6. Population inference from samples

lan Lundberg Cornell Info 6751: Causal Inference in Observational Settings Fall 2022

8 Sep 2022

Definition of potential outcomes

- ▶ $\{Y_i^1, Y_i^0\}$ are potential outcomes. When $A_i = 1$, then Y_i^1 is factual and Y_i^0 is counterfactual. When $A_i = 0$, then these are reversed.
 - This is why potential, not necessarily counterfactual.
- ▶ Y^a is the outcome of a randomly sampled unit assigned to treatment value a. In itself, it is not an average over a group—that would be $E(Y^a)$.

$$E(Y | A = 1) > E(Y | A = 0)$$

- Descriptive
- Outcomes were higher, on average, for those who got the treatment

$$\mathsf{E}(Y^1) > \mathsf{E}(Y^0)$$

- ► Causal
- ▶ The treatment (1 vs 0) increases outcomes, on average

$$Y_i^1 > Y_i^0$$
 for all i

- Causal
- ▶ The treatment (1 vs 0) increases the outcome for every unit

Observational Claims Causal Claims

Observational Evidence Causal Evidence

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"...all causal inference is based on assumptions that cannot be derived from observations alone," (Greenland, Pearl, & Robins 1999, p. 47)

Observational Claims

Causal Claims

Observational Evidence

-Causal Evidence

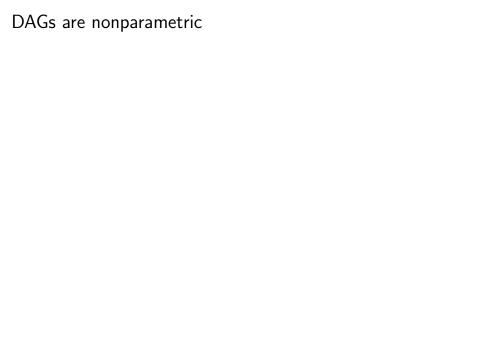
"...all causal inference is based on assumptions that cannot be derived from observations alone," (Greenland, Pearl, & Robins 1999, p. 47)

There is no causal evidence. There is only observational evidence, which speaks to causal claims under assumptions.

Learning goals for today

At the end of class, you will be able to:

- 1. Understand DAGs more fully
 - ► DAGs are nonparametric
 - ► DAGs are hard to learn from data
- 2. Generalize from a sample to a population
 - ► Encode sampling assumptions in DAGs



$$X \xrightarrow{A \to Y} Y$$

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This does **not** mean

$$Y = \beta_0 + \beta_1 X + \beta_2 A + \epsilon$$

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This does mean

- ► $A = f(X, U_A)$ for some function f()
- ► $Y = g(X, A, U_Y)$ for some function g()

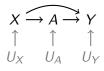
$$\begin{array}{ccc}
X \xrightarrow{A} \xrightarrow{A} & Y \\
\uparrow & \uparrow & \uparrow \\
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which allows that

- ► The effect of A may depend on X (heterogeneity)
- ightharpoonup E(Y | X, A) may be a nonlinear function of each input

$$X \xrightarrow{A \to Y} Y$$

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This tells us:

$$\underbrace{\mathsf{E}(Y^a \mid X = x)}_{\mathsf{Causal Quantity}} = \underbrace{\mathsf{E}(Y \mid A = a, X = x)}_{\mathsf{Statistical Quantity}}$$

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Right statement:

- ▶ Among everyone with X = x and A = a,
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These are two different sets of people

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Once the DAG gives us the above,

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Once the DAG gives us the above, we can use **any** prediction function for the statistical part.

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- \blacktriangleright $\{\beta, \gamma\}$ are "main effects"
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- \blacktriangleright $\{\beta, \gamma\}$ are "main effects"
- \blacktriangleright η is an "interaction": the effect of A varies by X
- ▶ Key assumption: $A \perp \epsilon$, or A is "exogenous"

That requires us to do **both** causal reasoning **and** statistical reasoning simultaneously.

DAGs support causal reasoning before statistical reasoning

$$X \xrightarrow{A} Y$$

$$E(Y^a \mid X = x) = E(Y \mid A = a, X = x)$$
Causal Quantity Statistical Quantity

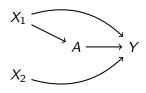
Let's pause to discuss this.

Learning goals for today

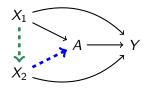
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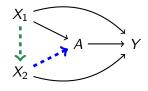
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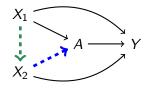


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Can data tell us whether the dashed edges exist?

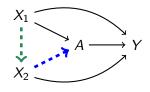
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► In the absence of both edges,

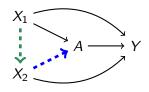
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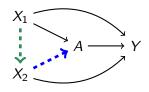
▶ In the absence of both edges, $X_1 \perp \!\!\! \perp X_2$ and $X_2 \perp \!\!\! \perp A$

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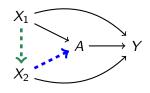
- ▶ In the absence of both edges, $X_1 \perp \!\!\! \perp X_2$ and $X_2 \perp \!\!\! \perp A$
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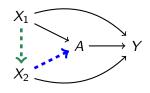
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Will data replace human researchers?

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Example: Does the unobserved U confound treatment assignment?



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A and Y are associated either way. The absence of U is a completely untestable assumption.

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As a general rule, the DAG encodes substantive theory (made by a human)

rather than data (crunched by a computer)

Some academic history of DAGs

- ► Historical roots in path models in the 1920s
 - ▶ Wright, S. (1921). Correlation and causation. Part I: Method of path coefficients. Journal of Agricultural Research, 20(7), 557-585.
- ► Linear path models in the 1960s
 - ▶ Duncan, O. D. (1966). Path analysis: Sociological examples. American Journal of Sociology, 72(1), 1-16.
- ► Landmark contributions: Pearl, Greenland, Robins
 - (assigned) Greenland, S., Pearl, J., & Robins, J. M. (1999). Causal diagrams for epidemiologic research. Epidemiology, 37-48
 - ► Pearl, J. (2000). Causality. Cambridge University Press.
 - ▶ Pearl, J., & Mackenzie, D. (2018). The Book of Why: The New Science of Cause and Effect. Basic Books.
- ► More accessible introduction for social scientists
 - ► Morgan, S. L., & Winship, C. (2015). Counterfactuals and Causal Inference. Cambridge University Press.

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$\mathsf{Sample} \to \mathsf{Population}$

Fun example

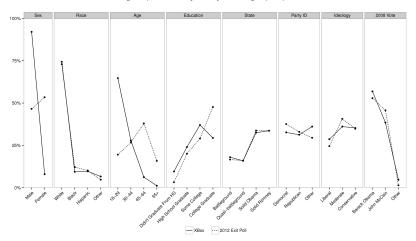
Wang, Rothschild, Goel, & Gelman

Survey of **Xbox users** to forecast the 2012 election!³

³Wang, W., Rothschild, D., Goel, S., & Gelman, A. (2015). Forecasting elections with non-representative polls. International Journal of Forecasting, 31(3), 980-991.

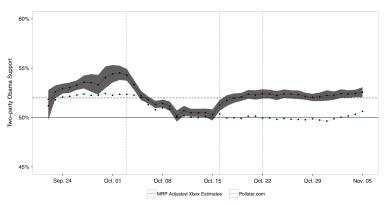
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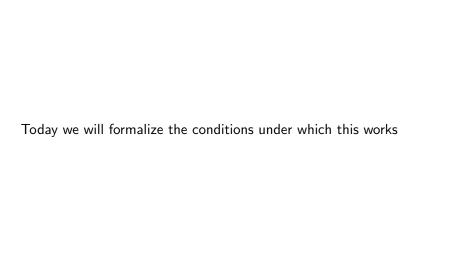
W. Wang et al. / International Journal of Forecasting 31 (2015) 980-991

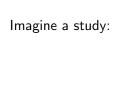


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Can we draw conclusions about the population of U.S. voters?

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Yes! A probability sample

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Iffy. Almost a probability sample

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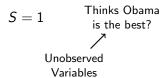
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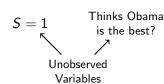
Big worry:

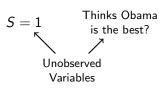
Do we believe that selection into the sample is independent of Obama support?



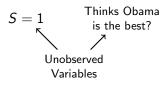
S = 1 Thinks Obama is the best?



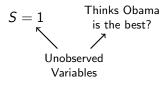




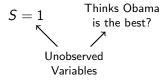
Income



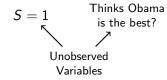
Income Party ID



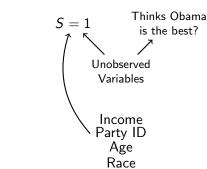
Income Party ID Age



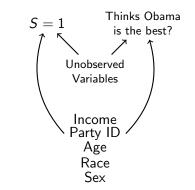
Income Party ID Age Race

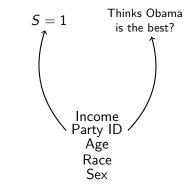


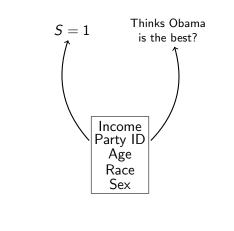
Income Party ID Age Race Sex

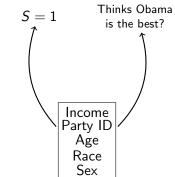


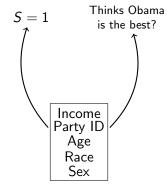
Sex





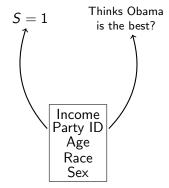






1. Split into sample subgroups

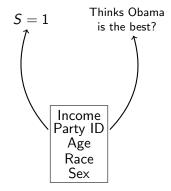
(in sample)



- 1. Split into sample subgroups
- 2. Take the mean Obama support

(in sample)

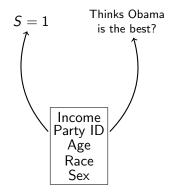
(in sample)



(in sample) (in sample)

If this is the case:

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- 3. Find each subgroup size in all voter records (in population)

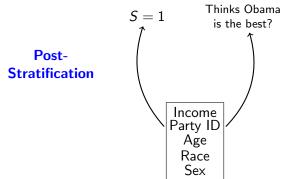


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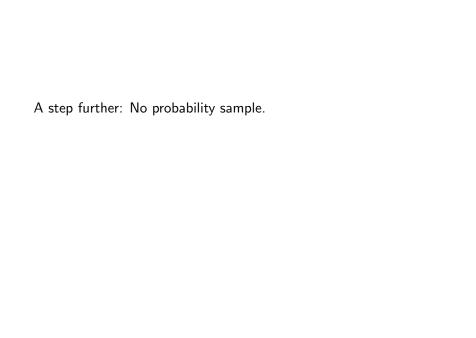


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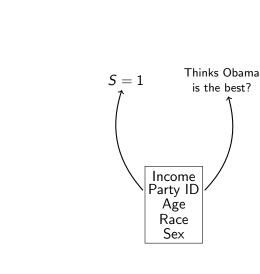


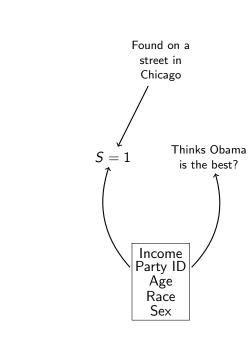
A step further: No probability sample.
We sample random passers-by on the streets of Chicago.

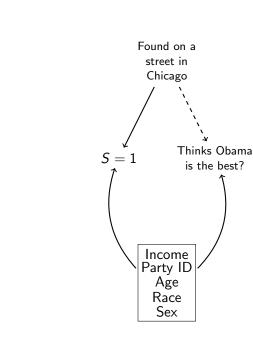
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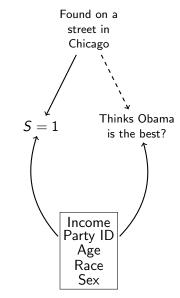
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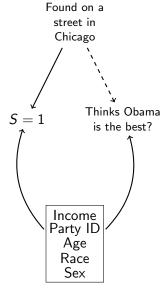


Credibility depends on causal assumptions

— what causes sample inclusion?

— what causes the outcome?

Need conditional independence.



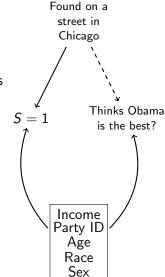
Credibility depends on causal assumptions

— what causes sample inclusion?

— what causes the outcome?

Need conditional independence.

These assumptions belong in a DAG



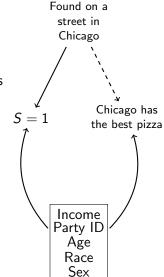
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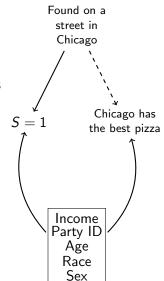
Credibility depends on causal assumptions

- what causes sample inclusion?
- what causes the outcome?

Need conditional independence.

These assumptions belong in a DAG

The DAG requires theory about the particular question





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► Have I identified the causal effect well in my sample? and also about **external validity**

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- ► Have I identified the causal effect well in my sample? and also about external validity
 - ▶ Does my sample speak to the population of interest?

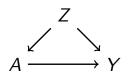
We often care about internal validity

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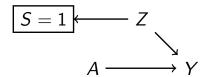
The authors combine these to discuss target validity

Westreich et al. 2019, Fig 1 (modified)

Nonexchangeability for internal validity due to confounding



Nonexchangeability for external validity due to sampling bias



Learning goals for today

At the end of class, you will be able to:

- 1. Understand DAGs more fully
 - ► DAGs are nonparametric
 - ► DAGs are hard to learn from data
- 2. Generalize from a sample to a population
 - ► Encode sampling assumptions in DAGs

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at calendly.com/ianlundberg/office-hours Come say hi!