

20. Principal Stratification (Part 1)

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Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

2 Nov 2022

Learning goals for today

At the end of class, you will be able to:

1. Define principal strata
2. Understand how they address a post-treatment problem
3. Make assumptions to draw inference about a latent stratum

A hypothetical setting

- ▶ Suppose you have a new exercise program $A = 1$ vs $A = 0$

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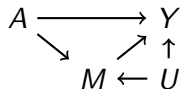
- ▶ Suppose you have a new exercise program $A = 1$ vs $A = 0$
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- ▶ But only some of the people survive $M = 1$

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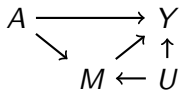
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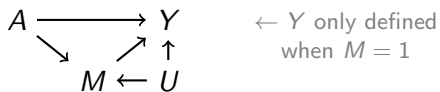
How would you analyze this?

A hypothetical setting: Common solutions



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A hypothetical setting: Common solutions



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A hypothetical setting: Common solutions



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5. New approach: **Principal stratification**

Principal stratification: The big idea

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But before the treatment is assigned, there are 4 types of people:

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4. Would survive if and only if they do not receive the program

▶ $M_i^1 = 0, M_i^0 = 1$

The effect on Y is only defined for stratum 1.

Principal stratification: The difficulty

Four principal strata

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- Survivors in the program Mix of 1 and 3

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|--------------------------------|----------------|
| ▶ Survivors in the program | Mix of 1 and 3 |
| ▶ Survivors not in the program | Mix of 1 and 4 |

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- ▶ Survivors in the program Mix of 1 and 3
- ▶ Survivors not in the program Mix of 1 and 4
- ▶ Die in the program

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We observe the sets

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| ▶ Survivors in the program | Mix of 1 and 3 |
| ▶ Survivors not in the program | All 1 Mix of 1 and 4 |
| ▶ Die in the program | All 2 Mix of 2 and 4 |
| ▶ Die without the program | Mix of 2 and 3 |

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What to do with the sets that are still mixed?

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What to do with the sets that are still mixed? **Bounds**

Principal Stratification: Exercise

You will learn more by doing bounds yourself!

The rest of class will be a pen-and-paper [exercise](#)

tinyurl.com/PrinStratEx

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References

Original paper

- ▶ Frangakis, C. E., & Rubin, D. B. (2002). [Principal stratification in causal inference](#). Biometrics, 58(1), 21-29.

Good recent summary (assigned on syllabus)

- ▶ Page, L. C., Feller, A., Grindal, T., Miratrix, L., & Somers, M. A. (2015). [Principal stratification: A tool for understanding variation in program effects across endogenous subgroups](#). American Journal of Evaluation, 36(4), 514-531.

Good resource for bounds

- ▶ Imai, K. (2008). [Sharp bounds on the causal effects in randomized experiments with “truncation-by-death”](#). Statistics & Probability Letters, 78(2), 144-149.

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!