

^a4. The **Hilbert matrix** of order n is defined by $a_{ij} = (i + j - 1)^{-1}$ for $1 \leq i, j \leq n$. It is often used for test purposes because of its ill-conditioned nature. Define $b_i = \sum_{j=1}^n a_{ij}$. Then the solution of the system of equations $\sum_{j=1}^n a_{ij}x_j = b_i$ for $1 \leq i \leq n$ is $\mathbf{x} = [1, 1, \dots, 1]^T$. Verify this. Select some values of n in the range $2 \leq n \leq 15$, solve the system of equations for \mathbf{x} using procedures *Gauss* and *Solve*, and see whether the result is as predicted. Do the case $n = 2$ by hand to see what difficulties occur in the computer.