

## 23. Measurement error

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Cornell Info 6751: Causal Inference in Observational Settings  
Fall 2022

10 Nov 2022

# Learning goals for today

At the end of class, you will be able to:

1. Present measurement error in a DAG
2. Formalize when measurement error undermines identification
3. Apply these ideas in a hypothetical research design

# Broad issue: Measurement error

This lecture closely follows

Hernán, M. A., & Cole, S. R. (2009).

[Invited commentary: Causal diagrams and measurement bias.](#)

American Journal of Epidemiology, 170(8), 959-962.

# Plan for class: Your own example

We will discuss ideas using an example from Hernán & Cole 2009.  
For each idea, each group will apply it to their own example.

**Task:** In pairs or triples, construct your own running example

Each unit (e.g., person) in your study should report

- ▶ A treatment  $A$ , reported with error
- ▶ An outcome  $Y$ , reported with error

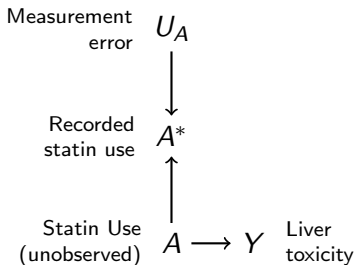
You choose how to define  $A$  and  $Y$

Statin Use  
(unobserved)  $A \longrightarrow Y$  Liver  
toxicity

**Augment this DAG**

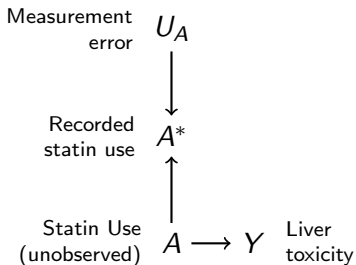
Recorded statin use  $A^*$   
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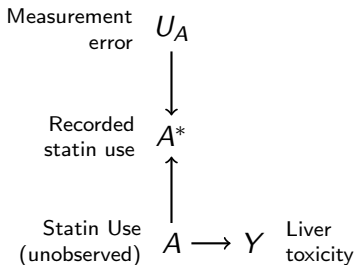


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By what path(s) are  $A^*$  and  $Y$  associated?

If  $\{A^*, Y\}$  are associated, does this imply  $A \rightarrow Y$ ?



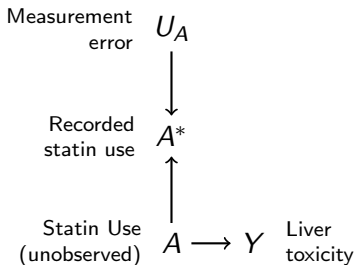


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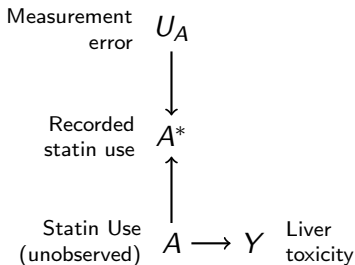
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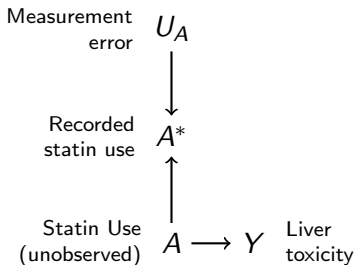
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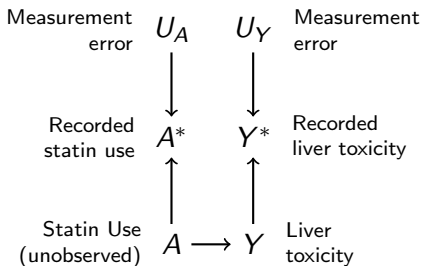
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In your example: How could you have measurement error like this?



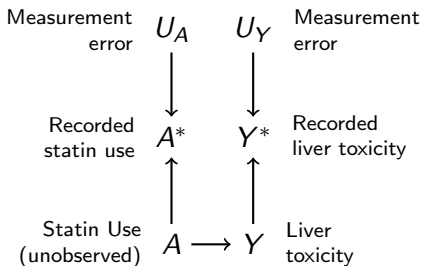
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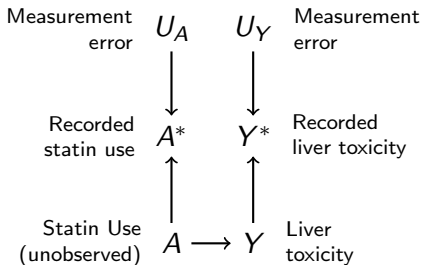
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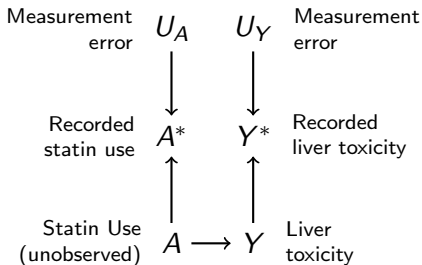


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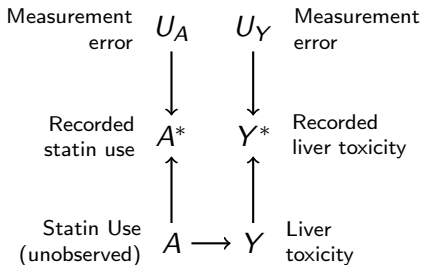
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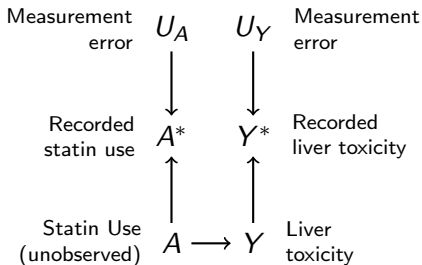
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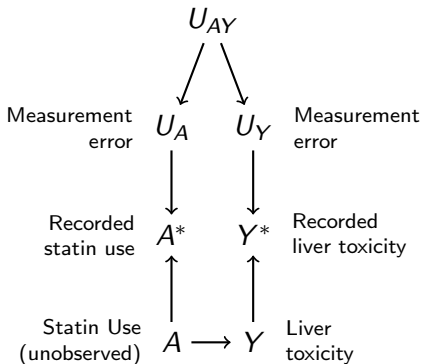
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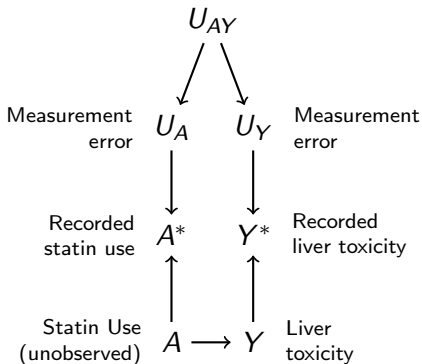
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Reports are retrospective,  
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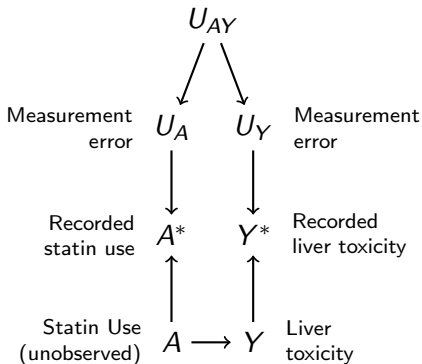
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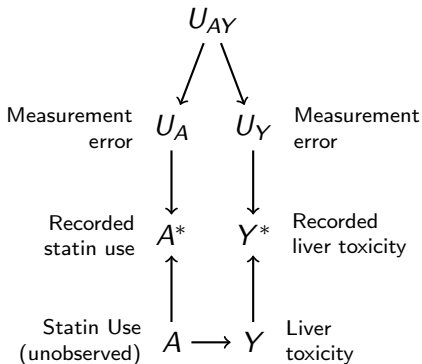
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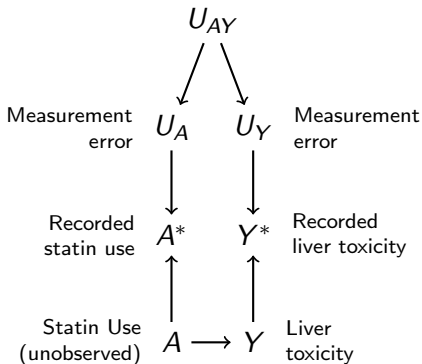
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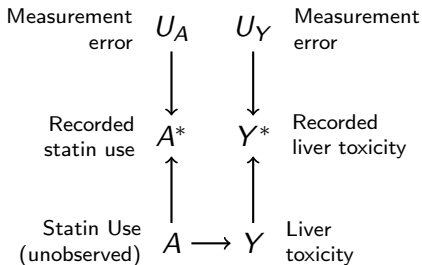
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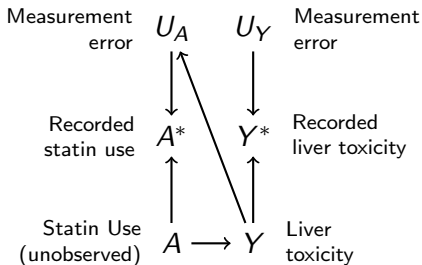
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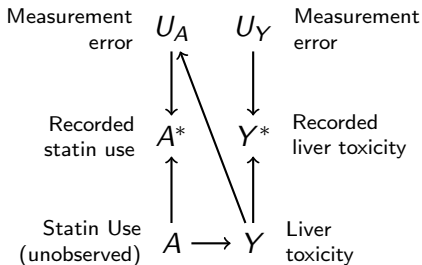
When someone has liver toxicity, the doctor asks different questions about statin use





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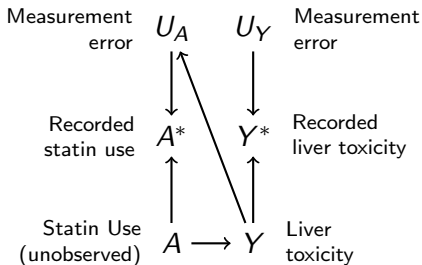
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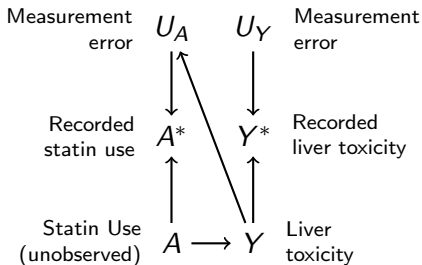
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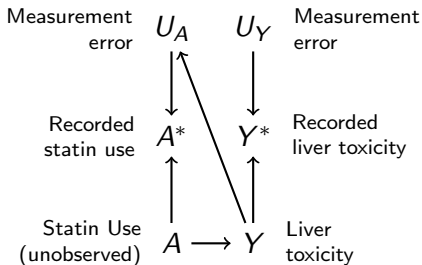
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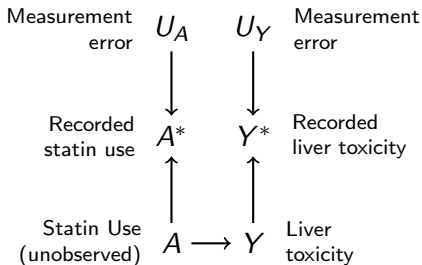
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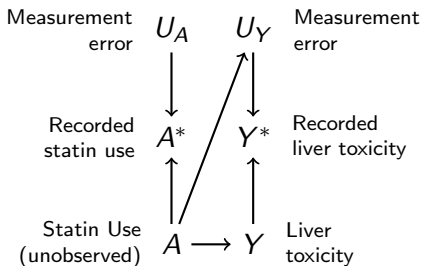
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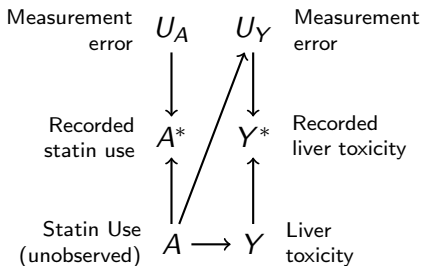
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Physicians watch liver toxicity more closely among statin users



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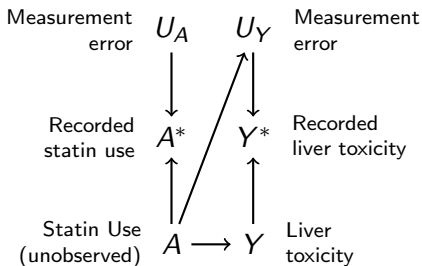


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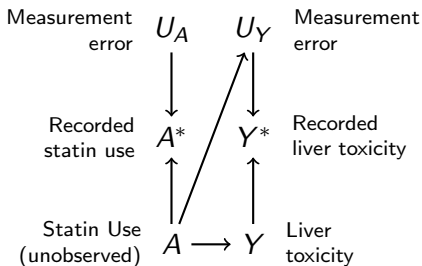
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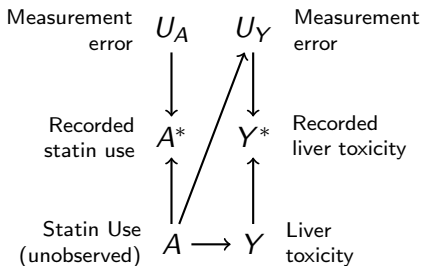
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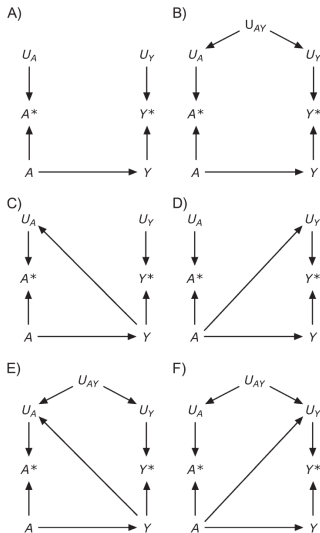
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# Hernan & Cole Fig 2



**Figure 2.** A structural classification of measurement error.

Hernán & Cole illustration:

Does body mass index (BMI) affect health outcomes?

Weight  $W$  and height  $H$  affect the risk of a condition  $A$   
which affects a health outcome  $Y$

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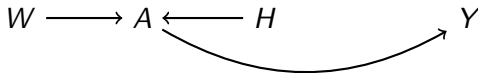
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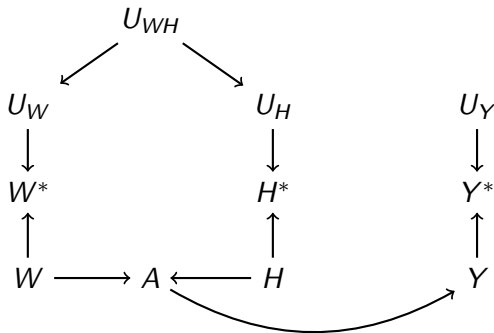
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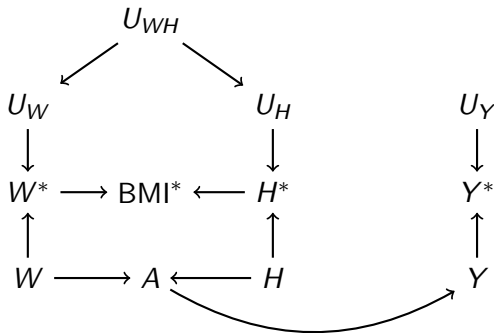
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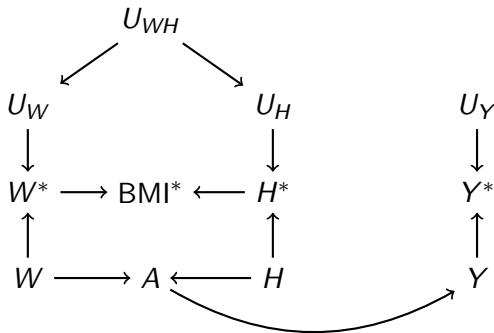
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No. There is no causal path.

## Examples to discuss

[tinyurl.com/MeasurementExercise](https://tinyurl.com/MeasurementExercise)

# Learning goals for today

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1. Present measurement error in a DAG
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3. Apply these ideas in a hypothetical research design

Let me know what you are thinking

[tinyurl.com/CausalQuestions](https://tinyurl.com/CausalQuestions)

Office hours TTh 11am-12pm and at  
[calendly.com/ianlundberg/office-hours](https://calendly.com/ianlundberg/office-hours)  
Come say hi!