26. Instrumental variables

Ian Lundberg
Cornell Info 6751: Causal Inference in Observational Settings
Fall 2022

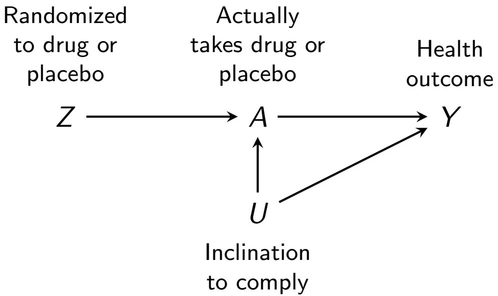
22 Nov 2022

Learning goals for today

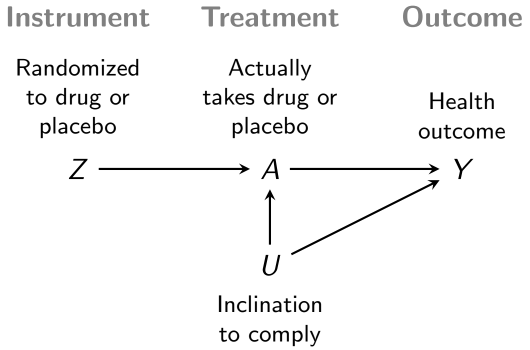
At the end of class, you will be able to:

1. Understand the logic of instrumental variables
2. Derive the average effect among compliers in experiments with noncompliance
3. Recognize the pitfalls of IVs in observational settings

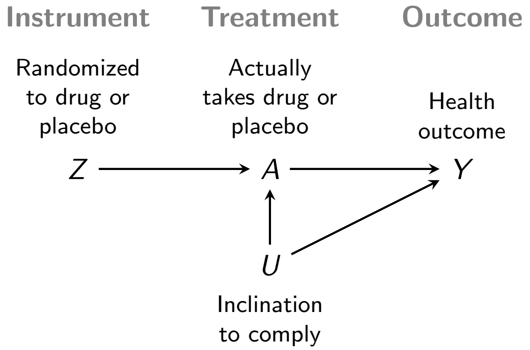
Instrumental variables: Experiment with noncompliance



Instrumental variables: Experiment with noncompliance



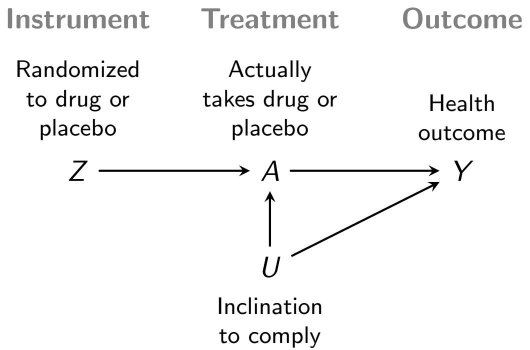
Instrumental variables: Experiment with noncompliance



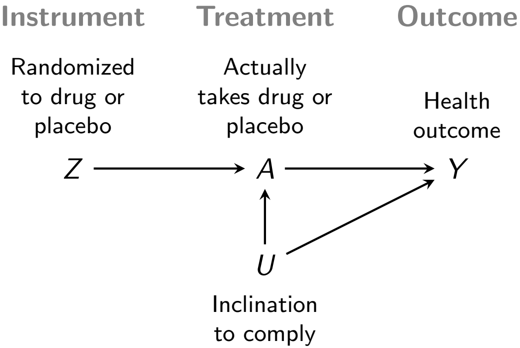
Two ideas

1. Intent to treat effect
2. Average effect among compliers

Instrumental variables: 1) Intent to treat effect

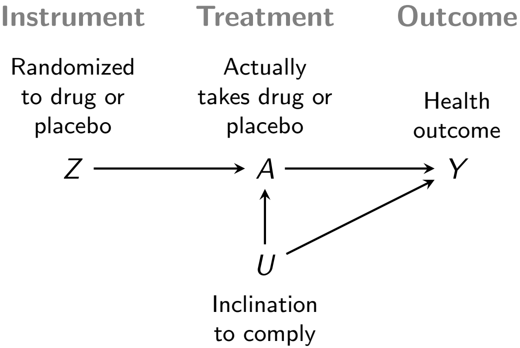


Instrumental variables: 1) Intent to treat effect



Ignore A. What is the effect of Z on Y?

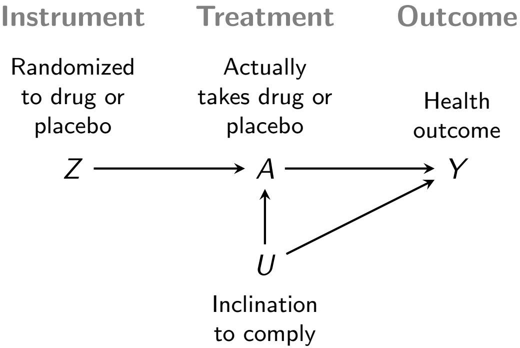
Instrumental variables: 1) Intent to treat effect



Ignore A. What is the effect of Z on Y?

$$\mathrm{E}(\underbrace{Y^1}_{\text{Outcome under } Z=1} - \underbrace{Y^0}_{\text{Outcome under } Z=0})$$

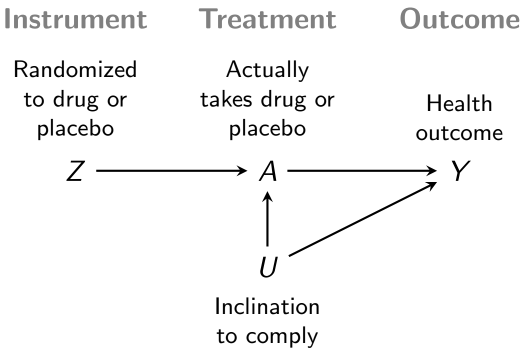
Instrumental variables: 1) Intent to treat effect



Ignore A. What is the effect of Z on Y?

$$\mathrm{E}(\underbrace{Y^1}_{\text{Outcome under } Z=1} - \underbrace{Y^0}_{\text{Outcome under } Z=0}) \underset{\text{By Positivity Consistency Exchangeability for } Z}{=}$$

Instrumental variables: 1) Intent to treat effect



Ignore A. What is the effect of Z on Y?

$$E(Y^1 - Y^0) = E(Y \mid Z = 1) - E(Y \mid Z = 0)$$

Outcome under $Z = 1$

Outcome under $Z = 0$

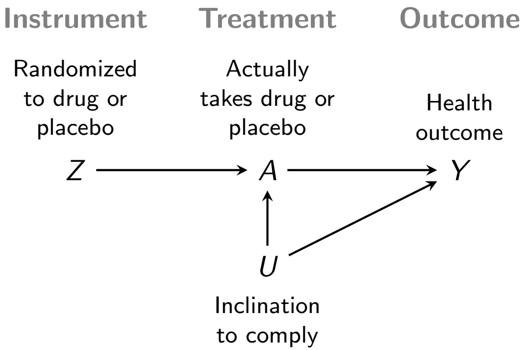
By Positivity Consistency Exchangeability for Z

Mean difference in observable outcomes

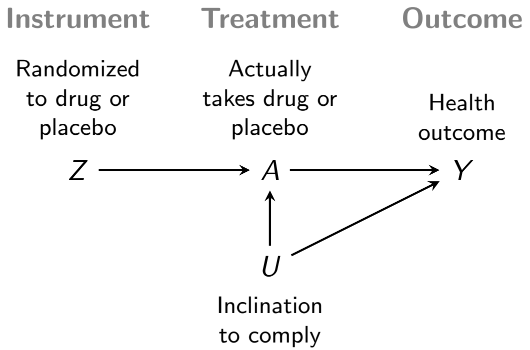
Instrumental variables: 2) Average effect among compliers

Imbens, G., & Angrist, J. (1994). Identification and estimation of local average treatment effects. Econometrica, 62(2), 467-475.

Instrumental variables: 2) Average effect among compliers

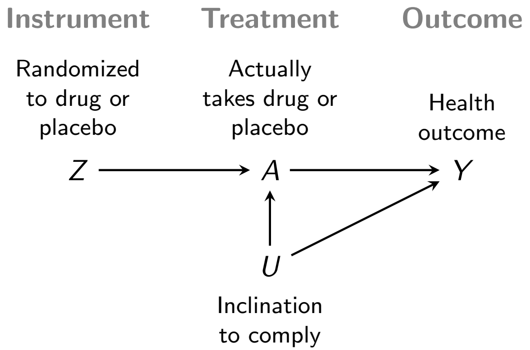


Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

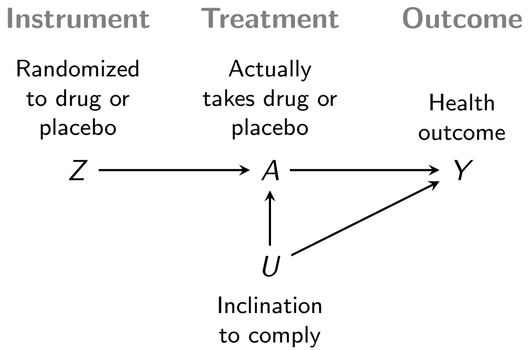
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \to Y$

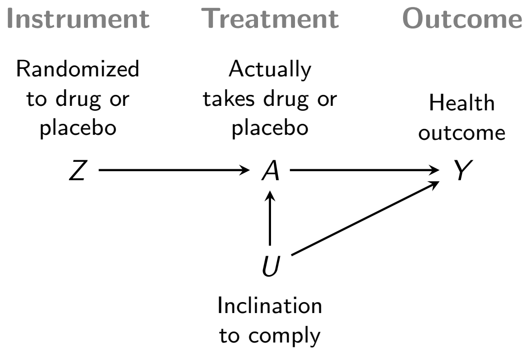
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \to Y$ (we just did)

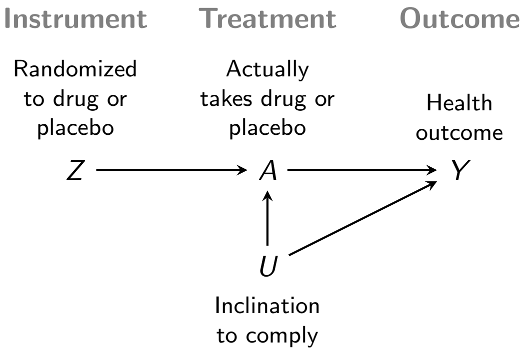
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \rightarrow Y$ (we just did)
2. Study the effect of $Z \rightarrow A$

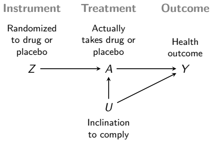
Instrumental variables: 2) Average effect among compliers



Key insight: The effect of Z on Y operates entirely through A

1. Study the effect of $Z \to Y$ (we just did)
2. Study the effect of $Z \to A$
3. Learn about $A \to Y$ since $Z \to Y$ is $Z \to A \to Y$

Instrumental variables: 2) Average effect among compliers



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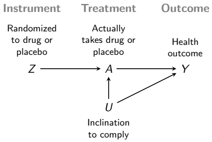
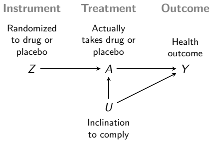


The effect $Z \rightarrow A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

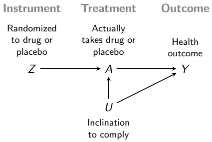
Instrumental variables: 2) Average effect among compliers



The effect $Z \to A$ has four **principal strata**:
latent sets of people who respond to Z a particular way

Compliers $\quad A^{Z=0} = 0 \quad A^{Z=1} = 1 \quad$ (follow assignment)

Instrumental variables: 2) Average effect among compliers



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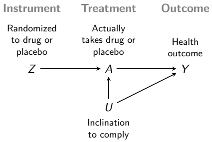
Compliers	$A^{Z=0} = 0$	$A^{Z=1} = 1$	(follow assignment)
Always takers	$A^{Z=0} = 1$	$A^{Z=1} = 1$	(always take treatment)

Instrumental variables: 2) Average effect among compliers



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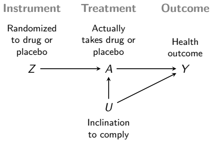

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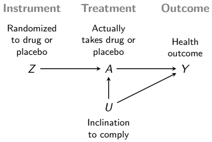
Defiers	$A^{Z=0} = 1$	$A^{Z=1} = 0$	(defy assignment)

Instrumental variables: 2) Average effect among compliers



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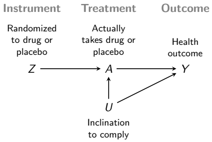
Defiers	$A^{Z=0} = 1$	$A^{Z=1} = 0$	(defy assignment)

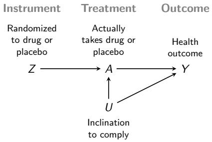
Discuss: In which strata is the effect $Z \rightarrow Y$ zero?

Instrumental variables: 2) Average effect among compliers



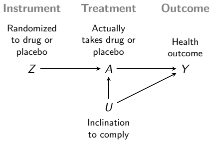
Among **always takers** and **never takers**,

Instrumental variables: 2) Average effect among compliers



Among **always takers** and **never takers**,
Z does not affect A

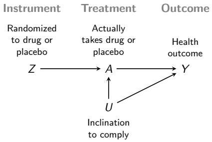
Instrumental variables: 2) Average effect among compliers



Among **always takers** and **never takers**,
Z does not affect A

Z only affects Y through A

Instrumental variables: 2) Average effect among compliers

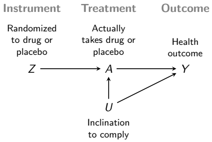


Among **always takers** and **never takers**,
Z does not affect A

Z only affects Y through A

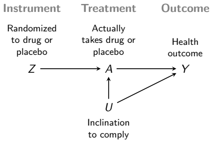
In these strata, Z does not affect Y

Instrumental variables: 2) Average effect among compliers



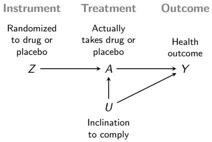
Among **compliers**,

Instrumental variables: 2) Average effect among compliers



Among **compliers**,
$Z = 1$ implies $A = 1$ and
$Z = 0$ implies $A = 0$

Instrumental variables: 2) Average effect among compliers



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In these strata, $Z = A$

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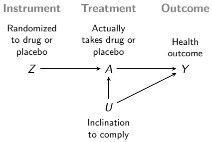

Among **compliers**,
$Z = 1$ implies $A = 1$ and
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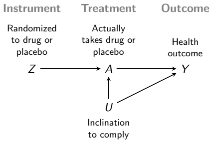
In these strata, $Z = A$

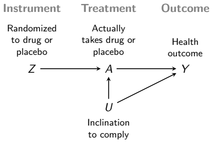
$Z \rightarrow Y$ and $A \rightarrow Y$ are the same

Instrumental variables: 2) Average effect among compliers



Among **defiers**,

Instrumental variables: 2) Average effect among compliers



Among **defiers**,
$Z = 1$ implies $A = 0$ and
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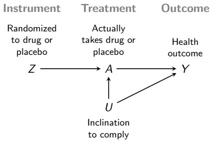
Instrumental variables: 2) Average effect among compliers



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$Z = 1$ implies $A = 0$ and
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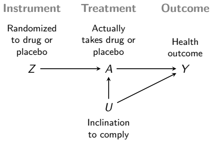
In these strata, $Z = 1 - A$

Instrumental variables: 2) Average effect among compliers

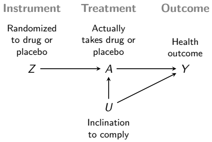


Among **defiers**,
$Z = 1$ implies $A = 0$ and
$Z = 0$ implies $A = 1$

In these strata, $Z = 1 - A$

$Z \to Y$ and $A \to Y$ are the same magnitude
but have opposite signs

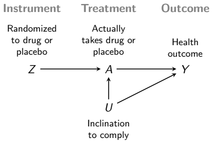
Instrumental variables: 2) Average effect among compliers



Four principal strata

Compliers	$(Z \to A) = +1$	$(Z \to Y) = (A \to Y)$
Always takers	$(Z \to A) = 0$	$(Z \to Y) = 0$
Never takers	$(Z \to A) = 0$	$(Z \to Y) = 0$
Defiers	$(Z \to A) = -1$	$(Z \to Y) = -(A \to Y)$

Instrumental variables: 2) Average effect among compliers



Four principal strata

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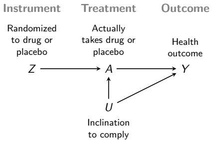
Defiers	$(Z \rightarrow A) = -1$	$(Z \rightarrow Y) = -(A \rightarrow Y)$

Assume **no defiers** in the population

Instrumental variables: 2) Average effect among compliers



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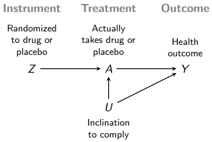

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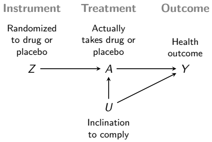
Discuss a hypothetical.

Instrumental variables: 2) Average effect among compliers



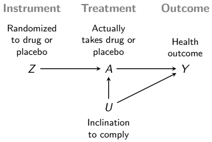
Four principal strata

$$
\begin{array}{rll}
\text{Compliers} & (Z \rightarrow A) = +1 & (Z \rightarrow Y) = (A \rightarrow Y) \\
\text{Always takers} & (Z \rightarrow A) = 0 & (Z \rightarrow Y) = 0 \\
\text{Never takers} & (Z \rightarrow A) = 0 & (Z \rightarrow Y) = 0
\end{array}
$$

Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Instrumental variables: 2) Average effect among compliers



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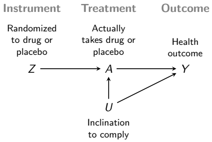
Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of $Z \to Y$ among compliers is 0.6

Instrumental variables: 2) Average effect among compliers



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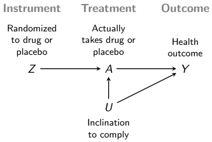
Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of $Z \to Y$ among compliers is 0.6

What is the average effect of $Z \to Y$ in the population?

Instrumental variables: 2) Average effect among compliers



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Discuss a hypothetical.

Population is 50% compliers, 25% always takers, 25% never takers

Average effect of $Z \to Y$ among compliers is 0.6

What is the average effect of $Z \to Y$ in the population? **0.3**

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$E(Y^{Z=1} - Y^{Z=0})$$

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$\begin{aligned}&\mathrm{E}(Y^{Z=1} - Y^{Z=0})\\&= \sum_s \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = s)\underbrace{\mathrm{P}(S = s)}_{\substack{\text{Denote}\\ \pi_s}}\end{aligned}$$

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$
\begin{aligned}
&\mathrm{E}(Y^{Z=1} - Y^{Z=0}) \\
&= \sum_s \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = s) \underbrace{\mathrm{P}(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\
&= \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}} \\
&\qquad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Always-Taker})\pi_{\text{Always-Taker}} \\
&\qquad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Never-Taker})\pi_{\text{Never-Taker}} \\
&\qquad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Defier})\pi_{\text{Defier}}
\end{aligned}
$$

Instrumental variables: 2) Average effect among compliers

Deriving the general case:

$$
\begin{aligned}
&\mathrm{E}(Y^{Z=1} - Y^{Z=0}) \\
&= \sum_s \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = s) \underbrace{\mathrm{P}(S = s)}_{\substack{\text{Denote} \\ \pi_s}} \\
&= \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}} \\
&\qquad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Always-Taker})\pi_{\text{Always-Taker}} \qquad (= 0) \\
&\qquad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Never-Taker})\pi_{\text{Never-Taker}} \qquad (= 0) \\
&\qquad + \mathrm{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Defier})\pi_{\text{Defier}} \qquad (= 0)
\end{aligned}
$$

Instrumental variables: 2) Average effect among compliers

$$\text{E}(Y^{Z=1} - Y^{Z=0}) = \text{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Instrumental variables: 2) Average effect among compliers

$$\underbrace{}_{\text{Average effect of instrument}} = \underbrace{}_{\text{Average among compliers}} \times \underbrace{}_{\text{Proportion who comply}}$$

$$\text{E}(Y^{Z=1} - Y^{Z=0}) = \text{E}(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Instrumental variables: 2) Average effect among compliers

Average effect of instrument	$=$	Average among compliers	$\times$	Proportion who comply

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Among compliers, $(Z \to Y) = (A \to Y)$.

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Rearrange to get the complier average treatment effect

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$$\mathrm{E}(Y^{Z=1} - Y^{Z=0}) = \mathrm{E}(Y^{A=1} - Y^{A=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

Rearrange to get the complier average treatment effect

$$\mathrm{E}(Y^{A=1} - Y^{A=0} \mid S = \text{Complier}) = \frac{\mathrm{E}(Y^{Z=1} - Y^{Z=0})}{\pi_{\text{Complier}}}$$

Instrumental variables: 2) Average effect among compliers

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$$E(Y^{Z=1} - Y^{Z=0}) = E(Y^{Z=1} - Y^{Z=0} \mid S = \text{Complier})\pi_{\text{Complier}}$$

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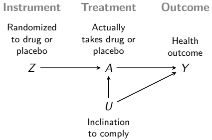
Rearrange to get the complier average treatment effect

$$\begin{aligned} E(Y^{A=1} - Y^{A=0} \mid S = \text{Complier}) &= \frac{E(Y^{Z=1} - Y^{Z=0})}{\pi_{\text{Complier}}} \\ &= \frac{E(Y^{Z=1} - Y^{Z=0})}{E(A^{Z=1} - A^{Z=0})} \end{aligned}$$

Instrumental variables: 2) Average effect among compliers



Summary: Average treatment effect among compliers

1. Estimate the average effect of Z on Y
2. Estimate the average effect of Z on A
3. Divide (1) by (2)

Assumptions that got us there:

1. Instrument Z is unconfounded (exogeneity)
2. $Z \rightarrow Y$ is entirely mediated by M (exclusion restriction)
3. No defiers (monotonicity)

where (3) is an assumption outside of the DAG

Two equivalent estimators

Two equivalent estimators

We have been studying the **Wald estimator**

$$\frac{E(Y \mid Z = 1) - \mathrm{E}(Y \mid Z = 0)}{\mathrm{E}(A \mid Z = 1) - \mathrm{E}(A \mid Z = 0)}$$

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A numerically equivalent estimator is **two-stage-least-squares**:

1. Regress A on Z
2. Predict $\hat{A}$
   - Intuition: Isolates Z-generated variation in A
3. Regress Y on $\hat{A}$

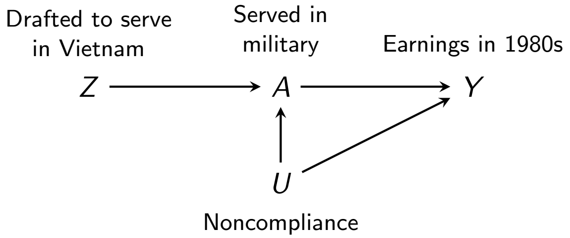
An observational settings that is **clean**

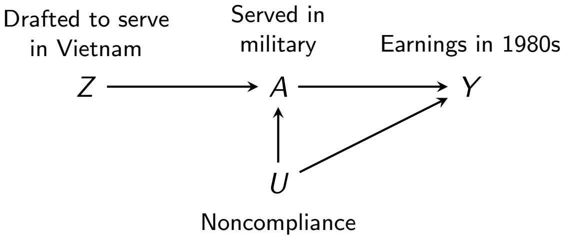
1970 RANDOM SELECTION SEQUENCE, BY MONTH AND DAY

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	305	086	108	032	330	249	093	111	225	359	019	129
2	159	144	029	271	298	228	350	045	161	125	034	328
3	251	297	267	083	040	301	115	261	049	244	348	157
4	215	210	275	081	276	020	279	145	232	202	266	165
5	101	214	293	269	364	028	188	054	082	024	310	056
6	224	347	139	253	155	110	327	114	006	087	076	010
7	306	091	122	147	035	085	050	168	008	234	051	012
8	199	181	213	312	321	366	013	048	184	283	097	105
9	194	338	317	219	197	335	277	106	263	342	080	043
10	325	216	323	218	065	206	284	021	071	220	282	041
11	329	150	136	014	037	134	248	324	158	237	046	039
12	221	068	300	346	133	272	015	142	242	072	066	314
13	318	152	259	124	295	069	042	307	175	138	126	163
14	238	004	354	231	178	356	331	198	001	294	127	026
15	017	089	169	273	130	180	322	102	113	171	131	320
16	121	212	166	148	055	274	120	044	207	254	107	096
17	235	189	033	260	112	073	098	154	255	288	143	304
18	140	292	332	090	278	341	190	141	246	005	146	128
19	058	025	200	336	075	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	063	192	185	135
21	186	363	334	062	250	060	027	291	204	243	156	070
22	337	290	265	316	326	247	153	339	160	117	009	053
23	118	057	256	252	319	109	172	116	119	201	182	162
24	059	236	258	002	031	358	023	036	195	196	230	095
25	052	179	343	351	361	137	067	286	149	176	132	084
26	092	365	170	340	357	022	303	245	018	007	309	173
27	355	205	268	074	296	064	289	352	233	264	047	078
28	077	299	223	262	308	222	088	167	257	094	281	123
29	349	285	362	191	226	353	270	061	151	229	099	016
30	164	---	217	208	103	209	287	333	315	038	174	003
31	211	---	030	---	313	---	193	011	---	079	---	100

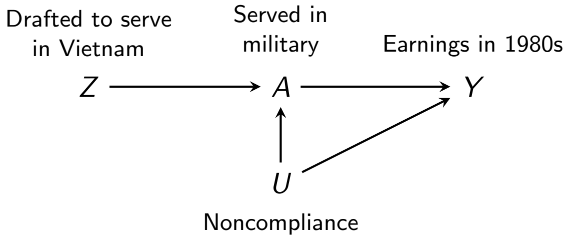
https://www.historynet.com/whats-your-number/

https://commons.wikimedia.org/wiki/File:
1969_draft_lottery_photo.jpg


Drafted to serve in Vietnam
Served in military
Earnings in 1980s
Z
A
Y
U
Noncompliance

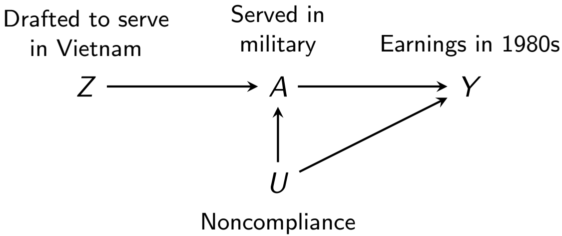


This is credible



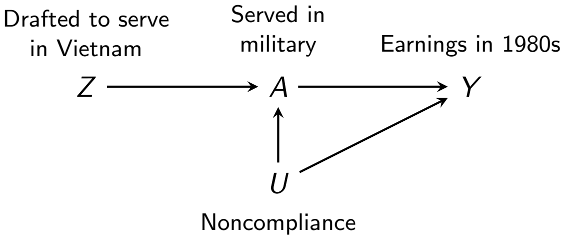
This is credible

- Exogeneity: Randomly selected draft numbers



This is credible

- Exogeneity: Randomly selected draft numbers
- Exclusion: Being drafted affects earnings only through service



This is credible

- Exogeneity: Randomly selected draft numbers
- Exclusion: Being drafted affects earnings only through service
- Monotonicity: No one joins the military in defiance of not being drafted

Angrist, J. D. (1990). Lifetime earnings and the Vietnam era draft lottery: Evidence from Social Security administrative records. The American Economic Review, 313-336.

Observational settings that are **less straightforward**

A difficult example: Genes as instruments

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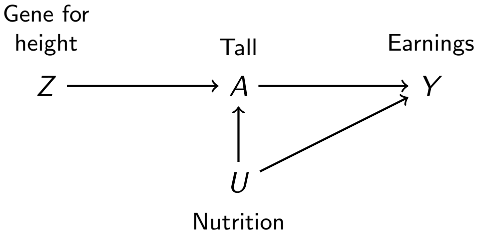
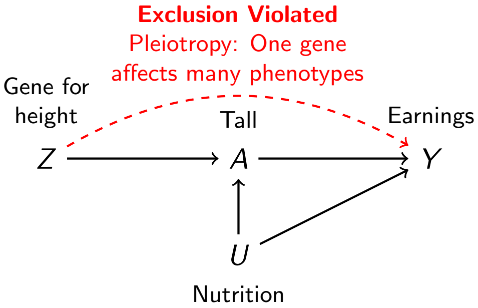
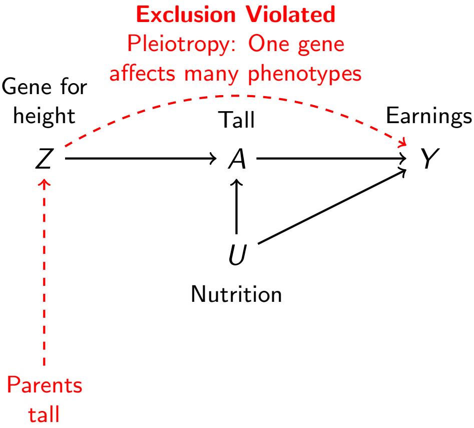


A difficult example: Genes as instruments



A difficult example: Genes as instruments



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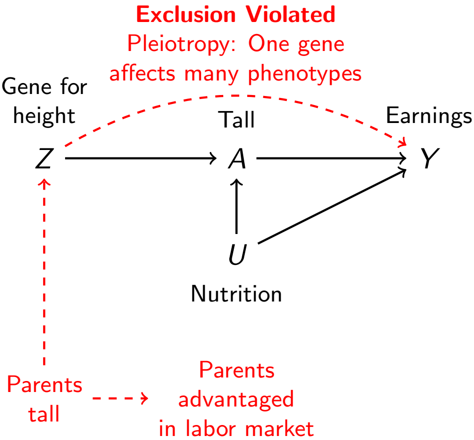

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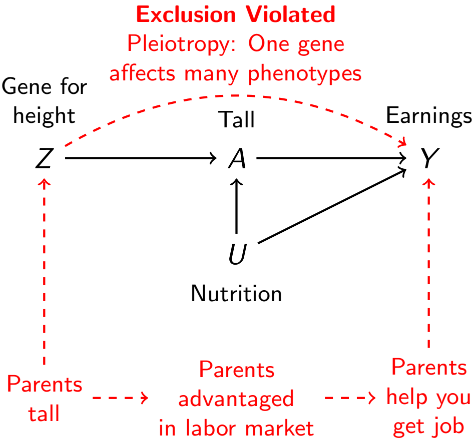

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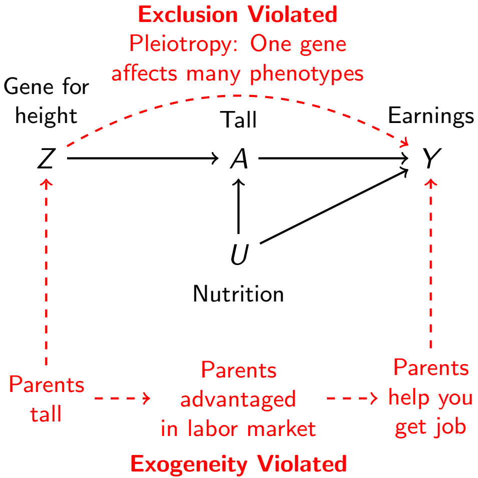
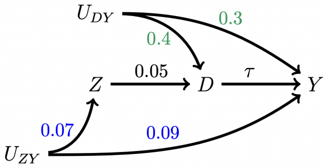


Warning: IV is very sensitive to assumptions

Fig 5 from Felton, C., & Stewart, B. M. (2022). Handle with Care: A Sociologist's Guide to Causal Inference with Instrumental Variables. SocArXiv.



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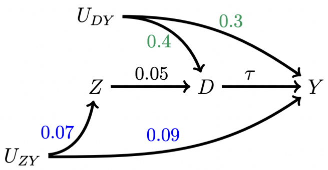


Asymptotic Bias of OLS

$$= 0.3 \times 0.4 = \mathbf{0.12}$$

Asymptotic Bias of IV

$$= (0.07 \times 0.09)/0.05 = \mathbf{0.126}$$

Examples where things are very hard

Examples where things are very hard

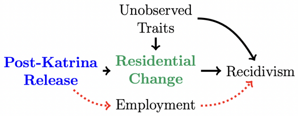
From Table 1 in Felton, C., & Stewart, B. M. (2022). Working paper. Handle with Care: A Sociologist's Guide to Causal Inference with Instrumental Variables.

1. Kirk (2009)[1] studies recidivism among parolees
   - Z: Parolee released before or after Hurricane Katrina
   - A: Parolee returns to home neighborhood upon release
   - Y: Recidivism

[1] Kirk, D. S. (2009). A natural experiment on residential change and recidivism: Lessons from Hurricane Katrina. American Sociological Review, 74(3), 484-505.

From Table 1 in Felton, C., & Stewart, B. M. (2022). Working paper.

Handle with Care: A Sociologist's Guide to Causal Inference with Instrumental Variables.

Study	Causal Structure	Identification Assumptions
Kirk (2009)		(1) Post-Katrina release induces moves away from a parolee's home neighborhood. (2) Post-Katrina release affects recidivism *only through* residential change and not, e.g., employment or police resources. (3) The timing of release shares no common causes with recidivism. (4) Post-Katrina release *never discourages* residential change.

2. Laidley & Conley (2018)[2] study variation in sunlight exposure across days for children observed repeatedly

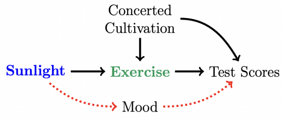
- Z: Sunlight
- A: Exercise
- Y: Test scores

[2] Laidley, T., & Conley, D. (2018). The effects of active and passive leisure on cognition in children: Evidence from exogenous variation in weather. Social Forces, 97(1), 129-156.

From Table 1 in Felton, C., & Stewart, B. M. (2022). Working paper.

Handle with Care: A Sociologist's Guide to Causal Inference with Instrumental Variables.

Laidley and Conley (2018)



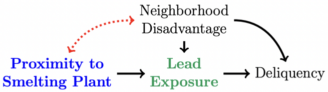
(1) More sunlight causes kids to exercise more. (2) Sunlight affects test scores *only through* exercise and not, e.g., mood. (3) When person fixed effects are included, within-person sunlight variation shares no common causes with test scores. (4) Sunnier weather *never discourages* exercise.

3. Sampson & Winter (2018)[3] study child development
  - Z: Proximity to a smelting plant
  - A: Lead exposure
  - Y: Delinquent behavior

[3] Sampson, R. & Winter, A. (2018). Poisoned development: Assessing childhood lead exposure as a cause of crime in a birth cohort followed through adolescence. Criminology, 56(2), 269-301.

From Table 1 in Felton, C., & Stewart, B. M. (2022). Working paper.

Handle with Care: A Sociologist's Guide to Causal Inference with Instrumental Variables.

Sampson and Winter (2018)		(1) Proximity to a smelting plant increases lead exposure. (2) Proximity affects deliquency *only through* lead exposure. (3) Proximity shares no common causes—such as neighborhood disadvantage—with deliquency. (4) Moving closer to a smelting plant *never* causes someone to experience less lead exposure.

4. Harding et al. (2018)[4] study defendants in criminal court.
   - Z: Which judge is assigned for the trial
   - A: Sentence of incarceration
   - Y: Employment after release

[4] Harding, D. J., Morenoff, J. D., Nguyen, A. P., & Bushway, S. D. (2018). Imprisonment and labor market outcomes: Evidence from a natural experiment. American Journal of Sociology, 124(1), 49-110.

From Table 1 in Felton, C., & Stewart, B. M. (2022). Working paper.

Handle with Care: A Sociologist's Guide to Causal Inference with Instrumental Variables.

Harding et al. (2018)

(1) Judge assignment affects the probability of being incarcerated. (2) Judge assignment affects employment *only through* incarceration. (3) Judge assignment shares no common causes with future employment. (4) If assigned to a more lenient judge, a defendant will *never* receive a harsher sentence.

Summary

Instrumental variables requires strong assumptions

- Exogenous instrument (e.g., randomization)
- Mediated fully through the treatment
- Under monotonicity (e.g., no defiers)

Works well in randomized studies with noncompliance.

In observational settings, keep the experimental ideal in mind!
Usefulness depends on how close it is to that ideal.

Complications we have not addressed today

- Proxy instruments
- Non-binary instruments
- Non-binary treatments

Learning goals for today

At the end of class, you will be able to:

1. Understand the logic of instrumental variables
2. Derive the average effect among compliers in experiments with noncompliance
3. Recognize the pitfalls of IVs in observational settings

Let me know what you are thinking

tinyurl.com/CausalQuestions

Office hours TTh 11am-12pm and at
calendly.com/ianlundberg/office-hours
Come say hi!