20. Principal Stratification (Part 1)

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

2 Nov 2022

Learning goals for today

At the end of class, you will be able to:

1. Define principal strata
2. Understand how they address a post-treatment problem
3. Make assumptions to draw inference about a latent stratum

A hypothetical setting

- Suppose you have a new exercise program $A = 1$ vs $A = 0$

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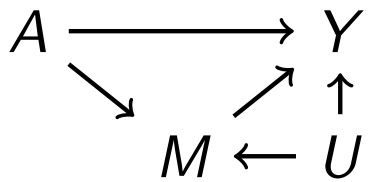
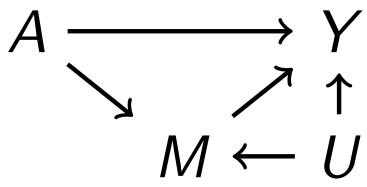


← Y only defined when $M = 1$

How would you analyze this?

A hypothetical setting: Common solutions



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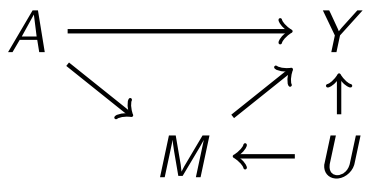

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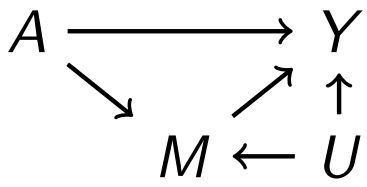

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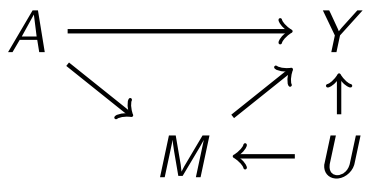

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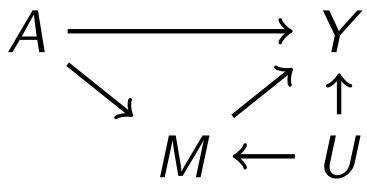

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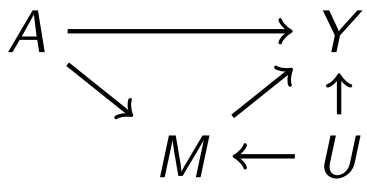

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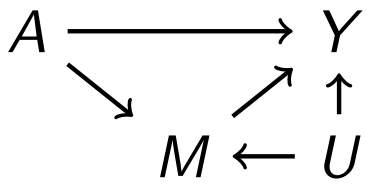

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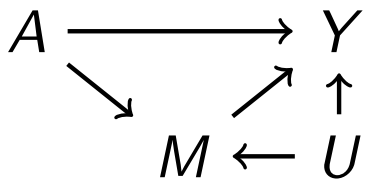

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5. New approach: **Principal stratification**

Principal stratification: The big idea

Survival M is a **post-treatment** variable.

But before the treatment is assigned, there are 4 types of people:

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2. Would not survive regardless of treatment

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4. Would survive if and only if the do not receive the program
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The effect on Y is only defined for stratum 1.

Principal stratification: The difficulty

Four principal strata

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Difficulty: We don't observe the strata. We observe:

- Survivors in the program Mix of 1 and 3

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- Survivors in the program — Mix of 1 and 3
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- Survivors not in the program — Mix of 1 and 4

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Difficulty: We don't observe the strata. We observe:

- Survivors in the program — Mix of 1 and 3
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- Die in the program

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- Die in the program — Mix of 2 and 4

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- Die in the program — Mix of 2 and 4
- Die without the program

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- Die without the program — Mix of 2 and 3

Assumption: Monotonicity

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We observe the sets

- Survivors in the program — Mix of 1 and 3
- Survivors not in the program — All 1 ~~Mix of 1 and 4~~
- Die in the program — All 2 ~~Mix of 2 and 4~~
- Die without the program — Mix of 2 and 3

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What to do with the sets that are still mixed?

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What to do with the sets that are still mixed? **Bounds**

Principal Stratification: Exercise

You will learn more by doing bounds yourself!

The rest of class will be a pen-and-paper exercise

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3. Make assumptions to draw inference about a latent stratum

References

Original paper

- Frangakis, C. E., & Rubin, D. B. (2002). Principal stratification in causal inference. Biometrics, 58(1), 21-29.

Good recent summary (assigned on syllabus)

- Page, L. C., Feller, A., Grindal, T., Miratrix, L., & Somers, M. A. (2015). Principal stratification: A tool for understanding variation in program effects across endogenous subgroups. American Journal of Evaluation, 36(4), 514-531.

Good resource for bounds

- Imai, K. (2008). Sharp bounds on the causal effects in randomized experiments with "truncation-by-death". Statistics & Probability Letters, 78(2), 144-149.

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!