

23. Measurement error

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

10 Nov 2022

Learning goals for today

At the end of class, you will be able to:

1. Present measurement error in a DAG
2. Formalize when measurement error undermines identification
3. Apply these ideas in a hypothetical research design

Broad issue: Measurement error

This lecture closely follows

Hernán, M. A., & Cole, S. R. (2009).

[Invited commentary: Causal diagrams and measurement bias.](#)

American Journal of Epidemiology, 170(8), 959-962.

Plan for class: Your own example

We will discuss ideas using an example from Hernán & Cole 2009.
For each idea, each group will apply it to their own example.

Task: In pairs or triples, construct your own running example

Each unit (e.g., person) in your study should report

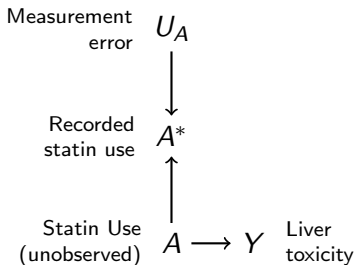
- ▶ A treatment A , reported with error
- ▶ An outcome Y , reported with error

You choose how to define A and Y

Statin Use
(unobserved) $A \longrightarrow Y$ Liver
toxicity

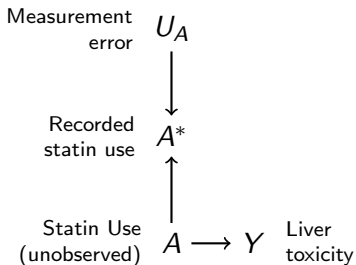
Recorded statin use A^*
has haphazard errors

Statin Use (unobserved) $A \rightarrow Y$ Liver toxicity



Augment this DAG

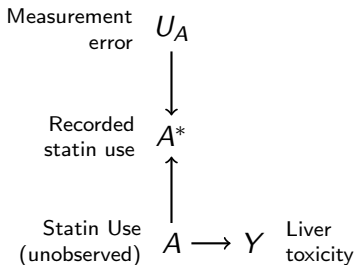
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Question

By what path(s) are A^* and Y associated?

If $\{A^*, Y\}$ are associated, does this imply $A \rightarrow Y$?

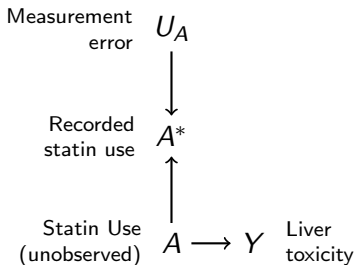


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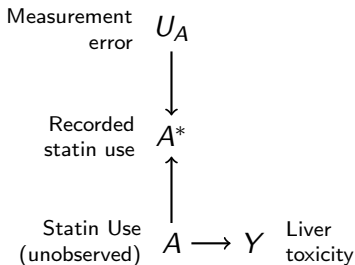
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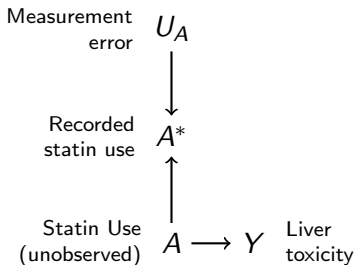
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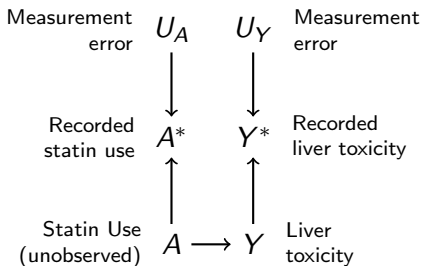
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In your example: How could you have measurement error like this?



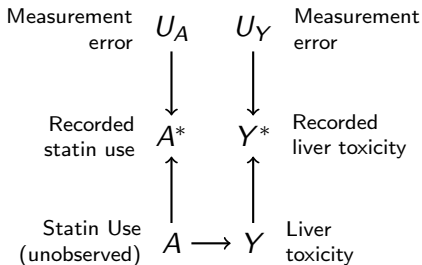
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Recorded liver toxicity Y^*
has haphazard errors



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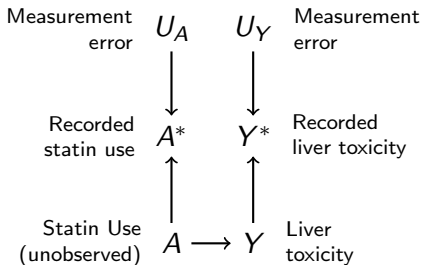
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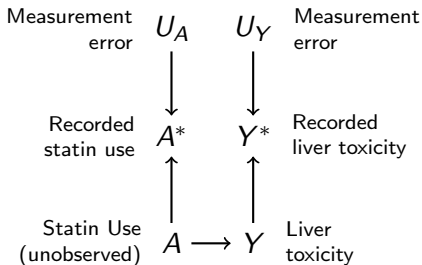


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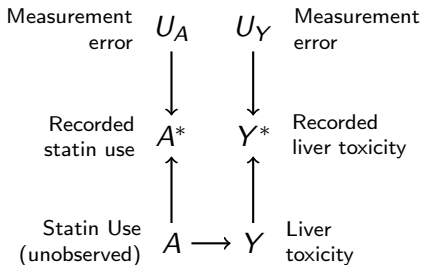
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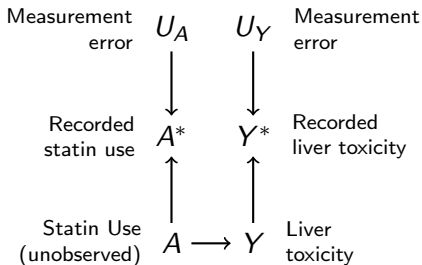
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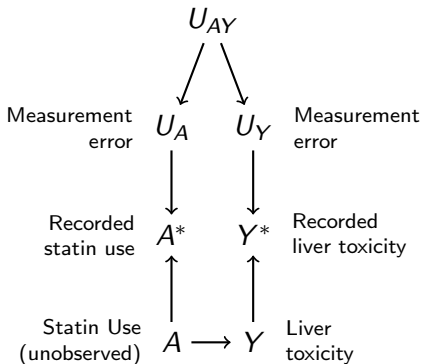
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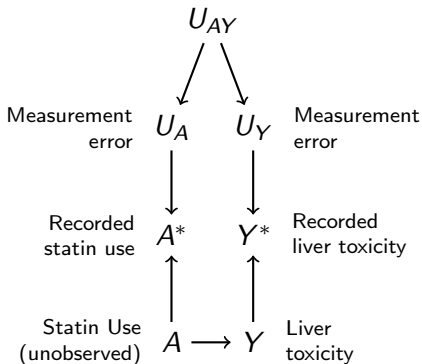
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Reports are retrospective, and people who over-report statin use tend to over-report liver toxicity



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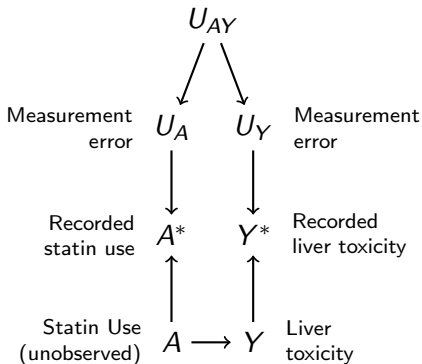
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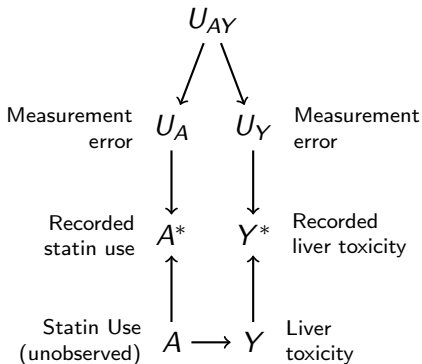
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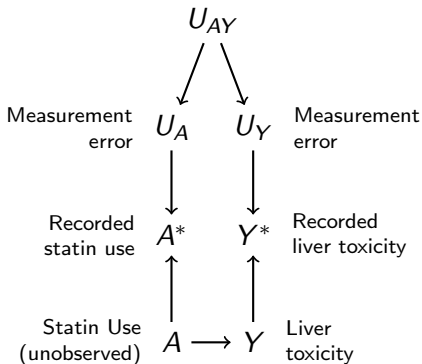
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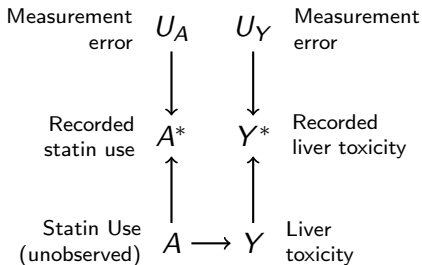
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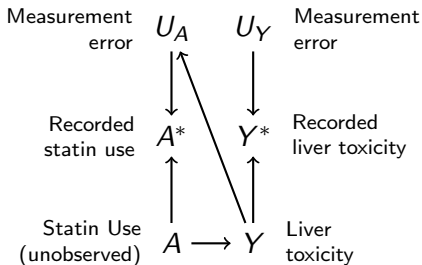
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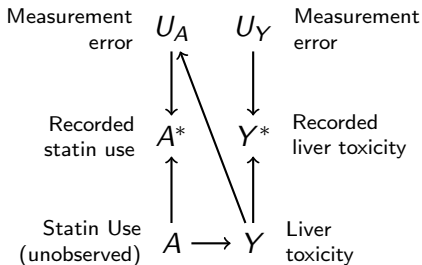
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When someone has liver toxicity, the doctor asks different questions about statin use



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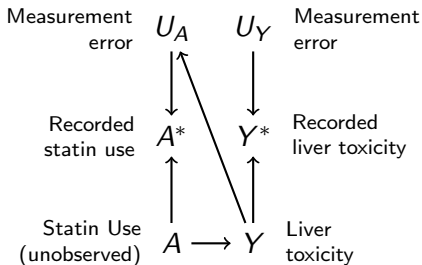
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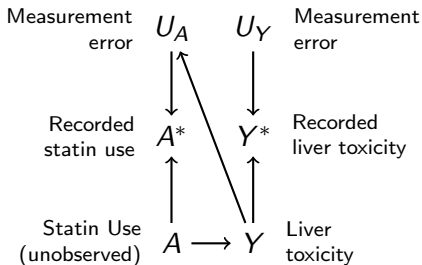
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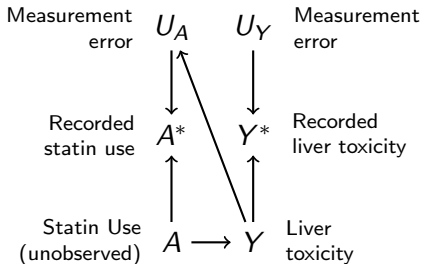
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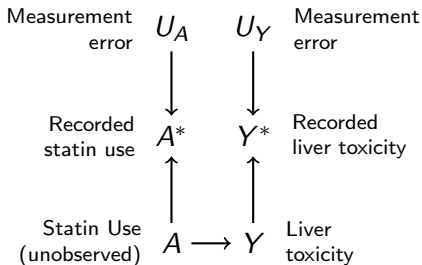
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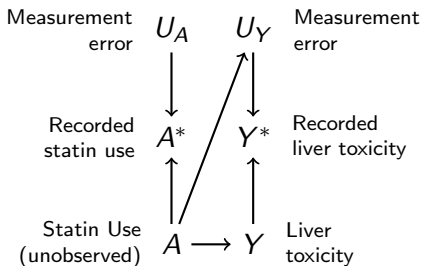
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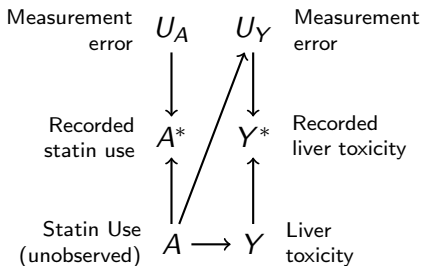
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Physicians watch liver toxicity more closely among statin users



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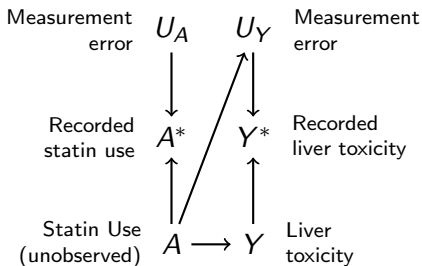
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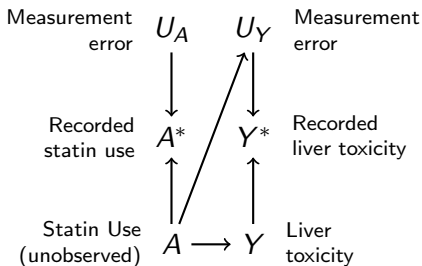
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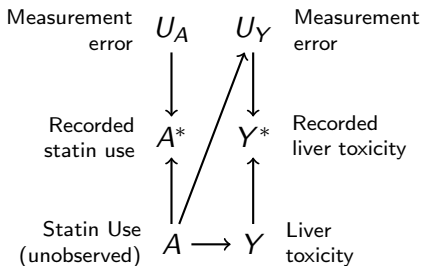
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Hernan & Cole Fig 2

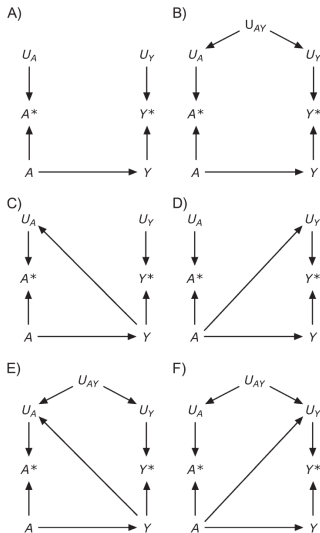


Figure 2. A structural classification of measurement error.

Hernán & Cole illustration:

Does body mass index (BMI) affect health outcomes?

Weight W and height H affect the risk of a condition A
which affects a health outcome Y

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Body Mass Index (BMI) is a function of measured weight W^* and height H^*

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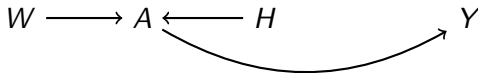
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Does BMI affect Y ?

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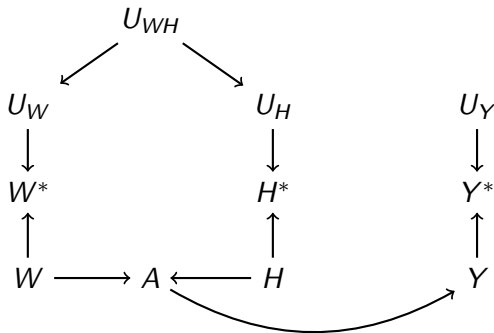
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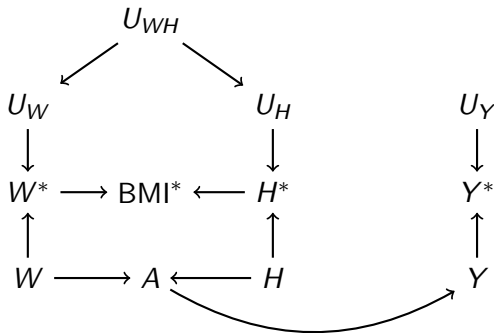
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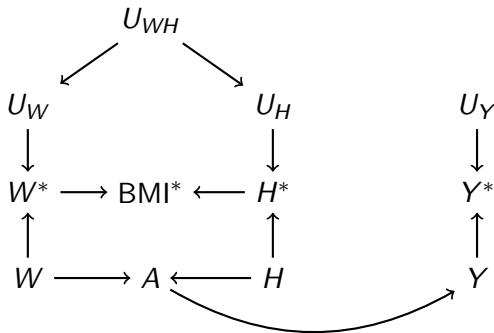
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No. There is no causal path.

Examples to discuss

tinyurl.com/MeasurementExercise

Learning goals for today

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1. Present measurement error in a DAG
2. Formalize when measurement error undermines identification
3. Apply these ideas in a hypothetical research design

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!