhood Exposure History

Note: NH = Neighborhood

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Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!