Assignment 1.

Due Friday, Jan. 14.

Reading: Ch 1. Secs. 2.1-2.9.

1. Use MATLAB (or a programming language of your choice) to evaluate the second order accurate approximation

$$u''(x) \approx \frac{u(x+h) + u(x-h) - 2u(x)}{h^2}$$

for $u(x) = \sin x$ and $x = \pi/6$. Try $h = 10^{-1}, 10^{-2}, \ldots, 10^{-16}$, and make a table of values of h, the computed finite difference quotient, and the error. Explain your results.

2. Use the formula in the previous exercise with $h = 0.2$, $h = 0.1$, and $h = 0.05$ to approximate $u''(x)$, where $u(x) = \sin x$ and $x = \pi/6$. Use one step of Richardson extrapolation, combining the results from $h = 0.2$ and $h = 0.1$, to obtain a higher order accurate approximation. Do the same with the results from $h = 0.1$ and $h = 0.05$. Finally do a second step of Richardson extrapolation, combining the two previously extrapolated values, to obtain a still higher order accurate approximation. Make a table of the computed results and their errors. What do you think is the order of accuracy after one step of Richardson extrapolation? How about after two?

3. Using Taylor series, derive the error term for the approximation

$$u'(x) \approx \frac{1}{2h}[-3u(x) + 4u(x+h) - u(x+2h)].$$

4. Consider a forward difference approximation for the second derivative of the form

$$u''(x) \approx Au(x) + Bu(x+h) + Cu(x+2h).$$

Use Taylor's theorem to determine the coefficients A, B, and C that give the maximal order of accuracy and determine what this order is.

5. Consider the two-point boundary value problem

$$u'' + 2xu' - x^2 u = x^2, \quad u(0) = 1, \quad u(1) = 0.$$

Let $h = 1/4$ and explicitly write out the difference equations, using centered differences for all derivatives.