16. Treatments in many time periods. What to do.

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

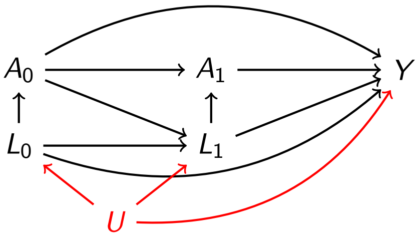
18 Oct 2022

Learning goals for today

At the end of class, you will be able to:

1. Reason about the sequential ignorability assumption
2. Apply inverse probability weighting to treatments over time

Identification: The adjustment set



A joint adjustment set for $\bar{A}$ is doomed

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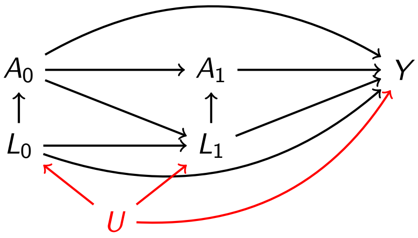

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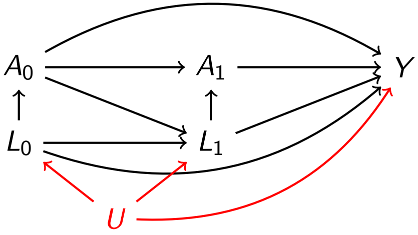

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- What happens if you adjust for L_1?
  - You block a causal path: $A_0 \to \boxed{L_1} \to Y$
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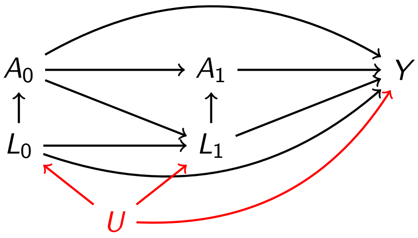

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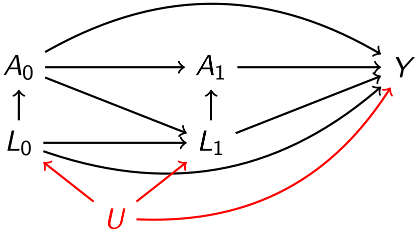

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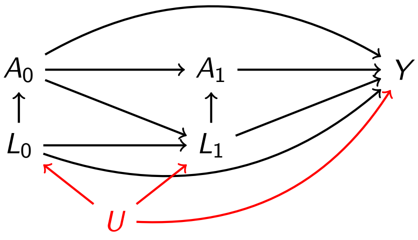

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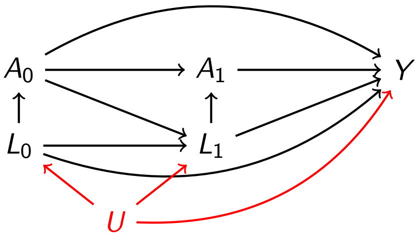

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What to do? [Class Exercise]

Generalizing the class exercise

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An ideal case: The **sequentially randomized experiment**

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1. randomize treatment at time 0, then

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4. randomize treatment at time k

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Then you can estimate $\text{E}(Y^{a_1,\dots,a_k})$ by $\text{E}(Y \mid \vec{A} = \vec{a})$.

Generalizing the class exercise: Conditional assignments

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A **sequential conditionally randomized experiment**

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Generalizing the class exercise: Conditional assignments

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   - Measure covariates $\vec{L}_0$

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3. ...
4. Repeat up to time k

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Then you can estimate $\text{E}(Y^{a_1,\dots,a_k})$ by the methods to come

Notation

- $\bar{A}_k = (A_0, A_1, \ldots, A_k)$ treatments up to time k
- $\bar{L}_k = (L_0, L_1, \ldots, L_k)$ confounders up to time k
- $g()$ treatment strategy
- Y^g potential outcome under that strategy

Identification: Sequential ignorability

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Potential outcome under assignment rule g	is independent of	treatment at time k	given	treatments up to $k-1$ followed rule g	and confounders up to time k
Y^g	$\perp\!\!\!\perp$	A_k	$\mid$	$\bar{A}_{k-1} = g(\bar{A}_{k-2}, \bar{L}_{k-1})$,	$\bar{L}_k$

for all assignment rules g and time periods $k = 1, \ldots, K$

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\begin{tikzcd}
A_0 \arrow[r] \arrow[dr] \arrow[rr, bend left] & A_1 \arrow[r] & Y \\
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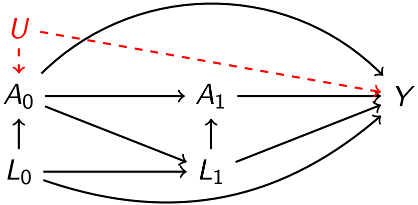

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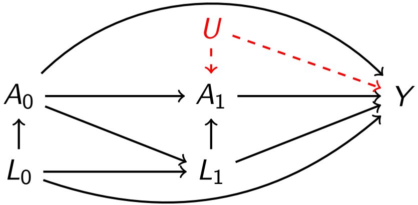

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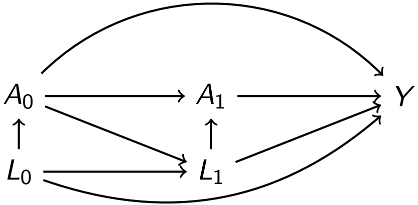

Holds by design in sequentially randomized experiments.
Holds by assumption in observational studies.

Estimation: Two strategies

1. Inverse probability weighting ($+$ marginal structural models)
2. Structural nested mean models (coming next class)

Inverse probability weighting: DAG motivation

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We observe data from this model

$$
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Inverse probability weighting: DAG motivation

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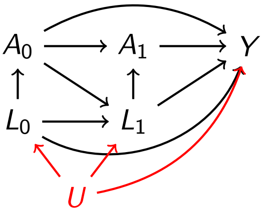

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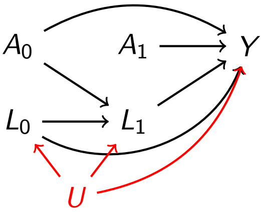

Inverse probability weighting

In time 0, define an inverse probability of treatment weight such that $A_0 \perp\!\!\!\perp L_0$ in the weighted pseudo-population

$$W^{A_0} = \frac{1}{\mathrm{P}(A_0 \mid L_0)}$$

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Define the overall weight as the product

$$W^{\bar{A}} = \prod_{k=0}^{K} \frac{1}{\mathrm{P}(A_k \mid \bar{A}_{k-1}, \bar{L}_k)}$$

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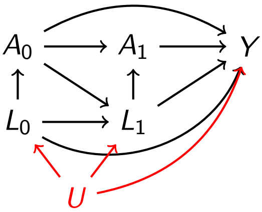
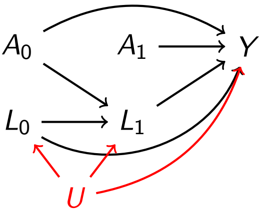


to this pseudo-population



Real example: Neighborhood disadvantage

Wodtke et al. 2011

Wodtke, G. T., Harding, D. J., & Elwert, F. (2011).
Neighborhood effects in temporal perspective: The impact of long-term exposure to concentrated disadvantage on high school graduation.
American Sociological Review, 76(5), 713-736.

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  - poverty
  - unemployment
  - welfare receipt
  - female-headed households
  - education
  - occupational structure

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This 5-value treatment is "neighborhood disadvantage"

Real example: Neighborhood disadvantage

Wodtke et al. 2011

Neighborhoods are experienced over time:

$$\bar{a}$$

is a trajectory of neighborhood disadvantage over ages $2, 3, \ldots, 17$

The authors study the effect of neighborhood disadvantage,

$$\begin{aligned} E(Y_{\bar{a}} - Y_{\bar{a}'}) &= E(Y_{\bar{a}}) - E(Y_{\bar{a}'}) \\ &= P(Y_{\bar{a}} = 1) - P(Y_{\bar{a}'} = 1), \end{aligned} \qquad (1)$$

Example:
$\bar{a}$ is residence in the most advantaged neighborhood each year
and
$\bar{a}'$ is residence in the most disadvantaged neighborhood each year

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Wodtke et al. 2011

Problem: Neighborhoods A_1 shape family characteristics L_2, which confound where people live in the future A_2

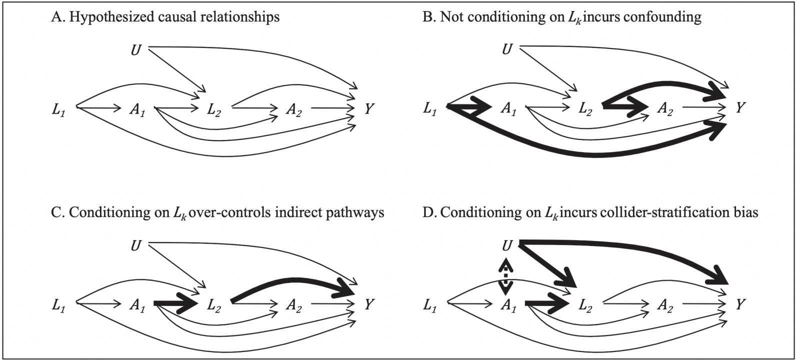

Figure 1. Causal Graphs for Exposure to Disadvantaged Neighborhoods with Two Waves of Follow-up

Note: A_k = neighborhood context, L_k = observed time-varying confounders, U = unobserved factors, Y = outcome.

Table 2. Time-Dependent Sample Characteristics

Variable	Blacks (n = 834)			Nonblacks (n = 1,259)		
	Age 1	Age 10	Age 17	Age 1	Age 10	Age 17
NH disadvantage index, percent						
1st quintile	3.48	3.60	3.48	13.34	19.14	20.65
2nd quintile	3.24	3.72	6.00	19.46	18.67	21.84
3rd quintile	5.28	5.88	7.79	26.13	23.27	22.48
4th quintile	14.87	18.11	18.47	26.13	23.99	21.13
5th quintile	73.14	68.71	64.27	14.93	14.93	13.90
FU head's marital status, percent						
Unmarried	33.93	44.84	52.04	5.88	11.36	15.09
Married	66.07	55.16	47.96	94.12	88.64	84.91
FU head's employment status, percent						
Unemployed	27.22	32.61	33.09	8.10	8.02	9.69
Employed	72.78	67.39	66.91	91.90	91.98	90.31
Public assistance receipt, percent						
Did not receive AFDC	81.06	75.66	82.37	96.27	96.19	97.93
Received AFDC	18.94	24.34	17.63	3.73	3.81	2.07
Homeownership, percent						
Do not own home	69.66	53.48	50.12	40.19	22.32	20.73
Own home	30.34	46.52	49.88	59.81	77.68	79.27
FU income in $1,000s, mean	19.68	25.04	27.45	32.59	46.65	57.50
FU head's work hours, mean	30.08	26.82	27.51	42.65	40.84	40.68
FU size, mean	5.75	5.32	4.81	4.22	4.69	4.33
Cum. residential moves, mean	.32	2.48	3.64	.32	2.16	3.02

Note: NH = neighborhood; FU = family unit. Statistics reported for children not lost to follow-up before age 20 (first imputation dataset).

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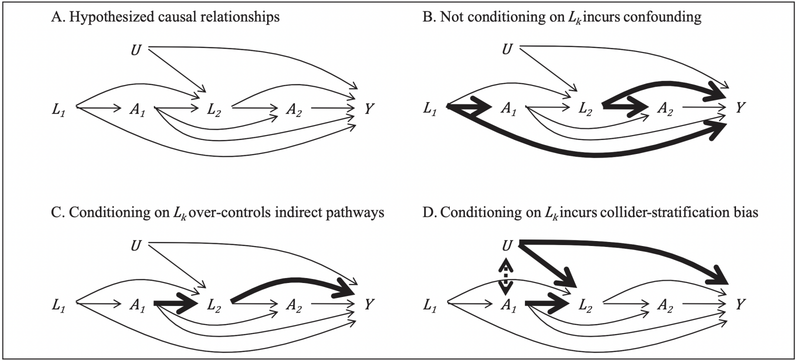

Figure 1. Causal Graphs for Exposure to Disadvantaged Neighborhoods with Two Waves of Follow-up

Note: A_k = neighborhood context, L_k = observed time-varying confounders, U = unobserved factors, Y = outcome.

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Wodtke et al. 2011

Solution: MSM-IPW

$$w_i = \prod_{k=1}^{K} \frac{1}{P(A_k = a_{ki} \mid \bar{A}_{k-1} = \bar{a}_{(k-1)i}, \bar{L}_k = \bar{l}_{ki})}. \quad (4)$$

Also with stabilized weights

$$sw_i = \prod_{k=1}^{K} \frac{P(A_k = a_{ki} \mid \bar{A}_{k-1} = \bar{a}_{(k-1)i}, L_0 = l_0)}{P(A_k = a_{ki} \mid \bar{A}_{k-1} = \bar{a}_{(k-1)i}, \bar{L}_k = \bar{l}_{ki})}, \quad (5)$$

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Problem: Huge number of treatments

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- 5 levels of neighborhood disadvantage

Real example: Neighborhood disadvantage

Wodtke et al. 2011

Problem: Huge number of treatments

- 5 levels of neighborhood disadvantage
- 16 time periods

Real example: Neighborhood disadvantage

Wodtke et al. 2011

Problem: Huge number of treatments

- 5 levels of neighborhood disadvantage
- 16 time periods
- $16^5 = 1,048,576$ possible treatment vectors $\vec{A}$

Digression: Marginal structural models in dynamic settings

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Recall that when A takes many values, we can fit a marginal structural model

- Example: $\mathsf{E}(Y^a) = \alpha + \beta a$
- Estimate by $\mathsf{E}^{\mathsf{PP}}(Y \mid A = a)$ where E^{PP} is the expectation in the pseudopopulation weighted so that treatment is independent of confounders.

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Wodtke et al. 2011

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MSMs also apply in dynamic settings where $\vec{A}$ is a vector over time.

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MSMs also apply in dynamic settings where $\vec{A}$ is a vector over time.

From Wodtke et al. 2011:

$$logit(P(Y_{\bar{a}} = 1)) = \theta_0 + \theta_1\left(\sum_{k=1}^{16} a_k / 16\right). \quad (2)$$

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MSMs also apply in dynamic settings where $\vec{A}$ is a vector over time.

From Wodtke et al. 2011:

$$logit(P(Y_{\bar{a}} = 1)) = \theta_0 + \theta_1 \left(\sum_{k=1}^{16} a_k / 16 \right). \quad (2)$$

Interpretation: $\bar{a}$ is duration-weighted exposure

Results: Neighborhood disadvantage

Wodtke et al. 2011

Results: Neighborhood disadvantage

Wodtke et al. 2011

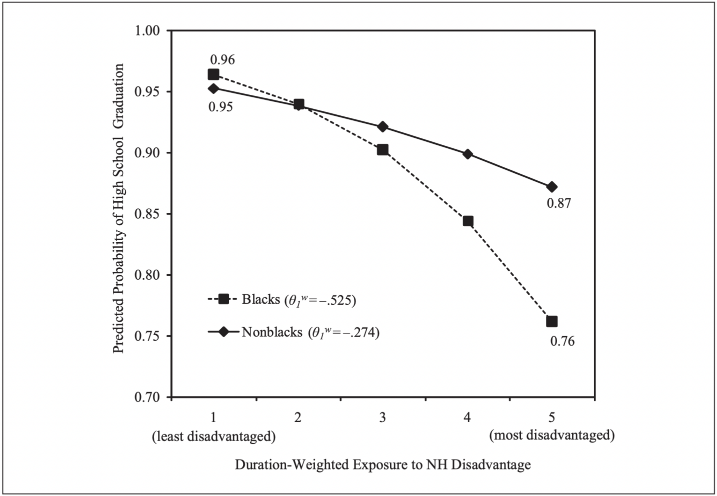

Figure 3. Predicted Probability of High School Graduation by Neighborhood Exposure History

Note: NH = Neighborhood

Learning goals for today

At the end of class, you will be able to:

1. Reason about the sequential ignorability assumption
2. Apply inverse probability weighting to treatments over time

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!