Assignment 5.

Due Wednesday, Feb. 23.

Reading: Chapter 3 in the text plus supplemental material on finite elements in 2D.

1. Write a code to solve Poisson's equation on the unit square with Dirichlet boundary conditions:
$$u_{xx} + u_{yy} = f(x, y), \quad 0 < x, y < 1$$
$$u(x, 0) = u(x, 1) = u(0, y) = u(1, y) = 1.$$
Take $f(x, y) = x^2 + y^2$, and demonstrate numerically that your code achieves second order accuracy. [Note: If you do not know an analytic solution to a problem, one way to check the code is to solve the problem on a fine grid and pretend that the result is the exact solution, then solve on coarser grids and compare your answers to the fine grid solution. However, you must be sure to compare solution values corresponding to the same points in the domain.]

2. Now use the 9-point formula with the correction term described in Sec. 3.5 to solve the same problem as in the previous exercise. Again take $f(x, y) = x^2 + y^2$, and numerically test the order of accuracy of your code by solving on a fine grid, pretending that is the exact solution, and comparing coarser grid approximations to the corresponding values of the fine grid solution. [Note: You probably will not see the 4th-order accuracy described in the text. Can you explain why?]

3. We have discussed using finite element methods to solve elliptic PDE's such as
$$\triangle u = f \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega,$$
with *homogeneous* Dirichlet boundary conditions. How could you modify the procedure to solve the *inhomogeneous* Dirichlet problem:
$$\triangle u = f \quad \text{in } \Omega, \quad u = g \quad \text{on } \partial\Omega,$$
where g is some given function? Derive the equations that you would need to solve to compute, say, a continuous piecewise bilinear approximation for this problem when Ω is the unit square $(0, 1) \times (0, 1)$.