14. Marginal Structural Models

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

6 Oct 2022

Learning goals for today

At the end of class, you will be able to:

1. Gain efficiency with marginal structural models
2. Recognize how that gain comes through information sharing
3. Understand stabilized weights

From $A \in \{0, 1\}$ to $A \in \{1, 2, 3, 4, 5\}$

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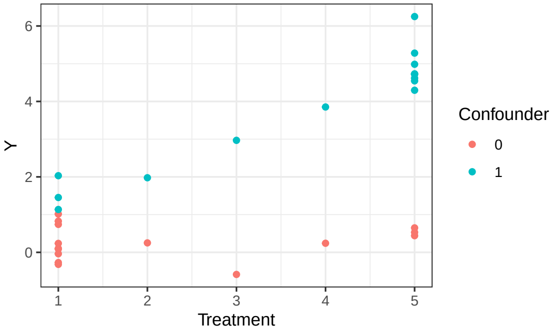

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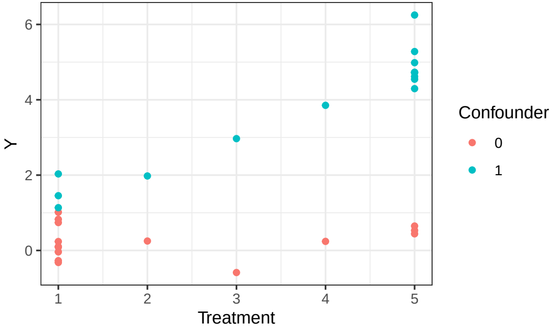

How to estimate $\mathrm{E}(Y^3)$?

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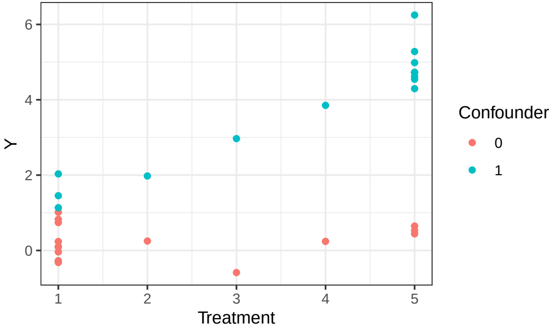

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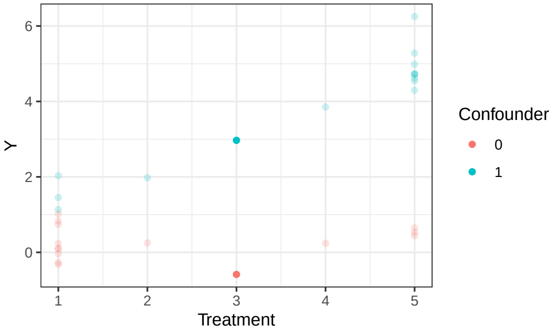

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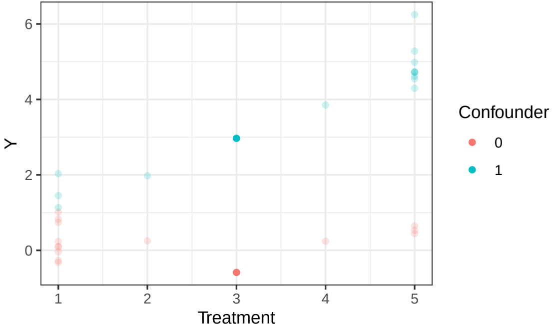

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1) Restrict to $A = 3$
2) Take inverse probability weighted average

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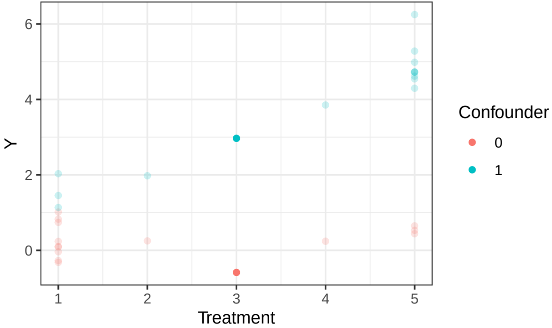

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But only 2 units! High variance!

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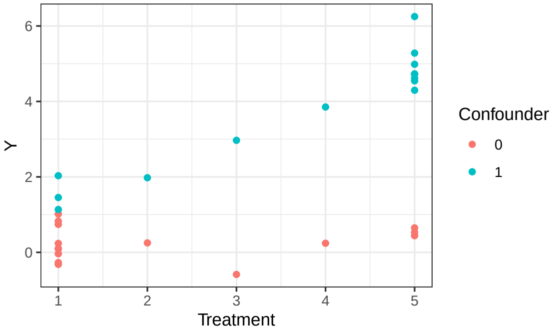

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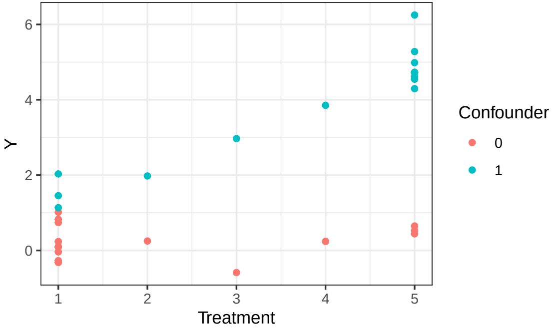

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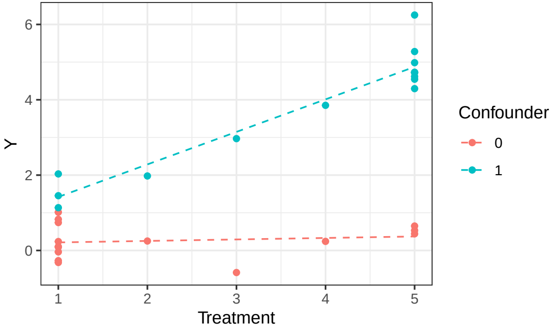

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1) Fit a model for $E(Y \mid A, L)$

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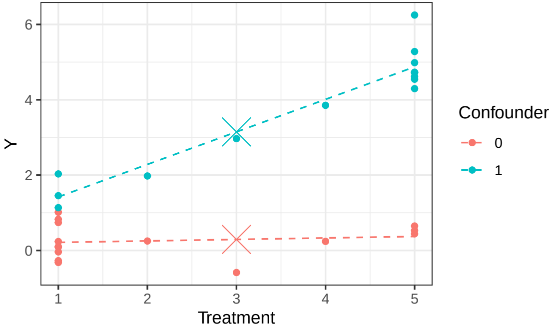

How to estimate $E(Y^3)$?

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1) Fit a model for $E(Y \mid A, L)$
2) Predict at $A = 3$ in each group

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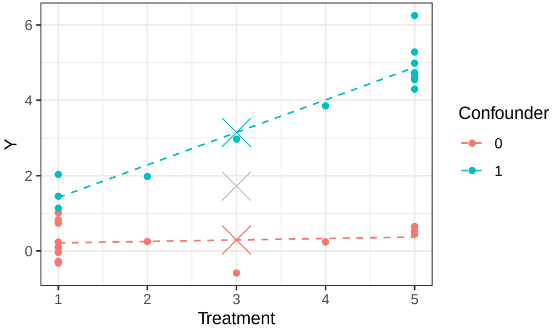

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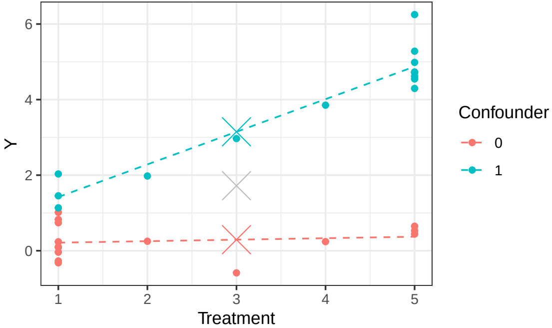

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But so much modeling!

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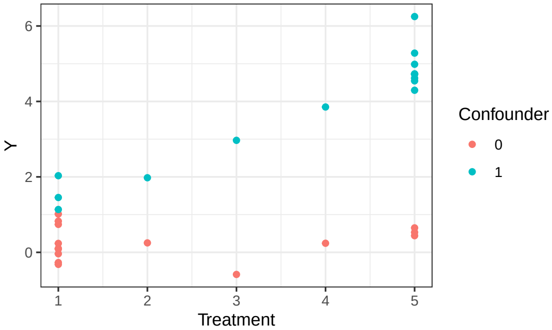

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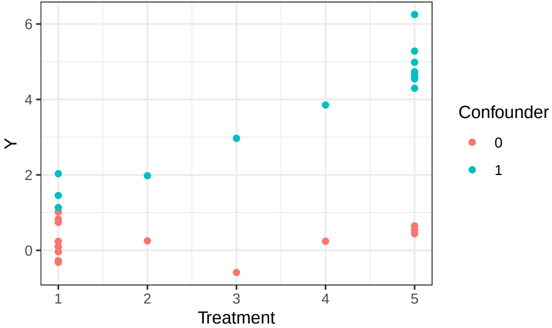

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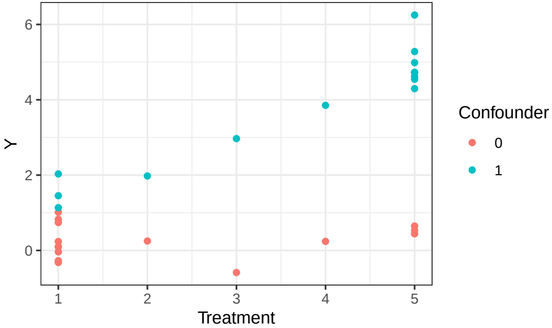

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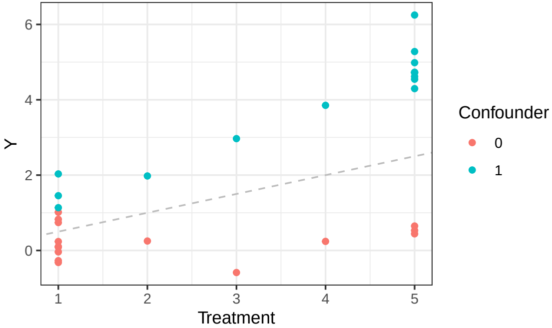

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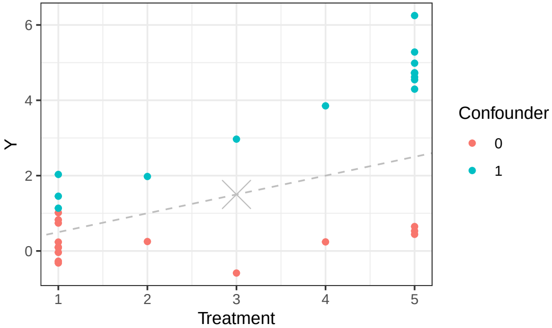

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Reweight to a pseudo-population

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- Effectively: Remove the dashed edge

In our pseudo-population, the mean given $A = a$ equals the expected outcome under an intervention to set $A = a$

$$\mathrm{E}_{\text{PseudoPopulation}}(Y \mid A = a) = \mathrm{E}(Y^a)$$

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To estimate:

$$E(Y^a) = E_{\text{PsuedoPopulation}}(Y \mid A = a) = \alpha + \beta a$$

This is OLS weighted to the pseudo-population

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4. Estimate $\hat{\mathrm{E}}(Y^a)$: Weighted regression of Y on A, using $\hat{w}$

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This yields efficiency gains only for when the model for $\mathrm{E}(Y^a)$ is not saturated (Hernán & Robins p. 158)

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When A is continuous, pooling information over A is appealing.

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See Hernán & Robins 12.4.

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Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!