# Assignment 4.

Due Friday, Feb. 11.

Reading: Chapter 2 in the text plus supplemental material on finite elements.

1. (finite elements) Use the Galerkin finite element method with continuous piecewise linear basis functions to solve the problem

$$-\frac{d}{dx}\left((1+x^2)\frac{du}{dx}\right) = f(x), \quad 0 \le x \le 1,$$

$$u(0) = 0, \quad u(1) = 0.$$

   (a) Derive the matrix equation that you will need to solve for this problem.

   (b) Write a code to solve this set of equations. You can test your code on a problem where you know the solution by choosing a function $u(x)$ that satisfies the boundary conditions and determining what $f(x)$ must be in order for $u(x)$ to satisfy the differential equation. Try $u(x) = x(1-x)$. Then $f(x) = 2(3x^2 - x + 1)$.

   (c) Try several different values for the mesh size $h$. Based on your results, what would you say is the order of accuracy of the Galerkin method with continuous piecewise linear basis functions?

   (d) Now try a nonuniform mesh spacing, say, $x_i = (i/(m+1))^2$, $i = 0, 1, \ldots, m+1$. Do you see the same order of accuracy, if $h$ is defined as the maximum mesh spacing, $\max_i(x_{i+1} - x_i)$?

   (e) Suppose the boundary conditions were $u(0) = a$, $u(1) = b$. Show how you would represent the approximate solution $\hat{u}(x)$ as a linear combination of hat functions and how the matrix equation in part (a) would change.

2. (spectral methods, `chebfun`) Download the package `chebfun` from `www.chebfun.org`. This package works with functions that are represented (to machine precision) as sums of Chebyshev polynomials. It can solve 2-point boundary value problems using spectral methods. Use `chebfun` to solve the same problem as in the previous exercise and check the $L_2$-norm and the $\infty$-norm of the error.