25. Future treatments as proxies for confounding

Class guest: Felix Elwert
University of Wisconsin, Madison

Ian Lundberg
Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

17 Nov 2022

Learning goals for today

At the end of class, you will be able to:

1. Reason about when future treatments can proxy for unmeasured confounding

Note that this class is based on:

Elwert, F., & Pfeffer, F. T. (2022). The future strikes back: Using future treatments to detect and reduce hidden bias. *Sociological Methods & Research*, 51(3), 1014-1051.

A few things we've recently covered

Hernán & Cole 2009

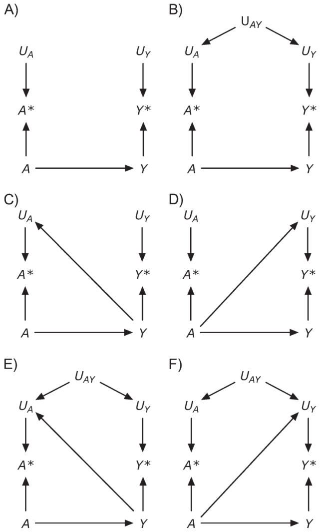
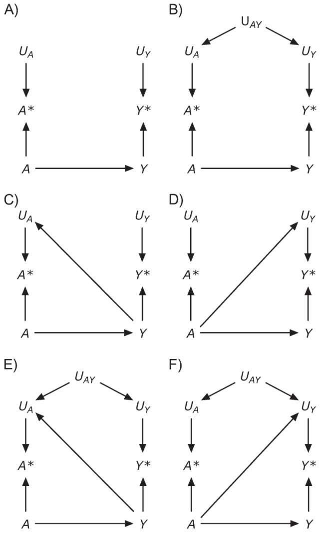


Figure 2. A structural classification of measurement error.

Hernán & Cole 2009



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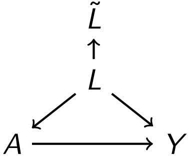

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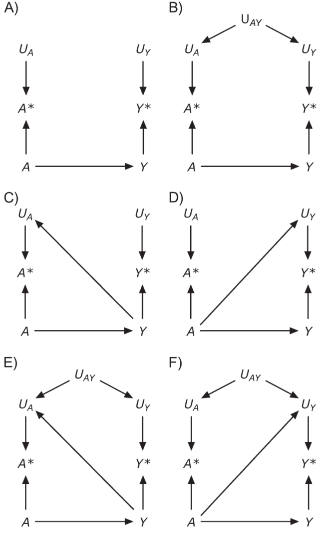

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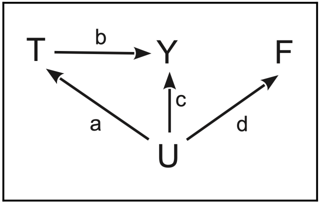
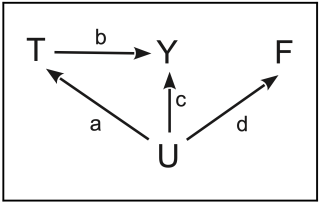
Elwert & Pfeffer 2022



Elwert & Pfeffer 2022

If you measure U

$$E(Y \mid A, U) = \alpha + \beta A + \gamma U$$



Elwert & Pfeffer 2022

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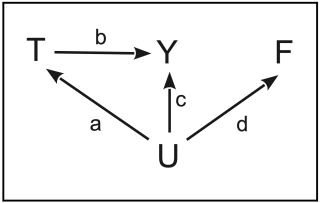
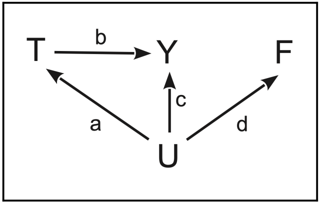


$$\begin{aligned}
\beta &= \frac{\mathrm{Cov}(T, Y) - \mathrm{Cov}(U, Y)\mathrm{Cov}(U, A)}{1 - [\mathrm{Cov}(U, A)]^2} \\
&= \frac{\big(b + ac\big) - \big((c + ab)a\big)}{1 - a^2} \\
&= \frac{b + ac - ac - a^2 b}{1 - a^2} \\
&= \frac{b(1 - a^2)}{1 - a^2} \\
&= b
\end{aligned}$$

Elwert & Pfeffer 2022

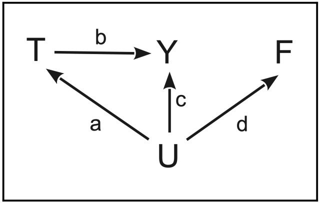
If you don't measure U



Elwert & Pfeffer 2022

If you don't measure U

$$\mathsf{E}(Y \mid A, F) = \alpha + \beta A + \gamma F$$



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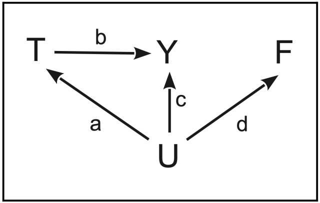

$$\begin{aligned}
\beta &= \frac{\mathrm{Cov}(A, Y) - \mathrm{Cov}(\tilde{L}, Y)\mathrm{Cov}(\tilde{L}, A)}{1 - \left[\mathrm{Cov}(\tilde{L}, A)\right]^2} \\
&= \frac{\big(b + ac\big) - \big((cd + abd)ad\big)}{1 - a^2d^2} \\
&= \frac{b + ac - acd^2 - a^2bd^2}{1 - a^2d^2} \\
&= \frac{b(1 - a^2d^2) + ac(1 - d^2)}{1 - a^2d^2} \\
&= b + \underbrace{ac}_{\substack{\text{Bias}\\\text{without}\\\text{control}}} \underbrace{\frac{1 - d^2}{1 - a^2d^2}}_{\substack{\text{Bias}\\\text{Multiplier}\\|M|<1}}
\end{aligned}$$

Elwert & Pfeffer 2022

If you don't measure U

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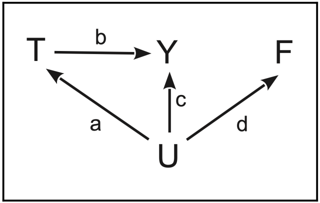

Control Estimator

$$\beta = b + \underbrace{ac}_{\substack{\text{Bias}\\\text{without}\\\text{control}}} \underbrace{\frac{1-d^2}{1-a^2d^2}}_{\substack{\text{Bias}\\\text{Multiplier}\\|M|<1}}$$

Elwert & Pfeffer 2022

If you don't measure U

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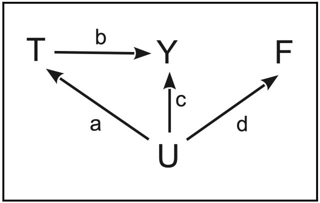

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Difference (Mayer) Estimator

$$\beta - \gamma = b + \underbrace{ac}_{\substack{\text{Bias}\\ \text{without}\\ \text{control}}} \underbrace{\frac{a-d}{a-a^2d}}_{\substack{\text{Bias}\\ \text{Multiplier}\\ |M|}}$$

Elwert & Pfeffer 2022

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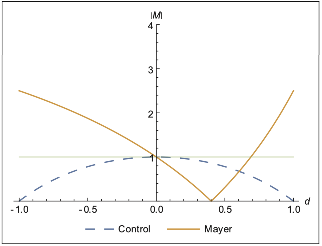

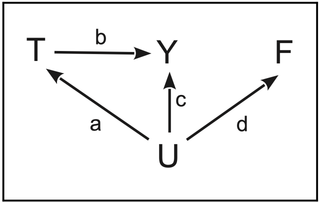

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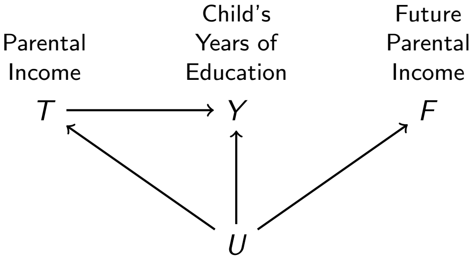
Parts of the paper we have not yet covered

Empirical example

What is the effect of log parental income on years of education?



Panel Study of Income Dynamics. Born 1956–1968. ($n = 1,513$)

- T log family income averaged at child age 13–17
- Y years of education by age 24
- F log family income at child age 25–29

Table 3. Estimating the Causal Effect of Parental Income on Children's Years of Education With and Without Future Treatments.

	(1)	(2)	(3)	(4)
Coefficients				
T: Parental income	.448	.319	.185	.118
	(.039)***	(.049)***	(.039)***	(.041)**
F: Future parental income		.274		.202
		(.088)**		(.076)**
X: Controls			Yes	Yes
Difference in coefficients				
$T - F$		.045		−.084
		(.126)		(.098)
Test of equality of coefficients on *T*: *p* values				
Model (1) versus (2): .0006				
Model (1) versus (3): .0000				
Model (3) versus (4): .0006				
Model (1) versus (4): .0000				
N	1,513	1,513	1,513	1,513

Note. Standardized OLS regression coefficients (standard errors in parentheses); weighted. Significance tests for the difference between coefficients across models are using seemingly unrelated regression.

Statistical significance at $^{\dagger}p < .10$. $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$ (two-tailed test).

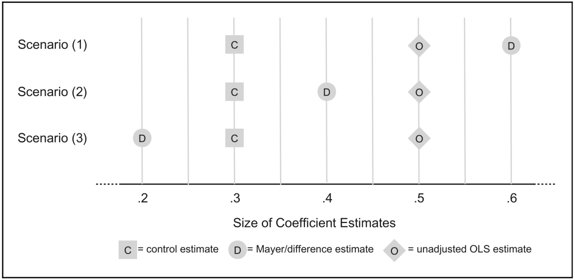
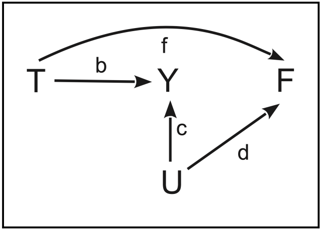
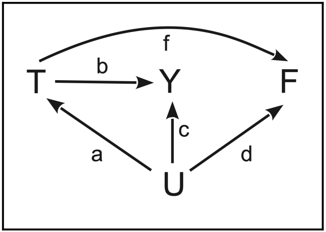


Figure 4. Illustration of the heuristic for choosing between estimates. The relative position of the control (*C*), difference (*D*), and unadjusted OLS (*O*) estimates can help the analyst decide between alternative estimates. In data generated by Figure 2, the location of the control estimate indicates the direction of unadjusted OLS bias (in this example, upward bias). In scenarios (1) and (2), the control estimate is preferred. In scenario (3), additional assumptions are needed to decide between the control and difference estimates.

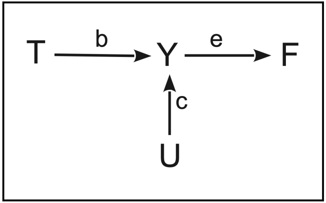
Challenge 1: True State Dependence



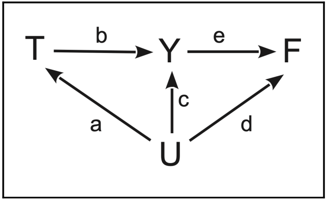
Challenge 2: Confounded True State Dependence



Challenge 3: Unconfounded Study with Selection



Challenge 4: Confounded Study with Selection



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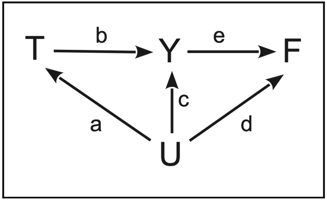

Table 2. Performance of the Control Estimator and the Mayer/Difference Estimator in the Presence of Selection and Weak to Moderate Path Parameters, $|p| < .5$.

	Bias With	
Selection (e)	Control Estimator	Mayer/Difference Estimator
Selection $(e \neq 0)$	Negligibly amplified or weakly reduced (see below)	Mostly amplified
Mild selection $(|e| \lesssim 0.3)$	Weakly reduced	Mostly amplified

Nonparametric results

Previous results relied on a linear path model.
Next results rely only on a DAG (nonparametric).

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Next results rely only on a DAG (nonparametric).

Motivating (but incomplete) intuition:

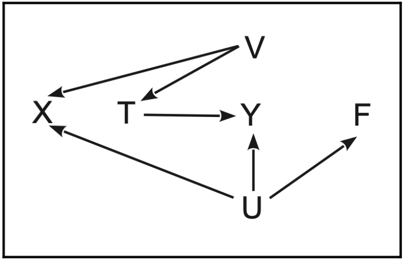
A future treatment F cannot affect an outcome Y.

If the outcome Y is related to F,
then there must be unobserved confounding.

Nonparametric Example 1.

Does a relationship between Y and F imply confounding?

Is F related to Y?
Is $T \to Y$ confounded given X?



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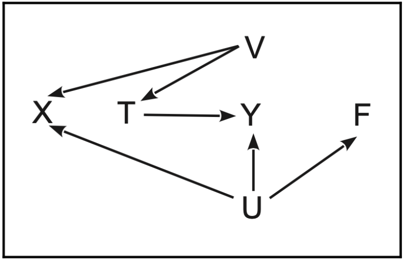

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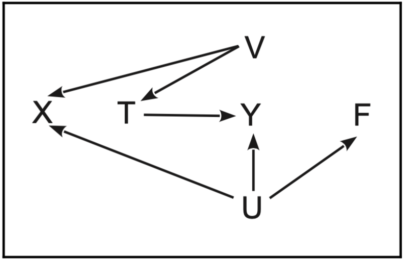

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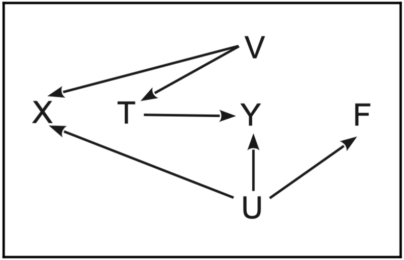

A conditional relationship between Y and F implies confounding.

Nonparametric results: Example 2.

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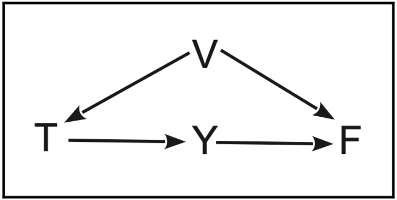

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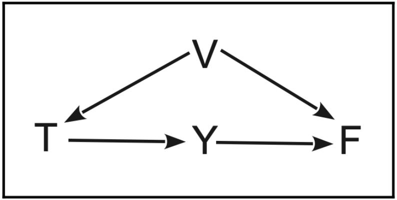

Nonparametric results: Example 2.

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Is F related to Y? Yes. $Y \leftarrow V \rightarrow F$

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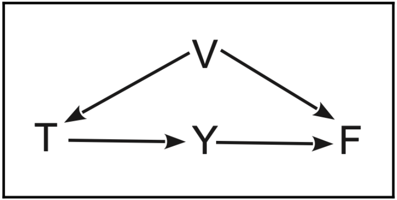

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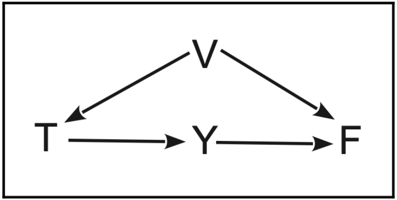

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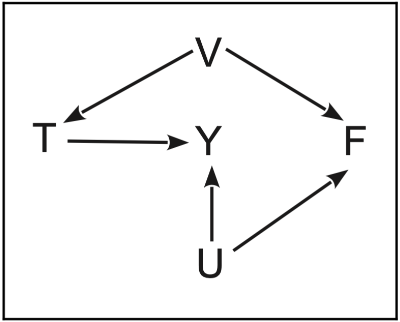

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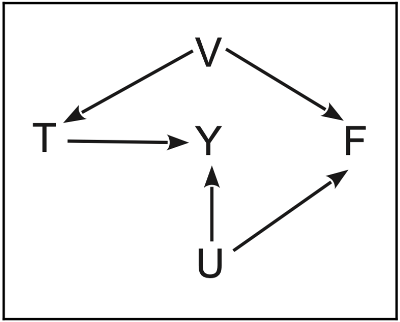

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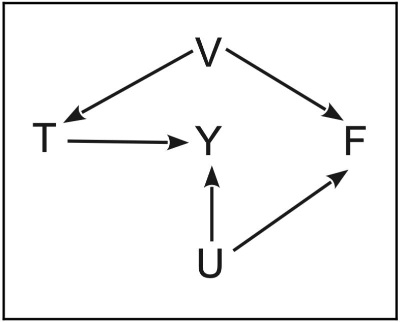

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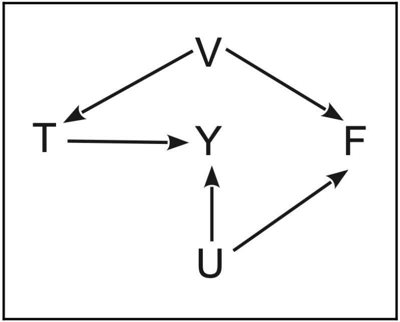


A conditional relationship between Y and F does not imply confounding.

Two nonparametric results: A formal answer

What does the relationship between F and Y tell us about confounding?

Two nonparametric results: A formal answer

What does the relationship between F and Y tell us about confounding?

$$
\begin{tikzcd}
X \arrow[r] \arrow[rr, bend left] \arrow[rrr, bend left] \arrow[drr, bend right] & T \arrow[r] & Y & F \\
& & V \arrow[ul] \arrow[u] \arrow[ur] &
\end{tikzcd}
$$

Result 11. (Requires Assumption 1)

$$F \perp\!\!\!\perp Y \mid \{T, X\} \qquad \to \qquad \{Y^0, Y^1\} \perp\!\!\!\perp T \mid X$$

Result 12. (Requires Assumptions 1–3)

$$F \not\perp\!\!\!\perp Y \mid \{T, X\} \qquad \to \qquad \{Y^0, Y^1\} \not\perp\!\!\!\perp T \mid X$$

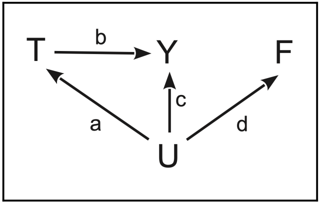
Assumption 1. There exists some unobserved V such that $V \to T$ and $V \not\perp\!\!\!\perp F \mid \{T, X\}$

Assumption 2. All unobserved causes of F also cause T

Assumption 3. Y does not directly or indirectly cause F

Discussion: Applied examples

The usefulness of future treatments relies heavily on this DAG.



Exercise. Discuss the plausibility of this DAG in applied cases.

tinyurl.com/FutureTreatmentsExamples

Group 1

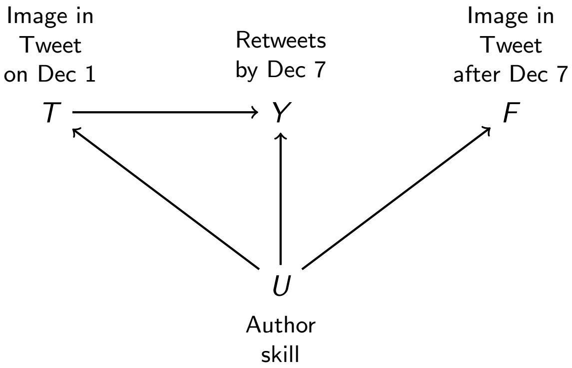
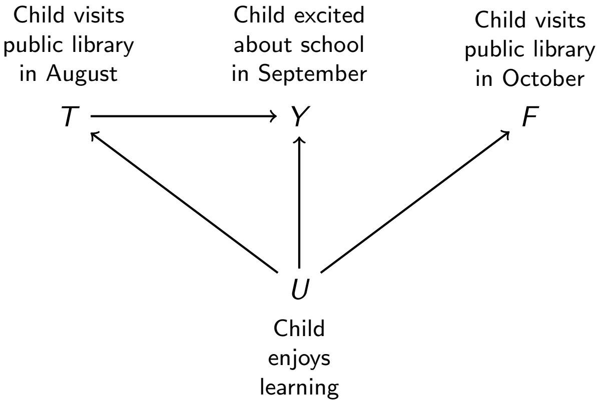
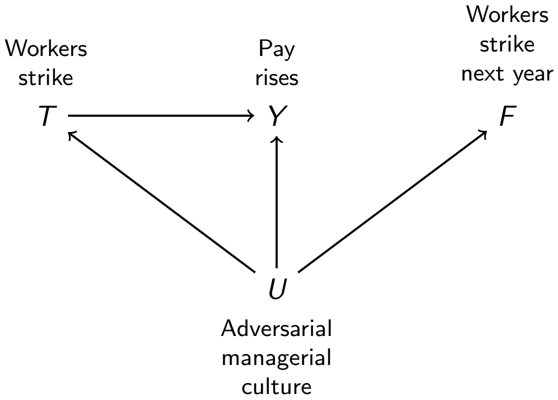


Image in
Tweet
on Dec 1
T
Retweets
by Dec 7
Y
Image in
Tweet
after Dec 7
F
U
Author
skill

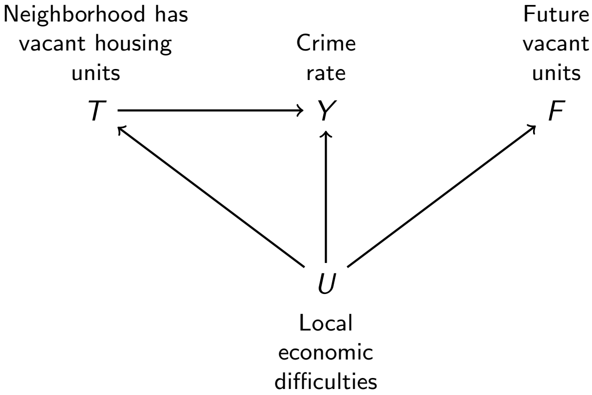
Group 2


Child visits
public library
in August
Child excited
about school
in September
Child visits
public library
in October
T
Y
F
U
Child
enjoys
learning

Group 3


Workers
strike
T
Pay
rises
Y
Workers
strike
next year
F
U
Adversarial
managerial
culture

Group 4


Neighborhood has
vacant housing
units
Crime
rate
Future
vacant
units
T
Y
F
U
Local
economic
difficulties

Learning goals for today

At the end of class, you will be able to:

1. Reason about when future treatments can proxy for unmeasured confounding

Note that this class is based on:

Elwert, F., & Pfeffer, F. T. (2022). The future strikes back: Using future treatments to detect and reduce hidden bias. *Sociological Methods & Research*, 51(3), 1014-1051.

Feedback from class: tinyurl.com/CausalQuestions

No office hours today

Invitation: Cornell Sociology Department Colloquium

Friday, 3-4:15pm, Uris G08

Felix Elwert, University of Wisconsin, Madison

Rearranging the Desk Chairs: A Large Randomized Field Experiment on the Effects of Close Contact on Interethnic Relations